Linear Algebra and its Applications

Assignment

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1. $A+B+C=1$
$A+2B+4C=-1$
$A+3B+9C=1$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$Ax = b$$

$$[A\ b] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & -1 \\ 1 & 3 & 9 & 1 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & -2 \\ 0 & 2 & 8 & 0 \end{bmatrix} \quad R_3 \to R_3 - 2R_2$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & 2 & 4 \end{bmatrix}$$

$2C = 4 \Rightarrow C = 2$
$B + 3C = -2 \Rightarrow B = -8$
$A + B + C = 1 \Rightarrow A = 7$

$\therefore$ Equation of parabola : $\boxed{y = 7 - 8x + 2x^2}$

2. $$A = \begin{bmatrix} 2 & 5 & 2 & -5 \\ 4 & 12 & 3 & -14 \\ -10 & -29 & -5 & 38 \\ 10 & 21 & 21 & -6 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 + 5R_1 \\ R_4 \to R_4 - 5R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & -4 & 5 & 13 \\ 0 & -4 & 11 & 19 \end{bmatrix} \quad \begin{matrix} R_3 \to R_3 + 2R_2 \\ R_4 \to R_4 + 2R_2 \end{matrix}$$

$$\sim \begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 9 & 11 \end{bmatrix} \quad R_4 \to R_4 - 3R_3$$

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$$\sim \begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 0 & -4 \end{bmatrix} = U \qquad L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -5 & -2 & 1 & 0 \\ 5 & -2 & 3 & 1 \end{bmatrix}$$

$\therefore \quad A = LU$

$$\begin{bmatrix} 2 & 5 & 2 & -5 \\ 4 & 12 & 3 & -14 \\ -10 & -29 & -5 & 38 \\ 10 & 21 & 21 & -6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -5 & -2 & 1 & 0 \\ 5 & -2 & 3 & 1 \end{bmatrix} \begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 0 & -4 \end{bmatrix}$$

3. $T(x,y,z) = (x+2y-z,\ y+z,\ x+y-2z)$

(i) $T(1,0,0) = (1,0,1) = 1\begin{bmatrix}1\\0\\0\end{bmatrix} + 0\begin{bmatrix}0\\1\\0\end{bmatrix} + 1\begin{bmatrix}0\\0\\1\end{bmatrix}$

$T(0,1,0) = (2,1,1) = 2\begin{bmatrix}1\\0\\0\end{bmatrix} + 1\begin{bmatrix}0\\1\\0\end{bmatrix} + 1\begin{bmatrix}0\\0\\1\end{bmatrix}$

$T(0,0,1) = (-1,1,-2) = -1\begin{bmatrix}1\\0\\0\end{bmatrix} + 1\begin{bmatrix}0\\1\\0\end{bmatrix} - 2\begin{bmatrix}0\\0\\1\end{bmatrix}$

$$\therefore \quad T = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$

(ii) $[T \ b] = \begin{bmatrix} 1 & 2 & -1 & x \\ 0 & 1 & 1 & y \\ 1 & 1 & -2 & z \end{bmatrix}$ $\quad R_3 \to R_3 - R_1$

$\sim \begin{bmatrix} 1 & 2 & -1 & x \\ 0 & 1 & 1 & y \\ 0 & -1 & -1 & z-x \end{bmatrix}$ $\quad R_3 \to R_3 + R_2$

$\sim \begin{bmatrix} 1 & 2 & -1 & x \\ 0 & 1 & 1 & y \\ 0 & 0 & 0 & -x+y+z \end{bmatrix}$

Basis for $C(A) = \{(1,0,1), (2,1,1)\}$ $\quad \dim(C(A)) = 2$

$C(A)$ is a 2-D plane in R^3

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Basis for $C(A^T) = \{(1,2,-1), (0,1,1)\}$ $\dim(C(A^T)) = 2$

$C(A^T)$ is a 2-D plane in R^3

Reducing T to row reduced form:

$$\sim \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \qquad R_1 \to R_1 - 2R_2$$

$$\sim \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Special solution : $\{(3,-1,1)\}$ $\dim(N(A)) = 1$

