

Day 4-STATS

- ① Central Limit Theorem. ✓
- ② Probability. ✓
- ③ Permutation And combination ✓
- ④ Covariance, Pearson Correlation, Spearman Rank Correlation. } ✓

⑤ Bernoulli Distribution

⑥ Binomial Distribution

⑦ Power law (Pareto Distribution).

⑧ Central Limit Theorem

$$\begin{matrix} n < 30 \\ \downarrow \downarrow \\ n \geq 30 \\ \uparrow \end{matrix}$$

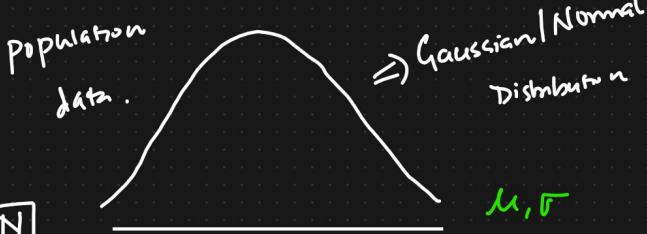
Size of sample

The larger the value the better

$$\begin{matrix} \uparrow \\ n \end{matrix}$$

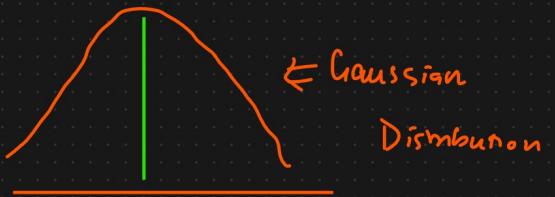
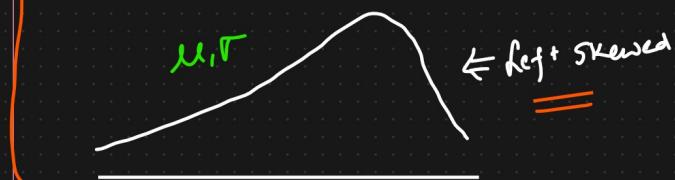
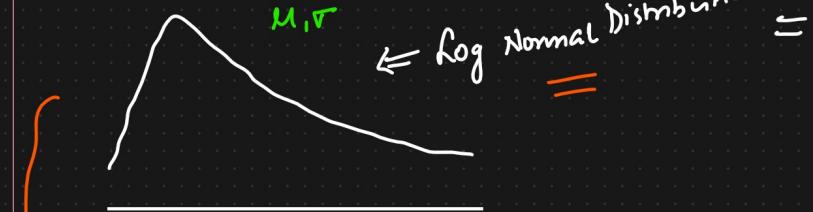
$$\begin{matrix} \uparrow \\ m \end{matrix}$$

No. of samples



$$\begin{aligned} \rightarrow S_1 &\rightarrow \{x_1, x_2, x_3, \dots, x_n\} \rightarrow \bar{x}_1 = \bar{s}_1 \\ \rightarrow S_2 &\rightarrow \{x_3, x_4, \dots, x_1, \dots, x_n\} \rightarrow \bar{x}_2 = \bar{s}_2 \\ \rightarrow S_3 &\rightarrow \{x_n, x_1, \dots, x_{n-1}\} \rightarrow \bar{x}_3 = \bar{s}_3 \\ &\vdots \quad \vdots \\ &\bar{x}_m = \bar{s}_m \end{aligned}$$

Sampling with replacement



10 different region

$$n > 30$$

Size of shark through out the world? \rightarrow Assumptions

$N < 30$

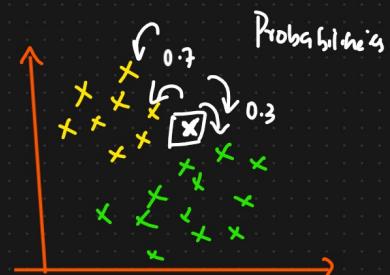
$m \uparrow \uparrow \uparrow$

② Probability: Probability is a measure of the likelihood of an event

Eg: Tossing a fair coin $P(H) = 0.5$ $P(T) = 0.5$

Strongly → coin $P(H) = 1$
unfair coin

Strong
Basic
↑



Rolling a Dice

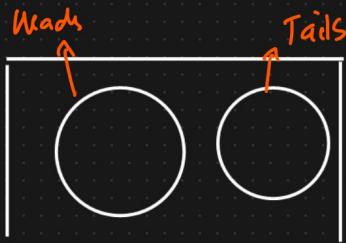
$$P(1) = \frac{1}{6} \quad P(2) = \frac{1}{6} \quad P(3) = \frac{1}{6}$$

① Mutually Exclusive Events

Two events are mutually exclusive if they cannot occur at the same time

① Tossing a coin

② Rolling a dice



② Non Mutual Exclusive Events

Two events can occur at the same time.



