

Pingala Series

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Abstract—This manual provides a simple introduction to Transforms

1 JEE 2019

Let

$$a_n = \frac{\alpha^n - \beta^n}{\alpha - \beta}, \quad n \geq 1 \quad (1.1)$$

$$b_n = a_{n-1} + a_{n+1}, \quad n \geq 2, \quad b_1 = 1 \quad (1.2)$$

Verify the following using a python code.

1.1

$$\sum_{k=1}^n a_k = a_{n+2} - 1, \quad n \geq 1 \quad (1.3)$$

1.2

$$\sum_{k=1}^{\infty} \frac{a_k}{10^k} = \frac{10}{89} \quad (1.4)$$

1.3

$$b_n = \alpha^n + \beta^n, \quad n \geq 1 \quad (1.5)$$

1.4

$$\sum_{k=1}^{\infty} \frac{b_k}{10^k} = \frac{8}{89} \quad (1.6)$$

2 PINGALA SERIES

2.1 The *one sided* Z-transform of $x(n)$ is defined as

$$X^+(z) = \sum_{n=0}^{\infty} x(n)z^{-n}, \quad z \in \mathbb{C} \quad (2.1)$$

2.2 The *Pingala* series is generated using the difference equation

$$x(n+2) = x(n+1) + x(n), \quad x(0) = x(1) = 1, n \geq 0 \quad (2.2)$$

Generate a stem plot for $x(n)$.

2.3 Find $X^+(z)$.

2.4 Find $x(n)$.

2.5 Sketch

$$y(n) = x(n-1) + x(n+1), \quad n \geq 0 \quad (2.3)$$

2.6 Find $Y^+(z)$.

2.7 Find $y(n)$.

3 POWER OF THE Z TRANSFORM

3.1 Show that

$$\sum_{k=1}^n a_k = \sum_{k=0}^{n-1} x(n) = x(n) * u(n-1) \quad (3.1)$$

3.2 Show that

$$a_{n+2} - 1, \quad n \geq 1 \quad (3.2)$$

can be expressed as

$$[x(n+1) - 1]u(n) \quad (3.3)$$

3.3 Show that

$$\sum_{k=1}^{\infty} \frac{a_k}{10^k} = \frac{1}{10} \sum_{k=0}^{\infty} \frac{x(k)}{10^k} = \frac{1}{10} X^+(10) \quad (3.4)$$

3.4 Show that

$$\alpha^n + \beta^n, \quad n \geq 1 \quad (3.5)$$

can be expressed as

$$w(n) = (\alpha^{n+1} + \beta^{n+1})u(n) \quad (3.6)$$

and find $W(z)$.

3.5 Show that

$$\sum_{k=1}^{\infty} \frac{b_k}{10^k} = \frac{1}{10} \sum_{k=0}^{\infty} \frac{y(k)}{10^k} = \frac{1}{10} Y^+(10) \quad (3.7)$$

3.6 Solve the JEE 2019 problem.