Advanced Numerical Techniques

January 21, 2019

1 LAB 2

1.1 Question 2.a

$$\theta'' = \lambda\theta$$

wrt:

$$\theta(0) = 1$$
$$\theta'(1) = 0$$

```
In [162]: import numpy as np
          import pandas as pd
```

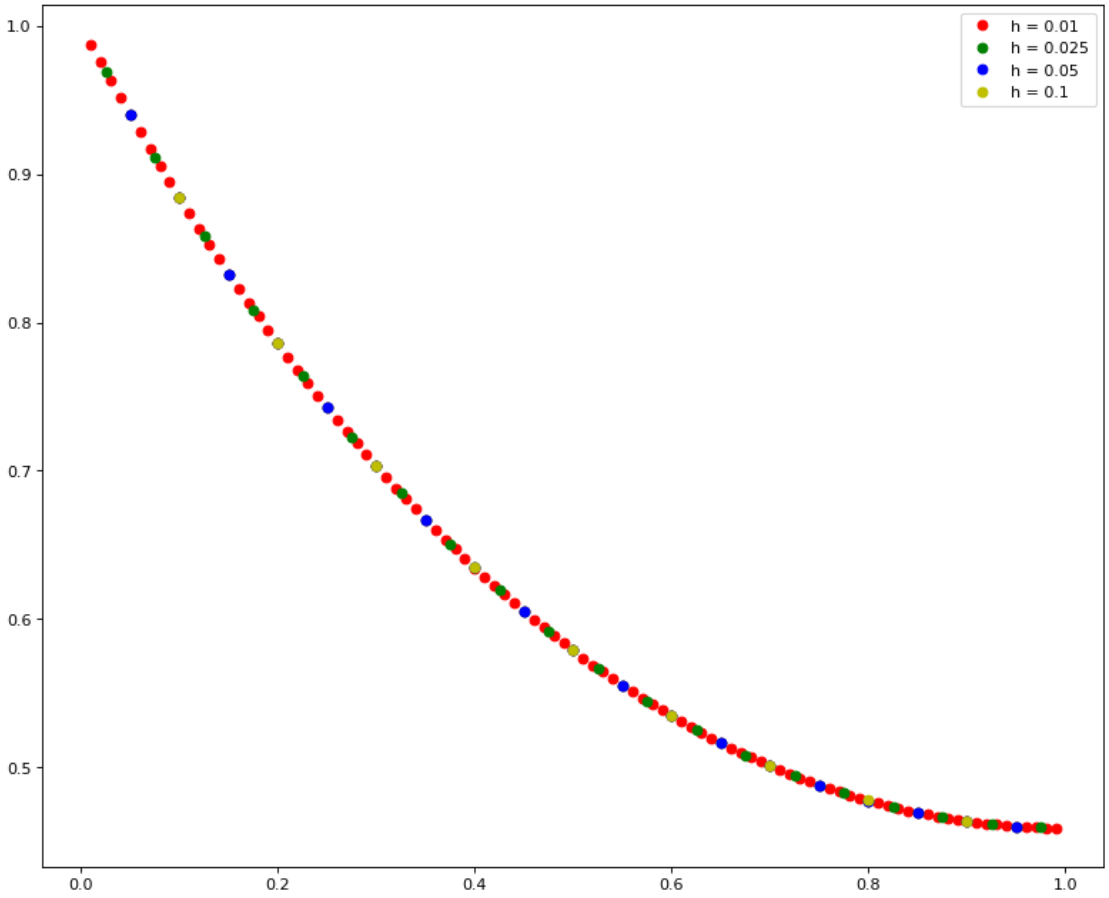
```
In [ ]: x1 = 0
        x2 = 1
        h = 0.25
```

```
In [163]: def thomas_(a,b,c,d):
              c_ = np.zeros(c.size)
              d_ = np.zeros(d.size)

              c_[0] = c[0]/b[0]
              d_[0] = d[0]/b[0]

              for i in range(1, c.shape[0]-1):
                  c_[i] = c[i]/(b[i] - a[i]*c_[i-1])

              for i in range(1, d.shape[0]):
                  d_[i] = (d[i] - a[i]*d_[i-1])/(b[i] - a[i]*c_[i-1])

              return [c_, d_]


          def main_(h=0.25):
              n = int((x2-x1)/h)

              a = np.zeros(n-1)
              b = np.zeros(n-1)
```

```
    c = np.zeros(n-1)
    d = np.zeros(n-1)
    x_f = np.zeros(n-1)

    b[0] = -1*( 2/(h*h) + 2 )
    c[0] = ( 1/(h*h) )
    d[0] = -1*(1/(h*h))
    x_f[0] = x1+h

    for i in range(1, n-2):
        x_f[i] = x1+(i+1)*h
        a[i] = ( 1/(h*h) )

    for i in range(1, n-2):
        b[i] =-1*( 2 + 2/(h*h) )

    for i in range(1, n-2):
        c[i] = ( 1/(h*h) )

    a[-1] = ( 2/(3*h*h) )
    b[-1] = -1*(2/(3*h*h) + 2 )
    x_f[-1] = x2-h

    c_, d_ = thomas_(a,b,c,d)
    res = np.zeros(n-1)

    res[-1] = d_[-1]
    for i in range(n-2):
        res[n-3-i] = d_[n-3-i] - res[n-2-i]*c_[n-3-i]

    return [res, x_f]
```

In [167]:
```
a_1, x_1 = main_(0.01)
a_2, x_2 = main_(0.025)
a_3, x_3 = main_(0.05)
a_4, x_4 = main_(0.1)
```

In [168]:
```
import matplotlib.pyplot as plt
from matplotlib.pyplot import figure
figure(num=None, figsize=(12, 10), dpi=80, facecolor='w', edgecolor='k')

plt.plot(x_1, a_1, 'ro', label = 'h = 0.01')
plt.plot(x_2, (a_2),  'go', label = 'h = 0.025')
plt.plot(x_3, (a_3),  'bo', label = 'h = 0.05')
plt.plot(x_4, (a_4), 'yo', label = 'h = 0.1')
plt.legend(loc='best')
plt.show()
```



In [ ]:

1.2 Question 2.b

$$y'' - 2xy' - 2y = -4x$$

Subject to:

$$y(0) - y'(0) = 0$$
$$2y(1) - y'(1) = 1$$

```
In [125]: x1 = 0
          x2 = 1
          h = 0.1
```

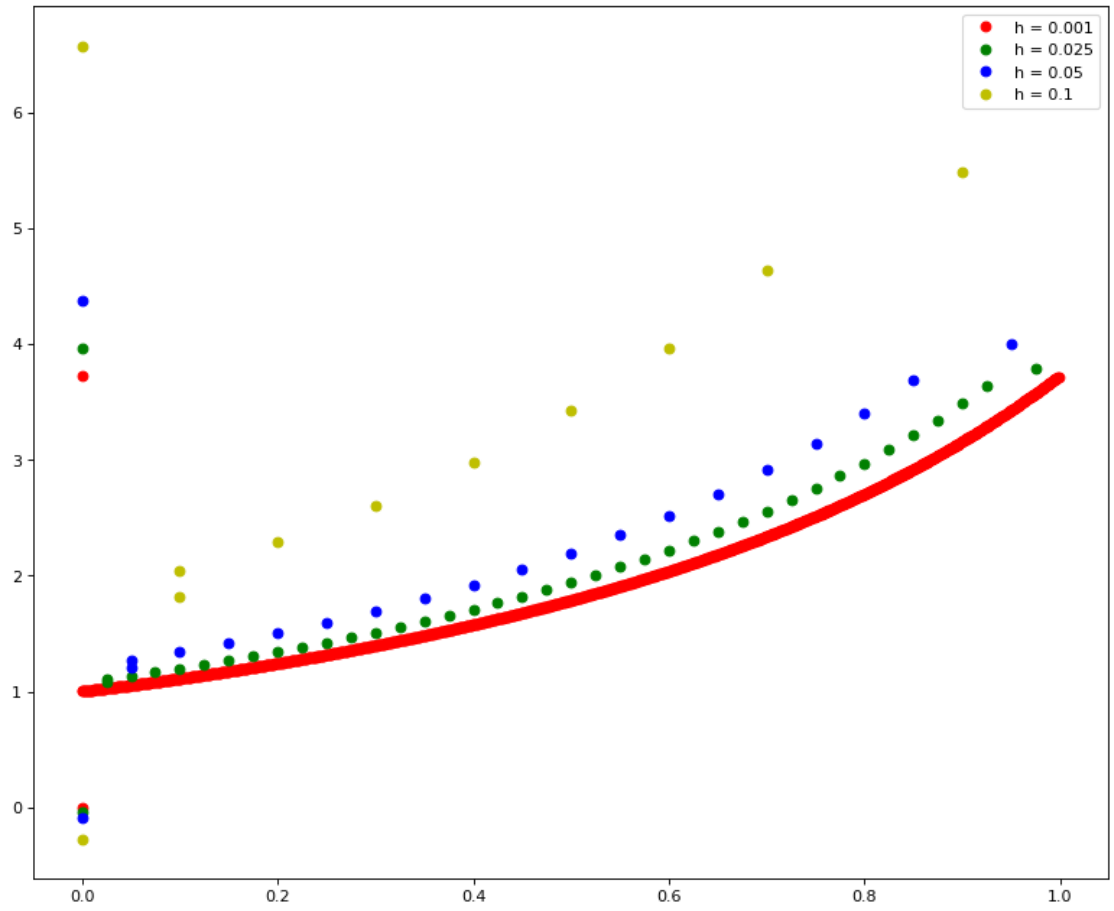
```
In [153]: def thomas_(a,b,c,d):
              c_ = np.zeros(c.size)
              d_ = np.zeros(d.size)

              c_[0] = c[0]/b[0]
```

```
    d_[0] = d[0]/b[0]

    for i in range(1, c.shape[0]-1):
        c_[i] = c[i]/(b[i] - a[i]*c_[i-1])

    for i in range(1, d.shape[0]):
        d_[i] = (d[i] - a[i]*d_[i-1])/(b[i] - a[i]*c_[i-1])

    return [c_, d_]


def main_(h=0.1):
    n = int((x2-x1)/h)

    x_f = np.zeros(n+1)
    x[0] = 0
    x[-1] = 1

    a = np.zeros(n-1)
    b = np.zeros(n-1)
