Cooperative Game Theory with MATLAB and Mathematica

From Fundamentals to Advanced Computational Techniques

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3rd February 2025

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Preface

The book describes how the reader can use our software tools for MATLAB© and *Mathematica*© for her/his research or teaching. It is divided into three parts. The first part deals with the prerequisites of Cooperative Game Theory, *Mathematica*©, and MATLAB©. To this end, the first chapter is thought as a quick refresher of the essentials of cooperative game theory, the chapters devoted to *Mathematica*© and MATLAB© are designed to familiarize even readers with little or no background in these languages with their semantics and syntax for mastering the underlying programming concepts and techniques. These concepts and techniques are illustrated on code snippets to cope with some basic game theoretical problems. In contrast, the second part develops how our MATLAB© toolbox can be applied to study solution concepts and properties for TU games with and without a trivial coalition structure. Before its usage on selected examples is explained, the game theoretical background is extensively introduced and discussed. Important fundamental aspects and techniques of a computer-based analysis with our MATLAB© toolbox are described and are further panned out as a basis for the reader's explorations. Particular subjects like the plotting of solutions as well as the investigation of the axiomatization (principles of distributive justice) of selected solution concepts by our software tool are comprehensively discussed and developed. Leading then into the discussion of solving game theoretical hard problems by parallel as well as GPU computation as the backbone of High-Performance Computing. To end up in object-oriented programming for retrieving and modifying game data to ensure a consistent computation environment for doing game theory by a unified computer-based approach. The last part retrieves the study of solution concepts and properties for TU games with a trivial coalition structure with *Mathematica*©. These are the building blocks to direct the reader to the theory of industrial cooperation while focusing on the partition function approach that enables us to study the stability of cartel agreements.

Analogously to the previous part, we revive for our *Mathematica*© package the essentials of High-Performance Computing while establishing its parallel computational features. The remaining two chapters deal with the transfer of game data between these two worlds of programming languages. Finally, additional background information is provided in various appendices.

Karlsruhe, Germany
February 2025

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About this Book

Nowadays, numerical computation and computer algebra systems are common standards for scientists in various fields. Although algorithmic game theory is a very dynamic field of research, computer-based approaches to analyzing cooperative games are still uncommon. This book fills this gap while describing our software tools for MATLAB© and *Mathematica*© for doing interactively cooperative game theory on a computer in a uniform environment. Before a wide variety of examples are introduced, the computational and game theoretical aspects are extensively discussed and explained. Finally, moving to parallel- as well as GPU computation, which are at the cutting edge of High-Performance Computing for studying game theoretical hard problems. The book is suitable for readers with little or no background in MATLAB© or *Mathematica*© due to its emphasis on carefully introducing their programming concepts and techniques for self-study. In this respect, it essentially neither requires a special previous background in Game Theory, apart from mathematical maturity, nor programming knowledge. However, the book covers thoroughly a course in cooperative game theory with side payments on an advanced level, whether MATLAB© or *Mathematica*© is used or not. Due to this general approach to discussing the introduced algorithms, the software tools described can also be transferred to other programming languages without much additional effort, such as `Python` or `R`, to name just two other software systems that are widely used in the academic world.

The book is addressed to scientists, graduate students, advanced undergraduates, and programmers who want to study several aspects of cooperative games with side payments (TU games) by means of a computer-aided approach with the software systems MATLAB© and *Mathematica*©. The novelty of the book lies in the fact that it supports the theoretical presentation of concepts from cooperative game theory through well-selected game examples with a complexity level that could not be examined without software support. Enhancing our theoretical understanding through faster access to new ideas that would otherwise not be accessible to us. To illuminate this itinerary of knowledge further, not only sophisticated numerical examples are discussed, but also a selection of algorithms for addressing large-scale issues and their implementations into both software systems. Providing the reader with the necessary skills to follow his own fluctuating but nevertheless promising journey of knowledge.

To prepare the reader for this journey, the present book provides, after having discussed through a first step the underlying theory, computer-aided means to model and impose solution analysis on transferable utility games (TU games) using MATLAB and *Mathematica*. The book is divided into three main parts and an appendix. We start off with a part-wise general overview of the book and then move over to a more detailed chapter-wise summary structured by parts. The software tools accompanying the book are available online. At the end of this book's itinerary, one will find a brief overview of the software companions.

