ASEN 6412 Uncertainty Quantification

Midterm Exam

Instructor: Alireza Doostan

Due: Wednesday 04/03/24 at 8:00 am on GradeScope

Name:

1. **The exam duration is 180 minutes. When you start the exam, the 180 will start. In addition to 180 min, you have 10 minutes to scan and upload your exam on GradeScope.**
2. **For coding questions, please include your codes as a .pdf attachment to the rest of your submission.**
3. You may use any material related to the course (notes, slides, HW solutions, etc.), but please do not use any online material or books (including those that are references for this class).
4. Please do not discuss the exam problems with anyone else.
5. Please answer each question asked as briefly as possible; please do not redefine the problems but explain any assumption you make. Contradicting answers will be graded based on the incorrect answer.
6. Late exams will not receive full grade.

Problem 1. **[15 points]** Let random variable $X \sim U[-1, 1]$. Consider random variables $Y_1 := X$ and $Y_2 := (3X^2 - 1)$. Using a proper reasoning, answer the following questions:

1. What are the expectations of Y_1 and Y_2?
2. Are Y_1 and Y_2 uncorrelated?

Problem 2. **[35 points]** The initial concentration of a chemical species is measured in the field 200 times and reported below. We would like to model this initial concentration as a **non-Gaussian** random variable and draw 1000 samples of that ransom variable that are *consistent* with the data. Propose an approach to accomplish this, write a code (in your language of choice) to generate 1000 new samples, and compare the resulting histogram with that of the data. **Note:** *Several approaches can be used for this modeling, so please stretch your thinking and use an approach that is easier to code up.*

3.26 6.8 3.79 2.45 2.67 2.88 5.54 4.63 4.13 3.02 4.1 1.53 1.96 2.96 2.69 4.26 2.57 6.92 1.7 3.13 3.15 2.68 3.14 5.3 2.29 1.71 3.54 2.1 3.31 2.97 4.12 2.92 4.36 3.62 1.96 4.56 2.04 3.8 3.04 2.38 2.37 1.89 2.16 2.37 3.16 7.35 1.77 6.08 5.08 2.93 1.62 3.01 1.93 2.79 1.67 1.7 1.72 1.83 2.45 4.01 6.22 2.25 4.65 3.39 1.64 2.69 2.28 1.57 2.14 2.19 2.78 2.44 1.81 2.58 2.44 2.6 3.23 1.71 1.92 4.77 3.43 2.92 2.77 2.5 3.89 2.08 2.09 2.89 2.9 2.95 2.19 1.99 3.52 3.9 3.51 3.99 1.96 2.31 2.51 6.56 3 3.6 3.41 1.83 1.85 1.92 2.72 1.96 2.98 3.16 2.36 3.46 2.44 3.35 1.6 3.28 4.21 1.84 2.67 2.47 2.14 3.06 1.93 2.66 1.41 3.7 5.07 1.74 9.51 2.24 3.27 2.68 3.65 1.93 4.46 4.19 1.83 3.44 3.11 2.24 1.36 3.19 3.94 4.64 2.72 3.33 1.88 2.24 1.93 4.82 1.73 7.51 5.4 3.45 2.54 2.05 1.59 2.6 1.38 7.4 1.73 2.41 3.54 2.23 3.69 5.41 4.18 4.07 2.75 4.6 4.04 2.82 2.25 3 2.18 2.01 5.32 2.27 1.85 2.24 2.2 1.7 1.64 4.67 2.36 5.22 2.5 2.71 1.68 2.5 4.73 1.64 2.78 4.88 2.93 3.97 1.45 3.14 2.25 4.04

Problem 3. **[20 points]** Write a code that uses Monte Carlo simulation to compute the probability of getting a sum of 8 from a pair of fair dice. Show the convergence of your estimate as a function of number of Monte Carlo samples relative to the reference value.

Problem 4. **[10 points]** True or False? Please answer the following questions with justifications that are **no more than two sentences**.

1. The Karhunen-Loeve (KL) expansion is only applicable to second-order Gaussian stochastic processes.
2. Any truncated KL expansion of a WSS process is also WSS.
3. In general, unbiased parameter estimators are preferred.
4. Maximum likelihood estimation requires data to be statistically independent.

Problem 5. **[25 points]** The solution to the ODE model $u_t = -ku$ with initial solution $u(t) = u_0$ is given by $u(t) = u_0 \exp(-kt)$. We would like to model the decay rate $k > 0$ as a suitable **uniform** random variable and quantify the mean and variance of u at $t = 10$ sec using stochastic collocation.

1. Please explain what quadrature grid you would suggest using and how you would compute the desired mean and variance. Please use equations to explain your answer; **no calculations or coding needed.**
2. Now let u_0 follow a **Gaussian** distribution. Can stochastic collocation be used to compute the mean and variance of u at $t = 10$ sec? Please answer this question in no more than three sentences.

Note: *Notice that* 5 *bonus points are considered (sum of the points is 105).*