# Assignment 1

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***Problem 3b, ICSE 10 2019:***

M and N are two points on the X axis and Y axis respectively. P (3, 2) divides the line segment MN in the ratio 2 : 3.

Find:

1) The coordinates of M and N
2) Slope of the line MN

***Solution:***

The various parameters involved in this question are listed in Table (I):

| **Parameter** | **Symbol/Formula** | **Value** |
|---|---|---|
| standard vector 1 | $\mathbf{e_1}$ | $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ |
| standard vector 2 | $\mathbf{e_2}$ | $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ |
| position vector of point P | $\mathbf{P}$ | $3 \times \mathbf{e_1} + 2 \times \mathbf{e_2}$ |
| position vector of point M | $\mathbf{M}$ | $p \times \mathbf{e_1} + 0 \times \mathbf{e_2}$ |
| position vector of point N | $\mathbf{N}$ | $0 \times \mathbf{e_1} + q \times \mathbf{e_2}$ |

TABLE I

from section formula in vector form, we know that

$$\mathbf{P} = \frac{1 \times \mathbf{M} + k \times \mathbf{N}}{1 + k} \tag{1}$$

where k:1 is ratio in which point P divides the line joining M and N

Since P(3,2) divides M and N in ratio 2:3

So,$k = \frac{2}{3}$

Now,by applying section formula given in equation (1) to P on line MN, we get

$$\mathbf{P} = \frac{1 \times \mathbf{M} + \frac{2}{3} \times \mathbf{N}}{1 + \frac{2}{3}} \tag{2}$$

$$\implies 3 \times \mathbf{e_1} + 2 \times \mathbf{e_2} = \frac{1 \times (p \times \mathbf{e_1}) + \frac{2}{3} \times (q \times \mathbf{e_2})}{\frac{5}{3}} \tag{3}$$

$$\implies 3 \times \mathbf{e_1} + 2 \times \mathbf{e_2} = \frac{3 \times (p \times \mathbf{e_1}) + 2 \times (q \times \mathbf{e_2})}{5} \tag{4}$$

$$\implies 15 \times \mathbf{e_1} + 10 \times \mathbf{e_2} = 3p \times \mathbf{e_1} + 2q \times \mathbf{e_2} \tag{5}$$

$$\implies (15 - 3p) \times \mathbf{e_1} + (10 - 2q) \times \mathbf{e_2} = 0 \tag{6}$$

Now,coefficient of $\mathbf{e_1}$ and $\mathbf{e_2}$ in equation(6) should be independently equal to 0, So,

$$\begin{aligned} 3p - 15 = 0 \quad & 2q - 10 = 0 \\ \implies 3p = 15 \quad & \implies 2q = 10 \\ \implies p = 5 \quad & \implies q = 5 \end{aligned} \tag{7}$$

So,the vectors $\mathbf{M} = 5 \times \mathbf{e_1} + 0 \times \mathbf{e_2} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$ and $\mathbf{N} = 0 \times \mathbf{e_1} + 5 \times \mathbf{e_2} = \begin{pmatrix} 0 \\ 5 \end{pmatrix}$

therefore the points M and N would be **(5,0)** and **(0,5)** respectively.

Now,the vector

$$\mathbf{MN} = \mathbf{N} - \mathbf{M} \tag{8}$$

$$= (0 \times \mathbf{e_1} + 5 \times \mathbf{e_2}) - (5 \times \mathbf{e_1} + 0 \times \mathbf{e_2}) \tag{9}$$

$$= -5 \times \mathbf{e_1} + 5 \times \mathbf{e_2} \tag{10}$$

Now,we know that the slope of any vector is

$$= \frac{\text{coefficient of } \mathbf{e_2}}{\text{coefficient of } \mathbf{e_1}} \tag{11}$$

So,slope of $\mathbf{MN}$,

$$slope = \frac{5}{-5} \tag{12}$$

$$= \mathbf{-1} \tag{13}$$

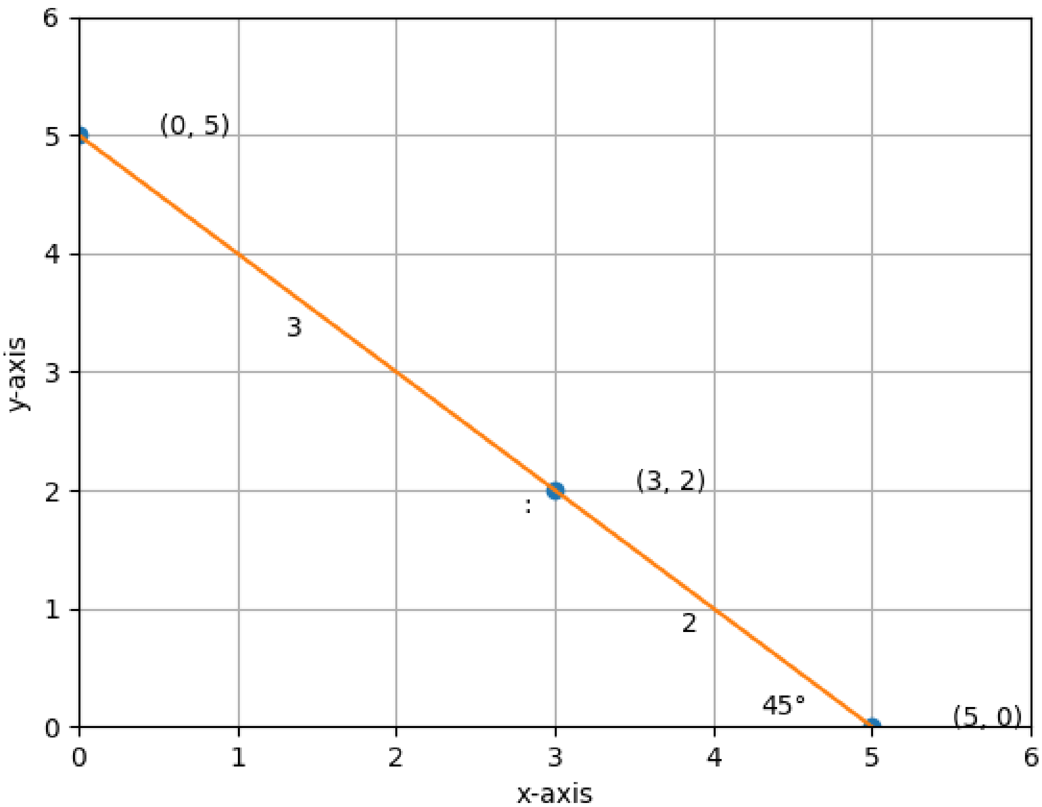


Fig. 1. graph representing M,N and P