- The preparatory Part I is composed of three chapters to formalize the reader with the essential principles of cooperative game theory with side payments (TU games), MATLAB, and *Mathematica*. To prepare readers, not all game classes treated by the book were covered in Chapter 1. We would like to point out that the first chapter only deals with those concepts/classes that are relevant to both Parts II and III that can be quickly grasped by a reader without possessing any good prior knowledge of Game Theory. Otherwise, we would have to immediately introduce quite extensive technical machinery without being able to explain the concepts in more detail with the help of our software tools, which will only be introduced in Parts II and III. The book aims to clarify the concepts of cooperative game theory, after their theoretical discussion, with the help of our programmed software tools, instead of bothering the reader with small tasks without making them aware of the entire complexity of the problem. Standard textbooks use small exercises for that, but with our software tools, much more complex problems can be discussed, and it expands rapidly the understanding, which we consider a real advantage of a computer-aided approach. For example, the intrinsic combinatorics in game theory concepts means that certain structures and properties of games cannot be illustrated in 3- or 4-person games. Very often, only games with at least five persons reveal enough structure to make the point clear. Nowadays, in the computer age, no one should be faced with having to create these structures and properties manually. This becomes particularly problematic if one wants to clarify the axiomatization of solution concepts. For this reason, we only outline the axiomatization for some solution concepts in Chapter 1. This topic is deepened in Chapter 7 with a whole set of MATLAB routines to examine the underlying axiomatization of a calculated solution.
- In contrast, Part II is composed of eight chapters to investigate TU games using our conceived software toolbox

for MATLAB. There, we mainly present the mathematical proofs, which will be discussed in detail in a further step using our designed MATLAB routines to deepen their understanding. Among others, it develops, after having imparted the theoretical background, how the software means designed for MATLAB can be applied to studying solution concepts and properties for TU games with and without a trivial coalition structure. Besides this, graphical extensions, real-word applications, axiomatization of selected solution concepts, object-oriented programming (OOP) techniques, multithreading as a tool of implicit parallelization, certain forms of explicit parallelization on multiple processors or for graphics processing units (GPUs), are discussed within a game theoretical context.

- However, Part III includes a total of five chapters to examine cooperative games with side payments using the `Wolfram Language`. It retrieves the study of solution concepts and properties for TU games with a trivial coalition structure under *Mathematica*, thereby presenting an alternative set of routines for scrutinizing TU games through multithreading and explicit parallelization. Note in this context that the theoretical foundation is treated in Chapter 4. There is no need to resume these fundamentals in Part III with *Mathematica*. It is only necessary to discuss the conceived *Mathematica* counterparts that allow us to study these topics with the `Wolfram Language` while referring to the mathematical results of the MATLAB section. In addition, this part focuses on cooperative games arising from industrial cooperation while discussing in the initial step the theoretical foundation, in order to move then along to the next step to establish how one can profit from parallel acceleration under *Mathematica* while investigating game issues. The remaining two chapters are devoted to the grand theme of cross-checking, thereby focusing in more detail on some fundamental methods for exchanging data, delegating, and running computer tasks into a different software environment without leaving the initial software platform.
- The book closes with the appendix, which provides some further background related to propositional logic, group theory, representation theory, numerical round-off errors, and our software tools.

As mentioned above, the book is divided into a preparatory, a MATLAB, and a *Mathematica* part. Readers who want to study the book through self-study and are familiar with these topics can skip the first part, thereby reading the remaining parts in isolation, simply following their preferences. However, readers not acquainted with these topics ought to read Chapter 1 first, and only then should they progress in accordance with their interests in reading either Chapter 2 or Chapter 3. To continue at all events with Chapter 4 of Part II, even if one is only interested in Part III. This is because the theoretical foundations necessary for understanding Part III are situated in this chapter. Though we have tried to cross-reference as much as possible in the text, not everything is referenceable in Part III.

In what follows, we provide a short synopsis of the upcoming chapters, structured by parts. We start with the chapter overview for the first part and end with a survey of the applied software platforms.

About Part I

The chapters of Part I have a preparatory nature; and can be skipped by informed readers. Each chapter can be read in isolation. However, novices in game theory ought to start with the first chapter.

Chapter 1 : This preparatory chapter is devoted to resuming the relevant background of cooperative game theory with side payments (TU games). In particular, it is conceived as a starter to refresh some fundamental principles of TU games. We formalize first treatments related to certain game classes, such as convexity, super-additivity, monotonicity, or zero-monotonicity. Most importantly, some solution concepts, such as the (pre-)kernel, (pre-)nucleolus, or the Shapely value, are resumed in formal but not rigorous technical treatments. Albeit these concepts are introduced formally, the emphasis is, however, more on providing the reader with the underlying intuition and motivation behind them. Technical aspects of game properties and solutions are more thoroughly examined in Part II or Part III.

