HOMEWORK-S CALC 2413 SPRING 2024

DR. HIMANSHU SINGH, PHD
DEPARTMENT OF MATHEMATICS
THE UNIVERSITY OF TEXAS AT TYLER

1. Reading Assignment

Go through Chapter 1 Section 1 from the prescribed course book.

2. Homework Policies and Practice Problems

2.1. **An important note.** This note in general is a homework policy[1] and is meant to be followed by students seriously. The practice problems for this homework are given after the immediate end of this note.

(1) This home-work is named as **Homework-S**.
(2) This HW is due by 26th Jan 2024 and worth of *31-points*.
(3) **You should submit your HW in-handwritten-in-person format on the standard-clean *Copy-Paper* on *26th Jan 2024*. Your attempts should be presented in a well-versed scientific manner**. This simply means that your attempts should have every necessary details that leads to your conclusion.
(4) Remember that, any other mode of submission of your homework such as emailing photos of your attempted homework etc., is absolutely not acceptable. Failing to submit the homework on a timely basis will in-turn leads to a $0-$point.

Question 1. *Do the following problems from the* Diagnostic Tests *from the course book available at page* `xvii-xxii`.

(1) *Following from* Diagnostic Tests (A)-Algebra
   (a) *All from problems 1-10*
(2) *Following from* Diagnostic Tests (B)-Analytic Geometry
   (a) *All from problems 1-5*
(3) *Following from* Diagnostic Tests (C)-Functions
   (a) *All from problems 1-7*
(4) *Following from* Diagnostic Tests (D)-Trigonometry
   (a) *All from problems 1-9*

Email address: `hsingh@uttyler.edu`

(Visiting Assistant Professor for Academic Year Aug 2023-May 2024) Department of Mathematics, The University of Texas at Tyler

Date: Due date 26th Jan 2024.

[1]subject to change based on the circumstantial situations!

<u>HOMEWORK-T</u> CALC 2413 SPRING 2024

DR. HIMANSHU SINGH, PHD
DEPARTMENT OF MATHEMATICS
THE UNIVERSITY OF TEXAS AT TYLER

1. Reading Assignment

Go through Chapter 1 Section(s) 1, 2 and 3 from the prescribed course book.

2. Homework Policies and Practice Problems

2.1. **An important note.** This note in general is a homework policy[1] and is meant to be followed by students seriously. The practice problems for this homework are given after the immediate end of this note.

(1) This home-work is named as **Homework-T**.
(2) This HW is due by 2nd Feb 2024 and worth of *100-points*.
(3) **You should submit your HW in-handwritten-in-person format on the standard-clean *Copy-Paper* on *2nd Feb 2024*. Your attempts should be presented in a well-versed scientific manner**. This simply means that your attempts should have every necessary details that leads to your conclusion. Please do not forget to staple your home-works.
(4) Remember that, any other mode of submission of your homework such as emailing photos of your attempted homework etc., is absolutely not acceptable. Failing to submit the homework on a timely basis will in-turn leads to a 0−point.

Question 1 (30-points)**.** *Do the following problems from the* Exercise 1.1.

(1) *1, 2, 21-24, 25-29, 30, 31-42, 59-64.*

Question 2 (30-points)**.** *Define a function* $f_I : \mathbb{R} \to \mathbb{R}$ *by* $f_I(x) = x$. *This function is called as an* identity function. *Additionally, define a function* $f_C : \mathbb{R} \to \mathbb{R}$ *by* $f_C(x) = C$ *where* C *is some constant in reals* $\mathbb{R}$. *This function is called as a* constant function.

(1) *Show that for these choice of functions,* $f_I \circ f_C = f_C \circ f_I$, *that is, in this case, the composition of two functions commute with each other.*

Date: Due date 2nd Feb 2024.

[1]subject to change based on the circumstantial situations!

