MATHEMATICS

SECTION A

January 9, 2024

1 Matrices

1. A is a square matrix with $\left|A\right| = 4$. Then find the value of $\left|A.\,(\text{adj}A)\right|$.

2. For the matrix $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$. Find $(A + A')$ and verify that it is a symmetric matrix.

3. Using elementary row transformations, find the inverse of the matrix $\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$

4. Using matrices, solve the following system of linear equations :

$$\begin{aligned} x + 2y - 3z &= -4 \\ 2x + 3y + 2z &= 2 \\ 3x - 3y - 4z &= 11 \end{aligned}$$

5. Using properties of determinants, find the value of k if $\begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix} = k\left|x^3 + y^3\right|$.

2 probability

6. In a multiple choice examination with three possible answers for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing ?

7. The probabilities of solving a specific problem independently by A and B are $\frac{1}{3}$ and $\frac{1}{5}$ respectively. If both try to solve the problem independently, find the probability that the problem is solved.

8. There are three coins. One is a coin having tails on both faces, another is a biased coin that comes up tails 70% of the time and the third is an unbiased coin. One of the coins is chosen at random and tossed, it shows tail. Find the probability that it was a coin with tail on both the faces.

3 Algebra

9. Prove that :

$$\cos^{-1}\left(\frac{12}{13}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \sin^{-1}\left(\frac{56}{65}\right)$$

4 vector

10. Find the vector equation of the plane which contains the line of intersection of the planes $\vec{r}.\left(\hat{i}+2\hat{j}+3\hat{k}\right)-4=0$, $\vec{r}.\left(2\hat{i}+\hat{j}-\hat{k}\right)+5=0$ and which is perpendicular to the plane $\vec{r}.\left(5\hat{i}+3\hat{j}-6\hat{k}\right)+8=0$.
11. Find the vector equation of a line passing through the point $(2,3,2)$ and parallel to the line $\vec{r}=\left(-2\hat{i}+3\hat{j}\right)\lambda\left(2\hat{i}-3\hat{j}+6\hat{k}\right)$. Also, find the distance between these two lines.
12. Find the value of **x** such that the four points with position vectors, $\mathbf{A}\left(3\hat{i}+2\hat{j}+\hat{k}\right)$, $\mathbf{B}\left(4\hat{i}+x\hat{j}+5\hat{k}\right)$, $\mathbf{C}\left(4\hat{i}+2\hat{j}-2\hat{k}\right)$, $\mathbf{D}\left(6\hat{i}+5\hat{j}-\hat{k}\right)$ are coplanar.
13. Find the vector equation of the plane determined by the points $\mathbf{A}(3,-1,2)$, $\mathbf{B}(5,2,4)$ and $\mathbf{C}(-1,-1,6)$. Hence, find the distance of the plane, thus obtained,from the origin.
14. Find the coordinates of the foot of the perpendicular **Q** drawn from $\mathbf{P}(3,2,1)$ to the plane $2x-y+z+1=0$. Also, find the distance **PQ** and the image of the point **P** treating this plane as a mirror.

5 Differentiation

15. A ladder $13m$ long is leaning against a vertical wall. The bottom of the ladder is dragged away from the wall along the ground at the rate of $2cm/sec$. How fast is the height on the wall decreasing when the foot of the ladder is $5m$ away from the wall ?
16. If $y=(\log x)^x+x^{\log x}$, find $\frac{dy}{dx}$.
17. If $x=\sin t, y=\sin pt$, prove that $\left(1-x^2\right)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+p^2y=0$.
18. Differentiate

$$\tan^{-1}\left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)$$

with respect to $\cos^{-1}x^2$.

19. Form the differential equation representing the family of curves $y=A\sin x$, by eliminating the arbitrary constant A .

6 Integration

20. Find the particular solution of the differential equation $\frac{dy}{dx}=\frac{xy}{x^2+y^2}$, given that $y=1$ when $x=0$.
21. Solve the differential equation $\frac{dy}{dx}=1+x^2+y^2+x^2y^2$, given that $y=1$ when $x=0$.
22. Prove that $\int_0^a f(x)\,dx=\int_0^a f(a-x)\,dx$, and hence evaluate $\int_0^1 x^2(1-x)^n\,dx$.

7 Linear forms

23. Using integration, find the area of the region bounded by the parabola $y^2=4x$ and the circle $4x^2+4y^2=9$.
24. Using the method of integration, find the area of the region bounded by the lines $3x-2y+1=0, 2x+3y-21=0$ and $x-5y+9=0$.
25. Using integration, find the area of the region bounded by the line $y=3x+2$, the x-axis and the ordinates $x=-2$ and $x=1$.
26. Using integration, find the area of the smaller region bounded by the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ and the line $\frac{x}{3}+\frac{y}{2}=1$.

8 Functions

27. Let $A = R - \{2\}$ and $B = R - \{1\}$. If $f : A \to B$ is a function defined by $f(x) = \frac{x-1}{x-2}$, show that f is one-one and onto. Hence, find f^{-1}.

28. Show that the relation S in the set $A = \{x \in Z : 0 \le x \le 12\}$ given by $S = \{(a, b) : a, b \in Z, a - b$ is divisible by 3 $\}$ is an equivalence relation.

9 Optimization

29. A dietician wishes to mix two types of food in such a way that the vitamin contents of the mixture contains at least $8\,units$ of vitamin A and $10\,units$ of vitamin C. Food I contains $2\,units/kg$ of vitamin A and $1\,unit/kg$ of vitamin C. It costs ₹50 $perkg$ to produce food I. Food II contains $1\,unit/kg$ of vitamin A and $2\,units/kg$ of vitamin C and it costs ₹70 $perkg$ to produce food II. Formulate this problem as a LPP to minimise the cost of a mixture that will produce the required diet. Also find the minimum cost.

30. A manufacturer produces nuts and bolts. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit of ₹ 35 per package of nuts and ₹ 14 per package of bolts. How many packages of each should be produced each day so as to maximise his profit, if he operates each machine for atmost 12 hours a day ? Convert it into an LPP and solve graphically.