Besides this, we present in the introduction to this chapter our interpretation of a TU game to efface with some common misunderstandings of cooperative game theory, notably, the prevalent opinion, that only binding agreements can be studied for cooperative games, caused probably by the fact that the conceptual aspects of cooperative game theory are neither well explained nor motivated in the literature while focusing merely on the technical aspect of results. The annoying and at the same time amazing consequence of this postulate is that it is unclear why one should care about solution concepts from cooperative game theory when they are enforceable by an unbiased arbitrator or a legal system. Why should one formalize proposals or counter-proposals, dominance, or stability issues when everything is binding? We shall learn that the wreaked conceptual damage of this postulate is dissolvable while considering the axiomatic foundation of solution concepts as a description of principles of distributive justice. Then, agreements can be non-binding and thereby stable simply because actors submit themselves by self-interest under the immanent rules of justice.

Chapter 2 : In this chapter, we are going to present some basic operations that will give us some understanding of how we can use the high-level programming language of MATLAB to investigate transferable utility games. Having introduced and discussed some basic operations of MATLAB, we apply our knowledge gained by writing in the first step some small routines to solve linear problems and drawing their solution sets in two- and three-dimensional graphics. In the next step, we deepen and enhance these skills while implementing routines to convert the binary-encoding of double-precision floating-point numbers to their equivalent decimal representations. Then, our attention is turned to the top-down design process. That is a formal approach to designing a computer program. Its design process is illustrated for an airport cost allocation problem borrowed from the literature. Having understood and mastered the program design of MATLAB through these examples, we introduce the toolbox approach to represent a coalitional game with transferable utility under MATLAB by the unique integer representations of coalitions. While understanding this, we shall discuss some alternative representation methods for a TU game from the literature. Specifically, we focus on a concise representation through the marginal contribution networks (MC-nets). Finally, we test our skills on some basic solution schemes from game theory, such as the standard solution, Shapley value, and the computation of the nucleolus for three-person games.

Chapter 3 : Similar to the foregoing chapter on MATLAB, we are going to present some basic operations that shall allow us to understand how we can use the versatile programming language of *Mathematica* to investigate transferable utility games. Having introduced and discussed some basic operations of *Mathematica*, we apply our knowledge gained from writing some small routines to get the game representation of weighted majority or bankruptcy games. Then, we turn our attention to some basic solution schemes like the Talmudic Rule (generalized contested garment principle), the computation of the nucleolus for three-person games while applying a unique formula, and the Shapley value. We close this chapter while resuming crucial aspects of optimization theory and the Nash equilibrium, which are of particular importance for studying cooperative games in partition function form.

About Part II

Part II deals with studying TU games by means of MATLAB. Readers not acquainted with this topic should have read Chapter 2 before continuing. Then, they should move on to Chapter 4 to study chiefly the modeling and solution sections. Readers familiar with this topic should also concentrate on this part in the first reading but may wish to skip the theoretical part while focusing primarily on handling the software means to study TU games. The remaining topics of Part II can then be studied in isolation.

Chapter 4 : Having mastered the preparatory part, we are in a position to clarify and extend game theoretical concepts by means of mathematical proofs. In addition, we shall observe how the knowledge of these concepts can be deepened using software-based means conceived in MATLAB. To this end, we start with the modeling part while establishing how game models can be described by means of our software tool. Then, the solution part is considered by representing several algorithms to compute the Shapley value, (pre-)nucleolus, elements of the (pre-)kernel, and much more. Not without first examining the theoretical background by providing numerous selected mathematical results that characterize game solutions and game properties. However, to elucidate their intrinsic complexity, programmed software is used to accentuate the underlying layers. Particularly, some selected and crucial game classes with a trivial coalition structure are presented. Then, the most common solution concepts and their corresponding algorithms are discussed, including how to determine them using user-defined MATLAB tools. Besides, the associated algorithms are introduced, discussed, and finally implemented into MATLAB code in conjunction with some command and code annotations. Instead of using only built-in solvers to get game solutions, we illustrate how one can achieve the same objective using MATLABs application programming interface (API) by calling external third-party solvers. Finally, we discuss how crucial game properties and convex solution sets can be analyzed using our software tool.

Chapter 5 : After we have carried out detailed studies related to transferable utility games (TU games) with a trivial coalition structure, we move forward to investigate in more detail TU games with non-trivial coalition structures. Notably, we shall see how the core, super-additive cover, (pre-)kernel, and (pre-)nucleolus must be generalized to cope with distributive arbitration (fairness standards) when non-trivial coalition structures are likely to prevail. In the same vein, we also study fairness and related values for coalition structures. Moreover, to advance our understanding of compliance with agreements, we shall investigate the stability of an agreement within a game setting, referring to the nucleolus, pre-kernel, and Shapley value. As in the foregoing Chapter 4, the rather technical analysis will be underpinned by software-aided means designed for our MATLAB toolbox to elucidate the arising combinatorial complexity issues.

