PHY 206 - Computational Thinking

Mini-Project

Coupled Anharmonic Oscillators

Team: Phi φ

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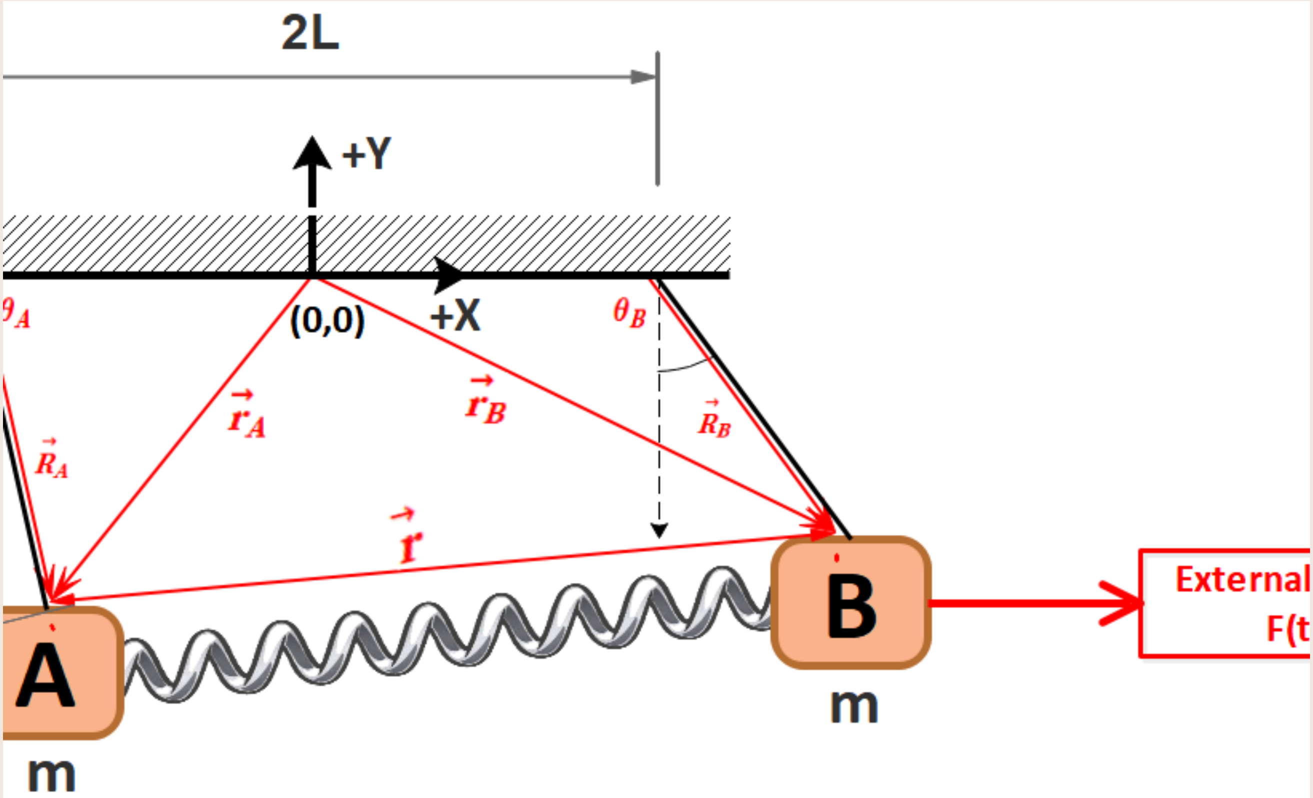
Investigation of motion of the given system taking variety of initial conditions.



All vector arrows are pointing for Center of Mass

In[3788]:=

§ Undamped without Driving Force -

Analysis of Dynamics

$$\vec{r}_A = L(\sin(\theta_A) - 1)\, i - L\cos(\theta_A)\, j$$
$$\vec{r}_B = L(\sin(\theta_B) + 1)\, i - L\cos(\theta_B)\, j$$
$$\vec{r} = (\vec{r}_A - \vec{r}_B) = L(2 + \sin(\theta_B) - \sin(\theta_A))\, i + L(\cos(\theta_A) - \cos(\theta_B))\, j$$
$$r = \left| \vec{r} \right|$$
$$\vec{F}_{\mathrm{KA}} = K(r - 2L)\,\hat{r} = K\left(1 - \frac{2L}{r}\right)\vec{r} = -\vec{F}_{\mathrm{KB}}$$

$$\vec{R}_A = \mathrm{L}\sin(\theta_A)\,\mathrm{i} - \mathrm{L}\cos(\theta_A)\,\mathrm{j}$$
$$\vec{R}_B = \mathrm{L}\sin(\theta_B)\,\mathrm{i} - \mathrm{L}\cos(\theta_B)\,\mathrm{j}$$
$$\vec{F}_A = \vec{T}_A + \mathrm{K}\left(1 - \frac{2L}{r}\right)\vec{r} - \mathrm{mg\,j}$$
$$\vec{F}_B = \vec{T}_B - \mathrm{K}\left(1 - \frac{2L}{r}\right)\vec{r} - \mathrm{mg\,j}$$
$$|\vec{\tau}_A| = |\vec{R}_A \times \vec{F}_A| = -\mathrm{mg\,L}\sin(\theta_A) + \mathrm{KL}^2\left(1 - \frac{2L}{r}\right)(2\cos(\theta_A) + \cos(\theta_A)\sin(\theta_B) - \cos(\theta_B)\sin(\theta_A))$$
$$\ddot{\theta}_A = -g/\mathrm{L}\sin(\theta_A) + \frac{K}{m}\left(1 - \frac{2L}{r}\right)(2\cos(\theta_A) - \sin(\theta_A - \theta_B))$$
$$|\vec{\tau}_B| = |\vec{R}_B \times \vec{F}_B| = -\mathrm{mg\,L}\sin(\theta_B) + \mathrm{KL}^2\left(1 - \frac{2L}{r}\right)(2\cos(\theta_B) + \cos(\theta_B)\sin(\theta_A) - \cos(\theta_A)\sin(\theta_B))$$
$$\ddot{\theta}_B = -g/\mathrm{L}\sin(\theta_B) + \frac{K}{m}\left(1 - \frac{2L}{r}\right)(-2\cos(\theta_B) + \sin(\theta_A - \theta_B))$$

Non-Dimensionalization

$$t \to \sqrt{\frac{L}{g}}\, t \quad , \quad r \to Lr$$

$$\ddot{\theta}_A = -\sin(\theta_A) + K\frac{L}{\mathrm{mg}}\left(1 - \frac{2}{r}\right)(2\cos(\theta_A) - \sin(\theta_A - \theta_B))$$

$$\ddot{\theta}_B = -\sin(\theta_B) + K\frac{L}{\mathrm{mg}}\left(1 - \frac{2}{r}\right)(-2\cos(\theta_B) + \sin(\theta_A - \theta_B))$$

Module for above Dynamic

In[3789]:=

```
Clear[ini, tf, k, c, nMax];
dynamics1[F_, x0_, tf_, k_, nMax_] :=
  Module[{h, datalist, prev, r1, r2, r3, r4, next},
   Clear[h, datalist, prev, r1, r2, r3, r4, next];
   h = (tf - x0[[1]]) / nMax // N;
   For[datalist = {x0},
    Length[datalist] ≤ nMax,
    AppendTo[datalist, next],
    prev = Last[datalist];
    r1 = Through[F @@ prev];
    r2 = Through[F @@ (prev + h/2 r1)];
    r3 = Through[F @@ (prev + h/2 r2)];
    r4 = Through[F @@ (prev + h r3)];
    next = prev + h/6 (r1 + 2 r2 + 2 r3 + r4);
   ];
   Return[datalist];
  ];

comX1[data_] := Module[{comx},
   Clear[comx];
   comx = Table[{data[[n, 1]], (Sin[data[[n, 2]]] + Sin[data[[n, 3]]])/2},
     {n, 1, Length[data]}];
   Return[comx];
  ];
comY1[data_] := Module[{comy},
   Clear[comy];
    comy = Table[{data[[n, 1]], 1/2 (- (Cos[data[[n, 2]]] + Cos[data[[n, 3]]]))},
     {n, 1, Length[data]}];
   Return[comy];
  ];
```

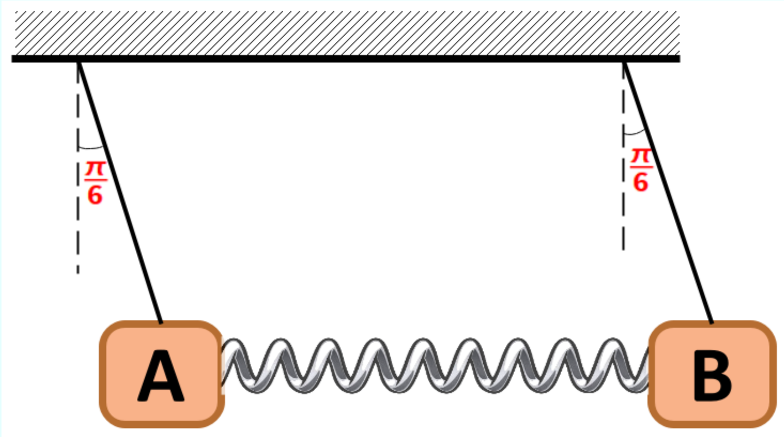
In[3793]:=
```
Clear[c, k];
r[θ1_, θ2_] = √((2 + Sin[θ2] - Sin[θ1])^2 + (Cos[θ1] - Cos[θ2])^2);
(* Distance between blocks *)
id[t_, θ1_, θ2_, θ1dot_, θ2dot_] = 1;
θ1dot[t_, θ1_, θ2_, θ1dot_, θ2dot_] = θ1dot;
θ2dot[t_, θ1_, θ2_, θ1dot_, θ2dot_] = θ2dot;
w1dot[t_, θ1_, θ2_, θ1dot_, θ2dot_] =
  c k (1 - 2/r[θ1, θ2]) (2 Cos[θ1] - Sin[θ1 - θ2]) - Sin[θ1];
w2dot[t_, θ1_, θ2_, θ1dot_, θ2dot_] =
  c k (1 - 2/r[θ1, θ2]) (-2 Cos[θ2] + Sin[θ1 - θ2]) - Sin[θ2];
```

In[3799]:=

In[3800]:=

Case 1 : Same Initial Angular Displacement

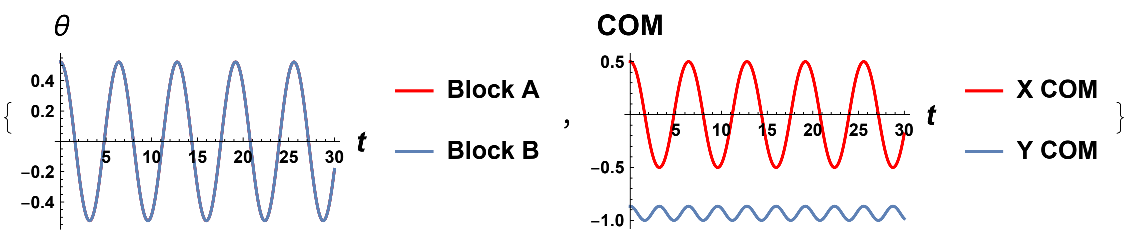
If both blocks initially displaced through same angle then both blocks should move identically as simple pendulum independet of spring.



