

UNIT-4

Exercises
Practicals

Binaryzation:- It is thresholding technique in image processing that simplifies an image by converting its grayscale pixels values (ranging from 0 to 255) into binary values. (It)

- * It is widely used as preprocessing step in tasks like OCR, document scanning & object segmentation
- * Main goal is to separate foreground from background
- * It's based on idea that important visual information is captured only at levels - on/off, black/white

Formulae

Let $I(x,y)$ be the grayscale intensity of pixel (x,y) . Let T be the threshold.

The Binary Image $B(x,y)$ be given by-

$$B(x,y) = \begin{cases} 1 & \text{if } I(x,y) > T \\ 0 & \text{otherwise} \end{cases}$$

Types of Binarization

1) Global Thresholding

- A single threshold T is applied to the whole image.
- * Simple and fast
- * Not suitable for images with uneven lighting or shadows

Formula:- $B(x,y) = \begin{cases} 1 & \text{if } I(x,y) > T \\ 0 & \text{if } I(x,y) \leq T \end{cases}$

Adaptive Thresholding

- * The threshold $T(x,y)$ is computed locally for each pixel based on intensity of neighbouring pixels
- * Better for non-uniform illumination shadows

Common approaches

- * Mean Adaptive Thresholding $T(x,y) = \text{avg}$ surrounding pixels
- * Gaussian Adaptive Thresholding - weighted using a Gaussian window

Otsu's Method

- An automatic global thresholding that minimizes intra-class variance and maximizes inter class variance

2] Basic set theory

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1) what is set (First Principles)

- ↳ A set is a fundamental concept in mathematics - its collection of distinct objects.
- ↳ Sets are usually written as:-

$$A = \{a, b, c, d\}$$

meaning set A containing elements
 $\{a, b, c, d\}$

- The order does not matter in a set
duplicates are not counted.
- If an element x belongs to set A , we write :-

$$x \in A$$

- If x doesn't belong to A , we write:-

$$x \notin A$$

2) Why set theory is important in Image Processing

Images are made up of pixels can be considered as elem. in a set. Many image processing operations can be designed / described as Ops on set of pixels

e.g :- foreground pixels of binary image can be viewed as a set A

* Bgrd pixel form another set B

Many ops like morphological op's, region analysis & based on manipulating these pixels

a) Union : In img processing if A & B
represent set of pixel then union
regions represent comb. of pixels in both regions

b) Intersection : common pixels of both sets -
like overlap of 2 shapes

c) Difference ($A - B$)
↳ contains elem in A & not in B
 $A - B$ ↳ subtracting pixel from one shape from other

d) Compliment (A^c)
↳ The compliment of A cont all elem not in A
↳ compliment of foregrd is bgrd & vice versa

4) Set Theory of Binary Images

Morphology
* Consider a binary image as set of points
 $A = \{(x, y)\} : \text{pixel at } (x, y)$

A cont. all foregrd pixels
Bgrd pixels are the compliment of A

S Erosion: Shrinks foreground set A by B

Dilation: Expands foreground set A by B

6 Maths of Morphological Ops.

$$A \ominus B = \{ z : B_z \subseteq A \}$$

B_z is B translated by z

• Dilation of set A by B

$$A \oplus B = \{ z : (B^{\text{ref}})_z \cap A \neq \emptyset \}$$

$\leftarrow B^{\text{ref}} = \text{ref of } B$

Intuitive Explan'n

* Imagine the image

Set A = all points where obj exist

Set B = small gray

Erosion

Morphology means: study of shape & structure

In image processing, it refers to operations that process images based on shapes.

Binary Image: set of foreground (white) pixels

e.g. $A = \{ (x,y) : \text{pixel at } (x,y) = 1 \}$

Erosion :-

Goal:- shrink white regions, remove boundary pixels
useful for removing noise.

Intuition:- Math: $A \ominus B = \{ z \in E : B \subseteq$

(1)

Ans * Erosion never adds pixel

$$* A \ominus B \subseteq A$$

2 Dialation

Grow the white region add boundary pixel useful for bridging gap

$$A \oplus B = \{ z \in E : B^{\text{ref}} \cap A \neq \emptyset \}$$

* Dialation never reduce white pixels:

$$A \subseteq A \oplus B$$

3 Opening ($A \circ B$)

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Goal :- Remove small white noise details
preserving main shape.