Chapter 6 : Having treated transferable utility games (TU games) with trivial and non-trivial coalition structures, we shall address the question of how one can depict three-dimensional convex solutions for four-person games, such as the core, strong ϵ-cores, the Weber set, or visualize the geometric properties of the nucleolus or kernel by means of our software tool. In a further step, we move on to study simple games and then the political decision-making process through some real-world examples through weighted majority games. Especially, we are investigating the political decision-making process of the UN Security Council, the minimum homogeneous representation of weighted majority games, and their implication on the shape of the (pre-)kernel solution.

However, all these approaches have measured the power of individuals in a political decision-making process without taking into account the formation, control, and domination of a network of relationships to influence a voting outcome and institutions to serve one's interests. For incorporating such an underlying mesh of relations into the measurement of power, a network is specified by an undirected graph in order to obtain a game representation in characteristic function form from which such an analysis is feasible. This kind of power analysis is exemplified by Florentine marriage and business relationships during the early Renaissance. The chapter closes by discussing some procedures to handle numerical errors when seeking a pre-kernel point.

Chapter 7 : The axiomatic foundation of cooperative solution concepts is studied more deeply by relying on their theoretical results and exemplifying crucial aspects by means of computer-aided software tools. The purpose of such an analysis is twofold: On the oneăhand, to verify a set of solution characteristics for a better understanding of a solution itself and gain more intuitions, and on the other hand, to check on the correctness of a solution obtained by floating-point arithmetic. Notably, we can check if a solution vector satisfies several consistency properties. For instance, the pre-nucleolus should fulfill the Davis-Maschler reduced game property, and the Shapley value should meet the Hart and Mas-Colell reduced game property. In this respect, a number of software-aided tools were presented that support an axiomatic analysis of solutions.

Chapter 8 : As a particular application of axiomatizing solution theory, we focus within this chapter on the modiclus, also known under the name modified nucleolus. The modiclus simultaneously considers, besides the primal power, also the preventive power (dual power) of the game to permit actors in a bargaining situation to enforce their claims. This allows us to present and discuss numerous alternative axiomatizations of the modiclus that refer to the primal and preventive power of coalitions. The immanent charm of our proposed alternative characterizations of the modiclus or the anti-pre-nucleolus can be attributed to the fact that we can now refer to the preventive power of a coalition without leaving the original game context and introducing the dual game. To show the pertinence of the derived results, we demonstrate that the modiclus is equal to the anti-pre-nucleolus for the class of PS, weighted graph, and modest bankruptcy games.

Chapter 9 : This chapter focuses on the axiomatization of related solution concepts of the modiclus. Alike those, both the modified and proper modified pre-kernel take the primal as well as the preventive power (dual power) of coalitions during a stylized bargaining scenario into account. Even though it was developed as an auxiliary solution concept to investigate the modiclus, no effective computation methods were designed to support this analysis. We are closing this gap and presenting some algorithms for their computation. Before, we shall resume their theoretical foundation while providing some alternative axiomatization for both solution concepts.

Chapter 10: In the course so far, we have focused on sequential computing only. This form of computing can process one instruction at a time. Consequently, the next instruction is solely executable at the moment when the previous one has been completed. No set of instructions can run concurrently. By contrast, parallelization is a particular form of computation in which the initial problem is decomposable into separate and independent sub-problems for processing these subtasks at the same time on the computer system. We provide procedures for how one can conduct a computer-based analysis for transferable utility games with MATLAB using parallel and GPU computing. During this chapter, we provide a set of procedures allowing us to benefit from parallel acceleration while investigating game issues with our toolbox.

Chapter 11: The MATLAB programming language also offers the opportunity to create object-oriented programs. Relying on object-oriented programming (OOP) techniques allows us to encapsulate data and operations within a set of objects designed to ensure that a user operates within a clearly defined and consistent environment. This avoids unintentionally changing data and performing a relatively sophisticated list of tasks properly. Data types that permit both encapsulated manipulation of data and user-defined operations on them are called classes. In OOP terminology, a class comprises similar objects. Each TU game has identical attributes, like a characteristic function and a player set. These attributes constitute the objects that form the class of TU games. Though TU games have identical essential characteristics, they have different external manifestations, inducing distinct game properties, and are consequently different objects from the same game class. Now, OOP allows us to create this kind of abstract data type by defining a class with the mentioned attributes to compose a class object for a TU game. This allows for carrying out only some pre-defined and segregated data manipulations on a class object. To familiarize ourselves with OOP, we provide a short synopsis of the essential features of OOP with MATLAB. Then, we discuss how the resulting OOP-class objects are applicable to investigate TU games in a clearly defined and consistent computing environment.

