

Assignment 1

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Problem 3b, ICSE 10 2019:

M and N are two points on the X axis and Y axis respectively. P (3, 2) divides the line segment MN in the ratio 2 : 3.

Find:

- (i) the coordinates of M and N
- (ii) slope of the line MN.

Solution:

let \mathbf{P} be the position vector of point P

$$\text{so, } \mathbf{P} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

M and N be the position vector of point M and N respectively

Since M and N are points on x and y axis respectively

$$\text{So, let } \mathbf{M} = \begin{pmatrix} x \\ 0 \end{pmatrix} \text{ and } \mathbf{N} = \begin{pmatrix} 0 \\ y \end{pmatrix}$$

from section formula in vector form, we know that

$$\mathbf{P} = \frac{\mathbf{M}b + \mathbf{N}a}{a + b}$$

where a:b is ratio in which point P divides the line joining M and N

Since P(3,2) divides M and N in ratio 2:3

So, a=2,b=3

Now, by applying section formula to P on line MN, we get

$$\begin{aligned} \mathbf{P} &= \frac{\mathbf{M} * 3 + \mathbf{N} * 2}{3 + 2} \\ \rightarrow \begin{pmatrix} 3 \\ 2 \end{pmatrix} &= \frac{3 \begin{pmatrix} x \\ 0 \end{pmatrix} + 2 \begin{pmatrix} 0 \\ y \end{pmatrix}}{5} \\ \rightarrow 5 \begin{pmatrix} 3 \\ 2 \end{pmatrix} &= \begin{pmatrix} 3x \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 2y \end{pmatrix} \\ \rightarrow \begin{pmatrix} 15 \\ 10 \end{pmatrix} &= \begin{pmatrix} 3x \\ 2y \end{pmatrix} \end{aligned}$$

So,

$$3x = 15 \quad 2y = 10$$

$$\rightarrow x = 5 \quad \rightarrow y = 5$$

So, the points M and N would be **(5,0)** and **(0,5)** respectively.

Now, we know that the slope of any line AB is

$$\text{slope} = \frac{y_A - y_B}{x_A - x_B}$$

So, slope of line MN is

$$\text{slope} = \frac{0 - 5}{5 - 0}$$

$$\rightarrow \text{slope} = \mathbf{-1}$$