* first erode :- shrink all white obj;
Then dilate :- restore them (big ones
would come back)

Operation :-

$$A \circ B = (A \ominus B) \oplus B$$

Use Case :-

- Smooth Boundaries
- Remove small blobs
- Disconnect thin Bridges

~~Prep~~ ~~Obj~~
~~→ Res~~

Closing $A \bullet B$

↳ fill small Blackholes in Obj region

First dilate :- expand all white
objects

Then erode :- shrink them back

$$A \bullet B = (A \oplus B) \ominus B$$

Use:-

- (1) Full range
- (2) Smooth boundaries

#9 COLOR IMG REPRESENT.

→ A color image is not just matrix of no. its per pixel desc. of how human perceive color. But computers can't see color so we represent color model which break down each pixel's color into components

1) RGB (Red Green Blue)

Perception of color :-

Our eye is sensitive has 3 types of cone cells, each sensitive to:-

- Red (R)
- Green (G)
- Blue (B)

#1 RGB model

→ Each pixel is stored as a triplet (R, G, B) range from 0 to 255 [i.e.]

$$\text{Red} = (255, 0, 0)$$

$$\text{Green} = (0, 255, 0)$$

$$\text{Blue} = (0, 0, 255)$$

$$\text{White} = (255, 255, 255)$$

$$\text{Black} = (0, 0, 0)$$

Q HSV (Hue Sat Value)

③
Perception
Formation

More human intuitive color

Hue \rightarrow color type ($0 \text{--} 360^\circ$)

Saturation: intensity/purity ($0 \text{=} \text{gray}$)

Value = brightness ($0 \text{=} b \text{ -- } 1 \text{=} \text{full color}$)

RGB \leftrightarrow HSV

$$V = \max(R, G, B)$$

$$S = \begin{cases} 0 & \text{if } V=0 \\ \frac{V - \min(R, G, B)}{V} & \end{cases}$$

$$H = \begin{cases} 0 & \text{if } \Delta=0 \\ \end{cases}$$

$$H = \begin{cases} 0 & \text{if } \Delta=0 \\ 60 \times \left(\frac{G-B}{\Delta} + 1.6 \right) & V=R \\ 60 \times \left(\frac{B-R}{\Delta} + 2 \right) & V=G \\ 60 \times \left(\frac{R-G}{\Delta} + 4 \right) & V=B \end{cases}$$

$$\Delta = V - \min(R, G, B)$$

Mathematical model

RGB

$$L_{\text{memory}} = 3 \cdot \text{width} \cdot \text{height} \cdot 8 \text{ bits}$$

RGB \rightarrow Grayscale

$$\text{Gray} = 0.2989 \cdot R + 0.5896 \cdot G + 0.1115 \cdot B$$

$$\text{eg } R=0.8 \quad G=0.6 \quad B=0.2$$

$$C_{\max} = \max(R, G, B) \\ = 0.8$$

$$C_{\min} = \min(R, G, B) \\ = 0.2$$

$$\Delta = C_{\max} - C_{\min} \\ = 0.8 - 0.2 = 0.6$$

$$V = C_{\max}$$

$$S = \begin{cases} 0 & \text{if } C_{\max} = 0 \\ \frac{\Delta}{C_{\max}} & = \frac{0.6}{0.8} = 0.75 \end{cases}$$

$$\text{Hue} = \begin{cases} 0 & \text{if } \Delta = 0 \\ 60 \cdot \frac{(G-B) \cdot 1.6}{\Delta} & C_{\max} = R \end{cases}$$

$$\left(\frac{0.9}{0.6}\right) \cdot 1.6 \quad 60 \cdot \frac{(B-R) + 2}{\Delta} \quad C_{\max} = G$$

$$\left(\frac{2}{3} \cdot 1.6\right) \quad 60 \cdot \frac{(R-G) + 1}{\Delta} \quad B$$

Color Image Processing

- color image processing is branch of image processing that handles image in colors not just grayscale.
- grayscale has just one channel (just intensity)
- A color image typically has 3 channels: RGB, HSV etc.

Each pixel carries more information - not just how bright it is but what color it is.

Why Color?

