Pingala Series

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Contents

***Abstract*—This manual provides a simple introduction to Transforms**

1 JEE 2019

Let

$$a_n = \frac{\alpha^n - \beta^n}{\alpha - \beta}, \quad n \geq 1 \tag{1.1}$$

$$b_n = a_{n-1} + a_{n+1}, \quad n \geq 2, \quad b_1 = 1 \tag{1.2}$$

Verify the following using a python code.

1.1

$$\sum_{k=1}^{n} a_k = a_{n+2} - 1, \quad n \geq 1 \tag{1.3}$$

Solution: Download and run the following python program.

```
https://github.com/himanshukumargupta11012/
    EE3900_assignments/blob/main/pingala/
    ques_1/1.1.py
```

1.2

$$\sum_{k=1}^{\infty} \frac{a_k}{10^k} = \frac{10}{89} \tag{1.4}$$

Solution: Download and run the following python program.

```
https://github.com/himanshukumargupta11012/
    EE3900_assignments/blob/main/pingala/
    ques_1/1.1.py
```

$$b_n = \alpha^n + \beta^n, \quad n \geq 1 \tag{1.5}$$

1.3

$$\sum_{k=1}^{\infty} \frac{b_k}{10^k} = \frac{8}{89} \tag{1.6}$$

2 Pingala Series

2.1 The *one sided* Z-transform of $x(n)$ is defined as

$$X^+(z) = \sum_{n=0}^{\infty} x(n) z^{-n}, \quad z \in \mathbb{C} \tag{2.1}$$

2.2 The *Pingala* series is generated using the difference equation

$$x(n+2) = x(n+1) + x(n), \quad x(0) = x(1) = 1, n \geq 0 \tag{2.2}$$

Generate a stem plot for $x(n)$.

2.3 Find $X^+(z)$.

Solution:

$$X^+(z) = \sum_{n=0}^{\infty} x(n)z^{-n} \tag{2.3}$$

$$= x(0) + x(1) + \sum_{n=2}^{\infty}\{x(n-1) + x(n-2)\}z^{-n} \tag{2.4}$$

$$= 1 + z^{-1} + \sum_{n=2}^{\infty} x(n-1)z^{-n} + \sum_{n=2}^{\infty} x(n-2)z^{-n} \tag{2.5}$$

$$= 1 + z^{-1} + \sum_{m=1}^{\infty} x(m)z^{-m-1} + \sum_{p=0}^{\infty} x(p)z^{-p-2} \tag{2.6}$$

$$= 1 + z^{-1} + z^{-1}\{\sum_{m=0}^{\infty} x(m)z^{-m} - 1\} \tag{2.7}$$

$$= +z^{-2}\sum_{p=0}^{\infty} x(p)z^{-p} \tag{2.8}$$

$$= 1 + z^{-1} + z^{-1}\{X^+(z) - 1\} + z^{-2}X^+(z) \tag{2.9}$$

$$= 1 + z^{-1}X^+(z) + z^{-2}X^+(z) \tag{2.10}$$

$$\implies X^+(z) = 1 + z^{-1}X^+(z) + z^{-2}X^+(z) \tag{2.11}$$

$$\implies X^+(z) = \frac{1}{1 - z^{-1} + z^{-2}} \tag{2.12}$$

2.4 Find $x(n)$.
2.5 Sketch

$$y(n) = x(n-1) + x(n+1), \quad n \geq 0 \tag{2.13}$$

2.6 Find $Y^+(z)$.
2.7 Find $y(n)$.

3 Power of the Z transform

3.1 Show that

$$\sum_{k=1}^{n} a_k = \sum_{k=0}^{n-1} x(n) = x(n) * u(n-1) \tag{3.1}$$

3.2 Show that

$$a_{n+2} - 1, \quad n \geq 1 \tag{3.2}$$

can be expressed as

$$[x(n+1) - 1]\, u(n) \tag{3.3}$$

3.3 Show that

$$\sum_{k=1}^{\infty} \frac{a_k}{10^k} = \frac{1}{10}\sum_{k=0}^{\infty} \frac{x(k)}{10^k} = \frac{1}{10}X^+(10) \tag{3.4}$$

3.4 Show that

$$\alpha^n + \beta^n, \quad n \geq 1 \tag{3.5}$$

can be expressed as

$$w(n) = \left(\alpha^{n+1} + \beta^{n+1}\right) u(n) \tag{3.6}$$

and find $W(z)$.
3.5 Show that

$$\sum_{k=1}^{\infty} \frac{b_k}{10^k} = \frac{1}{10}\sum_{k=0}^{\infty} \frac{y(k)}{10^k} = \frac{1}{10}Y^+(10) \tag{3.7}$$

3.6 Solve the JEE 2019 problem.