

Cooperative Game Theory with MATLAB and Mathematica

From Fundamentals to Advanced Computational Techniques

Holger I. MEINHARDT *

3rd February 2025

Preface

The book describes how the reader can use our software tools for MATLAB[®] and *Mathematica*[®] for her/his research or teaching. It is divided into three parts. The first part deals with the prerequisites of Cooperative Game Theory, *Mathematica*[®], and MATLAB[®]. To this end, the first chapter is thought as a quick refresher of the essentials of cooperative game theory, the chapters devoted to *Mathematica*[®] and MATLAB[®] are designed to familiarize even readers with little or no background in these languages with their semantics and syntax for mastering the underlying programming concepts and techniques. These concepts and techniques are illustrated on code snippets to cope with some basic game theoretical problems. In contrast, the second part develops how our MATLAB[®] toolbox can be applied to study solution concepts and properties for TU games with and without a trivial coalition structure. Before its usage on selected examples is explained, the game theoretical background is extensively introduced and discussed. Important fundamental aspects and techniques of a computer-based analysis with our MATLAB[®] toolbox are described and are further panned out as a basis for the reader's explorations. Particular subjects like the plotting of solutions as well as the investigation of the axiomatization (principles of distributive justice) of selected solution concepts by our software tool are comprehensively discussed and developed. Leading then into the discussion of solving game theoretical hard problems by parallel as well as GPU computation as the backbone of High-Performance Computing. To end up in object-oriented programming for retrieving and modifying game data to ensure a consistent computation environment for doing game theory by a unified computer-based approach. The last part retrieves the study of solution concepts and properties for TU games with a trivial coalition structure with *Mathematica*[®]. These are the building blocks to direct the reader to the theory of industrial cooperation while focusing on the partition function approach that enables us to study the stability of cartel agreements.

Analogously to the previous part, we revive for our *Mathematica*[®] package the essentials of High-Performance Computing while establishing its parallel computational features. The remaining two chapters deal with the transfer of game data between these two worlds of programming languages. Finally, additional background information is provided in various appendices.

Karlsruhe, Germany
February 2025

Holger Ingmar Meinhardt

About this Book

Nowadays, numerical computation and computer algebra systems are common standards for scientists in various fields. Although algorithmic game theory is a very dynamic field of research, computer-based approaches to analyzing cooperative games are still uncommon. This book fills this gap while describing our software tools for MATLAB[®] and *Mathematica*[®] for doing interactively cooperative game theory on a computer in a uniform environment. Before a wide variety of examples are introduced, the computational and game theoretical aspects are extensively discussed and explained. Finally, moving to parallel- as well as GPU computation, which are at the cutting edge of High-Performance Computing for studying game theoretical hard problems. The book is suitable for readers with little or no background in MATLAB[®] or *Mathematica*[®] due to its emphasis on carefully introducing their programming concepts and techniques for self-study. In this respect, it essentially neither requires a special previous background in Game Theory, apart from mathematical maturity, nor programming knowledge. However, the book covers thoroughly a course in cooperative game theory with side payments on an advanced level, whether MATLAB[®] or *Mathematica*[®] is used or not. Due to this general approach to discussing the introduced algorithms, the software tools described can also be transferred to other programming languages without much additional effort, such as Python or R, to name just two other software systems that are widely used in the academic world.

The book is addressed to scientists, graduate students, advanced undergraduates, and programmers who want to study several aspects of cooperative games with side payments (TU games) by means of a computer-aided approach with the software systems MATLAB[®] and *Mathematica*[®]. The novelty of the book lies in the fact that it supports the theoretical presentation of concepts from cooperative game theory through well-selected game examples with a complexity level that could not be examined without software support. Enhancing our theoretical understanding through faster access to new ideas that would otherwise not be accessible to us. To illuminate this itinerary of knowledge further, not only sophisticated numerical examples are discussed, but also a selection of algorithms for addressing large-scale issues and their implementations into both software systems. Providing the reader with the necessary skills to follow his own fluctuating but nevertheless promising journey of knowledge.

To prepare the reader for this journey, the present book provides, after having discussed through a first step the underlying theory, computer-aided means to model and impose solution analysis on transferable utility games (TU games) using MATLAB and *Mathematica*. The book is divided into three main parts and an appendix. We start off with a part-wise general overview of the book and then move over to a more detailed chapter-wise summary structured by parts. The software tools accompanying the book are available online. At the end of this book's itinerary, one will find a brief overview of the software companions.

- The preparatory Part I is composed of three chapters to formalize the reader with the essential principles of cooperative game theory with side payments (TU games), MATLAB, and *Mathematica*. To prepare readers, not all game classes treated by the book were covered in Chapter 1. We would like to point out that the first chapter only deals with those concepts/classes that are relevant to both Parts II and III that can be quickly grasped by a reader without possessing any good prior knowledge of Game Theory. Otherwise, we would have to immediately introduce quite extensive technical machinery without being able to explain the concepts in more detail with the help of our software tools, which will only be introduced in Parts II and III. The book aims to clarify the concepts of cooperative game theory, after their theoretical discussion, with the help of our programmed software tools, instead of bothering the reader with small tasks without making them aware of the entire complexity of the problem. Standard textbooks use small exercises for that, but with our software tools, much more complex problems can be discussed, and it expands rapidly the understanding, which we consider a real advantage of a computer-aided approach. For example, the intrinsic combinatorics in game theory concepts means that certain structures and properties of games cannot be illustrated in 3- or 4-person games. Very often, only games with at least five persons reveal enough structure to make the point clear. Nowadays, in the computer age, no one should be faced with having to create these structures and properties manually. This becomes particularly problematic if one wants to clarify the axiomatization of solution concepts. For this reason, we only outline the axiomatization for some solution concepts in Chapter 1. This topic is deepened in Chapter 7 with a whole set of MATLAB routines to examine the underlying axiomatization of a calculated solution.
- In contrast, Part II is composed of eight chapters to investigate TU games using our conceived software toolbox

for MATLAB. There, we mainly present the mathematical proofs, which will be discussed in detail in a further step using our designed MATLAB routines to deepen their understanding. Among others, it develops, after having imparted the theoretical background, how the software means designed for MATLAB can be applied to studying solution concepts and properties for TU games with and without a trivial coalition structure. Besides this, graphical extensions, real-word applications, axiomatization of selected solution concepts, object-oriented programming (OOP) techniques, multithreading as a tool of implicit parallelization, certain forms of explicit parallelization on multiple processors or for graphics processing units (GPUs), are discussed within a game theoretical context.

- However, Part III includes a total of five chapters to examine cooperative games with side payments using the **Wolfram Language**. It retrieves the study of solution concepts and properties for TU games with a trivial coalition structure under *Mathematica*, thereby presenting an alternative set of routines for scrutinizing TU games through multithreading and explicit parallelization. Note in this context that the theoretical foundation is treated in Chapter 4. There is no need to resume these fundamentals in Part III with *Mathematica*. It is only necessary to discuss the conceived *Mathematica* counterparts that allow us to study these topics with the **Wolfram Language** while referring to the mathematical results of the MATLAB section. In addition, this part focuses on cooperative games arising from industrial cooperation while discussing in the initial step the theoretical foundation, in order to move then along to the next step to establish how one can profit from parallel acceleration under *Mathematica* while investigating game issues. The remaining two chapters are devoted to the grand theme of cross-checking, thereby focusing in more detail on some fundamental methods for exchanging data, delegating, and running computer tasks into a different software environment without leaving the initial software platform.
- The book closes with the appendix, which provides some further background related to propositional logic, group theory, representation theory, numerical round-off errors, and our software tools.

As mentioned above, the book is divided into a preparatory, a MATLAB, and a *Mathematica* part. Readers who want to study the book through self-study and are familiar with these topics can skip the first part, thereby reading the remaining parts in isolation, simply following their preferences. However, readers not acquainted with these topics ought to read Chapter 1 first, and only then should they progress in accordance with their interests in reading either Chapter 2 or Chapter 3. To continue at all events with Chapter 4 of Part II, even if one is only interested in Part III. This is because the theoretical foundations necessary for understanding Part III are situated in this chapter. Though we have tried to cross-reference as much as possible in the text, not everything is referenceable in Part III.

In what follows, we provide a short synopsis of the upcoming chapters, structured by parts. We start with the chapter overview for the first part and end with a survey of the applied software platforms.

About Part I

The chapters of Part I have a preparatory nature; and can be skipped by informed readers. Each chapter can be read in isolation. However, novices in game theory ought to start with the first chapter.

Chapter 1 : This preparatory chapter is devoted to resuming the relevant background of cooperative game theory with side payments (TU games). In particular, it is conceived as a starter to refresh some fundamental principles of TU games. We formalize first treatments related to certain game classes, such as convexity, super-additivity, monotonicity, or zero-monotonicity. Most importantly, some solution concepts, such as the (pre-)kernel, (pre-)nucleolus, or the Shapely value, are resumed in formal but not rigorous technical treatments. Albeit these concepts are introduced formally, the emphasis is, however, more on providing the reader with the underlying intuition and motivation behind them. Technical aspects of game properties and solutions are more thoroughly examined in Part II or Part III.

Besides this, we present in the introduction to this chapter our interpretation of a TU game to efface with some common misunderstandings of cooperative game theory, notably, the prevalent opinion, that only binding agreements can be studied for cooperative games, caused probably by the fact that the conceptual aspects of cooperative game theory are neither well explained nor motivated in the literature while focusing merely on the technical aspect of results. The annoying and at the same time amazing consequence of this postulate is that it is unclear why one should care about solution concepts from cooperative game theory when they are enforceable by an unbiased arbitrator or a legal system. Why should one formalize proposals or counter-proposals, dominance, or stability issues when everything is binding? We shall learn that the wreaked conceptual damage of this postulate is dissolvable while considering the axiomatic foundation of solution concepts as a description of principles of distributive justice. Then, agreements can be non-binding and thereby stable simply because actors submit themselves by self-interest under the immanent rules of justice.

- Chapter 2 :** In this chapter, we are going to present some basic operations that will give us some understanding of how we can use the high-level programming language of MATLAB to investigate transferable utility games. Having introduced and discussed some basic operations of MATLAB, we apply our knowledge gained by writing in the first step some small routines to solve linear problems and drawing their solution sets in two- and three-dimensional graphics. In the next step, we deepen and enhance these skills while implementing routines to convert the binary-encoding of double-precision floating-point numbers to their equivalent decimal representations. Then, our attention is turned to the top-down design process. That is a formal approach to designing a computer program. Its design process is illustrated for an airport cost allocation problem borrowed from the literature. Having understood and mastered the program design of MATLAB through these examples, we introduce the toolbox approach to represent a coalitional game with transferable utility under MATLAB by the unique integer representations of coalitions. While understanding this, we shall discuss some alternative representation methods for a TU game from the literature. Specifically, we focus on a concise representation through the marginal contribution networks (MC-nets). Finally, we test our skills on some basic solution schemes from game theory, such as the standard solution, Shapley value, and the computation of the nucleolus for three-person games.
- Chapter 3 :** Similar to the foregoing chapter on MATLAB, we are going to present some basic operations that shall allow us to understand how we can use the versatile programming language of *Mathematica* to investigate transferable utility games. Having introduced and discussed some basic operations of *Mathematica*, we apply our knowledge gained from writing some small routines to get the game representation of weighted majority or bankruptcy games. Then, we turn our attention to some basic solution schemes like the Talmudic Rule (generalized contested garment principle), the computation of the nucleolus for three-person games while applying a unique formula, and the Shapley value. We close this chapter while resuming crucial aspects of optimization theory and the Nash equilibrium, which are of particular importance for studying cooperative games in partition function form.

About Part II

Part II deals with studying TU games by means of MATLAB. Readers not acquainted with this topic should have read Chapter 2 before continuing. Then, they should move on to Chapter 4 to study chiefly the modeling and solution sections. Readers familiar with this topic should also concentrate on this part in the first reading but may wish to skip the theoretical part while focusing primarily on handling the software means to study TU games. The remaining topics of Part II can then be studied in isolation.

- Chapter 4 :** Having mastered the preparatory part, we are in a position to clarify and extend game theoretical concepts by means of mathematical proofs. In addition, we shall observe how the knowledge of these concepts can be deepened using software-based means conceived in MATLAB. To this end, we start with the modeling part while establishing how game models can be described by means of our software tool. Then, the solution part is considered by representing several algorithms to compute the Shapley value, (pre-)nucleolus, elements of the (pre-)kernel, and much more. Not without first examining the theoretical background by providing numerous selected mathematical results that characterize game solutions and game properties. However, to elucidate their intrinsic complexity, programmed software is used to accentuate the underlying layers. Particularly, some selected and crucial game classes with a trivial coalition structure are presented. Then, the most common solution concepts and their corresponding algorithms are discussed, including how to determine them using user-defined MATLAB tools. Besides, the associated algorithms are introduced, discussed, and finally implemented into MATLAB code in conjunction with some command and code annotations. Instead of using only built-in solvers to get game solutions, we illustrate how one can achieve the same objective using MATLABs application programming interface (API) by calling external third-party solvers. Finally, we discuss how crucial game properties and convex solution sets can be analyzed using our software tool.
- Chapter 5 :** After we have carried out detailed studies related to transferable utility games (TU games) with a trivial coalition structure, we move forward to investigate in more detail TU games with non-trivial coalition structures. Notably, we shall see how the core, super-additive cover, (pre-)kernel, and (pre-)nucleolus must be generalized to cope with distributive arbitration (fairness standards) when non-trivial coalition structures are likely to prevail. In the same vein, we also study fairness and related values for coalition structures. Moreover, to advance our understanding of compliance with agreements, we shall investigate the stability of an agreement within a game setting, referring to the nucleolus,

pre-kernel, and Shapley value. As in the foregoing Chapter 4, the rather technical analysis will be underpinned by software-aided means designed for our MATLAB toolbox to elucidate the arising combinatorial complexity issues.

Chapter 6 : Having treated transferable utility games (TU games) with trivial and non-trivial coalition structures, we shall address the question of how one can depict three-dimensional convex solutions for four-person games, such as the core, strong ϵ -cores, the Weber set, or visualize the geometric properties of the nucleolus or kernel by means of our software tool. In a further step, we move on to study simple games and then the political decision-making process through some real-world examples through weighted majority games. Especially, we are investigating the political decision-making process of the UN Security Council, the minimum homogeneous representation of weighted majority games, and their implication on the shape of the (pre-)kernel solution.

However, all these approaches have measured the power of individuals in a political decision-making process without taking into account the formation, control, and domination of a network of relationships to influence a voting outcome and institutions to serve one's interests. For incorporating such an underlying mesh of relations into the measurement of power, a network is specified by an undirected graph in order to obtain a game representation in characteristic function form from which such an analysis is feasible. This kind of power analysis is exemplified by Florentine marriage and business relationships during the early Renaissance. The chapter closes by discussing some procedures to handle numerical errors when seeking a pre-kernel point.

Chapter 7 : The axiomatic foundation of cooperative solution concepts is studied more deeply by relying on their theoretical results and exemplifying crucial aspects by means of computer-aided software tools. The purpose of such an analysis is twofold: On the one hand, to verify a set of solution characteristics for a better understanding of a solution itself and gain more intuitions, and on the other hand, to check on the correctness of a solution obtained by floating-point arithmetic. Notably, we can check if a solution vector satisfies several consistency properties. For instance, the pre-nucleolus should fulfill the Davis-Maschler reduced game property, and the Shapley value should meet the Hart and Mas-Colell reduced game property. In this respect, a number of software-aided tools were presented that support an axiomatic analysis of solutions.

Chapter 8 : As a particular application of axiomatizing solution theory, we focus within this chapter on the modiclus, also known under the name modified nucleolus. The modiclus simultaneously considers, besides the primal power, also the preventive power (dual power) of the game to permit actors in a bargaining situation to enforce their claims. This allows us to present and discuss numerous alternative axiomatizations of the modiclus that refer to the primal and preventive power of coalitions. The immanent charm of our proposed alternative characterizations of the modiclus or the anti-pre-nucleolus can be attributed to the fact that we can now refer to the preventive power of a coalition without leaving the original game context and introducing the dual game. To show the pertinence of the derived results, we demonstrate that the modiclus is equal to the anti-pre-nucleolus for the class of PS, weighted graph, and modest bankruptcy games.

Chapter 9 : This chapter focuses on the axiomatization of related solution concepts of the modiclus. Alike those, both the modified and proper modified pre-kernel take the primal as well as the preventive power (dual power) of coalitions during a stylized bargaining scenario into account. Even though it was developed as an auxiliary solution concept to investigate the modiclus, no effective computation methods were designed to support this analysis. We are closing this gap and presenting some algorithms for their computation. Before, we shall resume their theoretical foundation while providing some alternative axiomatization for both solution concepts.

Chapter 10: In the course so far, we have focused on sequential computing only. This form of computing can process one instruction at a time. Consequently, the next instruction is solely executable at the moment when the previous one has been completed. No set of instructions can run concurrently. By contrast, parallelization is a particular form of computation in which the initial problem is decomposable into separate and independent sub-problems for processing these subtasks at the same time on the computer system. We provide procedures for how one can conduct a computer-based analysis for transferable utility games with MATLAB using parallel and GPU computing. During this chapter, we provide a set of procedures allowing us to benefit from parallel acceleration while investigating game issues with our toolbox.

Chapter 11: The MATLAB programming language also offers the opportunity to create object-oriented programs. Relying on object-oriented programming (OOP) techniques allows us to encapsulate data and oper-

ations within a set of objects designed to ensure that a user operates within a clearly defined and consistent environment. This avoids unintentionally changing data and performing a relatively sophisticated list of tasks properly. Data types that permit both encapsulated manipulation of data and user-defined operations on them are called classes. In OOP terminology, a class comprises similar objects. Each TU game has identical attributes, like a characteristic function and a player set. These attributes constitute the objects that form the class of TU games. Though TU games have identical essential characteristics, they have different external manifestations, inducing distinct game properties, and are consequently different objects from the same game class. Now, OOP allows us to create this kind of abstract data type by defining a class with the mentioned attributes to compose a class object for a TU game. This allows for carrying out only some pre-defined and segregated data manipulations on a class object. To familiarize ourselves with OOP, we provide a short synopsis of the essential features of OOP with MATLAB. Then, we discuss how the resulting OOP-class objects are applicable to investigate TU games in a clearly defined and consistent computing environment.

About Part III

Part III deals with investigating a TU game by means of *Mathematica*. Readers not acquainted with this topic should have read Chapter 3 before continuing. Then, they should proceed to Chapter 4 to study mainly the theoretical foundations discussed in detail there. Readers familiar with this topic can safely skip this part and go straight to Chapter 12, thereby focusing primarily on handling software means to study TU games. The remaining chapters of Part III are treatable in isolation.

- Chapter 12:** This chapter discusses an alternative set of software routines to study TU games with *Mathematica* instead of MATLAB. For both software tools, there is a basic set of functions that overlap in their functionality, i.e., they provide the same features. However, they are based on different programming paradigms and are therefore based on diverse operating principles. Though one might have implemented in both programming languages the same algorithm, the coding in both programming environments had to differ while adjusting to the particularities of the language. Therefore, it is relatively unlikely to produce the same programming failure in both environments. Thus, getting in both environments an identical answer increases the reliability of the returned result. We continue to cover the core features by letting the reader see the differences between game analyses performed under MATLAB and those under *Mathematica*. It will be shown how the (pre-)kernel elements, the (pre-)nucleolus, the modiclus, the modified and proper modified pre-kernel, the Shapley value, Lorenz solution, Dutta-Ray solution, excess payoffs, the τ -value, χ -value, Gately point, the vertices of a core, and much more is computable. Moreover, we shall demonstrate the verification of some interesting game properties, such as convexity, average-convexity, or super-additivity with *Mathematica*.
- Chapter 13:** In this chapter, we are focusing on the study of industrial cooperation using software tools conceived for *Mathematica*. Similar to the foregoing procedures, in the initial step, the underlying theory is extensively discussed before the reader is introduced to the procedure to analyze problems arising from industrial cooperation using computer-aided tools conceived with *Mathematica*. That is to say, we focus on the computation of Cournot, Bertrand, and Stackelberg equilibria, as well as on the computation of the associated TU-oligopoly games arising from post-merger equilibria like those of γ -, δ -, as well as s -type characteristic function forms.
- Chapter 14:** Having understood the essentials of sequential computing under *Mathematica*, we head over in this chapter to the parallel implementation of computer-aided analysis of transferable utility games with conceived tools for the **Wolfram Language**. Typically, users perform sequential symbolic processing with *Mathematica* most of their time. Besides this, there are also very computationally and data-intensive calculations in game theory that request, instead of sequentially processing, a parallelization of the tasks in which the initial problem is decomposable into separate and independent sub-problems. We show how the user can profit under *Mathematica* from a performance gain by processing separate and independent sub-tasks simultaneously.
- Chapter 15:** Cross-checking results should be part of a scientist's normal toolbox. In this chapter, we are introducing tools for achieving this by calling *Mathematica* functions designed for studying TU games from a running MATLAB session. For this purpose, we discuss some fundamental methods whose functionality we are already acquainted with MATLAB but whose handling of the results differs due to some *Mathematica* specifics.
- Chapter 16:** Here, we study the reverse procedure of how MATLAB functions are accessible from a running *Mathematica* session to study TU games. We shall demonstrate how selected toolbox functions are easily

usable from within *Mathematica* to offset numerical disadvantages that cannot be compensated for by parallel processing. In addition, it extends the functionality of our *Mathematica* software tool by granting access to functional primitives that are not implemented yet with the *Wolfram Language*. It allows the application of the rich prototyping capabilities of the *Wolfram Language* to conceive algorithms for studying game properties or axiomatizations without leaving the original modeling framework by accessing extra functionality for easy verification of new ideas without taking unnecessary detours by laboriously creating identical game models from scratch in a different modeling environment.

Appendix

This part deals with some additional material related to cooperative game theory. For instance, we provide chapters to discuss some concepts of mathematical logic and group theory. Both fields are extensively applied in the founding work of game theory by [von Neumann and Morgenstern \(1944\)](#), but they are often treated superficially and even incorrectly in the literature. Each chapter of the appendix is addressable in isolation.

- Chapter A:** This first chapter of the appendix focuses on the computational study of benchmarking in order to present the computational behavior of a method in getting an element of the pre-kernel. For this purpose, we resume the reported executing time of [Meinhardt \(2013\)](#) for different profiles for serial and parallel computation of an element of the pre-kernel, based on Algorithm A.0.1. Though the data generated with MATLAB Release 2013a are somewhat outdated, they nevertheless provide some comparison in terms of the fast development in computer science. To set the point related to the reported timings in Chapter 10, where we have deployed up to 40 logical cores compared to the conducted test at that epoch in parallel on compute nodes with eight logical cores only.
- Chapter B:** We present a detailed treatment of propositional logic that should allow the reader to grasp crucial logical arguments applied in several proof techniques given in the literature. Propositional logic can be subsumed as a method of formalizing inferences for analyzing reasoning; it is, therefore, at the heart of conducting mathematical proofs. For that purpose, we resume some proof methods and then move along on the basis of the Deduction Theorem (Herbrand Theorem, 1930) to the question under which circumstances a statement is classifiable as non-provable. To answer this question, we need to leave the framework of propositional logic while making some borrowings from predicate logic (first-order logic).
- Chapter C:** The first section of this appendix reviews crucial aspects of floating-point representation, allowing us to grasp rounding-errors and other kinds of numerical errors caused by machine arithmetic which are captured in greater detail in the second section. These are indispensable concepts for estimating the quality of a computed result.
- Chapter D:** This appendix reviews specific definitions, concepts, and aspects of group theory. An abstract group allows us to capture the properties of mathematical objects through classification. In this sense, group theory and group algebra are the building blocks for identifying similarities and invariances to classify mathematical objects. Classifications can also be useful in studying games, characterizing solutions and game properties, or performing a stability analysis of solutions. Notably, the study of the pre-kernel in Subsection 4.4.3 pursued this approach by conducting an economic stability analysis by means of the positive general linear groups. We present a concise remainder of group theory that should allow the reader to grasp crucial but abbreviated arguments applied within some proofs of the book.
- Chapter E:** In this chapter, we will expose the essential ingredients of representation theory and apply them to the analysis of cooperative games with externalities (games in partition function form). Representation theory is the study of the nature and extent to which a given group acts on a vector space. The action of group elements endows, by means of a homomorphism, a structure on the vector space, a representation of the group, from which a classification of the underlying mathematical objects is obtainable. Applying representation techniques to the theory of cooperative games with externalities allows us to decompose the vector space of games in partition function under the group action of the symmetric group into a direct sum of irreducible subspaces from which symmetric linear solutions can be analyzed concisely and elegantly. To make the point, we review some essential results of the work of [Sánchez-Pérez \(2014, 2017\)](#) to discuss a direct sum decomposition of the game space into invariant subspaces. Exemplarily, we present for $n = 4$ and $n = 5$ by means of character theory even a decomposition into the irreducible submodules.
- Chapter F:** In the course of this appendix, we shall provide some instructions to install the MATLAB toolbox *MatTuGames*. In the initial step, we have to discuss the requirements for running the toolbox properly. Moreover, we also present some add-ons for the toolbox and provide their installation instructions.

The software listed within the requirements section is considered a recommendation. Nothing of the mentioned software tools is really needed to run the toolbox in a basic operating mode. However, to get full functionality out of the toolbox and for accuracy reasons, we strongly recommend its installation. For the impatient user, we provide a quick guide on how to get started and some fundamental operations.

Chapter G: This appendix provides some instructions to install the *Mathematica* package **TuGames**. In particular, we shall focus on a custom installation. Furthermore, we additionally deal with some information on how one can run the package in parallel or run the parallel version of our package and how one can use the **Cddmathlink** library to profit from the graphical features of our package. For the users who are even interested in generating some movies to illustrate, for instance, the geometric properties of the (pre-)nucleolus and (pre-)kernel, we also make sure of running the **Cddmathlink** library, even in parallel. Furthermore, we also give some pieces of information w.r.t. **MatLink**, which is needed to call our MATLAB toolbox **MatTuGames** within a running *Mathematica* session. For the impatient user, we provide a brief guide to getting started and some basic operations.

Chapter H: In Subsections 2.8.4 and 3.5.8, we shall discuss a MATLAB, respectively, a *Mathematica* program for computing the nucleolus for three-person zero-normalized super-additive games using the formula invented by [Leng and Parlar \(2010\)](#). For comparison reasons, we will present their Maple code for its computation in this appendix.

About the Software

We have selected MATLAB[®] and *Mathematica*[®] to conduct computer-based approaches to analyzing cooperative games. We will now give the user some insights into our motives for focusing on these tools. Both software platforms are carefully designed, implemented, maintained, powerful, and reliable. In addition, they have a high-level programming language, powerful graphical capabilities, and relatively fast routines for numerical and symbolic computations. They support parallel computation on multicore processors or general-purpose computation on the graphics processing unit(s) (GPGPU). Furthermore, their functionality is expandable by easily integrating external libraries from C/C++, FORTRAN, Python, or even third-party solvers from different vendors, for example. Besides, they are easy to learn, so one can make quick progress in studying one's own problems. For that reason, they have attracted a great community of users, providing lots of resources, and consequently, it is relatively easy to find advice for arising troubles. All that makes them the prevalent remedy in the academic world and an ideal tool for prototyping to test the preliminaries of ideas. This must be seen in the context that the featuring of creating through their high-level programming language concise codes, enabling, therefore, the users to primarily focus on the concepts of cooperative games, putting the implementation aspects of mathematical tools aside.

MATLAB[®] Toolbox for Cooperative Game Theory

For the reader who is mainly interested in the computational aspects of TU games using MATLAB[®], we offer the software tool **MatTuGames**, which provides the ability to apply enhanced numerical problem-solving for cooperative games with transferable utility. This MATLAB[®] toolbox is downloadable under the following URL:

<http://www.mathworks.com/matlabcentral/fileexchange/35933-mattugames>

or alternatively at the URL:

<https://github.com/himeinhardt/MatTuGames>

As an auxiliary software means to study combinatorial and group theoretical aspects of cooperative games, we have conceived the toolbox **MatRep** under MATLAB[®] to provide a set of functions for studying representation in symmetric groups.

<https://mathworks.com/matlabcentral/fileexchange/62142-matrep-a-matlab-representation-theory-toolbox-symmetric-groups>

The designed MATLAB[®] toolboxes are compatible with MATLAB[®] R2024a or prior releases.

Mathematica[®] packages for Cooperative Game Theory

The reader who is more familiar with *Mathematica*[®] can use the software package **TuGames**. The latest version is **TUG-3.1.4**, compatible with *Mathematica*[®] version 12.0 or higher, and is downloadable from the following URL:

<https://github.com/himeinhardt/TuGames>

As an additional software means for doing industrial cooperation, we also offer the *Mathematica*[®] package **TuOlig**, which needs at least **TUG-3.1.2**.

<https://github.com/himeinhardt/TuOlig>

The developed *Mathematica*[®] packages are compatible with *Mathematica*[®] 14.1, or prior versions, but not before

version 12.0.

We have used our MATLAB[®] and *Mathematica*[®] software tools to create the graphics for the book.