    c = np.zeros(n-1)
    d = np.zeros(n-1)

    b[0] = -1*( (2/(h*h) + 2) - (1/(h*h) + (x1+h)/h)*(4/(2*h+3)) )
    c[0] = ( (1/(h*h) - (x1+h)/h) - (1/(h*h) + (x1+h)/h)*(1/(2*h+3)) )
    d[0] = -4 * (x1+h)
    x_f[1] = x1+h

    for i in range(1, n-2):
        x_i = x1+(i*h)
        x_f[i+1] = x_i
        a[i] = ( 1/(h*h) + x_i/h )


    for i in range(1, n-2):
        x_i = x1+(i*h)
        b[i] = -1 * ( 2/(h*h) +2 )

    for i in range(1, n-2):
        x_i = x1+(i*h)
        c[i] = ( 1/(h*h) - x_i/h )


    for i in range(1, n-2):
        x_i = x1+(i*h)
        d[i] = -4 * (x_i)

    x_n = x2-h
```

```
    x_f[-2] = x_n
    a[-1] = ( (1/(h*h) + x_n/h) + (1/(h*h) - x_n/h)*(1/(4*h-3)) )
    b[-1] = -1 * ( (2/(h*h) + 2) + (1/(h*h) - x_n/h)*(4/(4*h-3)) )
    d[-1] = -4*x_n - ( 2*h/(4*h-3))*(1/(h*h) - x_n/h )


    c_, d_ = thomas_(a,b,c,d)
    res = np.zeros(n-1)

    res[-1] = d_[-1]
    for i in range(n-2):
        res[n-3-i] = d_[n-3-i] - res[n-2-i]*c_[n-3-i]

    res_final = np.zeros(n+1)

    for i in range(1,n):
        res_final[i] = res[i-1]

    res_final[0] = 4*(res[0]-res[1])/(2*h+3)
    res_final[-1] = (2*h - 4*res[-1] + res[-2])/(4*h-3)

    return [res_final, x_f]
```

