Advanced Numerical Techniques

April 7, 2019

1 LAB 7

2 PDE

2.1 Question 7

2.1.1 Solve by Crank-Nicholson Scheme.

Parabolic PDE, $B^2 - AC = 0$. Heat Equation.

$$u_t = u_{xx}$$

Initial Condition:

$$u(x, 0) = \sin(\pi x)$$

Boundary Condition:

$$u(0, t) = 0$$

$$u(1, t) = 0$$

$$\delta t = \frac{1}{96}, \quad \text{Experiment with different } \delta x.$$

```
In [1]: import numpy as np
        import pandas as pd
```

```
In [2]: x1 = 0
        x2 = 1
```

```
In [117]: def thomas_(a,b,c,d):
              c_ = np.zeros(c.size)
              d_ = np.zeros(d.size)

              c_[0] = c[0]/b[0]
              d_[0] = d[0]/b[0]

              for i in range(1, c.shape[0]-1):
```

```
        c_[i] = c[i]/(b[i] - a[i]*c_[i-1])

    for i in range(1, d.shape[0]):
        d_[i] = (d[i] - a[i]*d_[i-1])/(b[i] - a[i]*c_[i-1])

    return [c_, d_]


def main_(dx=None):
    dt = 1/96
    r = dt/(dx*dx)
    n = int((x2-x1)/dx)

    x_f = np.zeros(n+1)
    u_j_n = np.zeros(n+1)

    for i in range(n+1):
        x_f[i] = i*dx
        u_j_n[i] = np.sin(np.pi*x_f[i])

    u_j_n[0]=u_j_n[-1]=0
    print("Value of r = ", r)

    flag=10
    while flag!=0:

        a = np.zeros(n-1)
        b = np.zeros(n-1)
        c = np.zeros(n-1)
        d = np.zeros(n-1)

        #print(u_j_n)
        #print(-1 * u_j_n[1] - (r/2)*(u_j_n[2]-2*u_j_n[1]+u_j_n[0]))

        for i in range(n-1):
            a[i] = r/2
            b[i] = -1 * r
            c[i] = r/2-1
            d[i] = -1 * u_j_n[i+1] - (r/2)*(u_j_n[i+2] - 2*u_j_n[i+1] + u_j_n[i])

        #print(d)

        a[0] = 0
        c[-1] = 0

        c_, d_ = thomas_(a,b,c,d)
        res1 = np.zeros(n-1)
```

```
                res1[-1] = d_[-1]
                for i in range(n-2):
                    res1[n-3-i] = d_[n-3-i] - res1[n-2-i]*c_[n-3-i]

                res = np.zeros(n+1)
                for i in range(n-1):
                    res[i+1] = res1[i]

                flag = flag-1

                u_j_n = res

            return [u_j_n, x_f]
```

```
In [118]: a_1, x_1 = main_(0.05)
          a_2, x_2 = main_(0.01)
          a_3, x_3 = main_(0.001)
          # a_4, x_4 = main_(0.1)
```

```
Value of r =  4.166666666666665
Value of r =  104.16666666666666
Value of r =  10416.666666666666
```

```
In [126]: print(pd.DataFrame(np.column_stack((x_1, a_1)), columns=["x", "u(x,t)"]))
          print()
          print(pd.DataFrame(np.column_stack((x_2, a_2)), columns=["x", "u(x,t)"]))
