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Pratical no. 1

Limits and continuity.

① $\lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]$

② $\lim_{y \to 0} \left[\frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \right]$

③ $\lim_{x \to \pi/6} \left[\frac{\cos x - \sqrt{3}\sin x}{\pi - 6x} \right]$

④ $\lim_{x \to \infty} \left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \right]$

⑤ Examine the following functions continuity at a given point

(i) $f(x) = \frac{\sin 2x}{\sqrt{1-\cos x}}$ for $0 < x \le \frac{\pi}{2}$

$= \frac{\cos x}{\pi - 2x}$ for $\frac{\pi}{2} < x < \pi$ } at $x = \frac{\pi}{2}$

(ii) $f(x) = \frac{x^2-9}{x-3}$ $0 < x < 3$

$= x+3$ $3 \le x < 6$

$= \frac{x^2-9}{x+3}$ $6 \le x < 9$ } at $x = 3$ & $x = 6$

⑥ Find value of k so that the function is continuous at given point

(1) $f(x) = \frac{1-\cos 4x}{x^2}$ $x < 0$

$= k$ $x = 0$ } at $x = 0$

② $f(x) = (\sec^2 x)^{\cot^2 x}$ $x \ne 0$

$= k$ $x = 0$ } at $x = 0$

③ $f(x) = \frac{\sqrt{3} - \tan x}{\pi - 3x}$ $x \ne \frac{\pi}{3}$

$= k$ $x = \frac{\pi}{3}$ } at $x = \frac{\pi}{3}$

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Discuss the continuity of the following function which of these functions have a remarkable discontinuity & Redefine the function so as to ~~dife~~ remove the discontinuity.

$$f(x) = \begin{cases} \dfrac{1-\cos 3x}{x \tan x} & x \neq 0 \\ = 9 & x = 0 \end{cases} \quad \text{at } x = 0$$

$$f(x) = \begin{cases} \dfrac{(e^{3x}-1)\sin x^\circ}{x^2} & x \neq 0 \\ = \dfrac{\pi}{60} & x = 0 \end{cases} \quad \text{at } x = 0$$

If $f(x) = \dfrac{e^{x^2} - \cos x}{x^2}$ for $x \neq 0$ is continuous at $x = 0$ find $f(0)$

If $f(x) = \dfrac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$ for $x \neq \dfrac{\pi}{2}$ is continuous at

at $x = \dfrac{\pi}{2}$ find $f\left(\dfrac{\pi}{2}\right)$

1. $$\lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]$$

$$= \lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \times \frac{\sqrt{a+2x} + \sqrt{3x}}{\sqrt{a+2x} + \sqrt{3x}} \times \frac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{3a+x} + 2\sqrt{x}} \right]$$

$$= \lim_{x \to a} \frac{(a+2x-3x)\,(\sqrt{3a+x} + 2\sqrt{x})}{(3a+x-4x)\,(\sqrt{a+2x} + \sqrt{3x})}$$

$$= \lim_{x \to a} \frac{(a-x)\,(\sqrt{3a+x} + 2\sqrt{x})}{(3a-3x)\,(\sqrt{a+2x} + \sqrt{3x})}$$

$$= \frac{1}{3} \lim_{x \to a} \frac{(a-x)\,(\sqrt{3a+x} + 2\sqrt{x})}{(a-x)\,(\sqrt{a+2x} + \sqrt{3x})}$$

$$= \frac{1}{3} \frac{\sqrt{3a+a} + 2\sqrt{a}}{\sqrt{a+2a} + \sqrt{3a}}$$

$$= \frac{1}{3} \times \frac{\sqrt{4a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}}$$

$$= \frac{1}{3} \times \frac{2\sqrt{a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}}$$

$$= \frac{1}{3} \times \frac{\overset{2}{\cancel{4}}\sqrt{a}}{\cancel{2}\sqrt{3a}}$$

$$= \frac{2}{3\sqrt{3}}$$

2. $$\lim_{y \to 0} \left[\frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \right]$$

$$= \lim_{y \to 0} \left[\frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \times \frac{\sqrt{a+y} + \sqrt{a}}{\sqrt{a+y} + \sqrt{a}} \right]$$

$$= \lim_{y \to 0} \frac{\cancel{a} + \cancel{y} - \cancel{a}}{\cancel{y}\sqrt{a+y}\,(\sqrt{a+y} + \sqrt{a})}$$

$$= \lim_{y \to 0} \frac{1}{\sqrt{a+y}\,(\sqrt{a+y} + \sqrt{a})}$$

$$= \frac{1}{\sqrt{a+0}\,(\sqrt{a+0} + \sqrt{a})} = \frac{1}{\sqrt{a}\,(\sqrt{a} + \sqrt{a})} = \frac{1}{2a}$$

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3) $\lim_{x \to \pi/6} \dfrac{\cos x - \sqrt{3}\sin x}{\pi - 6x}$

By substituting $x - \pi/6 = h$

$x = h + \pi/6$

where $h \to 0$

$$= \lim_{h \to 0} \frac{\cos(h + \pi/6) - \sqrt{3}\sin(h + \frac{\pi}{6})}{\pi - 6(h + \pi/6)}$$

using

$\cos(A+B) = \cos A.\cos B - \sin A \sin B$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$$= \lim_{h \to 0} \frac{\cos h \cos \pi/6 - \sin h \sin \pi/6 - \sqrt{3}\sin h \cos \pi/6 + \cos h \sin \pi/6}{\pi - 6\left(\frac{6h + \pi}{6}\right)}$$

$$= \lim_{h \to 0} \frac{\cos h.\frac{\sqrt{3}}{2} - \sin h \frac{1}{2} - \sqrt{3}\left(\sin h \sqrt{3}/2 + \cos h \frac{1}{2}\right)}{\cancel{\pi} - 6h + \cancel{\pi}}$$

$$= \lim_{h \to 0} \frac{\cos h \frac{\sqrt{3}}{2} - \frac{\sin h}{2} - \frac{\sin 3h}{2} - \cos h \frac{\sqrt{3}}{2}}{-6h}$$

$$= \lim_{h \to 0} \frac{-\sin 4h/2}{-6h}$$

$$= \lim_{h \to 0} \frac{\sin 4h}{12}$$

$$= \lim_{h \to 0} \frac{\sin h}{3} = \frac{1}{3} \lim_{h \to 0} \frac{\sin h}{h} = \frac{1}{3} \times 1 = \frac{1}{3}$$

4] $\lim\limits_{x\to\infty}\left[\dfrac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}}\right]$

By rationalizing Numerator & Denominator both

$= \lim\limits_{x\to\infty}\left[\dfrac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}} \times \dfrac{\sqrt{x^2+5}+\sqrt{x^2+3}}{\sqrt{x^2+5}+\sqrt{x^2-3}} \times \dfrac{\sqrt{x^2+3}+\sqrt{x^2+1}}{\sqrt{x^2+3}+\sqrt{x^2+1}}\right]$

$= \lim\limits_{x\to\infty}\left[\dfrac{(x^2+5-x^2+3)(\sqrt{x^2+3}+\sqrt{x^2+1})}{(x^2+3-x^2-1)(\sqrt{x^2+5}+\sqrt{x^2-3})}\right]$

$= \lim\limits_{x\to\infty} \dfrac{8}{2}\dfrac{(\sqrt{x^2+3}+\sqrt{x^2+1})}{(\sqrt{x^2+5}+\sqrt{x^2-3})}$

$= 4 \lim\limits_{x\to\infty} \dfrac{\sqrt{x^2(1+3/x^2)}+\sqrt{x^2(1+1/x^2)}}{\sqrt{x^2(1+5/x^2)}+\sqrt{x^2(1-3/x^2}}$

After applying limit we get,

$= 4$ //

5] $f(x) = \dfrac{\sin 2x}{\sqrt{1-\cos^2 x}}$ for $0 < x \leq \pi/2$

$= \dfrac{\cos x}{\pi - 2x}$ for $\dfrac{\pi}{2} < x < \pi$ } at $x = \pi/2$

$f(\pi/2) = \dfrac{\sin 2(\pi/2)}{\sqrt{1-\cos 2(\pi/2)}}$ $\therefore f(\pi/2) = 0$

f at $x = \dfrac{\pi}{2}$ define

ii] $\lim\limits_{x\to\pi/2^+} f(x) = \lim\limits_{x\to\pi/2^+} \dfrac{\cos x}{\pi - 2x}$

By substituting method weget

$x - \dfrac{\pi}{2} = h$

$\therefore x = h + \pi/2$ where $h \to 0$

$$= \lim_{h\to 0} \frac{\cos(h+\pi/2)}{\pi - 2(h+\pi/2)}$$

$$= \lim_{h\to 0} \frac{\cos(h+\pi/2)}{\pi - 2\left(\frac{2h+\pi}{2}\right)}$$

$$= \lim_{h\to 0} \frac{\cos(h+\pi/2)}{-2h}$$ using $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$$= \lim_{h\to 0} \frac{\cos h \cdot \cos \pi/2 - \sin h \sin \pi/2}{-2h}$$

$$= \lim_{h\to 0} \frac{\cos h \cdot 0 - \sin h}{-2h}$$

$$= \lim_{h\to 0} \frac{-\sin h}{-2h}$$

$$= \frac{1}{2} \lim_{h\to 0} \frac{\sin h}{h} = \frac{1}{2}$$

$$\lim_{x\to \pi/2^-} f(x) = \lim_{x\to \pi/2^-} \frac{\sin 2x}{\sqrt{1-\cos 2x}}$$

$$= \lim_{x\to \pi/2^-} \frac{2\sin x \cdot \cos x}{\sqrt{2\sin^2 x}}$$ using $\sin 2x = 2\sin x \cdot \cos x$

$$= \lim_{x\to \pi/2^-} \frac{2\sin x \cdot \cos x}{\sqrt{2}\,\sin x}$$

$$= \lim_{x\to \pi/2} \frac{2\cos x}{\sqrt{2}}$$

$$= \frac{2}{\sqrt{2}} \lim_{x\to \pi/2^-} \cos x$$

$\therefore$ L.H.L $\neq$ R.H.L

$\therefore$ f is not continuous at $x = \pi/2$

iii] $f(x) = \left.\begin{array}{ll} \dfrac{x^2-9}{x-3} & 0 < x < 3 \\ = x+3 & 3 \le x \le 6 \\ = \dfrac{x^2-9}{x+3} & 6 \le x < 9 \end{array}\right\}$ at $x = 3$ & $x = 6$

at $x = 3$

$f(3) = \dfrac{x^2-9}{x-3} = 0$

f at $x = 3$ define

$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} x+3$

$f(3) = x+3 = 3+3 = 6$

f is define at $x = 3$

$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (x+3) = 6$

$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} \dfrac{x^2-9}{x-3} = \dfrac{(x-3)(x+3)}{(x-3)}$

$\therefore$ L.H.L = R.H.L

f is continuous at $x = 3$

for $x = 6$

$f(6) = \dfrac{x^2-9}{x+3} = \dfrac{36-9}{6+3} = \dfrac{27}{9} = 3$

2. $\lim_{x \to 6^+} \dfrac{x^2-9}{x+3}$

$\lim_{x \to 6^+} \dfrac{(x-3)(x+3)}{(x+3}$

$\lim_{x \to 6^+} (x-3) = 6-3 = 3$

$\lim_{x \to 6^-} x+3 = 3+6 = 9$

$\therefore$ L.H.L $\neq$ R.H.L

function is not continuous at $x = 6$

6 i) $f(x) = \dfrac{1-\cos 4x}{x^2}$ $\quad x < 0$ $\Big\}$ at $x = 0$

$= k$ $\quad x = 0$

sol^n f is continuous at $x = 0$.

$\lim_{x \to 0} f(x) = f(0)$

$\lim_{x \to 0} \dfrac{1-\cos 4x}{x^2} = k$

$\lim_{x \to 0} \dfrac{2\sin^2 2x}{x^2} = k$

$2 \lim_{x \to 0} \dfrac{\sin^2 x}{x^2} = k$

$2 \lim_{x \to 0} \left(\dfrac{\sin 2x}{x}\right)^2 = k$

$2(2^2) = k$

$\therefore k = 8$

ii) $f(x) = (\sec^2 x)^{\cos^2 x}$ $\quad x \neq 0$ $\Big\}$ at $x = 0$

$= k$ $\quad x = 0$

sol^n:

$f(x) = (\sec^2 x)^{\cot^2 x}$

using

$\sec^2 x - \tan^2 x$ - $\sec^2 x = 1$

$\sec^2 x = 1 + \tan^2 x$

$\cot^2 x = \dfrac{1}{\tan^2 x}$

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$\therefore \lim_{x \to 0} (\sec^2 x)^{\cot^2 x}$

$\lim_{x \to 0} (1 + \tan^2 x)^{\frac{1}{\tan^2 x}}$

we, know that

$\lim_{x \to 0} (1 + px)^{\frac{1}{px}} = e$

$\therefore k = e$

iii) $f(x) = \dfrac{\sqrt{3} - \tan x}{\pi - 3x} \quad x \neq \pi/3$
$= k \quad x = \pi/3$ } at $x = \pi/3$

$x - \dfrac{\pi}{3} = h \quad \therefore x = h + \dfrac{\pi}{3}$ where $h \to 0$

$f\left(\pi/3 + h\right) = \dfrac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$

$= \lim_{h \to 0} \dfrac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$

Using,

$\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A . \tan B}$

$= \lim_{h \to 0} \dfrac{\sqrt{3} - \dfrac{\tan \pi/3 + \tan h}{1 - \tan \pi/3 . \tan h}}{\pi - \pi - 3h}$

$= \lim_{h \to 0} \dfrac{\dfrac{\sqrt{3}(\cancel{1} - \tan \pi/3 . \tan h) - (\tan \pi/3 + \tan h)}{1 - \tan \pi/3 . \tan h}}{-3h}$

$= \lim_{h \to 0} \dfrac{(\sqrt{3} - \sqrt{3}.\sqrt{3} - \tan h) - (\sqrt{3} + \tan h)}{1 - \sqrt{3} . \tan h}$

$$= \lim_{h\to 0} \frac{\dfrac{(\sqrt{3} - 3 - \tan h) - (\sqrt{3} + \tan h)}{1 - \sqrt{3}\cdot\tan h}}{-3h}$$

$$= \lim_{h\to 0} \frac{\dfrac{(\cancel{\sqrt{3}} - 3\tan h - \cancel{\sqrt{3}} - \tan h)}{1 - \sqrt{3}\tan h}}{-3h}$$

$$= \lim_{h\to 0} \frac{-4\tan h}{-3h(1 - \sqrt{3}\tan h)}$$

$$= \lim_{h\to 0} \frac{4\tan h}{3h(1 - \sqrt{3}\tan)}$$

$$= \frac{4}{3} \lim_{h\to 0} \frac{\tan h}{h} \cdot \lim_{h\to 0} \frac{1}{(1 - \sqrt{3}\tan h)}$$

$$= \frac{4}{3} \frac{1}{(1 - \sqrt{3}(0))} = \frac{4}{3}\left(\frac{1}{1}\right) = \frac{4}{3}$$

7. $f(x) = \left.\begin{cases} \dfrac{1 - \cos 3x}{x \tan x} & x \neq 0 \\ = 9 & x = 0 \end{cases}\right\}$ at $x = 0$

$$f(x) = \frac{1 - \cos 3x}{x \tan x}$$

$$= \lim_{x\to 0} \frac{2\sin^2 3/2\, x}{x \tan x}$$

$$= \lim_{x\to 0} \frac{\dfrac{2\sin^2 \frac{3x}{2}}{x^2} \times x^2}{x \tan x / x^2 \times x^2}$$

$$= 2 \lim_{x\to 0} \frac{\left(\frac{3}{2}\right)^2}{1} = 2 \times \frac{9}{4} = \frac{9}{2}$$

$\lim_{x \to 0} f(x) = 9/2 \qquad 9 \neq f(0)$

$\therefore$ f is not continuous at $x=0$

Redefine function

$$f(x) = \begin{cases} \dfrac{1-\cos 3x}{x \tan x} & x \neq 0 \\ \dfrac{9}{2} & x = 0 \end{cases}$$

Now $\lim_{x \to 0} f(x) = f(0)$

f has removal discontinuity at $x=0$

(ii) $$f(x) = \begin{cases} \dfrac{(e^{3x}-1)\sin x^{\circ}}{x^2} & x \neq 0 \\ \dfrac{\pi}{6} & x = 0 \end{cases} \quad \text{at } x = 0$$

$$= \lim_{x \to 0} \frac{(e^{3x}-1)\sin\left(\frac{\pi x}{180}\right)}{x^2}$$

$$= \lim_{x \to 0} \frac{e^{3x}-1}{x} \cdot \lim_{x \to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$

$$= \lim_{x \to 0} 3 \cdot \frac{e^{3x}-1}{3x} \lim_{x \to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$

$$= 3 \lim_{x \to 0} \frac{e^{3x}-1}{3x} \lim_{x \to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$

$$= 3 \log_e \frac{\pi}{180} = \frac{\pi}{6} = f(0)$$

f is continuous at $x=0$

f. $$f(x) = \frac{e^{x^2} - \cos x}{x^2} \qquad x = 0$$

is continuous at $x=0$

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$\therefore$ given

f is continuous at $x=0$

$$\lim_{x\to 0} f(x) = f(0)$$

$$= \lim_{x\to 0} \frac{e^{x^2}-\cos x}{x^2} = f(0)$$

$$= \lim_{x\to 0} \frac{e^{x^2}-\cos x - 1 + 1}{x^2}$$

$$= \lim_{x\to 0} \frac{(e^{x^2}-1) + (1-\cos x)}{x^2}$$

$$= \lim_{x\to 0} \frac{e^{x^2}-1}{x^2} + \lim_{x\to 0} \frac{1-\cos x}{x^2}$$

$$= \log e + \lim_{x\to 0} \frac{2\sin^2 x/2}{x^2}$$

$$= \log e + 2\lim_{x\to 0} \left(\frac{\sin x/2}{x}\right)^2$$

Multiply with 2 on Num & Denominator

$$= 1 + 2 \times \frac{1}{4} = \frac{3}{2} = f(0)$$

9. $f(x) = \dfrac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x} \qquad x \neq \pi/2$

$f(0)$ is continuous at $x = \pi/2$

$$= \lim_{x\to \pi/2} \frac{\sqrt{2}-\sqrt{1+\sin x}}{\cos^2 x} \times \frac{\sqrt{2}+\sqrt{1+\sin x}}{\sqrt{2}+\sqrt{1+\sin x}}$$

$$= \lim_{x\to \pi/2} \frac{2-1+\sin x}{\cos^2 x(\sqrt{2}+\sqrt{1+\sin x})}$$

$$= \lim_{x\to \frac{\pi}{2}} \frac{1}{(1-\sin x)(\sqrt{2}+\sqrt{1+\sin x})}$$

$$= \frac{1}{2(\sqrt{2}+\sqrt{2})} = \frac{1}{2\sqrt{2}\sqrt{2})} = \frac{1}{4\sqrt{2}}$$

$$\therefore f(\pi/2) = \frac{1}{4\sqrt{2}}$$

06/02/2020

Pratical - 2

Derivative

Q1 Show that the following function defined from $\mathbb{R}$ to $\mathbb{R}$ are differentiable.

i) $\cot x$

$f(x) = \cot x$

$$f(o) = \lim_{x \to a} \frac{f(x) - f(a)}{x-a}$$

$$= \lim_{x \to a} \frac{\cot x - \cot a}{x - a}$$

$$= \lim_{x \to a} \frac{1/\tan x - 1/\tan a}{x - a}$$

$$= \lim_{x \to 0} \frac{\tan a - \tan x}{(x-a) \tan x \cdot \tan a}$$

Put $x - a = h$

$x = a + h$

as $x \to a$, $h \to 0$

$$f^*(h) = \lim_{h \to 0} \frac{\tan a - \tan (a+h)}{(a+h-a) \tan (a+h) \tan a}$$

$$= \lim_{h \to 0} \frac{\tan a - \tan (a+h)}{h \times \tan(a+h) \tan a}$$

formula ; $\tan (A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

$\tan A - \tan B = \tan (A-B) \; (1 + \tan A \tan B)$

$$= \lim_{h \to 0} \frac{\tan (a-a-h) - (1 + \tan a \tan(a+h))}{h \times \tan (a+h) \tan a}$$

$$= \lim_{h\to 0} -\frac{\tan h}{h} \times \frac{1 + \tan a \tan(a-h)}{\tan(a-h)\tan a}$$

$$= -1 \times \frac{1 + \tan^2 a}{\tan^2 a}$$

$$= -\frac{\sec^2 a}{\tan^2 a}$$

$$= -\frac{1}{\cos^2 a} \times \frac{\cos^2 a}{\sin^2 a} \qquad = -\operatorname{cosec}^2 a$$