Bag of Marbles

④ Picking randomly a card from a deck of cards, two events "heart" and "king" can be selected.

Mutual Exclusive Event

① What is the probability of coin landing on heads or tails



Addition Rule for mutual exclusive events

$$P(A \text{ or } B) = P(A) + P(B)$$

$$= \frac{1}{2} + \frac{1}{2} = 1$$

② What is the probability of getting 1 or 6 or 3 while rolling a dice?

$$P(1 \text{ or } 6 \text{ or } 3) = P(1) + P(6) + P(3)$$

$$= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$$

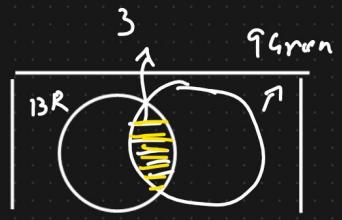
Non Mutual Exclusive Event

Bag of Marbles : 10 Red, 6 Green, 3 (R&G) R/G

① When picking randomly from a bag of marbles what is the probability of choosing a marble that is red or green?



Non mutual Exclusive



Addition Rule for Non Mutual Exclusive Event

$$P(A \text{ or } B) = P(A) + P(B) - \boxed{P(A \text{ and } B)}$$

$$= \frac{13}{19} + \frac{9}{19} - \frac{3}{19} = \frac{19}{19} = \underline{\underline{1}}.$$

Deck of cards \rightarrow What is the probability of choosing Q or Queen

$$P(Q \text{ or Queen}) = P(Q) + P(\text{Queen}) - P(Q \text{ and Queen})$$

$$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \boxed{\frac{16}{52}}$$

Multiplication Rule

④ Dependent Events : Two events are dependent if they affect one another

Bag of marbles ○ ○ ○ X
○ ○ ○

$$\Rightarrow P(W) = \frac{4}{7} \longrightarrow P(Y) = \frac{3}{6}$$

\uparrow
white
1 marble

⑤ What is the probability of rolling a "5" and then a "3" with a normal 6 sided dice?

Ans) $P(1) = \frac{1}{6}$ $P(2) = \frac{1}{6}$ $P(3) = \frac{1}{6}$ $P(4) = \frac{1}{6}$

Independent Dependent ↗

Multiplication Rule for Independent Events

$$P(A \text{ and } B) = P(A) * P(B)$$

$$= \frac{1}{6} * \frac{1}{6} = \boxed{\frac{1}{36}}$$

$P(A \text{ or } B) \Rightarrow$

- Mutual Exclusive
- Non Mutual Exclusive

$\overbrace{P(K \text{ or } M)}^{\text{non mutual}} = P(A) + P(B) - \boxed{P(A \text{ and } B)} \rightarrow \text{Non Mutual Exclusive.}$

$P(A \text{ or } B) = P(A) + P(B) \quad [\text{Mutual Exclusive}]$

Dependent and Independent Events

Event A Event B

$P(A \text{ and } B) = P(A) * P(B)$

Tossing a coin $\in \{H, T\}$

$P(H) = 0.5 \quad P(T) = 0.5$

②



\Rightarrow Dependent Events

Probability of drawing a "Orange" and then drawing a "Yellow"

marble from the bag?

Ans)

$$P(O) = \frac{4}{7} \quad \boxed{P(Y/O)} \quad \left(\frac{3}{6} = \frac{1}{2} \right) \quad \text{conditional probability}$$

Orange Marble

Naive Bayes

$$P(O \text{ and } Y) = P(O) * \boxed{P(Y/O)}$$

$$= \frac{4}{7} * \frac{3}{6} = \frac{4}{7} * \frac{1}{2} = \frac{2}{7} \approx 0.286$$

Permutation

School of Children

$$\begin{array}{c}
 \text{Sneakers, 5 star} \\
 \text{= } \boxed{\frac{5 \times 4 \times 3}{60 \text{ ways}}} \Rightarrow \text{Permutation} \\
 \left\{ \text{Dairy Milk, Kit Kat, Milky Bar, } \right\}
 \end{array}$$

