Bayes Nets Formalized

A Bayes net (also called a belief network) is an augmented directed acyclic graph, represented by the pair V , E where:

- V is a set of vertices.
- E is a set of directed edges joining vertices. No loops of any length are allowed.

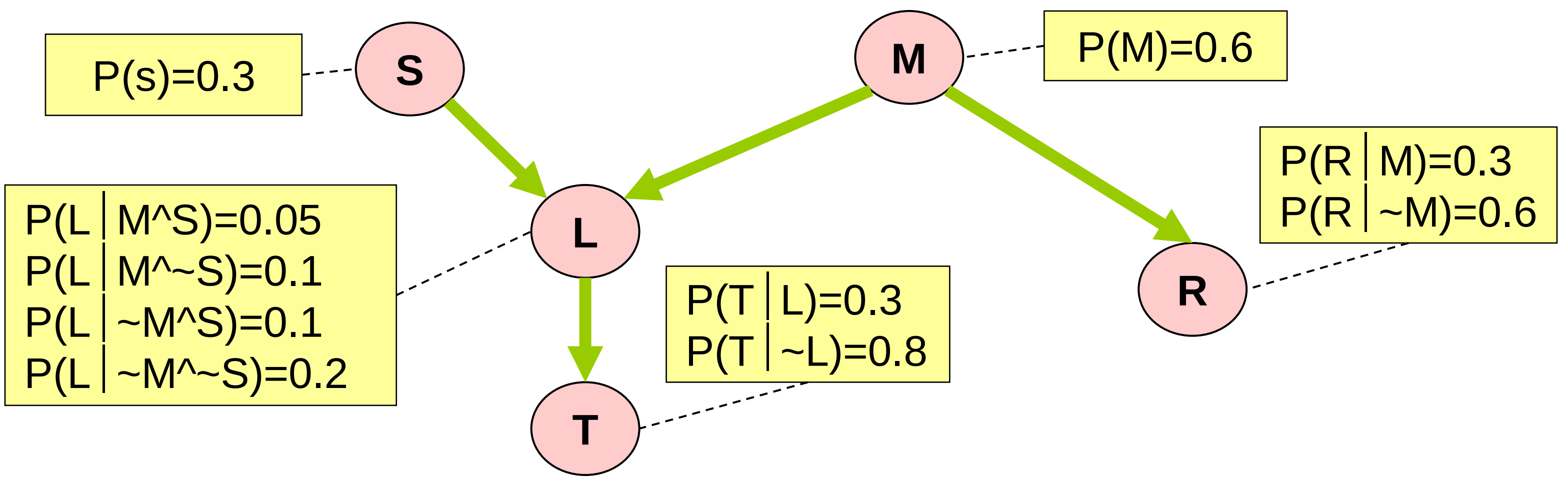
Each vertex in V contains the following information:

- The name of a random variable
- A probability distribution table indicating how the probability of this variable's values depends on all possible combinations of parental values.

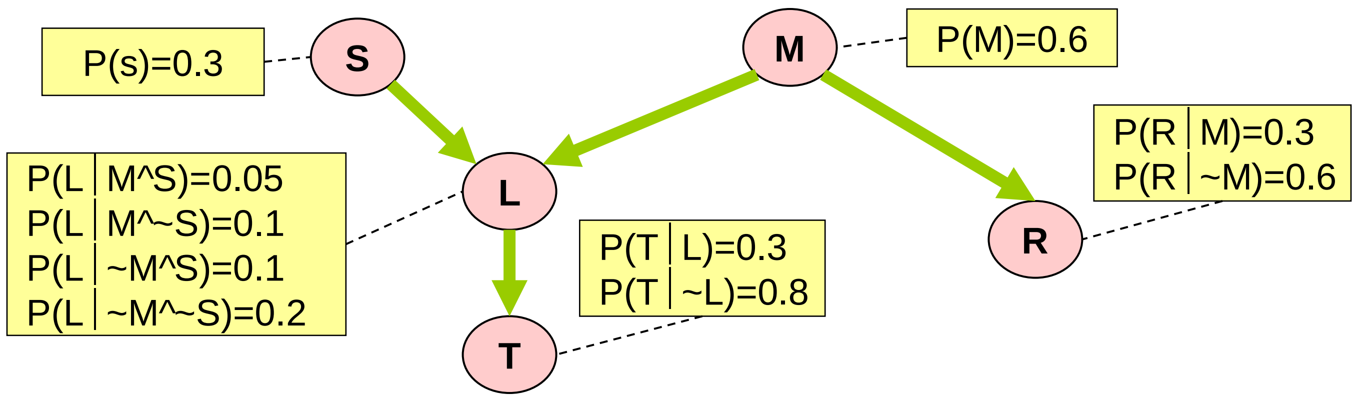
Computing a Joint Entry

How to compute an entry in a joint distribution?

E.G: What is P(S ^ ~M ^ L ~R ^ T)?



Computing with Bayes Net



P(T ^ ~R ^ L ^ ~M ^ S) =

P(T | ~R ^ L ^ ~M ^ S) * P(~R ^ L ^ ~M ^ S) =

P(T | L) * P(~R ^ L ^ ~M ^ S) =

P(T | L) * P(~R | L ^ ~M ^ S) * P(L^~M^S) =

P(T | L) * P(~R | ~M) * P(L^~M^S) =

P(T | L) * P(~R | ~M) * P(L | ~M^S)*P(~M^S) =

P(T | L) * P(~R | ~M) * P(L | ~M^S)*P(~M | S)*P(S) =

P(T | L) * P(~R | ~M) * P(L | ~M^S)*P(~M)*P(S).

The general case

$P(X_1=x_1 \wedge X_2=x_2 \wedge \ldots. X_{n-1}=x_{n-1} \wedge X_n=x_n) =$

$P(X_n=x_n \wedge X_{n-1}=x_{n-1} \wedge \ldots. X_2=x_2 \wedge X_1=x_1) =$

$P(X_n=x_n \mid X_{n-1}=x_{n-1} \wedge \ldots. X_2=x_2 \wedge X_1=x_1) * P(X_{n-1}=x_{n-1} \wedge \ldots. X_2=x_2 \wedge X_1=x_1) =$

$P(X_n=x_n \mid X_{n-1}=x_{n-1} \wedge \ldots. X_2=x_2 \wedge X_1=x_1) * P(X_{n-1}=x_{n-1} \mid \ldots. X_2=x_2 \wedge X_1=x_1) *$

$P(X_{n-2}=x_{n-2} \wedge \ldots. X_2=x_2 \wedge X_1=x_1) =$

:

:

=

$$\prod_{i=1}^{n} P\Big(\big(X_i = x_i\big) \Big| \big(\big(X_{i-1} = x_{i-1}\big) \wedge \ldots \big(X_1 = x_1\big)\big)\Big)$$

$$=$$

$$\prod_{i=1}^{n} P\Big(\big(X_i = x_i\big) \Big| \text{Assignments of Parents}\big(X_i\big)\Big)$$

So any entry in joint pdf table can be computed. And so **any conditional probability** can be computed.

Where are we now?

Step 1: Compute P(R ^ T ^ ~S)

Sum of all the rows in the Joint that match R ^ T ^ ~S

Step 2: Compute P(~R ^ T ^ ~S)

Sum of all the rows in the Joint that match ~R ^ T ^ ~S

Step 3: Return

$$\frac{P(R \wedge T \wedge \sim S)}{P(R \wedge T \wedge \sim S) + P(\sim R \wedge T \wedge \sim S)}$$

…or building Bayes nets.