$f(a) = \cos^2 a$ $\quad \therefore f$ is differentiable $\forall\, a \in R$

ii] cosec x

$f(x) = \operatorname{cosec} x$

$$f(a) = \lim_{x\to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x\to a} \frac{\operatorname{cosec} x - \operatorname{cosec} a}{x - a}$$

$$= \lim_{x\to a} \frac{1/\sin x - 1/\sin a}{x - a}$$

$$= \lim_{x\to a} \frac{\sin a - \sin x}{(x-a)\sin a \sin x}$$

Put $x - a = h$

$\therefore x = a + h$ $\quad$ as $x \to a$, $h \to 0$

$$f(h) = \lim_{h\to 0} \frac{\sin a - \sin(a+h)}{(a+h-a)\sin a \sin(a+h)}$$

$\therefore \sin C - \sin C = 2\cos\left(\frac{C+D}{2}\right) \sin\left(\frac{C-D}{2}\right)$

$$= \lim_{h\to 0} \frac{2\cos\left(\frac{a+a+h}{2}\right) \cdot \sin\left(\frac{a-a-h}{2}\right)}{h \times \sin a \sin(a+h)}$$

$$= \lim_{h\to 0} \frac{-\sin h/2}{h/2} \times \frac{1}{2} \times \frac{2\cos\left(\frac{2a+h}{2}\right)}{\sin a \sin(a+h)}$$

$$= \frac{-1}{2} \times \frac{2\cos\left(\frac{2a+0}{2}\right)}{\sin(a+0)}$$

$$= -\frac{\cos a}{\sin^2 a} = -\cot a \quad \operatorname{cosec} a$$

iii) $\sec x$

$$f(x) = \sec x$$

$$f(a) = \lim_{x\to a} \frac{f(x) - f(a)}{x-a}$$

$$= \lim_{x\to a} \frac{\sec x - \sec a}{x-a}$$

$$= \lim_{x\to 0} \frac{\frac{1}{\cos x} - \frac{1}{\cos a}}{x-a}$$

$$= \lim_{x\to a} \frac{\cos a - \cos x}{(x-a)\cos a \cos x}$$

Put $x - a = h$

$x = a+h$ as $x \to a$, $h \to 0$

$$f(h) = \lim_{h\to 0} \frac{\cos a - \cos(a+h)}{h \times \cos a \cos(a+h)}$$

$\because$

$$-2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$$

$$= \lim_{h\to 0} \frac{-2\sin\left(\frac{a+a+h}{2}\right)\sin\left(\frac{a-a-h}{2}\right)}{h \times \cos a \cos(a+h)}$$

$$= \lim_{h\to 0} \frac{-2\sin\left(\frac{2a+h}{2}\right)\sin\frac{-h}{2}}{\cos a \cos(a+h) \times \frac{-h}{2}} \times \frac{-1}{2}$$

$$= \frac{-1}{2} \times \frac{-2\sin\left(\frac{2a+0}{2}\right)}{\cos a \cos(a+0)}$$

$$= \frac{-1}{2} \times -2 \frac{\sin a}{\cos a \cos a} = \tan a \sec a$$

Q.

2. If $f(x) = 4x+1 \quad x \le 2$
$\quad = x^2+5 \quad x > 0$ at $x = 2$, then find function is differentiable or not

sol^n :

L.H.D :

$$f'(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x+1-(4 \times 2+1)}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x+1-9}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x-8}{x+2}$$

$$= \lim_{x \to 2^-} \frac{4(x-2)}{x-2} = 4$$

$f'(2^-) = 4$

R.H.D

$$f'(2^+) = \lim_{x \to 2^+} \frac{x^2+5-9}{x-2}$$

$$= \lim_{x \to 2^+} \frac{x^2-4}{x-2}$$

$$= \lim_{x \to 2^+} \frac{(x+2)(x-2)}{(x-2)}$$

$$= 2+2 = 4$$

$\therefore$ R.H.D = L.H.D

$\therefore$ f is differentiable at $x = 2$

3] If $f(x) = 4x+7 \quad x < 3$
$\quad = x^2+3x-1 \quad x \ge 3$ at $x = 3$ then find f is differentiable or not?

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Sol^n :-

R.H.D :-

$$f(3^+) = \lim_{x \to 3^+} \frac{f(x) - f(3)}{x-3}$$

$$= \lim_{x \to 3^+} \frac{x^2+3x+1 - (3^2+3\times3+1}{x-3}$$

$$= \lim_{x \to 3^+} \frac{x^2+3x-18}{x-3}$$

$$= \lim_{x \to 3^+} \frac{x^2+6x-3x-18}{x-3}$$

$$= \lim_{x \to 3^+} \frac{x(x+6)-3(x+6)}{x-3}$$

$$= \lim_{x \to 3^+} \frac{(x-3)(x+6)}{(x-3)} = 3+6 = 9$$

$$f(3^+) = 9$$

$$L.H.D = f(3^-) = \lim_{x \to 3^-} \frac{f(x) - f(3)}{x-3}$$

$$= \lim_{x \to 3^-} \frac{4x+7-12}{x-3}$$

$$= \lim_{x \to 3^-} \frac{4x-12}{x-3}$$

$$= \lim_{x \to 3^-} \frac{4\cancel{(x-3)}}{\cancel{(x-3)}}$$

$$f'(3^-) = 4$$

$\therefore$ L.H.D $\neq$ R.H.D

f is not different cable at $x=3$

4) If $f(x) = 8x - 5 \quad x \le 2$

$= 3x^2 - 4x + 7 \quad x > 2$ at $x = 2$ then find f is differentiable or not

sol^n

$f(2) = 8 \cdot 2 - 5 = 16 - 5 = 11$

$R \cdot H \cdot D = f(2^+) = \lim_{x \to 2^+} \frac{f(x) - f(2)}{(x-2)}$

$= \lim_{x \to 2^+} \frac{3x^2 - 4x + 7 - 11}{x - 2}$

$= \lim_{x \to 2^+} \frac{3x^2 (x-2) + 2(x-2)}{x-2}$

$= \lim_{x \to 2^+} \frac{(3x+2)(x-2)}{(x-2}$

$= 3 \times 2 + 2 = 8$

$L \cdot H \cdot D = f(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x-2}$

$= \lim_{x \to 2^-} \frac{8x - 5 - 11}{x - 2}$

$= \lim_{x \to 2^-} \frac{8x - 16}{x-2}$

$= \lim_{x \to 2^-} \frac{8\cancel{(x-2)}}{\cancel{(x-2)}}$

$= 8$

$\therefore L \cdot H \cdot D = R \cdot H \cdot D$

f is differentiable at $x = 3$

AL
19/12/19

Pratical - 3

Topic : Application of Derivative

1] Find the intervals in which function is increasing or decreasing

(i) $f(x) = x^3 - 5x - 11$

(ii) $f(x) = x^2 - 4x$

(iii) $f(x) = 2x^3 + x^2 - 20x + 4$

(iv) $f(x) = x^3 - 27x + 5$

(v) $f(x) = 69 - 24x - 9x^2 + 2x^3$

2] Find the intervals in which function is concave upwards

(i) $Y = 3x^2 - 2x^3$

(ii) $Y = x^4 - 6x^3 + 12x^2 + 5x + 7$

(iii) $Y = x^3 - 27x + 5$

(iv) $Y = 69 - 24x$

(v) $Y = 2x^3 + x^2 - 20x + 4$

Solution :-

Q1. i] $f(x) = x^3 - 5x - 11$

$\therefore f'(x) = 3x^2 - 5$

$\therefore$ ~~f is increasing~~ iff $f'(x) > 0$

$3x^2 - 5 > 0$

$3(x^2 - 5/3) > 0$

$(x - \sqrt{5/3})(x + \sqrt{5/3}) > 0$

+   −   +

$-\sqrt{5/3}$ $\sqrt{5/3}$

$x \in (-\infty, -\sqrt{5/3}) \cup (\sqrt{5/3}, \infty)$

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and f is decreasing iff $f'(x) < 0$

$\therefore 3x^2 - 5 < 0$

$\therefore 3(x^2 - 5/3) < 0$

$\therefore (x - \sqrt{5/3})(x + \sqrt{5/3}) < 0$

$+ \quad \sqrt{5/3} \quad - \quad \sqrt{5/3} \quad +$

$x \in (-\sqrt{5/3}, \sqrt{5/3})$

$f(x) = x^2 - 4x$

$f'(x) = 2x - 4$

$\therefore f(x)$ is increasing iff $f'(x) > 0$

$\therefore 2x - 4 > 0$

$\therefore 2(x-2) > 0$

$x - 2 > 0$

$x \in (2, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 2x - 4 < 0$

$2(x-2) < 0$

$x - 2 < 0$

$x \in (-\infty, 2)$

$f(x) = 2x^3 + x^2 - 20x + 4$

$\therefore f'(x) = 6x^2 + 2x - 20$

$\therefore f$ is increasing iff $f'(x) > 0$

$6x^2 + 2x - 20 > 0$

$2(3x^2 + x - 10) > 0$

$3x^2 + x - 10 > 0$

$3x^2 + 6x - 5x - 10 > 0$

$3x(x+2) - 5(x+2) > 0$

$\therefore (x+2)(3x-5) > 0$

$+ \quad - \quad +$

$-2 \quad 5/3$

$x \in (-\infty, -2) \cup (5/3, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 6x^2 + 2x - 20 < 0$

$\therefore 2(3x^2 + x - 10) < 0$

$3x^2 + x - 10 < 0$

$3x(x+2) - 5(x+2) < 0$

$(x+2)(3x-5) < 0$

$+ \quad - \quad +$

$-2 \quad 5/3$

$x \in (-2, 5/3)$

4] $f(x) = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$\therefore f$ is increasing iff $f'(x) > 0$