Contents

Foreword	i
Preface	iii
About this Book	v
Acknowledgments	xiii
List of Figures	xxiii
List of Tables	xxv
List of Results	xxvii
List of Algorithms	xxxv
List of Definitions and Examples	xxxv
I A First Course in Cooperative Game Theory, MATLAB, and MATHEMATICA	1
1 A Refresher to Cooperative Game Theory	3
1.1 Cooperative Games in Characteristic Function Form	5
1.1.1 The Pre-Imputation Set	5
1.1.2 A first Glance on Game Classes	6
1.2 Some Solution Concepts	6
1.2.1 The Kernel and Pre-Kernel	7
1.2.2 The Nucleolus and Pre-Nucleolus	7
1.2.3 The Shapley Value	9
1.2.4 The Core and the Strong ϵ -Cores	10
1.2.5 The Anti-Core	14
1.2.6 Consistency of the Core and the Pre-Kernel	15
1.2.7 Stable Sets	17
1.2.8 The Bargaining Set $\mathcal{M}(N, v)$	20
1.2.9 Computing the Nucleolus for Three-Person Games	20
1.3 Crucial Game Classes and Concepts	24
1.3.1 Classes of Bankruptcy Games	25
1.3.2 Simple Games	26
1.3.3 The Potential of a Game	27
1.4 Bibliographical Notes	28
2 Basic Operations of MATLAB	29
2.1 Some Fundamentals of MATLAB	29
2.1.1 Variable Assignment, Names and Types	30
2.1.2 Operators and Expressions	35
2.2 Vectors, Matrices and Multidimensional Arrays	64
2.2.1 Initializing Arrays	66
2.2.2 Vector and Matrix Operations	68
2.2.3 Linear Indexing and by Position	69
2.2.4 Basic Linear Algebra	71

2.2.5	Multidimensional Arrays	92
2.2.6	Subarrays	93
2.3	Data Types: Cell and Structure Arrays	95
2.3.1	Cell Arrays	95
2.3.2	Structures	98
2.4	Essentials of MATLAB Graphics	100
2.4.1	Two-Dimensional Graphics	100
2.4.2	Three-Dimensional Graphics	103
2.4.3	Saving and Printing Figures	108
2.5	Fundamentals of Program Design	109
2.5.1	Top-Down Design Process	109
2.5.2	Modular Program	120
2.5.3	Error Types	121
2.6	User-defined Functions	128
2.6.1	Structure of an M-Function	128
2.6.2	Variable Scope	131
2.6.3	Variable Passing	132
2.6.4	Option Passing	132
2.6.5	Local Functions	133
2.6.6	Function Handles and Anonymous Functions	133
2.6.7	Function Functions	136
2.6.8	Command-Function Duality	138
2.7	Flow of Control	139
2.7.1	Branching Statement: if-Construct	139
2.7.2	Branching Statement: switch-Construct	150
2.7.3	Exception Handling: try/catch-Construct	155
2.7.4	Counted Loop: for-Construct	159
2.7.5	Vectorization of Code	165
2.7.6	Conditional Loop: while-Construct	168
2.7.7	Recursion	175
2.8	Studying TU Games with MATLAB: Some Essential Insights	180
2.8.1	Defining a TU Game: Fundamentals	180
2.8.2	Standard Solution	189
2.8.3	Shapley Value	190
2.8.4	Nucleolus for Three-Person Games	194
3	Basic Operations of Mathematica	199
3.1	Some Survival Rules	199
3.1.1	Be Careful with Products	202
3.1.2	Be Careful with Compound Symbols	203
3.1.3	How to use Brackets correctly	205
3.1.4	How to use the Operators Set, SetDelayed, and Equal correctly	206
3.1.5	How to use the operators Rule and ReplaceAll correctly	208
3.1.6	Be Careful in Defining Equations	209
3.1.7	Distinguish Functions and Value Variables	211
3.2	The Nature of Expressions	213
3.2.1	Internal Form of Expressions	213
3.2.2	Every Input is an Expression	214
3.2.3	Changing the Head of an Expression	215
3.2.4	A First Look at the Evaluation of Expressions	216
3.2.5	Non-Atomic Expressions	216
3.2.6	Atomic Expression	217
3.2.7	Compound Expression	220
3.2.8	The Global Rule Base	221
3.2.9	How Expressions are evaluated	221
3.2.10	Controlling the Evaluation	224
3.2.11	Pattern and Matching	228

3.3	Rewrite Rules	234
3.3.1	Built-in and User-Defined Functions	234
3.3.2	Creating Rewrite Rules	234
3.3.3	Transformation Rules	247
3.4	Flow of Control	251
3.4.1	Branching Functions: The If command	252
3.4.2	Branching Functions: The Which command	254
3.4.3	Branching Functions: The Switch command	254
3.4.4	Iteration by Do and While Loop	255
3.4.5	Recursion	260
3.4.6	Comparison of Computation Time: While Loop versus Recursion	261
3.4.7	Pure Functions	262
3.5	Studying TU Games with Mathematica: Some Essential Insights	263
3.5.1	Weighted Majority Games	263
3.5.2	Bankruptcy Problems	271
3.5.3	The Greedy Bankruptcy Game: Some Code	278
3.5.4	Contested Garment Principle	279
3.5.5	Generalized Contested Garment Principle	281
3.5.6	Defining a TU Game	284
3.5.7	Standard Solution	287
3.5.8	Nucleolus for Three-Person Games	288
3.5.9	Shapley Value	293
3.6	Optimization	294
3.6.1	Reconsidering Optimization Theory	294
3.6.2	Classical Optimization Problems	299
3.6.3	Mathematical Programming	309
3.7	Computing a Nash Equilibrium	318
3.7.1	Nash Equilibrium	318
3.7.2	Existence of a Nash Equilibrium: The Theoretical Background	319
3.7.3	Uniqueness of the Nash Equilibrium	322
3.7.4	Coding Example for Getting a Nash Equilibrium	323
3.7.5	Bertrand-Nash Equilibrium	323
3.7.6	Cournot-Nash Equilibrium	330

II Cooperative Game Theory with MATLAB

339

4	TU Games with a trivial Coalition Structure	341
4.1	A Short Overview of the Software Tool	341
4.2	Some Preliminaries	342
4.3	Defining a TU Game	348
4.3.1	Two basic Examples	348
4.3.2	A more comprehensive Example	354
4.3.3	Market and totally balanced Games	356
4.3.4	Convex Games	367
4.3.5	Almost-Convex Games	379
4.3.6	Average-Convex Games	381
4.3.7	Assignment Games	389
4.3.8	Permutation Games	391
4.3.9	Flow Games	395
4.3.10	Weighted Graph Games	403
4.3.11	Minimum Cost Spanning Tree Games	411
4.4	Some Solution Concepts	429
4.4.1	The Shapley Value	429
4.4.2	The Tau-Value	434
4.4.3	The Pre-Kernel and Kernel	437
4.4.4	The Pre-Nucleolus and Nucleolus	463

4.4.5	The weighted (Pre-)Kernel and (Pre-)Nucleolus	476
4.4.6	The Anti-Pre-Kernel and Anti-Pre-Nucleolus	486
4.4.7	The Least Square (Pre-)Nucleolus	495
4.5	Third-Party Solvers	497
4.6	Some Game Properties and Convex Solution Sets	504
4.6.1	The Core of a TU Game and related Subjects	504
4.6.2	Investigation of Game Properties	506
4.6.3	The Core of a TU Game Reconsidered	509
4.6.4	The Anti-Core	511
5	TU Games with non-trivial Coalition Structures	521
5.1	Solutions for a Coalition Structure	521
5.1.1	The Core and Super-Additive Cover	524
5.1.2	The Kernel and Pre-Kernel	530
5.1.3	The Nucleolus and Pre-Nucleolus	545
5.2	Fairness and Related Values for a Coalition Structure	550
5.2.1	The Solidarity and related Values	551
5.2.2	The Owen and related Values	553
5.2.3	The Myerson and related Values	555
5.2.4	The Position Value	560
5.3	Replication of some Game Solutions	562
5.3.1	Replication of the Nucleolus by Characterization Sets	562
5.3.2	Replication of a Pre-Kernel Element	587
5.3.3	Replication of the Shapley Value	591
6	Graphical Extensions, and Real-World Applications	599
6.1	External Libraries and Graphical Features	599
6.1.1	Plotting the Core	600
6.1.2	Plotting the Core-Cover	601
6.1.3	Plotting the Weber Set	604
6.1.4	Plotting a strong ϵ -Core	604
6.1.5	The Geometric Property of the (Pre-)Kernel	606
6.2	Simple Game	609
6.3	Simple Weighted Majority Game	614
6.3.1	The UN Security Council	614
6.3.2	Minimum Homogeneous Representation and Center Solution	622
6.3.3	Star-Shapedness of the (Pre-)Kernel	637
6.3.4	Florentine Marriage and Business Relations	647
6.4	Handling Numerical Errors When Seeking a Pre-Kernel Point	682
7	Axiomatization of Selected Solutions	689
7.1	General Discussion of Some Solution Properties	689
7.2	Checking Some Basic Properties	693
7.3	Reduced Game Properties	697
7.4	Converse Reduced Game Properties	704
7.5	Reconfirmation Property	708
7.6	ISRG Property	709
8	Axiomatization and Computation of the Modiclus	713
8.1	Definition and Computation of the Modiclus	715
8.2	Wedge Games via the Modiclus	716
8.3	Discussion of its Axiomatization	722
8.4	Alternative Axiomatization of the Modiclus: A First Approach	751
8.5	The Derived Game Property	775
8.6	Market Side, Power Index, and Corners	779
8.7	The Anti-Derived Game Property	785
8.8	Coincidence of Modiclus and Anti-Pre-Nucleolus: Some Game Examples	788
8.9	Checking an Extended Kohlberg Criterion	809

8.10	Computing the Anti-Modiclus	809
9	The Axiomatization of the Proper Modified and Modified Pre-Kernel	813
9.1	Definition of the Modified and Proper Modified Pre-Kernel	814
9.2	Axiomatization of the Modified Pre-Kernel	823
9.3	Alternative Axiomatization of the Modified Pre-Kernel	827
9.4	Indirect Method to compute the Modiclus	833
10	Parallel and GPU Computing	837
10.1	Parallelization	837
10.1.1	CPU Acceleration	838
10.1.2	GPU Acceleration	851
10.2	Accelerating Performance	859
10.2.1	Amdahl's Law	859
10.2.2	Testing Parallel Scalability	860
10.3	Using the Parallel Computing Toolbox	862
10.4	(Average-) Convexity and Consistency Reconsidered	868
10.5	Replication of a Pre-Kernel Element Reconsidered	872
10.6	Replication of the Shapley Value Reconsidered	874
10.7	Weighted Majority Games: Game Solutions Reconsidered	875
10.8	For-Loop versus Parfor-Loop	879
11	OOP-Class TuGame	883
11.1	OOP Classes and Objects	883
11.1.1	Object-Oriented Programming Terminology	883
11.1.2	M-Class Definitions	884
11.1.3	M-Class Inheritance	885
11.2	Features of the OOP-Class TuGame	886
11.3	OOP-Class TuProp	894
11.4	OOP-Class TuSol and p_TuSol	895
11.5	OOP-Class TuMCnets	902
11.6	OOP-Class TuVal and p_TuVal	907
11.7	OOP-Class TuCore	918
11.8	OOP-Class TuVert	923
11.9	OOP-Class TuRep	925
11.10	OOP-Class TuShRep	931
11.11	OOP-Class p_TuCons and p_TuKcons	936
III	Cooperative Game Theory with Mathematica	945
12	The Mathematica Package TuGames	947
12.1	Package TuGames: An Overview	947
12.2	Getting Started with TuGames	948
12.3	How to define Games?	948
12.4	Checking Game Properties	950
12.5	Computing Game Solutions	953
12.6	Graphical Features	956
12.6.1	The Core for Three-Person Games	956
12.6.2	The Core for Four-Person Games	960
12.6.3	Animate Geometric Property of the Kernel/Nucleolus	963
12.7	Defining more complex Games	966
12.7.1	Getting Started	967
12.7.2	Weighted Majority	967
12.7.3	Modest Bankruptcy	969
12.7.4	Probability Game	971
12.8	Non-linear Method to Compute the (Pre-)Nucleolus	974
12.8.1	Some Definitions and Results	974

12.8.2	Extending the Results to the Pre-Nucleolus	977
12.8.3	Calculating the (Pre-)Nucleolus	978
13	Analysis of Industrial Cooperation with TuOligopoly	985
13.1	Cournot Cooperative Oligopoly Games	987
13.1.1	Quantity Competition under a Homogeneous Good	988
13.1.2	Quantity Competition under Product Differentiation	988
13.1.3	Monopoly in Cournot Models	990
13.1.4	Pre-Merger Equilibrium in Cournot Models	992
13.1.5	Characteristic Function Forms	993
13.1.6	Super-modularity of linear Cooperative Oligopoly Games	995
13.1.7	Partition Function Approach	1004
13.1.8	Post-Merger Equilibrium in Cournot Models	1006
13.1.9	Cournot Model: Two Firms	1006
13.1.10	Cournot Model: Four Firms	1011
13.1.11	Aggregation Across Firms: A Fallacy	1019
13.2	Bertrand Cooperative Oligopoly Games	1033
13.2.1	Price Competition under Product Differentiation	1033
13.2.2	Monopoly in Bertrand Models	1035
13.2.3	Pre-Merger Equilibrium in Bertrand Models	1036
13.2.4	Post-Merger Equilibrium in Bertrand Models	1038
13.2.5	Bertrand Model: Two Firms	1038
13.2.6	Bertrand-Dixit Model: Four Firms	1042
14	Parallel Computing with the Mathematica Package TuGames	1047
14.1	Parallel Computing with Mathematica	1047
14.1.1	Launching Local Kernels	1048
14.1.2	Testing Parallel Scalability	1049
14.1.3	Parallel Evaluation	1051
14.2	Parallel Computing with the Package: An Overview	1055
14.3	Parallel Computing: A more Advanced Approach	1059
15	Calling TuGames from MATLAB	1065
15.1	Starting the MathKernel from Matlab	1065
15.2	Generating TU Games with Mathematica	1066
15.3	Studying Game Properties	1069
15.4	Computing Game Solutions and Analyzing its Properties	1070
16	Calling the MATLAB Toolbox MatTuGames from Mathematica	1077
16.1	Getting Started	1077
16.2	Defining a Game under MATLAB	1078
16.3	Computing Game Solutions under Matlab	1079
IV	Appendix	1085
A	Pre-Kernel Computation Timing Tables	1087
B	Propositional Logic	1091
B.1	Conditional Statements and Truth Tables	1091
B.2	Various Proof Techniques	1092
B.2.1	The direct Proof	1092
B.2.2	The contrapositive Proof	1092
B.2.3	The Indirect Proof	1093
B.3	The Deduction Theorem	1094
B.3.1	Incorrect Application of the Deduction Theorem	1094
B.3.2	Incorrect Application of the Indirect Proof	1095

C	Computational Errors	1101
C.1	Floating-Point Representation	1101
C.2	Kinds of Numerical Errors	1103
C.2.1	Absolute/Relative Error	1103
C.2.2	Rounding Error	1106
C.2.3	Loss of Significance	1107
C.2.4	Truncation Error	1109
C.2.5	Propagation Error	1109
C.2.6	Overflow/Underflow	1110
C.2.7	Negligible Addition	1111
C.2.8	Error Magnification	1111
D	A Quick Overview to Group Theory	1113
D.1	Some Fundamentals of Group Theory	1113
D.2	A Sketched Application of Group Theory	1119
E	A Quick Overview to Representation Theory	1123
E.1	Some Fundamentals of Representation Theory	1123
E.2	Games in Partition Function Form	1130
E.3	A Decomposition Example of the Space of Games with Externalities	1130
F	Installing the MATLAB Toolbox MatTuGames	1135
F.1	Installation	1135
F.1.1	Requirements	1135
F.1.2	General Instruction	1136
F.1.3	Quick Start with MatTuGames	1136
F.1.4	More Sophisticated Instruction	1138
G	Installing the MATHEMATICA Package TuGames	1141
G.1	Installation	1141
G.1.1	Custom Installation	1141
G.1.2	Requirements	1142
G.1.3	First Steps with TuGames	1142
G.1.4	Running the Package in Parallel	1144
G.1.5	MATLink and MatTuGames	1145
G.1.6	The Cddmathlink-Library	1145
G.1.7	Running the cddmathlink libraries in Parallel	1148
G.1.8	Trouble Shooting	1149
G.1.9	Documentation of the Package	1149
H	Maple Program of Leng and Parlar	1151
H.1	Maple Code to Compute the Nucleolus for Three Person Games	1151
	Bibliography	1153
	Author Index	1167
	Subject Index	1171

List of Figures

1.1	The core of a three-person zero-normalized game	12
1.2	The strong 10-core of a three-person zero-normalized game	13
1.3	The least core of a three-person zero-normalized game	13
2.1	Two-Dimensional plots of Game Data (left) and Payoff Data using a Marker (right)	101
2.2	Two-Dimensional plots of Data Points using Point (left) and Diamond Symbol (right) as Markers	102
2.3	Two-Dimensional plots of Data Points without (left) and with Legend (right)	103
2.4	Subplot (left) and Three-Dimensional Plot of Data Points (right)	104
2.5	Perpendicular Projection to the Solution Plane	106
2.6	Structure of a Two-Part Tariff Scheme for Operating an Airport	112
2.7	Comparison of Pricing Policies	119
2.8	Comparison of Cost Coverage Policies	120
2.9	Modular Programming Structure of an Airport Problem	121
2.10	Comparison of Price and Cost Coverage Policies with 65 Types	153
2.11	Parameterized Pricing Policies for 35 Types	162
2.12	Comparison of Price and Cost Coverage Policies with 52 Types	168
2.13	Solow Capital Growth Model	175
3.1	Talmudic Rule Payoffs to the Claimants	284
3.2	Best reply curves of firms: Intersection point is the Bertrand-Nash Equilibrium	327
3.3	Best reply curves of firms: Intersection point (black dot) is the Cournot-Nash Equilibrium	334
4.1	Airport Game: Imputation Set, Core, Nucleolus, and Shapley value.	386
4.2	Airport Cost Game: Anti-Imputation Set, Anti-Core, Anti-Nucleolus, and Shapley value	386
4.3	Airport Game: Imputation Set, Weber Set, Nucleolus, and Shapley value.	387
4.4	Airport Cost Game: Anti-Imputation Set, Weber Set, Anti-Nucleolus, and Shapley value	387
4.5	Average-Convex: Imputation Set, Core, Nucleolus, and Shapley value.	388
4.6	Almost Average-Convex: Imputation Set, Core, Nucleolus, and Shapley value	388
4.7	Flow Problem of 3 persons and 7 nodes	396
4.8	Flow Problem of 13 persons and 7 nodes	398
4.9	Weighted Graph Problem with 6 persons and 14 edges	403
4.10	Cost Spanning Graph	414
4.11	MATLAB Plot	414
4.12	Minimal Tree	416
4.13	MATLAB Plot	416
4.14	Anti-Core of the M.C.S.T. Game	419
4.15	Minimal Tree	425
4.16	Cost Spanning Graph	514
4.17	MATLAB Plot	514
4.18	M.C.S.T. Prob. $\langle N_0, \mathbf{C}_{N_0} \rangle$	517
4.19	M.C.S.T. Prob. $\langle S_0^1, \mathbf{C}_{S_0^1}^1 \rangle$	517
4.20	M.C.S.T. Prob. $\langle S_0^2, \mathbf{C}_{S_0^2}^2 \rangle$	517
5.1	The core, imputation set, and pre-nucleolus of game v_{sa} with c.s. $gP\{10\}$	531
6.1	The core, Shapley value, and pre-nucleolus of game bv using <i>CorePlot()</i>	601
6.2	The core, Shapley value, and pre-nucleolus of game bv using <i>CddCorePlot()</i>	602
6.3	The core cover, Shapley value, and pre-nucleolus of game bv using <i>CddCoreCoverPlot()</i>	604
6.4	The Weber set, the core, Shapley value, and pre-nucleolus of bv using <i>CddWeberSetPlot()</i>	605
6.5	The Strong 114-core, core, Shapley value, and pre-nucleolus of bv using <i>CddStrongCorePlot()</i>	606

6.6	The Strong 130-core, the core, Shapley value, and pre-nucleolus of bv	608
6.7	The Least-core, the core, Shapley value, and pre-nucleolus of bv	608
6.8	Florentine Elite Marriages 1395-1434 ¹⁴	652
6.9	Florentine Elite Marriages 1395-1434: Priori Union Between Albizzi and Guadagni ¹⁷	669
6.10	Florentine Elite Business Relations 1395-1434 ¹⁸	674
8.1	Flow Problem Representation that arises from a Set of Assignments	717
10.1	Parallel Scalability Test	861
11.1	Class Hierarchy TuGame	886
11.2	Class Hierarchy TuProp	894
11.3	Class Hierarchy TuSol	896
11.4	Class Hierarchy p_TuSol	896
11.5	Class Hierarchy TuMCnets	903
11.6	Class Hierarchy TuVal and p_TuVal	907
11.7	Class Hierarchy TuCore	919
11.8	Class Hierarchy TuVert	923
11.9	Class Hierarchy TuRep	926
11.10	Class Hierarchy p_TuRep	926
11.11	Class Hierarchy TuShRep	931
11.12	Class Hierarchy p_TuShRep	931
11.13	Class Hierarchy p_TuCons and p_TuKcons	936
13.1	Best reply curves of firms: Intersection point is the Cournot-Nash Equilibrium	1010
13.2	Shapley value, Nucleolus, Imputation Set, and Core of the Cournot γ -TU-oligopoly Game	1018
13.3	Shapley value, Nucleolus, Imputation Set, and Core of the Cournot δ -TU-oligopoly Game	1018
13.4	Individual Best Reply Functions of Trust $\{1, 2, 3\}$ and Firm 4	1022
13.5	Individual Best Reply Functions of Trust $\{1, 2, 3\}$ and Firm 5	1022
13.6	Best Reply Functions of Trust $\{1, 2, 3\}$ and Outsider 4	1023
13.7	Best Reply Functions of Trust $\{1, 2, 3\}$ and Outsider 5	1024
13.8	Shapley value, Nucleolus, Imputation Set, and Core of the Bertrand-Dixit γ -TU-oligopoly Game	1045
14.1	Mathematica Parallel Scalability Test	1050
14.2	Parallel Scalability Test of Function ParaPreKernelElement for 12-Person Game	1063
14.3	Comparing Parallel Scalability Test with Amdahls Law for $p = 0.67$	1063
G.1	Starting Page of the Documentation	1150
G.2	Getting Started Page	1150

List of Tables

2.5.1	Airport Cost Allocation Problem	112
2.5.2	Airport Cost Allocation Problem	118
2.8.1	Short Sided Market Game	182
2.8.2	Applied Rules MC-Nets Representation for a TU Game	186
2.8.3	Wedge Production Game	189
4.3.1	Seven-Person Game	354
4.3.2	Optimal Trading Decisions	359
4.3.3	Market Game	359
4.3.4	Lucas' 10-Game with no Stable Set	362
4.3.5	Airport Savings and Cost Game	372
4.3.6	Game where Pre-Kernel imposes the same Partition	378
4.3.7	Four-Person Almost-Convex Game	379
4.3.8	Average-Convex Game, but not Almost-Convex	381
4.3.9	Four-Person Almost- and Average-Convex Game	381
4.3.10	Almost- and Average-Convex, but not Veto-Rich Game	382
4.3.11	Average-Convex Game violating Theorem 4.3.15	382
4.3.12	Almost Average-Convex Game	383
4.3.13	Assignment Game	392
4.3.14	Permutation Game	392
4.3.15	Game of the Desert Water Problem	415
4.4.1	MCST and Cost Savings Game	443
4.4.2	Negative Entry for the Nucleolus	467
4.4.3	Four-Person Game	475
4.5.1	Third-Party and MATLAB Solvers	503
6.3.1	Minimal Representations of all Partition Games for $n \in [4, 8]$	642
6.3.2	Florentine Marriage Relations	651
6.3.3	Florentine Business Relations	651
6.3.4	Solution Matrix of Game SV1	667
6.3.5	Solution Matrix of Game SV1w	667
6.3.6	Solution Matrix of Game SV1t	668
6.3.7	Solution Matrix of Game SV2	668
6.3.8	Solution Matrix of Game SV2w	673
6.3.9	Solution Matrix of Game SV2t	674
6.3.10	Parameter Settings	680
6.3.11	Solution Matrix of Game SV3	680
6.3.12	Solution Matrix of Game SV3w	681
6.3.13	Solution Matrix of Game SV3t	681
6.3.14	Variances of Power Distribution	681
7.1.1	Four-Person Super-Additive Game	691
7.1.2	Three-Person Reduced Game $v_{R,x}$	692
7.1.3	Three-Person Super-Additive Game	692
7.3.1	Subgame $\langle \{1, 3, 4\}, v_{\{1,3,4\}} \rangle$	704
7.3.2	Subgame $\langle \{2, 3, 4\}, v_{\{2,3,4\}} \rangle$	704
7.4.1	Subgame $\langle \{1, 2, 4\}, v_{\{1,2,4\}} \rangle$	707
7.4.2	Subgame $\langle \{2, 3, 4\}, v_{\{2,3,4\}} \rangle$	707
9.1.1	Bertrand-Shubik Oligopoly TU Game	819

10.2.1	Amdahl's Law	860
11.5.1	Wedge Production Game	904
12.8.1	5-person TU game v	979
12.8.2	A Four-Person TU-Game v	982
14.3.1	Amdahl's Law vs. Parallel Test	1063
A.0.1	Tests conducted on HP XC3000 with <i>PreKernel()</i> Version 0.3	1088
A.0.2	Tests conducted on HP XC3000 with <i>p_PreKernel()</i> Version 0.3	1088
A.0.3	Tests conducted on HP XC3000 with <i>PreKernel()</i> Version 0.2	1089
A.0.4	Tests conducted on HP XC3000 with <i>p_PreKernel()</i> Version 0.2	1089
C.2.1	Relative Error Norms of the Iterations	1105

List of Theorems

Chapter 1

1.2.1	Theorem (Single-Valuedness of the (Pre-)Nucleolus)	8
1.2.2	Theorem (Axiomatization of the Pre-Nucleolus (Sobolev (1975)))	9
1.2.3	Theorem (Standard Solution)	10
1.2.4	Theorem (Axiomatization Shapley Value)	10
1.2.5	Theorem ((Pre-)Nucleolus Core Element)	11
1.2.6	Theorem (The Core of Balanced TU Games is Non-Empty (Bondareva (1963); Shapley (1967)))	14
1.2.7	Theorem (The Core satisfies RGP (Peleg (1986)))	16
1.2.8	Theorem (The Pre-Kernel satisfies RGP (Peleg (1986)))	16
1.2.9	Theorem (The Core satisfies RCP (Hwang and Sudhölter (2001)))	16
1.2.10	Theorem (The Core satisfies CRGP (Peleg (1986)))	17
1.2.11	Theorem (The Pre-Kernel satisfies CRGP (Peleg (1986)))	17
1.2.12	Theorem (Coincidence of The Core and the Set of Undominated Allocations (Driessen (1988)))	18
1.2.1	Corollary (Relation between the Core and Stable Set (Driessen (1988)))	19
1.2.13	Theorem (Stable Set for Convex Games)	19
1.2.14	Theorem (Stable Set of a Composite Game)	19
1.2.15	Theorem (Coincidence between Core and Bargaining Set (Maschler et al. (1972)))	20
1.2.16	Theorem (Leng and Parlar (2010) Thm. 1)	21
1.2.17	Theorem (Leng and Parlar (2010) Thm. 2)	21
1.3.1	Theorem (Convexity of Bankruptcy Games)	25
1.3.2	Theorem (Core of an Essential Constant-Sum Game)	26

Chapter 2

2.2.1	Theorem (Characterization of a Non-Homogeneous System of Linear Equations)	72
2.2.1	Proposition (Characterization of a Square Matrix)	73
2.2.2	Proposition (Properties of the Determinant)	84
2.2.2	Theorem (Orthogonal Factorization of a Symmetric Matrix)	85
2.2.3	Proposition (QR Factorization)	85
2.2.4	Proposition (LU Factorization)	88
2.2.5	Proposition (Cholesky Factorization)	90
2.5.1	Proposition (Shapley Value of an Airport Cost Game)	113
2.5.2	Proposition (Nucleolus of an Airport Cost Game)	114
2.5.1	Remark (Concave Airport Cost Game)	114

Chapter 3

3.5.1	Theorem (Consistent Solution of a Bankruptcy Problem (Aumann and Maschler (1985)))	282
3.5.2	Theorem (Consistent Garment Principle Coincides with the Nucleolus (Aumann and Maschler (1985)))	282
3.6.1	Theorem (Necessary Condition for Relative Extrema)	295
3.6.1	Proposition (Characterization of Definiteness)	295
3.6.2	Theorem (Sufficient Condition for Relative Extremum)	296
3.6.3	Theorem (KKT necessary conditions)	298
3.6.4	Theorem (KKT sufficient conditions)	299
3.6.5	Theorem (Alternative Characterization of Definiteness)	310
3.6.6	Theorem (Quasi-Convex Function)	311
3.6.1	Remark (Necessary and Sufficient Condition for Relative Extremum)	311
3.6.7	Theorem (Sufficient Condition for Relative Extremum)	311
3.7.1	Lemma (Strategy Profile Constituting a Nash Equilibrium)	319
3.7.1	Remark (Interpretation Uhc)	319
3.7.1	Theorem (Closed Uhc Correspondence)	320