…tial …

…pone…

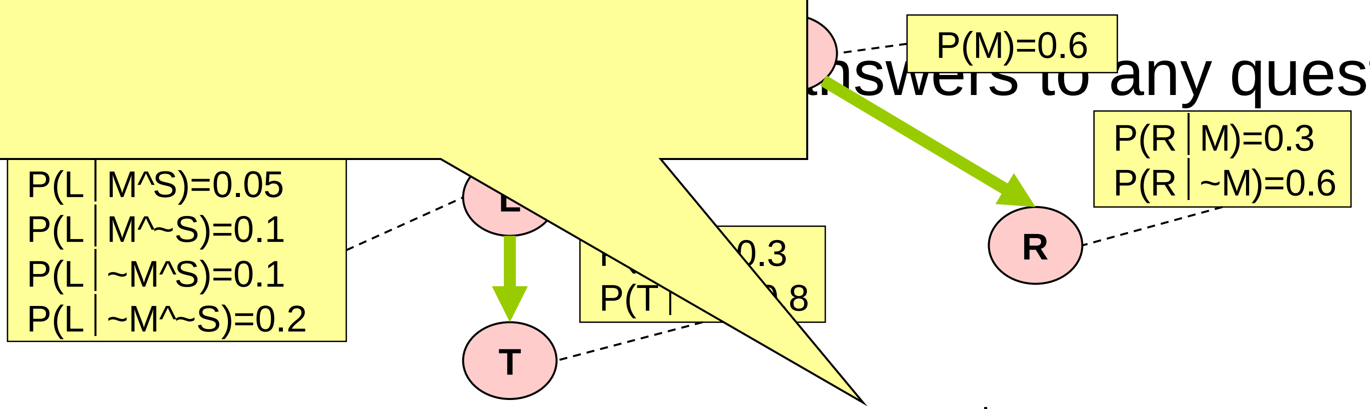
…nod…

…ties of any given assignment

…bles. And we can do it in time

…nodes.

…nswers to any questions.



E.G. What could we do to compute P(R | T,~S)?

Where are we now?

Step 1: Compute P(R ^ T ^ ~S)

Step 2: Compute P(~R ^ T ^ ~S)

Step 3: Return

$$\frac{P(R \wedge T \wedge \sim S)}{P(R \wedge T \wedge \sim S) + P(\sim R \wedge T \wedge \sim S)}$$

Sum of all the rows in the Joint that match R ^ T ^ ~S

4 joint computes

Sum of all the rows in the Joint that match ~R ^ T ^ ~S

4 joint computes

Each of these obtained by the “computing a joint probability entry” method of the earlier slides

...or building Bayes nets.

...ties of any given ass...

...bles...

...node...

...nsw...

P(L | M^S)=0.05	
P(L | M^~S)=0.1	
P(L | ~M^S)=0.1	
P(L | ~M^~S)=0.2	

P(T ... 0.3

P(T ... 0.8

P(R \| ~M)=0.6

L

T

R

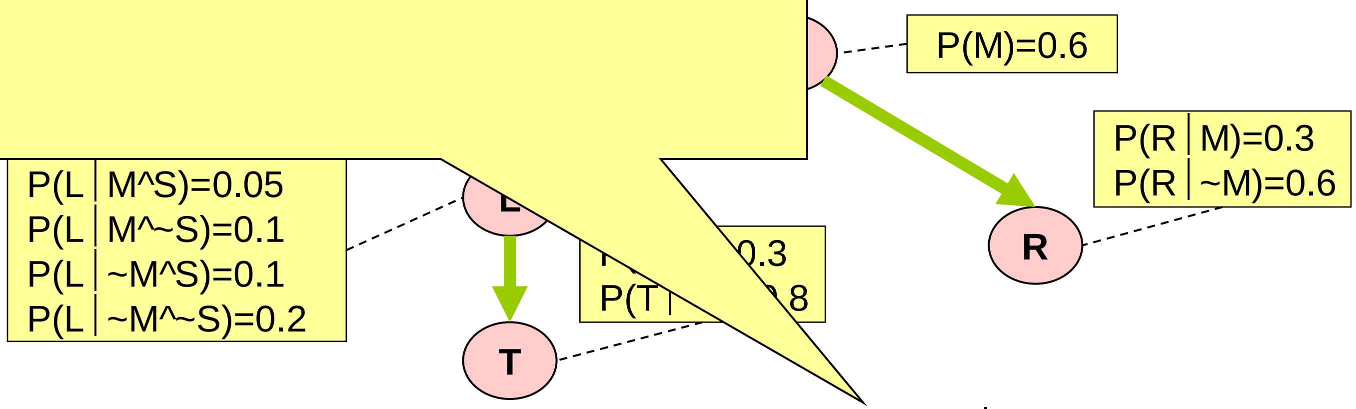
E.G. What could we do to compute P(R | T,~S)?

Step 1: Compute P(R ^ T ^ ~S)

Step 2: Compute P(~R ^ T ^ ~S)

Step 3: Return

$$\frac{P(R \wedge T \wedge \sim S)}{P(R \wedge T \wedge \sim S) + P(\sim R \wedge T \wedge \sim S)}$$



E.G. What could we do to compute P(R | T,~S)?