ADVANCED NUMERICAL TECHNIQUES

January 14, 2019

1 Related Code

Question: Compute values of given boundary value problem with h= 0.1, 0.05 and 0.01. Compare with exact solution i.e. $y = \frac{1}{2}\log x^2$.

DE

$$x^2y'' + xy' = 1$$

$y(1) = 0 \qquad y(1.4) = 0.0566$

Use Thomas algorithm to solve tridiagonal problem. Find 2nd order solution for each h.

```
In [45]: import numpy as np
         import argparse
```

```
In [46]: x1 = 1
         x2 = 1.4
         y1 = 0
         y2 = 0.0566
         h = 0.01
```

```
In [69]: def thomas_(a,b,c,d):
             c_ = np.zeros(c.size)
             d_ = np.zeros(d.size)

             c_[0] = c[0]/b[0]
             d_[0] = d[0]/b[0]

             for i in range(1, c.shape[0]-1):
                 c_[i] = c[i]/(b[i] - a[i]*c_[i-1])

             for i in range(1, d.shape[0]):
                 d_[i] = (d[i] - a[i]*d_[i-1])/(b[i] - a[i]*c_[i-1])

             return [c_, d_]


         def main_(h=0.1):
             n = int((x2-x1)/h)+1
```

```
    a = np.zeros(n-1)
    b = np.zeros(n-1)
    c = np.zeros(n-1)
    d = np.zeros(n-1)

    for i in range(n-1):
        b[i] = -2/np.power(h,2)

    for i in range(n-1):
        a[i] = 1/np.power(h,2) - 1/(2*h*(x1+(i+1)*h))

    for i in range(n-1):
        c[i] = 1/np.power(h,2) + 1/(2*h*(x1+(i+1)*h))

    d[0] = 1/np.power(x1+h,2) - a[0]*y1
    for i in range(n-3):
        d[i+1] = 1/(np.power(x1+(i+2)*h, 2))
    d[-1] = 1/np.power(x2-2*h, 2) - c[-1]*y2

    c_, d_ = thomas_(a,b,c,d)
    res = np.zeros(n-1)

    res[-1] = d_[-1]
    for i in range(n-2):
        res[n-3-i] = d_[n-3-i] - res[n-2-i]*c_[n-3-i]

    pred = np.zeros(res.shape)

    for i in range(n-1):
        pred[i] = np.power(np.log(x1+h*(i+1)),2)/2

    return [res, pred]

#     for i in range(n-1):
#         print('{0:.5f}'.format(np.power(np.log(x1+h*(i+1)),2)/2) , '{0:.5f}'.format(r
#     print()
```

2 COMPARISON

Now we compare the predicted and exact solution of values with each h.

```
In [71]: h_1_res, h_1_pred = main_(0.1)
         h_2_res, h_2_pred = main_(0.05)
         h_3_res, h_3_pred = main_(0.01)
```

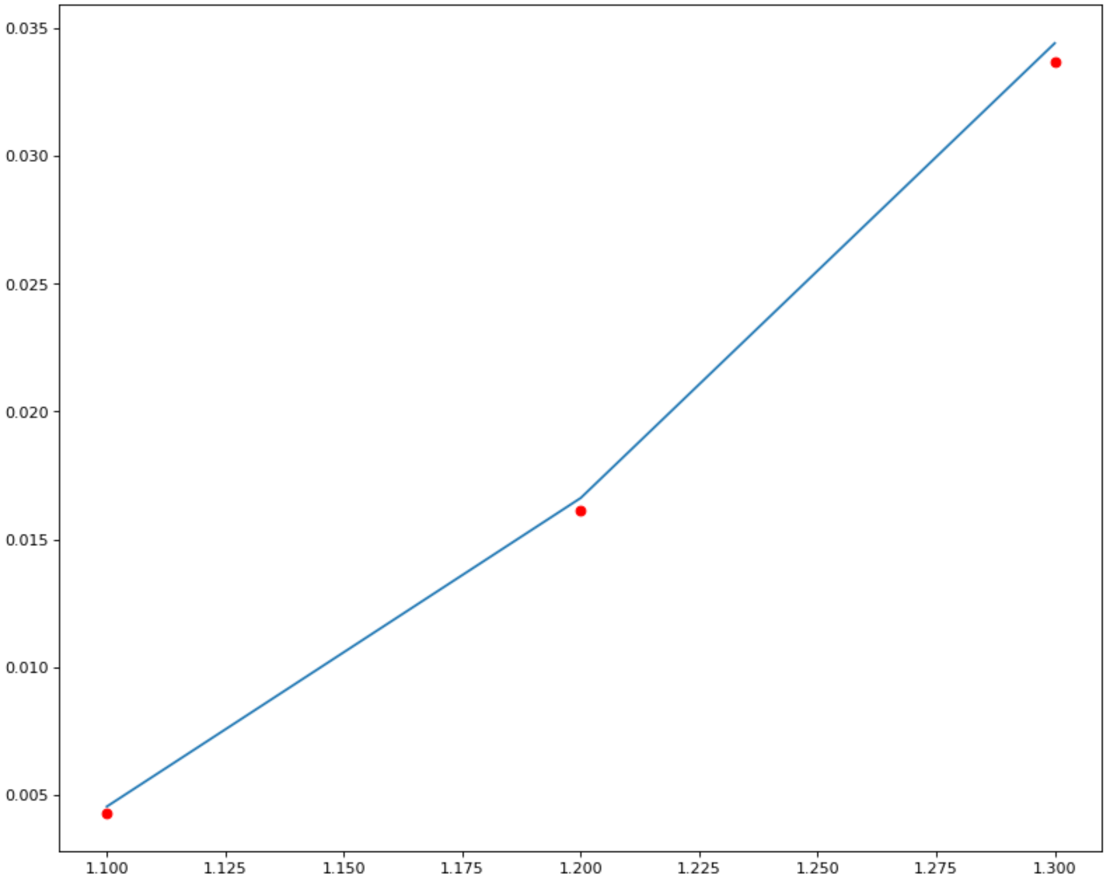
```
In [119]: import pandas as pd
          print(pd.DataFrame(np.column_stack((x1[1:-1], h_1_res, h_1_pred)), columns=["values",
          print()
```

```
        print(pd.DataFrame(np.column_stack((x2[1:-1], h_2_res, h_2_pred)), columns=["values",
        print()
        print(pd.DataFrame(np.column_stack((x3[1:-1], h_3_res, h_3_pred)), columns=["values",
```

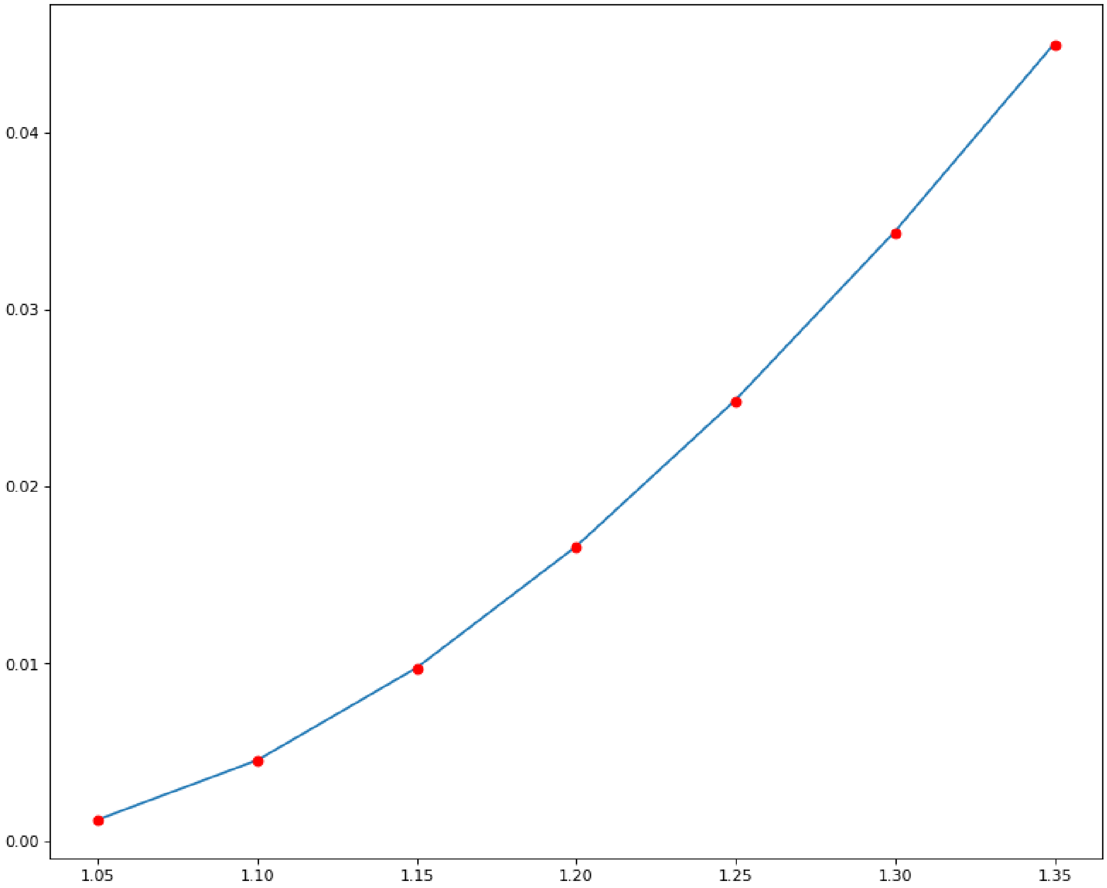
```
   values  predicted     exact
0     1.1   0.004294  0.004542
1     1.2   0.016120  0.016621
2     1.3   0.033666  0.034418

   values  predicted     exact
0    1.05   0.001179  0.001190
1    1.10   0.004519  0.004542
2    1.15   0.009730  0.009767
3    1.20   0.016569  0.016621
4    1.25   0.024830  0.024897
5    1.30   0.034336  0.034418
6    1.35   0.044935  0.045031

    values  predicted     exact
0     1.01   0.000049  0.000050
1     1.02   0.000196  0.000196
2     1.03   0.000436  0.000437
3     1.04   0.000768  0.000769
4     1.05   0.001189  0.001190
5     1.06   0.001697  0.001698
6     1.07   0.002288  0.002289
7     1.08   0.002960  0.002962
8     1.09   0.003712  0.003713
9     1.10   0.004540  0.004542
10    1.11   0.005444  0.005446
11    1.12   0.006420  0.006422
12    1.13   0.007466  0.007469
13    1.14   0.008582  0.008584
14    1.15   0.009764  0.009767
15    1.16   0.011011  0.011014
16    1.17   0.012322  0.012325
17    1.18   0.013694  0.013698
18    1.19   0.015126  0.015130
19    1.20   0.016617  0.016621
20    1.21   0.018164  0.018168
21    1.22   0.019767  0.019771
22    1.23   0.021423  0.021427
23    1.24   0.023132  0.023136
24    1.25   0.024892  0.024897
25    1.26   0.026701  0.026706
26    1.27   0.028559  0.028565
27    1.28   0.030465  0.030470
28    1.29   0.032416  0.032421
```

```
29    1.30   0.034412  0.034418
30    1.31   0.036452  0.036457
31    1.32   0.038534  0.038540
32    1.33   0.040657  0.040664
33    1.34   0.042821  0.042828
34    1.35   0.045025  0.045031
35    1.36   0.047267  0.047273
36    1.37   0.049546  0.049553
37    1.38   0.051862  0.051869
38    1.39   0.054213  0.054220
```

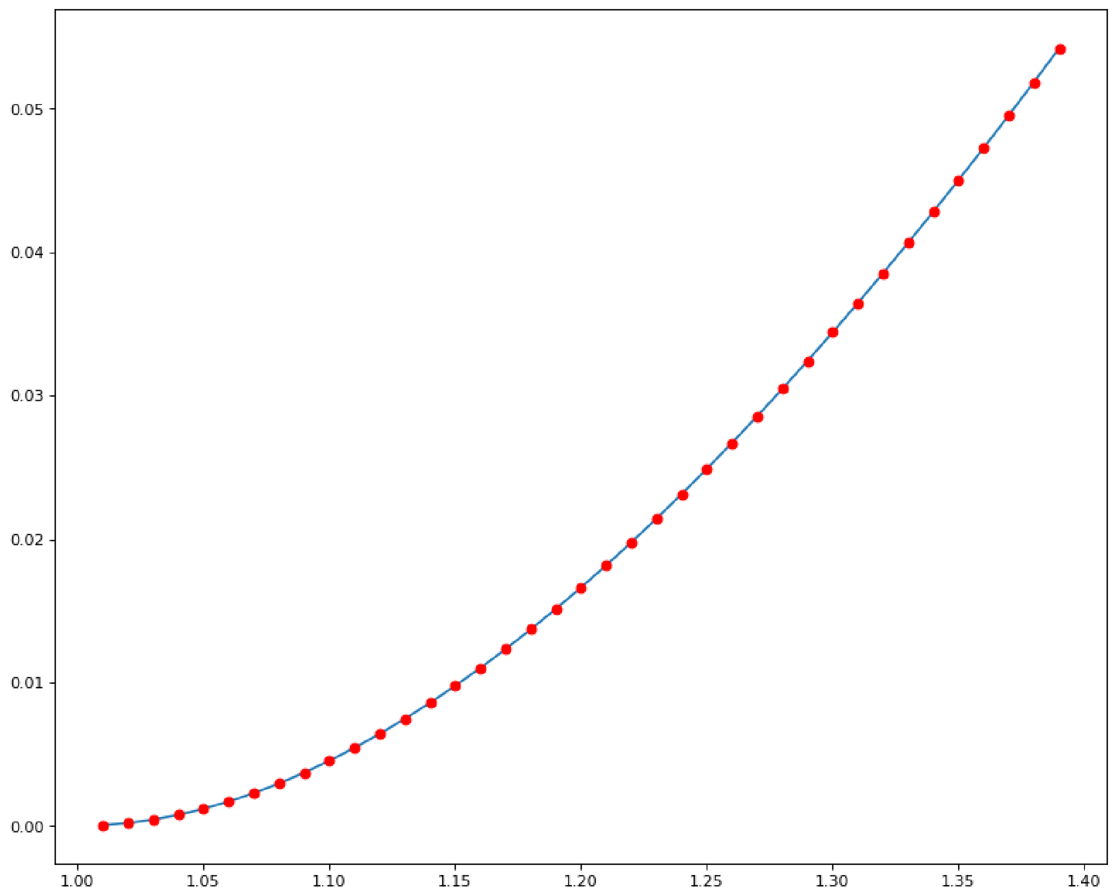
```
In [115]: import matplotlib.pyplot as plt
          from matplotlib.pyplot import figure
          figure(num=None, figsize=(12, 10), dpi=80, facecolor='w', edgecolor='k')
          x1 = np.arange(1,1.5,.1)
          plt.plot(x1[1:-1], h_1_pred)
          plt.plot(x1[1:-1], h_1_res, 'ro')
          plt.show()
```



```
In [114]: import matplotlib.pyplot as plt
          from matplotlib.pyplot import figure
          figure(num=None, figsize=(12, 10), dpi=80, facecolor='w', edgecolor='k')
          x2 = np.arange(1,1.45,.05)
          plt.plot(x2[1:-1], h_2_pred)
          plt.plot(x2[1:-1], h_2_res, 'ro')
          plt.show()
```



```
In [113]: import matplotlib.pyplot as plt
          from matplotlib.pyplot import figure
          figure(num=None, figsize=(12, 10), dpi=80, facecolor='w', edgecolor='k')
          x3 = np.arange(1,1.41,.01)
          plt.plot(x3[1:-1], h_3_pred)
          plt.plot(x3[1:-1], h_3_res, 'ro')
          plt.show()
```



```
In [ ]:
```