# Assignment 1

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***Problem 3b, ICSE 10 2019:***

M and N are two points on the X axis and Y axis respectively. P (3, 2) divides the line segment MN in the ratio 2 : 3.

Find:

1) The coordinates of M and N
2) Slope of the line MN

***Solution:***

The various parameters involved in this question are listed in Table (I):

| Parameter | Symbol/Formula | Value |
|---|---|---|
| standard vector 1 | $\mathbf{e_1}$ | $\begin{pmatrix}1\\0\end{pmatrix}$ |
| standard vector 2 | $\mathbf{e_2}$ | $\begin{pmatrix}0\\1\end{pmatrix}$ |
| position vector of point P | $\mathbf{P}$ | $3\times\mathbf{e_1}+2\times\mathbf{e_2}$ |
| position vector of point M | $\mathbf{M}$ | $p\times\mathbf{e_1}+0\times\mathbf{e_2}$ |
| position vector of point N | $\mathbf{N}$ | $0\times\mathbf{e_1}+q\times\mathbf{e_2}$ |

TABLE I

from section formula in vector form, we know that

$$\mathbf{P}=\frac{1\times\mathbf{M}+k\times\mathbf{N}}{1+k} \tag{1}$$

where k:1 is ratio in which point P divides the line joining M and N

Since P(3,2) divides M and N in ratio 2:3

So,$k=\frac{2}{3}$

Now,by applying section formula given in equation (1) to P on line MN, we get

$$\mathbf{P}=\frac{1\times\mathbf{M}+\frac{2}{3}\times\mathbf{N}}{1+\frac{2}{3}} \tag{2}$$

$$\implies 3\times\mathbf{e_1}+2\times\mathbf{e_2}=\frac{1\times(p\times\mathbf{e_1})+\frac{2}{3}\times(q\times\mathbf{e_2})}{\frac{5}{3}} \tag{3}$$

$$\implies 3\times\mathbf{e_1}+2\times\mathbf{e_2}=\frac{3\times(p\times\mathbf{e_1})+2\times(q\times\mathbf{e_2})}{5} \tag{4}$$

$$\implies 15\times\mathbf{e_1}+10\times\mathbf{e_2}=3p\times\mathbf{e_1}+2q\times\mathbf{e_2} \tag{5}$$

$$\implies (15-3p)\times\mathbf{e_1}+(10-2q)\times\mathbf{e_2}=0 \tag{6}$$

Now,coefficient of $\mathbf{e_1}$ and $\mathbf{e_2}$ in equation(6) should be independently equal to 0, So,

$$\begin{aligned} 3p-15=0 &\quad 2q-10=0 \\ \implies 3p=15 &\quad \implies 2q=10 \\ \implies p=5 &\quad \implies q=5 \end{aligned} \tag{7}$$

So,the vectors $\mathbf{M}=5\times\mathbf{e_1}+0\times\mathbf{e_2}=\begin{pmatrix}5\\0\end{pmatrix}$ and $\mathbf{N}=0\times\mathbf{e_1}+5\times\mathbf{e_2}=\begin{pmatrix}0\\5\end{pmatrix}$

therefore the points M and N would be **(5,0)** and **(0,5)** respectively.

Now,the vector

$$\mathbf{MN}=\mathbf{N}-\mathbf{M} \tag{8}$$
$$=(0\times\mathbf{e_1}+5\times\mathbf{e_2})-(5\times\mathbf{e_1}+0\times\mathbf{e_2}) \tag{9}$$
$$=-5\times\mathbf{e_1}+5\times\mathbf{e_2} \tag{10}$$

Now,we know that the slope of any vector is

$$=\frac{\text{coefficient of }\mathbf{e_2}}{\text{coefficient of }\mathbf{e_1}} \tag{11}$$

So,slope of $\mathbf{MN}$,

$$slope=\frac{5}{-5} \tag{12}$$
$$=\mathbf{-1} \tag{13}$$

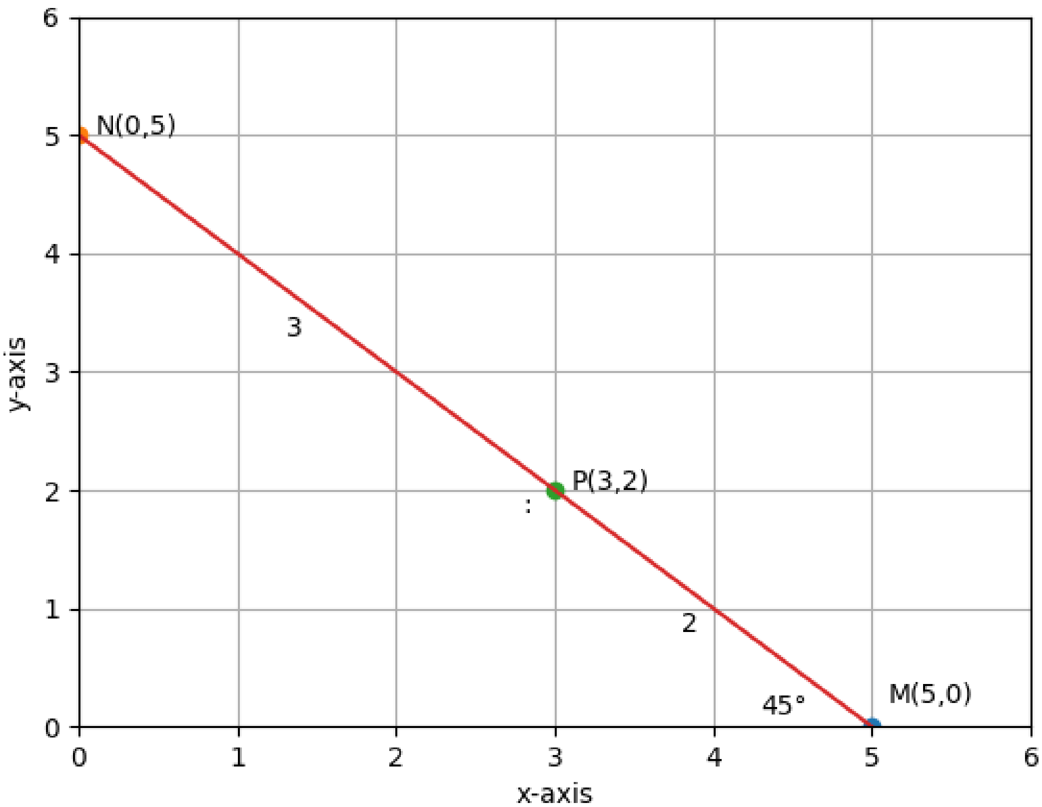


Fig. 1. graph representing M,N and P