

# Assignment 12

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## Question : Ex. 9.42 , Papoulis

The process  $x(t)$  is WSS with  $E\{x(t)\} = 5$  and  $R_{xx}(\tau) = 25 + 4e^{-2|\tau|}$ . If  $y(t) = 2x(t) + 3x'(t)$ , find  $\eta_y$ ,  $R_{yy}(\tau)$  and  $S_{yy}(\omega)$ .

# Solution

$$y(t) = 2x(t) + 3x'(t)$$

$$\eta_x = 5$$

So, the process  $y(t)$  is the output of system  $H(s) = 2 + 3s$  with input  $x(t)$ .

Hence,

$$n_y = n_x H(0) = 5 \times 2 \quad (1)$$

$$\begin{aligned} C_{xx}(\tau) &= R_{xx}(\tau) - |\eta_x|^2 \\ &= 25 + 4e^{-2|\tau|} - 5^2 \\ &= 4e^{-2|\tau|} \end{aligned} \quad (2)$$

$$\begin{aligned}
 R_{yy}(\tau) &= E \{y(t+\tau)y(t)\} \\
 &= E \{(2x(t+\tau) + 3x'(t+\tau))(2x(t) + 3x'(t))\} \\
 &= 4E \{x(t+\tau)x(t)\} + 6E \{x(t+\tau)x'(t)\} \\
 &\quad + 6E \{x'(t+\tau)x(t)\} + 9E \{x'(t+\tau)x'(t)\} \\
 &= 4R_{xx} + 0 \\
 &= 100 + 16e^{-2|\tau|}
 \end{aligned} \tag{3}$$

$$\begin{aligned}
 S_{yy}(\omega) &= S_{xx}(\omega) |H(\omega)|^2 \\
 &= \frac{16}{4 + \omega^2} (4 + 9\omega^2)
 \end{aligned} \tag{4}$$