In [154]:
```
a_1, x_1 = main_(0.001)
a_2, x_2 = main_(0.025)
a_3, x_3 = main_(0.05)
a_4, x_4 = main_(0.1)
```

In [161]:
```
import matplotlib.pyplot as plt
from matplotlib.pyplot import figure
figure(num=None, figsize=(12, 10), dpi=80, facecolor='w', edgecolor='k')

plt.plot(x_1, a_1, 'ro', label = 'h = 0.001')
plt.plot(x_2, (a_2),  'go', label = 'h = 0.025')
plt.plot(x_3, (a_3),  'bo', label = 'h = 0.05')
plt.plot(x_4, (a_4), 'yo', label = 'h = 0.1')
plt.legend(loc='best')
plt.show()
```



In [ ]:

1.3 Question 2.c

$$y'' + y' = 0$$

wrt:

$$y(0) = 0$$
$$y(1) = 1$$

```
In [33]: x1 = 0
         x2 = 1
         h = 0.25
```

```
In [175]: def thomas_(a,b,c,d):
              c_ = np.zeros(c.size)
              d_ = np.zeros(d.size)

              c_[0] = c[0]/b[0]
```

```
    d_[0] = d[0]/b[0]

    for i in range(1, c.shape[0]-1):
        c_[i] = c[i]/(b[i] - a[i]*c_[i-1])

    for i in range(1, d.shape[0]):
        d_[i] = (d[i] - a[i]*d_[i-1])/(b[i] - a[i]*c_[i-1])

    return [c_, d_]


def main_(h=0.25):
    n = int((x2-x1)/h)

    a = np.zeros(n-1)
    b = np.zeros(n-1)
    c = np.zeros(n-1)
    d = np.zeros(n-1)
    x_f = np.zeros(n-1)

    b[0] = ( 1- 2/(h*h) )
    c[0] = ( 1/(h*h) )
    x_f[0] = x1+h

    for i in range(1, n-2):
        x_f[i] = x1+(i+1)*h
        a[i] = ( 1/(h*h) )

    for i in range(1, n-2):
        b[i] =( 1 - 2/(h*h) )

    for i in range(1, n-2):
        c[i] = ( 1/(h*h) )

    a[-1] = ( 1/(h*h) )
    b[-1] = ( 1 -  2/(h*h) )
    d[-1] = -1/(h*h)
    x_f[-1] = x2-h

    c_, d_ = thomas_(a,b,c,d)
    res = np.zeros(n-1)

    res[-1] = d_[-1]
    for i in range(n-2):
        res[n-3-i] = d_[n-3-i] - res[n-2-i]*c_[n-3-i]

    pred = np.zeros(res.shape)
```

```
            for i in range(1, n):
                x_i = x1+i*h
                pred[i-1] = np.sin(x_i)/np.sin(1)

            return [res, pred, x_f]
```

