Assignment 1

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Problem 3b, ICSE 10 2019:

M and N are two points on the X axis and Y axis respectively. P (3, 2) divides the line segment MN in the ratio 2 : 3.

Find:

1) The coordinates of M and N
2) Slope of the line MN

Solution:

The various parameters involved in this question are listed in Table (I):

Parameter	Symbol/Formula	Value
standard vector 1	$\mathbf{e_1}$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
standard vector 2	$\mathbf{e_2}$	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$
position vector of point P	$\mathbf{P}$	$3 \times \mathbf{e_1} + 2 \times \mathbf{e_2}$
position vector of point M	$\mathbf{M}$	$p \times \mathbf{e_1} + 0 \times \mathbf{e_2}$
position vector of point N	$\mathbf{N}$	$0 \times \mathbf{e_1} + q \times \mathbf{e_2}$

TABLE I

from section formula in vector form, we know that

$$\mathbf{P} = \frac{1 \times \mathbf{M} + k \times \mathbf{N}}{1 + k} \tag{1}$$

where k:1 is ratio in which point P divides the line joining M and N

Since P(3,2) divides M and N in ratio 2:3

So, $k = \frac{2}{3}$

Now, by applying section formula given in equation (1) to P on line MN, we get

$$\mathbf{P} = \frac{1 \times \mathbf{M} + \frac{2}{3} \times \mathbf{N}}{1 + \frac{2}{3}}$$

$$\rightarrow 3 \times \mathbf{e_1} + 2 \times \mathbf{e_2} = \frac{1 \times (p \times \mathbf{e_1} + 0 \times \mathbf{e_2}) + \frac{2}{3} \times (0 \times \mathbf{e_1} + q \times \mathbf{e_2})}{\frac{5}{3}}$$

$$\rightarrow 3 \times \mathbf{e_1} + 2 \times \mathbf{e_2} = \frac{3 \times (p \times \mathbf{e_1} + 0 \times \mathbf{e_2}) + 2 \times (0 \times \mathbf{e_1} + q \times \mathbf{e_2})}{5}$$

$$\rightarrow 15 \times \mathbf{e_1} + 10 \times \mathbf{e_2} = 3p \times \mathbf{e_1} + 0 \times \mathbf{e_2} + 0 \times \mathbf{e_1} + 2q \times \mathbf{e_2}$$

$$\rightarrow (15 - 3p) \times \mathbf{e_1} + (10 - 2q) \times \mathbf{e_2} = 0 \tag{2}$$

Now, coefficient of $\mathbf{e_1}$ and $\mathbf{e_2}$ in equation(2) should be independently equal to 0, So,

$$3p - 15 = 0 \quad 2q - 10 = 0$$

$$\rightarrow 3p = 15 \quad \rightarrow 2q = 10$$

$$\rightarrow p = 5 \quad \rightarrow q = 5 \tag{3}$$

So, the vectors $\mathbf{M} = 5 \times \mathbf{e_1} + 0 \times \mathbf{e_2} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$ and

$\mathbf{N} = 0 \times \mathbf{e_1} + 5 \times \mathbf{e_2} = \begin{pmatrix} 0 \\ 5 \end{pmatrix}$

therefore the points M and N would be **(5,0)** and **(0,5)** respectively.

Now, the vector

$$\mathbf{MN} = \mathbf{N} - \mathbf{M} \tag{4}$$

$$= (0 \times \mathbf{e_1} + 5 \times \mathbf{e_2}) - (5 \times \mathbf{e_1} + 0 \times \mathbf{e_2}) \tag{5}$$

$$= -5 \times \mathbf{e_1} + 5 \times \mathbf{e_2} \tag{6}$$

$$\tag{7}$$

Now, we know that the slope of any vector is

$$= \frac{\text{coefficient of } \mathbf{e_2}}{\text{coefficient of } \mathbf{e_1}} \tag{8}$$

So, slope of $\mathbf{MN}$,

$$slope = \frac{5}{-5} \tag{9}$$

$$= \textbf{-1} \tag{10}$$