With permutation, order matters

$$\begin{array}{c}
 \left\{ \begin{array}{ccc} DM & KK & MB \end{array} \right\} \quad \left\{ \begin{array}{c} \end{array} \right\} \quad \left\{ \begin{array}{c} \end{array} \right\} \quad \left\{ \begin{array}{c} \end{array} \right\} \\
 \left\{ \begin{array}{ccc} KK & DM & MB \end{array} \right\} \quad \left\{ \begin{array}{c} \end{array} \right\} \quad \left\{ \begin{array}{c} \end{array} \right\} \quad \left\{ \begin{array}{c} \end{array} \right\} \\
 \left\{ \begin{array}{c} \end{array} \right\} \quad \left\{ \begin{array}{c} \end{array} \right\} \quad \left\{ \begin{array}{c} \end{array} \right\} \quad \left\{ \begin{array}{c} \end{array} \right\} \\
 \text{Possible Arrangements} \quad n=5
 \end{array}$$

$$n_p = \frac{n!}{(n-r)!} = \frac{5!}{(5-3)!}$$

$$\therefore \frac{5 \times 4 \times 3 \times 2!}{2!} = \boxed{\underline{60}}$$

$N = \text{Total No. of Objects}$

γ = # of subsection

* Combination

Repetition will not occur

{ Dm KR MB }

Unique Combination

$X \{ MB \quad KC \quad DM \} \leftarrow$

$$n \choose r = \frac{n!}{r!(n-r)!} = \frac{5!}{3!(2)!} = \frac{5 \times 4 \times 3 \times 2 \times 1}{3! \times 2} = 10/1$$

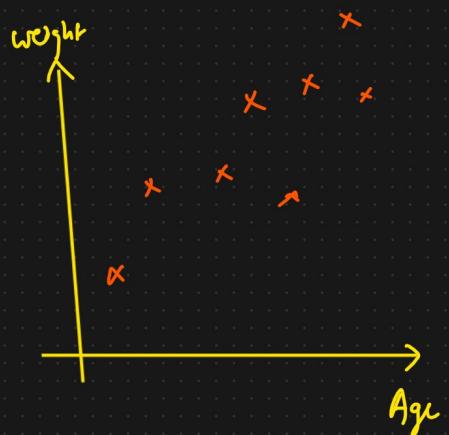
DREAM 11

(*) Covariance ✓

X	Y	Weight
Age		
12	40	
13	45	
15	48	
17	60	
18	62	
$\bar{x} = 15$	$\bar{y} = 51$	

{Feature Selection}

Age ↑	Weight ↑
Age ↓	Weight ↓



Quantify the relationship

x & y using mathematical
question

$$\text{Cov}(x, y) = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{n-1}$$

$$s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1}$$

$$s^2 = \frac{\sum (x_i - \bar{x})(x_i - \bar{x})}{n-1}$$

\Downarrow

$\boxed{2y} \Rightarrow +ve$ $\text{Cov}(x, x) \Leftarrow$

$$\boxed{\text{Cov}(x, x) = \text{Var}(x)} \Leftarrow$$

+ve Covariance

$x \uparrow$	$y \uparrow$
$x \downarrow$	$y \downarrow$

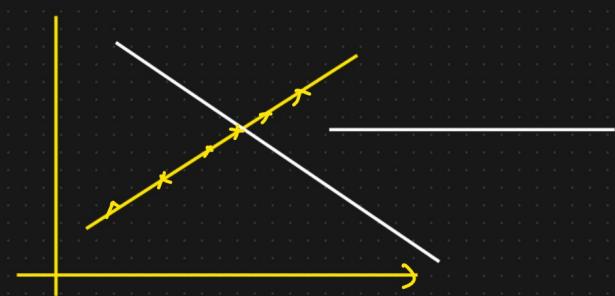
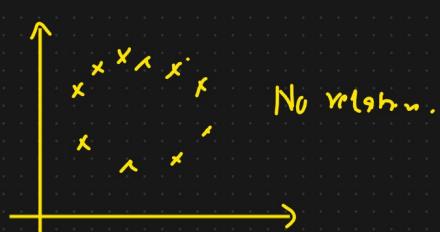
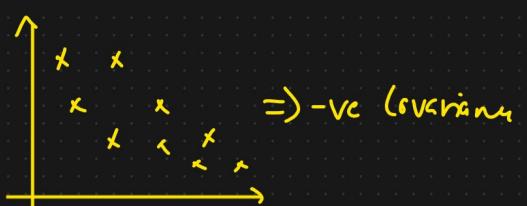
Covariance = 0

