Assignment 1

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Problem 3b, ICSE 10 2019:

M and N are two points on the X axis and Y axis respectively. P (3, 2) divides the line segment MN in the ratio 2 : 3.

Find:

1) The coordinates of M and N
2) Slope of the line MN

Solution:

The various parameters involved in this question are listed in Table (I):

Parameter	**Symbol/Formula**	**Value**
standard vector 1	$\mathbf{e_1}$	$\begin{pmatrix}1\\0\end{pmatrix}$
standard vector 2	$\mathbf{e_2}$	$\begin{pmatrix}0\\1\end{pmatrix}$
position vector of point P	$\mathbf{P}$	$3\times\mathbf{e_1}+2\times\mathbf{e_2}$
position vector of point M	$\mathbf{M}$	$p\times\mathbf{e_1}+0\times\mathbf{e_2}$
position vector of point N	$\mathbf{N}$	$0\times\mathbf{e_1}+q\times\mathbf{e_2}$

TABLE I

from section formula in vector form, we know that

$$\mathbf{P} = \frac{1\times\mathbf{M}+k\times\mathbf{N}}{1+k} \tag{1}$$

where k:1 is ratio in which point P divides the line joining M and N

Since P(3,2) divides M and N in ratio 2:3

So,$k=\frac{2}{3}$

Now,by applying section formula given in equation (1) to P on line MN, we get

$$\mathbf{P} = \frac{1\times\mathbf{M}+\frac{2}{3}\times\mathbf{N}}{1+\frac{2}{3}}$$

$$\implies 3\times\mathbf{e_1}+2\times\mathbf{e_2} = \frac{1\times(p\times\mathbf{e_1})+\frac{2}{3}\times(q\times\mathbf{e_2})}{\frac{5}{3}}$$

$$\implies 3\times\mathbf{e_1}+2\times\mathbf{e_2} = \frac{3\times(p\times\mathbf{e_1})+2\times(q\times\mathbf{e_2})}{5}$$

$$\implies 15\times\mathbf{e_1}+10\times\mathbf{e_2} = 3p\times\mathbf{e_1}+0\times\mathbf{e_2}+0\times\mathbf{e_1}+2q\times\mathbf{e_2}$$

$$\implies (15-3p)\times\mathbf{e_1}+(10-2q)\times\mathbf{e_2}=0 \tag{2}$$

Now,coefficient of $\mathbf{e_1}$ and $\mathbf{e_2}$ in equation(2) should be independently equal to 0, So,

$$3p-15=0 \quad 2q-10=0$$
$$\implies 3p=15 \quad \implies 2q=10$$
$$\implies p=5 \quad \implies q=5 \tag{3}$$

So,the vectors $\mathbf{M}=5\times\mathbf{e_1}+0\times\mathbf{e_2}=\begin{pmatrix}5\\0\end{pmatrix}$ and $\mathbf{N}=0\times\mathbf{e_1}+5\times\mathbf{e_2}=\begin{pmatrix}0\\5\end{pmatrix}$

therefore the points M and N would be **(5,0)** and **(0,5)** respectively.

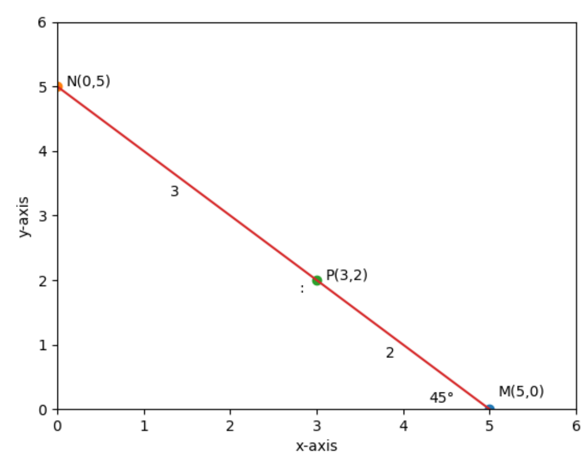

Fig. 1. graph representing M,N and P

Now,the vector

$$\mathbf{MN} = \mathbf{N}-\mathbf{M} \tag{4}$$
$$= (0\times\mathbf{e_1}+5\times\mathbf{e_2})-(5\times\mathbf{e_1}+0\times\mathbf{e_2}) \tag{5}$$
$$= -5\times\mathbf{e_1}+5\times\mathbf{e_2} \tag{6}$$
$$\tag{7}$$

Now,we know that the slope of any vector is

$$=\frac{\text{coefficient of }\mathbf{e_2}}{\text{coefficient of }\mathbf{e_1}} \tag{8}$$

So,slope of $\mathbf{MN}$,

$$\begin{aligned} slope &= \frac{5}{-5} \quad (9) \\ &= \textbf{-1} \quad (10) \end{aligned}$$