3.7.2	Remark (Characterization of Euclidean Space)	320
3.7.2	Lemma (Uhc image of a compact set)	320
3.7.3	Lemma (Product Set)	320
3.7.2	Theorem (Uhc of the multivalued function)	320
3.7.3	Theorem (Maximum Value Theorem (Weierstrass))	320
3.7.4	Theorem (Berge's Maximum Theorem)	320
3.7.3	Remark (Applicability of Berge's Maximum Theorem)	321
3.7.5	Theorem (Kakutani's Fixed Point Theorem)	321
3.7.4	Lemma (Characterization of the Reaction Correspondence)	321
3.7.6	Theorem (Existence of Nash Equilibrium)	321
3.7.4	Remark (Properties of Reaction Correspondence)	321
3.7.7	Theorem (Contraction Mapping Theorem)	322
3.7.8	Theorem (Uniqueness of Nash Equilibrium)	322
Chapter 4		
4.3.1	Theorem (Non-Empty Core for Market Games (Shapley and Shubik (1969)))	357
4.3.1	Corollary (The Associated Subgame is a Market Game (Shapley and Shubik (1969)))	358
4.3.2	Theorem (Market Game is Totally Balanced (Shapley and Shubik (1969)))	358
4.3.2	Corollary (Market Game is Super-Additive)	358
4.3.1	Proposition (Composed Market Game)	360
4.3.3	Theorem (Characterizing Non-Empty Core by Balanced Cover (Shapley and Shubik (1969)))	361
4.3.4	Theorem (Characterizing Totally Balancedness (Shapley and Shubik (1969)))	361
4.3.3	Corollary (Coincidence of Totally Balanced game with Totally Balanced Cover (Shapley and Shubik (1969)))	362
4.3.2	Proposition (Domination-Equivalent of a Balanced TU game (Shapley and Shubik (1969)))	362
4.3.1	Remark (Closed-End Fund (CEF))	364
4.3.4	Corollary (Characterization of Utility Function (Shapley and Shubik (1969)))	364
4.3.1	Lemma (Characterization of a Market Game through a Direct Market)	365
4.3.2	Lemma (Totally Balanced Cover is a Market Game (Shapley and Shubik (1969)))	366
4.3.5	Theorem (Totally Balanced Game is a Market Game (Shapley and Shubik (1969)))	367
4.3.6	Theorem (Equivalence of Market Game and Totally Balanced Game (Shapley and Shubik (1969)))	367
4.3.7	Theorem (Convexity)	368
4.3.8	Theorem (Weber Set)	373
4.3.9	Theorem (Characterization Convex Game)	373
4.3.10	Theorem (Shapley (1971))	374
4.3.11	Theorem (Convexity of the Davis-Maschler Reduced Game (Maschler et al. (1972)))	375
4.3.3	Lemma (Characterization of Davis-Maschler Reduced Games for Convex Games (Maschler et al. (1972)))	376
4.3.4	Lemma (Implication for a Collection of Coalitions when it contains a Partition (Arin and Iñarra (1998)))	376
4.3.12	Theorem (Collection of Coalitions contains Partition (Arin and Katsev (2013)))	377
4.3.5	Lemma (Identical (Anti-)Partition)	378
4.3.13	Theorem (Single-valuedness of the Pre-Kernel (Maschler et al. (1972)))	378
4.3.5	Corollary (Coincidence of Pre-Kernel with Pre-Nucleolus)	379
4.3.14	Theorem (Single-valuedness of the Pre-Kernel for Almost-Convex Games)	380
4.3.6	Corollary (Single-valuedness of the Kernel for Almost-Convex Games (Getán et al. (2012)))	380
4.3.15	Theorem (Extension of Single-valuedness of the Pre-Kernel)	380
4.3.16	Theorem (Kernel of Veto-Rich Games coincides with the Nucleolus (Arin and Feltkamp (1997)))	381
4.3.17	Theorem (Assignment Game is a Permutation Game (Curiel (1997)))	393
4.3.18	Theorem (Permutation Games are Totally Balanced (Curiel (1997)))	393
4.3.19	Theorem (Equivalence of Game Classes)	395
4.3.20	Theorem (Max-Flow, Min-Cut)	400
4.3.21	Theorem (Convexity of Weighted Graph Games)	403
4.3.22	Theorem (Shapley value of Weighted Graph Games (Deng and Papadimitriou (1994)))	404
4.3.2	Remark (PS Property)	404
4.3.23	Theorem (Inclusion Result of PS Games)	404

4.3.24	Theorem (Coincidence of Shapley value and Pre-Nucleolus (Kar et al. (2009)))	404
4.3.7	Corollary (Coincidence of Shapley value and Nucleolus (Deng and Papadimitriou (1994)))	405
4.3.3	Remark (Minimum Cost Tree Solution)	414
4.3.25	Theorem (Minimum Cost Tree Solution in the Anti-Core of M.C.S.T. Game (Granot and Huberman (1981)))	417
4.3.4	Remark (Core vs. Anti-Core of M.C.S.T. Games)	417
4.3.8	Corollary (Empty Core of M.C.S.T. Game)	417
4.3.5	Remark (Blocking Mutual Cooperation for Cost Games)	417
4.3.9	Corollary (Anti-Nucleolus Unique Intersection Point (Granot and Huberman (1984)))	418
4.3.6	Remark (Nucleolus vs. Anti-Nucleolus)	418
4.3.3	Proposition (Unique Irreducible Form of the M.C.S.T. Problem)	420
4.3.7	Remark (Concavity vs. Convexity)	421
4.3.26	Theorem (Irreducible Form Induces Concave Game (Aarts and Driessen (1993)))	421
4.3.8	Remark	422
4.4.1	Theorem (Indirect Function (Martinez-Legaz (1996)))	439
4.4.1	Lemma (Lower Bound on Transfers (Meseguer-Artola (1997)))	439
4.4.1	Proposition (The Indirect Function Characterizes the Pre-Kernel (Meseguer-Artola (1997)))	439
4.4.1	Corollary (The Set of Global Minima Characterizes the Pre-Kernel (Meinhardt (2013)))	440
4.4.2	Proposition (The Indirect Function Characterizes the Kernel (Meseguer-Artola (1997)))	453
4.4.2	Theorem (Characterization of the Pre-Nucleolus with Balanced Collections)	469
4.4.3	Theorem (Characterization of the Pre-Nucleolus by Property I)	469
4.4.4	Theorem (Characterization of the Pre-Nucleolus by Property II)	469
4.4.1	Remark (Possible Flaws with a Proof by Contradiction)	471
4.4.5	Theorem (Characterization of the Nucleolus by Weak Property I (Kohlberg (1971)))	473
4.4.6	Theorem (Characterization of the Nucleolus by Weak Property II (Kohlberg (1971)))	473
4.4.7	Theorem (Characterization of the Weighted Pre-Nucleolus with Balanced Collections)	478
4.4.8	Theorem (Characterization of the Dual Pre-Kernel (Funaki and Meinhardt (2006)))	487
4.4.2	Remark (Anti-Pre-Kernel)	487
4.4.9	Theorem (Characterization of the Dual Pre-Nucleolus (Funaki and Meinhardt (2006)))	491
4.4.10	Theorem (Characterization of the Anti-Pre-Nucleolus by a Balanced Collection)	492
4.4.11	Theorem (Characterization of the Anti-Pre-Nucleolus by Anti-Property I)	493
4.4.12	Theorem (Characterization of the Anti-Pre-Nucleolus by Anti-Property II)	495
4.4.13	Theorem (Characterization of the Anti-Nucleolus by Weak Anti-Property I)	495
4.4.14	Theorem (Characterization of the Anti-Nucleolus by Weak Anti-Property II)	495
4.6.1	Theorem (Anti-(Pre-)Nucleolus Anti-Core Element)	512
4.6.2	Theorem (Cartesian Product of Anti-Cores (Granot and Huberman (1981)))	518
4.6.1	Remark (Anti-Monotonic Cover of M.C.S.T. Games)	518
4.6.3	Theorem (Cartesian Product of Anti-Nucleoli (Granot and Huberman (1981)))	520
4.6.2	Remark (Anti-Nucleolus of an M.C.S.T. Game)	520

Chapter 5

5.1.1	Theorem (Characterization of Super-Additive Games)	526
5.1.2	Theorem (Core with Coalition Structure)	526
5.1.3	Theorem (Properties of a Core with Coalition Structure)	529
5.1.1	Proposition (The Indirect Function Characterizes the Pre-Kernel with C.S. (Meseguer-Artola (1997)))	531
5.1.1	Corollary (Pre-Kernel Set for a C.S. and the Set of Global Minima)	532
5.1.2	Proposition (Partition of the Set $\text{dom } h$)	533
5.1.1	Remark (Note to the Proof (Meinhardt (2013)))	534
5.1.3	Proposition (Finite Cardinality of the Collection of Equivalence Classes)	534
5.1.4	Proposition (Quadratic Function)	534
5.1.5	Proposition (Least Squares)	536
5.1.1	Lemma (Coincidence of the Functions h and h_{γ_k})	538
5.1.2	Corollary (Restriction of Function h on Equivalence Class)	538
5.1.2	Lemma (Equality of Function Values on Equivalence Classes)	538
5.1.6	Proposition (Objective Function Composed)	538
5.1.3	Lemma (Characterization of Solution Vector (Meinhardt (2013)))	538

5.1.4	Lemma (Induced Relationships of a Solution Vector (Meinhardt (2013)))	538
5.1.7	Proposition (Matrix \mathbf{P} is Positive Semi-Definite (Meinhardt (2013)))	539
5.1.8	Proposition (\mathbf{P} is an Orthogonal Projection Operator (Meinhardt (2013)))	539
5.1.4	Theorem (Orthogonal Projection Method)	540
5.1.5	Theorem (Iteration Steps to Determine a Pre-Kernel Element with C.S.)	543
5.1.6	Theorem (Characterization of the Pre-Nucleolus for a C.S.)	546
5.1.7	Theorem (Characterization of the Pre-Nucleolus by Property I w.r.t. a C.S.)	546
5.1.8	Theorem (Characterization of the Pre-Nucleolus by Property II w.r.t. a C.S.)	546
5.2.1	Proposition (Union Stable System \mathcal{F} (Algaba et al. (2001)))	558
5.3.1	Theorem (Characterization of Pre-Nucleolus of Game with C.F. by Property I (Maschler (1992)))	564
5.3.2	Theorem (Characterization of Nucleolus of Game with C.F. by Property I (Maschler (1992)))	565
5.3.3	Theorem (Characterization Set for the Nucleolus (Granot et al. (1998)))	565
5.3.1	Lemma (Balanced Collection of Coalitions Formation Restrictions (Granot et al. (1998)))	565
5.3.1	Corollary (C-Set Characterization of Coalition Formation Restrictions (Granot et al. (1998)))	567
5.3.4	Theorem (C-Set for the Nucleolus (Huberman (1980)))	571
5.3.5	Theorem (C-Set for the Nucleolus (Solymosi and Sziklai (2016)))	572
5.3.6	Theorem (C-Set for the Nucleolus for Monotonic Games (Solymosi and Sziklai (2016)))	573
5.3.7	Theorem (Induced Relationships of (Dual-)Essential Sets for Bankruptcy Game)	575
5.3.1	Remark (Note to the Proof)	576
5.3.2	Lemma (Irreducible Saturated Coalition has no Partition (Granot et al. (1998)))	586
5.3.8	Theorem (C-Set for the Nucleolus (Granot et al. (1998)))	586
5.3.9	Theorem (Calculation of the Nucleolus in Strongly Polynomial Time (Granot et al. (1998)))	587
5.3.10	Theorem (Replication of the Pre-Kernel (Meinhardt (2013)))	588
5.3.11	Theorem (Non-Empty Interior of Payoff Equivalence Class Replicates a Singleton Pre-Kernel (Meinhardt (2024)))	588
Chapter 6		
6.2.1	Theorem (Non-Emptiness of the Core with a Veto Player)	613
6.3.1	Lemma (Reduction Lemma (Peleg et al. (1994)))	625
6.3.1	Theorem (Pre-Kernel without Equivalent Steps and Null-Players (Peleg and Rosenmüller (1992)))	625
6.3.2	Theorem (Center Solution is an Element of the Pre-Kernel (Sudhölter (1996)))	625
6.3.3	Theorem (Compatible Simple Game Representation is Homogeneous Representation (Rosenmüller (1987)))	626
6.3.1	Corollary (Pre-Kernel of Truncated Game)	632
6.3.1	Remark	633
6.3.2	Corollary (Single-valuedness of the Pre-Kernel of a Simple Game with a Homogeneous Representation)	635
6.3.2	Lemma (Pre-Kernel determined on the Basis of the Minimal Winning Coalitions (Sudhölter (1996)))	637
6.3.1	Proposition (Star-Shaped Pre-Kernel for a Homogeneous Standard Game (Sudhölter (1996)))	637
6.3.4	Theorem (Center Solution of a Homogeneous Standard Game (Sudhölter (1996)))	637
6.3.3	Corollary (Maximum Surplus attained by a Minimal Winning Coalition (Peleg (1966)))	642
6.3.5	Theorem (Degenerate Star-Shaped Pre-Kernel (Sudhölter (1996)))	642
6.3.2	Remark (Alternative Argument to Conclude on Single-Valuedness of the Pre-Kernel)	643
6.3.6	Theorem (Single-valued Pre-Kernel for Simple Games)	644
Chapter 7		
7.3.1	Lemma (Lorenz Maximality w.r.t. the Core (Hougaard et al. (2001)))	700
7.3.1	Theorem (The Lorenz set satisfies RGP (Hougaard et al. (2001)))	702
7.3.2	Theorem (The Core satisfies HMS-WRGP (Dietzenbacher and Sudhölter (2021)))	703
7.4.1	Theorem (The Core satisfies HMS-CRGP (Dietzenbacher and Sudhölter (2021)))	706
7.5.1	Theorem (Axiomatization of the Pre-Nucleolus (Orshan and Sudhölter (2003)))	708
Chapter 8		
8.2.1	Theorem (Characterization of a Flow Game)	717
8.3.1	Proposition (Pre-Nucleolus of the Dual Cover of a Game (Sudhölter (1997a)))	724

8.3.1	Theorem (Coincidence of the Modiclus and Restriction of the Pre-Nucleolus of the Dual Cover (Sudhölter (1997a); Peleg and Sudhölter (2007)))	724
8.3.1	Corollary (The Complaint Vector in relation to the Dual Cover and Bi-Complaint Vector)	725
8.3.1	Remark (Dual Cover)	725
8.3.2	Corollary (Symmetric Pre-Nucleolus of the Dual Extension (Sudhölter (1997a)))	726
8.3.3	Corollary (The Complaint Vector in relation to the Dual Extension and Bi-Complaint Vector)	727
8.3.2	Remark (Symmetric Pre-Nucleolus)	727
8.3.3	Remark (LED-Stability)	729
8.3.4	Corollary (LED and Core Existence)	729
8.3.1	Lemma (Large Excess Difference of Shift Game (Sudhölter (1997a)))	729
8.3.2	Lemma (Pre-Nucleolus of Shift Game (Sudhölter (1997a)))	730
8.3.4	Remark (Reasonableness of Core Solutions)	731
8.3.5	Remark (Excess Comparability Cover Game (Sudhölter (1997a)))	732
8.3.2	Theorem (Modiclus of Constant-Sum Game (Sudhölter (1996); Peleg and Sudhölter (2007)))	734
8.3.3	Lemma (Reduced Game of Dual Cover (Rosenmüller and Sudhölter (2004)))	734
8.3.4	Lemma (Induced Properties of LED (Sudhölter (1997a)))	735
8.3.5	Corollary (Induced Properties of LED)	737
8.3.6	Remark (Relationship between the Modiclus and Pre-nucleolus)	737
8.3.6	Corollary (Pre-Imputation is Reasonable on Both Sides)	737
8.3.5	Lemma (ECC Game and LED (Sudhölter (1997a,b)))	737
8.3.7	Corollary (Reduced Game and LED)	738
8.3.6	Lemma (ECC Game and t -Shift Dual Cover)	738
8.3.7	Remark (Modiclus is Reasonable on Both Sides (Sudhölter (1997a,b)))	739
8.3.2	Proposition (Shifted Games by LED and the Modiclus)	739
8.3.8	Corollary (Shifted Games by LED and the Modiclus)	739
8.3.7	Lemma (Replication of a Solution Vector (Meinhardt (2013)))	740
8.3.8	Lemma (Critical Bound to Remain in an Ellipsoid by an ij -Transfer (Meinhardt (2013)))	740
8.3.3	Proposition (No Transversal Vector Space of Balanced Excesses to Map a Pre-Kernel Element (Meinhardt (2024)))	740
8.3.4	Proposition (Extension of Proposition 8.3.3 (Meinhardt (2024)))	740
8.3.5	Proposition (No Non-Transversal Vector Space of Balanced Excesses to Map a Pre-Kernel Element (Meinhardt (2024)))	740
8.3.3	Theorem (Replication of a Singleton Pre-Kernel (Meinhardt (2024)))	740
8.3.4	Theorem (ECC Game and Replication of the Modiclus (Meinhardt (2024)))	741
8.3.8	Remark (Replication of the Modiclus)	741
8.3.9	Lemma (Substitution of PO by LEDCONS)	742
8.3.9	Remark (EC is not Essential)	742
8.3.10	Lemma (SIVA and DCP Imply AN (Sudhölter (1997a)))	742
8.3.11	Lemma (Strong Self-Duality)	743
8.3.12	Lemma (Reasonableness by Primal Power)	743
8.3.13	Lemma	743
8.3.14	Lemma (Substitution of DCP by DRP)	744
8.3.5	Theorem (Axiomatization of the Modiclus (Sudhölter (1997a)))	744
8.3.9	Corollary (Modified Axiomatization of the Modiclus)	745
8.3.10	Remark (LEDCONS and the Pre-Nucleolus)	749
8.3.15	Lemma (Axioms to Imply LEDCONS)	751
8.4.1	Theorem (ARGP of the Anti-Pre-Nucleolus)	752
8.4.1	Remark (Relation between LED and SED)	753
8.4.1	Lemma (Equivalent SED Representation)	753
8.4.2	Lemma (SED and Core Solutions)	754
8.4.3	Lemma (The Anti-Pre-Nucleolus of Shifted Games)	755
8.4.1	Proposition (Anti-Pre-Nucleolus of the Dual Floor of a Game)	755
8.4.2	Theorem (Coincidence of the Modiclus and Restriction of the Anti-Pre-Nucleolus of the Dual Floor)	755
8.4.1	Corollary (Symmetric Anti-Pre-Nucleolus of the Primal Extension)	757
8.4.2	Remark (Note to the Proof of Corollary 8.4.1)	757
8.4.3	Remark (SED and the Modiclus)	758

8.4.3	Theorem (Constant-Sum Games: Modiclus and Anti-Pre-Nucleolus)	758
8.4.4	Lemma (Anti-Reduced Game of Dual Floor)	758
8.4.5	Lemma (Induced Properties of SED)	759
8.4.2	Corollary (SED, ECF and AR game)	761
8.4.4	Remark (Relationship between the Modiclus and Anti-Pre-nucleolus)	761
8.4.3	Corollary (Pre-Imputation is Reasonable on Both Sides)	761
8.4.6	Lemma (ECF Game and PO)	761
8.4.4	Corollary (Anti-Reduced Game and SED)	762
8.4.5	Corollary	762
8.4.7	Lemma (ECF Game and t -Shift Dual Floor)	762
8.4.2	Proposition (Shifted Games by SED and the Modiclus)	762
8.4.6	Corollary (Shifted Games by SED and the Modiclus)	763
8.4.5	Remark (Dissection of the Anti-Pre-Nucleolus)	763
8.4.4	Theorem (The Modiclus of Convex Games)	765
8.4.6	Remark (The Modiclus of Super-Additive Games)	765
8.4.7	Remark (Inapplicable Deduction Theorem)	765
8.4.7	Corollary (Core Inclusion Result of ECF Game)	766
8.4.5	Theorem (Orthogonal Projection Method)	766
8.4.8	Lemma (Replication of a Solution Vector)	766
8.4.9	Lemma (Critical Bound to Remain in an Ellipsoid by an ij -Transfer)	767
8.4.8	Corollary (No Transversal Vector Space of Balanced Excesses to Map an Anti-Pre-Kernel Element)	767
8.4.9	Corollary (Extension of Corollary (8.4.8))	767
8.4.10	Corollary (No Non-Transversal Vector Space of Balanced Excesses to Map an Anti-Pre-Kernel Element)	767
8.4.11	Corollary (Replication of a Singleton Anti-Pre-Kernel)	768
8.4.12	Corollary (ECF Game and Replication of the Modiclus (Meinhardt (2024)))	768
8.4.10	Lemma (Substitution of PO by SEDCONS)	769
8.4.11	Lemma (Substitution of AN by DFP)	769
8.4.13	Corollary (Substitution of AN by PRP)	769
8.4.12	Lemma (Reasonableness by Preventive Power)	769
8.4.13	Lemma	769
8.4.14	Lemma (Substitution of DFP by PRP)	770
8.4.14	Corollary (Substitution of DCP by DFP)	771
8.4.15	Corollary (Substitution of DRP by PRP)	771
8.4.15	Lemma (Substitution of LEDCONS by SEDCONS)	771
8.4.16	Corollary	771
8.4.16	Lemma (Substitution of EC by REC)	771
8.4.6	Theorem (Modiclus: First Alternative Axiomatization)	772
8.4.17	Corollary (Modiclus: Modified Alternative Axiomatization)	772
8.4.8	Remark (SEDCONS and the Anti-Pre-Nucleolus)	772
8.4.9	Remark (Modiclus and SED/SEDCONS)	773
8.4.17	Lemma (Substitution of PO by ARGP)	775
8.4.18	Lemma (Axioms to imply SEDCONS)	775
8.5.1	Lemma (Derived Game and t -Shifted Reduced Game)	776
8.5.2	Lemma (Properties of Derived Game)	777
8.5.1	Corollary (Derived Game and LED)	777
8.5.3	Lemma (Modiclus and Pre-Nucleolus of the Derived Game (Rosenmüller and Sudhölter (2004)))	777
8.5.4	Lemma (Substitution of PO by DGP)	778
8.5.5	Lemma (Substitution of LEDCONS and EC by DGP)	778
8.5.2	Corollary	779
8.5.1	Theorem (Modiclus: Second Alternative Axiomatization)	779
8.7.1	Lemma (Anti-Derived Game and t -Shifted Anti-Reduced Game)	786
8.7.2	Lemma (Properties of an Anti-Derived Game)	786
8.7.1	Corollary (Anti-Derived Game and SED)	787
8.7.3	Lemma (Modiclus and Anti-Pre-Nucleolus of the Anti-Derived Game)	787
8.7.4	Lemma (Substitution of PO by ADGP)	787

8.7.5	Lemma (Substitution of REC and SEDCONS by ADGP)	788
8.7.2	Corollary	788
8.7.1	Theorem (Modiclus: Third Alternative Axiomatization)	788
8.8.1	Corollary (Modiclus of PS Game)	789
8.8.2	Corollary (Modiclus of Weighted Graph Game)	789
8.8.1	Lemma (Properties of Greedy Bankruptcy Game (Driessen (1998)))	790
8.8.1	Theorem (Driessen (1998))	790
8.8.1	Proposition (Anti-Pre-Nucleolus of Bankruptcy Games)	791
8.8.1	Remark (Composed Constant-Sum Games)	791
8.8.2	Lemma (Modiclus and Anti-Pre-Nucleolus)	792
8.8.3	Corollary (Anti-Pre-Nucleolus and Core)	793
8.8.2	Remark (Alternative Proof to Corollary 8.8.3)	793
8.8.2	Proposition (Replication of an Anti-Pre-Kernel Element (Meinhardt (2024)))	794
8.8.3	Remark (Preserving Properties of the Anti-Pre-Nucleolus)	794
8.8.4	Remark (Anti-Pre-Nucleolus for Convex Games)	794
8.8.2	Theorem (Modiclus of Bankruptcy Games)	794
8.8.4	Corollary (Modiclus of Bankruptcy Games)	794
8.9.1	Lemma (Bi-Balancedness of the Modiclus)	809
8.10.1	Theorem (Coincidence of Modiclus and Anti-Modiclus)	810
Chapter 9		
9.1.1	Lemma (Relationship among the Pre-Kernel and the Proper Modified Pre-Kernel (Sudhölter (1997b)))	816
9.1.1	Remark (Sudhölter (1997b))	817
9.1.2	Lemma (LED Implications on the (Modified) Pre-Kernel (Sudhölter (1997b)))	817
9.1.2	Remark (LED Implications on the (Modified) Pre-Kernel)	818
9.1.3	Remark (Note to the Proof of Lemma 9.1.2)	818
9.1.1	Corollary (LED Implication on Different Kinds of Maximum Surpluses)	822
9.1.4	Remark (LED on TU Games)	822
9.1.2	Corollary (Modified Pre-Kernel meets LEDCONS and EC (Sudhölter (1997b)))	822
9.1.3	Corollary	823
9.2.1	Theorem (Axiomatization Modified Pre-Kernel (Sudhölter (1997b)))	823
9.2.1	Remark (Note to the Proof Found in the Literature)	824
9.3.1	Lemma (Pre-Kernel of a Shift Game)	828
9.3.2	Lemma (Relationship between the Pre-Kernel of the Derived Game and Proper Modified Pre-Kernel)	828
9.3.1	Corollary (NE, COV, PO, ETP, and DGP imply EC and LEDCONS)	828
9.3.2	Corollary	829
9.3.3	Lemma (NE, COV, PO, ETP, CDGP imply LEDCOCONS)	829
9.3.3	Corollary (Proper Modified Pre-Kernel Meets DGP)	829
9.3.1	Theorem (Alternative Axiomatization of the Modified Pre-Kernel)	829
Chapter 12		
12.4.1	Proposition (Super-Additivity of a TU Game)	951
12.4.2	Proposition (Monotonicity of a TU Game)	951
12.5.1	Theorem (Bisection Property of the Kernel)	954
12.5.2	Theorem (Bisection Property of the Pre-Kernel)	954
12.5.1	Corollary (Directional Improvement Method of Pre-Kernel Iteration (Meinhardt (2013)))	954
12.8.1	Theorem ((p, k) -Converging Method to the Nucleolus (Kido (2004)))	975
12.8.2	Theorem (Equivalence of the Property II modified (Kido (2008)))	976
12.8.3	Theorem ((p, k) -Converging Method to the Pre-Nucleolus (Kido (2004)))	977
Chapter 13		
13.1.1	Lemma (Price Monotonicity Relationship (Driessen and Meinhardt (2010)))	997
13.1.2	Lemma (Induced Price Monotonicity Relationship (Driessen and Meinhardt (2010)))	997
13.1.1	Proposition (Monotonicity Property (Driessen and Meinhardt (2010)))	998
13.1.3	Lemma (Revenue Regularity (Driessen and Meinhardt (2010)))	998

13.1.2	Proposition (Monotonicity of a TU Oligopoly Game (Driessen and Meinhardt (2010)))	1000
13.1.1	Theorem (Convexity of a TU Oligopoly Game (Zhao (1999b)))	1001
13.1.2	Theorem (Convexity of a TU Oligopoly Game (Driessen and Meinhardt (2010)))	1001
Chapter A		
A.0.1	Proposition (Quadratic Function)	1087
Chapter B		
B.2.1	Proposition (Direct Proof Method)	1092
B.2.2	Proposition (Contrapositive Proof Method)	1093
B.2.3	Proposition (Contrapositive Reformulation)	1093
B.2.4	Proposition (Indirect Proof Method)	1093
B.3.1	Lemma (Misuse of the Deduction Theorem (Shapley and Shubik (1969)))	1098
Chapter C		
C.2.1	Theorem (Relative error without guard digit (Goldberg (1991)))	1108
C.2.2	Theorem (Relative error with guard digit (Goldberg (1991)))	1108
Chapter D		
D.1.1	Proposition (Uniqueness of the Identity)	1114
D.1.2	Proposition (Uniqueness of Inverse)	1114
D.1.1	Remark (Properties of Maps)	1115
D.1.1	Lemma (Bijectivity of Composite Map)	1115
D.1.3	Proposition (Group under Composition of Maps)	1115
D.1.4	Proposition (Subgroup Characterization)	1116
D.1.5	Proposition (Abelian Subgroup)	1116
D.1.6	Proposition (Induced Equivalence Relation)	1117
D.1.7	Proposition (Right Coset forms an Equivalence Class)	1118
D.1.1	Corollary (Alternative Characterization of an Equivalence Class by Right Cosets)	1118
D.1.8	Proposition (Bijection between Subgroup and Coset)	1118
D.1.1	Theorem (Lagrange)	1118
D.1.9	Proposition (A Homomorphism Result)	1119
D.2.1	Proposition (Positive General Linear Group (Meinhardt (2013)))	1120
D.2.1	Corollary (Group Action Induced (Meinhardt (2013)))	1121
Chapter E		
E.1.1	Proposition (Kernel/Image is \mathbf{G} -Submodule)	1125
E.1.1	Theorem (Schur's Lemma)	1125
E.1.1	Corollary (Schur's Lemma for Matrices)	1126
E.1.2	Proposition (Orthogonal Complement is \mathbf{G} -Submodule)	1126
E.1.3	Proposition (Decomposition in Irreducible Subspaces)	1126
E.1.2	Theorem (Maschke)	1126
E.1.3	Theorem (Complete Reducibility)	1127
E.1.4	Proposition (Properties of a Character)	1127
E.1.4	Theorem (Character Relations of the First Kind)	1128
E.1.2	Corollary (Complete Reducibility of a Matrix Representation)	1128
E.1.5	Proposition (Decomposable Group Algebra)	1128
E.1.5	Theorem (Frobenius Reciprocity)	1129
E.3.1	Theorem (Decomposable Game Space (Sánchez-Pérez (2017)))	1131
E.3.1	Lemma (Game Solution Satisfying LIN and SYM (Sánchez-Pérez (2017)))	1132
E.3.1	Corollary (Dimension of LIN and SYM Game-Solution Space (Sánchez-Pérez (2017)))	1134
E.3.2	Corollary (Dimension of LIN, SYM, and EFF Game-Solution Space (Sánchez-Pérez (2017)))	1134
E.3.1	Proposition (Dimension Relation between Game Solution Spaces (Sánchez-Pérez (2017)))	1134

