Circuits and Transforms

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***Abstract*—This manual provides a simple introduction to Transforms**

1 Definitions

1. The unit step function is

$$u(t) = \begin{cases} 1 & t > 0 \\ \frac{1}{2} & t = 0 \\ 0 & t < 0 \end{cases} \tag{1.1}$$

2. The Laplace transform of $g(t)$ is defined as

$$G(s) = \int_{-\infty}^{\infty} g(t) e^{-st} \, dt \tag{1.2}$$

2 Laplace Transform

1. In the circuit, the switch S is connected to position P for a long time so that the charge on the capacitor becomes $q_1 \, \mu C$. Then S is switched to position Q. After a long time, the charge on the capacitor is $q_2 \, \mu C$.

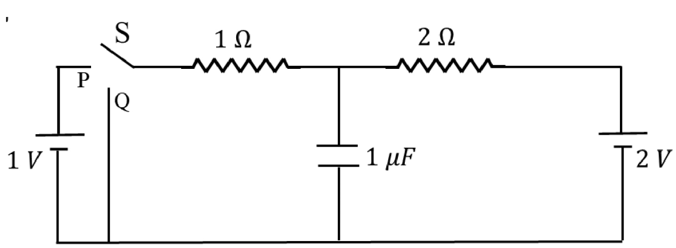
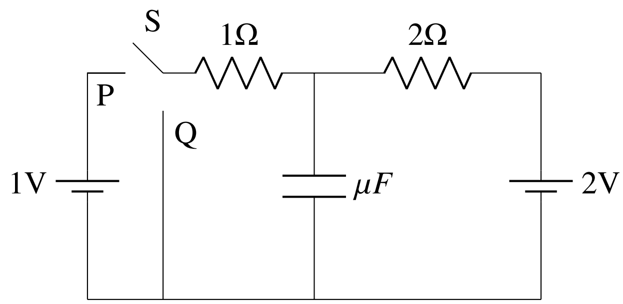
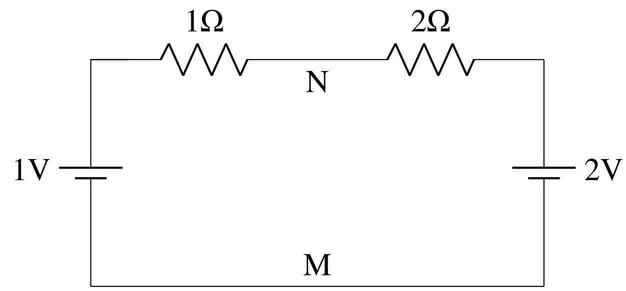


Fig. 2.1

2. Draw the circuit using latex-tikz.
Solution:



3. Find q_1.
Solution: On connecting S to P for a long time make the capacitor fully charged. So,it will behave like disconnected wire. So, circuit will look like



So, on applying KVL we get current as,

$$i = 1/3 \tag{2.1}$$

in counter-clockwise direction
So, PD across the capacitor i.e. between M and N is given by,

$$V_Q = 1 + \frac{1}{3} \times 1 \tag{2.2}$$

$$= \frac{4}{3} \tag{2.3}$$

Now, charge on capacitor will be

$$q_1 = V_Q C_0 \tag{2.4}$$

$$= \frac{4}{3} \mu C \tag{2.5}$$

4. Show that the Laplace transform of $u(t)$ is $\frac{1}{s}$ and find the ROC.
Solution: Laplace of u(t) is given by

$$U(t) = \int_{-\infty}^{\infty} u(t) e^{-st} \, dt \tag{2.6}$$

$$= \int_{0}^{\infty} e^{-st} \, dt \tag{2.7}$$

$$= \left. \frac{e^{-st}}{-s} \right]_0^{\infty} \tag{2.8}$$

$$= \frac{1}{s} \tag{2.9}$$

Since e^{-st} is defined at $t \to \infty$ only for s¿0

So, ROC will be $s > 0$

5. Show that

$$e^{-at}u(t) \overset{\mathcal{H}}{\longleftrightarrow} L\frac{1}{s+a}, \quad a > 0 \tag{2.10}$$

and find the ROC.

Solution:

$$\mathcal{L}\left[e^{-at}u(t)\right] = \int_{-\infty}^{\infty} e^{-at}u(t)e^{-st}\,dt \tag{2.11}$$

$$= \int_{-\infty}^{\infty} e^{-at}u(t)e^{-st}\,dt \tag{2.12}$$

$$= \int_{0}^{\infty} e^{-(a+s)t}\,dt \tag{2.13}$$

$$= \left.\frac{e^{-(s+a)t}}{-(s+a)}\right]_0^{\infty} \tag{2.14}$$

$$= \frac{1}{s+a} \tag{2.15}$$

Since $e^{-(s+a)t}$ is defined at $t \to \infty$ only for s¿-a

So, ROC will be $s > -a$

6. Now consider the following resistive circuit transformed from Fig. **??** where

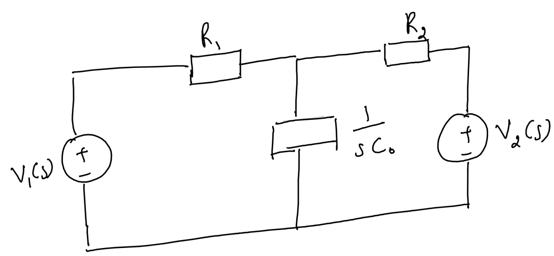


Fig. 2.2: hbhb

$$u(t) \overset{\mathcal{H}}{\longleftrightarrow} LV_1(s) \tag{2.16}$$

$$2u(t) \overset{\mathcal{H}}{\longleftrightarrow} LV_2(s) \tag{2.17}$$

Find the voltage across the capacitor $V_{C_0}(s)$.

Solution: Given that $V_1(s) = \frac{1}{s}$ and $V_2(s) = \frac{2}{s}$

7. Find $v_{C_0}(t)$. Plot using python.
8. Verify your result using ngspice.
9. Obtain Fig. **??** using the equivalent differential equation.

3 Initial Conditions

1. Find q_2 in Fig. **??**.
2. Draw the equivalent s-domain resistive circuit when S is switched to position Q. Use variables R_1, R_2, C_0 for the passive elements. Use latex-tikz.
3. $V_{C_0}(s) = ?$
4. $v_{C_0}(t) = ?$ Plot using python.
5. Verify your result using ngspice.
6. Find $v_{C_0}(0-), v_{C_0}(0+)$ and $v_{C_0}(\infty)$.
7. Obtain the Fig. in problem **??** using the equivalent differential equation.

4 Bilinear Transform

1. In Fig. **??**, consider the case when S is switched to Q right in the beginning. Formulate the differential equation.
2. Find $H(s)$ considering the ouput voltage at the capacitor.
3. Plot $H(s)$. What kind of filter is it?
4. Using trapezoidal rule for integration, formulate the difference equation by considering

$$y(n) = y(t)|_{t=n} \tag{4.1}$$

5. Find $H(z)$.
6. How can you obtain $H(z)$ from $H(s)$?