Assignment 8

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May 30, 2022

Contents

Question

Let x be a negative binomial random variable with parameters r and p. Show that as $p \to 1$ and $r \to \infty$ such that $r\left(1-p\right) \to \lambda$, a constant, then

$$\Pr\left(x = n + r\right) \to e^{-\lambda}\frac{\lambda^n}{n!}$$

where n=0,1,2,...

Solution

We know that

$$\Pr\left(X = k\right) = \binom{k-1}{r-1} p^r q^{k-r} \tag{1}$$

where k=r,r+1,...

Let k=n+r so that

$$\Pr\left(X = n + r\right) = \binom{n + r - 1}{r - 1} p^r q^n$$

where n=0,1,2,...

$$= \frac{(n + r - 1)!}{(n)!\,(r - 1)!} p^r \left(1 - p\right)^n$$

$$= \frac{(n + r - 1)(n + r - 2)...(r)}{n!} \frac{\left\{r(1 - p)\right\}^n p^r}{r^n}$$

$$= \frac{\lambda^n}{n!} \left\{\left(1 + \frac{n - 1}{r}\right) ... (1)\right\} \left\{1 - \frac{r\left(1 - p\right)}{r}\right\}^r$$

$$= \frac{\lambda^n}{n!} \left\{\prod_{k=1}^{n} \left(1 + \left(n - k\right) r\right)\right\} \left(1 - \frac{\lambda}{r}\right)^r \tag{2}$$

Now,

$$\lim_{r\to\infty} \Pr(X = n + r) = \frac{\lambda^n}{n!}\left\{\lim_{r\to\infty}\prod_{k=1}^{n}\left(1 + \frac{n-k}{r}\right)\right\}\lim_{r\to\infty}\left(1 - \frac{\lambda}{r}\right)^r$$
$$= \frac{\lambda^n}{n!} \times 1 \times e^{-\lambda} \tag{3}$$

So,

$$\Pr(x = n + r) \to e^{-\lambda}\frac{\lambda^n}{n!}$$

as $p \to 1$ and $r \to \infty$

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E:\>python assignment_8_AI1110.py
lmda**5*exp(-lmda)/120
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Figure: python output