List of Algorithms

2.5.1	Method for Computing the Nucleolus of an Airport Cost Allocation Game	116
4.3.1	Prim's Algorithm for finding a minimal tree	413
4.3.2	Kruskal's Algorithm for finding a minimal tree	413
4.4.1	Method for Computing the Potential of a TU Game	430
4.4.2	Method for Computing the Shapley Value by the P-Potential	431
4.4.3	Procedure to seek a Pre-Kernel Element	442
4.4.4	Procedure to seek a Kernel Element	454
4.4.5	Method for Computing the Pre-Nucleolus	463
4.4.6	Algorithm for verifying if a pre-imputation is the pre-nucleolus	472
4.4.7	Algorithm for verifying if an imputation is the nucleolus	474
4.4.8	Algorithm for verifying if an imputation is the nucleolus (\mathcal{B}_0 -Property)	474
4.4.9	Method for Computing the weighted Pre-Nucleolus for Weight System P^N	477
4.4.10	Procedure to seek a weighted Pre-Kernel Element for Weight System P^N	482
4.4.11	Procedure to seek an Anti-Pre-Kernel Element	489
4.4.12	Method for Computing the Anti-Pre-Nucleolus	491
4.4.13	Method for Computing the Least Square Nucleolus	496
4.5.1	Seeking for a Pre-Kernel Element (Quadratic Programming)	499
4.5.2	Seeking for a Pre-Kernel Element (The Least Squares/Conic Optimization)	501
5.1.1	Procedure to seek a Pre-Kernel Element for a Coalition Structure	542
5.1.2	Method for Computing the Pre-Nucleolus for a Coalition Structure	545
5.1.3	Algorithm for verifying if a pre-imputation is the pre-nucleolus for a coalition structure	547
5.3.1	Method for Computing the Pre-Nucleolus of Game with Coalition Formation Restrictions	568
6.3.1	Algorithm for finding the minimum homogeneous representation	627
8.1.1	Method for Computing the Modiclus	716
8.9.1	Algorithm for verifying if a pre-imputation is the modiclus	809
8.10.1	Method for Computing the Anti-Modiclus	810
12.5.1	Procedure to seek a Pre-Kernel Element (Directional Improvement)	955
12.8.1	Non-linear Method to Compute the Nucleolus	976
12.8.2	Non-linear Method to Compute the Pre-Nucleolus	978
A.0.1	Procedure to seek a Pre-Kernel Element (Quadratic Programming)	1087
C.2.1	Quadratic Iterative Method of Getting the Pseudo-Inverse	1104
C.2.2	Method for Approximating the Exponential Function at x	1109

List of Definitions and Examples

Chapter 1

1.2.1	Example (Shapley Value not a Core Element)	11
1.2.2	Example (Strong ϵ -Core)	12
1.2.1	Definition (A Domination Concept across Payoffs)	18
1.2.2	Definition (Stable Sets)	18
1.2.3	Definition (Composite Game)	19
1.2.3	Example (Non-Empty Core (Thm. 1.2.17 Case 3))	22
1.2.4	Example (Non-Empty Core (Thm. 1.2.17 Case 4))	23
1.2.5	Example (Non-Empty Core (Thm. 1.2.17 Case 5))	24
1.3.1	Example (Manual Computation of the Shapley Value)	27

Chapter 2

2.2.1	Example (Solving Non-Singular System of Linear Equations)	74
2.2.2	Example (Solving Underdetermined System of Linear Equations)	78
2.2.3	Example (Solving Overdetermined System of Linear Equations)	81
2.2.4	Example (Computing Moore-Penrose (3×4)-Matrix)	83
2.2.5	Example (Computing Moore-Penrose (4×3)-Matrix)	84
2.2.6	Example (QR Factorization of an Overdetermined System)	86
2.2.7	Example (Singular Value Factorization of an Underdetermined System)	87
2.2.8	Example (LU Factorization of a Non-Singular System)	89
2.2.9	Example (Cholesky Factorization of a Symmetric System)	90
2.4.1	Example (Projection of 4-Dimensional Data onto a 3-Dimensional Simplex)	104
2.5.1	Definition (Airport Capital Cost Game)	113
2.8.1	Definition (Canonical Order onto the Power Set)	181
2.8.1	Example (Unique Integer Representations vs. Canonical Order)	181
2.8.2	Example (MC-Nets representation)	186

Chapter 3

3.7.1	Definition (Nash Equilibrium)	319
3.7.2	Definition (Uhc Correspondence)	319
3.7.3	Definition (Lhc Correspondence)	320
3.7.4	Definition (Continuous Correspondence)	320
3.7.5	Definition (Closed Graph)	320
3.7.6	Definition (Contraction Mapping)	322

Chapter 4

4.3.1	Definition (Market Situation)	356
4.3.2	Definition (Market Game)	357
4.3.1	Example (4-Person Market Game)	359
4.3.3	Definition (Balanced Cover of a TU Game)	361
4.3.4	Definition (Totally Balanced Cover of a TU Game)	361
4.3.2	Example (10-Person Game with No Stable Set, but with a Core)	362
4.3.5	Definition (Direct Market)	364
4.3.6	Definition (Partially Ordered Set)	367
4.3.7	Definition (Lattice)	367
4.3.3	Example (Lattices)	367
4.3.4	Example (Airport Problem)	371
4.3.5	Example (Anti-Partition)	376
4.3.6	Example (Separating Collection)	376
4.3.7	Example (Not a Separating Collection)	377

4.3.8	Example (Imposing the same Partition)	378
4.3.9	Example (4-Person Average-Convex Game)	382
4.3.8	Definition (Flow Game)	395
4.3.9	Definition (Generalized Permutation)	397
4.3.10	Definition (Minimum Cut of a Flow Problem)	400
4.3.11	Definition (PS Property)	404
4.3.12	Definition (COV)	404
4.4.1	Definition (Collection of Coalitions Attaining a Level of Excess)	468
4.4.2	Definition (Property I)	469
4.4.3	Definition (Property II)	469
4.4.4	Definition (Weak Property I)	472
4.4.5	Definition (Weak Property II)	473
4.4.6	Definition (Collection of Coalitions Attaining a Level of Weighted Excess)	477
4.4.7	Definition (Collection of Coalitions Attaining a Level of Dissatisfaction)	492
4.4.8	Definition (Anti-Property I)	493
4.4.9	Definition (Anti-Property II)	495
4.4.10	Definition (Weak Anti-Property I)	495
4.4.11	Definition (Weak Anti-Property II)	495
Chapter 5		
5.1.1	Definition (Super-Additive Cover)	525
5.1.2	Definition (Collection of Coalitions w.r.t. a C.S. Attaining a Level of Excess)	546
5.1.3	Definition (Property I w.r.t. a C.S.)	546
5.1.4	Definition (Property II w.r.t. a C.S.)	546
5.3.1	Definition (Permissible Collection of Coalitions Achieving a Level of Excess)	563
5.3.2	Definition (Property I with Coalition Formation Restrictions)	563
5.3.3	Definition (Property I and C.F.)	563
5.3.4	Definition (C-Set)	565
5.3.5	Definition (Essential Sets)	571
5.3.6	Definition (Dually Essential Sets)	571
5.3.7	Definition (Saturated Coalition (Granot et al. (1998)))	586
5.3.8	Definition (Irreducible Saturated Coalition (Granot et al. (1998)))	586
Chapter 6		
6.2.1	Definition (Simple Game)	609
6.2.2	Definition (Simple TU Game)	610
6.2.3	Definition (Shapley-Shubik Power Index (Shapley and Shubik (1954)))	612
6.3.1	Definition (Compatible Simple Game Representation)	626
6.3.2	Definition (Natural Representation of a Simple Game)	626
6.3.1	Example (Not Homogeneous Standard Simple Game)	637
6.3.2	Example (Star-Shaped Pre-Kernel)	641
6.3.3	Definition (Partition Game)	642
6.3.3	Example (Six-Person Simple Game with a Minimum Homogeneous Representation)	643
6.3.4	Example (Seven-Person Projective Game)	643
6.3.5	Example (Single-valued Pre-Kernel)	644
Chapter 7		
7.1.1	Example (DM-RGP)	691
7.1.2	Example (HMS-RGP)	692
7.3.1	Example (HMS-Weak RGP)	703
7.4.1	Example (HMS-CRGP)	706
Chapter 8		
8.2.1	Definition (DM-RGP)	719
8.3.1	Definition (Dual Cover)	724
8.3.2	Definition (LED and Diverse Game Properties)	728
8.3.3	Definition (Diverse Solution Properties)	731
8.3.4	Definition (SDCP)	744

8.3.5	Definition (Modified LEDCONS)	750
8.4.1	Definition (Anti-RGP)	752
8.4.2	Definition (SED and Diverse Game Properties)	753
8.4.3	Definition (SEDCONS and REC)	754
8.4.4	Definition (Dual Floor)	755
8.4.5	Definition (PRP and DFP)	755
8.4.6	Definition (SDFP)	770
8.4.7	Definition (Modified SEDCONS)	773
8.5.1	Definition (DM-DGP)	778
8.7.1	Definition (Anti-Derived TU Game)	786
8.9.1	Definition (Collection of Coalitions Attaining a Level of Bi-Excess)	809
Chapter 9		
9.1.1	Definition (Maximum (Proper) Modified Surplus)	814
9.1.2	Definition (Self-Dual Modifications of the Pre-Kernel)	814
9.2.1	Definition (COCONS)	823
9.2.2	Definition (LED COCONS)	823
9.3.1	Definition (Converse DGP)	827
9.3.2	Definition (Strong Converse DGP)	831
Chapter 11		
11.5.1	Example (MC-Nets Representation)	903
Chapter 12		
12.8.1	Definition (Modified Coalition Array (Kido (2008)))	975
12.8.2	Definition (Generating of a Modified Coalition Array (Kido (2008)))	975
12.8.3	Definition ($((v, \mathbf{x})$ -Balancedness (Kido (2008)))	975
12.8.4	Definition ($((v, \mathbf{x})$ -Maximal Balancedness (Kido (2008)))	975
12.8.5	Definition (Property II Modified (Kido (2008)))	976
Chapter B		
B.3.1	Example (Wrong Indirect Proof Method)	1097
B.3.2	Example (Wrong Indirect Proof Method Reconsidered)	1097
Chapter C		
C.2.1	Definition (Absolute/Relative Error)	1103
C.2.2	Definition (Significance of Digits)	1103
C.2.3	Definition (Significance of Decimal Places)	1103
C.2.4	Definition (Number of Significant Digits)	1103
C.2.5	Definition (Least Significant Digit)	1104
Chapter D		
D.1.1	Definition (Group)	1113
D.1.1	Example (Groups under Addition)	1114
D.1.2	Example (Groups under Multiplication)	1114
D.1.3	Example (Permutation Group)	1115
D.1.2	Definition (Abelian)	1115
D.1.3	Definition (Relation on a Set)	1117
D.1.4	Definition (Equivalence Relation)	1117
D.1.5	Definition (Equivalence Class)	1117
D.1.6	Definition (Cosets)	1117
D.1.7	Definition (Group Action)	1118
D.2.1	Definition (Connected Group)	1120
Chapter E		
E.1.1	Definition (Ring)	1123
E.1.2	Definition (R -Module)	1123
E.1.3	Definition (\mathbf{G} -Module)	1124

E.1.4	Definition (Invariant Subspace)	1125
E.1.5	Definition (Irreducible Space)	1125
E.1.6	Definition (Character)	1127
E.1.7	Definition (Class Function)	1127
E.1.8	Definition (Group Algebra)	1127
E.1.9	Definition (Partition of a positive Integer)	1130

Bibliography

- H. Aarts and T. Driessen. The Irreducible Core of a Minimum Cost Spanning Tree Game. *ZOR - Methods and Models of Operations Research*, 38:163174, 1993. doi: 10.1007/BF01414212. URL <https://doi.org/10.1007/BF01414212>.
- E. Algaba, J. M. Bilbao, P. Borm, and J. J. López. The Position Value for Union Stable Systems. *Mathematical Methods of Operations Research*, 52:221–236, 2000.
- E. Algaba, J. M. Bilbao, P. Borm, and J. J. López. The Myerson Value for Union Stable Structures. *Mathematical Methods of Operations Research*, 54:359–371, 2001.
- E. Algaba, J. M. Bilbao, and J. J. López. The Position Value in Communication Structures. *Mathematical Methods of Operations Research*, 59:465–477, 2004.
- E. Algaba, V. Fragnelli, and J. Sánchez-Soriano, editors. *Handbook of the Shapley Value*. Series in Operations Research. Chapman and Hall/CRC, New York, 1 edition, 2020. doi: 10.1201/9781351241410. URL <https://doi.org/10.1201/9781351241410>.
- Ch. D. Aliprantis and K. C. Border. *Infinite Dimensional Analysis: A Hitchhiker's Guide*. Springer-Verlag, Berlin, 1994.
- J. M. Alonso-Mejide, F. Ferreira, M. Álvarez Mozos, and A. A. Pinto. Two New Power Indices based on Winning Coalitions. *Journal of Difference Equations and Applications*, 17(7):1095–1100, 2011. doi: 10.1080/10236190903200677. URL <https://doi.org/10.1080/10236190903200677>.
- J.M. Alonso-Mejide, F. Carreras, M.G. Fiestras-Janeiro, and G. Owen. A Comparative Axiomatic Characterization of the BanzhafOwen Coalitional Value. *Decision Support Systems*, 43(3):701–712, 2007. ISSN 0167-9236. doi: <https://doi.org/10.1016/j.dss.2006.11.008>. URL <https://www.sciencedirect.com/science/article/pii/S0167923606002028>. Integrated Decision Support.
- Y. Altman. *Accelerating MATLAB Performance*. CRC Press, Boca Raton, FL. (USA), 1 edition, 2015.
- E.D. Andersen, C. Roos, and T. Terlaky. On Implementing a Primal-Dual Interior-Point Method for Conic Quadratic Optimization. *Mathematical Programming*, 95(2):249 – 277, 2002. doi: 10.1007/s10107-002-0349-3. URL <https://doi.org/10.1007/s10107-002-0349-3>.
- J. Arin and V. Feltkamp. The Nucleolus and Kernel of Veto-Rich Transferable Utility Games. *International Journal of Game Theory*, 26:61–73, 1997. URL <https://doi.org/10.1007/BF01262513>.
- J. Arin and E. Iñarra. A Characterization of the Nucleolus for Convex Games. *Games and Economic Behaviour*, 23:12–24, 1998.
- J. Arin and I. Katsev. The Coincidence of the Kernel and Nucleolus of a Convex Game: An Alternative Proof. Technical report, University of Basque Country, 2013.
- S. Arora and B. Barak, editors. *Computational Complexity: A Modern Approach*. Cambridge University Press, Cambridge, 2009. doi: <https://doi.org/10.1017/CBO9780511804090>. URL <https://www.cambridge.org/core/books/computational-complexity/3453CAFD EB0B4820B186FE69A64E1086>.
- St. Attaway. *MATLAB: A Practical Introduction to Programming and Problem Solving*. Butterworth-Heinemann/Elsevier, Oxford, UK., 5 edition, 2019.
- R. J. Aumann. Acceptable Points in General Cooperative n -Person Games. In Tucker and Luce, editors, *Contributions to the theory of games IV, Annals of Mathematics Studies Vol. 40*, pages 287–324, 1959.
- R. J. Aumann. A Survey on Cooperative Games without Side Payments. In M. Shubik, editor, *Essays in Mathematical Economics in Honor of Oskar Morgenstern*, pages 3–27, Princeton, 1961. Princeton University Press.
- R. J. Aumann. Agreeing to Disagree. *The Annals of Statistics*, 4(6):1236–1239, 1976. ISSN 00905364. URL <http://www.jstor.org/stable/2958591>.
- R. J. Aumann and J. H. Drèze. Cooperative Games with Coalition Structures. *International Journal of Game Theory*, 3:217–237, 1974.
- R. J. Aumann and M. Maschler. Game Theoretic Analysis of a Bankruptcy Problem from the Talmud. *Journal of Economic Theory*, 36: 195–213, 1985.
- J. F. Banzhaf. Weighted Voting Doesn't Work. *Rutgers Law Review*, 19:221–236, 1965.
- R.J. Barro and X. Sala-I-Martin. *"Economic Growth"*. McGraw-Hill, New York, 1 edition, 1995.
- E.N. Barron. *"Game Theory: An Introduction"*. John Wiley & Sons, Hoboken, 2 edition, 2013.
- B. Barry. Is it Better to Be Powerful or Lucky?: Part I. *Political Studies*, 28(2):183–194, 1980a. doi: 10.1111/j.1467-9248.1980.tb01244.x. URL <https://doi.org/10.1111/j.1467-9248.1980.tb01244.x>.

- B. Barry. Is it Better to Be Powerful or Lucky? Part II. *Political Studies*, 28(3):338–352, 1980b. doi: 10.1111/j.1467-9248.1980.tb00473.x. URL <https://doi.org/10.1111/j.1467-9248.1980.tb00473.x>.
- G. Bergantiños and J. Vidal-Puga. A Review of Cooperative Rules and their Associated Algorithms for Minimum-Cost Spanning Tree Problems. *SERIEs*, 12:73–100, 2021.
- C. G. Bird. On Cost Allocation for a Spanning Tree: A Game Theoretic Approach. *Networks*, 6(4):335–350, 1976. doi: <https://doi.org/10.1002/net.3230060404>. URL <https://onlinelibrary.wiley.com/doi/abs/10.1002/net.3230060404>.
- O.J. Blanchard and S. Fischer. *"Lectures on Macroeconomics"*. MIT-Press, Cambridge, Massachusetts, 1 edition, 1989.
- O. N. Bondareva. Some Applications of Linear Programming Methods to the Theory of Cooperative Games. *Problemy Kibernet*, 10: 119–139, 1963. in Russian.
- K. C. Border. *Fixed Point Theorems with Applications to Economics and Game Theory*. Cambridge University Press, Cambridge, UK, 1985.
- P. Borm, G. Owen, and SH. Tijs. On the Position Value for Communication Situations. *SIAM Journal of Discrete Mathematics*, 5: 305–320, 1992.
- St. Boyd and L. Vandenberghe. *Convex Optimization*. Cambridge University Press, Cambridge, eight reprint edition, 2010.
- E. Bozzo, M. Franceschet, and F. Rinaldi. Vulnerability and Power on Networks. *Network Science*, 3(2):196226, 2015. doi: 10.1017/nw.s.2015.8.
- R.L. Breiger and Ph. E. Pattison. Cumulated Social Roles: The Duality of Persons and their Algebras. *Social Networks*, 8(3):215 – 256, 1986. ISSN 0378-8733. doi: [https://doi.org/10.1016/0378-8733\(86\)90006-7](https://doi.org/10.1016/0378-8733(86)90006-7). URL <http://www.sciencedirect.com/science/article/pii/0378873386900067>.
- S. Béal, M. Manea, E. Rémila, and Ph. Solal. *Games with Identical Shapley Values*, page 93:110. In Algaba et al. (2020), 1 edition, 2020. doi: 10.1201/9781351241410. URL <https://doi.org/10.1201/9781351241410>.
- Sylvain Béal, Eric Rémila, and Philippe Solal. A Decomposition of the Space of TU-games Using Addition and Transfer Invariance. *Discrete Applied Mathematics*, 184:1–13, 2015. ISSN 0166-218X. doi: <https://doi.org/10.1016/j.dam.2014.12.019>. URL <https://www.sciencedirect.com/science/article/pii/S0166218X14005496>.
- E. Calvo and E. Gutiérrez. A Value for Cooperative Games with a Coalition Structure. Technical report, University of Valencia, Spain, 2012.
- E. Calvo and E. Gutiérrez. The Shapley-Solidarity Value for Games with a Coalition Structure. *International Game Theory Review*, 15 (1):1350002–1–1350002–24, 2013.
- L. Carpenste, B. Casas-Méndez, I. García-Jurado, and A. van den Nouveland. The Shapley Valuation Function for Strategic Games in which Players Cooperate. *Mathematical Methods of Operations Research*, 63:435–442, 2006.
- M. Carter. *Cooperative Games*, pages 167–191. In Varian (1993), 1993.
- G. Chalkiadakis, E. Elkind, and M. Wooldridge. *Computational Aspects of Cooperative Game Theory*, volume 16 of *Synthesis Lectures on Artificial Intelligence and Machine Learning*. Morgan and Claypool Publisher, Heidelberg/Berlin, 2012.
- Ch. Chang and T. S. H. Driessen. (Pre)Kernel Catchers for Cooperative Games. *OR Spectrum*, 17(1):pp 23–26, 1995.
- Ch. Chang and Ch. Hu. A Non-Cooperative Interpretation of the Kernel. *International Journal of Game Theory*, 46(1):185–204, 2017. doi: 10.1007/s00182-016-0529-7. URL <https://doi.org/10.1007/s00182-016-0529-7>.
- Ch. Chang and Ch. Y. Kan. The Bound of the Kernel. *Mathematical Social Sciences*, 25:87–93, 1992.
- Ch. Chang and Ch.-H. Lian. Some Results on (Pre)Kernel Catchers and the Coincidence of the Kernel with Prekernel. *International Game Theory Review*, 4(3):201–211, 2002.
- Chih Chang and Ying-Chih Tseng. On the Coincidence Property. *Games and Economic Behavior*, 71(2):304 – 314, 2011. ISSN 0899-8256. doi: <https://doi.org/10.1016/j.geb.2010.04.010>. URL <http://www.sciencedirect.com/science/article/pii/S0899825610000710>.
- St. J. Chapman. *MATLAB Programming for Engineers*. Brooks/Cole, Pacific Grove, CA. (USA), 2 edition, 2002.
- W. Cheney and D. Kincaid. *Numerical Mathematics and Computing*. Thomson Higher Education, Belmont, 6th edition, 2008.
- A. C. Chiang. *Fundamental Methods of Mathematical Economics*. McGraw-Hill, Singapore, third. edition, 1984.
- A. Claus and D. J. Kleitman. Cost Allocation for a Spanning Tree. *Networks*, 3(4):289–304, 1973. doi: <https://doi.org/10.1002/net.3230030402>. URL <https://onlinelibrary.wiley.com/doi/abs/10.1002/net.3230030402>.
- R. H. Coase. The Marginal Cost Controversy. *Economica*, 13(51):169–182, 1946. ISSN 00130427, 14680335. URL <http://www.jstor.org/stable/2549764>.
- J. S. Coleman. *Control of Collectivities and the Power of a Collectivity to Act*. Gordon and Breach, New York, 1 edition, 1971.

- I. Curiel. *Cooperative Game Theory and Applications*, volume 16 of *Theory and Decision Library: Series C*. Kluwer Acad. Publ., Boston, 1997.
- M. Davis and M. Maschler. The Kernel of a Cooperative Game. *Naval Research Logistic Quarterly*, 12:223–259, 1965.
- R. de Roover. *The Rise and Decline of the Medici Bank 1397-1494*. The Norton Library, New York, USA, 1966.
- J. Deegan and E.W. Packel. A New Index of Power for Simple n-Person Games. *International Journal of Game Theory*, 7:113–123, 1978. doi: 10.1007/BF01753239. URL <https://doi.org/10.1007/BF01753239>.
- P. Dehez. On Harsanyi Dividends and Asymmetric Values. *International Game Theory Review*, 19(03):1750012–1–36, 2017. doi: 10.1142/S0219198917500128. URL <https://doi.org/10.1142/S0219198917500128>.
- P. Dehez. Sharing a Collective Probability of Success. *Mathematical Social Sciences*, 123:122–127, 2023. ISSN 0165-4896. doi: <https://doi.org/10.1016/j.mathsocsci.2023.03.006>. URL <https://www.sciencedirect.com/science/article/pii/S0165489623000343>.
- P. Dehez. Cooperative Product Games. *International Game Theory Review*, 26(01):2350016, 2024. doi: 10.1142/S0219198923500160. URL <https://doi.org/10.1142/S0219198923500160>.
- P. Dehez and P. M. Pacini. *Comparing the Nucleolus and the Shapley Value of 3-player Transferable Utility Games*. Center for Operations Research and Econometrics (CORE-LIDAM), UCLouvain, Louvain-la-Neuve, Belgium, January 2025.
- X. Deng and C. H. Papadimitriou. On the Complexity of Cooperative Solution Concepts. *Mathematics of Operations Research*, 19: 257–266, 1994.
- J. Derks and J. Kuipers. On the Core and the Nucleolus of Routing Games. Discussion Paper Report M92-07, Department of Mathematics, University of Maastricht, 1992.
- J. J. M. Derks and H. Haller. Weighted Nucleoli. *International Journal of Game Theory*, 28:173–187, 1999.
- B. Dietzenbacher and P. Sudhölter. HartMas-Colell Consistency and the Core in Convex Games. *International Journal of Game Theory*, pages 1–17, 2021. ISSN 0254-5330. doi: 10.1007/s00182-021-00798-6. URL <https://doi.org/10.1007/s00182-021-00798-6>.
- A. Dixit. A Model of Duopoly suggesting a Theory of Entry Barriers. *Bell Journal of Economics*, 10:20–32, 1979.
- A. K. Dixit. *Optimization in Economic Theory*. Oxford University Press, Oxford, 2 edition, 1990.
- T. Driessen. *Contributions to the Theory of Cooperative Games: The τ -Value and k -Convex Games*. Ph.d thesis, University of Nijmegen, Nijmegen, 1985.
- T. Driessen. *Cooperative Games, Solutions and Applications*. Kluwer Academic Publishers, Dordrecht, 1988.
- T. Driessen. The Greedy Bankruptcy Game: An Alternative Game Theoretic Analysis of a Bankruptcy Problem. In L.A. Petrosjan and V.V. Mazalov, editors, *Game Theory and Applications*, volume IV, pages 45–61, Commack, New York, 1998. Nova Science Publishers Inc.
- T. Driessen and H. Meinhardt. (Average-) Convexity of Common Pool and Oligopoly TU-games. *International Game Theory Review*, 3: 141–158, 2001.
- T. Driessen and H. Meinhardt. Convexity of Oligopoly Games without Transferable Technologies. *Mathematical Social Sciences*, 50(1): 102–126, 2005.
- T. S. H. Driessen and H. I. Meinhardt. On the Supermodularity of Homogeneous Oligopoly Games. *International Game Theory Review (IGTR)*, 12(04):309–337, 2010. doi: 10.1142/S0219198910002702. URL <https://doi.org/10.1142/S0219198910002702>.
- T. A. Driscoll and R. J. Braun. *Fundamentals of Numerical Computation*. SIAM, Philadelphia PA, USA, 1th edition, 2018.
- P. Dubey and L. S. Shapley. Mathematical properties of the banzhaf power index. *Mathematics of Operations Research*, 4(2):99–131, 1979. ISSN 0364765X, 15265471. URL <http://www.jstor.org/stable/3689345>.
- H. Dym. *Linear Algebra in Action*, volume 78. American Mathematical Society, Providence, Rhode Island, Graduate Studies in Mathematics edition, 2007.
- H.-D. Ebbinghaus, J. Flum, and W. Thomas. *Einführung in die mathematische Logik*. Spektrum Akademischer Verlag, Berlin/Heidelberg, 5 edition, 2007.
- L. Elden, L. Wittmeyer-Koch, and H. B. Nielsen. *Introduction to Numerical Computation*. Studentlitteratur AB, Lund, Sweden, 2004.
- E. Elkind, L. A. Goldberg, P. W. Goldberg, and M. Wooldridge. A Tractable and Expressive Class of Marginal Contribution Nets and Its Applications. *Mathematical Logic Quarterly*, 55(4):362–376, 2009. doi: 10.1002/malq.200810021. URL <https://onlinelibrary.wiley.com/doi/10.1002/malq.200810021>.
- A. Estévez-Fernández, M.G. Fiestras-Janeiro, M.A. Mosquera, and E. Sánchez-Rodríguez. A Bankruptcy Approach to the Core Cover. Technical Report TI 2012-012/1, VU University Amsterdam, 2012.
- U. Faigle and M. Grabisch. Linear Transforms, Values and Least Square Approximation for Cooperation Systems. Technical report, Panthéon-Sorbonne, Université Paris 1, 2014.