In[3801]:=

```
ini = {0, π / 6, π / 6, 0, 0};    (* Initials: time, θ1, θ2, θ̇1, θ̇2 *)
tf = 30;
k = 2;                           (* Spring Constant *)
c = 1;                           (* L/mg *)
nMax = 1000;

data = dynamics1[{id, θ1dot, θ2dot, w1dot, w2dot}, ini, tf, k, nMax];

comx = comX1[data];
comy = comY1[data];

List[Show[ListPlot[data[[ ;; , {1, 2}]], Joined → True, PlotStyle → Red,
    AxesLabel → {Style[t, 15], Style[θ, 15]}, PlotLegends → {"Block A"}],
   ListPlot[data[[ ;; , {1, 3}]], Joined → True, PlotLegends → {"Block B"}]],
  Show[ListPlot[comx, Joined → True, PlotRange → All, PlotStyle → Red,
    AxesLabel → {Style[t, 15], Style[COM, 15]}, PlotLegends → {"X COM"}],
   ListPlot[comy, Joined → True, PlotLegends → {"Y COM"}], PlotRange → All]]
```

Out[3807]=



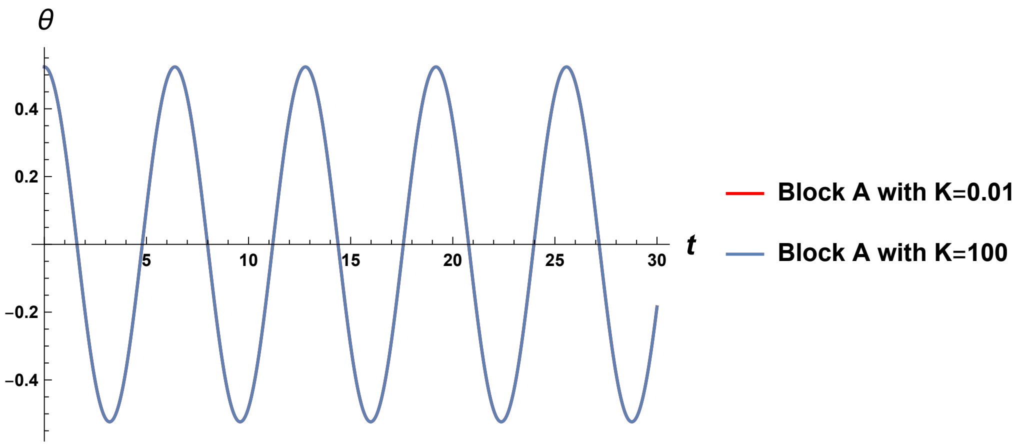
- *From graph it is clear that both blocks have identical angular displacement as both are overlapping.*

In[3808]:=

Now we will confirm independence of spring

In[3809]:=
```
dataK1 = dynamics1[{id, θ1dot, θ2dot, w1dot, w2dot}, ini, tf, 0.01, nMax];
(* K=0.01 *)
dataK2 = dynamics1[{id, θ1dot, θ2dot, w1dot, w2dot}, ini, tf, 100, nMax];
(* K=100 *)

Show[ListPlot[dataK1[[ ;; , {1, 2}]], Joined → True, PlotStyle → Red,
  AxesLabel → {Style[t, 15], Style[θ, 15]}, PlotLegends → {"Block A with K=0.01"}],
 ListPlot[dataK2[[ ;; , {1, 2}]], Joined → True,
  PlotLegends → {"Block A with K=100"}]]
```

Out[3811]=



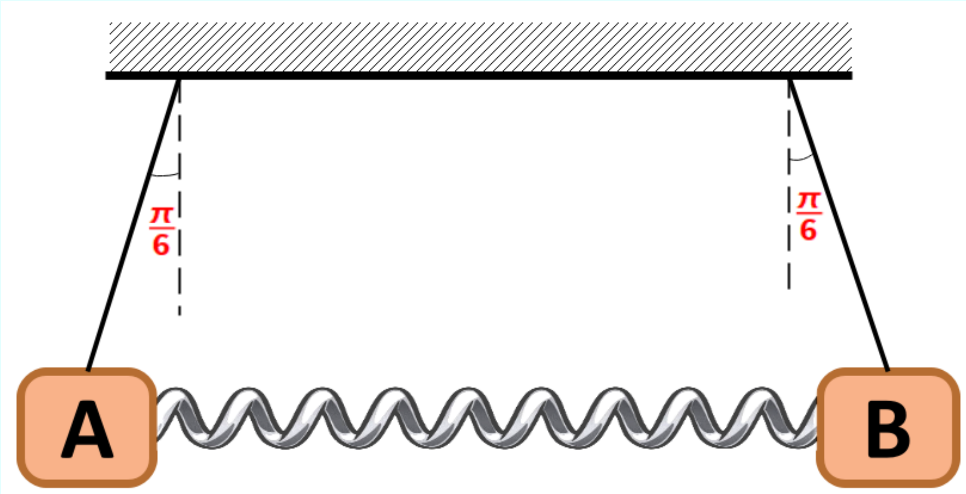
- *From above graph it is clear motion of blocks is independent of spring as angular displacement of both (k = 0.01 & k = 100) are overlapping.*

In[3812]:=

In[3813]:=

Case 2 : Equal and Opposite Angular Displacement

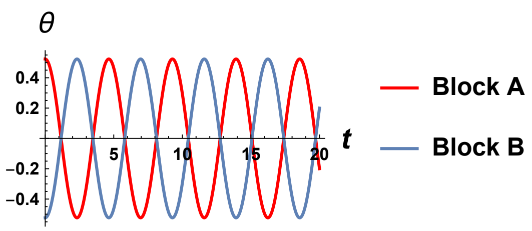
If both blocks initially displaced through same angle in opposite direction then system would be symmetrical and both blocks will reach their maxima and minima at same time with phase difference of π.



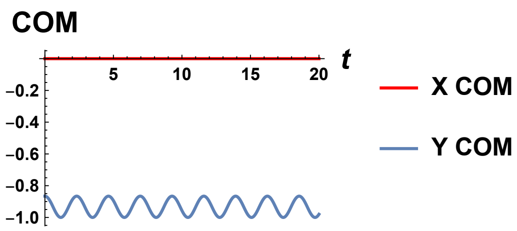
In[3814]:=

```
ini = {0, π / 6, -π / 6, 0, 0};   (* initials: time, θ1, θ2, θ̇1, θ̇2 *)
tf = 20;
k = 0.5;                         (* spring constant *)
c = 1;                           (* L/mg *)
nMax = 1000;

data = dynamics1[{id, θ1dot, θ2dot, w1dot, w2dot}, ini, tf, k, nMax];

comx = comX1[data];
comy = comY1[data];

List[Show[ListPlot[data[[ ;; , {1, 2}]], Joined → True, PlotStyle → Red,
    AxesLabel → {Style[t, 15], Style[θ, 15]}, PlotLegends → {"Block A"}],
   ListPlot[data[[ ;; , {1, 3}]], Joined → True, PlotLegends → {"Block B"}]],
  Show[ListPlot[comx, Joined → True, PlotRange → All, PlotStyle → Red,
    AxesLabel → {Style[t, 15], Style[COM, 15]}, PlotLegends → {"X COM"}],
   ListPlot[comy, Joined → True, PlotLegends → {"Y COM"}, PlotRange → All]]]
```

Out[3820]= {



,



}

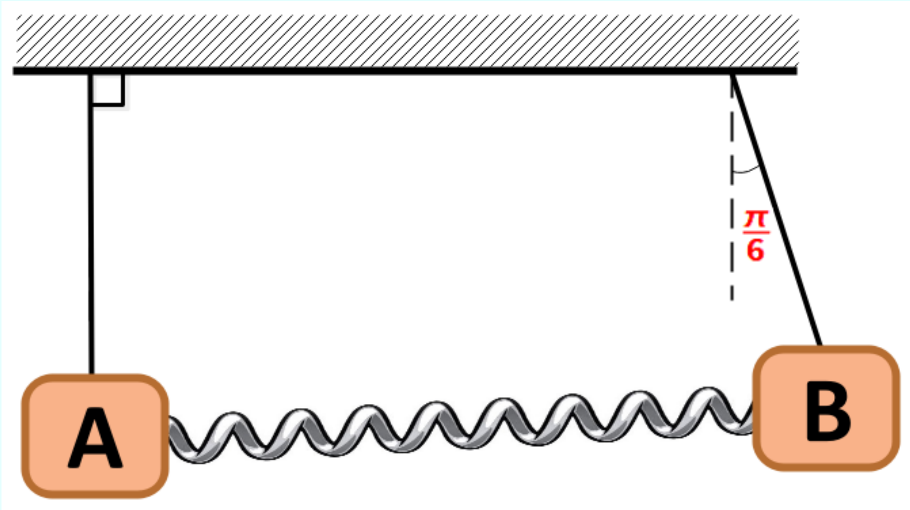
Observe in this case there is restoring spring force in addition to gravitational so time period should be lesser than previous case and which is *clearly justified by both curves.* T for previous case (> 6) and here (< 5).

In[3821]:=

In[3822]:=

Case 3 : Only one block is initially displaced

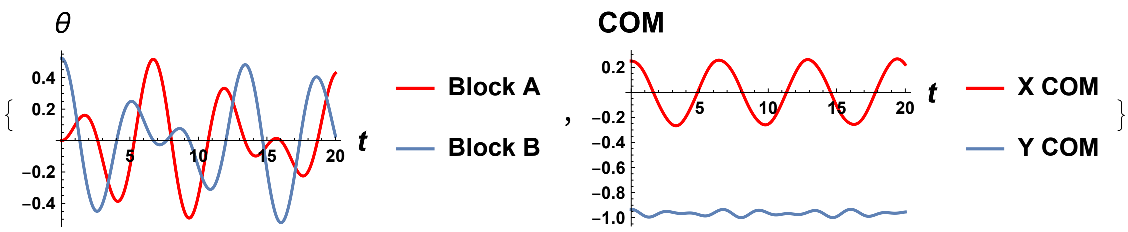
In this case Energy & Momentum of displaced block will transfer to other block as a result there will be decrement in amplitude of displaced block and simultaneously increment in other block and this pattern will repeat.