- color provides more info. to distinguish objects, regions, & textures.
- you can't rip tomatoes from a green vine in grayscale.

Basic Processes in CV

↳ Color enhancement refers to improving appearance, visual perception or interpretability of a color image by adjusting its color related features.

↳ Real world images suffer from:

- poor lighting
- lower contrast
- under exposure / over exposure
- color bias

Techniques

① Contrast & Brightness Adjustment

$$\rightarrow I'(x,y) = \alpha I(x,y) + \beta$$

$I(x,y)$ = original pixel value at position (x,y) for RGB

$\alpha > 1 \rightarrow$ Inc. contrast

β = control brightness

Contrast = how different is from each color

② Histogram Equalization

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- Theory
- Redistributions pixels intensities to flatten the histogram to increase contrast
 - Applied separately to each of R,G,B

- Problem
- Doing this independently might shift color unnaturally

$cdf(i) \in$ cumulative distribution function of pixel histogram

$$c.g. \text{ value} \in \frac{cdf(i) - cdf_{min}}{(M \times N) - cdf_{max}} \times L$$

$M \times N \in$ image size
 $L \in$ no. of intensity levels

③ Saturatⁿ Adj (HSL or HSV)

S↑ → color pop more
Decrease S

④ Colour Normalizatⁿ

① compute mean of R, G, B

$$\bar{M}_R, \bar{M}_G, \bar{M}_B$$

② find global average

$$M_{gray} = \frac{\bar{M}_R + \bar{M}_G + \bar{M}_B}{3}$$

$$R' = R \cdot \frac{M_{gray}}{M_R}$$

$$G' = G \cdot \frac{M_{gray}}{M_G}$$

$$B' = B \cdot \frac{M_{gray}}{M_B}$$

⑤ Gamma Correctn

$$I'(x, y) = I(x, y)^r$$

$r \geq 1$:- lightens image

$r < 1$ Darkens

~~Color Restoration - Correcting color distortions due to non-linearities or color balance~~

Color Seg : Partitioning of an image into regions based on color similarity.

Spatial filtering

Every Image is a 2D funcn of intensity.
In digital images, intensity values are sampled and quantised into pixels.

A spatial filter modifies the intensity of each pixel by taking account of the neighbour pixels.

Steps for spatial filtering

- 1) choose filter kernel eg (3×3 matrix)
- 2) slide over kernel over each pixel
- 3) At each position compute sum of products of kernel values and pixel values

This operatⁿ is called convolution
or correlation

$-g(x,y)$

for gray scale image

$$g(x,y) = \sum_{i=-K}^K \sum_{j=-K}^K f(x+i, y+j) h(i,j)$$

$f(x,y)$ = original image

$h(i,j)$ = filter mask or kernel

$g(x,y)$ = output image.

$K \quad K$

$$g_c(x,y) = \sum_{i=-K}^K \sum_{j=-K}^K f_c(x+i, y+j) h(i,j)$$

$c \in \{R, G, B\}$

Types of Spatial Filtering

3) Filtering in RGB Vector Space

$$\vec{p}'(x,y) = [R(x,y), G(x,y), B(x,y)]$$

- vector mean (averaging RGB vector)
- Vector median (reducing noise)
- Euclidean dist for filtering

Working with RGB Images Vector

Digital Image are typically represented in RGB model i.e. each pixel having 3 values R, G, B

$$\vec{P} = [R, G, B]^T$$

→ Instead of processing R, G, B indirectly we process them jointly as a vector this is what is meant by "working in RGB vector space"

Vector Rep = $P_i = \begin{bmatrix} R \\ G \\ B \end{bmatrix}$

$$\|\vec{P}_i\| = \sqrt{R_i^2 + G_i^2 + B_i^2}$$

$$D(P_i, P_j) = P_i - P_j$$

Angle (color similarity based on direction)

Types of Vector Based Filtering

A) Vector mean filter

Used for smoothing or blurring

$$P_{\text{mean}} = \frac{1}{N} \sum_{i=1}^N P_i$$

- each pixel is averaged out over a neighbourhood.

- result is new RGB filter that replaces central pixel

B) Vector Median filter

$$\hat{P}_{\text{median}} = \arg \left(\min \sum_{i=1}^N |P_i - P_j| \right)$$