(1) Given,

Random Sample = $x_1\ x_2\ x_3\ -\ -\ -\ -\ x_n$ from a normal distribution have mean $\theta_1$ & variance $\theta_2$

We know PDF of normal distribution:-

$$f(x, \theta_1, \theta_2) = \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x-\theta_1)^2}{2\theta_2}}$$

Likelihood function:-

$$L(\theta_1, \theta_2) = \prod_{i=1}^{n} f(x_i, \theta_1, \theta_2)$$

Log Likelihood function:-

$$\ln L(\theta_1, \theta_2) = -\frac{n}{2} \ln(2\pi) - \frac{n}{2} \ln(\theta_2) - \frac{1}{2\theta_2} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

$$\Rightarrow \frac{\partial \ln L(\theta_1, \theta_2)}{\partial \theta_1} \Rightarrow \frac{1}{\theta_2} \sum_{i=1}^{n} (x_i - \theta_1) = 0$$

$$\Rightarrow \sum_{i=1}^{n} x_i = n\theta_1$$

$$\theta_1 = \frac{1}{n} \sum_{i=1}^{n} x_i$$

So, MLE for $\theta_1$ is sample mean.

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Now differentiating w.r.t to $\theta_2$

$$\frac{\partial \ln L(\theta_1, \theta_2)}{\partial \theta_2} = \frac{-n}{2\theta_2} + \frac{1}{2\theta_2^2} \sum_{i=1}^{n} (x_i - \theta_1)^2 = 0$$

$$\theta_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

So, MLE for $\theta_2$ is sample variance.

(2) Given,
$m \rightarrow$ known tve integer
$\theta \rightarrow$ unknown parameter $\in (0, 1)$

We know PMF for binomial distribution:

$$f(x, m, \theta) = {}^{m}C_x \, \theta^x (1-\theta)^{m-x}$$

Liklihood function for sample:-

$$L(\theta) = \prod_{i=1}^{n} f(x_i, m, \theta)$$

Taking log both sides

$$\ln L(\theta) = \sum_{i=1}^{n} \left[ \ln({}^{m}C_x) + x_i \ln(\theta) + (m - x_i) \ln(1-\theta) \right]$$

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$$\frac{\partial \ln L(\theta)}{\partial \theta} = \sum_{i=1}^{n} \left[ \frac{x_i}{\theta} - \frac{m - x_i}{1-\theta} \right] = 0$$

$$\Rightarrow \sum_{i=1}^{n} \frac{x_i}{\theta} - \frac{n.m - \sum_{i=1}^{n} x_i}{1-\theta} = 0$$

$$\Rightarrow \theta = \frac{\sum_{i=1}^{n} x_i}{n.m}$$

$$\theta = \frac{\sum_{i=1}^{n} x_i}{n} \times \frac{1}{m}$$

$\therefore$ MLE for $\theta$ is sample mean divided by known no. 'm'.