In[3823]:=

```
ini = {0, 0, π / 6, 0, 0};   (* initials: time, θ1, θ2, θ̇1, θ̇2 *)
tf = 20;
k = 0.5;                     (* spring constant *)
c = 1;                       (* L/mg *)
nMax = 1000;

data = dynamics1[{id, θ1dot, θ2dot, w1dot, w2dot}, ini, tf, k, nMax];

comx = comX1[data];
comy = comY1[data];

List[Show[ListPlot[data[[ ;; , {1, 2}]], Joined → True, PlotStyle → Red,
    AxesLabel → {Style[t, 15], Style[θ, 15]}, PlotLegends → {"Block A"}],
   ListPlot[data[[ ;; , {1, 3}]], Joined → True, PlotLegends → {"Block B"}]],
  Show[ListPlot[comx, Joined → True, PlotRange → All, PlotStyle → Red,
    AxesLabel → {Style[t, 15], Style[COM, 15]}, PlotLegends → {"X COM"}],
   ListPlot[comy, Joined → True, PlotLegends → {"Y COM"}], PlotRange → All]]
```

Out[3831]=



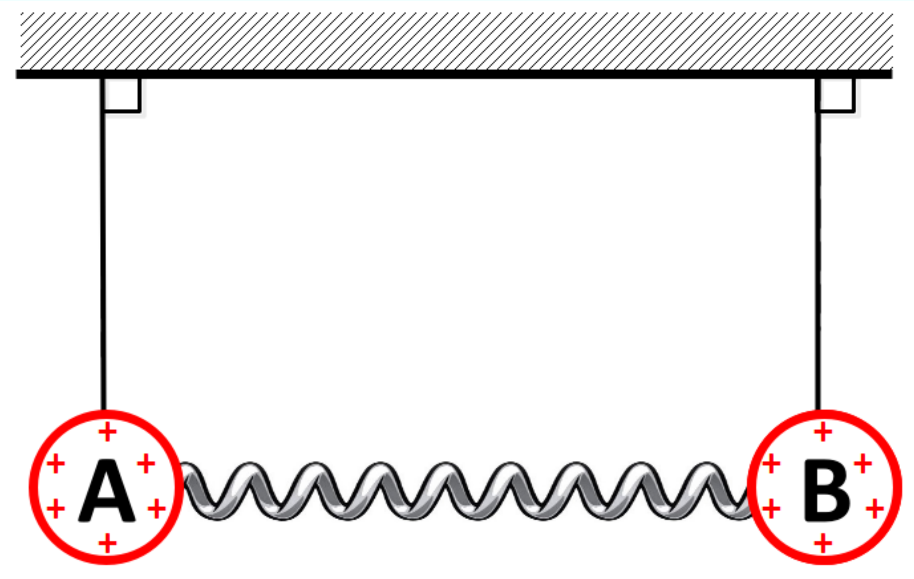
- *So, when one block achieves its maxima other achieves its minima.*

In[3832]:=

In[3833]:=

Case 4 : Some initial charges are given

In this case motion will initiate due to interaction between electric fields.
(Here electrodynamics part is neglected)



$$\vec{F}_A = \vec{T}_A + [\, K \left(1 - \frac{2L}{r}\right) - \frac{q_1 . q_2}{4\pi\epsilon r^3}]\, \vec{r} - mg\, j$$
$$\vec{F}_B = \vec{T}_B + [\, -K \left(1 - \frac{2L}{r}\right) + \frac{q_1 . q_2}{4\pi\epsilon r^3}]\, \vec{r} - mg\, j$$

$$|\vec{\tau}_A| = |\vec{R}_A \times \vec{F}_A| = -mg\,L \sin(\theta_A) + [\, KL^2 \left(1 - \frac{2L}{r}\right) - \frac{q_1 . q_2}{4\pi\epsilon r^3} L^2]\, (\, 2\cos(\theta_A) + \cos(\theta_A)\sin(\theta_B) - \cos(\theta_B) \sin(\theta_A)\,)$$
$$\ddot{\theta}_A = \frac{-g}{L} \sin(\theta_A) + [\, \frac{K}{m}\left(1 - \frac{2L}{r}\right) - \frac{q_1 . q_2}{4\pi\epsilon\, m\, r^3} L^2]\, (\, 2\cos(\theta_A) - \sin(\theta_A - \theta_B)\,)$$

$$|\vec{\tau}_B| = |\vec{R}_B \times \vec{F}_B| = -mg\,L \sin(\theta_B) + [\, KL^2 \left(1 - \frac{2L}{r}\right) - \frac{q_1 . q_2}{4\pi\epsilon r^3} L^2]\, (\, 2\cos(\theta_B) + \cos(\theta_B)\sin(\theta_A) - \cos(\theta_A) \sin(\theta_B)\,)$$
$$\ddot{\theta}_B = \frac{-g}{L} \sin(\theta_B) + [\, \frac{K}{m}\left(1 - \frac{2L}{r}\right) - \frac{q_1 . q_2}{4\pi\epsilon\, m\, r^3} L^2]\, (\, -2\cos(\theta_B) + \sin(\theta_A - \theta_B)\,)$$

Non-Dimensionalization

$$t \to \sqrt{\frac{L}{g}}\, t \qquad r \to Lr$$

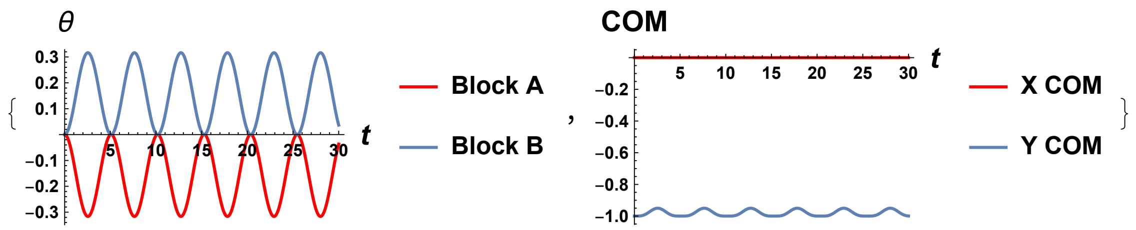
$$\ddot{\theta}_A = -\sin(\theta_A) + [\, K \frac{L}{mg}\left(1 - \frac{2}{r}\right) - \frac{1}{4\pi\epsilon L^3} \frac{L}{mg} \frac{q_1 . q_2}{r^3}]\, (\, 2\cos(\theta_A) - \sin(\theta_A - \theta_B)\,)$$

$$\ddot{\theta}_B = -\sin(\theta_B) + [\, K \frac{L}{mg}\left(1 - \frac{2}{r}\right) - \frac{1}{4\pi\epsilon L^3} \frac{L}{mg} \frac{q_1 . q_2}{r^3}]\, (\, -2\cos(\theta_B) + \sin(\theta_A - \theta_B)\,)$$

In[3834]:=
```
Clear[k, c, K, q1, q2];
wq1dot[t_, θ1_, θ2_, θ1dot_, θ2dot_] =
  (c k (1 - 2/r[θ1, θ2]) - K c (q1 q2)/(r[θ1, θ2])^3) (2 Cos[θ1] - Sin[θ1 - θ2]) - Sin[θ1];
wq2dot[t_, θ1_, θ2_, θ1dot_, θ2dot_] =
  (c k (1 - 2/r[θ1, θ2]) - K c (q1 q2)/(r[θ1, θ2])^3) (-2 Cos[θ2] + Sin[θ1 - θ2]) - Sin[θ2];
```

In[3837]:=
```
ini = {0, 0, 0, 0, 0};          (* initials: time, θ1, θ2, θ̇1, θ̇2 *)
tf = 30;
k = 0.1;                        (* spring constant *)
c = 1;                          (* L/mg *)
K = 10^9;                       (* K ∝ 1/(4πϵ) 1/L^3 *)
q1 = 10^-5;
q2 = 10^-4;
nMax = 1000;
data = dynamics1[{id, θ1dot, θ2dot, wq1dot, wq2dot}, ini, tf, k, nMax];

comx = comX1[data];
comy = comY1[data];

List[Show[ListPlot[data[[ ;; , {1, 2}]], Joined → True, PlotStyle → Red,
   AxesLabel → {Style[t, 15], Style[θ, 15]}, PlotLegends → {"Block A"}], ListPlot[
   data[[ ;; , {1, 3}]], Joined → True, PlotLegends → {"Block B"}], PlotRange -> All],
  Show[ListPlot[comx, Joined → True, PlotRange → All, PlotStyle → Red,
   AxesLabel → {Style[t, 15], Style[COM, 15]}, PlotLegends → {"X COM"}],
  ListPlot[comy, Joined → True, PlotLegends → {"Y COM"}], PlotRange → All]]
```

Out[3847]=



- ***(net F_{ext} = 0, x direction) and force on the system is symmetrical in x direction, so there is no displacement of COM in x direction***

  Here equilibrium of system will be at ($\theta_A = -\theta$ & $\theta_B = +\theta$)

To find equilibrium position (± θ) for blocks we have to solve couple of equations by applying condition (net F_{ext} = 0).

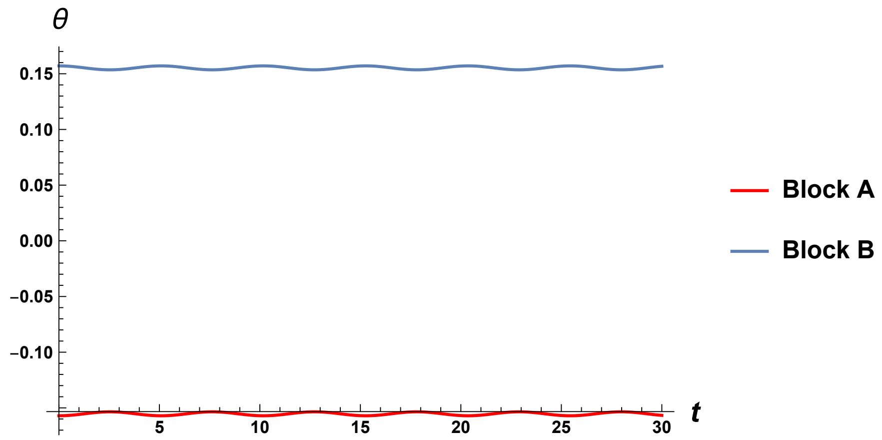
But, Observe that,
if we start with equilibrium condition then there will be no oscillation.
So, by manipulation of initial parameters we can find equilibrium position (± θ) for the system with dealing with equations.

