Assignment 1

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Exercise 3.4, Oppenheimer

Consider the z-transform $X(z)$ whose pole-zero plot is as shown in Figure 1.

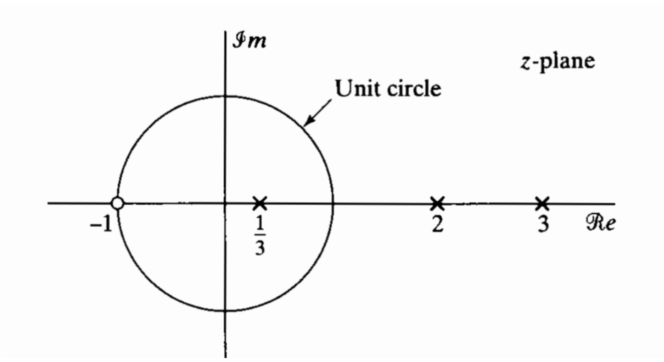

Fig. 1. Pole Zero plot of $X(n)$

1) Determine the region of convergence of $X(z)$ if it is known that fourier transform exists. For this case, determine whether the corresponding sequence $x[n]$ is right sided,left sided, or two sided.
Solution For given figure, 4 ROCs are possible which are

$$|x| < |\frac{1}{3}| \tag{1}$$

$$|\frac{1}{3}| < |x| < |2| \tag{2}$$

$$|2| < |x| < |3| \tag{3}$$

$$|3| < |x| \tag{4}$$

Now,since the fourier transform exist, so the unit circle must lie in ROC. So, only possible ROC will be $|2| < |x| < |3|$.
And since its bounded both side by poles only, so the sequence x[n] is two sided.

2) How many possible two-sided sequences have the pole-zero plot shown in Figure 1
Solution Among 4 ROCs above, we can see that only 2 are bounded both side by poles which are

$$|\frac{1}{3}| < |x| < |2| \tag{5}$$

$$|2| < |x| < |3| \tag{6}$$

So,there ar only 2 possible two-sided sequences

3) Is it possible for the pole-zero plot in Figure 1 to be associated with a sequence that is both stable and causal? If so, give the appropriate region of convergence.
Solution For stability, ROC must contain the unit circle. So, possible options is

$$|\frac{1}{3}| < |x| < |2| \tag{7}$$

For causality, the ROC must be outside of outermost pole which is 3.So, possible option is

$$|3| < |x| \tag{8}$$

Since nothing is common between two, so there is no possible signal which is both stable and causal.