

# Assignment 1

**Himanshu Kumar Gupta(AI21BTECH11012)**

**Problem 3b, ICSE 10 2019:**

M and N are two points on the X axis and Y axis respectively. P (3, 2) divides the line segment MN in the ratio 2 : 3.

Find:

- 1) The coordinates of M and N
- 2) Slope of the line MN

**Solution:**

The various parameters involved in this question are listed in Table (I):

Parameter	Symbol	Value
standard vector 1	$\mathbf{e}_1$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
standard vector 2	$\mathbf{e}_2$	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$
position vector of point P	$\mathbf{P}$	$\begin{pmatrix} 3 \\ 2 \end{pmatrix}$
position vector of point M	$\mathbf{M}$	$\begin{pmatrix} p \\ 0 \end{pmatrix}$
position vector of point N	$\mathbf{N}$	$\begin{pmatrix} 0 \\ q \end{pmatrix}$

TABLE I

from section formula in vector form, we know that

$$\mathbf{P} = \frac{1 \times \mathbf{M} + k \times \mathbf{N}}{1 + k} \quad (1)$$

where k:1 is ratio in which point P divides the line joining M and N

Since P(3,2) divides M and N in ratio 2:3

So,  $k = \frac{2}{3}$

Now, by applying section formula given in equation (1) to P on line MN, we get

$$\begin{aligned} \mathbf{P} &= \frac{1 \times \mathbf{M} + \frac{2}{3} \times \mathbf{N}}{1 + \frac{2}{3}} \\ \rightarrow \begin{pmatrix} 3 \\ 2 \end{pmatrix} &= \frac{1 \times \begin{pmatrix} p \\ 0 \end{pmatrix} + \frac{2}{3} \times \begin{pmatrix} 0 \\ q \end{pmatrix}}{\frac{5}{3}} \\ \rightarrow \begin{pmatrix} 3 \\ 2 \end{pmatrix} &= \frac{3 \times \begin{pmatrix} p \\ 0 \end{pmatrix} + 2 \times \begin{pmatrix} 0 \\ q \end{pmatrix}}{5} \end{aligned}$$

$$\rightarrow 5 \times \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 3p \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 2q \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 15 \\ 10 \end{pmatrix} = \begin{pmatrix} 3p \\ 2q \end{pmatrix} \quad (2)$$

So,

$$3p = 15 \quad 2q = 10$$

$$\rightarrow p = 5 \quad \rightarrow q = 5 \quad (3)$$

So, the vectors  $\mathbf{M} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$  and  $\mathbf{N} = \begin{pmatrix} 0 \\ 5 \end{pmatrix}$  therefore the points M and N would be (5,0) and (0,5) respectively.

So, the vector

$$\mathbf{MN} = \mathbf{N} - \mathbf{M} \quad (4)$$

$$= \begin{pmatrix} 0 - 5 \\ 5 - 0 \end{pmatrix} \quad (5)$$

$$= \begin{pmatrix} -1 \\ 1 \end{pmatrix} \quad (6)$$

$$= -1 \times \mathbf{e}_1 + 1 \times \mathbf{e}_2 \quad (7)$$

Now, we know that the slope of any vector is

$$= \frac{\text{coefficient of } \mathbf{e}_2}{\text{coefficient of } \mathbf{e}_1} \quad (8)$$

So, slope of MN,

$$\text{slope} = \frac{5}{-5} \quad (9)$$

$$= -1 \quad (10)$$