$N(A)$ is a line in R^3

Left Null space : Special solution : $\{(-1,1,1)\}$ $\dim(N(A^T))=$

$N(A^T)$ is a line in R^3

(iii) $|T - \lambda I| = 0$

$a+b+c = 0$

$$\begin{vmatrix} 1-\lambda & 2 & -1 \\ 0 & 1-\lambda & 1 \\ 1 & 1 & -2-\lambda \end{vmatrix} = 0$$

$ab+bc+ca = -3-1+1 = -3$

$abc = 0$

$\lambda^3 + (-3)\lambda = 0$

$\lambda = 0 \qquad \lambda^2 = 3$

$\boxed{\lambda = 0, \sqrt{3}, -\sqrt{3}}$ are the eigen vectors.

To find eigen values,

$$\lambda_1 = 0 \Rightarrow \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

This is the $N(A)$, null space of T.

$$x_1 = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$$

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$$\lambda_2 = \sqrt{3} \Rightarrow \begin{bmatrix} 1-\sqrt{3} & 2 & -1 \\ 0 & 1-\sqrt{3} & 1 \\ 1 & 1 & -2-\sqrt{3} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0 \qquad R_3 \to R_3 - \frac{1}{1-\sqrt{3}} R_1$$

$$\sim \begin{bmatrix} 1-\sqrt{3} & 2 & -1 \\ 0 & 1-\sqrt{3} & 1 \\ 0 & -\frac{(1+\sqrt{3})}{1-\sqrt{3}} & -2-\sqrt{3}+\frac{1}{1-\sqrt{3}} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0 \qquad R_3 \to R_3 + \frac{1+\sqrt{3}}{(1-\sqrt{3})^2} R_2$$

$$\sim \begin{bmatrix} 1-\sqrt{3} & 2 & -1 \\ 0 & 1-\sqrt{3} & 1 \\ 0 & 0 & \underbrace{-2-\sqrt{3}+\frac{1}{1-\sqrt{3}}+\frac{\sqrt{3}+1}{(1-\sqrt{3})^2}}_{0} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$(1-\sqrt{3})y + z = 0$$

$$y = \left(\frac{-1}{1-\sqrt{3}}\right) z$$

$$(1-\sqrt{3})x + 2y - z = 0$$

$$(1-\sqrt{3})x = z + \frac{2}{1-\sqrt{3}} z$$

$$x = \left(\frac{-\sqrt{3}}{1-\sqrt{3}}\right) z$$

$$\therefore \cancel{x_2 = \begin{bmatrix} (1+\sqrt{3}) \\ (1+\sqrt{3})/2 \end{bmatrix}} \qquad x_2 = \begin{bmatrix} -\sqrt{3}/(1-\sqrt{3}) \\ -1/(1-\sqrt{3}) \\ 1 \end{bmatrix}$$

$$\lambda_3 = -\sqrt{3} \Rightarrow \begin{bmatrix} 1+\sqrt{3} & 2 & -1 \\ 0 & 1+\sqrt{3} & 1 \\ 1 & 1 & -2+\sqrt{3} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0 \qquad R_3 \to R_3 - \frac{1}{1+\sqrt{3}} R_1$$

$$\begin{bmatrix} 1+\sqrt{3} & 2 & -1 \\ 0 & 1+\sqrt{3} & 1 \\ 0 & 1-\frac{2}{1+\sqrt{3}} & -2+\sqrt{3}+\frac{1}{1+\sqrt{3}} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0 \qquad R_3 \to R_3 - \frac{(-1+\sqrt{3})}{(1+\sqrt{3})^2} R_2$$

$$\begin{bmatrix} 1+\sqrt{3} & 2 & -1 \\ 0 & 1+\sqrt{3} & 1 \\ 0 & 0 & -2+\sqrt{3}+\frac{1}{1+\sqrt{3}}-\frac{(\sqrt{3}-1)}{(1+\sqrt{3})^2} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

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$(1+\sqrt{3})y + z = 0$ $\qquad$ $(1+\sqrt{3})x + 2\left(\frac{-1}{1+\sqrt{3}}\right)z - z = 0$

