Assignment 6

Himanshu Kumar Gupta (AI21BTECH11012)

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Contents

Ex. 4.24 , chapter 4 , Papoulis

A fair coin is tossed n times. Find n such that the probability that the number of heads is between 0.49n and 0.52n is at least 0.9.

Solution

Since the coin is fair. So,

$$p = q = 0.5 \tag{1}$$

No. of heads would be in between .49n and .52n. So,

$$k_1 = .49n \tag{2}$$

$$k_2 = .52n \tag{3}$$

Now,

$$\frac{k_2 - pn}{\sqrt{pqn}} = \frac{.52n - .5n}{\sqrt{.5 \times .5n}}$$
$$= \frac{.02n}{.5\sqrt{n}}$$
$$= .04\sqrt{n} \tag{4}$$

$$\frac{k_1 - pn}{\sqrt{pqn}} = \frac{.49n - .5n}{\sqrt{.5 \times .5n}}$$
$$= \frac{-.01n}{.5\sqrt{n}}$$
$$= -.02\sqrt{n} \tag{5}$$

So,

$$\begin{aligned} P(.49n \leq k \leq .52n) &= G(\frac{k_2 - pn}{\sqrt{pqn}}) - G(\frac{k_1 - pn}{\sqrt{pqn}}) \geq .9 \\ &= G(.04\sqrt{n}) - G(-.02\sqrt{n}) \geq .9 \\ &= G(.04\sqrt{n}) + G(.02\sqrt{n}) - 1 \geq .9 \\ &= G(.04\sqrt{n}) + G(.02\sqrt{n}) \geq 1.9 \end{aligned} \tag{6}$$

using table,we get that

$$\begin{aligned} .02\sqrt{n} &> 1.3 \\ \sqrt{n} &> 65 \\ n &> 65^2 \end{aligned} \tag{7}$$

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E:\>python assignment_6_AI1110.py

G(.04*math.sqrt(65**2)) + G(.02*math.sqrt(65**2)= 1.898538273906722
```

Figure: code output

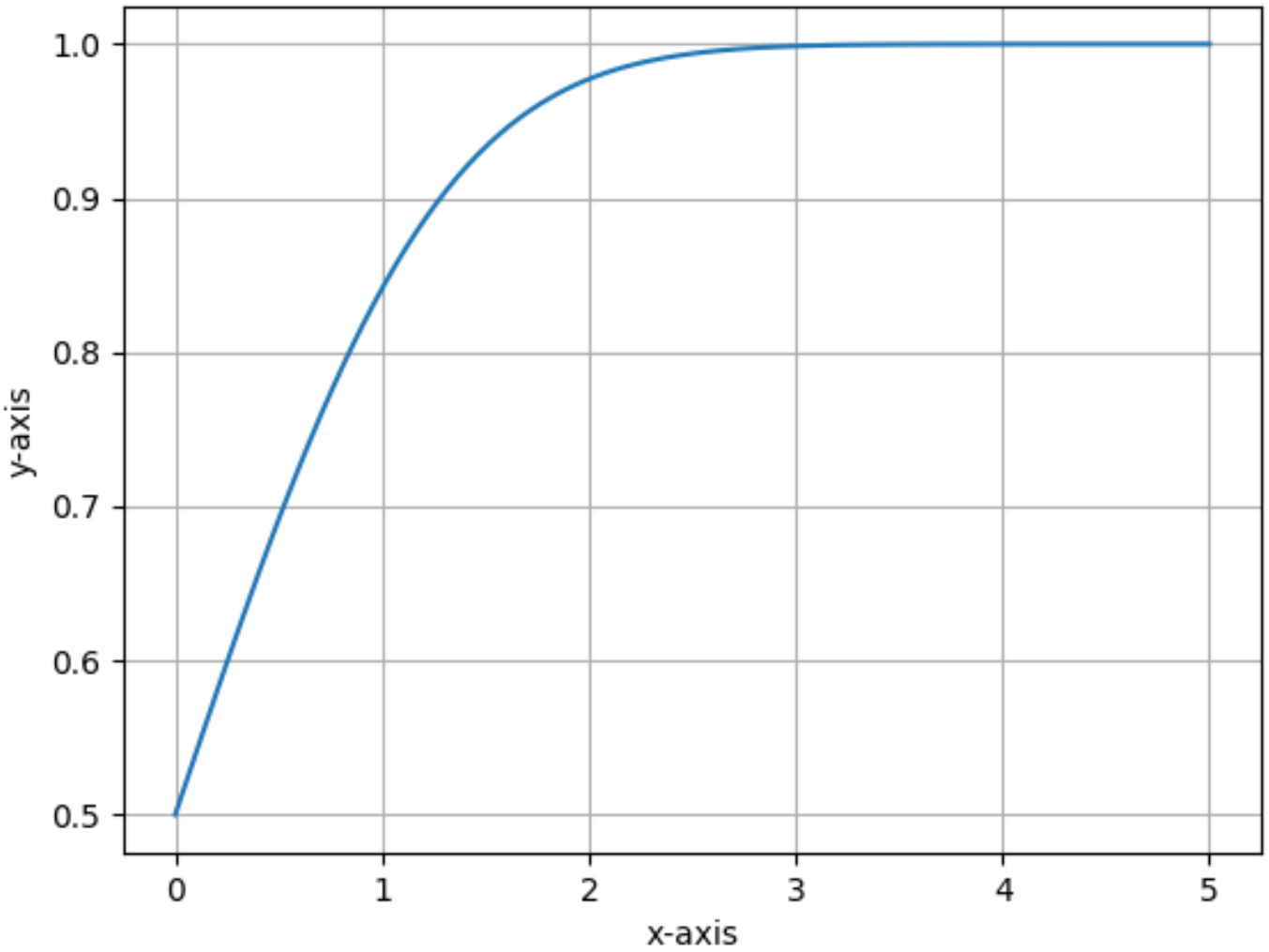

Figure: G(x) function