

Assignment 1

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Problem 3b, ICSE 10 2019:

M and N are two points on the X axis and Y axis respectively. P (3, 2) divides the line segment MN in the ratio 2 : 3.

Find:

- 1) The coordinates of M and N
- 2) Slope of the line MN

Solution:

The various parameters involved in this question are listed in Table (I):

Parameter	Symbol/Formula	Value
standard vector 1	\mathbf{e}_1	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
standard vector 2	\mathbf{e}_2	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$
position vector of point P	\mathbf{P}	$3 \times \mathbf{e}_1 + 2 \times \mathbf{e}_2$
position vector of point M	\mathbf{M}	$p \times \mathbf{e}_1 + 0 \times \mathbf{e}_2$
position vector of point N	\mathbf{N}	$0 \times \mathbf{e}_1 + q \times \mathbf{e}_2$

TABLE I

from section formula in vector form, we know that

$$\mathbf{P} = \frac{1 \times \mathbf{M} + k \times \mathbf{N}}{1 + k} \quad (1)$$

where k:1 is ratio in which point P divides the line joining M and N

Since P(3,2) divides M and N in ratio 2:3

$$\text{So, } k = \frac{2}{3}$$

Now, by applying section formula given in equation (1) to P on line MN, we get

$$\mathbf{P} = \frac{1 \times \mathbf{M} + \frac{2}{3} \times \mathbf{N}}{1 + \frac{2}{3}} \quad (2)$$

$$\Rightarrow 3 \times \mathbf{e}_1 + 2 \times \mathbf{e}_2 = \frac{1 \times (p \times \mathbf{e}_1) + \frac{2}{3} \times (q \times \mathbf{e}_2)}{\frac{5}{3}} \quad (3)$$

$$\Rightarrow 3 \times \mathbf{e}_1 + 2 \times \mathbf{e}_2 = \frac{3 \times (p \times \mathbf{e}_1) + 2 \times (q \times \mathbf{e}_2)}{5} \quad (4)$$

$$\Rightarrow 15 \times \mathbf{e}_1 + 10 \times \mathbf{e}_2 = 3p \times \mathbf{e}_1 + 2q \times \mathbf{e}_2 \quad (5)$$

$$\Rightarrow (15 - 3p) \times \mathbf{e}_1 + (10 - 2q) \times \mathbf{e}_2 = 0 \quad (6)$$

Now, coefficient of \mathbf{e}_1 and \mathbf{e}_2 in equation(6) should be independently equal to 0, So,

$$\begin{aligned} 3p - 15 &= 0 & 2q - 10 &= 0 \\ \Rightarrow 3p &= 15 & \Rightarrow 2q &= 10 \\ \Rightarrow p &= 5 & \Rightarrow q &= 5 \end{aligned} \quad (7)$$

So, the vectors $\mathbf{M} = 5 \times \mathbf{e}_1 + 0 \times \mathbf{e}_2 = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$ and

$$\mathbf{N} = 0 \times \mathbf{e}_1 + 5 \times \mathbf{e}_2 = \begin{pmatrix} 0 \\ 5 \end{pmatrix}$$

therefore the points M and N would be (5,0) and (0,5) respectively.

Now, the vector

$$\mathbf{MN} = \mathbf{N} - \mathbf{M} \quad (8)$$

$$= (0 \times \mathbf{e}_1 + 5 \times \mathbf{e}_2) - (5 \times \mathbf{e}_1 + 0 \times \mathbf{e}_2) \quad (9)$$

$$= -5 \times \mathbf{e}_1 + 5 \times \mathbf{e}_2 \quad (10)$$

Now, we know that the slope of any vector is

$$= \frac{\text{coefficient of } \mathbf{e}_2}{\text{coefficient of } \mathbf{e}_1} \quad (11)$$

So, slope of MN,

$$\text{slope} = \frac{5}{-5} \quad (12)$$

$$= -1 \quad (13)$$

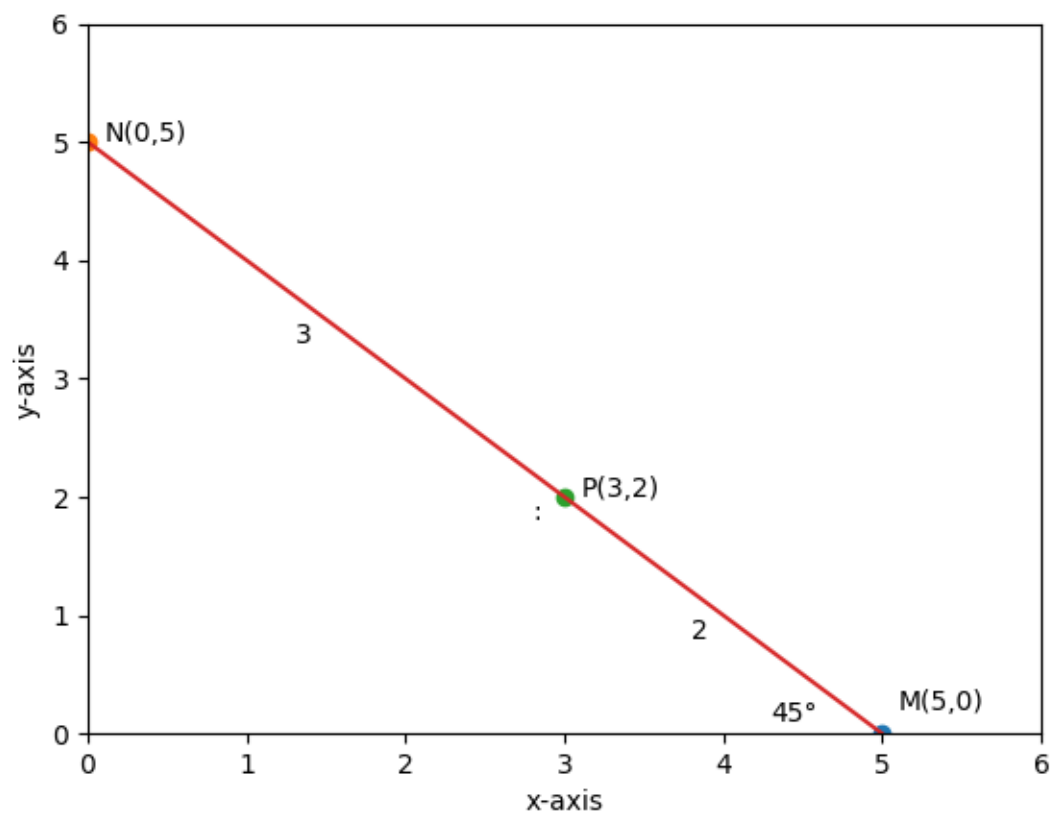


Fig. 1. graph representing M,N and P