Page No.:
Date:
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Parameter Evaluation

Q1 $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$

$x_1, x_2, x_3 \ldots x_n$ → sample of size n

$L(x_1, x_2, x_3 \ldots x_n) = f(x_1) \cdot f(x_2) \ldots f(x_n)$

$$\left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_1-\mu)^2}{2\sigma^2}}\right)\left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_2-\mu)^2}{2\sigma^2}}\right)$$

taking ln on both sides

$$\ln(L) = \frac{-n}{2}\ln(2\pi\sigma^2) + \sum_{i=1}^{n}\left(\frac{(x_i-\mu)^2}{2\sigma^2}\right) \quad \text{(I)}$$

taking partial derivative w.r.t μ of above eq

$$\frac{\partial \ln(L)}{\partial \mu} = 0 + \sum_{i=1}^{n} -\frac{2(x_i-\mu)}{2\sigma^2} = 0$$

$$\sum_{i=1}^{n}(x_i - \mu) = 0$$

$$n\bar{x}_n - n\mu = 0$$

$$\boxed{\bar{x} = \mu}$$

hence $\theta_1 = \bar{x}$

Now taking derivative wrt to σ^2

$$\frac{\partial \ln(L)}{\partial \sigma^2} = \frac{-n}{2\sigma^2} + \sum_{i=1}^{n} -\frac{(x_i-\mu)^2}{2\sigma^2} = 0$$

$$\cancel{\Rightarrow} \quad -n + \sum_{i=1}^{n} \frac{-(x_i-\mu)^2}{\sigma^2} = 0$$

$$\sum_{i=1}^{n} \frac{(x_i-\mu)^2}{\sigma^2} = n$$

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n} \frac{(x_i-\mu)^2}{\sigma^2}$$

$$\boxed{\theta_2 = \frac{1}{n}\sum_{i=1}^{n} (x_i-\mu)^2}$$

Q2. Binomial dist $= {}^nC_x\, \theta^{x_i}(1-\theta)^{n-x_i}$

$$L = \prod_{i=1}^{n} {}^nC_{x_i}\, \theta^{x_i}(1-\theta)^{n-x_i}$$

taking log on both sides

$$\log L = \sum_{i=1}^{n} \log({}^nC_{x_i}) + \log \theta^{x_i} + \log(1-\theta)^{n-x_i}$$

$$\log L = \sum_{i=1}^{n} \log({}^nC_{x_i}) + \log \theta^{\sum_{i=1}^{n} x_i} + \log(1-\theta)^{\sum_{i=1}^{n} n-x_i}$$

differentiate wrt to θ : $\frac{d\log(L)}{d\theta} = 0$

$$\frac{1}{\theta}\sum x_i - \frac{1}{1-\theta}\sum_{i=1}^{n} n - x_i = 0 \; ; \; \frac{1}{\theta}\sum x_i - \frac{n^2}{1-\theta} + \frac{1}{1-\theta}\sum x_i = 0$$

$$\frac{1}{\theta(1-\theta)}\sum x_i = \frac{n^2}{1-\theta} \Rightarrow \boxed{\theta = \frac{\sum x_i}{n^2}}$$