

Assignment 2

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Problem 1(viii), ICSE 12 2019:

Using L'Hospital's Rule, evaluate:

$$\lim_{x \rightarrow 0} \frac{8^x - 4^x}{4x}$$

Solution:

we know that,

if there is a function $f(x) = \frac{g(x)}{h(x)}$

then by L'Hospital's Rule

$$\lim_{x \rightarrow x_0} f(x) = \lim_{x \rightarrow x_0} \frac{g(x)}{h(x)} = \lim_{x \rightarrow x_0} \frac{g'(x)}{h'(x)} \quad (1)$$

so, by equation (1),

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{8^x - 4^x}{4x} &= \lim_{x \rightarrow 0} \frac{\frac{d(8^x - 4^x)}{dx}}{\frac{d(4x)}{dx}} \\ &= \lim_{x \rightarrow 0} \frac{8^x \log 8 - 4^x \log 4}{4} \end{aligned} \quad (2)$$

Now, putting value of $x=0$, we get

$$= \frac{\log 8 - \log 4}{4} \quad (4)$$

$$= \frac{\log 2}{4} \quad (5)$$