- D.S. Felsenthal. A Well-Behaved Index of a Priori P-Power for Simple N-Person Games. *Homo Oeconomicus*, 33:367–381, 2016. doi: 10.1007/s41412-016-0031-2. URL <https://doi.org/10.1007/s41412-016-0031-2>.
- V. Feltkamp. Alternative Axiomatic Characterizations of the Shapley and Banzhaf Values. *International Journal of Game Theory*, 24: 179–186, 1995. doi: 10.1007/BF01240041. URL <https://doi.org/10.1007/BF01240041>.
- L.R. Ford and D.R. Fulkerson. *Flows in Networks*. Princeton University Press, Princeton, 1962.
- A. Fronzetti Colladon and M. Naldi. Distinctiveness Centrality in Social Networks. *PLoS ONE*, 15(5), 2020. URL <https://doi.org/10.1371/journal.pone.0233276>.
- W. Fulton and J. Harris. *Representation Theory: A First Course*, volume 129 of *Graduate Texts in Mathematics*. Springer Publisher, Heidelberg, 2 edition, 2004.
- Y. Funaki. Upper and Lower Bounds of the Kernel and Nucleolus. *International Journal of Game Theory*, 15:121–129, 1986.
- Y. Funaki and H. I. Meinhardt. A Note on the Pre-Kernel and Pre-Nucleolus for Bankruptcy Games. *The Waseda Journal of Political Science and Economics*, 363:126–136, 2006.
- Y. Funaki and K. Yokote. *Several Bases of a Game Space and an Application to the Shapley Value*, pages 111–130. In [Algaba et al. \(2020\)](#), 1 edition, 2020. doi: 10.1201/9781351241410. URL <https://doi.org/10.1201/9781351241410>.
- D. Gale and H. Nikaido. The Jacobian Matrix and Global Univalence of Mappings. *Mathematische Annalen*, 159(2):1432–1807, 1965. doi: 10.1007/BF01360282. URL <https://doi.org/10.1007/BF01360282>.
- D. Gately. Sharing the Gains from Regional Cooperation: A Game Theoretic Application to Planning Investment in Electric Power. *International Economic Review*, 15(1):195–208, 1974. ISSN 00206598, 14682354. URL <http://www.jstor.org/stable/2526099>.
- G. Gaudet and St. W. Salant. Uniqueness of Cournot Equilibrium: New Results from Old Methods. *The Review of Economic Studies*, 58(2):399–404, 1991. ISSN 00346527, 1467937X. URL <http://www.jstor.org/stable/2297975>.
- R. J. Gaylord, S. N. Kamin, and P. R. Wellin. *An Introduction to Programming with Mathematica*. Telos/Springer, Santa Clara, CA, sec. edition, 1996.
- J. Getán, J. Izquierdo, J. Montes, and C. Rafels. The Bargaining Set and the Kernel of Almost-Convex Games. Technical report, University of Barcelona, Spain, 2012.
- D. Goldberg. What Every Computer Scientist Should Know About Floating-Point Arithmetic. *ACM Computing Surveys*, 23:5–48, 1991.
- J. González-Díaz and E. Sánchez-Rodríguez. A Natural Selection From the Core of a TU Game: The Core-Center. *International Journal of Game Theory*, 36:2746, 2007. doi: 10.1007/s00182-007-0074-5. URL <https://doi.org/10.1007/s00182-007-0074-5>.
- H.-G. Gräbe and M. Kofler. *Mathematica 6: Einführung, Grundlagen und Beispiele*. Scientific Tools. Pearson Studium, München, 5 edition, 2007.
- D. Granot and G. Huberman. Minimum Cost Spanning Tree Games. *Mathematical Programming*, 21:1–18, 1981. URL <https://doi.org/10.1007/BF01584227>.
- D. Granot and G. Huberman. The Relationship Between Convex Games and Minimum Cost Spanning Tree Games: A Case for Permutationally Convex Games. *SIAM Journal on Algebraic Discrete Methods*, 3(3):288–292, 1982. doi: 10.1137/0603029. URL <https://doi.org/10.1137/0603029>.
- D. Granot and G. Huberman. On the Core and Nucleolus of Minimum Cost Spanning Tree Games. *Mathematical Programming*, 29: 323347, 1984. URL <https://doi.org/10.1007/BF02592000>.
- D. Granot, F. Granot, and W. R. Zhu. Circular Network Games. Discussion paper, Faculty of Economics and Business Administration, UBC Vancouver, 1994.
- D. Granot, F. Granot, and W. R. Zhu. Characterization Sets for the Nucleolus. *International Journal of Game Theory*, 27:359–374, 1998.
- Z. Gu, E. Rothberg, and R. Bixby. Gurobi Software documentation. Technical report, Gurobi Optimization, Inc., Houston, Texas, 2024a. Gurobi Optimizer Reference Manual Version 12.0.
- Z. Gu, E. Rothberg, and R. Bixby. Gurobi Software program. Technical report, Gurobi Optimization, Inc., Houston, Texas, 2024b. Gurobi Optimizer Version 12.0.
- B. D. Hahn and D. T. Valentine. *Essential MATLAB for Engineers and Scientists*. Academic Press, Cambridge, MA. (USA), 7 edition, 2019.
- B. C. Hall. *Lie Groups, Lie Algebras, and Representations: An Elementary Introduction*, volume 222 of *Graduate Texts in Mathematics*. Springer-Verlag, 2003.
- S. Hart and M. Kurz. Endogenous Formation of Coalitions. *Econometrica*, 51(4):1047–1064, 1983.
- S. Hart and A. Mas-Colell. Potential, Value and Consistency. *Econometrica*, 57:589–614, 1989.

- D.A. Harville. *Matrix Algebra From a Statistician's Perspective*. Springer Publisher, New York, Heidelberg, 1997.
- M. Herceg, M. Kvasnica, C. N. Jones, and M. Morari. Multi-Parametric Toolbox 3.0. In *Proc. of the European Control Conference*, pages 502–510, Zurich, Switzerland, July 17–19 2013. European Control Conference. <http://control.ee.ethz.ch/~mpt>.
- L. Hernández-Lamonedá, R. Juárez, and F. Sánchez-Sánchez. Dissection of Solutions in Cooperative Game Theory Using Representation Techniques. *International Journal of Game Theory*, 35(3):395–426, Feb 2007. ISSN 1432-1270. doi: 10.1007/s00182-006-0036-3. URL <https://doi.org/10.1007/s00182-006-0036-3>.
- L. Hernández-Lamonedá, J. Sánchez-Pérez, and F. Sánchez-Sánchez. The Class of Efficient Linear Symmetric Values For Games in Partition Function Form. *International Game Theory Review*, 11(03):369–382, 2009. doi: 10.1142/S0219198909002364. URL <http://www.worldscientific.com/doi/abs/10.1142/S0219198909002364>.
- Ch. Hibbert. *The Rise and Fall of the House of Medici*. Harper Perennial, New York, USA, reprint 2003 edition, 1974.
- D. J. Higham and N. J. Higham. *MATLAB Guide*. SIAM, Philadelphia, PA. (USA), 1 edition, 2000.
- R. E. Hodel. *Introduction to Mathematical Logic*. Dover Publications, Mineola, New York, USA, republished first edition from 1995 edition, 2013.
- M. J. Holler. Forming Coalitions and Measuring Voting Power. *Political Studies*, 30:262–271, 1982.
- M. J. Holler. The Public Good Index: A Brief Introduction. *Finnish-German Yearbook of Political Economy*, 1:31–39, 2018.
- M. J. Holler and E. W. Packel. Power, Luck and the Right Index. *Journal of Economics*, 43:21–29, 1983.
- M. J. Holler and F. Rupp. Power in Networks: The Medici. *Homo Oeconomicus*, 38:5975, 2021. doi: 10.1007/s41412-021-00108-1. URL <https://doi.org/10.1007/s41412-021-00108-1>.
- H. Hotelling. The General Welfare in Relation to Problems of Taxation and of Railway and Utility Rates. *Econometrica*, 6(3):242–269, 1938. ISSN 00129682, 14680262. URL <http://www.jstor.org/stable/1907054>.
- D. Hou, G. Xu, P. Sun, and T. Driessen. The Shapley Value for the Probability Game. *Operations Research Letters*, 46(4):457–461, 2018. ISSN 0167-6377. doi: <https://doi.org/10.1016/j.orl.2018.06.004>. URL <https://www.sciencedirect.com/science/article/pii/S0167637718301500>.
- J. Hougaard, B. Peleg, and L. Thorlund-Petersen. On the Set of Lorenz-Maximal Imputations in the Core of a Balanced Game. *IJGT*, 30:147165, 2001. ISSN 0167-6687. doi: <https://doi.org/10.1007/s001820100070>. URL <https://link.springer.com/article/10.1007%2Fs001820100070>.
- J. L. Hougaard and A. Smilgins. Risk Capital Allocation with Autonomous Subunits: The Lorenz Set. *Insurance: Mathematics and Economics*, 67:151–157, 2016. ISSN 0167-6687. doi: <https://doi.org/10.1016/j.insmatheco.2015.12.002>. URL <https://www.sciencedirect.com/science/article/pii/S0167668715301712>.
- Cl. J. Huang and Ph. S. Crooke. *Mathematics and Mathematica for Economists*. Blackwell, Malden, Massachusetts, 1 edition, 1997.
- G. Huberman. *The Nucleolus and Essential Coalitions*, volume 28 of *Lecture Notes in Control and Information Sciences*. Elsevier, Amsterdam, 1980.
- J. F. Humphreys. *A Course in Group Theory*. Oxford University Press, 1996.
- Y.-A. Hwang and P. Sudhölter. Axiomatizations of the Core on the Universal Domain and Other Natural Domains. *International Journal of Game Theory*, 29:597–623, 2001.
- T. Ichiishi. Super-modularity: Applications to Convex Games and to the Greedy Algorithm for LP. *Journal of Economic Theory*, 25(2): 283–286, 1981. ISSN 0022-0531. doi: [https://doi.org/10.1016/0022-0531\(81\)90007-7](https://doi.org/10.1016/0022-0531(81)90007-7). URL <https://www.sciencedirect.com/science/article/pii/0022053181900077>.
- T. Ichiishi. *Microeconomic Theory*. Cambridge, MA., 1997.
- Publisher IEEE. IEEE Standard for Floating-Point Arithmetic. *IEEE Std 754-2019 (Revision of IEEE 754-2008)*, pages 1–84, 2019. doi: 10.1109/IEEESTD.2019.8766229.
- S. Ieong and Y. Shoham. Marginal Contribution Nets: A Compact Representation Scheme for Coalitional Games. In *Proceedings of the 6th ACM Conference on Electronic Commerce*, EC '05, page 193202, New York, NY, USA, 2005. Association for Computing Machinery. ISBN 1595930493. doi: 10.1145/1064009.1064030. URL <https://doi.org/10.1145/1064009.1064030>.
- E. Iñarra and J.M. Usategui. The Shapley Value and Average Convex Games. *International Journal of Game Theory*, 22:13–29, 1993.
- A. Isbell. A Class of Majority Games. *Quarterly Journal of Mathematics*, 7:183–187, 1956.
- A. Isbell. On the Enumeration of Majority Games. *Mathematical Tables and other Aids to Computation*, 13:21–28, 1959.
- Vocabulary ISO/IEC/IEEE. ISO/IEC/IEEE International Standard - Systems and software engineering-Vocabulary. *ISO/IEC/IEEE 24765:2017(E)*, pages 1–541, Aug 2017. doi: 10.1109/IEEESTD.2017.8016712.

- G. Jentzsch. Some Thoughts on the Theory of Cooperative Games. In M. Dresher, L.S. Shapley, and A.W. Tucker, editors, *Advances in Game Theory, Annals of Mathematical Studies*, number 52, pages 407–442. Princeton University Press, Princeton, 1964.
- M. Jerison. Russell on Gorman's Engel Curves: A Correction. *Economics Letters*, 43(2):171 – 175, 1993. ISSN 0165-1765. doi: [https://doi.org/10.1016/0165-1765\(93\)90032-8](https://doi.org/10.1016/0165-1765(93)90032-8). URL <http://www.sciencedirect.com/science/article/pii/0165176593900328>.
- M. Justman. Iterative Processes with 'Nucleolar' Restrictions. *International Journal of Game Theory*, 6:189–212, 1977.
- E. Kalai and E. Zemel. Generalized Network Problems Yielding Totally Balanced Games. *Operations Research*, 30(5):998–1008, 1982a. doi: 10.1287/opre.30.5.998. URL <https://doi.org/10.1287/opre.30.5.998>.
- E. Kalai and E. Zemel. Totally Balanced Games and Games of Flow. *Mathematics of Operations Research*, 7(3):476–478, 1982b. doi: 10.1287/moor.7.3.476. URL <https://doi.org/10.1287/moor.7.3.476>.
- M. Kaneko. Note on Transferable Utility. *International Journal of Game Theory*, 100(1):183–185, 1976. ISSN 1432-1270. doi: <https://doi.org/10.1007/BF01761600>. URL <https://link.springer.com/article/10.1007/BF01761600>.
- M. Kaneko and M. H. Wooders. *Utility Theories in Cooperative Games*, pages 1065–1098. Springer US, Boston, MA, 2004. ISBN 978-1-4020-7964-1. doi: 10.1007/978-1-4020-7964-1_6. URL https://doi.org/10.1007/978-1-4020-7964-1_6.
- A. Kar, M. Mitra, and S. Mutuswami. On the Coincidence of the Prenucleolus and the Shapley Value. *Mathematical Social Sciences*, 57(1):16 – 25, 2009. ISSN 0165-4896. doi: <https://doi.org/10.1016/j.mathsocsci.2008.08.004>. URL <http://www.sciencedirect.com/science/article/pii/S0165489608000851>.
- K. Kido. A nonlinear Approximation of the Nucleolus. In W. Takahashi and T. Tanaka, editors, *Proceedings of the International Conference on Nonlinear Analysis and Convex Analysis*, pages 307–317, Tokyo, 2004. Yokohama Publishers, Yokohama, Japan.
- K. Kido. Convergence Theorems for l_p -Norm Minimizers with respect to p . *Journal of Optimization Theory and Applications*, 125: 577–589, 2005.
- K. Kido. A Modified Kohlberg Criterion and a Nonlinear Method to compute the Nucleolus of a Cooperative Game. *Taiwanese Journal of Mathematics*, 12:1581–1590, 2008.
- N. L. Kleinberg and J. W. Weiss. Equivalent N-Person Games and the Null Space of the Shapley Value. *Mathematics of Operations Research*, 10(2):233–243, May 1985.
- J. Kleppe, H. Reijnierse, and P. Sudhölter. Axiomatizations of Symmetrically Weighted Solutions. *Ann. Oper. Res.*, 243:3753, 2016. URL <https://doi.org/10.1007/s10479-013-1494-1>.
- E. Kohlberg. On the Nucleolus of a Characteristic Function Game. *SIAM Journal of Applied Mathematics*, 20:62–66, 1971.
- E. Kohlberg. The Nucleolus as a Solution of a Minimization Problem. *SIAM, Journal of Applied Mathematics*, 23:34–39, 1972.
- A. Kopelowitz. Computation of the Kernels of Simple Games and the Nucleolus of N -Person Games. Technical report, RM 31, Research Program in Game Theory and Mathematical Economics, The Hebrew University of Jerusalem, 1967. mimeo.
- R. W. Krause and A. Caimo. Missing Data Augmentation for Bayesian Exponential Random Multi-Graph Models. volume 221, pages 63–72. International Workshop on Complex Networks, 2019. URL doi:10.21427/PME8-MT48.
- J. B. Kruskal. On the Shortest Spanning Subtree of a Graph and the Traveling Salesman Problem. *Proceedings of the American Mathematical Society*, 7(1):48–50, 1956. ISSN 00029939, 10886826. URL <http://www.jstor.org/stable/2033241>.
- J.-J. Laffont and J. Tirole, editors. *A Theory of Incentives in Procurement and Regulation*. The MIT Press, Cambridge, MA, 1993.
- V. Lakshmikantham and S. K. Sen. *Computational Error and Complexity in Science and Engineering*, volume 201 of *Mathematics in Science and Engineering*. Elsevier, Amsterdam, 2005.
- A. Lardon. The γ -Core in Cournot Oligopoly TU-Games with Capacity Constraints. *Theory and Decision*, 72(3):387–411, 2012. ISSN 0040-5833. doi: 10.1007/s11238-011-9256-5. URL <http://dx.doi.org/10.1007/s11238-011-9256-5>.
- P. Legros. *A Note on the Nucleolus of Three Person Games*. University of Paris, University of Paris, France, April 1981.
- P. V. Lekeas. Coalitional Beliefs in Cournot Oligopoly Tu Games. *International Game Theory Review*, pages 1–21, 2013. doi: 10.1142/S0219198913500047. URL <http://dx.doi.org/10.1142/S0219198913500047>.
- M. Leng and M. Parlar. Analytic Solution for the Nucleolus of a Three-Player Cooperative Game. *Naval Research Logistics (NRL)*, 57(7):667–672, 2010. doi: 10.1002/nav.20429. URL <https://onlinelibrary.wiley.com/doi/abs/10.1002/nav.20429>.
- N. Lindfield and J. Penny. *Numerical Methods Using MATLAB*. Prentice Hall, Upper Saddle River, NJ. (USA), 2 edition, 2000.
- S. Littlechild. A Simple Expression for the Nucleolus in a Simple Case. *International Journal of Game Theory*, 3:21–29, 1974a.
- S. Littlechild and K. G. Vaidya. The Propensity to disrupt and the Disruption Nucleolus of a Characteristic Function game. *International Journal of Game Theory*, 5:151–161, 1976. doi: <https://doi.org/10.1007/BF01753316>. URL <https://doi.org/10.1007/BF01753316>.
- S. C. Littlechild. A Simple Expression for the Nucleolus in a Special Case. *International Journal of Game Theory*, 3:21–29, 1974b. URL <https://doi.org/10.1007/BF01766216>.

- S. C. Littlechild and G. Owen. A Simple Expression for the Shapely Value in a Special Case. *Management Science*, 20(3):370–372, 1973. ISSN 00251909, 15265501. URL <http://www.jstor.org/stable/2629727>.
- S. C. Littlechild and G. Owen. A Further Note on the Nucleolus of the Airport Game. *International Journal of Game Theory*, 5(2): 91–95, 1976. URL <https://doi.org/10.1007/BF01753311>.
- S. C. Littlechild and G. F. Thompson. Aircraft Landing Fees: A Game Theory Approach. *The Bell Journal of Economics*, 8(1):186–204, 1977. ISSN 0361915X. URL <http://www.jstor.org/stable/3003493>.
- F. Lorenz. *"Lineare Algebra I"*. BI-Wissenschaftsverlag, Mannheim, 2 edition, 1988.
- S. Lorenzo-Freire. New characterizations of the Owen and BanzhafOwen Values Using the Intracoalitional Balanced Contributions Property. *TOP*, 25(3):579–600, 2017. doi: 10.1007/s11750-017-0446-3. URL <https://doi.org/10.1007/s11750-017-0446-3>.
- S. Lorenzo-Freire, J.M. Alonso-Mejide, B. Casas-Méndez, and M.G. Fiestras-Janeiro. Characterizations of the DeeganPackel and Johnston Power Indices. *European Journal of Operational Research*, 177(1):431 – 444, 2007. ISSN 0377-2217. doi: <https://doi.org/10.1016/j.ejor.2005.08.025>. URL <http://www.sciencedirect.com/science/article/pii/S0377221705009057>.
- W. F. Lucas. A Game with No Solution. *Bull. Amer. Math. Soc.*, 74:237–239, 1968.
- W. F. Lucas. A Proof that a Game may have not a Solution. *Bull. Amer. Math. Soc.*, 137:219–229, 1969.
- W. F. Lucas and M. Rabie. Games with No Solutions and Empty Cores. *Mathematics of Operations Research*, 7(4):491–500, 1982. ISSN 0364765X, 15265471. URL <http://www.jstor.org/stable/3689474>.
- D. Léonard and N. Van Long. *Optimal Control Theory and Static Optimization in Economics*. Cambridge University Press, Cambridge, second reprint edition, 1994.
- R. Maeder. *Programming in Mathematica*. Addison Wesley, Reading Massachusetts, 3 edition, 1996.
- R. Maeder. *Computer Science with Mathematica*. Cambridge University Press, Cambridge, UK, 1 edition, 2000.
- S. Mangano. *Mathematica Cookbook*. O'Reilly, Heidelberg, 2010.
- N. G. Mankiw. *"Macroeconomics"*. Worth, New York, 1 edition, 1992.
- J-E. Martinez-Legaz. Dual Representation of Cooperative Games based on Fenchel-Moreau Conjugation. *Optimization*, 36:291–319, 1996.
- A. Mas-Colell, M. Whinston, and J. Green. *Microeconomic Analysis*. Oxford University Press, Oxford, 1 edition, 1995.
- M. Maschler. An advantage of the Bargaining Set over the Core. *Journal of Economic Theory*, 13(2):184–192, 1976. ISSN 0022-0531. doi: [https://doi.org/10.1016/0022-0531\(76\)90013-2](https://doi.org/10.1016/0022-0531(76)90013-2). URL <https://www.sciencedirect.com/science/article/pii/0022053176900132>.
- M. Maschler. The Bargaining Set, Kernel and Nucleolus. In R. J. Aumann and S. Hart, editors, *Handbook of Game Theory*, volume 1, chapter 18, pages 591–668. Elsevier Science Publishers, Amsterdam, 1992.
- M. Maschler and B. Peleg. A Characterization, Existence Proof and Dimension Bounds for the Kernel of a Game . *Pacific Journal of Mathematics*, 18(2):289–328, 1966.
- M. Maschler, B. Peleg, and L.S. Shapley. The Kernel and Bargaining Set for Convex Games. *International Journal of Game Theory*, 1: 73–93, 1972.
- M. Maschler, B. Peleg, and L. S. Shapley. Geometric Properties of the Kernel, Nucleolus, and Related Solution Concepts. *Mathematics of Operations Research*, 4:303–338, 1979.
- M. Maschler, E. Solan, and Zamir Sh. *Game Theory*. Cambridge University Press, Cambridge, 1 edition, 2013.
- MathWorks. *Second-Order Cone Programming Algorithm - MATLAB & Simulink*. MathWorks Inc., Natick, Massachusetts, 2024.
- MATLAB. *version 24.2 (R2024b)*. The MathWorks Inc., Natick, Massachusetts, 2024.
- R. Meessen. Communication Games. Master's thesis, University of Nijmegen, the Netherlands, 1988. in Dutch.
- H. I. Meinhardt. Common Pool Games are Convex Games. *Journal of Public Economic Theory*, 2:247–270, 1999a.
- H. I. Meinhardt. Convexity and k -Convexity in Cooperative Common Pool Games. Discussion Paper 11, Institute for Statistics and Economic Theory, University Karlsruhe, Karlsruhe, 1999b.
- H. I. Meinhardt. *Cooperative Decision Making in Common Pool Situations*, volume 517 of *Lecture Notes in Economics and Mathematical Systems*. Springer, Heidelberg, 2002.
- H. I. Meinhardt. An LP Approach to Compute the Pre-Kernel for Cooperative Games. *Computers and Operations Research*, 33/2:pp 535–557, 2006.
- H. I. Meinhardt. *The Pre-Kernel as a Tractable Solution for Cooperative Games: An Exercise in Algorithmic Game Theory*, volume 45 of *Theory and Decision Library: Series C*. Springer Publisher, Heidelberg/Berlin, 2013. ISBN 978-3-642-39548-2. doi: 10.1007/978-3-642-39549-9. URL <https://doi.org/10.1007/978-3-642-39549-9>.

- H. I. Meinhardt. A Note on the Computation of the Pre-Kernel for Permutation Games. Technical Report MPRA-59365, Karlsruhe Institute of Technology (KIT), May 2014. URL <http://mpra.ub.uni-muenchen.de/59365/>.
- H. I. Meinhardt. The Incorrect Usage of Propositional Logic in Game Theory: The Case of Disproving Oneself. *ArXiv e-prints*, abs/1509.05883, 2015. URL <http://arxiv.org/abs/1509.05883>.
- H. I. Meinhardt. Finding the Nucleoli of Large Cooperative Games: A Disproof with Counter-Example. *CoRR*, abs/1603.00226, 2016a. URL <http://arxiv.org/abs/1603.00226>.
- H. I. Meinhardt. The Incorrect Usage of Propositional Logic in Game Theory: The Case of Disproving Oneself. *MPRA*, 75876, 2016b. URL <https://mpra.ub.uni-muenchen.de/75876/>. Revised Version.
- H. I. Meinhardt. Simplifying the Kohlberg Criterion on the Nucleolus: A Disproof by Oneself. Technical Report MPRA-77143, Karlsruhe Institute of Technology (KIT), March 2017a. URL <http://mpra.ub.uni-muenchen.de/77143/>.
- H. I. Meinhardt. Simplifying the Kohlberg Criterion on the Nucleolus: A Correct Approach. *ArXiv e-prints*, 2017b. URL <http://arxiv.org/abs/1706.08076>.
- H. I. Meinhardt. MatRep: A Matlab Toolbox for Representation Theory (Symmetric Groups), 2017c. URL https://de.mathworks.com/matlabcentral/fileexchange/62142-matrep-a-matlab-representation-theory-toolbox--symmetric-groups--?s_tid=prof_contriblnk.
- H. I. Meinhardt. A Manual of the Matlab Representation Theory Toolbox MatRep Version 0.1: A Quick Reference. Technical report, Karlsruhe Institute of Technology (KIT), Karlsruhe, Germany, 2017d. URL https://www.researchgate.net/publication/315661547_A_Manual_of_the_Matlab_Representation_Theory_Toolbox_MatRep_Version_01_A_Quick_Reference. Version 0.1.
- H. I. Meinhardt. *The Pre-Kernel as a Fair Division Rule for Some Cooperative Game Models*, pages 235–266. Springer International Publishing, Cham, 2018a. ISBN 978-3-319-61603-2. doi: 10.1007/978-3-319-61603-2_11. URL https://doi.org/10.1007/978-3-319-61603-2_11.
- H. I. Meinhardt. The Modiclus Reconsidered. Technical report, Karlsruhe Institute of Technology (KIT), Karlsruhe, Germany, 2018b. URL <http://dx.doi.org/10.13140/RG.2.2.32651.75043>.
- H. I. Meinhardt. Reconsidering Related Solutions of the Modiclus. Technical report, Karlsruhe Institute of Technology (KIT), Karlsruhe, Germany, 2018c. URL <http://dx.doi.org/10.13140/RG.2.2.27739.82729>.
- H. I. Meinhardt. Deduction Theorem: The Problematic Nature of Common Practice in Game Theory. *arXiv e-prints*, art. arXiv:1908.00409, Aug 2021a. URL <https://arxiv.org/abs/1908.00409v2>.
- H. I. Meinhardt. Disentangle the Florentine Families Network by the Pre-Kernel. Technical report, Karlsruhe Institute of Technology (KIT), Karlsruhe, Germany, 2021b. URL <http://dx.doi.org/10.13140/RG.2.2.10014.05447>.
- H. I. Meinhardt. MatTuGames: A Matlab Toolbox for Cooperative Game Theory, 2022. URL <http://www.mathworks.com/matlabcentral/fileexchange/74092-mattugames>.
- H. I. Meinhardt. TuGames: A Mathematica Package for Cooperative Game Theory, 2023. URL <https://github.com/himeinhardt/TuGames>.
- H. I. Meinhardt. On the Replication of the Pre-Kernel and Related Solutions. *Computational Economics*, 64:871946, 2024. URL <http://dx.doi.org/10.1007/s10614-023-10428-w>. Online available 2023.
- A. Meseguer-Artola. Using the Indirect Function to characterize the Kernel of a TU-Game. Technical report, Departament d'Economia i d'Història Econòmica, Universitat Autònoma de Barcelona, Nov. 1997. mimeo.
- P. Milgrom. An Axiomatic Characterization of Common Knowledge. *Econometrica*, 49(1):219–222, 1981. ISSN 00129682, 14680262. URL <http://www.jstor.org/stable/1911137>.
- MOSEK. The MOSEK Optimization Toolbox for MATLAB Manual. Version 10.2. Technical report, MOSEK ApS, Fruebjergvej 3, Boks 16, 2100 Copenhagen O, 2024. URL <http://docs.mosek.com/10.2/toolbox/index.html>.
- H. Moulin. Deterrence and Cooperation: A Classification of Two Person Games. *European Economic Review*, 15:179–193, 1981.
- H. Moulin. *Game Theory for the Social Sciences*. New York University Press, New York, 2 edition, 1986.
- H. Moulin. *Axioms of Cooperative Decision Making*. Econometric Society Monographs No. 15. Cambridge University Press, Cambridge, 1988.
- H. Moulin. *Fair Division and Collective Welfare*. MIT Press, Cambridge, Massachusetts, 2003.
- D. Mueller. A Cost Calculation Model for the Optimal Design of Size Ranges. *Journal of Engineering Design*, 22(7):467–485, 2011. doi: 10.1080/09544820903449218. URL <https://doi.org/10.1080/09544820903449218>.
- D. Mueller. *Investitionsrechnung und Investitionscontrolling*. Springer-Lehrbuch. Springer Publisher, Heidelberg, 2 edition, 2018.
- D. Mueller. *Betriebswirtschaftslehre für Ingenieure*. Springer-Lehrbuch. Springer Publisher, Heidelberg, 1 edition, 2020.