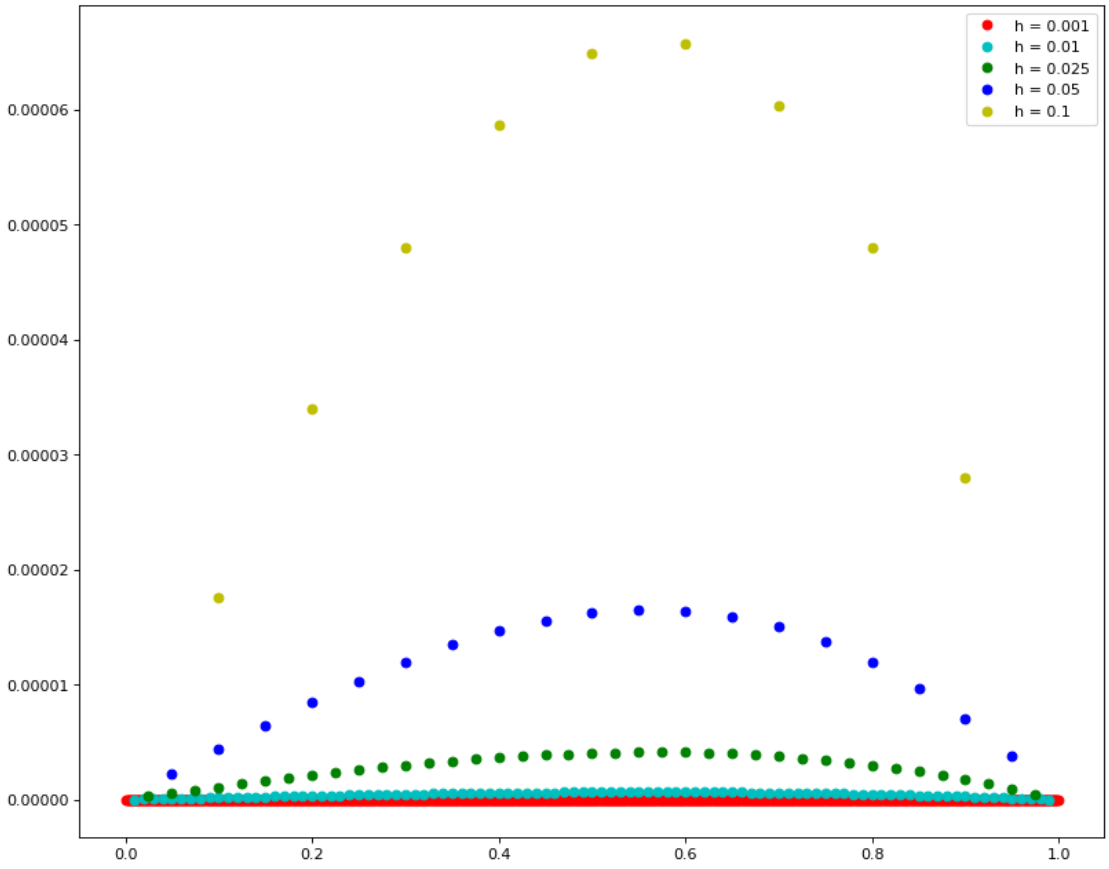
In [178]:
```
a_1, pred, x_1 = main_(0.001)
a_11, pred, x_11 = main_(0.01)
a_2, pred, x_2 = main_(0.025)
a_3, pred, x_3 = main_(0.05)
a_4, pred, x_4 = main_(0.1)
```

In [93]:

Out[93]:

	x	real	h=0.25	h=0.125	h=0.0625	h=0.03125
0	0.25	0.294014	0.294274	0.294078	0.294030	0.294018
1	0.50	0.569747	0.570156	0.569849	0.569772	0.569753
2	0.75	0.810056	0.810403	0.810142	0.810078	0.810062

In [180]:
```
import matplotlib.pyplot as plt
from matplotlib.pyplot import figure
figure(num=None, figsize=(12, 10), dpi=80, facecolor='w', edgecolor='k')

plt.plot(x_1, a_1-np.sin(x_1)/np.sin(1), 'ro', label = 'h = 0.001')
plt.plot(x_11, a_11-np.sin(x_11)/np.sin(1), 'co', label = 'h = 0.01')
plt.plot(x_2, (a_2)-np.sin(x_2)/np.sin(1),  'go', label = 'h = 0.025')
plt.plot(x_3, (a_3)-np.sin(x_3)/np.sin(1),  'bo', label = 'h = 0.05')
plt.plot(x_4, (a_4)-np.sin(x_4)/np.sin(1), 'yo', label = 'h = 0.1')
plt.legend(loc='best')
plt.show()
```


h = 0.001
h = 0.01
h = 0.025
h = 0.05
h = 0.1
0.00006
0.00005
0.00004
0.00003
0.00002
0.00001
0.00000
0.0
0.2
0.4
0.6
0.8
1.0

In [ ]: