Assignment 1

Himanshu Kumar Gupta(AI21BTECH11012)

Problem 3b, ICSE 10 2019:

M and N are two points on the X axis and Y axis respectively. P (3, 2) divides the line segment MN in the ratio 2 : 3.
Find:

1) The coordinates of M and N
2) Slope of the line MN

Solution:

let $\mathbf{P}$ be the position vector of point P
so, $\mathbf{P} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$
$\mathbf{M}$ and $\mathbf{N}$ be the position vector of point M and N respectively
Since M and N are points on x and y axis respectively
So,let $\mathbf{M} = \begin{pmatrix} x \\ 0 \end{pmatrix} and \quad \mathbf{N} = \begin{pmatrix} 0 \\ y \end{pmatrix}$

from section formula in vector form, we know that

$$\mathbf{P} = \frac{1 \times \mathbf{M} + k \times \mathbf{N}}{1 + k} \tag{1}$$

where k:1 is ratio in which point P divides the line joining M and N
Since P(3,2) divides M and N in ratio 2:3
So,$k = \frac{2}{3}$

Now,by applying section formula given in equation (1) to P on line MN, we get

$$\mathbf{P} = \frac{1 \times \mathbf{M} + \frac{2}{3} \times \mathbf{N}}{1 + \frac{2}{3}}$$

$$\rightarrow \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \frac{1 \times \begin{pmatrix} x \\ 0 \end{pmatrix} + \frac{2}{3} \times \begin{pmatrix} 0 \\ y \end{pmatrix}}{\frac{5}{3}}$$

$$\rightarrow \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \frac{3 \times \begin{pmatrix} x \\ 0 \end{pmatrix} + 2 \times \begin{pmatrix} 0 \\ y \end{pmatrix}}{5}$$

$$\rightarrow 5 \times \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 3x \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 2y \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 15 \\ 10 \end{pmatrix} = \begin{pmatrix} 3x \\ 2y \end{pmatrix} \tag{2}$$

So,

$$3x = 15 \quad 2y = 10$$

$$\rightarrow x = 5 \quad \rightarrow y = 5 \tag{3}$$

So,the points M and N would be **(5,0)** and **(0,5)** respectively.

So,the vector $\mathbf{MN} = \begin{pmatrix} 0 - 5 \\ 5 - 0 \end{pmatrix}$

Now,we know that the slope of any vector is

$$\frac{y - \text{component}}{x - \text{component}} \tag{4}$$

(5)

So,

$$slope = \frac{5}{-5} \tag{6}$$

$$= \textbf{-1} \tag{7}$$