- D. Mueller. *Investitionscontrolling: Entscheidungsfindung bei Investitionen II*. Springer-Lehrbuch. Springer Publisher, Heidelberg, 3 edition, 2022.
- J.R. Munkres. *Topology: A First Course*. Prentice-Hall, Englewood Cliffs, New Jersey, 1975.
- R. B. Myerson. Graphs and Cooperation in Games. *Mathematics of Operations Research*, 2(3):225–229, August 1977.
- R. B. Myerson. Conference Structures and Fair Allocation Rules. *International Journal of Game Theory*, 9(3):169–182, 1980.
- R. B. Myerson. *Game Theory: Analysis of Conflict*. Harvard University Press, Cambridge, Massachusetts, London, England, 1991.
- D. Müller and M. F. K. Pisch. An Improper Solution to the Flood Cost Sharing Problem. *Economics Letters*, 238:111706, 2024. ISSN 0165-1765. doi: <https://doi.org/10.1016/j.econlet.2024.111706>. URL <https://www.sciencedirect.com/science/article/pii/S0165176524001897>.
- J. F. Nash. Noncooperative Games. *Annals of Mathematics*, 54:289–295, 1951.
- H. Norde, K. H. Pham Do, and S. Tijs. Oligopoly Games with and without Transferable Technologies. *Mathematical Social Sciences*, 43:187–207, 2002.
- A. S. Nowak and T. Radzik. A Solidarity Value for n -Person Transferable Utility Games. *International Journal of Game Theory*, 23: 43–48, 1994.
- B. O'Neill. A Problem of Rights Arbitration from the Talmud. *Mathematical Social Sciences*, 2:345–371, 1982.
- G. Orshan. The Prenucleolus and the Reduced Game Property: Equal Treatment Replaces Anonymity. *International Journal of Game Theory*, 22:241–248, 1993.
- G. Orshan and P. Sudhölter. Reconfirming the Prenucleolus. *Mathematics of Operations Research*, 28:283–293, 2003.
- A. Ostmann. Classifying Three-Person Games. Technical Report 140, University of Bielefeld, 1984.
- A. Ostmann. On the Minimal Representation of Homogeneous Games. *International Journal of Game Theory*, 16(1):69–81, 1987.
- A. Ostmann. Limits of Rational Behavior in Cooperatively Played Normalform Games. In R. Tietz, W. Albers, and R. Selten, editors, *Bounded Rational Behavior in Experimental Games and Markets, Lecture Notes in Economics and Mathematical Systems 314*, pages 317–332, Berlin, 1988. Springer-Verlag.
- A. Ostmann. A Note on Cooperation in Symmetric Common Dilemmas. mimeo, 1994.
- A. Ostmann and H. I. Meinhardt. Non-Binding Agreements and Fairness in Commons Dilemma Games. *Central European Journal of Operations Research*, 15(1):63–96, 2007.
- A. Ostmann and H. I. Meinhardt. Toward an Analysis of Cooperation and Fairness That Includes Concepts of Cooperative Game Theory. In A. Biel, D. Eek, T. Garling, and M. Gustafson, editors, *New Issues and Paradigms in Research on Social Dilemmas*, pages 230–251. Springer-Verlag, 2008.
- A. Ostmann and M. Saboyá. Symmetric homogeneous local interaction. Technical report, Institut für Mathematische Wirtschaftsforschung, Universität Bielefeld., Bielefeld, 1999. URL <https://pub.uni-bielefeld.de/download/2909898/2909937>. Working Paper 314.
- J. Antonio Rivero Ostoic. Compositional Equivalence with Actor Attributes: Positional Analysis of the the Florentine Families Network. *CoRR*, abs/1804.09427, 2018. URL <http://arxiv.org/abs/1804.09427>.
- G. Owen. A Note on the Nucleolus. *International Journal of Game Theory*, 3(2):101–103, 1974. URL <https://doi.org/10.1007/BF01766395>.
- G. Owen. Values of Games with a priori Unions. In R. Henn and O. Moeschlin, editors, *Essays in Mathematical Economics and Game Theory*, pages 76–88. Springer-Verlag, New York, 1977.
- G. Owen. *Game Theory*. Academic Press, San Diego, 3 edition, 1995.
- J.F. Padgett. Open Elite? Social Mobility, Marriage, and Family in Florence, 1282–1494. *Renaissance Quarterly*, 63(2):357–411, 2010. ISSN 00344338, 19350236. URL <http://www.jstor.org/stable/10.1086/655230>.
- J.F. Padgett and C.K. Ansell. Robust Action and the Rise of the Medici, 1400–1434. *American Journal of Sociology*, 98:1259–1319, 1993.
- B. Peleg. On the Kernel of Constant-Sum Simple Games with Homogeneous Weights. *Illinois J. Math.*, 10(1):39–48, 03 1966. doi: 10.1215/ijm/1256055199. URL <https://doi.org/10.1215/ijm/1256055199>.
- B. Peleg. On the Reduced Game Property and its Converse. *International Journal of Game Theory*, 15:187–200, 1986.
- B. Peleg. On the Reduced Game Property and its Converse. A Correction. *International Journal of Game Theory*, 16:290, 1987.
- B. Peleg and J. Rosenmüller. The Least Core, Nucleolus, and Kernel of Homogeneous Weighted Majority Games. *Games and Economic Behavior*, 4(4):588–605, 1992. ISSN 0899-8256. doi: [https://doi.org/10.1016/0899-8256\(92\)90039-U](https://doi.org/10.1016/0899-8256(92)90039-U). URL <https://www.sciencedirect.com/science/article/pii/089982569290039U>.

- B. Peleg and P. Sudhölter. *Introduction to the Theory of Cooperative Games*, volume 34 of *Theory and Decision Library: Series C*. Springer-Verlag, Heidelberg, 2 edition, 2007.
- B. Peleg, J. Rosenmüller, and P. Sudhölter. The Kernel of Homogeneous Games with Steps. In N. Megiddo, editor, *Essays in Game Theory in Honor of Michael Maschler*, pages 163–192, Heidelberg, 1994. Springer Publisher.
- H. Peters. *Game Theory: A Multi-Leveled Approach*. Springer-Verlag, Heidelberg, 1 edition, 2008.
- H. Peters. *Game Theory: A Multi-Leveled Approach*. Springer-Verlag, Heidelberg, 2 edition, 2015.
- K. H. Pham Do. Water Resource Management: Challenges and Opportunities with Game Theory Approaches, 02 2022. URL <https://oxfordre.com/environmentalscience/view/10.1093/acrefore/9780199389414.001.0001/acrefore-9780199389414-e-763>.
- K. H. Pham Do, A. Dinar, and D. McKinney. Transboundary Water Management: Can Issue Linkage Help Mitigate Externalities? *International Game Theory Review*, 14(01):1250002, 2012. doi: 10.1142/S0219198912500028. URL <https://doi.org/10.1142/S0219198912500028>.
- N. Ploskas and N. Samaras. *GPU Programming in MATLAB*. Morgan Kaufmann/Elsevier, Cambridge, MA. (USA), 1 edition, 2016.
- N. Ploskas and N. Samaras. *Linear Programming Using MATLAB*, volume 127 of *Springer Optimization and its Applications*. Springer, Heidelberg, Germany, 1 edition, 2018.
- J. A. M. Potters, H. Reijnierse, and M. Ansing. Computing the Nucleolus by Solving a Prolonged Simplex Algorithm. *Mathematics of Operations Research*, 21(3):757–768, 1996.
- R. C. Prim. Shortest Connection Networks and some Generalizations. *The Bell System Technical Journal*, 36(6):1389–1401, 1957. doi: 10.1002/j.1538-7305.1957.tb01515.x.
- T.E.S. Raghavan and P. Sudhölter. The Modiclus and Core Stability. *International Journal of Game Theory*, 33:467–478, 2005. URL <https://doi.org/10.1007/s00182-005-0207-7>.
- H. Reijnierse and J. Potters. The \mathcal{B} -Nucleolus of TU-Games. *Games and Economic Behaviour*, 24:77–96, 1998.
- M. Richardson. On Finite Projective Games. *Proceedings of the American Mathematical Society*, 7(3):458–465, 1956. doi: 10.1090/S0002-9939-1956-0079543-2. URL <https://doi.org/10.1090/S0002-9939-1956-0079543-2>.
- R. Rockafellar. *Convex Analysis*. Princeton University Press, Princeton, 1970.
- D. Rosales. *Cooperative Product Games*, 2014. URL www.academia.edu/40176429. unpublished mimeo.
- J. S. Rose. *A Course on Group Theory*. Cambridge University Press, Cambridge, UK, 1978.
- J. B. Rosen. Existence and Uniqueness of Equilibrium Points for Concave N-Person Games. *Econometrica*, 33(3):520–534, 1965. ISSN 00129682, 14680262. URL <http://www.jstor.org/stable/1911749>.
- J. Rosenmüller. *The Theory of Games and Markets*. North-Holland, Amsterdam, 1981.
- J. Rosenmüller. Weighted majority games and the matrix of homogeneity. *Zeitschrift für Operations Research*, 28(5):123 – 141, 1984. doi: 10.1007/BF01920914. URL <https://doi.org/10.1007/BF01920914>.
- J. Rosenmüller. Homogeneous Games: Recursive Structure and Computation. *Mathematics of Operations Research*, 12(2):309–330, 1987. URL <https://www.jstor.org/stable/3689693>.
- J. Rosenmüller and P. Sudhölter. Cartels via the Modiclus. *Discrete Applied Mathematics*, 134(1):263 – 302, 2004. ISSN 0166-218X. doi: [https://doi.org/10.1016/S0166-218X\(03\)00227-0](https://doi.org/10.1016/S0166-218X(03)00227-0). URL <http://www.sciencedirect.com/science/article/pii/S0166218X03002270>.
- A.E. Roth. *The Shapley Value: Essays in Honor of Lloyd S. Shapley*. Cambridge University Press, Cambridge, 1988.
- L. M. Ruiz, F. Valenciano, and J. M. Zarzuelo. The Least Square Pre-Nucleolus and the Least Square Nucleolus. Two Values for TU Games based on the Excess Vector. *International Journal of Game Theory*, 25:113–134, 1996.
- T. Russell. On a Theorem of Gorman. *Economics Letters*, 11(3):223 – 224, 1983. ISSN 0165-1765. doi: [https://doi.org/10.1016/0165-1765\(83\)90139-8](https://doi.org/10.1016/0165-1765(83)90139-8). URL <http://www.sciencedirect.com/science/article/pii/0165176583901398>.
- T. Russell. Gorman Demand Systems and Lie Transformation Groups: A Reply. *Economics Letters*, 51(2):201 – 204, 1996. ISSN 0165-1765. doi: [https://doi.org/10.1016/0165-1765\(96\)00806-3](https://doi.org/10.1016/0165-1765(96)00806-3). URL <http://www.sciencedirect.com/science/article/pii/0165176596008063>.
- T. Russell and F. Farris. The Geometric Structure of some Systems of Demand Equations. *Journal of Mathematical Economics*, 22(4): 309 – 325, 1993. ISSN 0304-4068. doi: [https://doi.org/10.1016/0304-4068\(93\)90020-L](https://doi.org/10.1016/0304-4068(93)90020-L). URL <http://www.sciencedirect.com/science/article/pii/S030440689390020L>.
- T. Russell and F. Farris. Integrability, Gorman Systems, and the Lie Bracket Structure of the Real Line. *Journal of Mathematical Economics*, 29(2):183 – 209, 1998. ISSN 0304-4068. doi: [https://doi.org/10.1016/S0304-4068\(97\)00806-9](https://doi.org/10.1016/S0304-4068(97)00806-9). URL <http://www.sciencedirect.com/science/article/pii/S0304406897008069>.

- B. E. Sagan. *"The Symmetric Group: Representations, Combinatorial Algorithms, and Symmetric Functions"*, volume 203 of *Graduate Texts in Mathematics*. Springer Publisher, Heidelberg, 2 edition, 2000.
- N. Schlömer. Matlab2Tikz. Technical report, GitHub, 2022. URL <https://github.com/matlab2tikz/matlab2tikz>. Version 2022.
- D. Schmeidler. The Nucleolus of a Characteristic Function Game. *SIAM, Journal of Applied Mathematics*, 17:1163–1170, 1969.
- D. Schoch. netrankr: Analyzing Partial Rankings in Networks. Technical report, Manchester, UK, 2020a. URL <https://cran.r-project.org/web/packages/netrankr/index.html>. Version 0.3.
- D. Schoch. networkdata: Package of Different Network Datasets. Technical report, Manchester, UK, 2020b. URL <https://github.com/schochastics/networkdata>. Version 0.1.
- R. W. Sebesta. *Concepts of Programming Languages*. Pearson, 11 edition, 2016.
- S. K. Sen and S. S. Prabhu. Optimal Iterative Schemes for Computing the Moore-Penrose Matrix Inverse. *International Journal of Systems Science*, 7(8):847–852, 1976. doi: 10.1080/00207727608941969. URL <https://doi.org/10.1080/00207727608941969>.
- R. Serrano. Reinterpretation the Kernel. *Journal of Economic Theory*, 77:58–80, 1997.
- L. S. Shapley. A Value of N-Person Games. In H. W. Kuhn and A. W. Tucker, editors, *Contributions to the Theory of Games II*, volume 28, pages 307–317, Princeton, NJ., 1953. Annals of Mathematical Studies, Princeton University Press.
- L. S. Shapley. On Balanced Sets and Cores. *Naval Research Logistics Quarterly*, 14:453–460, 1967.
- L. S. Shapley. Cores of Convex Games,. *International Journal of Game Theory*, 1:11–26, 1971.
- L. S. Shapley and M. Shubik. A Method for Evaluating the Distribution of Power in a Committee System. *American Political Science Review*, 48:787–792, 1954.
- L. S. Shapley and M. Shubik. Quasi-Cores in a Monetary Economy with Nonconvex Preferences. *Econometrica*, 34(4):805–827, 1966. ISSN 00129682, 14680262. URL <http://www.jstor.org/stable/1910101>.
- L. S. Shapley and M. Shubik. On Market Games. *Journal of Economic Theory*, 1:9–25, 1969.
- L. S. Shapley and M. Shubik. An Assignment Game I: The Core. *International Journal of Game Theory*, 2:111–130, 1972.
- M. Shubik. *"Market structure and behavior"*. Harvard University Press, Cambridge, 1980.
- A. J. Sobolev. The Characterization of Optimality Principles in Cooperative Games by Functional Equations. In N. N. Vorobjev, editor, *Matematicheskie Metody v Sotsial'nykh Naukakh, Proceedings of the Seminar*, pages 94–151, Vilnius, 1975. Institute of Physics and Mathematics, Academy of Sciences of the Lithuanian SSR. (in Russian, English summary).
- T. Solymosi. The Kernel is in the Least Core for Permutation Games. *Central European Journal of Operations Research*, pages 1–15, March 2014. ISSN 1435-246X. doi: 10.1007/s10100-014-0342-y. URL <http://dx.doi.org/10.1007/s10100-014-0342-y>.
- T. Solymosi and T.E.S Raghavan. An Algorithm for Finding the Nucleolus of Assignment Games. *International Journal of Game Theory*, 6:94–151, 1994.
- T. Solymosi and B. Sziklai. Universal Characterization Sets for the Nucleolus in Balanced Games. Technical Report MT-DP – 2015/12, Corvinus University of Budapest, 2015.
- T. Solymosi and B. Sziklai. Characterization Sets for the Nucleolus in Balanced Games. *Operations Research Letters*, 44(4):520 – 524, 2016. ISSN 0167-6377. doi: <http://dx.doi.org/10.1016/j.orl.2016.05.014>. URL <http://www.sciencedirect.com/science/article/pii/S0167637716300414>.
- R.E. Stearns. Convergent Transfer Schemes for N-Person Games. *Transaction of the American Mathematical Society*, 134:449–459, 1968.
- U. Stein. *Objektorientierte Programmierung mit MATLAB*. Carl Hanser Verlag, Leipzig, 2016.
- B. Steinberg. *Representation Theory of Finite Groups: An Introductory Approach*. Universitext. Springer-Verlag, 2012.
- P. Sudhölter. Star-Shapedness of the Kernel of Homogeneous Games. *Mathematical Social Sciences*, 32:179–214, 1996.
- P. Sudhölter. The Modified Nucleolus: Properties and Axiomatizations. *International Journal of Game Theory*, 26(2):147–182, Jun 1997a. ISSN 1432-1270. doi: 10.1007/BF01295846. URL <https://doi.org/10.1007/BF01295846>.
- P. Sudhölter. Nonlinear Self Dual Solutions for TU-Games. In T. Parthasarathy, B. Dutta, J.A.M. Potters, T.E.S. Raghavan, D. Ray, and Sen A., editors, *Game Theoretical Applications to Economics and Operations Research*, volume 18 of *Theory and Decision Library: Series C*, pages 33–50, Boston, MA, 1997b. Springer.
- P. Sudhölter. *The Modified Nucleolus of a Cooperative Game*. Habilitation thesis, University of Bielefeld, Bielefeld, 1993.
- P. Sudhölter. The Modified Nucleolus as Canonical Representation of Weighted Majority Games. *Mathematics of Operations Research*, 21(3):734–756, 1996. ISSN 0166-218X. URL <http://www.jstor.org/stable/3690307>.

- F. Szidarovszky and S. Yakowitz. A New Proof of the Existence and Uniqueness of the Cournot Equilibrium. *International Economic Review*, 18(3):787–789, 1977. ISSN 00206598, 14682354. URL <http://www.jstor.org/stable/2525963>.
- J.G. Sánchez León. *Mathematica beyond Mathematics: The Wolfram Language in the Real World*. CRC Press, Boca Raton, US., 2017.
- J. Sánchez-Pérez. An Application of the Representations of Symmetric Groups to Characterizing Solutions of Games in Partition Function Form. *Operations Research and Decisions*, 2:97–122, 2014. URL <https://EconPapers.repec.org/RePEc:wut:journl:v:2:y:2014:p:97-122:id:1088>.
- J. Sánchez-Pérez. A Decomposition for the Space of Games with Externalities. *International Journal of Game Theory*, 46(1):205–233, Mar 2017. ISSN 1432-1270. doi: 10.1007/s00182-016-0530-1. URL <https://doi.org/10.1007/s00182-016-0530-1>.
- A. Takayama. *Analytical Methods in Economics*. Harvester Wheatsheaf, Hertfordshire, 1994.
- C. R. Taylor. The Long Side of the Market and the Short End of the Stick: Bargaining Power and Price Formation in Buyers', Sellers', and Balanced Markets. *The Quarterly Journal of Economics*, 110(3):837–855, 1995. ISSN 00335533, 15314650. URL <http://www.jstor.org/stable/2946701>.
- R. M. Thrall and W. F. Lucas. N-Person Games in Partition Function Form. *Naval Research Logistics*, pages 281–298, 1963. doi: 10.1002/nav.3800100126.
- S. H. Tijs. Bounds on the Core and the τ -value. In O. Moeschlin and D. Pallaschke, editors, *Game Theory and Mathematical Economics*, pages 123–132. North-Holland, Amsterdam, 1981.
- J. Tirole, editor. *The Theory of Industrial Organization*. The MIT Press, Cambridge, MA, 1988.
- D. M. Topkis. *Supermodularity and Complementarity*. Frontiers of Economic Research. Princeton University Press, Princeton, New Jersey, 1998.
- E. van Damme. *Stability and Perfection of Nash Equilibria*. Springer-Verlag, Heidelberg/Berlin, 2 edition, 1991.
- A. van den Nouweland, P. Borm, and SH. Tijs. Allocation Rules for Hypergraph Communication Situations. *International Journal of Game Theory*, 20:255–268, 1992.
- H. R. Varian. *Microeconomic Theory*. Norton, New York, 3 edition, 1992.
- H. R. Varian, editor. *Economic and Financial Modeling with Mathematica*. Telos/Springer, Santa Clara, CA, 1993.
- V.A. Vasil'ev. The Shapley Value for Cooperative Games of Bounded Polynomial Variation. *Optimizacija Vyp*, 17:5–27, 1975. in Russian.
- V.A. Vasil'ev. On a Class of Imputations in Cooperative Games. *Soviet Mathematics Dokladi*, 23:53–57, 1981. in Russian.
- X. Vives. *Oligopoly Pricing: Old Ideas and New Tools*. MIT Press, Cambridge, Massachusetts, 1999.
- J. von Neumann. Zur Theorie der Gesellschaftsspiele. *Mathematische Annalen*, 100(1):295–320, 1928. ISSN 1432-1807. doi: <https://doi.org/10.1007/BF01448847>. URL <https://link.springer.com/article/10.1007/BF01448847>.
- J. von Neumann and O. Morgenstern. *Theory of Games and Economic Behavior*. Princeton University Press, Princeton, 1944.
- Ing. Walter. *Die Strozzi: Eine Familie im Florenz der Renaissance*. Verlag C.H.Beck, München, Germany, 2011.
- X. H. Wang and J. Zhao. Merger Effects in Asymmetric and Differentiated Bertrand Oligopolies. *Mathematical Social Sciences*, 120: 37–49, 2022. ISSN 0165-4896. doi: <https://doi.org/10.1016/j.mathsocsci.2022.09.002>. URL <https://www.sciencedirect.com/science/article/pii/S0165489622000725>.
- R. J. Weber. *Probabilistic Values for Games*, pages 101–119. In Roth (1988), 1988.
- R. Webster. *Convexity*. Oxford University Press, Oxford, 1994.
- P. Wellin. *Programming with Mathematica: An Introduction*. Cambridge University Press, Cambridge, UK., 2013.
- P. Wellin. *Essentials of Programming in Mathematica*. Cambridge University Press, Cambridge, UK., 2016.
- Inc. Wolfram Research. Mathematica Edition: Version 14.1. Technical report, Wolfram Research, Inc., Champaign, Illinois, 2024.
- H. Wolsey. The Nucleolus and Kernel for Simple Games or Special Valid Inequalities for 0-1 Linear Integer Programs. *International Journal of Game Theory*, 5(4):227–238, 1976.
- D. Xue and Y. Chen. *Solving Applied Mathematical Problems with MATLAB*. CRC Press, 2010.
- W. Y. Yang, W. Cao, T-S. Chung, and J. Morris. *"Applied Numerical Methods Using Matlab"*. John Wiley & Sons, Inc, 2005.
- K. Yokote, Y. Funaki, and Y. Kamijo. Linear Basis Approach to the Shapley Value. Working Paper 1303, Waseda University, 2013.
- E. Zeidler, W. Hackbusch, and H. R. Schwarz. *Teubner-Taschenbuch der Mathematik*. Vieweg & Teubner, Stuttgart/Leipzig, 1996.
- J. Zhao. β -Core Existence Result and its Application to Oligopoly Markets. *Games and Economic Behavior*, 27:153–168, 1999a.

- J. Zhao. A Necessary and Sufficient Condition for the Convexity in Oligopoly Games. *Mathematical Social Sciences*, 37:189–204, 1999b.
- J. Zhao. The Existence of TU α -Core in Normal Form Games. *International Journal of Game Theory*, 28:25–34, 1999c.
- J. Zhao. The Relative Interior of Base Polyhedron and the Core. *Economic Theory*, 18:635–648, 2001.
- J. Zhao. Estimating Merging Costs by Merger Preconditions. *Theory and Decision*, 66:373–399, 2009.
- J. Zhao. An Open Problem on Inverse Matrices from Industrial Organization, and a Partial Solution. *Linear Algebra and its Applications*, 437(1):294–306, 2012. ISSN 0024-3795. doi: <https://doi.org/10.1016/j.laa.2012.02.019>. URL <https://www.sciencedirect.com/science/article/pii/S0024379512001632>.
- J. Zhao. Merger Profitability in All Linear Oligopolies with Four Firms. Discussion paper, Department of Economics, University of Saskatchewan, Saskatoon, 2016. mimeo.
- J. Zhao. TU Oligopoly Games and Industrial Cooperation. In Corchon and Marini, editors, *Handbook of Game Theory and Industrial Organization*, volume I, page 392422. Edward Elgar Publisher, Cheltenham, 2018.

Author Index

*!?

Álvarez Mozos, M., [619](#), [654](#)

A

Aarts, H., [421](#), [422](#)
 Algaba, E., [28](#), [412](#), [557](#), [560](#), [561](#), [907](#)
 Aliprantis, Ch. D., [320](#), [322](#)
 Alonso-Mejide, J. M., [619](#), [620](#), [654](#), [659](#)
 Altman, Y., [837](#), [838](#), [841](#), [851](#)
 Andersen, E. D., [500](#)
 Ansell, C. K., [648](#), [664](#), [672](#)
 Ansing, M., [584](#)
 Antonio Rivero Ostoic, J., [648](#)
 Arin, J., [375–377](#), [381](#), [647](#)
 Arora, S., [185](#)
 Attaway, St., [29](#), [37](#), [67](#), [100](#), [118](#), [884](#), [898](#)
 Aumann, R. J., [279](#), [318](#), [552](#), [994](#)

B

Banzhaf, J. F., [611](#)
 Barak, B., [185](#)
 Barro, R. J., [170](#)
 Barron, E.N., [21](#), [28](#)
 Barry, B., [618](#)
 Bergantiños, G., [412](#), [417](#)
 Bilbao, J. M., [557](#), [560](#), [561](#), [907](#)
 Bird, C. G., [412](#), [420](#), [422](#)
 Bixby, R., [502](#)
 Blanchard, O. J., [169](#), [170](#)
 Bondareva, O. N., [14](#)
 Border, K. C., [320–322](#)
 Borm, P., [557](#), [560](#), [561](#), [915](#)
 Boyd, St., [294](#), [298](#), [299](#), [309](#), [498](#), [500](#)
 Bozzo, E., [648](#)
 Braun, R. J., [1111](#)
 Breiger, R.-L., [648](#), [653](#), [662](#), [664](#), [671](#), [674](#), [676](#)
 Béal, S., [592](#), [595](#), [1113](#), [1123](#)

C

Caimo, A., [648](#)
 Calvo, E., [554](#)
 Cao, W., [29](#), [57](#), [121](#), [443](#), [682](#), [1102](#), [1110](#)
 Carpentente, L., [10](#)
 Carreras, F., [659](#)
 Carter, M., [272](#), [285](#), [293](#), [947](#), [956](#), [1142](#)
 Casas-Méndez, B., [10](#), [620](#)
 Chalkiadakis, G., [184](#)
 Chang, Ch., [4](#), [7](#), [8](#), [405](#), [641](#)
 Chapman, St. J., [29](#), [109](#), [132](#)
 Chen, Y., [29](#)
 Cheney, W., [1106](#)
 Chiang, A. C., [84](#), [299](#), [309](#)
 Chung, T.-S., [29](#), [57](#), [121](#), [443](#), [682](#), [1102](#), [1110](#)
 Claus, A., [412](#)
 Coase, R. H., [111](#)
 Coleman, J. S., [611](#)
 Crooke, Ph. S., [199](#), [294](#), [299](#), [306](#), [309](#), [312](#), [316](#)
 Curiel, I., [389](#), [413](#)

D

Davis, M., [7](#), [8](#), [20](#), [438](#)
 Deegan, J., [618](#)
 Dehez, P., [20](#), [349](#), [371](#), [373](#), [967](#), [971](#), [1053](#)
 Deng, X., [184](#), [403](#), [405](#)
 Derks, J., [476](#), [584](#)
 de Roover, R., [649](#), [662](#), [677](#)
 Dietzenbacher, B., [703](#)
 Dinar, A., [1005](#)
 Dixit, A. K., [294](#), [989](#), [1034](#)
 Driessen, T. S. H., [5](#), [8](#), [18](#), [20](#), [28](#), [118](#), [272](#), [278](#), [281](#), [348](#), [370](#), [374](#), [421](#), [422](#), [434](#), [505](#), [641](#), [790](#), [971](#), [988](#), [990](#), [994](#), [996](#), [1001](#), [1099](#)
 Driscoll, T. A., [1111](#)
 Drèze, J. H., [552](#)
 Dubey, P., [611](#), [612](#)
 Dym, H., [65](#)

E

Ebbinghaus, H.-D., [471](#), [765](#), [1020](#), [1091](#)
 Elden, L., [57](#), [1102](#)
 Elkind, E., [184](#)
 Estévez-Fernández, A., [603](#)

F

Faigle, U., [429](#), [593](#), [595](#), [865](#)
 Farris, F., [1019](#)
 Felsenthal, D. S., [620](#)
 Feltkamp, V., [381](#)
 Ferreira, F., [619](#), [654](#)
 Fiestras-Janeiro, M. G., [603](#), [620](#), [659](#)
 Fischer, S., [169](#), [170](#)
 Flum, J., [471](#), [765](#), [1020](#), [1091](#)
 Ford, L. R., [400](#)
 Fragnelli, V., [28](#)
 Franceschet, M., [648](#)
 Fronzetti Colladon, A., [648](#)
 Fulkerson, D. R., [400](#)
 Fulton, W., [1123](#), [1127](#), [1129](#)
 Funaki, Y., [429](#), [486](#), [491](#), [591](#), [595](#), [641](#), [713](#), [1113](#), [1123](#)

G

Gale, D., [1015](#)
 García-Jurado, I., [10](#)
 Gatelly, D., [955](#)
 Gaudet, G., [1015](#)
 Gaylord, R. J., [199](#)
 Getán, J., [379](#), [383](#)
 Goldberg, D., [1102](#), [1107](#)
 Goldberg, L. A., [184](#)
 Goldberg, P. W., [184](#)
 González-Díaz, J., [374](#)
 Grabisch, M., [429](#), [593](#), [595](#), [865](#)
 Granot, D., [417](#), [421](#), [422](#), [514](#), [562](#), [567](#), [584](#), [587](#)
 Granot, F., [562](#), [567](#), [584](#), [587](#)
 Green, J., [111](#), [299](#), [309](#), [321](#), [356](#)
 Gräbe, H.-G., [199](#), [200](#), [262](#)
 Gu, Z., [502](#)
 Gutiérrez, E., [554](#)

H

Hackbusch, W., 57, 1102
 Hahn, B. D., 29, 36, 100, 118
 Hall, B. C., 1120, 1123
 Haller, H., 476
 Harris, J., 1123, 1127, 1129
 Hart, S., 293, 429, 554, 689, 699, 806, 867, 1005, 1019, 1123
 Harville, D. A., 73, 82, 322
 Hernández-Lamonedá, L., 763, 1113
 Hibbert, Ch., 650, 661, 662, 672
 Higham, D. J., 29
 Higham, N. J., 29
 Hodel, R. E., 1091
 Holler, M. J., 618, 620, 648, 654, 662, 668, 670
 Hotelling, H., 110
 Hou, D., 971
 Hougaard, J., 700, 701
 Hu, Ch., 4, 7, 8
 Hu, G., 971
 Huang, Cl. J., 199, 294, 299, 306, 309, 312, 316
 Huberman, G., 417, 421, 422, 514, 563, 571
 Humphreys, J. F., 1113
 Hwang, Y.-A., 16, 709

I

Ichiishi, T., 111, 321
 Jeong, S., 184
 Isbell, A., 623, 641, 645
 Izquierdo, J., 379, 383
 Iñarra, E., 376, 381, 505, 647, 950

J

Jentzsch, G., 994
 Jerison, M., 1019
 Justman, M., 477
 Juárez, R., 763, 1113

K

Kalai, E., 395, 717
 Kamijo, Y., 429, 591, 595
 Kamin, S. N., 199
 Kan, Ch. Y., 641
 Kaneko, M., 4, 8, 10, 357
 Kar, A., 404
 Katsev, I., 375, 377, 647
 Kido, K., 495, 974, 975
 Kincaid, D., 1106
 Kleinberg, N. L., 562, 591, 1123
 Kleitman, D. J., 412
 Kleppe, J., 479, 483, 563
 Kofler, M., 199, 200, 262
 Kohlberg, E., 464, 468, 472, 809
 Kopelowitz, A., 464, 649, 665
 Krause, R. W., 648
 Kruskal, J.-B., 412
 Kuipers, J., 584
 Kurz, M., 554, 806, 1005, 1019, 1123