$\therefore 3(x^2 - 9) > 0$

$\therefore (x-3)(x+3) > 0$

$+ \quad - \quad +$

$-3 \quad 3$

$x \in (-\infty, -3) \cup (3, \infty)$

and f is decreasing iff $f'(x) < 0$

$3x^2 - 27 < 0$

$3(x^2-9) < 0$

$(x-3)(x+3) < 0$

$+ \quad - \quad +$

$-3 \quad 3$

$\therefore x \in (-3, 3)$

a) $f(x) = 2x^3 - 9x^2 - 24x + 69$

$f'(x) = 6x^2 - 18x - 24$

$\because f$ is increasing iff $f'(x) > 0$

$\therefore 6x^2 - 18x - 24 > 0$

$\therefore 6(x^2 - 3x - 4) > 0$

$\therefore x^2 - 4x + x - 4 > 0$

$x(x-4) + 1(x-4) > 0$

$(x-4)(x+1) > 0$

+ − +

−1 4

$\therefore x \in (-\infty, -1) \cup (4, \infty)$

and f is decreasing iff $f'(x) < 0$

$6x^2 - 18x - 24 < 0$

$6(x^2 - 3x - 4) < 0$

$x^2 - 4x + x - 4 < 0$

$x(x-4) + 1(x-4) < 0$

$(x-4)(x+1) < 0$

+ − +

−1 4

$\therefore x \in (-1, 4)$

2] i) $y = 3x^2 - 2x^3$

$\because f(x) = 3x^2 - 2x^3$

$\therefore f'(x) = 6x - 6x^2$

$f''(x) = 6 - 12x$

f is concave upward iff $f''(x) > 0$

$(6 - 12x) > 0$

$12(6/12 - x) > 0$

$x - \frac{1}{2} > 0$

$x > \frac{1}{2}$

$\therefore f''(x) > 0$

$\therefore x \in (\frac{1}{2}, \infty)$

2] $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

$f'(x) = 4x^3 - 18x^2 + 24x + 5$

$f''(x) = 12x^2 - 36x + 24$

f is concave upward iff $f''(x) > 0$

$\therefore 12x^2 - 36x + 24 > 0$

$\therefore 12(x^2 - 3x + 2) > 0$

$\therefore x^2 - 2x - x + 2 > 0$

$\therefore (x-2)(x-1) > 0$

$+ \quad - \quad +$

$1 \quad 2$

$x \in (-\infty, 1) \cup (2, \infty)$

3] $y = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$f''(x) = 6x$

f is concave upward iff $f''(x) > 0$

$\therefore 6x > 0$

$x > 0$

$\therefore x \in (0, \infty)$

4] $y = 69 - 24x - 9x^2 + 2x^3$

$f'(x) = 6x^2 - 18x - 24$

$f''(x) = 12x - 18$

7.5

f is concave upward iff $f''(x) > 0$

$\therefore \quad 12x - 18 > 0$

$\therefore \quad 12(x - 18/12) > 0$

$x - 3/2 > 0$

$\therefore \quad x > 3/2 \qquad \therefore \quad x \in (3/2, \infty)$

c) $y = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$f''(x) = 12x + 2$

f is concave upward iff $f''(x) > 0$

$\therefore \quad f''(x) > 0$

$\therefore \quad 12x + 2 > 0$

$12(x + 1/6) > 0$

$x + 1/6 > 0$

$x < -1/6$

$\therefore \quad f''(x) \not> 0$

$\therefore$ there exist no interval.

Pratical no. 4

Q1:-

i] $f(x) = x^2 + 16/x^2$

$f'(x) = 2x - 32/x^3$

Now consider,

$f'(x) = 0$

$\therefore 2x - 32/x^3 = 0$

$2x = 32/x^3$

$x^4 = 32/2$

$x^4 = 16$

$x = \pm 2$

$f''(x) = 2 + 96/x^4$

$f''(2) = 2 + 96/2^4$

$= 2 + 96/16$

$= 2 + 6$

$= 8 > 0$

$\therefore$ f has minimum value at $x = 2$

$\therefore f(-2) = 2^2 + 16/2^2$

$= 4 + 16/4$

$= 8$

$f''(-2) = 2 + 96/(-2)^4$

$= 2 + 96/16$

$= 8 > 0$

$\therefore$ f has minimum value at $x = -2$

$\therefore$ function reaches minimum value at $x = 2$ and $x = -2$

ii] $f(x) = 3 - 5x^3 + 3x^5$

$f'(x) = -15x^2 + 15x^4$

Consider:-

$f'(x) = 0$

$-15x^2 + 15x^4 = 0$

$15x^4 = 15x^2$

$x^2 = 1$

$x = \pm 1$

$\therefore f''(x) = -30x + 60x^3$

$f(1) = -30 + 60$

$= 30 > 0$

f has minimum value at $x = 1$

$\therefore f(1) = 3-5(1)^3 + 3(1)^5$
$= 6-5$
$= 1$

$f''(-1) = -30(-1) + 60(-1)^3$
$= 30 - 60$
$= -30 < 0$

$\therefore$ f has maximum value at $x = -1$

$\therefore f(-1) = 3 - 5(-1)^3 + 3(-1)^5$
$= 3+5-3$
$= 5$

$\therefore$ f has the maximum value at 5 at $x=-1$ and has the minimum value 1 at $x=1$

iii) $f(x) = x^3 - 3x^2 + 1$

$\therefore f'(x) = 3x^2 - 6x$

Consider,

$f'(x) = 0$

$\therefore 3x^2 - 6x = 0$

$3x(x-2) = 0$

$\therefore 3x = 0$ or $x-2=0$

$x = 0$ or $x = 2$

$f''(x) = 6x - 6$

$f''(0) = 6(0) - 6$
$= -6 < 0$

$\therefore$ f has maximum value at $x = 0$

$\therefore f(0) = (0)^3 - 3(0)^2 + 1$
$= 1$

$f''(2) = 6(2) - 6$
$= 12 - 6$
$= 6 > 0$

$\therefore$ f has minimum value at $x = 2$

$f(2) = (2)^3 - 3(2)^2 + 1$
$= 8 - 3(4) + 1$
$= 9 - 12$
$= -3$

$\therefore$ f has maximum value 1 at $x=0$ and f has minimum value -3 at $x=2$

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4) $f(x) = 2x^3 - 3x^2 - 12x + 1$

$f'(x) = 6x^2 - 6x - 12$

Consider,

$f'(x) = 0$

$\therefore 6x^2 - 6x - 12 = 0$

$6(x^2 - x - 2) = 0$

$x^2 - x - 2 = 0$

$x^2 + x - 2x - 2 = 0$

$x(x+1) - 2(x+1) = 0$

$(x-2)(x+1) = 0$

$x = 2$ or $x = -1$

$\therefore f''(x) = 12x - 6$

$f''(2) = 12(2) - 6$

$= 24 - 6$

$= 18 > 0$

f has maximum value at

$x = 2$

$f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 1$

$= 16 - 12 - 24 + 1$

$= -19$

$f''(-1) = 12(-1) - 6$

$= -18 < 0$

f has maximum value at

$x = -1$

$\therefore f(-1) = 2(-1)^3 - 3(-1)^2 - 12(-1) + 1$

$= -2 - 3 + 12 + 1$

$= 8$

$\therefore f$ has maximum value 8 at

$x = -1$ and f has

minimum value -19 at

$x = 2$

Q7:- $f(x) = x^3 - 3x^2 - 55x + 9.5$ $x = 0 \rightarrow$ given

$f'(x) = 3x^2 - 6x - 55$

By Newton's Method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 0 + \frac{9.5}{55}$$

$$x_1 = 0.1727$$

$$f(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1725) + 9.5$$
$$= 0.0051 - 0.0895 - 9.4985 + 9.5$$
$$= -0.0829$$

$$f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$$
$$= 0.0895 - 1.0362 - 55$$
$$= -55.9467$$

$$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$
$$= 0.1727 - \frac{0.0829}{55.9467}$$
$$= 0.1712$$

$$f(x_2) = (0.1712)^3 - 3(0.1712)^2 - 55(0.1712) + 9.5$$

$= 0.0089 - 0.0879 - 9.416 + 9.5$

$= 0.0011$

$f'(x_2) = 3(0.1712)^2 - 6(0.1712) - 55$

$= 0.0879 - 1.0272 - 55$

$= -55.9393$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 0.1712 - \frac{0.0011}{55.9393}$$

$= 0.1712$

The root of the equation is 0.1712

ii] $f(x) = x^3 - 4x - 9$ $\quad [2, 3]$

$f'(x) = 3x^2 - 4$

$f(2) = 2^3 - 4(2) - 9$

$= 8 - 8 - 9$

$= -9$

$f(3) = 3^3 - 4(3) - 9$

$= 27 - 12 - 9$

$= 6$

Let $x_0 = 3$ be the initial approximation

$\therefore$ By Newton's Method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

8

$$= 3 - \frac{6}{23}$$

$$= 2.7392$$

$$f(x_1) = (2.7392)^3 - 4(2.7392) - 9$$

$$= 20.5578 - 10.9568 - 9$$

$$= 0.596$$

$$f'(x_1) = 3(2.7392)^2 - 4$$

$$= 22.5096 - 4$$

$$= 18.5096$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 2.7392 - \frac{0.596}{18.5096}$$

$$= 2.7071$$

$$f(x_2) = (2.7071)^3 - 4(2.7071) - 9$$

$$= 0.0102$$

$$f'(x_2) = 3(2.7071)^2 - 4$$

$$= 17.9851$$

$$\therefore x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 2.7071 - \frac{0.0102}{17.9851}$$

$$\approx 2.7015$$

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$f(x_3) = (2.7015)^3 - 4(2.7015) - 9$
$= -0.0901$

$f'(x_3) = 3(2.7015)^2 - 4$
$= 17.8943$

$x_4 = 2.7015 + \frac{0.0901}{17.8943}$

$= 2.7065$

$f(x) = x^3 - 1.8x^2 - 10x + 17 \qquad [1, 2]$
$f'(x) = 3x^2 - 3.6x - 10$

$f(1) = (1)^3 - (1.8)(1)^2 - 10(1) + 17$
$= 6.2$

$f(2) = (2)^3 - (1.8)(2)^2 - 10(2) + 17$
$= -2.2$

Let $x_0 = 2$ be initial approximation
By Newton's method

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= 2 - \frac{2.2}{5.2} = 2 - 0.4230$
$= 1.577$

$$f(x_1) = (1.577)^3 - 1.8(1.577)^2 - 10(1.577) + 17$$
$$= 0.6755$$

$$f'(x_1) = 3(1.577)^2 - 3.6(1.577) - 10$$
$$= -8.2164$$

$$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$
$$= 1.577 + \frac{0.6755}{8.2164}$$
$$= 1.577 + 0.0822$$
$$= 1.6592$$

$$f(x_2) = (1.6592)^3 - 1.8(1.6592)^2 - 10(1.6592) + 17$$
$$= 0.0204$$

$$f'(x_2) = 3(1.6592)^2 - 3.6(1.6592) - 10$$
$$= -7.7143$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$
$$= 1.6592 + \frac{0.0204}{7.7143}$$
$$= 1.6618$$

$$f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) + 17$$
$$= 0.0004$$

$$f'(x_3) = 3(1.6618)^2 - 3.6(1.6618) - 10$$

$$= 8.2847 - 5.9824 - 10$$

$$= -7.6977$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$= 1.8618 + \frac{0.0004}{7.6977}$$

$$= 1.6618$$

The root of equation is 1.6618

02

Pratical no. 5

Integration

Q1 :-

i] $\int \frac{dx}{\sqrt{x^2+2x-3}}$

$= \int \frac{1}{\sqrt{x^2+2x-3}} \, dx$

$= \int \frac{1}{\sqrt{x^2+2x-1-4}} \, dx$

$= \int \frac{1}{\sqrt{(x+1)^2-4}} \, dx = \int \frac{1}{\sqrt{(x+1)^2-(2)^2}} \, dx$

$\because a^2+2ab+b^2 = (a+b)^2$

substitute

$x+1 = t$

$dx = \frac{1}{t} \, dt$ where $t=1, tx=1$

$\int \frac{1}{\sqrt{t^2-4}} \, dt$

$= \log \cancel{(} |t + \sqrt{t^2-4}|)$ $[\because \int \frac{1}{\sqrt{x^2-a^2}} dx = \log |x+\sqrt{x^2-a^2}|$

$= \log (|x+1+\sqrt{(x+1)^2-4}|)$

$= \log \left(1x + 1 + \sqrt{x^2 - 2x - 3}\ 1 \right) + c$

i) $\int (4e^{x} + 1)\, dx$

$I = \int (4e^{3x} + 1)\, dx$

$= \int 4 e^{3x} dx + \int dx$

$= 4 \int e^{3x} dx + \int dx$

$= \frac{4e^{3x}}{3} + x \qquad \left[\because \int e^{ax} dx = \frac{1}{a} e^{ax}\right]$

$= \frac{4e^{3x}}{3} + x + C$ //

ii) $\int 2x^2 - 3 \sin(x) + 5\sqrt{x}\, dx$

$I = \int 2x^2 - 3 \sin x + 5\sqrt{x}\, dx$

$= \int 2x^2 - 3 \sin x + 5x^{1/2}\, dx$

$= \int 2x^2\, dx - 3\int \sin x\, dx + 5\int x^{1/2} dx$

$= \frac{2x^3}{3} + 3\cos x + \frac{10\sqrt[3]{x}}{3} + C$

$= \frac{2x^3 + 10\sqrt[3]{x}}{3} + 3\cos x + C$

4) $\int \frac{x^3 + 3x + 4}{\sqrt{x}}\, dx$

$I = \int \frac{x^3 + 3x + 4}{\sqrt{x}}\, dx$

$= \int \left(\frac{x^3}{\sqrt{x}} + \frac{3x}{\sqrt{x}} + \frac{4}{\sqrt{x}} \right) dx$

$$= \int \frac{x^3}{x^{1/2}}\,dx + \int \frac{3x}{x^{1/2}}\,dx + 4\int x^{-1/2}\,dx$$

$$= \frac{x^{5/2+1}}{5/2+1} + 3\,\frac{x^{1/2+1}}{1/2+1} + 4\,\frac{x^{-1/2+1}}{-1/2+1}$$

$$= \frac{x^{7/2}}{7/2} + \frac{3x^{3/2}}{3/2} + \frac{4x^{1/2}}{1/2}$$

$$= \frac{2x^{7/2}}{7} + 2x^{3/2} + 8x^{1/2}$$

5] $\int t^7 \times \sin(2t^4)\,dt$

put $u = 2t^4$

$du = 2 \times 4t^3$

$$= \int t^7 \times \sin(2t^4) \times \frac{1}{2\times 4t^3}\,du$$

$$= \int t^4 \sin(2t^4) \times \frac{1}{2\times 4}\,du$$

$$= \int t^4 \sin(2t^4) \times \frac{1}{8}\,du$$

$$= t^4 \times \frac{\sin(2t^4)}{8}\,du$$

substitute t^4 with $\frac{u}{2}$

$$= \int \frac{u/2 \times \sin(2\,u/2)}{8}\,du$$

$$= \int \frac{\frac{u/2 \times \sin(u)}{2}}{8}\,du$$

$$= \frac{1}{16} \int u \times \sin(u)\, du$$

$$= \frac{1}{16} \left(u \times (-\cos(u)) - \int -\cos(u)\, du\right)$$

$$= \frac{1}{16} \times \left(u \times (-\cos u)\right) + \int \cos(u)\, du$$

$$= \frac{1}{16} \times \left(u \times (-\cos u)) + \sin(u)\right)$$

Resubstituting $u = 2t^4$

$$= \frac{1}{16} \times [2t^4 \times [-\cos(2t^4) + \sin(2t^4))$$

$$= \frac{-t^4 \times \cos(2t^4)}{8} + \frac{\sin(2t^4)}{16} + C$$

6) $\int \sqrt{x}\,(x^2-1)\, dx$

$$I = \int \sqrt{x}\,(x^2-1)\, dx$$

$$= \int x^{1/2}(x^2-1)\, dx$$

$$= \int x^{5/2} - x^{1/2}\, dx$$

$$= \frac{x^{5/2+1}}{5/2+1} - \frac{x^{1/2+1}}{1/2+1}$$

$$= \frac{2x^{7/2}}{7} - \frac{2x^3}{3} + C$$

7) $\int \frac{\cos x}{3\sqrt{\sin(x)^3}} \, dx$

$I = \int \frac{\cos x}{3\sqrt{\sin(x)^3}} \, dx$

$= \frac{\cos x}{\sin(x)^{3/2}} \, dx$

put $t = \sin x$

$dt = \cos x \, dx$

$= \frac{1}{(t)^{2/3}} \, dt$

$= (t)^{-2/3} \, dt$

$= \frac{t^{-2/3+1}}{-2/3+1}$

$= \frac{t^{1/3}}{1/3}$

$= 3\sqrt[3]{t} + C$

$= 3\sqrt[3]{\sin x} + C$

8) $\int e^{\cos^2 x} \sin 2x \, dx$

$I = \int e^{\cos^2 x} \sin 2x \, dx$

put $\cos^2 x = t$

$2\cos x(-\sin x)\,dx = dt$

$-\sin 2x\,dx = dt$

$\sin 2x\,dx = -dt$

$\therefore \int e^t(-dt)$

$= -\int e^t\,dt$ $\quad [\because \int e^x dx = e^x + c]$

Resubstituting $\cos^2 x = t$

$= -e^{\cos^2 x} + c$

(x) $\int \frac{x^2 - 2x}{x^3 - 3x^2 + 1}\,dx$

put $x^3 - 3x^2 + 1 = t$

$(3x^2 - 6x)\,dx = dt$

$3(x^2 - 2x)\,dx = dt$

$(x^2 - 2x)\,dx = \frac{dt}{3}$

$\therefore \int \left(\frac{1}{t}\right)\frac{dt}{3} = \frac{1}{3}\int \frac{1}{t}\,dt$

$= \frac{1}{3}\log|t| + c$ $\quad \left[\because \int \left(\frac{1}{x}\right)dx = \log|x| + c\right]$

Resubstituting $x^3 - 3x^2 + 1 = t$

$\therefore \frac{1}{3}\log|x^2 - 3x^2 + 1| + c$

27/01/2020

Pratical no. 6

Application of Integration & Numeric Integration

Q1. Find the length of the followings :-

i] $x = \sin t$; $y = 1-\cos t$ $\quad x \in [0, 2\pi]$

Sol^n :- length $= \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$

$dt = \int_0^{2\pi} \sqrt{(1-\cos t)^2 (\sin t)^2} \, dt$

$= \int_0^{2\pi} \sqrt{1 - 2\cos t + \cos^2 t + \sin^2 t} \; t\,dt = \sqrt{1-2\cos t}$

$= \int_0^{2\pi} \sqrt{2 - 2\cos t} \; dt$

$= \int_0^{2\pi} 2 \sin \frac{t}{2} \, dt$

$= \left[-4 \cos \frac{x}{2} \right]_0^{2\pi}$

$= (-4\cos \pi) + 4\cos 0$

$= 8$

1) $y = \sqrt{4 - x^2}$ $\quad x \in [-2, 2]$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{4 - x^2}} = \frac{-x}{\sqrt{4 - x^2}}$$

$$I = \int_{-2}^{2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_{-2}^{2} \sqrt{\frac{4 - x^2 + x^2}{4 - x^2}}\, dx$$

$$= \int_{-2}^{2} \frac{1}{2^2 - x^2}\, dx$$

$$= 2\left[\sin^{-1}(1) - \sin^{-1}(-1)\right]$$

$$= 2\left[\pi/2 + \pi/2\right] = 2\pi$$

3] $y = x^{3/2}$ on $[0, 4]$

$$\frac{dy}{dx} = \frac{3}{2} x^{1/2}$$

$$I = \int_0^4 \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_0^4 \sqrt{1 + \frac{d}{4}}\, dx$$

$$= \frac{1}{2}\left[\frac{(4+9x)^{3/2}}{3/2} \times \frac{1}{9}\right]_0^4$$

$$= \frac{1}{27}\left[(4+9x)^{3/2}\right]_0^4$$

$$= \frac{1}{27}\left[(4+0)^{3/2} - (4+36)^{3/2}\right]$$

$$= \frac{1}{27}\left(4^{3/2} - 8\right)$$

//

5]

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4) $x = 3\sin t \;, \quad y = 3\cos t \qquad t \in (0, 2\pi$

$\frac{dx}{dt} = 3\cos t \;; \quad \frac{dy}{dt} = -3\sin t$

$$I = \int_0^{2\pi} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{(3\cos t)^2 + (-3\sin t)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\, dt$$

$$= \int_0^{2\pi} 3\sqrt{x}\, dt$$

$$= 3\,[\sqrt{x}]_0^{2\pi}$$

$$= 6\pi$$

5) $x = \frac{1}{6}y^3 + \frac{1}{2}y \quad \text{on} \quad y \in (1, 2)$

$$\frac{dx}{dy} = \frac{y^2}{2} - \frac{1}{2}y^2$$

$$\frac{dx}{dy} = \frac{y^2 - 1}{2y^2}$$

$$I = \int_1^2 \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$= \int_1^2 \sqrt{1 + \left(\frac{y^4}{4y^4}\right)^2}\, dy$$

$$= \int_1^2 \sqrt{\frac{(y^4-1)^2 + 4y^4}{4y^4}}\, dy$$

$$= \int_1^2 \frac{y^4+1}{2y^2}\, dy$$

$$= \frac{1}{2}\int_1^2 y^2\, dy + \frac{1}{2}\int_1^2 y^{-2}\, dy$$

$$= \frac{1}{2}\left[\frac{y^3}{3} - \frac{y^{-1/2}}{1}\right]_1^2$$

$$= \frac{1}{2}\left[\frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 1\right]$$

$$= \frac{1}{2}\left[\frac{7}{3} + \frac{1}{2}\right]$$

$$= \frac{1}{2}\left[\frac{17}{6}\right] = \frac{17}{12}$$

$$\int_0^4 x^2\, dx \qquad n = 4$$

$$I = \frac{4-0}{4} = 1$$

x	0	1	2	3	4
y	0	1	4	9	16

$$\int_0^4 x^2 dx = \frac{2}{3}\left[(y_0 + y_n) + 4(y_1 + y_3)\, 2y_2\right]$$

$$= \frac{1}{3}\left[16 + 4(10) + 8\right]$$

$$= \frac{64}{3}$$

$$\int_0^4 x^2\, dx = 21.533$$

iii) $\int_0^{\pi/3} \sqrt{\sin x}\, dx$ with $n = 6$

$$i = \frac{\pi/3 - 0}{6} = \pi/18$$

x	0	$\pi/18$	$2\pi/18$	$4\pi/18$	$5\pi/18$	$6\pi/18$	$3\pi/18$
y	0	0.4167	0.58	0.80	0.87	0.93	0.70

$$\int_0^{\pi/3} \sqrt{\sin x} = \frac{1}{3}\left[y_0 + y_6 + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)\right]$$

$$= \pi/54 \times 12.1163$$

$$= \int_0^{\pi/3} \sqrt{\sin x}\, dx = 0.7049 \,//$$

Pratical no. 7

Topic :- Differential equation.

Q. solve

1) $x\dfrac{dy}{dx} + y = e^{x}$

2) $e^{x}\dfrac{dy}{dx} + 2e^{x}y = 1$

3) $x\dfrac{dy}{dx} = \dfrac{\cos x}{x} - 2y$

4) $x\dfrac{dy}{dx} + 3y = \dfrac{\sin x}{x^{2}}$

5) $e^{2}x\dfrac{dy}{dx} + 2e^{2x}y = 2x$

6) $\sec^{2}x\tan x\, y\, dx + \sec^{2}y\tan x\, dx = 0$

7) $\dfrac{dy}{dx} = \sin^{2}(x-y+1)$

8) $\dfrac{dy}{dx} = \dfrac{2x+3y-1}{6x+9y+6}$

1) $x\dfrac{dy}{dx} + y = e^x$

$\dfrac{dy}{dx} + \dfrac{1}{x} y = \dfrac{e^x}{x}$

$f(x) = 1/x \qquad Q(x) = e^x/x$

$I.F = e^{\int_0^1 1/x \, dx}$

$= e^{\ln x} = x$

$y\,(I.F) = \int Q(x)\,(I.F)\,dx + C$

$= \int \dfrac{e^x}{x} \times x\,dx + C$

$= \int e^x\,dx + C$

$= e^x dx + C$

2. $e^x \dfrac{dy}{dx} + 2e^x y = 1$

$\dfrac{dy}{dx} + 2y = e^{-x}$

$f(x) = 2 \qquad Q(x) = e^{-x}$

$\therefore I.F = e^{\int f(x)\,dx}$

$= e^{\int 2\,dx}$

$\therefore y\,(I.F) = \int Q(x)\,(I.F)\,dx + c$

$= \int e^{-x} \times e^{2x}\,dx + c$

$= \int e^{-x+2x}\,dx + c$

$= e^{x} + c$ //

3) $\frac{dy}{dx} + \frac{2}{x}\,y = \frac{\cos x}{x^2}$

$f(x) = \frac{2}{x}$ $\qquad Q(x) = \cos x / x^2$

$I.F = e^{\int f(x)\,dx}$

$= e^{\int 2/x\,dx}$

$= e^{\int \ln 2} \quad = x^2$

$y\,(I.F) = \int Q(x)\,(I.F)\,dx + c$

$= \int \frac{\cos^2}{x^2}\, x^2\,dx + c$

$= \int \cos x + c$

$= -\sin x + c$ //

4) $x\frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

$\frac{dy}{dx} + \frac{3}{x}\,y = \frac{\sin x}{x^2}$

$f(x) = \frac{3}{x}$ $\qquad Q(x) = \frac{\sin x}{x^2}$

$I.F = e^{\int f x \, dx}$

$= e^{\int 3/x \, dx}$

$= e^{\int \ln 3} \quad = x^3$

$y \, (I.F) = \int Q(x) \cdot (I.F) \, dx + C$

$= \int \frac{\sin x}{x^2} \times x^3 dx + C$

$= \int \sin x \, dx + C$

$= -\cos x + C$ //

5] $e^{2x} \frac{dy}{dx} + 2e^{2x} y = 2x$

$\frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$

$f(x) = 2 \qquad Q(x) = 2x / e^{2x}$

$I.F = e^{\int f(x) \, dx}$

$= e^{\int 2 dx} \quad = e^{2x}$

$y \, (I.F) = \int Q(x) \cdot (I.F) \, dx + C$

$= \int \frac{2x}{e^{2x}} \times e^{2x} dx + C$

$= \int 2x \, dx + C$

$= \cancel{2}x^2 + C$ //

$\sec^2 x \cdot \tan y \, dx + \sec^2 y \tan x \, dy = 0$

$\sec^2 x \cdot \tan y \, dx = -\sec^2 y \cdot \tan x \, dy$

$$\frac{\sec^2 x \, dx}{\tan x} = -\frac{\sec^2 y \, dy}{\tan y}$$

$$\therefore \int \frac{\sec^2 x \, dx}{\tan x} = -\frac{\sec^2 y \, dy}{\tan y}$$

$$= \log|\tan x| = -\log|\tan y| + 1$$

$$\therefore \log|\tan x| - |\tan y| = c$$

$$\tan x \cdot \tan y = c$$

$$\frac{dy}{dx} = \sin^2(x-y-1) - x$$

Differentiating both sides

$x - y - 1 = v$

$$\therefore 1 - \frac{dy}{dx} = \frac{dv}{dx}$$

$$1 - \frac{dv}{dx} = \frac{dy}{dx}$$

$$1 - \frac{dv}{dx} = \sin^2 v$$

$$\frac{dy}{dx} = \cos^2 v$$

$$\int \sec^2 v \, dv = \int dx$$

$$\tan v = x + c$$

$$\tan(x+y-1) = x + c$$

$$\frac{dy}{dx} = \frac{2x + 3y - 1}{6x + 9y + 6}$$

Put $2x + 3y = v$

$$\therefore \quad 2 + 3\frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$$

$$\frac{1}{3}\left(\frac{dv}{dx} - 2\right) = \frac{1}{3}\left(\frac{v-1}{v+2}\right)$$

$$\therefore \quad \frac{dv}{dx} = \frac{v-1}{v+2} + 2$$

$$\frac{dv}{dx} = \frac{v-1+2v+4}{v+2} = \frac{3v+3}{v+2} = 3\left(\frac{v+1}{v+2}\right)$$

$$\Rightarrow \int \frac{v+1}{v+1}\,dv + \int \frac{1}{v+1}\,dv = 3x + C$$

$$\Rightarrow \quad v + \log|v+1| = 3x + c$$

$$\Rightarrow \quad 2x + 3y + \log|2x + 3y + 1| = 3x + c$$

$$\Rightarrow \quad 3y = x - \log|2x + 3y + 1| + c$$

Praticol no. 8

Using Euler's Method.

1) $\frac{dy}{dx} = y + e^{x} - 2$, $y(0) = 0.5$ find $y(2)$

2) $\frac{dy}{dx} = 1 + y^2$, $y(0) = 0$, $h = 0.2$ find $y(1)$

3) $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$, $y(0) = 1$, $h = 0.2$ find $y(1)$

4) $\frac{dy}{dx} = 3x^2 + 1$, $y(1) = 2$ find $y(2)$
for $h = 0.5$ & $h = 0.25$

5) $\frac{dy}{dx} = \sqrt{xy} + 2$, $y(1) = 1$ find $y(1.2)$ with $h = 0.2$

sol

1) $f(x, y) = y + e^{x} - 2$ $y(0) = 2$ $h = 0.5$

x	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.1487	3.2231
2	1	3.2231	3.9414	5.1938
3	1.5	5.1938	7.6755	9.0185

$y(2) = 9.0315$

2] $\frac{dy}{dx} = 1 + y^2 = f(x, y)$ $\quad y(0) = 0 \quad h = 0.2$

x	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	1.04	0.405
2	0.4	0.408	1.1665	0.6413
3	0.6	0.6413	1.4113	0.9236
4	0.8	0.9236	1.8529	1.2942

$y(1) = 1.2942$ //

3] $f(x, y) = \sqrt{\frac{x}{y}}$ $\quad y(0) = 1 \quad h = 0.2$

x	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	0	1
1	0.2	1	0.4412	1.0894
2	0.4	1.0894	0.6059	1.2106
3	0.6	1.2106	0.704	1.3514
4	0.8	1.3314	0.7694	1.5053

$y(1) = 1.5053$ //

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4] $f(x,y) = 3x^2+1)$, $y(7)=2$ $h=0.5$

x	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
1	1	2	4	4
2	1.5	4	7.75	7.875

$y(2) = 7.875$

$h = 0.25$

x	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
1	1	2	4	4
2	1.25	3	5.6875	4.4219
3	1.5	4.4219	7.75	6.3594
4	1.75	6.3594	10.1875	8.9063

$y(1) = 8.9063$

5] $f(x,y) = \sqrt{xy} + 2$; $y(7) = 1$ $h = 0.2$

x	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
1	1	1	3	1.6

$y(1.2) = 1.6$

Pratical no. 9

Topic : Limits & Partial order Derivative.

1] Evaluate the following limits :

i) $\lim_{(x,y)\to(-4,-1)} \dfrac{x^3 - 3y + y^2 - 1}{xy + 5}$ (ii) $\lim_{(x,y)\to(2,0)} \dfrac{(y+1)(x^2+y^2-4x)}{x+3y}$

iii) $\lim_{(x,y,z)\to(1,1,1)} \dfrac{x^3 - y^2 z^2}{x^3 - x^2 y^2}$

2] find fx, fy for each of the following :

i) $f(x,y) = xy\,e^{x^2+y^2}$ ii) $f(x,y) = e^x \cos y$

iii) $f(x,y) = x^3 y^2 - 3x^2 y + y^3 + 1$

3] Using Definition find values of fx, fy at $(0,0)$ for : $f(x,y) = \dfrac{2x}{1+y^2}$

4] Find all second order partial derivatives of f. Also verify whether $fxy = fyx$

i) $f(x,y) = \dfrac{y^2 - xy}{x^2}$ ii) $f(x,y) = x^3 + 3x^2y^2 - \log(x^2+1)$

iii) $f(x,y) = \sin(xy) + e^{x+y}$

S.a.

5] find the linearization of $f(x,y)$ at given p

i] $f(x,y) = \sqrt{x^2+y}$ at $(1,1)$

2] $f(x,y) = 1-x+y\sin x$ at $\left(\frac{\pi}{2}, 0\right)$

3] $f(x,y) = \log x + \log y$ at $(1,1)$

$\Rightarrow$ Sol^n :-

Q1.

i] $\lim\limits_{(x,y)\to(-4,-1)} \dfrac{x^3-3y+y^2-1}{xy+5}$

$= \dfrac{(-4)^3-(3)(-1)+(-1)^2-1}{(-4)(-1)+5}$

$= \dfrac{-64+3+1-1}{4+5}$

$= \dfrac{-61}{9}$ //

ii] $\lim\limits_{(x,y)\to(2,0)} \dfrac{(y+1)(x^2+y^2-4x)}{x+3y}$

$= \dfrac{(0+1)(2^2+0^2-4(2))}{2+3(0)}$

$= \dfrac{1(-4)}{2} = -2$ //

iii] $\lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2z^2}{x^3-x^2y^2}$

$= \frac{(1)^2 - 1^2 . 1^2}{1^3 - 1^2.1.1}$

$= \frac{1-1}{1-1}$

$= 0$ → (solution not defined.

2]

i] $f(x,y.) = xy\,e^{x^2+y^2}$

$= xy\,e^{x^2}.e^{y^2}$

$\Rightarrow f_x = \frac{\partial x}{\partial x} . y . e^{x^2} . e^{y^2}$

$= ye^{y^2} . \frac{d}{dx} x . e^{x^2}$

$= ye^{y^2} \left[x \frac{\partial}{\partial x} e^{x^2} + e^{x^2} \frac{\partial}{\partial x} y \right]$

$= ye^{y^2} \left[2x^2 e^{x^2} + e^{x^2} \right]$

$= (2x^2+1)\, y e^{x^2+y^2}$

$\Rightarrow f_y = xe^{x^2} \frac{\partial}{\partial y} y . e^{y^2}$

$= xe^{x^2} \left[y \frac{\partial}{\partial y} e^{y^2} + e^{y^2} \frac{\partial}{\partial y} y \right]$

$= xe^{y^2} \left[2y^2 e^{y^2} + e^{y^2} \right]$

$= (2y^2+1)\, x e^{x^2+y^2}$

ii) $f(x,y) = e^x - \cos y$

$f_x = \frac{\partial}{\partial x} e^x - \cos y$

$= e^x - \cos y$

$f_y = \frac{\partial}{\partial y} e^x - \cos y$

$= e^x \cdot \sin y$

iii) $f(x,y) = x^3 y^2 - 3x^2 y + y^3 + 1$

$\Rightarrow f(x) = \frac{\partial}{\partial x}(x^3 y^2 - 3x^2 y + y^3 + 1)$

$= 3x^2 y^2 - 6xy$

$\Rightarrow f_y = \frac{\partial}{\partial y}(x^3 y^2 - 3x^2 y + y^3 + 1)$

$= 2x^3 y - 3x^2 + 3y^2$

3] i) $f(x,y) = \frac{2x}{1+y^2}$

$f(x) = \frac{1}{1+y^2} \frac{\partial}{\partial x}(2x) = \frac{2}{1+y^2}$

$f_x(0,0) = \frac{2}{1+(0)^2} = 2$

$\therefore f_y = 2x \frac{\partial}{\partial y}\left(\frac{1}{1+y^2}\right)$

$= 2x \times \frac{-1}{(1+y^2)^2} \times 2y$

$\frac{-4xy}{(1+y^2)^2}$

$f_y(0,0) = \frac{-4 \times 0 \times 0}{(1+(0)^2)^2} = 0$

$$f(x, y) = \frac{y^2 - xy}{x^2}$$

$$f(x) = \frac{\partial}{\partial x}\left(\frac{y^2 - xy}{x^2}\right)$$

$$= -2y^2 x^{-3} + \frac{y}{x^2}$$

$$= \frac{y}{x^2} - \frac{2y^2}{x^3}$$

$$f_y = \frac{2y - x}{x^2}$$

$$\therefore f_{xx} = \frac{\partial}{\partial x} f_x = \frac{\partial}{\partial x}\left(\frac{y}{x^2} - \frac{2y^2}{x^3}\right)$$

$$= \frac{6y^2}{x^2} - \frac{2y}{x^3}$$

$$\therefore f_{yy} = \frac{\partial}{\partial y} f_y = \frac{2}{x^2}$$

$$\therefore f_{xy} = \frac{\partial}{\partial y} f_x = \frac{\partial}{\partial y}\left(\frac{y}{x^2} - \frac{2y^2}{x^3}\right)$$

$$= \frac{1}{x^2} - \frac{4y}{x^3}$$

$$\therefore f_{xy} = f_{yx}$$

$$f(x,y) = x^3 + 3x^2 - \log(x^2+1)$$

$$f(x) = \frac{\partial}{\partial x}\left(x^3 + 3x^2y^2 - \log(x^2+1)\right)$$

$$= 3x^2 + 6xy^2 - \frac{2x}{x^2+1}$$

$$f_y = \frac{\partial}{\partial y}\left(x^3 + 3x^2y^2 - \log(x^2+1)\right)$$

$$= 0 + 6yx^2 - 0$$

$$= 6x^2y$$

$$f_{xx} = \frac{\partial}{\partial x}(f_x)$$

$$= \frac{\partial}{\partial x}\left(3x^2 + 6xy^2 - \frac{2x}{x^2+1}\right)$$

$$= 6x + 6y^2 - \frac{4x - 2x^2 + 2}{(x^2+2)^2}$$

$$f_{yy} = \frac{\partial}{\partial y}(f_y) = \frac{\partial}{\partial y}(6x^2y) = 6x^2$$

$$f_{xy} = \frac{\partial}{\partial y} f(x) = \frac{\partial}{\partial y}\left(3x^2 + 6xy^2 - \frac{2x}{x^2+1}\right)$$

$$= 12xy$$

$$f_{yx} = \frac{\partial}{\partial x} f_y = \frac{\partial}{\partial x}(6x^2y) = 12xy$$

$$\therefore f_{xy} = f_{yx}$$

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$$f(x, y) = \sin(xy) + e^{x+y}$$
$$= \sin(xy) + e^x . e^y$$

$$f(x) = \frac{\partial}{\partial x}(\sin(xy) + e^x . e^y)$$
$$= y \cos(xy) + e^x . e^y$$

$$f(y) = \frac{\partial}{\partial y}(\sin(xy) + e^x . e^y)$$
$$= x \cos(xy) + e^x . e^y$$

$$\therefore f_{xx} = \frac{\partial}{\partial x} f_x$$
$$= \frac{\partial}{\partial x} y \cos(xy) + e^x . e^y$$
$$= -y^2 \sin(yx) + e^x . e^y$$

$$\therefore f_{yy} = \frac{\partial}{\partial y} f_y$$
$$= \frac{\partial}{\partial y}(x \cos(xy) + e^x . e^y)$$
$$= -x^2 . \sin(xy) + e^x . e^y)$$

$$\therefore f_{xy} = \frac{\partial}{\partial y} f_x = \frac{\partial}{\partial y}(y \cos(xy) + e^x . e^y)$$
$$= -xy \sin(xy) + \cos(xy) + e^x . e^y$$

$\therefore f_{yx} = \frac{\partial}{\partial x} f_y$

$= \frac{\partial}{\partial x} (x \cos(xy) + e^x . e^y)$

$= -xy \sin(xy) + \cos(xy) + e^x . e^y$

$\therefore f_{xy} = f_{yx}$

8]

i] $f(x,y) = \sqrt{x^2+y^2}$ $\quad (a,b) = (1,1)$

$f(x) = \frac{1}{2\sqrt{x^2+y^2}} \times 2x$ $\quad f_y = \frac{2y}{2\sqrt{y^2+x^2}}$

$= \frac{x}{\sqrt{x^2+y^2}}$ $\quad = \frac{y}{\sqrt{y^2+x^2}}$

$f_x(1,1) = \frac{1}{\sqrt{2}}$ // $\quad = \frac{1}{\sqrt{2}}$ //

$\therefore L(x,y) = f(1,1) + f_x(1,1)(x-1) + f_y(1,1)(y-1)$

$= \sqrt{2} + \frac{x-1}{\sqrt{2}} + \frac{y-1}{\sqrt{2}}$

$= \sqrt{2} + \frac{x+y-2}{\sqrt{2}}$

$= \frac{x+4}{\sqrt{2}}$ //

$f(x,y) = 1 - x + y \sin x$ $\qquad (a,b) = (\pi/2, 0)$

$f(\pi/2, 0) = 1 - \frac{\pi}{2} + 0 \sin\frac{\pi}{2}$

$= \frac{2-\pi}{2}$

$f_x = -1 + y\cos x$

$f_x = (\pi/2, 0) = -1 + 0 \times \cos \pi/2$

$= -1 //$

$f_y = \sin x$

$f_y(\pi/2, 0) = \sin \pi/2$

$= 1 //$

$L(x,y) = f(\pi/2, 0) + f_x(\pi/2, 0)(x - \pi/2) + f_y(\pi/2, 0)(y - 0)$

$= \frac{2-\pi}{2} + (-1)\left(x - \frac{\pi}{2}\right) + 1(y)$

$= 1 - \frac{\pi}{2} - x + \frac{\pi}{2} + y$

$= 1 - x + y //$

$f(x,y) = \log x + \log y$ $\qquad (a,b) = (1,1)$

$f(1,1) = \log(1) + \log(1)$

$= 0$

$f(x) = \frac{1}{x}$ $\qquad f_y = \frac{1}{y}$

$f(1,1) = 1$ $\qquad f(1,1) = 1$

$\therefore L(x,y) = 0 + 1(x-1) + 1(y-1)$

$= x + y - 2 //$

Pratical no - 10

Find the decimal derivative of the following functions at given point & in the direction of given vector.

$f(x,y) = x + 2y - 3 \qquad a = (1, -1) \qquad u = 3i - j$

Here, $u = 3i - j$ is not a unit vector

$|\bar{u}| = \sqrt{3^2 + 1^2} = \sqrt{9+1} = \sqrt{10}$

unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{10}}(3, -1)$

$= \left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a) = f(1,-1) = (1) + 2(-1) - 3 = 1 - 2 - 3 = -4$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$= f\left(1 + \frac{3}{\sqrt{10}}\right), \left(-1, \frac{-h}{\sqrt{10}}\right) - 3$

$= 1 + \frac{3}{\sqrt{10}} - 2 - \frac{2h}{\sqrt{10}} - 3$

$f(a+hu) = -4 + \frac{h}{\sqrt{10}}$

$$D_u f(a) = \lim_{h \to 0} \frac{f(a+hu) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{-4 + h/\sqrt{10} + 4}{h}$$

$$D_u f(a) = \frac{1}{\sqrt{10}}$$

ii) $f(x) = 4^2 - 4x + 1$ $\quad a = (3,4)$ $\quad u = i + 5j$

Here, $u = i + 5j$ is not a unit vector

$|\bar{u}| = \sqrt{1^2 + 5^2} = \sqrt{26}$

unit along u is $\frac{u}{|u|} = \frac{1}{\sqrt{26}}(1, 5)$

$$= \left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$$

$f(a) = f(3,4) = 4^2 - 4(3) + 1) = 5$

$f(a+hu) = f(3,4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$

$$= f\left(3 + \frac{h}{\sqrt{26}}, 4 + \frac{5h}{\sqrt{26}}\right)$$

$$f(a+hu) = \left(u + \frac{5h}{\sqrt{26}}\right)^2 - 4\left(3 + \frac{h}{\sqrt{26}}\right) + 1$$

$$= \cancel{16} + \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1$$

$$= \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5$$

$$D_u f(a) = \lim_{h\to 0} \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5 - 5$$

$$= \frac{h\left(\frac{25h}{26} + \frac{36}{\sqrt{26}}\right)}{h}$$

$$\therefore D_u f(a) = \frac{25h}{26} + \frac{36}{\sqrt{26}}$$

2) Find gradiant vector for the following functions at given point.