(2) *Find the explicit value of either $f_I \circ f_C$ or $f_C \circ f_I$ and obviously it should be same.*
(3) *Demonstrate the failure of commutation of composition of two functions by choosing at-least one of the function of your choice.*

Question 3 (40-points). *Define $f(x) = \alpha x + \beta$ over reals, i.e. $\mathbb{R}$ with some constants α and β. Consider $n-$times composition of f with itself and denote it by f_n, that is $f_n = f \circ_1 f \circ_2 f \circ_3 \ldots \circ_n f$.*

(1) *Find f_2 and f_3 with $\alpha = 2$ and $\beta = -3$. This problem is from our book and we have gone through with it!*
(2) *Find an explicit expression of $f_n(x)$ in terms of α and β.*
(3) *Show that for some positive integer choices of n and m, $f_n = f_m$ if and only if $n = m$. This is a two-way direction proof, meaning first assume that for some distinct n and m, you have $f_n = f_m$, then conclude that $n = m$, and this is your first direction. For the second direction, assume that $n = m$ and then show that $f_n = f_m$.*
(4) *Observe the functional recursion given as $f_{n+1}(x) = \alpha^{n+1}x + f_n(\beta)$ for $n \geq 1$. Use it to find $f_{2024}(x)$ with $\alpha = 2$ and $\beta = -3$.*

Email address: `hsingh@uttyler.edu`

(Visiting Assistant Professor for Academic Year Aug 2023-May 2024) Department of Mathematics, The University of Texas at Tyler

<u>HOMEWORK-U</u> CALC 2413 SPRING 2024

DR. HIMANSHU SINGH, PHD
DEPARTMENT OF MATHEMATICS
THE UNIVERSITY OF TEXAS AT TYLER

1. Reading Assignment

Go through Chapter 1 Section(s) 4, 5 and 6 from the prescribed course book.

2. Homework Policies and Practice Problems

2.1. **An important note.** This note in general is a homework policy[1] and is meant to be followed by students seriously. The practice problems for this homework are given after the immediate end of this note.

(1) This home-work is named as **Homework-U**.
(2) This HW is due by 9th Feb 2024 and worth of *100-points*.
(3) **You should submit your HW in-handwritten-in-person format on the standard-clean *Copy-Paper* on *9th Feb 2024*. Your attempts should be presented in a well-versed scientific manner**. This simply means that your attempts should have every necessary details that leads to your conclusion. Please do not forget to staple your home-works.
(4) Remember that, any other mode of submission of your homework such as emailing photos of your attempted homework etc., is absolutely not acceptable. Failing to submit the homework on a timely basis will in-turn leads to a $0-$point.

Question 1 (36-points)**.** *Do the following problems from the* Exercise 1.4 *page 43-44.*

(1) *1-9, 10-28, 49-56*

Question 2 (10-points)**.** *Define* $S : \mathbb{R} \to [-1, 1]$ *via* $S(x) = \sin(x)$ *and lets recall that* $\lim_{\tau\to 0} \frac{S(\tau)}{\tau} = 1$. *We say* S_n *is the* $n-$*th composition of* S *with itself. Then,*

(1) *Find an explicit expression of* $S_n(\tau)$ *by using recursion relationship.*

Date: Due date 9th Feb 2024.

[1]subject to change based on the circumstantial situations!

(2) *If* $\lim_{\tau\to 0}\frac{S_n(x)}{\Lambda(\tau)} = 1$ *holds true for some function* $\Lambda(\tau)$*, then what possible choices of* $\Lambda(\tau)$ *you could consider so that the aforementioned limiting behavior is still true.*
(3) *Show that* $S_n(\tau)$ *is periodic with period* 2π*.*
(4) *Is* S_n *an odd function or even?*

Question 3 (10-points)**.** *Discuss the limit of* $\lim_{x\to 0} x^2 \sin\frac{1}{x} = 0$ *from Example-9 at Page 41 by the application of* the Squeeze theorem[2].

Question 4 (10-points)**.** *For values of* $\tau = 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$*, provide the computation of* $\sin\tau$ *and* $\cos\tau$ *at these values. Then based on these values collected, determine which one is an increasing function over* $\tau \in \left[0, \frac{\pi}{2}\right]$.

Question 5 (10-points)**.** *Find the limiting behavior of* $f(\tau) = \frac{\tau^n - 1}{\tau - 1}$ *as* τ *tends to* 1*. (Hint: Think of determining the pattern with* $n = 1, 2, 3$ *and so on!)*

Question 6 (10-points)**.** *From question 2, we recall function* $S(\tau) = \sin\tau$*. If* $\lim_{\tau\to 0}\frac{S(\tau)}{\tau} = 1$*, does it still holds true for* $\lim_{\tau\to 0}\frac{S^2(\tau)}{\tau} = 1$*? If no, then what value should be of this limit?*

Question 7 (14-points)**.** *For* $f_{[\#]}(\lambda) = \lambda^{\#}$*, where* # *is a natural number of your choice, i.e.* $\# \in \mathbb{Z}_+$*. In general, one can provide the domain and the range of function over reals i.e.* $\mathbb{R}$*. For this problem, we will restrict the domain of this function on an unit interval, i.e.* $I = [0, 1]$*. Then supply reasoning of following problems.*

(1) *If for* $f_{[\#]}$*, the domain is* I*, then what is its range? Say the determined range of* $f_{[\#]}$ *is represented by* $\mathcal{R}\left(f_{[\#]}\right)$*.*
(2) *After we have determined the range of this function that is* $\mathcal{R}\left(f_{[\#]}\right)$*, show for* $\lambda \in I$*, the function* $f_{[\#]}(\lambda)$ *is decreasing over* $\mathcal{R}\left(f_{[\#]}\right)$*.*
(3) *Let* F_τ *represents the* $\tau-$*times self-composition of* $f_{[\#]}$ *where* $\tau \in \mathbb{Z}_+$*. Then, find the explicit expression of* F_τ *at* $\lambda \in I$*. What is the range of* F_τ*. Show that* F *is also a decreasing function on its range.*
(4) *Without changing* I *and* $\mathcal{R}\left(f_{[\#]}\right)$*, what necessary changes one must do in the definition of* $f_{[\#]}$ *so that the new changed function is now increasing! (Hint: Think of* $f_{1/[\#]}$*)*

Email address: `hsingh@uttyler.edu`

(Visiting Assistant Professor for Academic Year Aug 2023-May 2024) Department of Mathematics, The University of Texas at Tyler

[2]from reading assignment of Chapter 1 Section 4.

HOMEWORK-D CALC 2413 SPRING 2024

DR. HIMANSHU SINGH, PHD
DEPARTMENT OF MATHEMATICS
THE UNIVERSITY OF TEXAS AT TYLER

1. Reading Assignment

Go through Chapter 2 Section(s) 2, 3 and 4 from the prescribed course book.

2. Homework Policies and Practice Problems

2.1. **An important note.** This note in general is a homework policy[1] and is meant to be followed by students seriously. The practice problems for this homework are given after the immediate end of this note.

(1) This home-work is named as **Homework-D**.
(2) This HW is due by 16th Feb 2024 and worth of *100-points*.
(3) **You should submit your HW in-handwritten-in-person format on the standard-clean *Copy-Paper* on *16th Feb 2024*. Your attempts should be presented in a well-versed scientific manner**. This simply means that your attempts should have every necessary details that leads to your conclusion. Please do not forget to staple your home-works.
(4) Remember that, any other mode of submission of your homework such as emailing photos of your attempted homework etc., is absolutely not acceptable. Failing to submit the homework on a timely basis will in-turn leads to a $0-$point.
(5) Problems present in blue are Optional. Here, in present context, Optional simply mean that these questions carries *less points weight-age*. Any other implication of Optional, be it direct or in-direct, in context to the present home-work shall not be drawn! These problems are meant to be turned in on the due date of this home-work and will also be graded as well.
(6) For this homework, lecture notes provided in class or uploaded on UT Tyler-Canvas, will surely be helpful.

Question 1 ($91-$points)**.** *Attempt following problems from course-book.*

(1) *From Chapter 1 Section §5 exercise*
    (a) *12-13,15-18, 29-30*
(2) *From Chapter 1 Section §6 exercise*
    (a) *13-33*
(3) *From Chapter 2 Section §1 exercise*
    (a) *3-6, 25-30*
(4) *From Chapter 2 Section §1 Example 3 at Page 77, Example 4, and 5 at Page 78.*

Date: Due date 16th Feb 2024.

[1]subject to change based on the circumstantial situations!

Question 2 (3−points; OPTIONAL). *Let $\mathcal{S}(\nu) = \sin\nu$ and $\mathcal{C}(\nu) = \cos\nu$. Recall the definition of* derivative of function *from Page-77 from our course-book.*

(1) *Find the derivative of $\mathcal{S}(\nu)$ and $\mathcal{C}(\nu)$ with respect to the independent variable ν by employing the derivative definition. The formulas $\mathcal{S}(\nu + w) = \mathcal{S}(\nu)\mathcal{C}(w) + \mathcal{S}(w)\mathcal{C}(\nu)$ and $\mathcal{C}(\nu + w) = \mathcal{C}(\nu)\mathcal{C}(w) - \mathcal{S}(\nu)\mathcal{S}(w)$ will be helpful here.*

(2) *Find the derivative of $\mathcal{S}^2(\nu)$ and $\mathcal{C}^2(\nu)$ with respect to the independent variable ν by employing the derivative definition. The formulas $\mathcal{S}^2(\nu) - \mathcal{S}^2(w) = \mathcal{S}(\nu + w)\mathcal{S}(\nu - w)$ and $\mathcal{C}^2(\nu) - \mathcal{C}^2(w) = \mathcal{S}(w - \nu)\mathcal{S}(\nu + w)$ will be useful here. Observe here that the derivatives sum of the aforementioned function is constant. What is that constant? Why we expect to have <u>that</u> constant?*

Question 3 (3−points; OPTIONAL). *List the points on real-line where trigonometric functions $\mathcal{S}(\nu)$ and $\mathcal{C}(\nu)$ is zero. Based on that, discuss the asymptotic behavior of* $\tan(\nu)$ *function. Also, demonstrate the asymptotic behavior of* $\cot\nu$ *function as well.*

Question 4 (3−points; OPTIONAL). *Find the equation of tangent line for curve $f(x) = x^n$ for some positive $n \in \mathbb{N}$ at $x-$coordinate $x = 1$. (Follow Example 1 on Page 73 and previous home-work for more help on this!)*

Email address: `hsingh@uttyler.edu`

(Visiting Assistant Professor for Academic Year Aug 2023-May 2024) DEPARTMENT OF MATHEMATICS, THE UNIVERSITY OF TEXAS AT TYLER

HOMEWORK-Y CALC 2413 SPRING 2024

DR. HIMANSHU SINGH, PHD
DEPARTMENT OF MATHEMATICS
THE UNIVERSITY OF TEXAS AT TYLER

1. Reading Assignment

Go through Chapter 2 Section(s) 2, 3, 4, 5 and 6 from the prescribed course book.

2. Homework Policies and Practice Problems

2.1. **An important note.** This note in general is a homework policy[1] and is meant to be followed by students seriously. The practice problems for this homework are given after the immediate end of this note.

(1) This home-work is named as **Homework-Y**.
(2) This HW is due by 23TH FEB 2024 and worth of *50-points.*
(3) **You should submit your HW in-handwritten-in-person format on the standard-clean *Copy-Paper* on *23th Feb 2024*. Your attempts should be presented in a well-versed scientific manner**. This simply means that your attempts should have every necessary details that leads to your conclusion. Please do not forget to staple your home-works.
(4) Remember that, any other mode of submission of your homework such as emailing photos of your attempted homework etc., is absolutely not acceptable. Failing to submit the homework on a timely basis will in-turn leads to a $0-$point.
(5) This home-work has only one problem with 4-sub-parts. The first three sub-parts are of $15-$points each and last part is of $5-$point.
(6) For this homework, lecture notes provided in class or uploaded on UT Tyler-Canvas, will surely be helpful.

Question 1 (50-points). *Consider $\mathcal{A}_{1,b_i} : \mathbb{R} \to \mathbb{R}$, a function which is an affine that is $\mathcal{A}_{1,b_i}(x) = x + b_i$ for some b_i. Here b_i can be thought*

Date: Due date 23th Feb 2024.
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as of $y-$intercept of the associated affine function $\mathcal{A}_{1,b_i}$. Let $\mathcal{P}_n(x)$ be the product of all $\mathcal{A}_{1,b_1}, \ldots, \mathcal{A}_{1,b_n}$, that is

$$\begin{aligned}\mathcal{P}_n(x) =& \prod_{i=1}^{n} \mathcal{A}_{1,b_i}(x)\\ =&\mathcal{A}_{1,b_1}(x) \cdots \mathcal{A}_{1,b_n}(x)\\ =& (x+b_1)\cdot(x+b_2)\cdots(x+b_n)\,.\end{aligned}$$

