

MATHEMATICS

SECTION A

January 6, 2024

1 Matrices

1. A is a square matrix with $|A| = 4$. Then find the value of $|A \cdot (\text{adj}A)|$.
2. For the matrix $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$, find $(A + A')$ and verify that it is a symmetric matrix.
3. Using elementary row transformations, find the inverse of the matrix

$$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}.$$

4. Using matrices, solve the following system of linear equations :

$$x + 2y - 3z = -4$$

$$2x + 3y + 2z = 2$$

$$3x - 3y - 4z = 11$$

5. Using properties of determinants, find the value of k if $\begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix} = k(x^3 + y^3)$.

2 probability

6. In a multiple choice examination with three possible answers for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing ?
7. The probabilities of solving a specific problem independently by A and B are $\frac{1}{3}$ and $\frac{1}{5}$ respectively. If both try to solve the problem independently, find the probability that the problem is solved.
8. There are three coins. One is a coin having tails on both faces, another is a biased coin that comes up tails 70% of the time and the third is an unbiased coin. One of the coins is chosen at random and tossed, it shows tail. Find the probability that it was a coin with tail on both the faces.

3 Algebra

9. Prove that :

$$\cos^{-1}\left(\frac{12}{13}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \sin^{-1}\left(\frac{56}{65}\right)$$

4 vector

10. Find the vector equation of the plane which contains the line of intersection of the planes $\vec{r} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) - 4 = 0$, $\vec{r} \cdot (2\hat{i} + \hat{j} - \hat{k}) + 5 = 0$ and which is perpendicular to the plane $\vec{r} \cdot (5\hat{i} + 3\hat{j} - 6\hat{k}) + 8 = 0$.
11. Find the vector equation of a line passing through the point $(2, 3, 2)$ and parallel to the line $\vec{r} = (-2\hat{i} + 3\hat{j})\lambda + (2\hat{i} - 3\hat{j} + 6\hat{k})$. Also, find the distance between these two lines.
12. Find the value of x such that the four points with position vectors, $\mathbf{A}(3\hat{i} + 2\hat{j} + \hat{k})$, $\mathbf{B}(4\hat{i} + x\hat{j} + 5\hat{k})$, $\mathbf{C}(4\hat{i} + 2\hat{j} - 2\hat{k})$, $\mathbf{D}(6\hat{i} + 5\hat{j} - \hat{k})$ are coplanar.
13. Find the vector equation of the plane determined by the points $\mathbf{A}(3, -1, 2)$, $\mathbf{B}(5, 2, 4)$ and $\mathbf{C}(-1, -1, 6)$. Hence, find the distance of the plane, thus obtained, from the origin.
14. Find the coordinates of the foot of the perpendicular \mathbf{Q} drawn from $\mathbf{P}(3, 2, 1)$ to the plane $2x - y + z + 1 = 0$. Also, find the distance \mathbf{PQ} and the image of the point \mathbf{P} treating this plane as a mirror.

5 Differentiation

15. A ladder $13m$ long is leaning against a vertical wall. The bottom of the ladder is dragged away from the wall along the ground at the rate of $2cm/sec$. How fast is the height on the wall decreasing when the foot of the ladder is $5m$ away from the wall?
16. If $y = (\log x)^x + x^{\log x}$, find $\frac{dy}{dx}$.
17. If $x = \sin t$, $y = \sin pt$, prove that $(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + p^2y = 0$.
18. Differentiate

$$\tan^{-1} \left(\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}} \right)$$

with respect to $\cos^{-1} x^2$.

6 Differential equation

19. Form the differential equation representing the family of curves $y = A \sin x$, by eliminating the arbitrary constant A .
20. Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{xy}{x^2+y^2}$, given that $y = 1$ when $x = 0$.
21. Solve the differential equation $\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$, given that $y = 1$ when $x = 0$.
22. Prove that $\int_0^a f(x) dx = \int_0^a f(a-x) dx$, and hence evaluate $\int_0^1 x^2 (1-x)^n dx$.

7 Integration

23. Using integration, find the area of the region bounded by the parabola $y^2 = 4x$ and the circle $4x^2 + 4y^2 = 9$.
24. Using the method of integration, find the area of the region bounded by the lines $3x - 2y + 1 = 0$, $2x + 3y - 21 = 0$ and $x - 5y + 9 = 0$.
25. Using integration, find the area of the region bounded by the line $y = 3x + 2$, the x -axis and the ordinates $x = -2$ and $x = 1$.
26. Using integration, find the area of the smaller region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line $\frac{x}{3} + \frac{y}{2} = 1$.

8 Functions

27. Let $A = \mathbb{R} - \{2\}$ and $B = \mathbb{R} - \{1\}$. If $f : A \rightarrow B$ is a function defined by $f(x) = \frac{x-1}{x-2}$, show that f is one-one and onto. Hence, find f^{-1} .
28. Show that the relation S in the set $A = \{x \in \mathbb{Z} : 0 \leq x \leq 12\}$ given by $S = \{(a, b) : a, b \in \mathbb{Z}, a - b \text{ is divisible by } 3\}$ is an equivalence relation.

9 Optimization

29. A dietician wishes to mix two types of food in such a way that the vitamin contents of the mixture contains at least 8 *units* of vitamin *A* and 10 *units* of vitamin *C*. Food *I* contains 2 *units/kg* of vitamin *A* and 1 *unit/kg* of vitamin *C*. It costs ₹50 *perkg* to produce food *I*. Food *II* contains 1 *unit/kg* of vitamin *A* and 2 *units/kg* of vitamin *C* and it costs ₹70 *perkg* to produce food *II*. Formulate this problem as a LPP to minimise the cost of a mixture that will produce the required diet. Also find the minimum cost.
30. A manufacturer produces nuts and bolts. It takes 1 hour of work on machine *A* and 3 hours on machine *B* to produce a package of nuts. It takes 3 hours on machine *A* and 1 hour on machine *B* to produce a package of bolts. He earns a profit of ₹ 35 per package of nuts and ₹ 14 per package of bolts. How many packages of each should be produced each day so as to maximise his profit, if he operates each machine for atmost 12 hours a day ? Convert it into an LPP and solve graphically.