$y = -1/1+\sqrt{3}\ z$

$x = \frac{\sqrt{3}}{1+\sqrt{3}}\ z$

$$\therefore\quad x_3 = \begin{bmatrix} \sqrt{3}/(1+\sqrt{3}) \\ -1/(1+\sqrt{3}) \\ 1 \end{bmatrix}$$

$\therefore$ Eigen vectors are $x_1 = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$, $x_2 = \begin{bmatrix} -\sqrt{3}/(1-\sqrt{3}) \\ -1/(1-\sqrt{3}) \\ 1 \end{bmatrix}$, $x_3 = \begin{bmatrix} \sqrt{3}/(1+\sqrt{3}) \\ -1/(1+\sqrt{3}) \\ 1 \end{bmatrix}$

(iv) $T = QR$. Using gram-schmidt process, $a = (1,0,1)$
$b = (2,1,1)$
$c = (-1,1,-2)$

- $q_1 = \frac{a}{\|a\|} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

- $B = b - (q_1^T b) q_1$

$q_1^T b = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} = 3/\sqrt{2}$

$B = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} - \frac{3}{(\sqrt{2})^2}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1 \\ -1/2 \end{bmatrix}$ $\qquad$ $\|B\| = \frac{2}{\sqrt{6}}$

$q_2 = \frac{B}{\|B\|} = \frac{1}{\sqrt{6}}\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$

- $C = c - (q_1^T c) q_1 - (q_2^T c) q_2$

$q_1^T c = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix} = -3/\sqrt{2}$

$q_2^T c = \frac{1}{\sqrt{6}}\begin{bmatrix} 1 & 2 & -1 \end{bmatrix}\begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix} = 3/\sqrt{6}$

$C = \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix} + \frac{3}{\sqrt{2}}\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} - \frac{3}{\sqrt{6}}\frac{1}{\sqrt{6}}\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = q_3$

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$$\therefore \quad Q = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{6} & 0 \\ 0 & 2/\sqrt{6} & 0 \\ 1/\sqrt{2} & -1/\sqrt{6} & 0 \end{bmatrix}$$

$$A = QR$$

$$Q^T A = Q^T Q R$$

$$\boxed{R = Q^T A}$$

$$R = Q^T A = \begin{bmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 1/\sqrt{6} & 2/\sqrt{6} & -1/\sqrt{6} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$

$$R = \begin{bmatrix} 1.414 & 2.121 & -2.121 \\ 0 & 1.224 & 1.224 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} \sqrt{2} & 3/\sqrt{2} & -3/\sqrt{2} \\ 0 & 3/\sqrt{6} & 3/\sqrt{6} \\ 0 & 0 & 0 \end{bmatrix}.$$

$$\therefore \quad A = QR$$

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{6} & 0 \\ 0 & 2/\sqrt{6} & 0 \\ 1/\sqrt{2} & -1/\sqrt{6} & 0 \end{bmatrix} \begin{bmatrix} \sqrt{2} & 3/\sqrt{2} & -3/\sqrt{2} \\ 0 & 3/\sqrt{6} & +3/\sqrt{6} \\ 0 & 0 & 0 \end{bmatrix}$$

4) $y = c + dx \Rightarrow$

$$4 = c - 4d$$
$$6 = c + d$$
$$10 = c + 2d$$
$$8 = c + 3d$$

$$\begin{bmatrix} 1 & -4 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}_{4\times 2} \begin{bmatrix} c \\ d \end{bmatrix}_{2\times 1} = \begin{bmatrix} 4 \\ 6 \\ 10 \\ 8 \end{bmatrix}_{4\times 1}$$

$$Ax = b.$$

$$A^T A \hat{x} = A^T b$$

$$A^T A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -4 & 1 & 2 & 3 \end{bmatrix}_{2\times 4} \begin{bmatrix} 1 & -4 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}_{4\times 2} = \begin{bmatrix} 4 & 2 \\ 2 & 30 \end{bmatrix}$$

$$A^T b = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -4 & 1 & 2 & 3 \end{bmatrix}_{2\times 4} \begin{bmatrix} 4 \\ 6 \\ 10 \\ 8 \end{bmatrix}_{4\times 1} = \begin{bmatrix} 28 \\ 34 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 2 \\ 2 & 30 \end{bmatrix} \begin{bmatrix} \hat{c} \\ \hat{d} \end{bmatrix} = \begin{bmatrix} 28 \\ 34 \end{bmatrix} \qquad R_2 \to R_2 - \frac{1}{2} R_1$$