L

Laffont, J.-J., 111
 Lakshmikantham, V., 57, 1101–1104, 1107
 Lardon, A., 1019, 1022
 Legros, P., 20, 194, 289
 Lekeas, P. V., 1005
 Leng, M., xi, 20, 194, 289, 292, 1151
 Lian, Ch.-H., 641
 Lindfield, N., 29

Littlechild, S. C., 110–113, 118, 151, 371, 584, 955
 Lorenz, F., 1123
 Lorenzo-Freire, S., 620
 Lucas, W. F., 362, 1005, 1130
 López, J. J., 557, 560, 561, 907
 Léonard, D., 294

M

Maeder, R., 199, 262
 Manea, M., 1113, 1123
 Mangano, S., 214, 1047
 Mankiw, N. G., 170
 Martínez-Legaz, J.-E., 438
 Mas-Colell, A., 111, 293, 299, 309, 321, 356, 429, 689, 699, 867
 Maschler, M., 7–9, 12, 20, 28, 181, 279, 375, 376, 378, 395, 438, 440, 469, 508, 533, 542, 564, 623, 641, 717, 739, 950, 953, 1019
 MathWorks, 500
 MATLAB, 29, 884
 McKinney, D., 1005
 Meessen, R., 560
 Meinhardt, H. I., x, 4, 5, 28, 370, 392, 398, 427, 438, 439, 442, 444, 471, 486, 491, 502, 532, 538, 542, 562, 563, 587, 588, 590, 639, 641, 643, 662, 670, 682, 713, 739, 740, 766, 789, 793, 865, 875, 926, 947, 953, 955, 987, 988, 990, 994, 996, 1001, 1019, 1020, 1023, 1087, 1094, 1113, 1120, 1136, 1141
 Meseguer-Artola, A., 439, 453, 531
 Milgrom, P., 318
 Mitra, M., 404
 Montes, J., 379, 383
 Morgenstern, O., x, 3, 615, 622, 994
 Morris, J., 29, 57, 121, 443, 682, 1102, 1110
 MOSEK, 500
 Mosquera, M. A., 603
 Moulin, H., 10, 20, 28, 289, 994
 Mueller, D., 111, 116, 271, 356, 511
 Munkres, J. R., 319, 320
 Mutuswami, S., 404
 Myerson, R. B., 10, 555

N

Naldi, M., 648
 Nash, J. F., 319, 321, 1019
 Nielsen, H. B., 57, 1102
 Nikaido, H., 1015
 Norde, H., 370, 988, 990, 995
 Nowak, A. S., 551

O

O'Neill, B., 272
 Orshan, G., 691, 708, 746
 Ostmann, A., 4, 5, 434, 623, 624, 662, 714, 987, 988, 995, 1113
 Owen, G., 10, 28, 110–113, 118, 362, 371, 464, 553, 560, 659, 915

P

Pacini, P. M., 20
 Packel, E. W., 618, 620
 Padgett, J. F., 648, 651, 664, 672
 Papadimitriou, C. H., 184, 403, 405
 Parlar, M., xi, 20, 194, 289, 292, 1151
 Pattison, Ph.-E., 648, 653, 662, 664, 671, 674, 676
 Peleg, B., 7, 9, 10, 12, 16, 20, 28, 375, 376, 378, 405, 438, 440, 464, 469, 508, 533, 546, 564, 609, 615, 623, 624, 641, 643, 660, 691, 700, 713, 734, 739, 751, 765, 809, 814, 823, 950, 953
 Penny, J., 29
 Peters, H., 27, 28, 373, 430, 469, 612
 Pham Do, K. H., 370, 988, 990, 995, 1005

Pinto, A. A., 619, 654
 Pisch, F.-K., 511
 Ploskas, N., 29, 852
 Potters, J. A. M., 584
 Prabhu, S. S., 1104
 Prim, R. C., 412
 Publisher IEEE, 29, 244, 251, 255, 1107

R

Radzik, T., 551
 Rafels, C., 379, 383
 Raghavan, T.E.S., 584, 713
 Reijnierse, H., 563, 584
 Reijnierse, H., 479, 483
 Richardson, M., 643
 Rinaldi, F., 648
 Rockafellar, R., 368, 438, 499
 Roos, C., 500
 Rosales, D., 967, 1053
 Rose, J. S., 1113
 Rosen, J. B., 1015
 Rosenmüller, J., 370
 Rosenmüller, J., 615, 623, 624, 626, 713, 716, 734, 779, 833
 Roth, A. E., 4, 28
 Rothberg, E., 502
 Ruiz, L. M., 495, 974, 980
 Rupp, F., 648, 654, 662, 668, 670
 Russel, T., 1019
 Rémila, E., 592, 595, 1113, 1123

S

Saboyá, M., 1113
 Sagan, B. E., 397, 1123, 1127, 1129
 Sala-I-Martin, X., 170
 Salant, St. W., 1015
 Samaras, N., 29, 852
 Schlömer, N., 174
 Schmeidler, D., 7, 463, 713
 Schoch, D., 653, 662, 669, 674
 Schwarz, H. R., 57, 1102
 Sebesta, R. W., 29, 121, 131, 214, 262, 884
 Sen, S. K., 57, 1101–1104, 1107
 Serrano, R., 4, 7, 8
 Shapley, L. S., 6, 7, 9, 11, 12, 14, 20, 293, 356–358, 364, 367, 374–376, 378, 389, 438, 440, 505, 508, 509, 533, 611, 612, 641, 691, 739, 746, 950, 953, 1098
 Shoham, Y., 184
 Shubik, M., 11, 356–358, 364, 389, 612, 990, 1034
 Smilgins, A., 701
 Sobolev, A. J., 9, 472, 691, 697, 745
 Solal, Ph., 592, 595, 1113, 1123
 Solan, E., 8, 28, 395, 469, 717, 1019
 Solymosi, T., 392, 563, 571, 572, 578, 584, 587
 Stearns, R. E., 649, 665
 Stein, U., 884, 898
 Steinberg, B., 1127, 1129
 Sudhölter, P., 10, 16, 28, 405, 464, 469, 479, 483, 546, 563, 564, 609, 615, 624, 637, 639, 641, 643, 660, 691, 703, 708, 709, 713, 716, 724, 734, 746, 751, 765, 779, 789, 809, 813, 818, 833
 Sun, P., 971
 Szidarovszky, F., 1015
 Sziklai, B., 563, 571, 572, 578, 587
 Sánchez-Rodríguez, E., 603
 Sánchez-Pérez, J., x, 1113, 1123, 1130
 Sánchez-Rodríguez, E., 374
 Sánchez-Soriano, J., 28
 Sánchez-Sánchez, F., 763, 1113

Sánchez León, J. G., 199, 1047

T

Takayama, A., 299, 309
 Taylor, C. R., 181
 Terlaky, T., 500
 Thomas, W., 471, 765, 1020, 1091
 Thompson, G. F., 110, 111, 151
 Thorlund-Petersen, L., 700
 Thrall, R. M., 1005, 1130
 Tijs, S., 370, 434, 560, 561, 915, 988, 990, 995
 Tirole, J., 111
 Topkis, D. M., 368, 375
 Tseng, Y.-C., 405

U

Usategui, J. M., 381, 505, 950

V

Vaidya, K. G., 955
 Valenciano, F., 495, 974, 980
 Valentine, D. T., 29, 36, 100, 118
 van Damme, E., 1019
 Vandenberghe, L., 294, 298, 299, 309, 498, 500
 van den Nouveland, A., 10, 561
 Van Long, N., 294
 Varian, H. R., 111, 294, 299, 300, 309, 356, 948, 955, 1019, 1026, 1042, 1142
 Vasil'ev, V. A., 370
 Vidal-Puga, J., 412, 417
 Vives, X., 188, 323, 988
 von Neumann, J., x, 3, 615, 622, 994

W

Walter, I., 662
 Wang, X. H., 990–992, 1006, 1034
 Weber, R. J., 373
 Webster, R., 14
 Weiss, J. W., 562, 591, 1123
 Wellin, P. R., 199
 Whinston, M., 111, 299, 309, 321, 356
 Wittmeyer-Koch, L., 57, 1102
 Wolfram Research, 199
 Wooders, M. H., 4
 Wooldridge, M., 184

X

Xue, D., 29

Y

Yakowitz, S., 1015
 Yang, W. Y., 29, 57, 121, 443, 682, 1102, 1110
 Yokote, K., 429, 591, 595, 1113, 1123

Z

Zamir, Sh., 8, 28, 395, 469, 717, 1019
 Zarzuelo, J. M., 495, 974, 980
 Zeidler, E., 57, 1102
 Zemel, E., 395, 717
 Zhao, J., 370, 985, 988–992, 994, 997, 1001, 1005, 1006, 1017, 1023, 1034
 Zhu, W. R., 562, 567, 584, 587

Subject Index

*!?

\Rightarrow , 249
 $\#$, 205, 262
 $\&$, 205, 262
 \rightarrow , 208, 247
 c -set, 565
 characterization set, 565
 dually essential, 571
 essential, 571
 d -contraction, 322
 l_p - k -nucleolus, 975
 l_p - k -pre-nucleolus, 977
 $/$, 300, 980
 $/.$, 208, 301
 $/;$, 242
 $:=$, 206
 $:>$, 249
 $=$, 206
 $==$, 206, 210
 $===$, 207, 252
 $[\]$, 217
 $_?EvenQ$, 232
 $_?OddQ$, 232

A

achange, 175
 additive_game, 408, 507, 798
 additivity, 10
 adjoint matrix, 65
 ADvalue, 552, 878, 911
 agreement
 compliance, 5
 obstruction, 4
 agreements
 binding, 4
 non-binding, 4
 AGame, 595
 airport cost allocation problem, 112
 nucleolus, 114
 Shapley value, 113
 airport_game, 372
 algebra
 $End(V)$, 1124
 $Hom(V, W)$, 1124
 $Mat_{r,d}$, 1124
 bilinear map, 1124
 direct sum, 1124
 eigenspace, 1125
 eigenvalue, 539, 1125
 eigenvector, 1125
 endomorphism ring, 1124
 general linear group, 1120, 1124
 group algebra, 1127, 1128
 Hermitian inner product space, 1127
 image, 1125
 irreducible space, 1125
 kernel, 1125
 module, 1123
 orthogonal complement, 1126
 ring, 1123
 tensor product, 1124
 trace, 1127

algorithm

 exponential time, 583
 nucleolus, 195
 polynomial time, 583
 pre-kernel, 442
 pre-kernel for a c.s., 542
 pre-nucleolus, 464
 strongly polynomial time, 583
 AllMarginalContributions, 374, 422
 AlmostAverageConvexQ, 383
 Amdahl's Law, 859, 1050
 analysis
 fairness, 3
 normative, 3
 positive, 3
 anti-balanced TU game, 15
 anti-core, 14, 511
 anti-kernel, 417, 487
 anti-nucleolus, 417, 490
 anti-outweighs, 487
 anti-pre-kernel, 487
 anti-pre-nucleolus, 490
 Anti-Property I, 493
 Anti-Property II, 495
 Anti_balancedCollectionQ, 494, 807
 Anti_Kernel, 426
 Anti_Modiclus, 810
 Anti_modiclusQ, 810
 Anti_Monotonic_Cover, 416
 Anti_Nucl, 418
 Anti_Nucl_llp, 425
 Anti_PreKernel, 490, 1074
 Anti_PrekernelQ, 492
 Anti_PreNucl, 492, 807
 AntiReduced_game_propertyQ, 752
 ApproxNuc, 980, 981
 ApproxPreNuc, 980, 981
 apu_SolidarityValue, 555, 911
 array, 29
 assignment_game, 390, 887
 Attributes
 Flat, 222
 HoldAllComplete, 224
 Listable, 222
 NumericFunction, 222
 OneIdentity, 222
 Orderless, 222
 Protected, 222
 average_convexQ, 383, 509, 1070
 AverageConvexQ, 979
 axiom
 Additivity (ADD), 690
 Anonymity (AN), 9, 690
 Associativity, 1113
 Closure, 1113
 Converse Consistency (COCONS), 823
 Converse Derived Game Property (CDGP), 827
 Converse Reduced Game Property (CRGP), 17, 690
 Covariance under Strategic Equivalence (COV), 9, 690
 Dual Cover Property (DCP), 731
 Dual Floor Property (DFP), 755
 Dual Replication Property (DRP), 731
 Efficiency (EFF), 690, 1132
 Equal Treatment Property (ETP), 690

Excess Comparability (EC), 731
 HM-Reduced Game Property (HMS-RGP), 690
 HM-Reduced Game Property set-valued (HMS-RGP, 690
 HMS-Converse Reduced Game Property (HMS-CRGP), 690
 HMS-Weak Reduced Game Property (HMS-WRGP), 690
 HMS-Weak Reduced Game Property set-valued (HMS-WRGP), 690
 Identity, 1114
 Imput. Saving Reduced Game Property (ISRGP), 691
 Individual Rationality (IR), 690
 Inverse, 1114
 Large Excess Difference Consistency (LEDCONS), 731, 750
 Large Excess Difference Converse Consistency (LED-COCONS), 823
 Linearity (LIN), 1132
 Nonemptiness (NE), 9, 690
 Null-Player Property (NP), 690
 Pareto Optimal (PO), 9, 690
 Primal Replication Property (PRP), 755
 Reasonable (RE), 731
 Reasonable from above (REAB), 731
 Reasonable from below (REBE), 731
 Reconfirmation Property (RCP), 16, 691
 Reduced Game Property (RGP), 9, 690
 Reverse Excess Comparability (REC), 754
 Single-Valuedness (SIVA), 9, 690
 Small Excess Difference Consistency (SEDCONS), 754, 773
 Strong Converse Derived Game Property (SCDGP), 831
 Strong Converse Reduced Game Property (SCRGP), 690
 Strong Dual Cover Property (SDCP), 744
 Strong Dual Floor Property (SDFP), 770
 Symmetry (SYM), 690, 1132
 Weak Reduced Game Property (WRGP), 690

B

B0_balancedCollectionQ, 473, 476
 backslash operation, 640
 balanced collection, 12
 balanced contributions property, 10
 balanced TU game, 12
 BalancedCollectionQ, 979
 balancedCollectionQ, 472, 476, 685
 bankruptcy game, 25
 bankruptcy situation, 25
 bankruptcy_game, 348, 863
 banzhaf, 612, 617, 877
 BanzhafOwenValue, 659
 bargaining range, 953
 bargaining set, 20
 coalition structure, 524
 BaryCenter, 722
 basis
 change, 1119
 equivalence class, 1121
 behavior
 standard, 4
 Belief
 δ -, 1004
 γ -, 1004
 breakup, 1004
 loyal, 1004
 bell_number, 658
 belongToAntiCoreQ, 418, 513
 belongToCoreQ, 511, 513, 617, 796, 1075
 belongToImputationSetQ, 523
 belongToWeberSetQ, 375
 Berge's Maximum Theorem, 320
 Best reply of a firm, 992, 1036

Best response set, 995
 best-response
 reaction correspondence, 319
 binary
 addition, 125
 arithmetic, 124
 division, 126
 multiplication, 126
 subtraction, 125
 bint_AssignmentGame, 390
 bottom-up design, 109
 bs_PreKernel(), 640

C

calculus system, 1094
 CanonicalOrder, 343
 Cartel
 see coalition, 988
 Cdd_CSCorePlot, 530
 Cdd_CSCoreVertices, 524
 Cdd_CSImputationVertices, 523
 Cdd_CSInterImpSupAddCover, 528
 Cdd_InterImpCSImpSet, 529
 CddAntiCoreVertices, 418, 513
 CddCfrCoreVertices, 566
 CddCfrNucl, 570
 CddCoreCoverPlot, 603
 CddCoreCoverVertices, 602
 CddCoreMovie, 606, 920
 CddCorePlot, 601, 920
 CddCoreQ, 910
 CddCoreVertices, 512, 601, 617, 1074
 CddImputationVertices, 523
 CddLeastCore, 605
 Cddmathlink, 957
 CDDMEX, 503
 CddStrongCorePlot, 605, 920
 CddWeberSetPlot, 604, 920
 center, 637, 660
 cfr_Kernel, 569
 cfr_kernelQ, 569
 cfr_nucl, 567
 cfr_PreKernel, 569
 cfr_PrekernelQ, 569
 cfr_PreNucl, 569
 CG-Principle
 see contested garment principle, 279
 character
 induced, 1129
 irreducible, 1127
 null-player, 624
 relations of the first kind, 1128
 step, 624
 sum, 624
 table, 1132
 Characteristic function
 α -, 188, 994
 β -, 994
 δ -, 1005
 γ -, 1005
 s-type, 995
 characteristic function, 5
 characteristic vector, 469, 809
 characterizationSetNucleolus, 566
 Characters, 218
 class function, 1127
 closed ball, 295
 closed neighborhood, 295
 closed-valued, 319
 CLP, 503
 clToMatlab, 346, 610

- coalition, 5
 - closure, 586
 - irreducible saturated, 586
 - saturated, 586
 - structure, 514, 521
 - vital, 573
- coalition formation restrictions, 563
- CoalitionSolidarity, 554, 879, 911
- Cobb-Douglas
 - see Function, 170, 304
- combination
 - conic, 499
- common knowledge, 318
- communication, 3
- communication structure, 555
- compact-valued, 319
- ComposeMarkets, 366
- Composition, 360
- compromiseAdmissibleQ, 603, 924
- compromiseStableQ, 603, 924
- ComputeRoot, 259, 260
- ComputeRoot2, 260
- concave TU game, 421
- concave_gameQ, 422
- concave_gameQ(), 373
- condition number, 1106
- Conditional Statements, 1091
- cone, 499
 - convex, 499
- cone program, 500
- conic optimization, 500
- consistency
 - reduced game property, 9, 690
 - weak reduced game property, 690
- consistent contested garment principle
 - generalized contested garment principle, 281
- consistent solution, 72
- constant elasticity of substitution
 - see Function, 304
- constant-sum game, 26
- ConstantSumQ, 506
- contested garment principle, 279
- Contraction Mapping, 322
- Contraction Mapping Theorem, 322
- contraposition, 1115
- converse consistency
 - converse reduced game property, 17, 690
 - HMS converse reduced game property, 690
- converse reduced game property, 17, 690
- Converse_DGP_Q, 830
- Converse_RGP_Q, 705
- convex TU game, 6, 368
- convex-valued, 319
- convex_gameQ, 406, 508, 868, 910
- convex_gameQ(), 373
- ConvexQ, 979
- core, 10
 - coalition structure, 524
 - least-, 12
 - strong ϵ -, 11
- CorePlot, 600, 920
- CoreQ, 979
- coreQ, 510, 613, 617
- CoreVertices, 510, 599
- correspondence
 - continuous, 320
 - lhc, 320
 - uhc, 319
- cost matrix, 412
- counter-objection, 20, 523
- counting_basis, 595
- COV_propertyQ, 409

- COV_propertyQ(), 696
- cp_kernel, 501
- cp_prekernel, 501
- CPLEX, 503
- cplex_cfr_nucl, 570
- cplex_kernel, 503
- cplex_prenucl, 685
- cplex_weightedPreNucl, 685
- critical point, 296
- critical_value1, 604
- critical_value2, 604
- critical_value_star, 605
- cs_balancedCollectionQ, 549
- cs_Banzhaf, 658
- cs_belongToCoreQ, 528
- cs_belongToImputationSetQ, 528
- cs_CoreQ, 524
- cs_GetNc, 550
- cs_GetPrk, 544
- cs_GetPrn, 550
- cs_PreKernel, 544
- cs_PrekernelQ, 544
- cs_PreNucl, 549
- cs_Weak_balancedCollectionQ, 549
- CVX, 503
- cvx_kernel, 503
- cycle in the decomposition, 573

D

- d-function, 322
- data type, 29
- DCP_propertyQ, 748
- decision-making
 - Rawlsian type, 4
- DecomposeGame, 592, 686, 693
- DecomposeVN, 594
- Deduction Theorem, 1091, 1094
- DeeganPackel, 619, 622
- derivative
 - directional, 76, 77, 80
- Derived_game_propertyQ, 779
- determinant
 - Gramian, 1120
- DetMarketSide, 780
- digit
 - guard, 1107
- DirectSumVN, 595
- distributive justice, 4
- division
 - left, 35
 - right, 35
- DixitGammaGame, 806, 808
- DM_Derived_game, 775
- DM_Reduced_game, 697, 720
- domination-equivalent, 362
- DRP_propertyQ, 747
- DTImatrix, 595
- dual game, 26
- dual problem, 77, 297
- dual_game, 350
- DualCover, 724
- DualEssentialSet, 572
- DuallySgnzSet, 575
- dummy player, 10

E

- e.f.s.q
 - ex falso sequitur quodlibet, 1095
- EC_propertyQ, 748
- ECCoverGame, 729, 800

ECFloorGame, 800
 efficiency, 10
 embedding coalition, 1130
 equal_treatmentQ, 695
 Equilibrium
 Bertrand, 323, 1036
 Cournot, 330, 992
 Nash, 319, 992, 1036
 existence, 319
 uniqueness, 322
 post-merger, 1005
 pre-merger, 992, 1036
 Stackelberg, 995
 subgame perfect, 15

error
 absolute computing, 1103
 absolute relative computing, 1103
 backward, 1111
 catastrophic cancellation, 122, 1108
 floating-point exception, 121
 forward, 1111
 index overflow, 123
 index underflow, 123
 integer overflow, 124
 logical, 124
 magnification, 122
 negligible addition, 122, 1111
 round-off, 122, 1106
 runtime, 122, 123
 syntax, 122
 truncation, 122, 1109
 underflow, 122, 1110
 unit round-off, 1107

EssentialQ, 506
 essentialSet, 571
 Euclidean inner product, 64
 Euclidean norm, 65, 1104
 ex falso sequitur quodlibet, 1095
 exact_game, 780
 exact_gameQ, 780
 excess, 7, 437
 excess code, 57, 1102
 extreme point, 294

F

fairness
 distributive justice, 4
 principles, 3
 filtration, 1122
 FindPreNucl, 401
 Fixed Point
 Kakutani, 321
 flow problem, 395
 flow_game, 396
 flow_probMinCut, 400
 forest of spanning trees, 412
 formatPowerSet, 345
 Frobenius Reciprocity, 1129
 Function
 α -characteristic, 188, 994
 β -characteristic, 994
 δ -characteristic, 1005
 γ -characteristic, 1005
 s -type-characteristic, 995
 CES, 304
 characteristic, 5
 Cobb-Douglas, 170, 304
 concave, 295
 convex, 295
 cost, 188, 330, 988
 distance, 322

Lagrange dual function, 297
 Lagrangian, 296, 309
 logarithmic barrier, 497
 payoff, 188, 330, 988
 profit, 188, 330, 988
 quadratic, 534, 1088
 revenue, 988

functional programming, 133, 214, 256, 262

G

Game
 Weighted Network Majority, 652
 game
 normal-form, 3
 NTU, 3
 partition function form, 3
 see TU game, 5, 284
 side payments, 4
 transferable utility, 4
 with non-transferable utility, 3
 with transferable utility, 3
 game_basis, 349
 gameToMama, 187, 348, 372, 391
 gameToMatlab, 348, 383
 GCG-Principle
 see generalized contested garment principle, 281
 see Talmudic Rule, 281
 generalized contested garment principle, 281
 generalized inverse matrix, 82
 generalized logarithm, 500
 genUnionStable, 559, 911
 genWeightSystem, 476
 getgame, 350, 1070
 getMinimalWinning, 647
 GetPartitions, 522
 getSymCostMatrix, 423
 global extremum, 294
 global maximizer, 294
 global maximum, 294
 global minimizer, 294
 global minimum, 294
 GLPK, 503
 Good
 differentiation, 324, 989
 homogeneous, 188, 330, 988
 Gorman form, 1019
 gputimeit, 856
 gradient, 77, 295
 graph
 closed, 320
 greedy bankruptcy game, 25, 795
 greedy_bankruptcy, 350
 gridMathematica, 1047
 grMaxFlowGame, 399
 group
 Abelian, 1115
 action, 1118
 automorphism, 1119
 center, 1116
 centralizer, 1116
 conjugacy class, 1127
 connected, 1120
 cyclic, 1118
 endomorphism, 1119
 faithful action, 1121
 free action, 1121
 general linear, 1120, 1124
 generator, 1118
 group, 1113
 group homomorphism, 1124
 homomorphism, 1119

isomorphism, 1119
 Lie, 1120
 matrix, 1120, 1124
 normal subgroup, 1117
 order of an element, 1117
 order of the group, 1117
 orthogonal, 1120
 permutation, 373, 742, 769, 1115, 1130
 positive general linear, 740, 1120
 special linear, 1120
 special orthogonal, 1120
 sub, 1120
 subgroup, 1115
 symmetric, 742, 769, 1115, 1130
 symmetry, 1121
 GUROBI, 503
 gurobi_cfr_nucl, 570
 gurobi_kernel, 502
 gurobi_prenucl, 685

H

hadamard_basis, 595
 Harsanyi dividends, 27, 349
 harsanyi_dividends, 349
 Herbrand Theorem, 1091, 1094
 Hessian matrix, 295, 325, 331, 1010
 diagonally dominant, 322
 HMS converse reduced game property, 690
 HMS-consistency
 see reduced game property (HMS), 690
 HMS-consistency set-valued
 see reduced game property (HMS), 690
 HMS-weak consistency
 see weak reduced game property (HMS), 690
 HMS_InputSavingReducedGame(), 710
 HMS_Reduced_game, 697
 holler, 618
 Holler-Packel Index, 618
 homogeneous_representationQ, 615
 homomorphism, 1121
 HSL, 503
 hypergraphQ, 561, 911

I

idempotent, 64
 identity matrix, 65
 ill-conditioned, 1106
 imputation
 coalition structure, 522
 imputation set, 5
 ImputationVertices, 437, 510
 ImputSavingReducedGame, 710
 inconsistent solution, 72
 indexing
 by position, 69
 linear, 69
 subscripted, 69
 indicator function, 13, 469, 809
 individual rationality, 5
 infix notation, 262
 inner product
 G-invariant, 1126
 Hermitian, 1127
 interior point, 295
 interior-point method, 498
 interpreted language, 29, 199
 IntersectionEssDualEssSets, 572
 IntersectionMaxCoalCorners, 781
 IPOPT, 503
 IrredAntiCore, 428

IrredCostMatrix, 420
 irreducible cost matrix, 420
 irreducible form, 420
 ISRG_propertyQ, 710

J

Johnston, 622

K

k_convexQ, 508
 Kakutani's Fixed Point Theorem, 321
 Kernel, 453, 502
 kernel, 7
 coalition structure, 530
 kernelQ, 502
 KKT condition
 see Optimization, 298
 Kruskal's algorithm, 413

L

Lagrange dual
 see Function, 77, 297
 Lagrange multiplier
 see Shadow Price, 77, 297
 Lagrangian
 see Function, 296, 309
 lambda calculus, 262
 lambda function, 262
 large excess difference, 729
 least-core, 12
 LED, 729
 LED_propertyQ, 730
 LedcoconsQ, 826
 legal recoverability, 4
 Lexicographical
 minimum, 444
 smallest coalitions, 443
 smallest coalitions for a c.s., 543
 LexOrder, 344
 lin_weightedPreKernel, 685
 linear system, 72
 local extreme point, 295
 local extremum, 295
 local maximizer, 295
 local maximum, 295
 local minimizer, 295
 local minimum, 295
 logically
 consistent, 1094
 inconsistent, 1094
 independent, 1094
 London Heathrow, 151, 167
 Lorenz maximal, 700
 Lorenz set, 700
 Lorenz solution, 700
 LorenzDom, 701
 LorenzMaxCoreQ, 701
 LorenzSol, 701
 Loss of Significance, 443
 lower hemi-continuous, 320
 LS_Nucl, 496
 LS_Nucl(), 980
 LS_PreNucl, 496
 LS_PreNucl(), 980
 LSNuc, 980
 LSPreNuc, 980

