SECTION A

January 4, 2024

1 Algebra

1. **A** is a square matrix with $\left|\mathbf{A}\right| = 4$. Then find the value of $\left|\mathbf{A}.\,(adj\mathbf{A})\right|$.
2. For the matrix $\mathbf{A} = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$, find $(\mathbf{A} + \mathbf{A}')$ and verify that it is a symmetric matrix.
3. Using elementary row transformations, find the inverse of the matrix
$$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}.$$
4. Using matrices, solve the following system of linear equations :
$$\begin{aligned} x + 2y - 3z &= -4 \\ 2x + 3y + 2z &= 2 \\ 3x - 3y - 4z &= 11 \end{aligned}$$
5. Using properties of determinants, find the value of **K** if $\begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix} = k\left(x^3 + y^3\right)$.
6. Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{xy}{x^2+y^2}$,given that $y = 1$ when $x = 0$.

2 probability

7. In a multiple choice examination with three possible answers for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing ?
8. The probabilities of solving a specific problem independently by **A** and **B** are $\frac{1}{3}$ and $\frac{1}{5}$ respectively. If both try to solve the problem independently, find the probability that the problem is solved.

9. There are three coins. One is a coin having tails on both faces, another is a biased coin that comes up tails 70% of the time and the third is an unbiased coin. One of the coins is chosen at random and tossed, it shows tail. Find the probability that it was a coin with tail on both the faces.

3 Trigonometry

10. prove that :

$$\cos^{-1}\left(\frac{12}{13}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \sin^{-1}\left(\frac{56}{65}\right)$$

4 vector and geometry

11. Find the vector equation of the plane which contains the line of intersection of the planes $\vec{r}.\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) - 4 = 0$,$\vec{r}.\left(2\hat{i} + \hat{j} - \hat{k}\right) + 5 = 0$ and which is perpendicular to the plane $\vec{r}.\left(5\hat{i} + 3\hat{j} - 6\hat{k}\right) + 8 = 0$.

12. Find the vector equation of a line passing through the point $(2, 3, 2)$ and parallel to the line $\vec{r} = \left(-2\hat{i} + 3\hat{j}\right) \lambda \left(2\hat{i} - 3\hat{j} + 6\hat{k}\right)$. Also, find the distance between these two lines.

13. Find the value of **x** such that the four points with position vectors, $A\left(3\hat{i} + 2\hat{j} + \hat{k}\right)$, $B\left(4\hat{i} + x\hat{j} + 5\hat{k}\right)$, $C\left(4\hat{i} + 2\hat{j} - 2\hat{k}\right)$, $D\left(6\hat{i} + 5\hat{j} - \hat{k}\right)$ are coplanar.

14. Find the vector equation of the plane determined by the points $\mathbf{A}(3, -1, 2)$, $\mathbf{B}(5, 2, 4)$ and $\mathbf{C}(-1, -1, 6)$. Hence, find the distance of the plane, thus obtained,from the origin.

15. Find the coordinates of the foot of the perpendicular **Q** drawn from $\mathbf{P}(3, 2, 1)$ to the plane $2x - y + z + 1 = 0$. Also, find the distance **PQ** and the image of the point **P** treating this plane as a mirror.

5 calculus

16. If $\mathbf{y} = (logX)^X + X^{logX}$, find $\frac{dy}{dx}$.

17. If $\mathbf{x} = \sin t, \mathbf{y} = \sin pt$, prove that $\left(1 - x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0$.

18. Differentiate

$$\tan^{-1}\left(\frac{\sqrt{1 + x^2} - \sqrt{1 - x^2}}{\sqrt{1 + x^2} + \sqrt{1 - x^2}}\right)$$

with respect to $\cos^{-1}\mathbf{x}^2$.

19. prove that $\int_0^a f(x)\,dx = \int_0^a f(a - x)\,dx$, and hence evaluate $\int_0^1 x^2(1 - x)^n\,dx$.

20. Form the differential equation representing the family of curves $y = A\sin x$, by eliminating the arbitrary constant **A** .

21. Solve the differential equation $\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$, given that $y = 1$ when $x = 0$.

22. Using integration, find the area of the region bounded by the parabola $y^2 = 4x$ and the circle $4x^2 + 4y^2 = 9$.

23. Using the method of integration, find the area of the region bounded by the lines $3x - 2y + 1 = 0, 2x + 3y - 21 = 0$ and $x - 5y + 9 = 0$.

24. Using integration, find the area of the region bounded by the line $y = 3x + 2$, the x-axis and the ordinates $\mathbf{x} = -2$ and $\mathbf{x} = 1$.

25. Using integration, find the area of the smaller region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line $\frac{x}{3} + \frac{y}{2} = 1$.

6 Relations and Functions

26. Let $\mathbf{A} = \mathbf{R} - \{2\}$ and $\mathbf{B} = \mathbf{R} - \{1\}$. If $f : \mathbf{A} \to \mathbf{B}$ is a function defined by $\mathbf{f}(x) = \frac{x-1}{x-2}$, show that **f** is one-one and onto. Hence, find $\mathbf{f}^{-1}$.

27. Show that the relation **S** in the set $\mathbf{A} = \{x \in Z : 0 \leq x \leq 12\}$ given by $\mathbf{S} = \{(a, b) : a, b \in \mathbf{Z}, |a - b|$ is divisible by 3 $\}$ is an equivalence relation.

7 Linear programming

28. A dietician wishes to mix two types of food in such a way that the vitamin contents of the mixture contains at least $8\,units$ of vitamin **A** and $10\,units$ of vitamin **C**. Food **I** contains $2\,units/kg$ of vitamin **A** and $1\,unit/kg$ of vitamin **C**. It costs ₹$50\,perkg$ to produce food **I**. Food **II** contains $1\,unit/kg$ of vitamin **A** and $2\,units/kg$ of vitamin **C** and it costs ₹$70\,perkg$ to produce food **II**. Formulate this problem as a LPP to minimise the cost of a mixture that will produce the required diet. Also find the minimum cost.

29. A manufacturer produces nuts and bolts. It takes 1 hour of work on machine **A** and 3 hours on machine **B** to produce a package of nuts. It takes 3 hours on machine **A** and 1 hour on machine B to produce a package of bolts. He earns a profit of ₹ 35 per package of nuts and ₹ 14 per package of bolts. How many packages of each should be produced each day so as to maximise his profit, if he operates each machine for atmost 12 hours a day ? Convert it into an LPP and solve graphically.

30. A ladder $13m$ long is leaning against a vertical wall. The bottom of the ladder is dragged away from the wall along the ground at the rate of $2cm/sec$. How fast is the height on the wall decreasing when the foot of the ladder is $5m$ away from the wall ?