And we have found out equilibrium position (± θ) to be approx. π / 20

In[3848]:=

```
ini = {0, -π / 20, π / 20, 0, 0};
data = dynamics1[{id, θ1dot, θ2dot, wq1dot, wq2dot}, ini, tf, k, nMax];
Show[ListPlot[data[[ ;; , {1, 2}]], Joined → True, PlotStyle → Red,
   AxesLabel → {Style[t, 15], Style[θ, 15]}, PlotLegends → {"Block A"}],
  ListPlot[data[[ ;; , {1, 3}]], Joined → True, PlotLegends → {"Block B"}],
  PlotRange -> All]
```

Out[3850]=



- *It is clear from above plot that equilibrium of system is approx. at* $\boldsymbol{\theta}_A = -\boldsymbol{\pi}/20$ & $\boldsymbol{\theta}_B = \boldsymbol{\pi}/20$.

In[3851]:=

§ Damped without Driving Force -

Analysis of Dynamics

$\vec{F}_A = \vec{T}_A + \mathrm{K}\left(1 - \frac{2L}{r}\right)\vec{r}$ - mg j $- b\,\vec{V}_A$

$\vec{F}_B = \vec{T}_B - \mathrm{K}\left(1 - \frac{2L}{r}\right)\vec{r}$ - mg j $- b\,\vec{V}_B$

$|\vec{\tau}_A| = |\vec{R}_A \times \vec{F}_A| = -\mathrm{mg\,L}\sin(\theta_A) + \mathrm{KL}^2\left(1 - \frac{2L}{r}\right)(\,2\cos(\theta_A) + \cos(\theta_A)\sin(\theta_B) - \cos(\theta_B)\sin(\theta_A)\,) - b\,\mathrm{LV}_B$

$$\ddot{\theta}_A = \frac{-g}{L}\sin(\theta_A) + \frac{K}{m}\left(1 - \frac{2L}{r}\right)(\,2\cos(\theta_A) - \sin(\theta_A - \theta_B)\,) - \frac{b\,V_A}{m\,L}$$

$|\vec{\tau}_B| = |\vec{R}_B \times \vec{F}_B| = -\mathrm{mg\,L}\sin(\theta_B) + [\,\mathrm{KL}^2\left(1 - \frac{2L}{r}\right) - b\,\frac{|\vec{V}_B|}{r}\,\mathrm{Sign}(V_{B_x})\,L^2\,]\,(\,2\cos(\theta_B) + \cos(\theta_B)\sin(\theta_A) - \cos(\theta_A)\sin(\theta_B)\,)$

$$\ddot{\theta}_B = \frac{-g}{L}\sin(\theta_B) + \frac{K}{m}\left(1 - \frac{2L}{r}\right)(\,-2\cos(\theta_B) + \sin(\theta_A - \theta_B)\,) - \frac{b\,V_B}{m\,L}$$

Non-Dimensionalization

$$t \to \sqrt{\frac{L}{g}}\,t \;,\quad r \to Lr \;,\quad V_A \to \sqrt{g\,L}\,V_A \;,\quad V_B \to \sqrt{g\,L}\,V_B$$

$$\ddot{\theta}_A = -\sin(\theta_A) + K\,\frac{L}{\mathrm{mg}}\left(1 - \frac{2}{r}\right)(\,2\cos(\theta_A) - \sin(\theta_A - \theta_B)\,) - \frac{b}{m}\sqrt{\frac{L}{g}}\,V_A$$

$$\ddot{\theta}_B = -\sin(\theta_B) + [\,K\,\frac{L}{\mathrm{mg}}\left(1 - \frac{2}{r}\right)(\,-2\cos(\theta_B) + \sin(\theta_A - \theta_B)\,) - \frac{b}{m}\sqrt{\frac{L}{g}}\,V_A$$

In[3852]:=

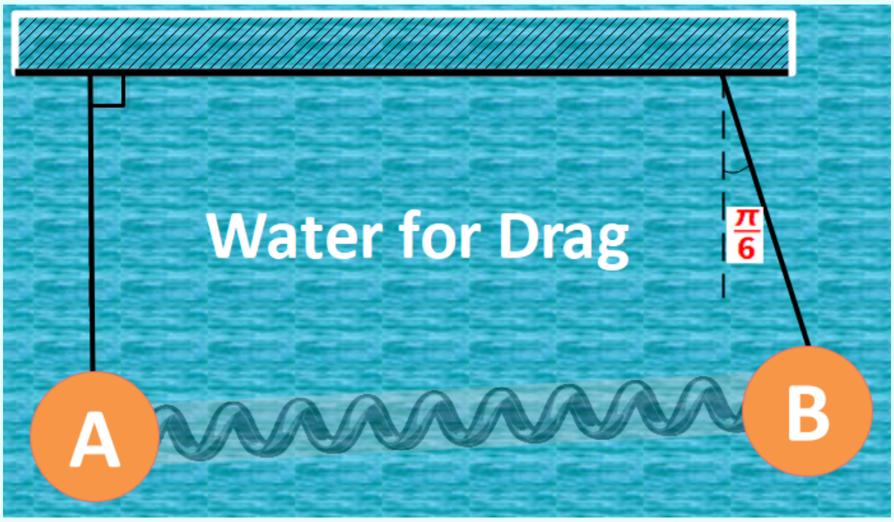
In[3853]:=

Case 5 : Damping due to Water drag.

In this case amplitude of blocks will decay due to Water drag opposite to velocity of blocks.

$$\vec{F}_{\text{drag}} = -6\,\pi\,\eta\,R\,\vec{v}$$

(Neglecting Buoyancy)



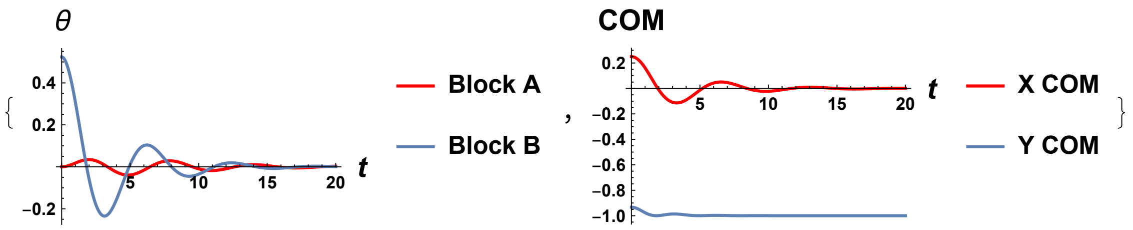
In[3854]:=

```
Clear[c, k, b];
wd1dot[t_, θ1_, θ2_, θ1dot_, θ2dot_] = c k (1 - 2/r[θ1, θ2]) (2 Cos[θ1] - Sin[θ1 - θ2]) -
   Sin[θ1] - b θ1dot[t, θ1, θ2, θ1dot, θ2dot] ;
wd2dot[t_, θ1_, θ2_, θ1dot_, θ2dot_] = c k (1 - 2/r[θ1, θ2]) (-2 Cos[θ2] + Sin[θ1 - θ2]) -
   Sin[θ2] - b θ2dot[t, θ1, θ2, θ1dot, θ2dot];
```

In[3857]:=

```
ini = {0, 0, π / 6, 0, 0};        (* initials: time, θ1, θ2, θ̇1, θ̇2 *)
tf = 20;
k = 0.1;                          (* spring constant *)
c = 1;                            (* L/mg *)
b = 0.5;                          (* b ∝ 6πηR *)
nMax = 1000;

data = dynamics1[{id, θ1dot, θ2dot, wd1dot, wd2dot}, ini, tf, k, nMax];

comx = comX1[data];  comy = comY1[data];

List[Show[ListPlot[data[[ ;; , {1, 2}]], Joined → True, PlotStyle → Red,
    AxesLabel → {Style[t, 15], Style[θ, 15]}, PlotLegends → {"Block A"}],
   ListPlot[data[[ ;; , {1, 3}]], Joined → True, PlotLegends → {"Block B"},
    PlotRange → All], PlotRange -> All],
  Show[ListPlot[comx, Joined → True, PlotRange → All, PlotStyle → Red,
    AxesLabel → {Style[t, 15], Style[COM, 15]}, PlotLegends → {"X COM"}], ListPlot[
    comy, Joined → True, PlotLegends → {"Y COM"}, PlotRange → All], PlotRange → All]]
```

Out[3865]=

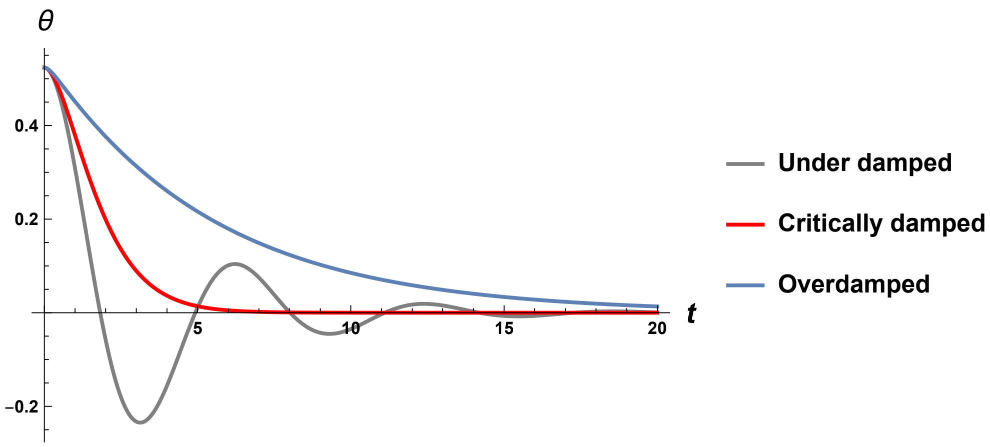


- *Critical damping provides the fastest approach to zero amplitude for in damped oscillation (Critical damping constant is approx. b=2)*
  On further increasing b it becomes overdamped, which is slowest approach to zero.

In[3866]:=

```
b = 2;
data1 = dynamics1[{id, θ1dot, θ2dot, wd1dot, wd2dot}, ini, tf, k, nMax];

b = 6;
 data2 = dynamics1[{id, θ1dot, θ2dot, wd1dot, wd2dot}, ini, tf, k, nMax];

Show[ListPlot[data[[ ;; , {1, 3}]], Joined → True, PlotStyle → {Thick, Gray},
   AxesLabel → {Style[t, 15], Style[θ, 15]}, PlotLegends → {"Under damped"},
   PlotRange → All], ListPlot[data1[[ ;; , {1, 3}]], Joined → True,
   PlotStyle → {Thick, Red}, AxesLabel → {Style[t, 15], Style[θ, 15]},
   PlotLegends → {"Critically damped"}, PlotRange → All],
  ListPlot[data2[[ ;; , {1, 3}]], Joined → True, PlotLegends → {"Overdamped"},
   PlotStyle → Thick, PlotRange → All], PlotRange → All]
```

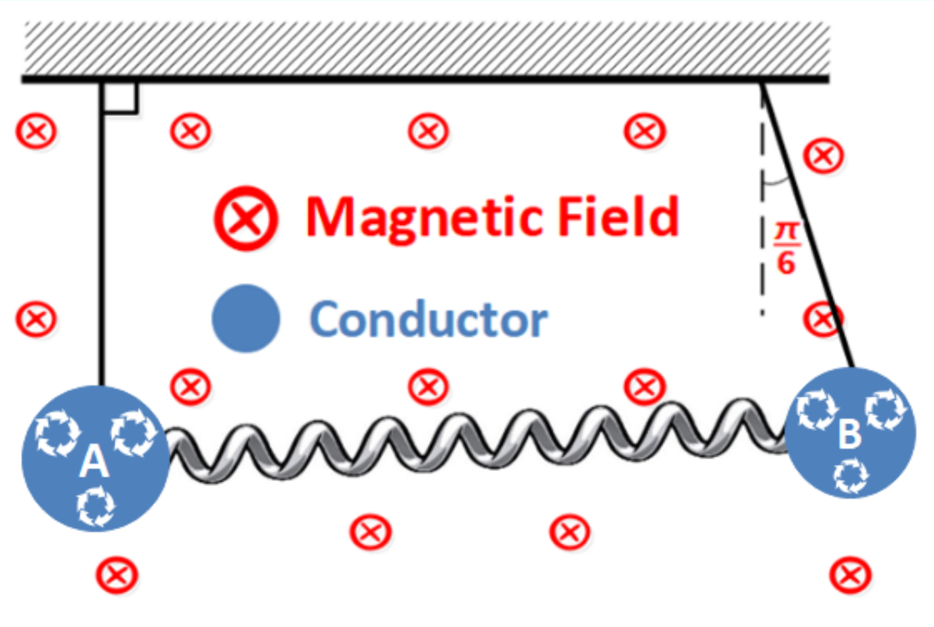
Out[3870]=



In[3871]:=

Case 6 : Damping due to Eddy currents.

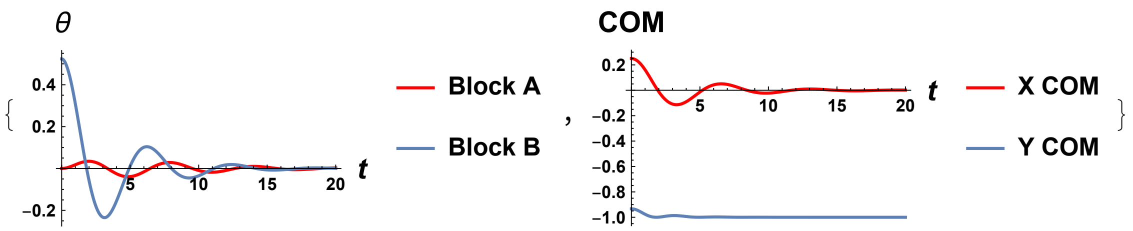
When a conductive material is subjected to a time-varying magnetic flux, eddy currents are generated in the conductor. These eddy currents circulate inside the conductor generating a magnetic field of opposite polarity as the applied magnetic field. The interaction of the two magnetic fields causes a force that resists the change in magnetic flux. However, due to the internal resistance of the conductive material, the eddy currents will be dissipated into heat and the motion will die out. *As the eddy currents are dissipated,energy is removed from the system,thus producing a damping effect.*



In[3872]:=

```
ini = {0, 0, π / 6, 0, 0};        (* initials: time, θ1, θ2, θ̇1, θ̇2 *)
tf = 20;
k = 0.1;                          (* spring constant *)
c = 1;                            (* L/mg *)
b = 0.5;
(* b ∝ coefficient of damping (resistance of material) *)
nMax = 1000;

data = dynamics1[{id, θ1dot, θ2dot, wd1dot, wd2dot}, ini, tf, k, nMax];

comx = comX1[data];  comy = comY1[data];

List[Show[ListPlot[data[[ ;; , {1, 2}]], Joined → True, PlotStyle → Red,
   AxesLabel → {Style[t, 15], Style[θ, 15]}, PlotLegends → {"Block A"}],
  ListPlot[data[[ ;; , {1, 3}]], Joined → True, PlotLegends → {"Block B"},
   PlotRange → All], PlotRange -> All],
 Show[ListPlot[comx, Joined → True, PlotRange → All, PlotStyle → Red,
   AxesLabel → {Style[t, 15], Style[COM, 15]}, PlotLegends → {"X COM"}], ListPlot[
   comy, Joined → True, PlotLegends → {"Y COM"}, PlotRange → All], PlotRange → All]]
```

Out[3878]=



In[3879]:=

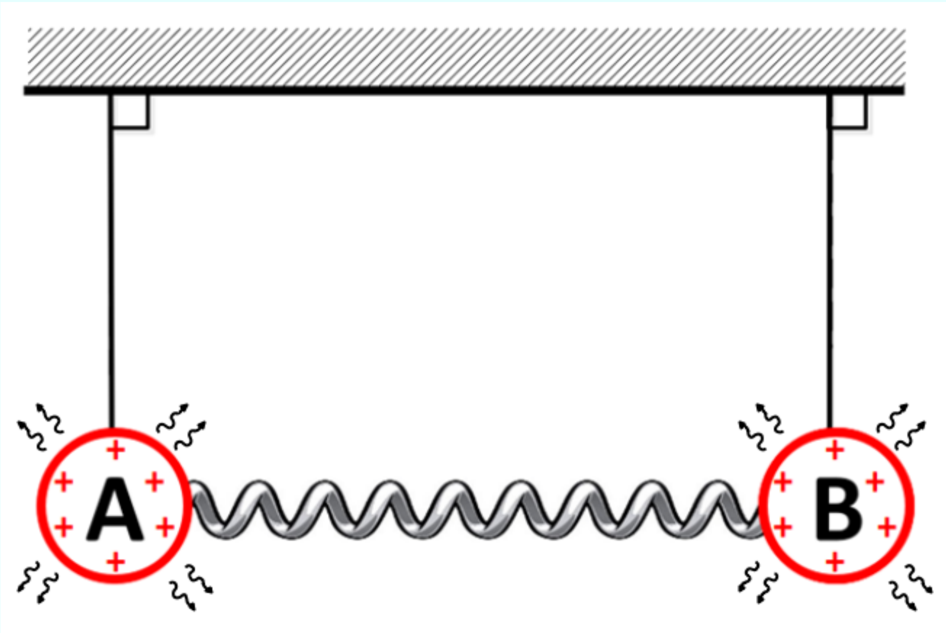
Case 7 : Radiation Damping due to accelerating charge

In radiation damping, energy of accelerating charges such as electrons is converted to ElectroMagnetic energy .

damping term is given by Larmor radiation rate $\propto \gamma\left(q^2 a^2\right)$

$$F_{\text{damping}} = \frac{\gamma a^2}{v}, \text{ where } \gamma \propto \text{charge}^2$$

Click here → * reference at end of notebook

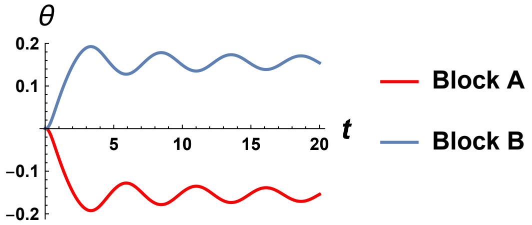


In[3880]:=
```
clear[c, k, K, γ, q1, q2];

ini = {0, 0, 0, 0.01, -0.01};       (* initials: time, θ1, θ2, θ̇1, θ̇2 *)
tf = 20;
k = 0.1;                            (* spring constant *)
c = 1;                              (* L/mg *)
γ = 200;                            (* b ∝ damping coefficient *)
q1 = 10^-5;
q2 = 10^-4;
K = 10^9;
nMax = 1000;

acc = {{w1dot @@ ini, w2dot @@ ini}};
wr1dot[t_, θ1_, θ2_, θ1dot_, θ2dot_] =
  (c k (1 - 2/r[θ1, θ2]) - K c (q1 q2)/(r[θ1, θ2])^3 - γ * 1/r[θ1, θ2]
      ((θ1dot[t, θ1, θ2, θ1dot, θ2dot])^3 + (Last[acc][[1]])^2/θ1dot[t, θ1, θ2, θ1dot, θ2dot]))
    (2 Cos[θ1] - Sin[θ1 - θ2]) - Sin[θ1];
wr2dot[t_, θ1_, θ2_, θ1dot_, θ2dot_] =
  (c k (1 - 2/r[θ1, θ2]) - K c (q1 q2)/(r[θ1, θ2])^3 + γ * 1/r[θ1, θ2]
      ((θ2dot[t, θ1, θ2, θ1dot, θ2dot])^3 + (Last[acc][[2]])^2/θ2dot[t, θ1, θ2, θ1dot, θ2dot]))
    (-2 Cos[θ2] + Sin[θ1 - θ2]) - Sin[θ2];
```

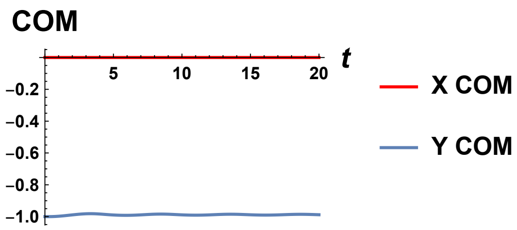
In[3890]:=

```
F = {id, θ1dot, θ2dot, wr1dot, wr2dot};
For[data = {ini}; h = tf/nMax,
  Length[data] ≤ nMax,
  AppendTo[data, next]; AppendTo[acc, nextacc],
  prev = Last[data];
  r1 = Through[F @@ prev];
  r2 = Through[F @@ (prev + h/2 r1)];
  r3 = Through[F @@ (prev + h/2 r2)];
  r4 = Through[F @@ (prev + h r3)];
  next = prev + h/6 (r1 + 2 r2 + 2 r3 + r4);
  nextacc = Through[{wr1dot, wr2dot} @@ prev];
 ];

comx = comX1[data]; comy = comY1[data];

List[Show[ListPlot[data[[ ;; , {1, 2}]], Joined → True, PlotStyle → Red,
   AxesLabel → {Style[t, 15], Style[θ, 15]}, PlotLegends → {"Block A"}],
  ListPlot[data[[ ;; , {1, 3}]], Joined → True, PlotLegends → {"Block B"},
   PlotRange → All], PlotRange -> All],
 Show[ListPlot[comx, Joined → True, PlotRange → All, PlotStyle → Red,
   AxesLabel → {Style[t, 15], Style[COM, 15]}, PlotLegends → {"X COM"}], ListPlot[
   comy, Joined → True, PlotLegends → {"Y COM"}, PlotRange → All], PlotRange → All]]
```

Out[3893]= {



,



}

In[3894]:=

§ Undamped with Driving Force -

Analysis of Dynamics

$$\vec{F}_A = \vec{T}_A + K\left(1 - \frac{2L}{r}\right)\vec{r} - mg\, j$$

$$\vec{F}_B = \vec{T}_B - K\left(1 - \frac{2L}{r}\right)\vec{r} - mg\, j + F(t)\, \hat{i}$$

$|\vec{\tau_A}| = |\vec{R}_A \times \vec{F}_A| = -mg\,L\sin(\theta_A) + KL^2\left(1 - \frac{2L}{r}\right)(2\cos(\theta_A) + \cos(\theta_A)\sin(\theta_B) - \cos(\theta_B)\sin(\theta_A))$

$$\ddot{\theta}_A = -g/L\sin(\theta_A) + \frac{K}{m}\left(1 - \frac{2L}{r}\right)(2\cos(\theta_A) - \sin(\theta_A - \theta_B))$$

$|\vec{\tau_B}| = |\vec{R}_B \times \vec{F}_B| = -mg\,L\sin(\theta_B) + F(t)\,L\cos\theta_B + KL^2\left(1 - \frac{2L}{r}\right)(2\cos(\theta_B) + \cos(\theta_B)\sin(\theta_A) - \cos(\theta_A)\sin(\theta_B))$

$$\ddot{\theta}_B = -g/L\sin(\theta_B) + \frac{(F(t)\cos\theta_B)}{mL} + \frac{K}{m}\left(1 - \frac{2L}{r}\right)(-2\cos(\theta_B) + \sin(\theta_A - \theta_B))$$

Non-Dimensionalization

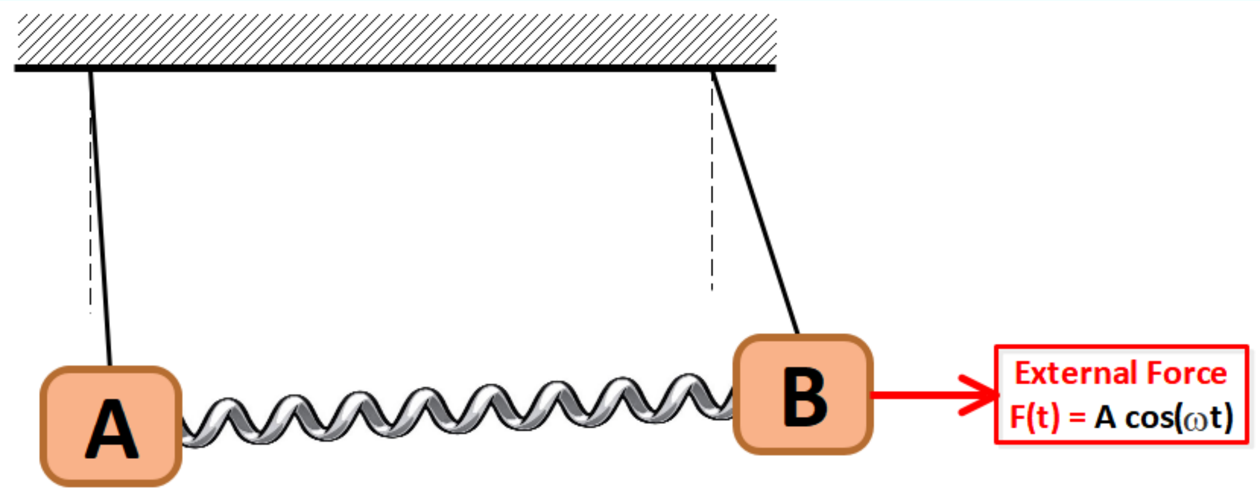
$$t \to \sqrt{\frac{L}{g}}\, t \qquad r \to Lr \qquad F(t) \;\blacksquare\; mg\,F(t)$$

$$\ddot{\theta}_A = -\sin(\theta_A) + K\frac{L}{mg}\left(1 - \frac{2}{r}\right)(2\cos(\theta_A) - \sin(\theta_A - \theta_B))$$

$$\ddot{\theta}_B = -\sin(\theta_B) + F(t)\cos\theta_B + K\frac{L}{mg}\left(1 - \frac{2}{r}\right)(-2\cos(\theta_B) + \sin(\theta_A - \theta_B))$$

In[3895]:=

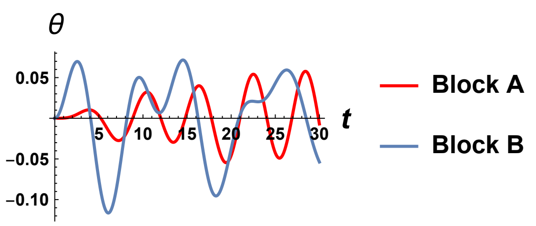
Case 8 : Sinusoidal Driving Force

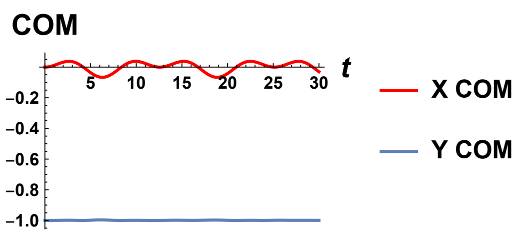


In[3896]:=

```
Clear[c, k, ω];
Fex[t_] = 0.05 * Cos[ω t];
wf1dot[t_, θ1_, θ2_, θ1dot_, θ2dot_] =
  c k (1 - 2/r[θ1, θ2]) (2 Cos[θ1] - Sin[θ1 - θ2]) - Sin[θ1];
wf2dot[t_, θ1_, θ2_, θ1dot_, θ2dot_] =
  c k (1 - 2/r[θ1, θ2]) (-2 Cos[θ2] + Sin[θ1 - θ2]) - Sin[θ2] + Fex[t] Cos[θ2];
```

In[3900]:=
```
ini = {0, 0, 0, 0, 0};        (* initials: time, θ1, θ2, θ̇1, θ̇2 *)
tf = 30;
k = 0.1;                      (* spring constant *)
c = 1;                        (* L/mg *)
ω = 0.5;                      (* Driving Force frequency *)

nMax = 1000;

data = dynamics1[{id, θ1dot, θ2dot, wf1dot, wf2dot}, ini, tf, k, nMax];

 comx = comX1[data];
comy = comY1[data];

List[Show[ListPlot[data[[ ;; , {1, 2}]], Joined → True, PlotStyle → Red,
   AxesLabel → {Style[t, 15], Style[θ, 15]}, PlotLegends → {"Block A"}], ListPlot[
   data[[ ;; , {1, 3}]], Joined → True, PlotLegends → {"Block B"}], PlotRange -> All],
 Show[ListPlot[comx, Joined → True, PlotRange → All, PlotStyle → Red,
   AxesLabel → {Style[t, 15], Style[COM, 15]}, PlotLegends → {"X COM"}],
  ListPlot[comy, Joined → True, PlotLegends → {"Y COM"}], PlotRange → All]]
```

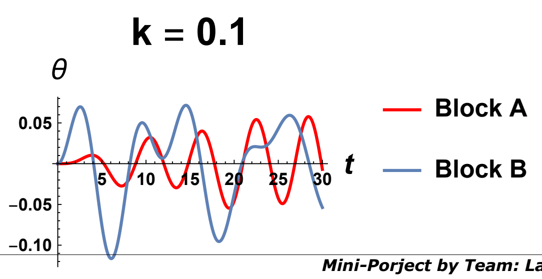
Out[3907]= {

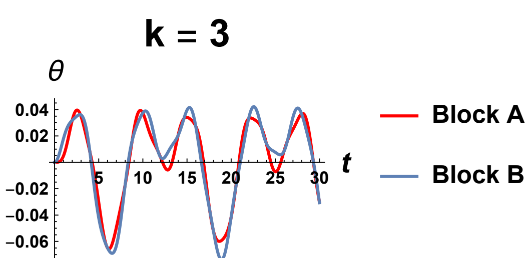
, 

}

Observe that if we increase spring constant then Block A will able to follow Block B.

In[3908]:=
```
k = 3;
datak2 = dynamics1[{id, θ1dot, θ2dot, wf1dot, wf2dot}, ini, tf, k, nMax];

List[Show[ListPlot[data[[ ;; , {1, 2}]], Joined → True, PlotStyle → Red,
   AxesLabel → {Style[t, 15], Style[θ, 15]}, PlotLegends → {"Block A"}],
  ListPlot[data[[ ;; , {1, 3}]], Joined → True, PlotLegends → {"Block B"}],
  PlotRange -> All, PlotLabel → Style["k = 0.1", 20]],
 Show[ListPlot[datak2[[ ;; , {1, 2}]], Joined → True, PlotStyle → Red,
   AxesLabel → {Style[t, 15], Style[θ, 15]}, PlotLegends → {"Block A"}],
  ListPlot[datak2[[ ;; , {1, 3}]], Joined → True, PlotLegends → {"Block B"}],
  PlotRange -> All, PlotLabel → Style["k = 3", 20]]]
```

Out[3910]= {

, 

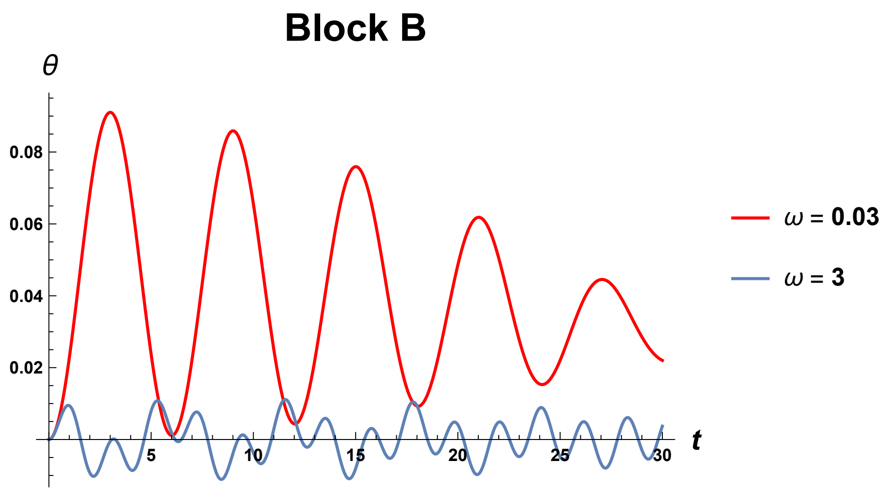
}

- *From above graph clearly our observation is justified.*

Observe if we increase frequency of external force then it will be difficult for Block B to follow its amplitude.

In[3911]:=
```
k = 0.1;
ω = 0.03;
dataw1 = dynamics1[{id, θ1dot, θ2dot, wf1dot, wf2dot}, ini, tf, k, nMax];

ω = 3;
dataw2 = dynamics1[{id, θ1dot, θ2dot, wf1dot, wf2dot}, ini, tf, k, nMax];

Show[ListPlot[dataw1[[ ;; , {1, 3}]], Joined → True, PlotStyle → Red,
  AxesLabel → {Style[t, 15], Style[θ, 15]}, PlotLegends → {"ω = 0.03"}],
 ListPlot[dataw2[[ ;; , {1, 3}]], Joined → True, PlotLegends → {"ω = 3"}],
 PlotRange -> All, PlotLabel → Style["Block B", 20]]
```

Out[3916]=

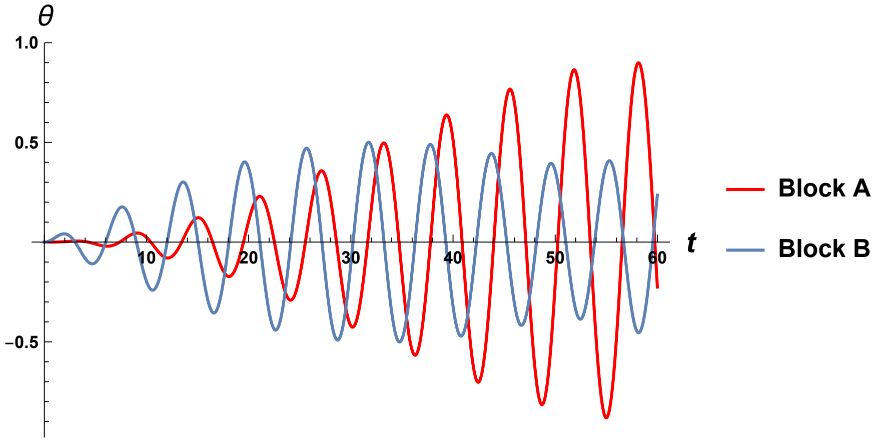


- *From above graph clearly there is significant difference in amplitude.*

Form Free oscillation we observed natural frequency of system was approx $\frac{2\pi}{6}$ so, if driving force will have near that frequency the system will achieve resonance.

In[3917]:=
```
tf = 60;
k = 0.1;
ω = 2 π / 6;
data = dynamics1[{id, θ1dot, θ2dot, wf1dot, wf2dot}, ini, tf, k, nMax];
Show[ListPlot[data[[ ;; , {1, 2}]], Joined → True, PlotStyle → Red,
  AxesLabel → {Style[t, 15], Style[θ, 15]}, PlotLegends → {"Block A"}],
 ListPlot[data[[ ;; , {1, 3}]], Joined → True, PlotLegends → {"Block B"}],
 PlotRange -> All]
```

Out[3921]=

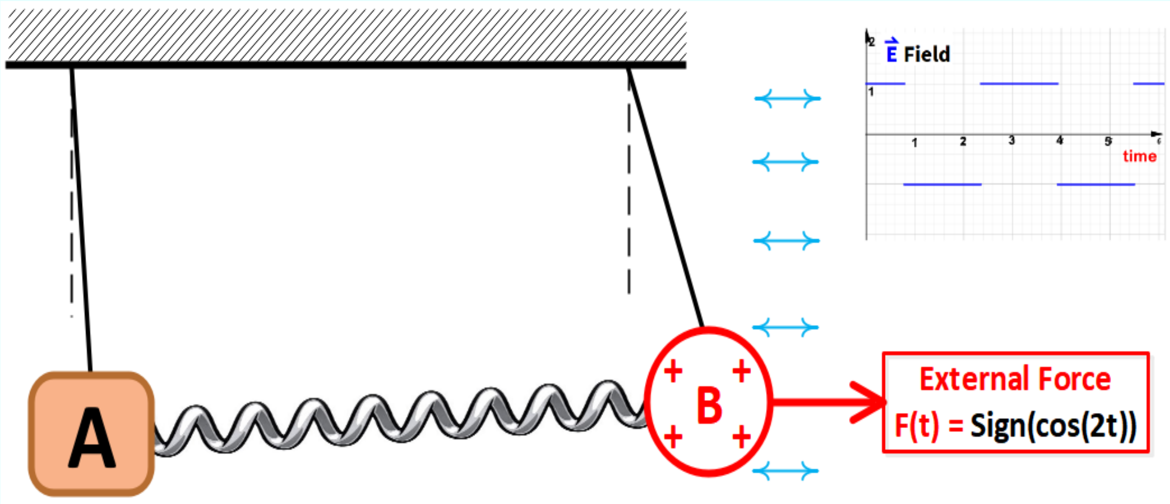


See External force with amplitude just of 0.05 unit make block A to oscillate with such a huge amplitude.
It’s in RESONANCE !!

In[3922]:=

Case 9 : Impulsive Electric Force

Here Driver is electric force by impulsive electric field.



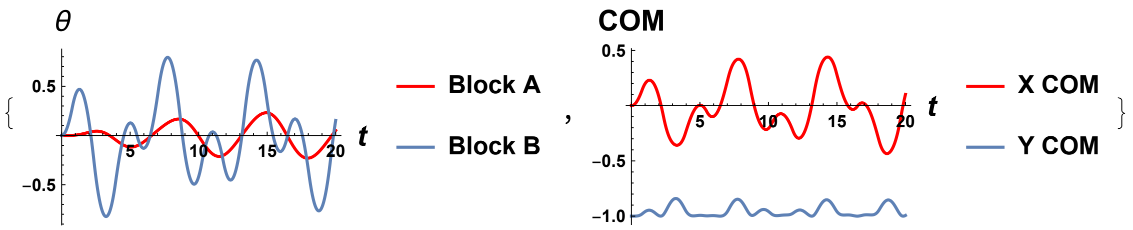
In[3923]:=

```
Clear[c, k, q];
Fex1[t_] = q Sign[Cos[2 t]];
wf2dot[t_, θ1_, θ2_, θ1dot_, θ2dot_] =
  c k (1 - 2/r[θ1, θ2]) (-2 Cos[θ2] + Sin[θ1 - θ2]) - Sin[θ2] + Fex1[t] Cos[θ2];
```

In[3926]:=

```
ini = {0, 0, 0, 0, 0};        (* initial time, θ1, θ2, θ̇1, θ̇2 *)
tf = 20;
k = 0.1;                       (* stiffness for spring *)
c = 1;                        (* L/mg *)
ω = 2;
q = 1;

nMax = 1000;

data = dynamics1[{id, θ1dot, θ2dot, wf1dot, wf2dot}, ini, tf, k, nMax];

 comx = comX1[data];
comy = comY1[data];

List[Show[ListPlot[data[[ ;; , {1, 2}]], Joined → True, PlotStyle → Red,
   AxesLabel → {Style[t, 15], Style[θ, 15]}, PlotLegends → {"Block A"}], ListPlot[
   data[[ ;; , {1, 3}]], Joined → True, PlotLegends → {"Block B"}], PlotRange -> All],
 Show[ListPlot[comx, Joined → True, PlotRange → All, PlotStyle → Red,
   AxesLabel → {Style[t, 15], Style[COM, 15]}, PlotLegends → {"X COM"}],
  ListPlot[comy, Joined → True, PlotLegends → {"Y COM"}], PlotRange → All]]
```

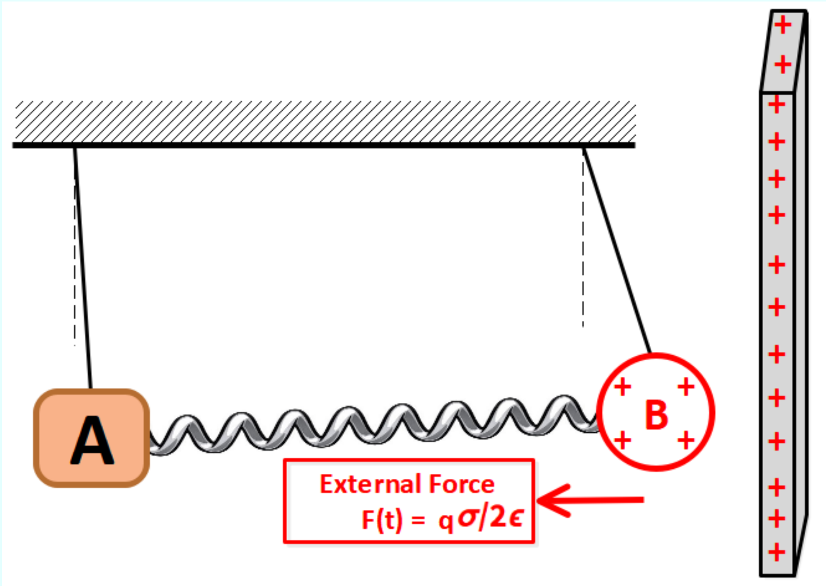
Out[3936]=



In[3937]:=

Case 10 : External Spring Force

Assuming wall is unformally charge and distribution remain unchanged and has very large dimensions as compared to blocks.



Here Electric Field is constant(acc to assumption described above) $E = \sigma / 2\,\epsilon$**. where** σ **is surface charge density.**

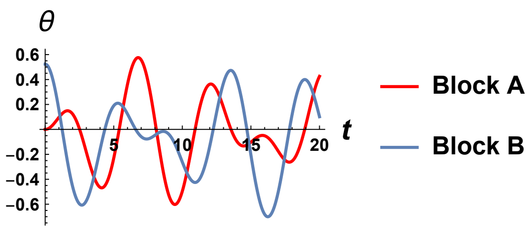
In[3938]:=

```
Clear[c, k, e, q, σ];
Fex2[t_] = e q σ;          (* e ∝ 1/ϵ *) (* σ is surface charge density *)
wef2dot[t_, θ1_, θ2_, θ1dot_, θ2dot_] =
  c k (1 - 2/r[θ1, θ2]) (-2 Cos[θ2] + Sin[θ1 - θ2]) - Sin[θ2] - e c q σ;
```

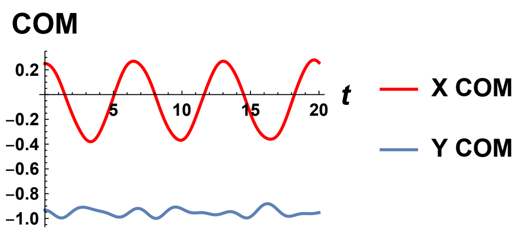
In[3941]:=

```
ini = {0, 0, π / 6, 0, 0};     (* initials: time, θ1, θ2, θ̇1, θ̇2 *)
tf = 20;
k = 0.5;                       (* spring constant *)
c = 1;                         (* L/mg *)
e = 10^9;
q = 10^-5;
σ = 10^-5;

nMax = 1000;

data = dynamics1[{id, θ1dot, θ2dot, wf1dot, wef2dot}, ini, tf, k, nMax];

 comx = comX1[data];
comy = comY1[data];

List[Show[ListPlot[data[[ ;; , {1, 2}]], Joined → True, PlotStyle → Red,
   AxesLabel → {Style[t, 15], Style[θ, 15]}, PlotLegends → {"Block A"}], ListPlot[
   data[[ ;; , {1, 3}]], Joined → True, PlotLegends → {"Block B"}], PlotRange -> All],
  Show[ListPlot[comx, Joined → True, PlotRange → All, PlotStyle → Red,
   AxesLabel → {Style[t, 15], Style[COM, 15]}, PlotLegends → {"X COM"}],
  ListPlot[comy, Joined → True, PlotLegends → {"Y COM"}], PlotRange → All]]
```