          print()
          print(pd.DataFrame(np.column_stack((x_3, a_3)), columns=["x", "u(x,t)"]))
```

```
       x    u(x,t)
0   0.00  0.000000
1   0.05  0.013352
2   0.10  0.029141
3   0.15  0.048360
4   0.20  0.070597
5   0.25  0.095570
6   0.30  0.122924
7   0.35  0.152104
8   0.40  0.182334
9   0.45  0.212616
10  0.50  0.241723
11  0.55  0.268183
12  0.60  0.290235
13  0.65  0.305754
14  0.70  0.312364
15  0.75  0.307875
16  0.80  0.289424
17  0.85  0.250073
```

```
18  0.90  0.195939
19  0.95  0.110200
20  1.00  0.000000

        x    u(x,t)
0    0.00  0.000000
1    0.01  0.009031
2    0.02  0.018027
3    0.03  0.027041
4    0.04  0.036099
5    0.05  0.045206
6    0.06  0.054362
7    0.07  0.063556
8    0.08  0.072778
9    0.09  0.082017
10   0.10  0.091260
11   0.11  0.100496
12   0.12  0.109713
13   0.13  0.118901
14   0.14  0.128050
15   0.15  0.137149
16   0.16  0.146189
17   0.17  0.155160
18   0.18  0.164052
19   0.19  0.172856
20   0.20  0.181562
21   0.21  0.190161
22   0.22  0.198643
23   0.23  0.206998
24   0.24  0.215217
25   0.25  0.223291
26   0.26  0.231209
27   0.27  0.238962
28   0.28  0.246542
29   0.29  0.253938
..    ...       ...
71   0.71  0.304959
72   0.72  0.298690
73   0.73  0.292064
74   0.74  0.285086
75   0.75  0.277759
76   0.76  0.270088
77   0.77  0.262076
78   0.78  0.253729
79   0.79  0.245052
80   0.80  0.236051
81   0.81  0.226732
82   0.82  0.217102
```

```
83   0.83  0.207168
84   0.84  0.196938
85   0.85  0.186420
86   0.86  0.175622
87   0.87  0.164553
88   0.88  0.153222
89   0.89  0.141638
90   0.90  0.129810
91   0.91  0.117744
92   0.92  0.105448
93   0.93  0.092926
94   0.94  0.080181
95   0.95  0.067219
96   0.96  0.054047
97   0.97  0.040680
98   0.98  0.027155
99   0.99  0.013547
100  1.00  0.000000

[101 rows x 2 columns]

          x    u(x,t)
0     0.000  0.000000
1     0.001  0.001101
2     0.002  0.002201
3     0.003  0.003301
4     0.004  0.004401
5     0.005  0.005501
6     0.006  0.006601
7     0.007  0.007700
8     0.008  0.008800
9     0.009  0.009899
10    0.010  0.010998
11    0.011  0.012097
12    0.012  0.013196
13    0.013  0.014295
14    0.014  0.015393
15    0.015  0.016492
16    0.016  0.017590
17    0.017  0.018688
18    0.018  0.019786
19    0.019  0.020884
20    0.020  0.021982
21    0.021  0.023079
22    0.022  0.024177
23    0.023  0.025274
24    0.024  0.026371
25    0.025  0.027468
```

