PROFESSIONAL DEVELOPMENT AGENDA
RESEARCH STATEMENT

DR. HIMANSHU SINGH, PHD

Background & Research approach

I am Himanshu Singh, currently employed as a *Visiting Assistant Professor* in the Department of Mathematics at the University of Texas, Tyler. My mathematical research lies in Applied Analysis and I hold expertise in *Operator Theory, Functional Analysis & Reproducing Kernel Hilbert Space* (RKHS). My research contribution finds direct application in understanding modern dynamical systems, machine learning and data science. Specifically, I focus on understanding of function theoretic operators over the underlying Hilbert spaces, in particular RKHS.

My PhD Dissertation research work

My PhD dissertation entitled as *Applied Analysis for Learning Architectures* explores about the question of largest possible extension of operators which can capture important patterns from the complex systems. My paper **Liouville weighted composition operators over the Fock space**[1] answers the existence of such operators.

To understand high-dimensional complex system, practitioners employ *Dynamic Mode Decomposition* (DMD), which in-fact, is a work-horse algorithm to identify governing features of complex data system. DMD is usually leveraged by the spectral properties of *Koopman operators* (or composition $\mathcal{K}_F$) where it is believed that these spectral properties will surely encodes the system. However, this encoding of the system works in heuristic sense and we refer this phenomena as *strong operator topology*. This limitation inhibits practitioners across the fields of mechanical engineers, biologists etc. to study complex dynamics.

To identity governing features of complex data, we employ Liouville operators $A_{\mathbf{f}}$ where its spectral properties surely encodes the system. This behaviour of spectral properties of Liouville operators is called as *norm convergence* and this is due to its compactness nature. In our investigation, we find that, a new introduction of symbol to Liouville operator resulting into $A_{\mathbf{f},\varphi}$ can make it the largest possible extension of operators which can surely encode the system. The combination of two aspects of Liouville operators, namely *the largest possible extension* and *norm convergence* makes it exciting and unique. The combination of these aspects is called as the *provable convergence guarantee phenomena*. Figure 2 intelligently summarizes about various types of spectral convergence in relation with respective operators.

My recent attention is on the possible extension of Liouville weighted composition operators. My initial research deals with simple single order dynamics however, real-life models can be more intricate. These intricate systems can involve higher order dynamics for which we may have to perform state-augmentation. It is expected that with the definition of higher order Liouville weighted composition operators, the state-augmentation can be avoided. These results has important implications, as identifying the higher-order dynamics is key for understanding how these complex models works. With these noteworthy results and conclusions, the consequences of these projects can easily capture the attention of mathematicians and engineers.

Snapshots Visualization The dynamic interface of Koopman modes (both real in Right and imaginary parts in Left) are provided as follows with limited data availability for the experiment of fluid flow across a cylinder.

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[1]joined work with Dr. Benjamin P. Russo & Dr. Joel A. Rosenfeld

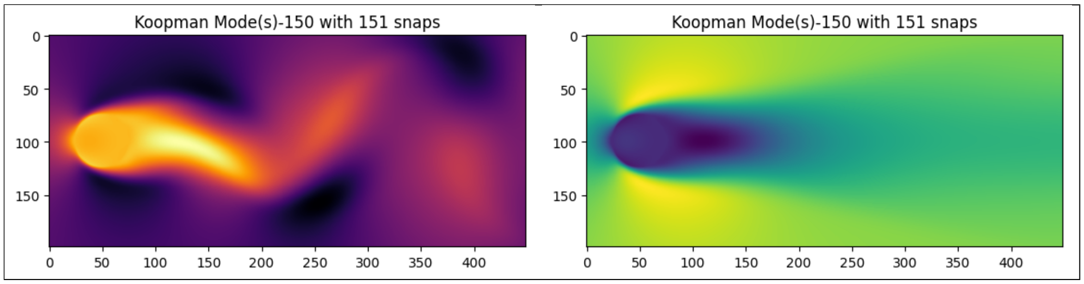

FIGURE 1. An example of Koopman Modes

This experimentation is executed in `julia` with suitable choices of color-maps for an apt data visualization.

FUTURE RESEARCH PLANS

Here, I have highlighted my upcoming research agendas that aligns with the present scope of ultimate needs.

- Learning architectures in Artificial Intelligence is greatly advanced by *machine learning* (cf: Figure 3). Machine learning models such as *neural networks, kernel regression or Gaussian processing*, to name a few, directly rely on the choice of mathematical functions. At any given point of instance, we always desire to construct such a mathematical functions which can outperforms typical-used-functions such as *Gaussian Radial Basis Function Kernel* (GRBF Kernel). I constructed the *Generalised Gaussian Radial Basis Function Kernel* (GGRBF Kernel) which yields excellent results in various machine learning routines. I investigated this function in paper '**An appointment with Reproducing Kernel Hilbert Space generated by Generalized Gaussian RBF as L^2-measure**' submitted to IEEE-TRANSACTIONS ON ARTIFICIAL INTELLIGENCE. Here, I constructed the Hilbert space theory for the GGRBF Kernel along with its application in machine learning routines, such as kernel regression, support vector machine and neural networks.
- My future research plans are around two research ideas that are in progress. First, I am constructing a new perspective in Hilbert space theory which is called as *equivalent norm representations*. We have evidence to support this theory for Hilbert spaces that are defined on an unit disc in complex plane. Secondly, I am interested in determining the spectral observables for the GGRBF Kernel. We are well-aware of the spectral observables for the GRBF kernel that are given by the Hermite orthonormal polynomials. My preliminary investigation to determine the spectral observables of GGRBF directs that we have to construct (similar) Hermite functions which are orthonormal against GGRBF as its weight in usual Lebesgue sense.

FIGURES

Here, I have compiled important and interesting figures from my research work.

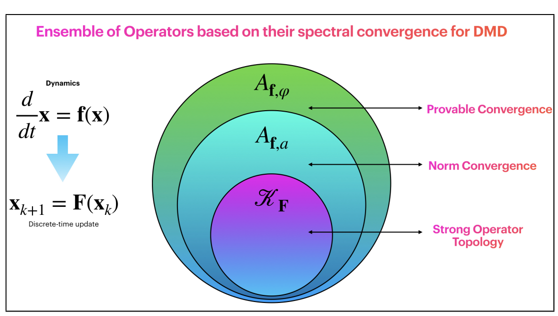

FIGURE 2. Spectral-convergence descriptions of operators

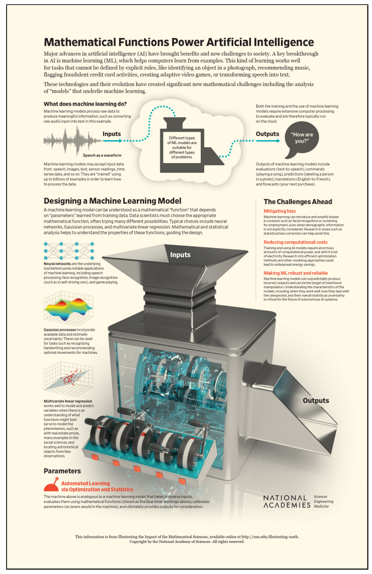

FIGURE 3. Artificial Intelligence powered by Mathematical Functions

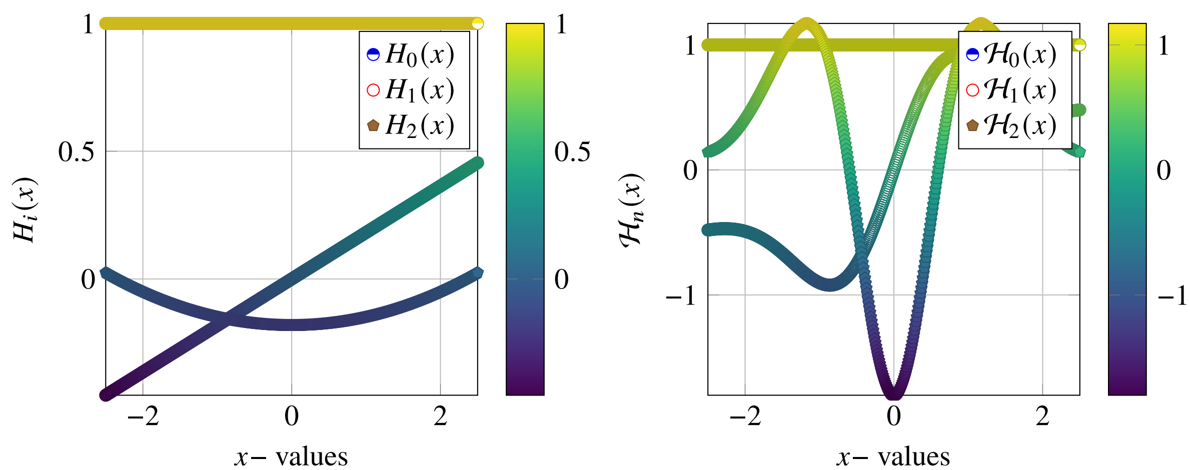

FIGURE 4. (LEFT) Spectral observables of GRBF Kernel; (RIGHT) Spectral observables of GGRBF Kernel.

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