Assignment 4

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Example 24 , class 12^{th} ***, CBSE:***

Two cards are drawn successively with replacement from a well-shuffled deck of 52 cards. Find the probability distribution of the number of aces.

Solution:

No. of aces is random variable.

So,let the random variable $X \in \{0,1,2\}$ denote the number of aces in the card drawing experiment.

We know that,

$$\Pr\left(X=i\right)=\frac{n\left(X=i\right)}{\sum_{i=0}^{2} n\left(X=i\right)} \tag{1}$$

where $i \in \{0,1,2\}$ and $n\left(X=i\right)$ is the frequency of getting i ace

Now, there are 52 cards and it's drawn 2 times. So,total cases would be 52×52.

So,

$$\begin{aligned}\sum_{i=0}^{2} n\left(X=i\right) &= 52 \times 52 \\ &= 2704 \end{aligned} \tag{2}$$

For X=2,

There are total 4 aces in deck of 52 cards and it's drawn 2 times and we need both as ace. So,total cases would be 4×4.

So,

$$\begin{aligned} n\left(X=2\right) &= 4 \times 4 \\ &= 16 \end{aligned} \tag{3}$$

For X=1,

There are 4 aces and 48 non-ace cards in deck of 52 cards and it's drawn 2 times and we need exactly 1 ace. So there are 2 possibility,first is that first card is ace and second one is not and the other case is first card is non-ace but other one is ace. So,total cases for this condition would be $4 \times 48 + 48 \times 4$.

So,

$$\begin{aligned} n\left(X=1\right) &= 4 \times 48 + 48 \times 4 \\ &= 384 \end{aligned} \tag{4}$$

For X=0,

There are 48 non-ace cards in deck of 52 cards and it's drawn 2 times.So,for being both non-ace card the number of cases would be 48×48.

So,

$$\begin{aligned} n\left(X=0\right) &= 48 \times 48 \\ &= 2304 \end{aligned} \tag{5}$$

Now,

From equation (1), probability of getting 2 aces,

$$\Pr\left(X=2\right)=\frac{n\left(X=2\right)}{\sum_{i=0}^{2} n\left(X=i\right)} \tag{6}$$

putting values from equations (2) and (3),

$$\Pr\left(X=2\right)=\frac{16}{2704} \tag{7}$$

$$= \textbf{.005917} \tag{8}$$

again,

from equation (1),probability of getting 1 ace,

$$\Pr\left(X=1\right)=\frac{n\left(X=1\right)}{\sum_{i=0}^{2} n\left(X=i\right)} \tag{9}$$

putting values from equations (2) and (4),

$$\Pr\left(X=1\right)=\frac{384}{2704} \tag{10}$$

$$= \textbf{.142} \tag{11}$$

again,

from equation (1),probability of getting 0 ace,

$$\Pr\left(X=0\right)=\frac{n\left(X=0\right)}{\sum_{i=0}^{2} n\left(X=i\right)} \tag{12}$$

putting values from equations (2) and (5),

$$\Pr\left(X=0\right)=\frac{2304}{2704} \tag{13}$$

$$= \textbf{.852} \tag{14}$$

i	0	1	2
Pr$(X=i)$	0.852	0.142	0.005917

TABLE I