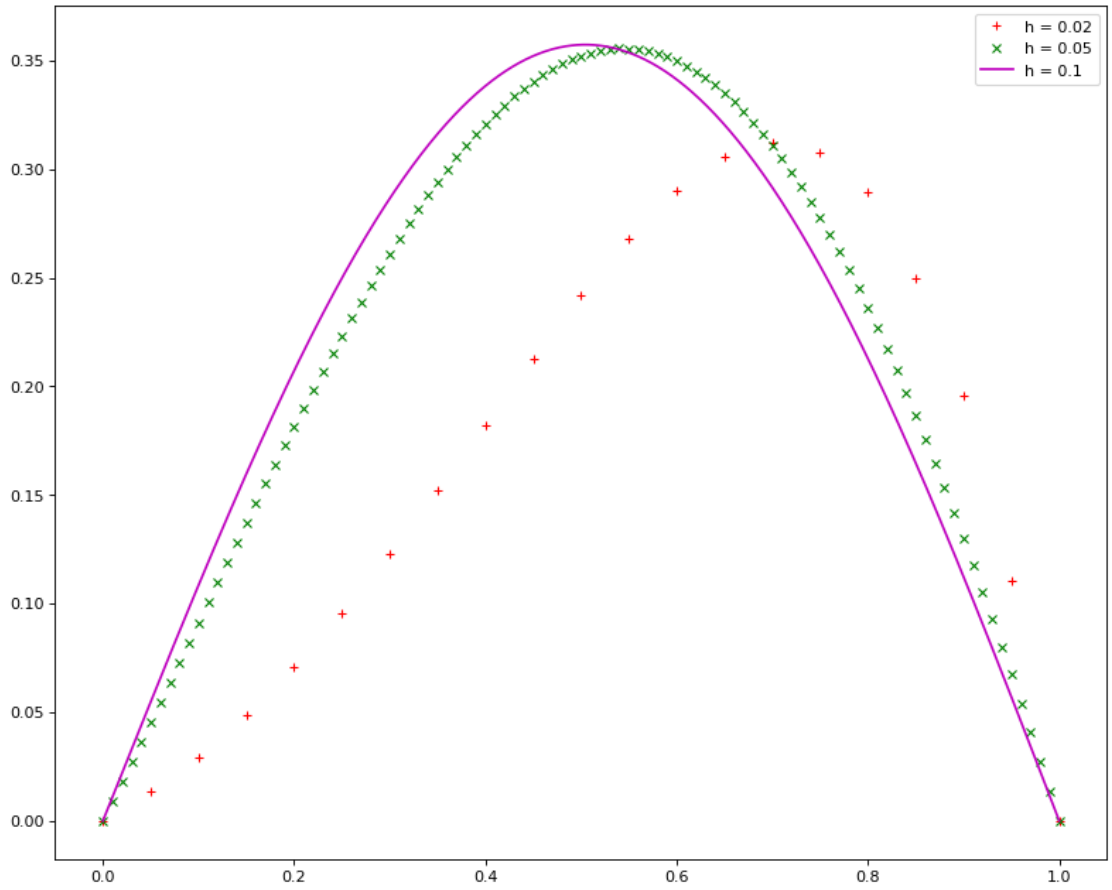
```
26    0.026  0.028564
27    0.027  0.029661
28    0.028  0.030757
29    0.029  0.031853
...     ...       ...
971   0.971  0.033179
972   0.972  0.032038
973   0.973  0.030897
974   0.974  0.029756
975   0.975  0.028614
976   0.976  0.027472
977   0.977  0.026329
978   0.978  0.025186
979   0.979  0.024043
980   0.980  0.022900
981   0.981  0.021756
982   0.982  0.020612
983   0.983  0.019468
984   0.984  0.018324
985   0.985  0.017179
986   0.986  0.016034
987   0.987  0.014889
988   0.988  0.013744
989   0.989  0.012599
990   0.990  0.011454
991   0.991  0.010308
992   0.992  0.009163
993   0.993  0.008017
994   0.994  0.006872
995   0.995  0.005726
996   0.996  0.004581
997   0.997  0.003435
998   0.998  0.002290
999   0.999  0.001145
1000  1.000  0.000000

[1001 rows x 2 columns]
```

```
In [124]: import matplotlib.pyplot as plt
          from matplotlib.pyplot import figure
          figure(num=None, figsize=(12, 10), dpi=80, facecolor='w', edgecolor='k')

          plt.plot(x_1, a_1, 'r+', label = 'h = 0.02')
          plt.plot(x_2, (a_2),  'gx', label = 'h = 0.05')
          plt.plot(x_3, (a_3),  'm-', label = 'h = 0.1')
          # plt.plot(x_4, (a_4), 'bx', label = 'h = 0.25')
          plt.legend(loc='best')
```

```
plt.show()
```



In [125]:
```
%%javascript
IPython.OutputArea.prototype._should_scroll = function(lines) {
    return false;
}
```

```
<IPython.core.display.Javascript object>
```

In [ ]: