CHE312- Heat Transfer and Its Applications

Group Assignment

Group 17

Prof: Dr. Harshwardhan H. Katkar

Group Members Name:

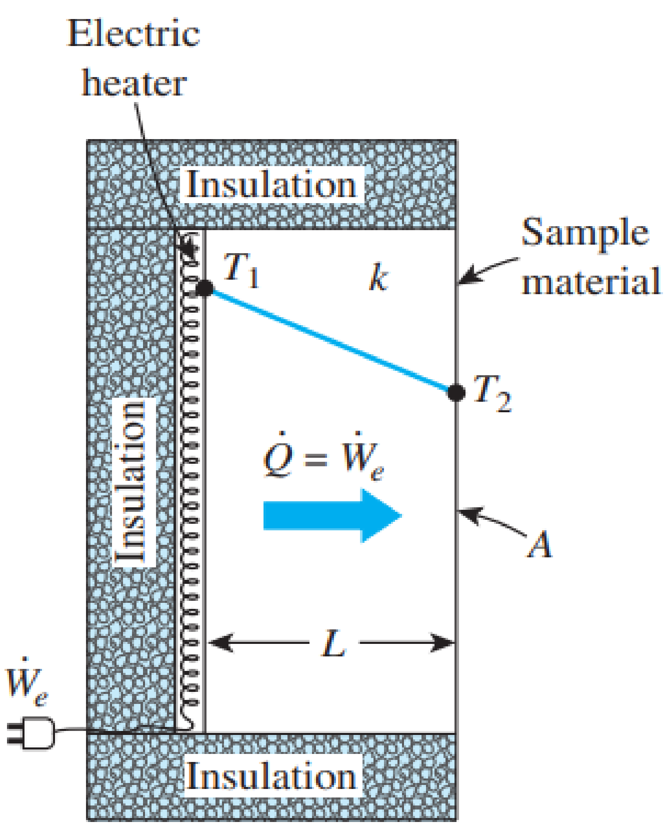
Riya Saini	200810
Kshipra Motikar	200598
Vedant Bang	201100
Rishabh	200788
Harish Kumar	200403
Ankur Kumar Raj	200141

Problem Statement:

The thermal conductivity of a slab can be measured using the setup discussed in the classroom. However, the procedure discussed in the classroom requires steady state temperatures. Device a protocol to estimate the thermal conductivity using the initial transient instead.

Solution:

For steady state temperatures we have, thermal conductivity



$$k = \frac{L}{A(T_1 - T_2)}\dot{Q}$$

We have a slab of known thickness and area. All the sides of the slab are well insulated so that no heat is transferred through surroundings and all the heat generated will be transferred through material. Then measure the temperature of two surfaces of material when steady state is reached and substitute them in the equation given below to calculate thermal conductivity.

$$\dot{Q}_{\text{cond}} = kA\,\frac{T_1 - T_2}{\Delta x} = -kA\,\frac{\Delta T}{\Delta x}$$

Now estimating thermal conductivity using the **initial transient** instead:
dQ/dt is not zero this time

We will consider two cases:

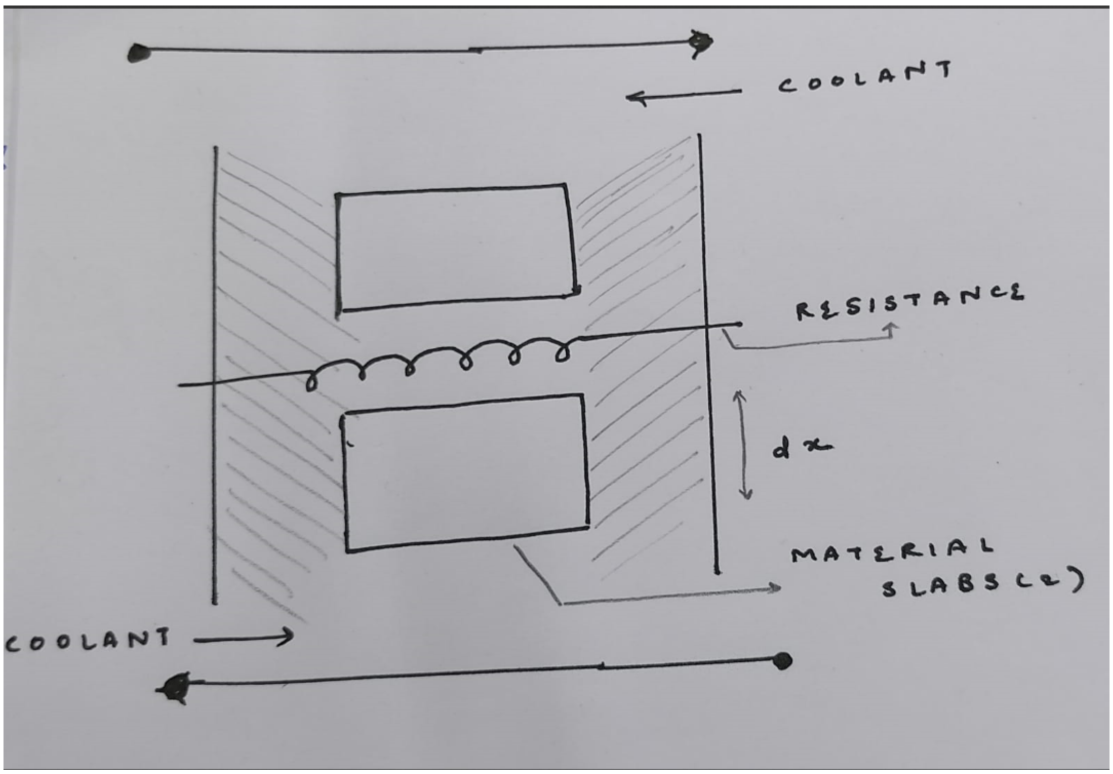
Case a - Q is time dependent

Case b - Temperature is time dependent

CASE A

THE SET UP FOR MEASUREMENT OF THERMAL CONDUCTIVITY

The setup consists of two material slabs and a wire having resistance **R** is placed between the slabs. Now a time dependent current **I(t)** is applied.



ANALYSIS :

The wire has a resistance and hence as the current is supplied, a potential difference(V) is created .

Therefore we are giving a power input to the system and the heat (Q) will be generated.

Q = V*I

A = cross sectional area ; k = thermal conductivity

$$Q = V * I$$
$$dQ = V\,dI$$
$$I = I(t)$$
$$dQ = V \frac{dI}{dt}$$
$$Q = = -kA \frac{dT}{dx}$$ (By Fourier's law of heat conduction)

let's say, $\frac{dI}{dt} = \alpha$

$$\therefore V\alpha = -kA \frac{dT}{dx}$$
$$V\alpha \int_0^L dx = -kA \int_{T_0}^{T_L} dT$$
$$V\alpha L = -kA(T_L - T_0)$$
$$k = \frac{V\alpha L}{A(T_0 - T_L)} = \frac{VL}{A(T_0 - T_L)}\left(\frac{dI}{dt}\right)$$

NOTES :

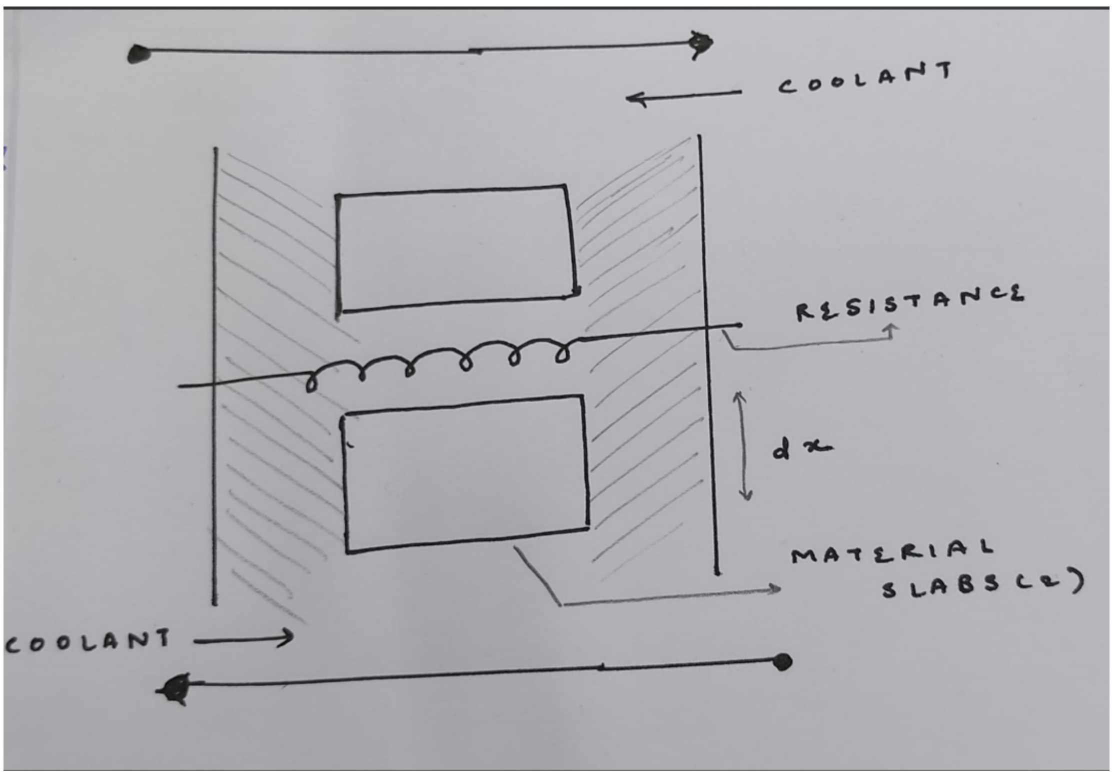
- Temperature difference across dx is time independent.
- The gradient is in one direction only

- The two blocks of same material are used and symmetry is obtained to minimize the loss in heat transfer. In the case of a single block, the results would be less accurate due to the huge amount of losses.

CASE B

Setup of the experiment:

The setup remains the same as the previous case: two material slabs and a wire having resistance **R** is placed between the slabs. Now a time dependent current **I(t)** is applied.



ANALYSIS :

The wire has a resistance and hence as the current is supplied, a potential difference(V) is created .

Therefore we are giving a power input to the system and the heat (Q) will be generated.

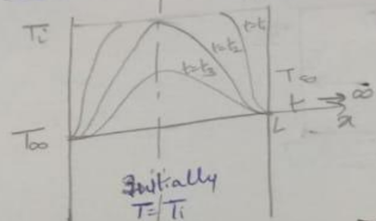
$Q = V*I$

A = cross sectional area ; k = thermal conductivity

As heat is transferred, temperature difference occurs which is dependent on both time and position.

We will consider the variation of temperature with time and position in one-dimensional.

Considering a slab of thickness $2L$ initially at uniform temperature T_i and $Q(t)$ heat is supplied continuously through electricity.



Transient temperature Profile

When the slab is first exposed to the surrounding medium at $T_\infty < T_i$ at $t=0$, the entire slab is at its initial temperature T_i. But the slab temperature at and near the surface starts to drop as a result of heat transfer from the slab to the surrounding medium.

Assumptions:

a) No heat generation.
b) Uniform initial temperature
c) thermal symmetry about the midplane
d) Heat transferred through electricity is dependent on time i.e $Q(t)$

Differential Equation.

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad -(1)$$

Boundary conditions

$$\frac{\partial T(0,t)}{\partial x} = 0 \quad \text{and} \quad \frac{-k\,\partial T(L,t)}{\partial x} = Q(t) \qquad -(2)$$

Initial condition

$$T(x,0) = T_i \qquad -(3)$$

Non-dimensionlizing the equation.

$$X = \frac{x}{L} \qquad \theta(x,t) = \frac{[T(x,t) - T_\infty]}{[T_i - T_\infty]}$$

We note that -

$$\frac{\partial \theta}{\partial X} = \frac{\partial \theta}{\partial (x/L)} = \frac{L}{T_i - T_\infty}\frac{\partial T}{\partial x} \; ; \quad \frac{\partial^2 \theta}{\partial X^2} = \frac{L^2}{T_i - T_\infty}\frac{\partial^2 T}{\partial x^2} \text{ and } \frac{\partial \theta}{\partial t} = \frac{1}{T_i - T_\infty}\frac{\partial T}{\partial t}$$

Rearranging the terms and substituting in eq. 1 and 2, we get

$$\frac{\partial^2\theta}{\partial x^2} = \frac{L^2}{\alpha}\frac{\partial\theta}{\partial t} \quad \text{and} \quad \frac{\partial\theta(1,t)}{\partial x} = \frac{L}{k}\phi(t)$$

We know, Dimensionless time $\tau = \frac{\alpha t}{L^2}$

Now, the formulation of the one-dimesional transient heat conduction problem in a slab can be expressed ~~as~~ in nondimensional form as

Dimensionless differential equation $\frac{\partial^2\theta}{\partial x^2} = \frac{\partial\theta}{\partial \tau}$

Dimensionless BC's: $\frac{\partial\theta(0,\tau)}{\partial x} = 0$ and $\frac{\partial\theta(1,\tau)}{\partial x} = \frac{L}{k}\phi(\tau)$

Dimensionless initial condition $\theta(x,0) = 1$

Solving this we can get θ and from there we can calculate thermal conductivity at initial transient

Notes:

- Temperature is time and position dependent
- The gradient is in one direction only
- The two blocks of same material are used and symmetry is obtained to minimize the loss in heat transfer. In the case of a single block, the results would be less accurate due to the huge amount of losses.
- No heat is generated