To make our theory more systematic, we consider the reciprocal of each $\mathcal{A}_{1,b_i}(x)$, that is

$$\mathcal{R}_{b_i}(x) = \frac{1}{\mathcal{A}_{1,b_i}(x)}.$$

Then provide the solution of following questions:

(1) *State all the $x-$points where $\mathcal{A}_{1,b_i}(x)$ is 0.*
(2) *Identify all the $x-$points where $\mathcal{R}_{b_i}(x)$ fails to be defined.*
(3) *What is the ratio of $\mathcal{P}_n'(x)$ with $\mathcal{P}_n(x)$, that is $\dfrac{\mathcal{P}_n'(x)}{\mathcal{P}_n(x)}$?*
(4) *What is the domain of $\dfrac{\mathcal{P}_n'(x)}{\mathcal{P}_n(x)}$?*

Email address: `hsingh@uttyler.edu`

(Visiting Assistant Professor for Academic Year Aug 2023-May 2024) Department of Mathematics, The University of Texas at Tyler

<u>HOMEWORK-A</u> CALC 2413 SPRING 2024

DR. HIMANSHU SINGH, PHD
DEPARTMENT OF MATHEMATICS
THE UNIVERSITY OF TEXAS AT TYLER

1. Reading Assignment

Go through Chapter 3 and Chapter 4 Section(s) 1 and 2 from the prescribed course book.

2. Homework Policies and Practice Problems

2.1. **An important note.** This note in general is a homework policy[1] and is meant to be followed by students seriously. The practice problems for this homework are given after the immediate end of this note.

(1) This home-work is named as **Homework-A**.
(2) This HW is due by 1st April 2024 and worth of *100-points*.
(3) **You should submit your HW in-handwritten-in-person format on the standard-clean *Copy-Paper* on *1st April 2024*. Your attempts should be presented in a well-versed scientific manner**. This simply means that your attempts should have every necessary details that leads to your conclusion. Please do not forget to staple your home-works.
(4) Remember that, any other mode of submission of your homework such as emailing photos of your attempted homework etc., is absolutely not acceptable. Failing to submit the homework on a timely basis will in-turn leads to a $0-$point.

Question 1. *If for some argument* Ξ*, the following relation holds:*

$$y(\Xi) = \tan^{-1}\left(\sqrt{\frac{1-\Xi}{1+\Xi}}\right), \tag{1}$$

find $y'(\Xi)$. *Your answer here should be present in the most simplified format.*

Date: Due date 1st April 2024.

[1]subject to change based on the circumstantial situations!

Question 2. *If* $\sin^{-1} x = \mathfrak{s}$*, then establish that* $\csc^{-1} \dfrac{1}{x} = \mathfrak{s}$.

Question 3. *If* $\mathbf{T}_1(\theta) = \dfrac{1 - \tan\theta}{1 + \tan\theta}$ *and* $\mathbf{T}_{\pi/4}(\theta) = \tan\left(\dfrac{\pi}{4} - \theta\right)$*, then show that*

$$(\mathbf{T}_1(\theta))' = \left(\mathbf{T}_{\pi/4}(\theta)\right)'. \tag{2}$$

Here, the derivative of respective quantities were taken with respect to θ*.*

Question 4. *Find the critical numbers of* $f(\theta) = 2\cos\theta + \sin^2\theta$*.*

Question 5. *Verify the* Rolle's Theorem *for* $f(\mu) = \sin\sqrt{-\mu^2 + 1}$ *over the domain* $[-1, 1]$ *with details.*

Question 6. *Compute the limit of following:*

$$\lim_{x\to\infty} \left\{ \frac{\sin^{-1}\left(\sqrt{\sin x^2}\right) + \cos^{-1}\left(\sqrt{\sin x^2}\right)}{e^x} \right\}. \tag{3}$$

Email address: `hsingh@uttyler.edu`

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HOMEWORK-L CALC 2413 SPRING 2024

DR. HIMANSHU SINGH, PHD
DEPARTMENT OF MATHEMATICS
THE UNIVERSITY OF TEXAS AT TYLER

1. Reading Assignment

Go through Chapter 4 Section 5 and Chapter 5 Section 3-Tables of Indefinite Integrals @Page 284 from the prescribed course book.

