Assignment 1

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Problem 3b, ICSE 10 2019:

M and N are two points on the X axis and Y axis respectively. P (3, 2) divides the line segment MN in the ratio 2 : 3.
Find:

1) The coordinates of M and N
2) Slope of the line MN

Solution:

let $\mathbf{P}$ be the position vector of point P
so, $\mathbf{P} = \begin{pmatrix}3\\2\end{pmatrix}$
$\mathbf{M}$ and $\mathbf{N}$ be the position vector of point M and N respectively
Since M and N are points on x and y axis respectively
So,let $\mathbf{M} = \begin{pmatrix}x\\0\end{pmatrix} and \quad \mathbf{N} = \begin{pmatrix}0\\y\end{pmatrix}$

from section formula in vector form, we know that

$$\mathbf{P} = \frac{1 \times \mathbf{M} + k \times \mathbf{N}}{1+k} \tag{1}$$

where k:1 is ratio in which point P divides the line joining M and N
Since P(3,2) divides M and N in ratio 2:3
So,$k = \frac{2}{3}$

Now,by applying section formula given in equation (1) to P on line MN, we get

$$\mathbf{P} = \frac{1 \times \mathbf{M} + \frac{2}{3} \times \mathbf{N}}{1+\frac{2}{3}}$$

$$\to \begin{pmatrix}3\\2\end{pmatrix} = \frac{1 \times \begin{pmatrix}x\\0\end{pmatrix} + \frac{2}{3} \times \begin{pmatrix}0\\y\end{pmatrix}}{\frac{5}{3}}$$

$$\to \begin{pmatrix}3\\2\end{pmatrix} = \frac{3 \times \begin{pmatrix}x\\0\end{pmatrix} + 2 \times \begin{pmatrix}0\\y\end{pmatrix}}{5}$$

$$\to 5 \times \begin{pmatrix}3\\2\end{pmatrix} = \begin{pmatrix}3x\\0\end{pmatrix} + \begin{pmatrix}0\\2y\end{pmatrix}$$

$$\to \begin{pmatrix}15\\10\end{pmatrix} = \begin{pmatrix}3x\\2y\end{pmatrix}$$

So,

$$3x = 15 \quad 2y = 10$$

$$\to x = 5 \quad \to y = 5$$

So,the points M and N would be **(5,0)** and **(0,5)** respectively.

So,the vector $\mathbf{MN} = \begin{pmatrix}0-5\\5-0\end{pmatrix}$

Now,we know that the slope of any vector is

$$\frac{y-\text{component}}{x-\text{component}}$$

So,

$$slope = \frac{5}{-5}$$
$$= \textbf{-1}$$