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$$\begin{bmatrix} 4 & 2 \\ 0 & 29 \end{bmatrix} \begin{bmatrix} \hat{c} \\ \hat{d} \end{bmatrix} = \begin{bmatrix} 28 \\ 20 \end{bmatrix}$$

$\hat{d} = \frac{20}{29}$ $4\hat{c} + 2\hat{d} = 28$

$\hat{c} = \frac{193}{29}$

$\therefore$ Best fit line, $\boxed{y = \frac{193}{29} + \frac{20}{29} x}$

5) The plane $x_1 + x_2 + 3x_3 + 4x_4 = 0$ can be viewed as the nullspace of $(1\ 1\ 3\ 4)$.

Row space of matrix $(1\ 1\ 3\ 4)$ is orthogonal to the nullspace, which is the plane.

$$\therefore \quad e = \begin{bmatrix} 1 \\ 1 \\ 3 \\ 4 \end{bmatrix}_{4\times 1}$$

$$\boxed{Q = (e^T e)^{-1} e e^T}$$

$e^T e = 27 \Rightarrow (e^T e)^{-1} = 1/27$

$$Q = \frac{1}{27} \begin{bmatrix} 1 \\ 1 \\ 3 \\ 4 \end{bmatrix} \begin{bmatrix} 1 & 1 & 3 & 4 \end{bmatrix} = \frac{1}{27} \begin{bmatrix} 1 & 1 & 3 & 4 \\ 1 & 1 & 3 & 4 \\ 3 & 3 & 9 & 12 \\ 4 & 4 & 12 & 16 \end{bmatrix}$$

$$P = I - Q = \frac{1}{27} \begin{bmatrix} 26 & -1 & -3 & -4 \\ -1 & 26 & -3 & -4 \\ -3 & -3 & 18 & -12 \\ -4 & -4 & -12 & 11 \end{bmatrix}$$

Alternate Method : $A = \begin{bmatrix} -1 & -3 & -4 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$P = A(A^T A)^{-1} A^T$

$Q = I - P$

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6) $A = \begin{bmatrix} a & 2 & 2 \\ 2 & a & 2 \\ 2 & 2 & a \end{bmatrix}$

Test for positive definite (determinant test),

$|a| > 0. \Rightarrow a > 0$

$\begin{vmatrix} a & 2 \\ 2 & a \end{vmatrix} > 0 \Rightarrow a^2 - 4 > 0 \Rightarrow a > 2.$
$a < -2$ ✗

$\begin{vmatrix} a & 2 & 2 \\ 2 & a & 2 \\ 2 & 2 & a \end{vmatrix} > 0 \Rightarrow a^3 - 12a + 16 > 0 \Rightarrow a > 2.$
$a < -4$ ✗

$\therefore$ a has to be greater than 2 for A to be positive definite.

$\Rightarrow$ Range of a : $(2, \infty)$

$f = 2(x_1^2 + x_2^2 + x_3^2 - x_1x_2 - x_2x_3)$

$= 2x_1^2 + 2x_2^2 + 2x_3^2 - 2x_1x_2 - 2x_2x_3$

$B = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$

7) $A = \begin{bmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{bmatrix}_{3\times2}$

$A_{3\times2} = U_{3\times3} \Sigma_{3\times2} V^T_{2\times2}$

$A^TA = \begin{bmatrix} -3 & 6 & 6 \\ 1 & -2 & -2 \end{bmatrix} \begin{bmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{bmatrix} = \begin{bmatrix} 81 & -27 \\ -27 & 9 \end{bmatrix}_{2\times2}$