1) $f(x,y) = x^y + y^x = a\ (1,1)$

$f(x) = y \cdot x^{y-1} + y^x \log y$

$f y = x^y \log x + x y^{x-1}$

$\nabla f(x,y) = (f x, f y)$

$= (y x^{y-1} + y^x \log y, x^y \log x + x y^{x-1})$

$f(1,1) = (1+0, 1+0)$

$= (1,1)$

(ii) $f(x,y) = (-\tan^{-1}x)y^2 \quad a = (1,-)$

$f(x) = \frac{1}{1+x^2} \cdot y^2 \quad f y = 2y - \tan^{-1}x$

$\nabla f(x, y) = f(x), f(y)$

$= \left(\frac{y^2}{1+x^2}, 2y \tan^{-1} x \right)$

$f(1, -1) = \left(\frac{1}{2}, \frac{\pi}{4}(-2) \right)$

$= \left(\frac{1}{2}, -\frac{\pi}{2} \right)$

(ii) $f(x, y, z) = xyz - e^{x+y+z}$ $\qquad a = (1, -1, 0)$

$f_x = yz - e^{x+y+z}$

$f_y = xz - e^{x+y+z}$

$f_z = xy - e^{x+y+z}$

$\nabla f(x, y, z) = f_x, f_y, f_z)$

$= yz - e^{x+y+z}, xz - e^{x+y+z}, xy - e^{x+y+z}$

$f(1, -1, 0) = ((-1)(0) - e^{(1+(-1)+0)}, 1 \cdot 0 - e^{(1+(-1)+0)}, (1)(-1) - e^{(1+(-1), 0)})$

$= (0 - e^0, 0 - e^0, -1 - e^0)$

$= (-1, -1, -2)$ //

Find the eq^n of tangent & normal to each of the following using corner at given pt.

1) $x^2 \cos y + e^{xy} = 2$ at $(1, 0)$

$f_x = \cos y \, 2x + e^{xy} y$

$f_y = x^2 (-\sin y) + e^{xy} . x$

$(x_0, y_0) = (1, 0)$

$\therefore x_0 = 1, \quad y_0 = 0$

eq^n of tangent

$f_x (x - x_0) + f_y (xy . y_0) = 0$

$f_x (x_0, y_0) = \cos 0 \, 2(1) + e^0$

$= (2)(1) + 0$

$= 2$

$f_y (x_0, y_0) = 1^2 (\sin 0) + e^0 . 1$

$= 0 + 1.1$

$= 1$

$2(x-1) + 1(y-0) = 0$

$2x - 2 + y = 0$

$2x + y - 2 = 0 \quad \ldots$ It is the required eq^n of tangent.

eq^n of normal

$= ax + by + c = 0$

$\therefore bx + ay + d = 0$

$1(1) + 2(y) + d = 0$

$1 + 2y + d = 0$ at $(1, 0)$

$1 + 2(0) + d = 0$

$d + 1 = 0$

$d = -1$

ii) $x^2 + y^2 - 2x + 3y + 2 = 0$ at $(2, -2)$

$f(x) = 2x + 0 - 2 + 0 + 0$

$= 2x - 2$

$f_y = 0 + 2y - 0 + 3 + 0$

$= 2y + 3$

$(x_0, y_0) = (2, -2)$ $\therefore$ $x_0 = 2,\ y_0 = -2$

$f_x(x_0, y_0) = 2(2) - 2 = 2$

$f_y(x_0, y_0) = 2(-2) + 3 = -1$

eq^n of tangent :-

$f_x(x - x_0) + f_y(y - y_0) = 0$

$2(x-2) + (-1(y+2)) = 0$

$2x - 2 - y - 2 = 0$

$2x - y - 4 = 0$ $\therefore$ It is required eq^n of tangent

eq^n of normal

$ax + by + c = 0$

$bx + ay + d = 0$

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Ans:

$= -1(x) + 2(y) + d = 0$ at $(2, -2)$

$-x + 2y + d = 0$

$-2 + 2(-2) + d = 0$

$-2 - 4 + d = 0$

$d = 6$ //

4] Find the eq^n of tangent & normal line to each of the following surface.

1) $x^2 - 2yz + 3y + xz = 7$ at $(2, 1, 0)$

$f_x = 2x - 0 + 0 + z$

$f_x = 2x + z$

$f_y = 0 - 2z + 3 + 0$

$= 2z + 3$

$f_z = 0 - 2y + 0 + x$

$= -2y + x$

$(x_0, y_0, z_0) = (2, 1, 0)$ $\therefore$ $x_0 = 2, \; y_0 = 1, \; z_0 = 0$

$f_x(x_0, y_0, z_0) = 2(2) + 0 = 4$

$f_y(x_0, y_0, z_0) = 2(0) + 3 = 3$

$f_z(x_0, y_0, z_0) = -2(1) + 2 = 0$

eq^n of tangent

$f_x(x - x_0) + f_y(y - y_0) + f_z(z - z_0) = 0$

$= 4(x-2) + 3(y-1) + 0(z-0) = 0$

$= 4x - 8 + 3y - 3 = 0$

$\therefore 4x + 3y - 11 = 0$

$\therefore$ This is required eq^n of tangent.

eq^n of normal at $(2, 3, -11)$

$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}$

$= \frac{x-2}{4} = \frac{y-4}{3} = \frac{z+11}{0}$

$\therefore 3xyz - x - y + z = 4$ at $(1, -1, 2)$

$3xyz - x - y + z + 4 = 0$ at $(1, -1, 2)$

$f_x = 3yz - 1 - 0 + 0 + 0$

$= 3yz - 1$

$f_y = 3xz - 0 - 1 + 0 + 0$

$= 3xz - 1$

$f_z = 3xy - 0 - 0 + 1 + 0$

$= 3xy + 1$

$(x_0, y_0, z_0) = (1, -1, 2)$ $\therefore x_0 = 1, y_0 = -1, z_0 = 2$

$f_x(x_0, y_0, z_0) = 3(-1)(2)(-1) = -7$

$f_y(x_0, y_0, z_0) = 3(+1)(2) - 1 = 5$

$f_z(x_0, y_0, z_0) = 3(1)(-1) + 1 = -2$

eq^n of tangent

$-7(x-1) + 5(y+1) - 2(z-2) = 0$

$-7x + 7 + 5y + 5 - 2z + 4 = 0$

$-7x + 5y - 2z + 16 = 0$ //

eq^n of normal at $(-7, 5, -2)$

$$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}$$

$$\frac{x-1}{7} = \frac{y+1}{5} = \frac{z-2}{-2}$$

Find the total maximal & minima for the following.

$f(x, y) = 3x^2 + y^2 - 3xy + 6x - 4y$

$f_x = 6x + 0 - 3y + 6 = 0$

$= 6x - 3y + 6$

$f_y = 0 + 2y - 3x + 0 - 4$

$= 2y - 3x - 4$

$f_x = 0$

$6x - 3y + 6 = 0$

$3(2x - y + 2) = 0$

$2x - y + 2 = 0$

$2x - y = -2 \quad \ldots (1)$

$f_y = 0$

$2y - 3x - 4 = 0$

$2y - 3x = 4$ ... (2)

multiply eq (1) with 2

$\therefore 4x - 2y = -4$

$2y - 3x = 4$

Substitute value of x in eq (1)

$2(0) - y = -2$

$y = 2$

$\therefore$ critical points are $(0, 2)$

$r = f_{xx} = 6$

$t = f_{yy} = 2$

$s = f_{xy} = -3$

Here, $r > 0$

$\therefore rt - s^2 = 6(2) - (-3)^2$

$= 12 - 9$

$= 3 > 0$

$\therefore f$ has maximum at $(0, 2)$

$3x^2 + y^2 - 3xy + 6x - 4y$ at $(0, 2)$

$3(0)^2 + (2)^2 - 3(0)(2) + 6(0) - 4(2)$

$= 0 + 4 - 0 + 0 - 8$

$= -4$

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(ii) $f(x,y) = 2x^4 + 3x^2y - y^2$

$f_x = 8x^3 + 6xy$

$f_y = 3x^2 - 2y$

$f_x = 0$

$\therefore \quad 8x^3 + 6xy = 0$

$\therefore \quad 2x(4x^2 + 3y) = 0$

$\therefore \quad 4x^2 + 3y = 0 \quad \ldots (1)$

$f_y = 0$

$3x^2 - 2y = 0 \quad \ldots (2)$

Multiply eq (1) with 3

(2) with 4

$\therefore \quad 12x^2 + 9y = 0$

$- \quad 12x^2 - 8y = 0$

$\therefore \quad y = 0$

substitute value of y in eq^n (1)

$4x^2 + 3(0) = 0$

$\therefore \quad 4x^2 = 0$

$x = 0$

critical point is $(0,0)$

$r = f_{xx} = 24x^2 + 6y$

$t = f_{yy} = 0 - 2 = -2$

$s = f_{xy} = 6x - 0 = 6x = 6(0) = 0$

r at $(0,0)$

$= 24(0) + 6(0) = 0$

$\therefore r = 0$

$rt - s^2 = (0-2) - s^2$

$= 0 - 0 - 0$

$\therefore r = 0$ & $rt - s^2 = 0$

(nothing to say)

$f(x, y)$ at $(0, 0)$

$\therefore 2(0)^4 + 3(0)^2 (0)(0)$

$= 0$ //

$f(x, y) = x^2 - y^2 + 2x + 8y - 70$

$f_x = 2x + 2$ $\quad f_y = -2y + 8$

$f_x = 0$ $\quad f_y = 0$

$\therefore 2x + 2 = 0$ $\quad -2y + 8 = 0$

$x = -1$ $\quad y = 4$

$\therefore$ critical point is $(-1, 4)$

$r = f_{xx} = 2$ $\quad t = f_{yy} = -2$ $\quad s = f_{xy} = 0$

$\therefore r > 0$

$rt - s^2 = 2(-2) - (0)^2 = -4 - 0 = -4 < 0$ //

$f(x, y)$ at $(-1, 4)$

$(-1)^2 - (4)^2 + 2(-1) + 8(4) - 70$

$= 1 + 16 - 2 + 32 - 70$

$= -23$ //

AK

11/01/2020