Out[3949]= {



,



}

In[3950]:=

§ Damped with Driving Force -

Analysis of Dynamics

$\vec{F}_A = \vec{T}_A + K\left(1 - \frac{2L}{r}\right)\vec{r}$ - mg j $- b\,\vec{V}_A$

$\vec{F}_B = \vec{T}_B$ - $K\left(1 - \frac{2L}{r}\right)\vec{r}$ - mg j $- b\,\vec{V}_B$ - $K'\,L\sin(\theta_B)\,\hat{i}$

$|\vec{\tau}_A| = |\vec{R}_A \times \vec{F}_A| =$ $-mg\,L\sin(\theta_A) + KL^2\left(1 - \frac{2L}{r}\right)(\,2\cos(\theta_A) + \cos(\theta_A)\sin(\theta_B) - \cos(\theta_B)\sin(\theta_A)\,) - b\,LV_B$

$$\ddot{\theta}_A = \frac{-g}{L}\sin(\theta_A) + \frac{K}{m}\left(1 - \frac{2L}{r}\right)(\,2\cos(\theta_A) - \sin(\theta_A - \theta_B)\,) - \frac{b\,V_A}{m\,L}$$

$|\vec{\tau}_B| = |\vec{R}_B \times \vec{F}_B| =$ $-mg\,L\sin(\theta_B) + [\,KL^2\left(1 - \frac{2L}{r}\right) - b\,\frac{|\vec{V}_B|}{r}\,\mathrm{Sign}(V_{B_x})\,L^2\,]\,(\,2\cos(\theta_B) + \cos(\theta_B)\sin(\theta_A) - \cos(\theta_A)\sin(\theta_B)\,)$ - $K'\,L^2\sin(\theta_B)\cos(\theta_B)$

$$\ddot{\theta}_B = \frac{-g}{L}\sin(\theta_B) + \frac{K}{m}\left(1 - \frac{2L}{r}\right)(\,-2\cos(\theta_B) + \sin(\theta_A - \theta_B)\,) - \frac{b\,V_B}{m\,L} - \frac{K'}{m}\sin(\theta_B)\cos(\theta_B)$$

Non-Dimensionalization

$$t \to \sqrt{\frac{L}{g}}\,t \;,\quad r \to Lr \;,\quad V_A \to \sqrt{g\,L}\,V_A \;,\quad V_B \to \sqrt{g\,L}\,V_B$$

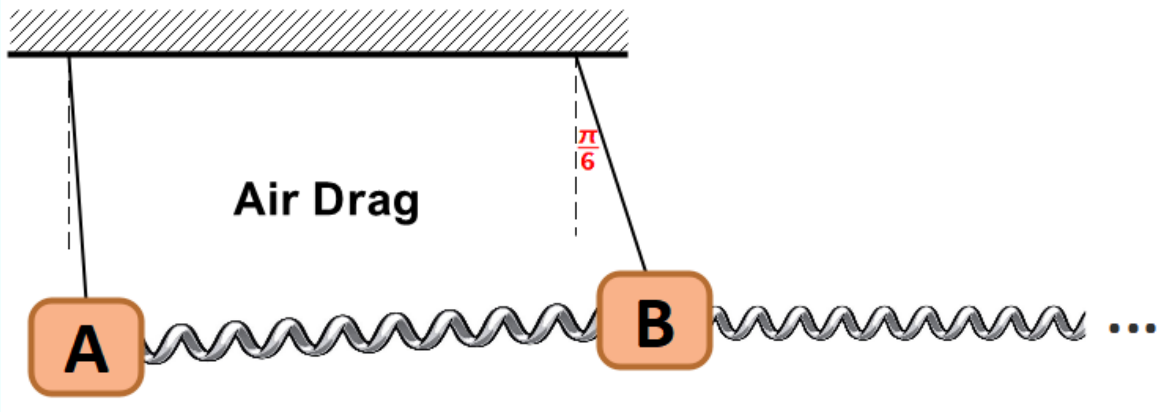
$$\ddot{\theta}_A = -\sin(\theta_A) + K\,\frac{L}{\mathrm{mg}}\left(1 - \frac{2}{r}\right)(\,2\cos(\theta_A) - \sin(\theta_A - \theta_B)\,) - \frac{b}{m}\sqrt{\frac{L}{g}}\,V_A$$

$$\ddot{\theta}_B = -\sin(\theta_B) + [\,K\,\frac{L}{\mathrm{mg}}\left(1 - \frac{2}{r}\right)(\,-2\cos(\theta_B) + \sin(\theta_A - \theta_B)\,) - \frac{b}{m}\sqrt{\frac{L}{g}}\,V_A - K'\,\frac{L}{\mathrm{mg}}\sin(\theta_B)\cos(\theta_B)$$

In[3951]:=

Case 11 : External Spring Force

Assuming external spring force always acting horizontally.
(It can by justified by assuming natural length of external spring >> L)

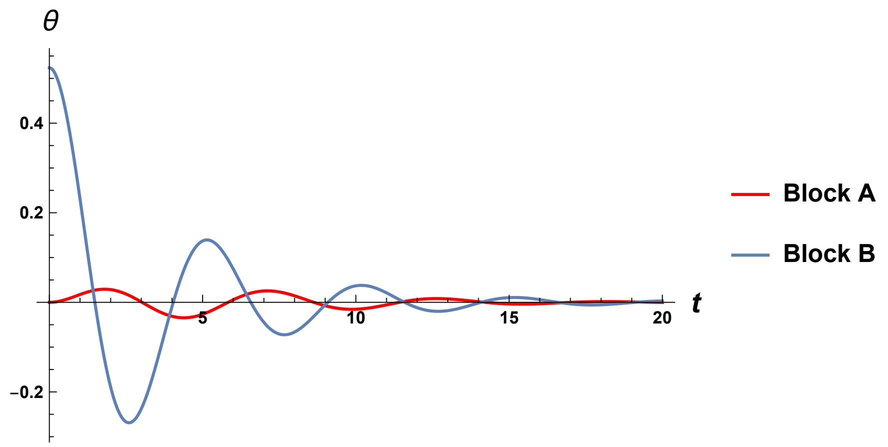


In[3952]:=

```
Clear[c, k, b, kex];
wfd1dot[t_, θ1_, θ2_, θ1dot_, θ2dot_] =
  c k (1 - 2 / r[θ1, θ2]) (2 Cos[θ1] - Sin[θ1 - θ2]) -
   Sin[θ1] - b * θ1dot[t, θ1, θ2, θ1dot, θ2dot];
wfd2dot[t_, θ1_, θ2_, θ1dot_, θ2dot_] =
  c k (1 - 2 / r[θ1, θ2]) (-2 Cos[θ2] + Sin[θ1 - θ2]) - Sin[θ2] -
   b * θ2dot[t, θ1, θ2, θ1dot, θ2dot] - kex c Sin[θ2] Cos[θ2];
```

In[3955]:=
```
ini = {0, 0, π / 6, 0, 0};        (* initials: time, θ1, θ2, θ̇1, θ̇2 *)
tf = 20;
k = 0.1;                          (* spring constant *)
kex = 0.5;                        (* external spring constant *)
c = 1;                            (* L/mg *)
b = 0.5;                          (* ∝ coefficient of damping *)
nMax = 1000;

data = dynamics1[{id, θ1dot, θ2dot, wfd1dot, wfd2dot}, ini, tf, k, nMax];

Show[ListPlot[data[[ ;; , {1, 2}]], Joined → True, PlotStyle → Red,
  AxesLabel → {Style[t, 15], Style[θ, 15]}, PlotLegends → {"Block A"},
  PlotRange → All], ListPlot[data[[ ;; , {1, 3}]], Joined → True,
  PlotLegends → {"Block B"}, PlotRange → All], PlotRange → All]
```

Out[3959]=



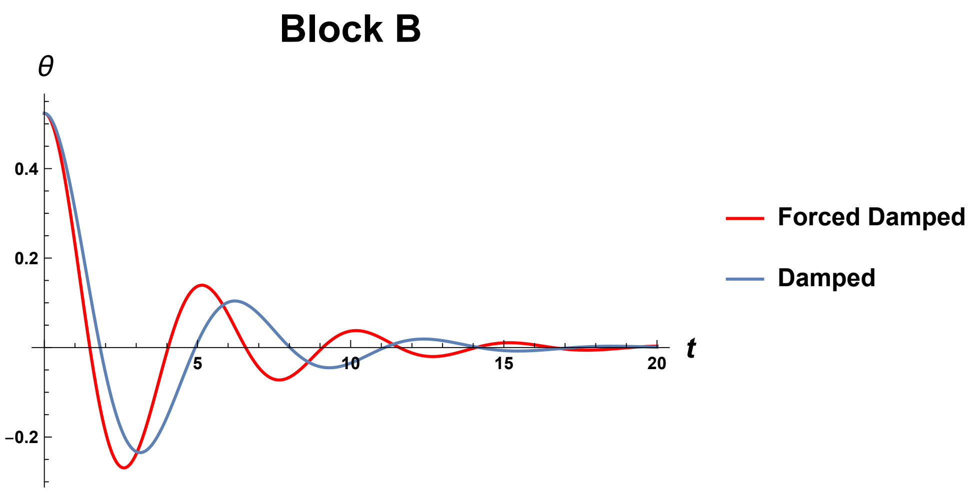
Observe that for Block B spring are almost in parallel combination so amplitude decay will be at slower rate than in damping without external spring force .

- To confirm this lets plot amplitude curve for block B in both cases

In[3960]:=

```
data1 = dynamics1[{id, θ1dot, θ2dot, wd1dot, wd2dot}, ini, tf, k, nMax];
Show[ListPlot[data[[ ;; , {1, 3}]], Joined → True,
  PlotStyle → Red, AxesLabel → {Style[t, 15], Style[θ, 15]},
  PlotLegends → {"Forced Damped"}, PlotRange → All],
 ListPlot[data1[[ ;; , {1, 3}]], Joined → True, PlotLegends → {"Damped"},
  PlotRange → All], PlotRange → All, PlotLabel → Style["Block B", 20]]
```

Out[3961]=



• From above graph clearly there is difference in decay rate of amplitude.

s

References :–

General Reference

https : // www.walter – fendt.de / html5 / phen / coupledpendula_en.htm

http : // demonstrations.wolfram.com / CoupledPendulumOscillations /

Case : 7

https : // www.researchgate.net / publication / 270 787 585 _Radiation _Damping _Force _ – _An _Alternative _Proposal

http : // www.feynmanlectures.caltech.edu / I_ 32. html