No relation
with x & y

-ve Covariance

$x \uparrow$	$y \downarrow$
$x \downarrow$	$y \uparrow$





X	Y
10	4
8	6
7	8
6	10
7.75	7

$$\text{Cov}(x, y) = \underline{\underline{-\text{ve}}}$$

$$= \left[(10 - 7.75)(4 - 7) + (8 - 7.75)(6 - 7) + (7 - 7.75)(8 - 7) + (6 - 7.75)(10 - 7) \right]$$

$$= -3.25$$

$$\begin{pmatrix} x \uparrow & y \downarrow \\ x \downarrow & y \uparrow \end{pmatrix}$$

$$\underline{\underline{}}$$

① Pearson Correlation Coefficient (-1 to 1)

$$\rho(x, y) = \frac{\text{Cov}(x, y)}{\sqrt{x} \cdot \sqrt{y}}$$

More the value towards +1

More +ve correlated it is

-1
negative correlated

Scale

-ve Covariance = +ve

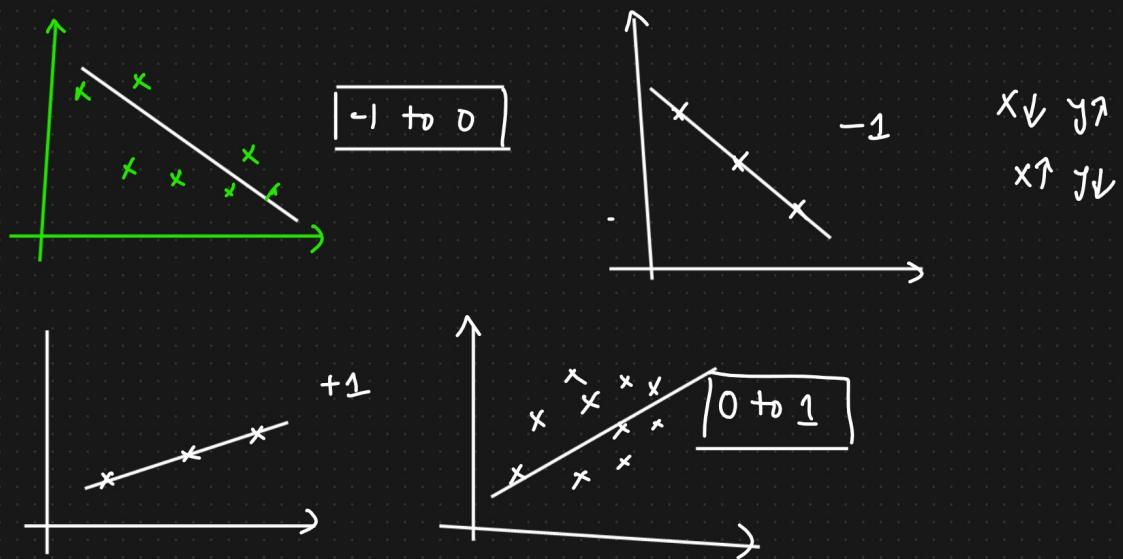
+ve

-ve

$$\begin{matrix} \curvearrowleft & & \\ x & y & z \end{matrix}$$

$$\begin{matrix} \curvearrowleft & \curvearrowright & \curvearrowright \\ x & y & z \end{matrix} \Rightarrow \boxed{0.7} \checkmark$$

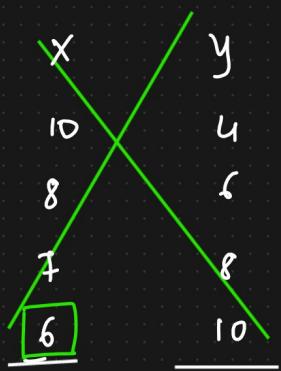
$$\boxed{0.5}$$



(4) Spearman Rank Correlation

$$\gamma_s = \frac{\text{Cov}(R(x), R(y))}{\sigma(R(x)) \sigma(R(y))}$$

Ascending Order



$R(x)$	$R(y)$
4	1
3	2
2	3
1	4

Spearman Rank Correlation

Ascending

Why this Correlation will be used?

$$0.95 \\ 95\% \\ \equiv$$



$$\begin{array}{c} \downarrow \\ +ve \\ -ve \\ \uparrow \end{array} \} \text{ Good} \\ 0.2 \quad 0.01$$

