Himanshu Vaishnav
23112042
Chemical Engineering

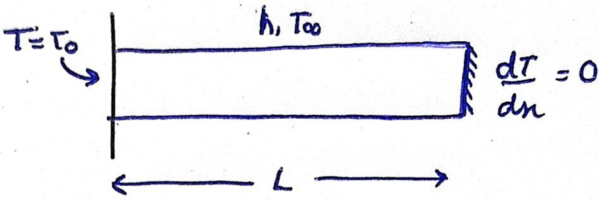
Computer Assignment - 1

Q-1 Solve the 1-D Rectangular fin problem with insulated tip for that following grid size, using any programming language.

(a) Grid spacing = 0.01 (Using TDMA & G-S)

Sol-1

Given, $\Delta x = 0.01$



- Governing Partial differential equation is:

$$\frac{\partial^2 T}{\partial x^2} - \left(\frac{KA}{hP}\right)^{-1}(T - T_\infty) = 0 \quad (1)$$

(energy equation for fin at steady state)

- Boundary Condition:

at $x=0$ $\quad T = T_0$

at $x = L$ $\quad \frac{dT}{dx} = 0$

- After Non-dimensionalizing the eq^n using $\theta = \frac{T - T_\infty}{T_0 - T_\infty}$ & $X = \frac{x}{L}$

we get $\frac{\partial^2 \theta}{\partial x^2} - (mL)^2 \theta = 0 \quad \left(m^2 = \frac{hP}{KA}\right) \quad (2)$

- Grid Discretization:

for a fin of unit length, $X = 1$, No of grid pt $= \frac{L}{\Delta x} = \frac{1}{0.01} = \boxed{100}$

for $M = 100 + 1$ (Initial pt) $= 101$ we need to solve $(M-1) = 101 - 1 = 100$ equations.

for $i = 1$ $\quad \theta_1 = 1$

for $i = 2 \dots M$, we use Central Difference on eq^n (2)

$$\frac{\theta_{i-1} - 2\theta_i + \theta_{i+1}}{(\Delta x)^2} - (mL)^2 \theta_i = 0$$

$$\theta_{i-1} - D\theta_i + \theta_{i+1} = 0$$

for $i = M$, After Applying Image point technique for fictitious $M+1$ pt.

$$2\theta_{M-1} - D\theta_i = 0$$

So, $\begin{cases} \theta_1 = 1 & i=1 \\ \theta_{i-1} - D\theta_i + \theta_{i+1} = 0 & i = 2 \ldots 100. \\ 2\theta_{M-1} - D\theta_M = 0 & i = 101 \end{cases}$ where

$D = 2 + m^2 L^2 \Delta x^2$

$= 2 + 4 \times (0.01)^2 = \boxed{2.0004.}$

Therefore, A Tridiagnol matrix of 100×100 will be formed.

$$\begin{bmatrix} 2.0004 & -1 & - & - & - & - \\ -1 & 2.0004 & -1 & - & - & \\ & & -1 & 2.0004 & -1 & \\ & & & \ddots & \ddots & \ddots \\ & & & & -2 & 2.0004 - 1 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \vdots \\ \theta_{100} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

Now, Its impossible to solve it by hand calcutator,

by using python code,

the output we get is

$$\begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \vdots \\ \theta_{100} \end{bmatrix} = \begin{bmatrix} 0.9834 \\ 0.9672 \\ 0.9514 \\ \vdots \\ 0.4205 \end{bmatrix}$$