Assignment 1

Himanshu Kumar Gupta(AI21BTECH11012)

Problem 3b, ICSE 10 2019:

M and N are two points on the X axis and Y axis respectively. P (3, 2) divides the line segment MN in the ratio 2 : 3.

Find:

1) The coordinates of M and N
2) Slope of the line MN

Solution:

Let $\mathbf{e_1} = \begin{pmatrix}1\\0\end{pmatrix}$ and $\mathbf{e_2} = \begin{pmatrix}0\\1\end{pmatrix}$ be the standard vectors

Let $\mathbf{P}$ be the position vector of point P

So, $\mathbf{P} = \begin{pmatrix}3\\2\end{pmatrix}$

$\mathbf{M}$ and $\mathbf{N}$ be the position vector of point M and N respectively

Since M and N are points on x and y axis respectively

So,let $\mathbf{M} = \begin{pmatrix}p\\0\end{pmatrix}$ and $\mathbf{N} = \begin{pmatrix}0\\q\end{pmatrix}$

from section formula in vector form, we know that

$$\mathbf{P} = \frac{1\times\mathbf{M} + k\times\mathbf{N}}{1+k} \tag{1}$$

where k:1 is ratio in which point P divides the line joining M and N

Since P(3,2) divides M and N in ratio 2:3

So,$k = \frac{2}{3}$

Now,by applying section formula given in equation (1) to P on line MN, we get

$$\mathbf{P} = \frac{1\times\mathbf{M} + \frac{2}{3}\times\mathbf{N}}{1+\frac{2}{3}}$$

$$\to \begin{pmatrix}3\\2\end{pmatrix} = \frac{1\times\begin{pmatrix}p\\0\end{pmatrix} + \frac{2}{3}\times\begin{pmatrix}0\\q\end{pmatrix}}{\frac{5}{3}}$$

$$\to \begin{pmatrix}3\\2\end{pmatrix} = \frac{3\times\begin{pmatrix}p\\0\end{pmatrix} + 2\times\begin{pmatrix}0\\q\end{pmatrix}}{5}$$

$$\to 5\times\begin{pmatrix}3\\2\end{pmatrix} = \begin{pmatrix}3p\\0\end{pmatrix} + \begin{pmatrix}0\\2q\end{pmatrix}$$

$$\to \begin{pmatrix}15\\10\end{pmatrix} = \begin{pmatrix}3p\\2q\end{pmatrix} \tag{2}$$

So,

$$3p = 15 \quad 2q = 10$$

$$\to p = 5 \quad \to q = 5 \tag{3}$$

So,the vectors $\mathbf{M} = \begin{pmatrix}5\\0\end{pmatrix}$ and $\mathbf{N} = \begin{pmatrix}0\\5\end{pmatrix}$ therefore the points M and N would be **(5,0)** and **(0,5)** respectively.

So,the vector

$$\mathbf{MN} = \mathbf{N} - \mathbf{M} \tag{4}$$

$$= \begin{pmatrix}0-5\\5-0\end{pmatrix} \tag{5}$$

$$= \begin{pmatrix}-1\\1\end{pmatrix} \tag{6}$$

$$= -1\times\mathbf{e_1} + 1\times\mathbf{e_2} \tag{7}$$

Now,we know that the slope of any vector is

$$= \frac{\text{coefficient of } \mathbf{e_2}}{\text{coefficient of } \mathbf{e_1}} \tag{8}$$

So,slope of $\mathbf{MN}$,

$$slope = \frac{5}{-5} \tag{9}$$

$$= \textbf{-1} \tag{10}$$