$\lambda^2 - 90\lambda = 0$

$\boxed{\lambda_1 = 90 \quad \lambda_2 = 0}$

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(5) $\lambda_1 = 90 \Rightarrow \begin{bmatrix} -9 & -27 \\ -27 & -81 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$ $\quad -9x = 27y$ $\boxed{x = -3y}$

$$x_1 = \begin{bmatrix} -3 \\ 1 \end{bmatrix} \Rightarrow V_1 = \begin{bmatrix} -3/\sqrt{10} \\ 1/\sqrt{10} \end{bmatrix}$$

$\lambda_2 = 0 \Rightarrow \begin{bmatrix} 81 & -27 \\ -27 & 9 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$ $\quad 81x = 27y$ $\boxed{3x = y}$

$$x_2 = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \Rightarrow V_2 = \begin{bmatrix} 1/\sqrt{10} \\ 3/\sqrt{10} \end{bmatrix}$$

$$V = \begin{bmatrix} -3/\sqrt{10} & 1/\sqrt{10} \\ 1/\sqrt{10} & 3/\sqrt{10} \end{bmatrix}_{2\times 2}$$

Singular values, $\sigma_1 = \sqrt{\lambda_1} = 3\sqrt{10}$

$\sigma_2 = \sqrt{\lambda_2} = 0$

$$\Sigma = \begin{bmatrix} 3\sqrt{10} & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}_{3\times 2}$$

Eigen values of AA^T are $90, 0, 0$

WKT, $Av = \sigma u$

$$\boxed{u_i = \frac{AV_i}{\sigma_i}}$$

$$u_1 = \frac{AV_1}{\sigma_1} = \frac{1}{3\sqrt{10}} \cdot \frac{1}{\sqrt{10}} \begin{bmatrix} -3 & 1 \\ +6 & -2 \\ 6 & -2 \end{bmatrix}\begin{bmatrix} -3 \\ 1 \end{bmatrix} = \frac{1}{30}\begin{bmatrix} 10 \\ -20 \\ -20 \end{bmatrix}$$

$$u_1 = \frac{1}{3}\begin{bmatrix} 1 \\ -2 \\ -2 \end{bmatrix}$$

u_2 and u_3 are orthogonal to u_1.

They will be in the null space of $\boxed{u_1^T x = 0}$

$$\frac{1}{3}x - \frac{2}{3}y - \frac{2}{3}z = 0$$

$\boxed{x - 2y - 2z = 0}$ (1) $\rightarrow$ u_2 and u_3 is in nullspace of (1)

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$\begin{bmatrix} 1 & -2 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

Basis $\Rightarrow a = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$, $b = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$

Applying gram-schmidt process to convert a and b to u_2 and u_3 respectively.

$$q_1 = \frac{a}{\|a\|} = \frac{1}{\sqrt{5}} \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} = u_2$$

$$E = b - (q_1^T b) q_1$$

$$q_1^T b = \frac{1}{\sqrt{5}} (2 \ 1 \ 0) \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} = 4/\sqrt{5}$$

$$E = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} - \frac{4}{5} \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2/5 \\ -4/5 \\ 1 \end{pmatrix} \qquad \|E\| = \frac{3\sqrt{5}}{5}$$

$$q_2 = \frac{E}{\|E\|} = \frac{1}{3\sqrt{5}} \begin{pmatrix} 2 \\ -4 \\ 5 \end{pmatrix} = u_3$$

$$\therefore \quad U = \begin{bmatrix} 1/3 & 2/\sqrt{5} & 2/3\sqrt{5} \\ -2/3 & 1/\sqrt{5} & -4/3\sqrt{5} \\ -2/3 & 0 & 5/3\sqrt{5} \end{bmatrix}_{3\times 3}$$

$$\therefore \quad A = U \Sigma V^T$$

$$\begin{bmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{bmatrix} = \begin{bmatrix} 1/3 & 2/\sqrt{5} & 2/3\sqrt{5} \\ -2/3 & 1/\sqrt{5} & -4/3\sqrt{5} \\ -2/3 & 0 & 5/3\sqrt{5} \end{bmatrix} \begin{bmatrix} 3\sqrt{10} & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} -3/\sqrt{10} & 1/\sqrt{10} \\ 1/\sqrt{10} & 3/\sqrt{10} \end{bmatrix}$$

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