

Assignment 7

Himanshu Kumar Gupta (AI21BTECH11012)

May 2022

Contents

1 Question

2 Solution

Question: Ex. 5.27 , chapter 5 , Papoulis

Show that if $U = [A_1, \dots, A_n]$ is a partition of S , then

$$E\{x\} = E\{x|A_1\}P(A_1) + \dots + E\{x|A_n\}P(A_n).$$

Solution

We know that,

$$E \{x\} = \int_{-\infty}^{\infty} x f(x) dx \quad (1)$$

and since there are n partitions in U , so

$$\begin{aligned} f(x) &= f(x|A_1) P(A_1) + \dots + f(x|A_n) P(A_n) \\ &= \sum_{j=1}^n f(x|A_j) P(A_j) \end{aligned} \quad (2)$$

putting expression of $f(x)$ from equation (2) to equation (1),

$$E\{x\} = \int_{-\infty}^{\infty} x \sum_{j=1}^n f(x|A_j) P(A_j) dx \quad (3)$$

from property of integration,

$$\begin{aligned} E\{x\} &= \sum_{j=1}^n \int_{-\infty}^{\infty} x f(x|A_j) P(A_j) dx \\ &= \sum_{j=1}^n P(A_j) \int_{-\infty}^{\infty} x f(x|A_j) dx \end{aligned} \quad (4)$$

we also know that,

$$E \{x|A_j\} = \int_{-\infty}^{\infty} xf(x|A_j) dx \quad (5)$$

putting expression from equation (5) to (4),

$$E \{x\} = \sum_{j=1}^n P(A_j) E \{x|A_j\} \quad (6)$$