UE18MA251: LINEAR ALGEBRA AND ITS APPLICATIONS

Unit 1 - Matrices and Gaussian Elimination Introduction, The Geometry of Linear Equations, Gaussian Elimination, Singular cases, Elimination Matrices, Triangular factors and Row Exchanges, Inverses and Transposes, Inverse by Gauss -Jordan method. Self Learning Component : Algebra of Matrices.

Class No.	Portions to be covered
1	Introduction to Linear Algebra
2-3	The Geometry of Linear Equations – Row and Column Pictures
4	Singular cases in two and three dimensions
5-6	Gaussian Elimination, The breakdown of elimination
7	**Scilab Class Number1 – Gaussian Elimination**
8	Elementary Matrices
9-10	Triangular Factors and Row Exchanges
11-12	Inverse by Gauss -Jordan Method, Transposes
13-14	**Scilab Class Number 2&3- LU Decomposition and Inverses**

Class Work Problems :

1	Explain the row approach to solve the system $2x - 3y = 3$, $x + 3y = 6$ with a neat diagram. What happens to the solution when the second equation is replaced by $x + 3y = -6$? **Answer**: Solution is $x = 3, y = 1$; New solution is $x = -1$, $y = -5/3$.
2	Explain the column approach to solve the system $x - 2y = 3$, $2x + y = 1$ with a neat diagram. What happens to the solution if the second equation is replaced by $2x - 4y = 5$? **Answer** : $x = 1$, $y = -1$. If the second equation is replaced by $2x - 4y = 5$ then the system is singular.
3	Solve the following systems of equations using Gaussian elimination: (i) $2x + y + 3z = 1$, $2x + 6y + 8z = 3$, $6x + 8y + 18z = 5$ **Answer** : $x = 3/10$, $y = 2/5$, $z = 0$ (ii) $3x + y - 6z = -10$, $2x + y - 5z = -8$, $6x - 3y + 3z = 0$ **Answer** : $x = k-2$, $y = 3k - 4$, $z = k$ where k is a scalar (iii) $x + z = 1$,$x + y + z = 2$, $x - y + z = 1$. What if the right hand side is (1, 2, 0) ? **Answer** : Inconsistent system ; $x = 1 - k$, $y = 1$, $z = k$, $k \in R$
4	Determine the values of a and b for which the system of equations $x + y + az = 2b$, $x + 3y + (2 + 2a)z = 7b$, $3x + y + (3 + 3a)z = 11b$ will have (i) unique nontrivial solution (ii) trivial solution (iii) no solution (iv) infinity of solutions. **Answer** :(i) $a \neq -5$ and any b (ii) $a \neq -5$ and $b = 0$ (iii) $a = -5$ and $b \neq 0$ (iv) $a = -5$ and $b=0$.
5	Let $A = \begin{bmatrix} 3 & -6 & 2 & -1 \\ -2 & 4 & 1 & 3 \\ 0 & 0 & 1 & 1 \\ 1 & -2 & 1 & 0 \end{bmatrix}$ and $b = (b_1, b_2, b_3, b_4)$. Use the method of Gaussian Elimination to find a condition on the components of b so that the system $Ax = b$ is consistent. When $b = (2, 1, 1, 1)$, if $(x, 0, 0, 1)$ is a solution of the system $Ax = b$ find x. **Answer** : $7b_4 - b_2 - 3b_1 = 0$ and $7b_3 - 3b_2 - 2b_1 = 0$.When $b = (2, 1, 1, 1)$ then $3x - 6y + 2z - t = 2$, $z + t = 1$, Solving we get $x = 1$.

