JEE-ADVANCED PI Q-9,10-

29 November 2021 14:04

Sol:- $L_1 : x\sqrt{2} + y - 1 = 0$

$L_2 : x\sqrt{2} - y + 1 = 0$

Let, the point P be.... (h, k)

ATQ,

$$d_{L_1} \cdot d_{L_2} = \lambda^2$$

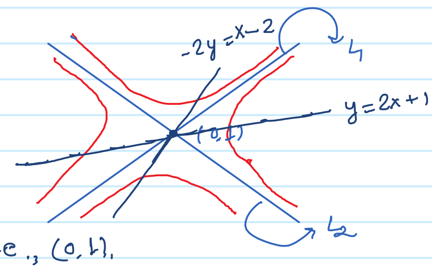
$$d_{L_1} = \frac{|h\sqrt{2} + k - 1|}{\sqrt{3}} \quad \text{and} \quad d_{L_2} = \frac{|h\sqrt{2} - k + 1|}{\sqrt{3}}$$

$$d_{L_1} \cdot d_{L_2} = \left|\frac{h\sqrt{2} + k - 1}{\sqrt{3}}\right| \left|\frac{h\sqrt{2} - k + 1}{\sqrt{3}}\right| = \lambda^2$$

$$\Rightarrow |h\sqrt{2} + k - 1||h\sqrt{2} - k + 1| = 3\lambda^2$$

Now, recall that, if the product of the distances of a point from two lines is constant, then the locus of point P is a hyperbola.

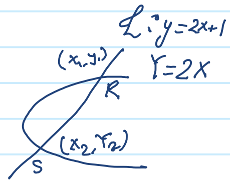
But, dealing with a shifted hyperbola is a NUT, HARD TO CRACK.



So, we try to shift our origin to the centre of the hyperbola i.e., $(0, 1)$.

So, $L_1' : X\sqrt{2} + Y = 0$

$L_2' : X\sqrt{2} - Y = 0$ } Lines - shifted $\begin{cases} X = x - 0 \\ Y = y - 1 \end{cases}$



Now, $|X\sqrt{2} + Y||X\sqrt{2} - Y| = 3\lambda^2$

$\Rightarrow |2X^2 - Y^2| = 3\lambda^2$ is the Locus of Point $P(h, k)$.

Also, it is given that, $d_{RS} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \sqrt{270}$

$$\Rightarrow \sqrt{(X_1 - X_2)^2 + (2X_1 - 2X_2)^2} = \sqrt{270}$$

$$\Rightarrow \sqrt{5}\,|X_1 - X_2| = \sqrt{270}$$

Now, $|2X^2 - Y^2| = 3\lambda^2$

$\Rightarrow |2X^2 - 4X^2| = 3\lambda^2$ (Intersection with $\mathcal{L}$)

$\Rightarrow 2X^2 = 3\lambda^2 \Rightarrow X = \pm\sqrt{\frac{3}{2}}\lambda \begin{cases} X_1 \\ X_2 \end{cases}$

So, $|X_1 - X_2| = 2\sqrt{\frac{3}{2}}\lambda$

$\therefore \sqrt{5} \times 2 \times \sqrt{\frac{3}{2}}\lambda = \sqrt{270} \Rightarrow \boxed{\lambda^2 = 9} \longrightarrow$ Q-9

If the line $\perp^r$ to $\mathcal{L}$ (say $\mathcal{L}'$) intersects C -

$\mathcal{L}' \equiv 2Y = -X$

$$d_{R'S'} = \sqrt{(X_1 - X_2)^2 + (Y_1 - Y_2)^2} = \sqrt{D}$$

$$\Rightarrow \sqrt{(X_1 - X_2)^2 + \frac{1}{4}(X_1 - X_2)^2} = \frac{\sqrt{5}}{2}|X_1 - X_2| = \sqrt{D}$$

$$\Rightarrow \sqrt{(x_1-x_2)^2 + \frac{1}{4}(x_1-x_2)^2} = \frac{\sqrt{5}}{2}\,|x_1-x_2| = \sqrt{D}$$

$$\because \quad C: \left|2x^2 - y^2\right| = 3a^2$$

$$\Rightarrow \left|2x^2 - \frac{1}{4}y^2\right| = 3 \times 9$$

$$\Rightarrow \frac{7}{4}x^2 = 27 \quad \Rightarrow \quad x = \pm\left(\sqrt{\frac{27}{7}}\right) \times 2$$

$$|x_1 - x_2| = 4\sqrt{\frac{27}{7}}$$

$$\therefore \frac{\sqrt{5}}{2}|x_1-x_2| = \frac{\sqrt{5}}{2} \times 4 \times \sqrt{\frac{27}{7}}$$

$$\sqrt{D} = \text{"} \quad \text{"}$$

$$\therefore D \simeq 77.142857 1425$$

$$\simeq \boxed{77.14} \longrightarrow Q\,10$$