2. Homework Policies and Practice Problems

2.1. **An important note.** This note in general is a homework policy[1] and is meant to be followed by students seriously. The practice problems for this homework are given after the immediate end of this note.

(1) This home-work is named as **Homework-L**.
(2) This HW is due by 8th April 2024 and worth of *100-points*.
(3) **You should submit your HW in-handwritten-in-person format on the standard-clean *Copy-Paper* on *8th April 2024*. Your attempts should be presented in a well-versed scientific manner**. This simply means that your attempts should have every necessary details that leads to your conclusion. Please do not forget to staple your home-works.
(4) Remember that, any other mode of submission of your homework such as emailing photos of your attempted homework etc., is absolutely not acceptable. Failing to submit the homework on a timely basis will in-turn leads to a $0-$point.

Question 1. *Work out on Chapter 4 Section 5 Examples 1, 2, 3 and 5.*

Email address: hsingh@uttyler.edu

(Visiting Assistant Professor for Academic Year Aug 2023-May 2024) Department of Mathematics, The University of Texas at Tyler

Date: Due date 8th April 2024.

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HOMEWORK-C CALC 2413 SPRING 2024

DR. HIMANSHU SINGH, PHD

1. Reading Assignment

Go through Chapter 5 Section 3-Tables of Indefinite Integrals @Page 284 from the prescribed course book.

2. Homework Policies and Practice Problems

2.1. **An important note.** This note in general is a homework policy[1] and is meant to be followed by students seriously. The practice problems for this homework are given after the immediate end of this note.

(1) This home-work is named as **Homework-c**. Note that its a small English letter c.
(2) This HW is due by 15th April 2024 and worth of *100-points*.
(3) **You should submit your HW in-handwritten-in-person format on the standard-clean *Copy-Paper* on *15th April 2024*. Your attempts should be presented in a well-versed scientific manner**. This simply means that your attempts should have every necessary details that leads to your conclusion. Please do not forget to staple your home-works.
(4) Remember that, any other mode of submission of your homework such as emailing photos of your attempted homework etc., is absolutely not acceptable. Failing to submit the homework on a timely basis will in-turn leads to a $0-$point.

Question 1. *Evaluate following:*

(1)
$$\int \left(\sin x + x^{10} + \frac{1}{x} + \sinh x\right) \, dx$$

(2)
$$\int_9^{16} \frac{7}{\sqrt{x}} \, dx$$

(3) *Find* $g'(x)$ *if* $g(x) = \displaystyle\int_{x^2}^{3} \cos t \, dt$.

(4)
$$\int_1^e \frac{\ln x}{x} \, dx$$

The above problems appeared in the Calc Fall 2023 Final exam.

Question 2. *Do Chapter 5 Section 3 Exercises-17, 22, 46, 47, 48 @Page 289.*

Question 3. *Do Chapter 5 Section 5 Exercises-1, 31, 46, 49, 65, 66 @Page 306-307.*

Date: Due date 15th April 2024.

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Question 4 (this will note be graded!)**.** *Evaluate* $\int abcde$*?*

Question 5. *Give a choice of a continuous function* f *on* $[0,1]$ *such that* $\int_0^1 f(x)dx = e$.

Question 6. *The following problem is based on the properties of definite integrals.*
Let $I_1(x) = \int_{\sin x}^{\cos x} f(t)dt$, *where* f *is a continuous function is single variable.*

(1) *Prove that* $I_1(-n\pi) = I_1(n\pi)$ *for any integer* n.
(2) *Show that* $I_1\left(\pi/2 - x\right) + I_1(x) = 0$.
(3) *Show that* $I_1\left(\pi/2 + x\right) + I_1(-x) = 0$.
(4) *Denote* $\mathcal{I}_1(x) = I_1(-x) - I_1(x)$. *Then prove that* $\mathcal{I}_1(-x) = -\mathcal{I}_1(x)$, *which means that* $\mathcal{I}_1(x)$ *is an odd function.*

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(Visiting Assistant Professor for Academic Year Aug 2023-May 2024) Department of Mathematics, The University of Texas at Tyler