6	Determine the equation of the polynomial y = f (x) of degree 2 whose graph passes through the points (1, 6) , (2, 3) and (3, 2). **Answer** : $y = 11 - 6x + x^2$.
7	Which three matrices E_{21} , E_{31} , E_{32} put A into upper triangular form where $A = \begin{bmatrix} 1 & 1 & 0 \\ 4 & 6 & 1 \\ -2 & 2 & 0 \end{bmatrix}$ Multiply those E's to get one matrix M that does elimination MA = U. **Answer** : $M = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 10 & -2 & 1 \end{bmatrix}$
8	Write down the elementary matrices E , F , G associated with the system of equations $2u + v + 3w = -1$, $4u + v + 7w = 5$, $-6u - 2v - 12w = -2$. Also find the LU decomposition of A. **Answer** : $L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -3 & -1 & 1 \end{bmatrix}$, $U = \begin{bmatrix} 2 & 1 & 3 \\ 0 & -1 & 1 \\ 0 & 0 & -2 \end{bmatrix}$
9	Find L and U for the matrix A = $\begin{bmatrix} 1 & -1 & 0 & 3 \\ 2 & -2 & 0 & 8 \\ -1 & -1 & 4 & -2 \\ -2 & -2 & 6 & -3 \end{bmatrix}$. Write down the permutation matrices, if any, used in the process of elimination. **Answer** : L = $\begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ -2 & 2 & 1 & 0 \\ 2 & 0 & 0 & 1 \end{bmatrix}$, U = $\begin{bmatrix} 1 & -1 & 0 & 3 \\ 0 & -2 & 4 & 1 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix}$ The permutation matrices used are P_{23} and P_{34}
10	Find PA = LDU factorization for $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 2 & 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 1 \end{bmatrix}$ **Answer** : PA = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & & \\ & 1 & \\ & & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ PB = $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & & \\ & -1 & \\ & & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

11 Find the symmetric factorization of A in the form $L D L^T$ and find conditions on a, b, c ,d to get A = LU with four pivots.

$$A = \begin{bmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{bmatrix}$$

12 Suppose A is a 4 x 4 identity matrix except for a vector v in column 2: Factor A into LU assuming $v_2 \neq 0$.

$$A = \begin{bmatrix} 1 & v_1 & 0 & 0 \\ 0 & v_2 & 0 & 0 \\ 0 & v_3 & 1 & 0 \\ 0 & v_4 & 0 & 1 \end{bmatrix}$$

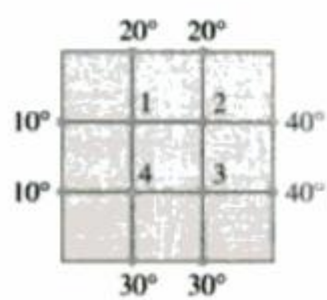
13 Use the Gauss – Jordan method to invert the following matrices

(i) $A = \begin{bmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$ (ii) $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}$

<u>Answer</u> : (i) $A^{-1} = \frac{1}{14}\begin{bmatrix} 3 & -1 & 5 \\ 5 & 3 & -1 \\ -1 & 5 & 3 \end{bmatrix}$ (ii) $A^{-1} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$

14 Producing x trucks and y planes requires x + 50 y tons of steel, 40 x + 1000 y pounds of rubber and 2x + 50 y months of labour. If the unit costs u, v , w are \$ 700 per ton, \$ 3 per pound and \$ 3000 per month, what are the values of one truck and one plane ? **<u>Answer</u>** : 6820 , 188000

15 Assume that the plate shown in the figure represents a cross section of a metal beam with negligible heat flow in the direction perpendicular to the plate. Let T1, T2 , T3 and T4 denote the temperatures at the four interior nodes of the mesh . The temperature at a node is approximately equal to the average of the four nearest nodes – to the right, left , above and below. For instance T1 = (10 + 20 + T2 + T4) / 4 or 4 T1 – T2 – T4 = 30. Write a system of 4 equations whose solution gives estimates for the temperatures T1, T2, T3 and T4.Hence find its solution.



16 Propane is a common gas used for cooking and home heating. Each molecule of propane is comprised of 3 atoms of carbon, and 8 atoms of hydrogen written as C_3H_8. When propane burns, it combines with oxygen gas O_2 to form carbon dioxide CO_2 and water H_2O. Balance the chemical equation $C_3H_8 + O_2 \rightarrow CO_2 + H_2O$ that describes this process.

<u>Answer</u> : $2\ C_3H_8 + 10\ O_2 \rightarrow 6\ CO_2 + 8\ H_2O$