M

m.c.s.t. game, 412
 m.c.s.t. problem, 412

- machine epsilon, 1103
- Map, 980
- map
 - bijjective, 1115
 - composite, 1115
 - injective, 1115
 - invertible, 1115
 - linear, 65
 - surjective, 1115
- marginal contribution, 27
- marginal contribution networks, 184
- marginal worth vector, 373
 - restricted, 703
- Market
 - Bertrand, 323, 1033
 - Cournot, 188, 330, 988
 - direct, 364
 - Oligopoly, 323
 - price, 188, 988
 - price of brand, 989
 - pure exchange economy, 356
- market_game(), 548
- MarketGameQ, 359, 548
- Maschke's Theorem, 1126
- Mathematica
 - \$ContextPath, 221
 - AbsoluteTiming, 275
 - Animate, 963
 - Apply, 214, 215, 301
 - AtomQ, 217
 - Attributes, 222
 - Blank, 202, 212, 254
 - Blanks, 228
 - Block, 243, 244
 - Cases, 231
 - Clear, 237
 - ClearAttributes, 226
 - CloseKernels, 1049
 - Complement, 277
 - Complex, 200, 219
 - Context, 201, 221
 - Context'Name, 201, 221
 - D, 209, 226, 300
 - Depth, 268
 - Distribute, 275
 - DistributeDefinitions, 1051
 - Do, 252, 255, 256
 - Dot, 301
 - Equal, 206
 - Evaluate, 224
 - Exp, 256
 - Extract, 266
 - Factorial, 200
 - False, 209
 - First, 301
 - Flatten, 203
 - For, 252, 255
 - FullForm, 199
 - FullSimplify, 210
 - Global', 201, 221
 - Head, 199
 - Hold, 222, 224
 - HoldAll, 224, 239
 - HoldFirst, 235
 - HoldForm, 215, 224
 - HoldPattern, 224
 - If, 251, 252
 - Im, 220
 - Integer, 200, 211, 219
 - IntegerDigits, 219
 - LaunchKernels, 1048
 - List, 199, 215, 301
 - Manipulate, 963, 965
 - Map, 219, 247, 256, 276, 300, 1051
 - MapThread, 204, 225, 248, 279
 - MatchQ, 228
 - MatrixForm, 206, 267
 - Max, 278
 - Min, 278
 - Mod, 200
 - Module, 243, 244, 273, 278
 - NSolve, 300
 - Options, 307
 - Outer, 266
 - ParallelEvaluate, 1056
 - Parallelize, 1051, 1052
 - ParallelMap, 1052
 - ParallelNeeds, 1056
 - ParallelTable, 965, 1051, 1052
 - ParallelTry, 1058
 - Part, 216
 - Plot, 226, 261
 - Plus, 200, 215
 - Power, 200, 229
 - PowerSet, 274
 - Pure Functions, 262
 - Quotient, 200
 - RandomInteger, 248
 - RandomReal, 236, 1054
 - Range, 275
 - Rational, 200, 219
 - Re, 220
 - Real, 200
 - RealDigits, 219
 - Remove, 237
 - Replace, 204
 - ReplaceAll, 208, 215, 247, 269, 301, 1009
 - ReplacePart, 216, 217
 - ReplaceRepeated, 251
 - Rule, 208, 215, 247, 249, 269, 301, 1009
 - RuleDelayed, 247, 249, 1009
 - SameQ, 207, 252, 275, 279
 - SeedRandon, 1054
 - SeedRandom, 235
 - Set, 201, 206, 234, 1056
 - SetDelayed, 201, 206, 238, 326, 1007, 1011, 1056
 - SetSharedVariable, 1057
 - Simplify, 210
 - Solve, 203, 257, 300
 - Sort, 276
 - String, 200
 - Subsets, 272, 275
 - Switch, 251, 254
 - Symbol, 200, 220
 - SymbolName, 220
 - Table, 248, 256, 965, 1051
 - TableForm, 267
 - Times, 200, 215
 - ToString, 218
 - Trace, 208, 216, 223, 248
 - TracePrint, 208
 - True, 207
 - Unevaluated, 224
 - Which, 251, 254, 268
 - While, 252, 256, 258
 - With, 243
- mathematical programming problem
 - see optimization problem, 296
- MATLAB
 - abs, 64
 - ans, 31
 - arrayfun, 852
 - bin2dec, 59, 141
 - blkdiag, 68

- bsxfun, 456, 852
- cd, 160
- cell, 95
- cell2mat, 98
- char, 49
- class, 34
- clear, 33
- colon, 38
- ctranspose, 39
- dec2bin, 53, 142
- det, 85
- diag, 67
- disp, 99
- double, 34
- eig, 91
- end, 41
- eps, 31
- exp, 31
- for, 139, 140, 159
- format long, 34
- format short, 34
- fprintf, 118
- gpuArray, 852
- if, 139
- index, 160
- Inf, 31
- input, 54
- intmax, 31
- intmin, 31
- inv, 75, 92
- issymmetric, 91
- ldivide, 40
- linprog, 384
- lt, 64
- lu, 89
- M-function, 128
- M-script file, 51
- minus, 35, 64
- mldivide, 35
- mpower, 35
- mrdivide, 35
- mtimes, 35
- namelengthmax, 33
- NaN, 31
- norm, 79
- null, 79
- numel, 164
- ones, 67
- pi, 31
- pinv, 83
- plot, 100
- plot3, 104
- plus, 35, 36
- pre-allocation, 93
- qr, 86
- randi, 43
- rank, 74
- rdivide, 40
- realmax, 31
- realmin, 31
- recursive M-function, 175
- save, 33
- single, 34
- size, 48
- str2func, 156
- string, 49
- struct, 98
- subplot, 103
- surf, 106
- svd, 87
- switch, 139, 150
- syms, 36
- tic/toc, 166
- times, 40
- title, 102
- tril, 68
- triu, 68
- true, 71
- try/catch, 155
- uminus, 35
- while, 139, 168
- whos, 32
- xlabel, 102
- ylabel, 102
- zeros, 67
- MATLink, 1077
- matrix, 64
 - adjoint, 65
 - basis, 1119
 - congruent, 1119
 - diagonally dominant, 322
 - generalized inverse, 82
 - Gram, 1120
 - Hessian, 325, 331, 1010
 - idempotent, 64
 - identity, 65
 - invertible, 72
 - isometric, 85
 - Moore-Penrose, 82
 - non-singular, 72
 - null, 65
 - orthogonal, 85
 - projection, 82
 - pseudo-inverse, 82
 - rectangular, 72
 - representation, 1124
 - self-adjoint, 65, 536
 - singular, 72
 - square, 73
 - symmetric, 536
 - transition, 1119
 - transpose, 65
- matrix factorization
 - Cholesky, 90
 - LU, 88
 - QR, 85
 - singular value, 87
- matrix theory, 64
- Maximum surplus, 443
- maximum surplus, 7
- Maximum surplus for a c.s., 543
- maximum value, 294
- Maximum Value Theorem, 320
- mcst_game, 414
- medium of exchange, 4
- method
 - copy_p_TuRep, 931
 - copy_p_TuShRep, 935
 - copyTuSol, 929, 931, 935
 - p_copyTuCons, 938
 - p_copyTuKcons, 942
 - p_copyTuSol, 938, 940, 942
 - p_setADvalue, 918
 - p_setAllSolutions, 902
 - p_setAllValues, 917, 918
 - p_setAntiPreKernel, 902
 - p_setApuSolidarity, 918
 - p_setBanzhaf, 902
 - p_setCoalitionSolidarity, 918
 - p_setCoalitionStructures, 915, 917, 918
 - p_setConverse_RGP, 938
 - p_setKConverse_RGP, 942
 - p_setKReconfirmation_property, 942
 - p_setKReducedGameProperty, 942

- p_setKStrConverse_RGP, 942
- p_setMyerson, 918
- p_setMyersonUS, 918
- p_setOwen, 918
- p_setPosition, 918
- p_setPositionHS, 918
- p_setPreKernel, 902
- p_setPreNuc, 902
- p_setReconfirmation_property, 938
- p_setReduced_game_property, 938
- p_setShapley, 902, 918
- p_setSolidarity, 918
- p_setSolidarityShapley, 918
- p_setTauValue, 902
- p_TuCons, 936, 938
- p_TuKcons, 942
- p_TuSol, 902
- p_TuVal, 907, 911, 918
- setADvalue, 918
- setAllSolutions, 899, 902, 907
- setAllValues, 917, 918
- setAllVertices, 924, 925
- setAntiPreKernel, 899, 902, 907
- setApuSolidarity, 918
- setBanzhaf, 899, 902, 907
- setCddCoverVertices, 925
- setCddImpVertices, 925
- setCddVertices, 925
- setCoalitionSolidarity, 918
- setCoalitionStructures, 912, 917, 918
- setImpVertices, 925
- setMyerson, 918
- setMyersonUS, 918
- setOwen, 918
- setPosition, 918
- setPositionHS, 918
- setPreKernel, 898, 902, 907
- setPreNuc, 899, 902, 907
- setReplicate_Prk, 927, 931
- setReplicate_Shapley, 932, 934, 935
- setShapley, 898, 902, 907, 918
- setSolidarity, 918
- setSolidarityShapley, 918
- setStrongCores, 920
- setTauValue, 899, 902, 907
- setVertices, 925
- TuCore, 919
- TuGame, 888
- TuMCnets, 907
- TuProp, 894
- TuRep, 925, 931
- TuShRep, 935
- TuSol, 896, 902
- TuVal, 907, 910, 911, 918
- TuVert, 923, 925
- min_game, 717
- min_homogrep(), 627
- minimal tree, 412
- minimal tree solution, 412
- minimal_winning, 615
- minimization a p -norm, 975
- Minimum cost spanning tree, 442
- minimum cost tree solution, 414
- minimum no-blocking payoff, 383
- minimum norm property, 82
- minimum representation, 623
- minimum value, 294
- minNoBlockPayoff, 384
- modest bankruptcy game, 795
- modest-bankruptcy game, 25
- Modiclus, 715, 718, 806
- modiclusQ, 718, 807
- modified coalition array, 975
- ModPreKernel, 815, 816
- ModPrekernelQ, 816
- modular program, 120
- monotone_gameQ, 508, 611
- monotonic TU game, 6
- MOSEK, 503
- msk_cfr_nucl, 570
- msk_kernel, 501, 502
- msk_prekernel, 501
- MTRCostMatrix, 416
- multivalued function
 - see correspondence, 320
- myaa, 109, 601
- MyersonValue, 557, 657, 911
- N**
 - Nash program, 4
 - negative definite, 295
 - negative semi-definite, 295
 - NetworkDeeganPackel, 655
 - NetworkJohnston, 655
 - NetworkMajorityGame, 655
 - NetworkPGL, 654
 - NetworkShapleyShubik, 656
 - Normal-form game, 188, 324, 330, 988
 - oligopoly, 188, 324, 330, 988
 - normalized condition, 82
 - norms
 - historical, 5
 - institutional, 5
 - social, 5
 - nucl, 464
 - nucl_llp, 402
 - nucleolus, 7, 292
 - three-person formula, 195
 - null matrix, 65
 - null space, 65
 - null-player, 26
 - NullPlayer_propertyQ, 694
 - nullShapley, 593
 - nullShapleyLB, 593
 - number
 - denormalized floating-point, 1102
 - denormals, 1102
 - fixed-decimal, 60
 - fixed-point, 60
 - floating-point, 1101
 - normalized floating-point, 56, 1101
- O**
 - OASES, 503
 - object-oriented programming, viii, 883
 - objection, 20, 523
 - oddeven_game, 581
 - offset binary, 57, 1102
 - ols_weightedPreKernel, 685
 - OOP-Class
 - p_TuCons, 936, 938
 - p_TuKcons, 936, 942
 - p_TuRep, 925
 - p_TuShRep, 931
 - p_TuSol, 895, 902
 - p_TuVal, 907, 918
 - TuACore, 918
 - TuCons, 936
 - TuCore, 918, 923
 - TuGame, 888
 - TuKcons, 936
 - TuMCnets, 902, 907

- TuProp, 894
- TuRep, 925, 931
- TuShRep, 931, 935
- TuSol, 895, 902
- TuVal, 907, 918
- TuVert, 923, 925
- open ball, 295
- open neighborhood, 295
- Optimization
 - weak duality, 297
 - classical optimization, 299
 - complementary slackness, 297
 - constraint qualification, 298
 - Karush-Kuhn-Tucker optimality conditions, 298
 - mathematical programming problem, 296
 - non-linear programming, 296
 - optimal duality gap, 297
 - optimization problem, 296
 - regularity condition, 298
 - strong duality, 297
- orbit, 1121
- orthogonal, 65
 - decomposition, 65
 - projection, 65
- Orthogonal projection, 442, 542
- outcome
 - fair, 4
 - rational, 4
- outer product, 65
- outweighs, 7
- OwenValue, 554, 658, 878, 911
- P**
 - p_ADvalue, 878
 - p_average_convexQ, 869
 - p_banzhaf, 877
 - p_CddTotallyBalancedQ, 402
 - p_CoalitionSolidarity, 879
 - p_convex_gameQ, 868
 - p_OwenValue, 878
 - p_PreKernel, 863, 870, 876, 877, 881
 - p_PrekernelQ, 864
 - p_Reduced_game_propertyQ, 871
 - p_replicate_prk, 872
 - p_replicate_Shapley, 874
 - p_ShapleyValue, 865
 - p_totallyBalancedQ, 402
 - p_union_stableQ, 910
 - p_weightedOwen, 879
 - pairwise equilibrium procedure, 7
 - Parallel Scalability
 - Matlab, 860
 - Mathematica, 1049
 - parallelization
 - explicit, 837
 - implicit, 837
 - parity_basis, 595
 - particular choice, 82
 - Partition function game, 1005
 - partition function game, 1130
 - decomposes into three summands, 1131
 - decomposition of the space, 1131
 - linear symmetric solution, 1132
 - solution mapping, 1130
 - space, 1131
 - symmetric, 1131
 - PartitionGameQ, 644
 - PartitionSL, 557
 - path, 412
 - payoff, 5
 - PermutationGame, 392, 523
 - PGI, 618
 - player, 5
 - PlotCostGraph, 414, 517
 - PModPreKernel, 815
 - PModPrekernelQ, 815
 - PositionValue, 560, 911
 - positive definite, 295
 - positive semi-definite, 295
 - Potential, 429, 430
 - potential, 27
 - Power Index
 - Holler and Packel, 618
 - Shapley-Shubik, 656
 - Power index
 - Banzhaf-Coleman, 612, 877
 - Deegan-Packel, 619, 621, 655
 - Holler, 654
 - Johnston, 622, 655
 - Public Good, 654
 - Shapley-Shubik, 618
 - PowerSet, 345, 353
 - pre-bargaining set
 - coalition structure, 524
 - pre-imputation
 - coalition structure, 523
 - pre-imputation set, 5
 - Pre-kernel, 442, 955
 - algorithm, 442
 - pre-kernel, 7
 - coalition structure, 530
 - Pre-kernel for a c.s., 542
 - pre-nucleolus, 8
 - Predicate, 232
 - predicate, 217
 - PreKernel, 452, 612, 622, 863, 870, 876, 880, 1072
 - PreKernelEqualsKernelQ, 979
 - PrekernelQ, 442, 455, 612, 622, 683, 864
 - PreNucl, 402, 464
 - PreNucl_llp, 401
 - PrenuclQ, 401
 - price-cost margin, 324
 - Prim's algorithm, 412
 - primal problem, 77, 297
 - principal minors, 295
 - Principle of Explosion, 1092, 1095
 - PrkEqsModPrkQ, 819
 - procedural programming, 255
 - product situation, 1053
 - Shapley value, 1053
 - product space
 - compact, 320
 - convex, 320
 - production_game_sq, 908
 - profit_matrix, 389
 - programming
 - functional, 133, 214, 256, 262
 - procedural, 255
 - projection, 64
 - matrix, 82
 - operator, 539
 - orthogonal, 65, 539
 - projective game, 643
 - proof method
 - contradiction, 1093
 - contrapositive, 1092
 - direct, 1092
 - indirect, 1093
 - proper coalition, 11
 - Property I, 469, 546, 563
 - Property II, 469, 546
 - Property II modified (IIM), 976
 - PropModPreKernel, 817

proposal, 7, 487
 PS property, 404
 ps_gameQ, 407
 pseudo-inverse matrix, 82
 Pseudo-inverse of matrix, 442
 psstar_game, 408
 Public Good Index
 Holler and Packel, 618

Q

QPBB, 503
 QPC, 503
 quotas, 434

R

reasonable set, 9
 REC_propertyQ, 772
 RecComputeRoot, 260
 Reconfirmation_propertyQ, 708
 rectangular matrix, 72
 Recursion, 175, 260
 reduced game property, 9, 690
 Hart and Mas-Colell (HMS), 690
 reduced game property set-valued
 Hart and Mas-Colell (HMS), 690
 Reduced_game_propertyQ, 698, 700, 721
 reference
 pass by, 132
 reflexive condition, 82
 replicate_nc, 580
 replicate_prk, 588
 replicate_Shapley, 592
 representation
 G -invariant, 1125
 G -module, 1124
 character, 1127
 relations of the first kind, 1128
 class function, 1127
 group algebra, 1127, 1128
 group homomorphism, 1124
 inner product space, 1126
 irreducible, 1127
 matrix, 1124
 normalized floating-point, 56
 number of conjugacy classes, 1128
 of finite group, 1124
 partition of a positive integer, 1130
 reducible, 1127
 standard, 1126
 submodule, 1125
 trivial, 1126
 unitary, 1126
 reversed normalized condition, 82
 ReverseMCNetsRep, 187
 Rewrite Rules, 234
 root_game, 384

S

satisfaction, 646
 SaveFrames, 607
 savings_game, 348
 Schur's Lemma, 1125
 SED_propertyQ, 773
 select_starting_pt, 684
 self-adjoint matrix, 65, 536
 self-concordant, 498
 semi_convexQ, 509
 separating_collectionQ, 377, 647
 set

 bounded, 319
 closed, 319
 compact, 319
 non-negatively homogeneous, 499
 of bases, 1121
 open, 319
 set-valued function
 see correspondence, 320
 sgnzSet, 574
 Shadow Price
 Lagrange multiplier, 77, 297
 Shapley value, 9, 443
 ShapleyQ, 407
 ShapleyValue, 406, 418, 429, 612, 618, 871, 876, 911
 ShapleyValueLB, 429
 ShapleyValueM, 429
 shiftGame, 733
 ShubikBertGammaGame, 819
 side conditions, 233
 side-payment, 6
 significance
 of decimal places, 1103
 of digits, 1103
 significand, 56, 1102
 coefficient, 56
 fraction, 56
 matissa, 56
 simple committee game
 simple game, 609
 simple game, 26
 simple_game, 611, 616
 singular matrix, 72
 Situation
 bankruptcy, 25
 Cournot, 188, 330, 988
 direct market, 364
 market, 356
 mcst, 442
 minimum cost spanning tree, 442
 oligopoly, 188, 330, 988
 wedge technology, 186
 small excess difference, 753
 snigSet, 579
 social dilemma, 5
 SOCP
 second-order cone program, 500
 SolidarityShapleyValue, 555, 911
 SolidarityValue, 551, 911
 Solow growth model, 169
 solow_capital, 170
 solution
 consistent, 72, 444, 543
 inconsistent, 72
 linear symmetric, 1132
 linear system of equations, 444, 543
 subgame perfect equilibrium, 15
 solution concept, 6
 anti-core, 14, 417, 511
 anti-imputation set, 487, 511
 anti-kernel, 417, 487
 anti-nucleolus, 417, 490
 anti-pre-kernel, 487
 anti-pre-nucleolus, 490
 bargaining set, 20
 bargaining set w.r.t. c.s., 524
 center solution, 624
 core, 10
 core w.r.t. c.s., 524
 improper, 511
 irreducible anti-core, 420
 irreducible core, 420
 kernel, 7, 953

- kernel for c.s., 530
 - least-core, 12
 - Lorenz, 700
 - Lorenz set, 700
 - Modiclus, 715
 - nucleolus, 7
 - pre-bargaining set w.r.t. c.s., 524
 - pre-kernel, 7, 442, 955
 - pre-kernel for a c.s., 542
 - pre-kernel for c.s., 530
 - pre-nucleolus, 8
 - quasi-core, 11
 - quota, 952
 - random order value, 373
 - self-dual, 26
 - set, 6
 - Shapley value, 9, 443
 - stable set, 4
 - stable sets, 18
 - standard solution, 10, 189, 287
 - strong ϵ -core, 11
 - super-additive cover, 525
 - value, 6
 - von Neumann-Morgenstern solutions, 18
 - Weber set, 373
 - solution scheme
 - see solution concept, 6
 - sortsets, 343, 1068
 - space
 - Hausdorff, 320
 - inner product, 1126
 - null, 65
 - regular, 320
 - vector, 64
 - spanning tree, 412
 - SplitSimpleGame(), 635
 - square matrix, 73
 - stabilizer, 1121
 - StandardSolution, 552
 - star, 637, 660
 - stationary point, 296
 - Strategic-form game
 - see normal form game, 330, 988
 - see normal-form game, 188, 324
 - StrConverse_DGP_Q(), 831
 - strict local maximum, 295
 - strict local minimum, 295
 - strong ϵ -core, 11
 - strong converse consistency
 - strong converse reduced game property, 690
 - strong converse reduced game property, 690
 - SubGame, 552
 - subgame perfect equilibrium, 15
 - Subtractive Cancellation, 443
 - super-additive cover, 525
 - super-additive TU game, 6
 - super_additiveQ, 363, 509
 - superadditive_cover, 525
 - superadditiveQ, 526
 - symmetricQ, 695
 - symmetry, 10
- T**
- Talmudic Rule, 281
 - Talmudic_Rule, 348, 452, 796, 865, 870
 - TauValue, 434, 613, 622
 - term-rewriting, 222, 234, 251
 - Rules, 222
 - rules, 222
 - TImatrix, 595
 - timeit, 166, 856
 - top-down design, 109
 - ToSimplex3d, 101, 104
 - totally anti-balanced TU game, 15
 - totally balanced TU game, 12
 - transfer, 6
 - transformation
 - linear, 64
 - Transformation Rule, 213, 247
 - Local Rewrite Rule, 208, 247
 - transpose matrix, 65
 - tree, 412
 - Trust
 - see coalition, 988
 - Truth Tables, 1091
 - TU game, 3, 5, 284
 - α -, 188, 994
 - β -, 994
 - δ -, 1005
 - γ -, 1005
 - s -type, 995
 - t -shift, 728
 - additive, 6, 504
 - airport cost, 113, 371
 - almost average-convex, 383
 - almost-convex, 379
 - anti-balanced, 15
 - anti-derived, 786
 - anti-monotonic cover, 416
 - anti-reduced, 752
 - assignment, 389
 - average-convex, 381, 951
 - balanced, 12, 358
 - bankruptcy, 25
 - concave, 371, 421
 - constant-sum, 26, 504
 - convex, 6, 368, 505, 950
 - cost savings, 371, 443
 - derived, 775
 - dual, 26, 505
 - dual cover, 724
 - dual extension, 723
 - dual floor, 755
 - dual probability, 972
 - essential, 8, 504
 - excess comparability cover, 728
 - excess comparability floor, 753
 - flow, 395
 - greedy bankruptcy, 25, 795
 - homogeneous standard, 624
 - improper, 511
 - inessential, 8, 504
 - m.c.s.t., 414
 - market, 357
 - minimum, 717
 - modest bankruptcy, 795
 - monotonic, 6, 504, 951
 - partition, 642
 - partition function, 1005, 1130
 - permutation, 391
 - positive, 370
 - primal extension, 723
 - probability, 971
 - product, 1053
 - PS, 404
 - reduced, 719
 - root, 383
 - rooted, 383
 - satellite, 625
 - shift, 728
 - simple, 26
 - strictly essential, 8, 26
 - subgame, 6, 13

- submodular, 371
- super-additive, 6, 504, 951
- super-modular, 505
- totally anti-balanced, 15
- totally balanced, 12, 358
- truncated, 624
- unanimity, 27, 349
- veto-rich, 381
- weakly super-additive, 505, 951
- wedge production, 186
- weighted graph, 403
- weighted majority, 26
- zero-monotonic, 11, 951
- zero-normalized, 8
- zero-one-normalized, 8
- TUG
 - AnimationKernelProperty2d, 957
 - AnimationKernelPropertyV6, 961, 963, 965
 - AntiPreKernelSolution, 973
 - AvConvexQ, 951, 968, 1061
 - BalancedCollectionQ, 969
 - ConcaveQ, 972
 - ConvexQ, 950, 968, 1057
 - CoreQ, 968, 973
 - CriticalValue, 963
 - DefineGame, 957, 961
 - DefineTuGame, 286
 - DualBankruptcy, 279
 - DualProbabilityGame, 972
 - FilledCoreV6, 957
 - GameMonotoneQ, 951
 - GatelyValue, 956
 - GreedyBankruptcy, 278
 - Kernel, 953, 961
 - LieBracket, 223
 - LowerCriticalVal, 963
 - ManipulateMode, 965
 - ModestBankruptcy, 272, 277, 969
 - Modiclus, 961
 - ModifiedNucleolus, 961, 973
 - MonotoneQ, 951
 - Nucleolus, 973
 - NucleolusThreePerson, 290
 - ParaAvConvexQ, 1061
 - ParaCharacteristicValues, 1060
 - ParaConvexQ, 1061
 - ParaPreKernel, 1062
 - ParaPreKernelElement, 1062
 - ParaProductGame, 1054
 - ParaTuGames, 1059
 - PlotCore3dV6, 961, 962
 - Potential, 293
 - PreKernelElement, 955, 973
 - PreKernelEqualsKernelQ, 973
 - PreKernelQ, 969
 - PreKernelSolution, 973, 1057, 1058
 - PreNucleolus, 973, 1058
 - ProbabilityGame, 971
 - ProductGame, 1054
 - Quota, 952
 - ShapleyValue, 955, 961
 - ShowCore, 964
 - ShowImputationSet, 964
 - ShowKernelCatcherV6, 961
 - StandardSolution, 287
 - StrongEpsCore2d, 957
 - StrongEpsCore3dV6, 961
 - SuperAdditiveQ, 951, 968
 - UpperCriticalVal, 963
 - VetoRichPlayers, 968
 - ViewNucleolusSol, 963
 - WeaklySuperAdditiveQ, 951, 968
 - WeightedMajority, 264, 270, 968
 - ZeroMonotoneQ, 951
 - tug_AntiPreKernel, 1074
 - tug_AvConvexQ, 1066, 1070
 - tug_BalancedCollectionQ, 1072, 1075
 - tug_BelongToCoreQ, 1075
 - tug_CollectionBalancedQ, 1069
 - tug_ConvexQ, 1065
 - tug_DetUCoord, 1069
 - tug_Mnuc, 1073
 - tug_Nuc, 1073
 - tug_PreKernel, 1071, 1076
 - tug_PreNuc, 1073
 - tug_ShapleyValue, 1070
 - tug_SymGameSizeK, 1067
 - tug_SymGameType2, 1066
 - tug_SymGameType3, 1067
 - tug_SymGameType4, 1067
 - tug_ZeroMonotoneQ, 1074
 - TuOlig
 - DeltaGame, 1017
 - DixitBertDeltaGame, 1044
 - DixitBertGammaGame, 1044
 - DixitBertrandParetoOpt, 1043
 - DixitBertrandPostMergerEq, 1044
 - DixitBertrandPreMergerEq, 1043
 - DixitPreMergerEq, 1043
 - GammaGame, 1016
 - NashEq, 323
 - OligDiagonallyDomQ, 328, 1010, 1015
 - PlotReactionCurves, 336
 - PlotReactionCurves2D, 329
 - PlotReactionCurves3D, 335
 - PostMergerEquilibrium, 1008, 1009
 - PreMergerEquilibrium, 1007, 1009
 - StackelbergEquilibrium, 1008, 1009
 - StypeGame, 1017
 - TuOptAux
 - DecVar, 307
 - Foc, 300
 - L, 300, 301
 - QuadraticForm, 307
 - QuadRep, 307
 - SolveOpt, 301, 302, 309
- U
 - ulps
 - units in the last place, 1107
 - unanimity basis, 349
 - underflow
 - gradual, 1110
 - undirected edge, 412
 - undirected graph, 185, 403
 - union stable structure, 558
 - union stable system, 557
 - union_stableQ, 559, 657, 911
 - UnionStableBasis, 558
 - upper hemi-continuous, 319
 - utility
 - interpersonal comparisons, 4, 7, 8, 10
 - theory, 4
 - utility function
 - quasi-linear, 4
 - UtopiaPayoff, 602
- V
 - value
 - pass by, 132
 - value of a game
 - Aumann-Drèze, 552, 878

- Banzhaf, 612, 617
 - Banzhaf with communication structure, 658
 - Banzhaf-Coleman, 612
 - Banzhaf-Owen, 659
 - Coalition Solidarity, 554, 879
 - Deegan-Packel, 619, 621, 655
 - Gately, 956
 - Holler, 654
 - Johnston, 622, 655
 - Myerson, 557, 657
 - of a coalition, 5
 - Owen, 554, 658, 878
 - Position, 560
 - Public Good, 654
 - random order, 373
 - Shapley, 9, 429, 443, 618, 871, 876, 955
 - Shapley-Shubik, 612, 656
 - Solidarity, 551
 - Solidarity Shapley, 555
 - Solidarity w.r.t. à priori unions, 555
 - Tau, 434, 622
 - weighted Owen, 879
 - weighted Shapley, 431
 - value_matrix, 393
 - variable
 - broadcast, 840
 - global, 132
 - local, 131
 - reduction, 840
 - scope, 131
 - sliced, 839
 - vclToMatlab, 351, 354, 475
 - vector
 - average marginal contributions, 10
 - characteristic, 13
 - coefficient, 65
 - column, 64
 - marginal contributions, 6
 - null, 65
 - vector space, 64
 - filtration, 1122
 - veto player, 26, 381
 - veto-rich player, 381
 - veto_players, 613, 616
 - veto_rich_players, 384
- W**
- walsh_basis, 595
 - weak Anti-Property I, 495
 - weak Anti-Property II, 495
 - weak reduced game property, 690
 - Hart and Mas-Colell (HMS), 690
 - Weak_balancedCollectionQ, 473
 - weakly_super_additiveQ, 363, 509
 - Weierstrass Theorem, 320
 - weighted graph problem, 185, 403
 - weighted majority game, 26
 - weighted_majority, 614, 875
 - weighted_truncated, 629
 - weightedBalancedCollectionQ, 478
 - weightedExcess, 479
 - weightedGraphGame, 406
 - weightedKernel, 482
 - weightedKernelQ, 483
 - weightedNucl, 482
 - weightedOwen, 879
 - weightedPreKernel, 482, 684
 - weightedPrekernelQ, 483
 - weightedPreNucl, 478
 - weightedShapley, 431
 - well-conditioned, 1106
 - winning player, 615, 624
 - winning_coalitions, 610, 616
 - winning_players(), 634
 - Wolfram Lightweight Grid Manager, 1047
 - worth
 - see value of a coalition, 5
 - WSysKernelQ, 483
 - WSysNuclQ, 480
 - WSysPreKernellQ, 483
 - WSysPreNuclQ, 479
- Z**
- zero_monotonicQ, 508, 611
 - ZeroMonotoneQ, 979
 - ZeroMonotonicQ, 979