

# Assignment 4

Himanshu Kumar Gupta (AI21BTECH11012)

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## Question : Example 24 , class 12<sup>th</sup> , CBSE

**Two cards are drawn successively with replacement from a well-shuffled deck of 52 cards. Find the probability distribution of the number of aces.**

# Solution

No. of aces is random variable.

So, let the random variable  $X \in \{0, 1, 2\}$  denote the number of aces in the card drawing experiment.

We know that,

$$\Pr(X = i) = \frac{n(X = i)}{\sum_{i=0}^2 n(X = i)} \quad (1)$$

where  $i \in \{0, 1, 2\}$  and  $n(X = i)$  is the frequency of getting  $i$  aces

Now, there are 52 cards and it's drawn 2 times. So, total cases would be  $52 \times 52$ .

So,

$$\begin{aligned}\sum_{i=0}^2 n(X = i) &= 52 \times 52 \\ &= 2704\end{aligned}\tag{2}$$

For  $X=2$ ,

There are total 4 aces in deck of 52 cards and it's drawn 2 times and we need both as ace. So, total cases would be  $4 \times 4$ .

So,

$$\begin{aligned} n(X = 2) &= 4 \times 4 \\ &= 16 \end{aligned} \tag{3}$$

For  $X=1$ ,

There are 4 aces and 48 non-ace cards in deck of 52 cards and it's drawn 2 times and we need exactly 1 ace. So there are 2 possibility, first is that first card is ace and second one is not and the other case is first card is non-ace but other one is ace. So, total cases for this condition would be  $4 \times 48 + 48 \times 4$ .

So,

$$\begin{aligned} n(X = 1) &= 4 \times 48 + 48 \times 4 \\ &= 384 \end{aligned} \tag{4}$$

For  $X=0$ ,

There are 48 non-ace cards in deck of 52 cards and it's drawn 2 times. So, for being both non-ace card the number of cases would be  $48 \times 48$ .

So,

$$\begin{aligned} n(X = 0) &= 48 \times 48 \\ &= 2304 \end{aligned} \tag{5}$$



Now,

From equation (1), probability of getting 2 aces,

$$\Pr(X = 2) = \frac{n(X = 2)}{\sum_{i=0}^2 n(X = i)} \quad (6)$$

putting values from equations (2) and (3),

$$\Pr(X = 2) = \frac{16}{2704} \quad (7)$$

$$= .005917 \quad (8)$$

again,

from equation (1), probability of getting 1 ace,

$$\Pr(X = 1) = \frac{n(X = 1)}{\sum_{i=0}^2 n(X = i)} \quad (9)$$

putting values from equations (2) and (4),

$$\Pr(X = 1) = \frac{384}{2704} \quad (10)$$

$$= .142 \quad (11)$$

again,

from equation (1), probability of getting 0 ace,

$$\Pr(X = 0) = \frac{n(X = 0)}{\sum_{i=0}^2 n(X = i)} \quad (12)$$

putting values from equations (2) and (5),

$$\Pr(X = 0) = \frac{2304}{2704} \quad (13)$$

$$= .852 \quad (14)$$

<b>i</b>	0	1	2
<b><math>\Pr(X = i)</math></b>	0.852	0.142	0.005917

Table: probability distribution table