About Part III

Part III deals with investigating a TU game by means of *Mathematica*. Readers not acquainted with this topic should have read Chapter 3 before continuing. Then, they should proceed to Chapter 4 to study mainly the theoretical foundations discussed in detail there. Readers familiar with this topic can safely skip this part and go straight to Chapter 12, thereby focusing primarily on handling software means to study TU games. The remaining chapters of Part III are treatable in isolation.

Chapter 12: This chapter discusses an alternative set of software routines to study TU games with *Mathematica* instead of MATLAB. For both software tools, there is a basic set of functions that overlap in their functionality, i.e., they provide the same features. However, they are based on different programming paradigms and are therefore based on diverse operating principles. Though one might have implemented in both programming languages the same algorithm, the coding in both programming environments had to differ while adjusting to the particularities of the language. Therefore, it is relatively unlikely to produce the same programming failure in both environments. Thus, getting in both environments an identical answer increases the reliability of the returned result. We continue to cover the core features by letting the reader see the differences between game analyses performed under MATLAB and those under *Mathematica*. It will be shown how the (pre-)kernel elements, the (pre-)nucleolus, the modiclus, the modified and proper modified pre-kernel, the Shapley value, Lorenz solution, Dutta-Ray solution, excess payoffs, the τ-value, χ-value, Gately point, the vertices of a core, and much more is computable. Moreover, we shall demonstrate the verification of some interesting game properties, such as convexity, average-convexity, or super-additivity with *Mathematica*.

Chapter 13: In this chapter, we are focusing on the study of industrial cooperation using software tools conceived for *Mathematica*. Similar to the foregoing procedures, in the initial step, the underlying theory is extensively discussed before the reader is introduced to the procedure to analyze problems arising from industrial cooperation using computer-aided tools conceived with *Mathematica*. That is to say, we focus on the computation of Cournot, Bertrand, and Stackelberg equilibria, as well as on the computation of the associated TU-oligopoly games arising from post-merger equilibria like those of γ-, δ-, as well as s-type characteristic function forms.

Chapter 14: Having understood the essentials of sequential computing under *Mathematica*, we head over in this chapter to the parallel implementation of computer-aided analysis of transferable utility games with conceived tools for the `Wolfram Language`. Typically, users perform sequential symbolic processing with *Mathematica* most of their time. Besides this, there are also very computationally and data-intensive calculations in game theory that request, instead of sequentially processing, a parallelization of the tasks in which the initial problem is decomposable into separate and independent sub-problems. We show how the user can profit under *Mathematica* from a performance gain by processing separate and independent sub-tasks simultaneously.

Chapter 15: Cross-checking results should be part of a scientist's normal toolbox. In this chapter, we are introducing tools for achieving this by calling *Mathematica* functions designed for studying TU games from a running MATLAB session. For this purpose, we discuss some fundamental methods whose functionality we are already acquainted with MATLAB but whose handling of the results differs due to some *Mathematica* specifics.

Chapter 16: Here, we study the reverse procedure of how MATLAB functions are accessible from a running *Mathematica* session to study TU games. We shall demonstrate how selected toolbox functions are easily usable from within *Mathematica* to offset numerical disadvantages that cannot be compensated for by parallel processing. In addition, it extends the functionality of our *Mathematica* software tool by granting access to functional primitives that are not implemented yet with the `Wolfram Language`. It allows the application of the rich prototyping capabilities of the `Wolfram Language` to conceive algorithms for studying game properties or axiomatizations without leaving the original modeling framework by accessing extra functionality for easy verification of new ideas without taking unnecessary detours by laboriously creating identical game models from scratch in a different modeling environment.

Appendix

This part deals with some additional material related to cooperative game theory. For instance, we provide chapters to discuss some concepts of mathematical logic and group theory. Both fields are extensively applied in the founding work of game theory by von Neumann and Morgenstern (1944), but they are often treated superficially and even incorrectly in the literature. Each chapter of the appendix is addressable in isolation.

Chapter A : This first chapter of the appendix focuses on the computational study of benchmarking in order to present the computational behavior of a method in getting an element of the pre-kernel. For this purpose, we resume the reported executing time of Meinhardt (2013) for different profiles for serial and parallel computation of an element of the pre-kernel, based on Algorithm A.0.1. Though the data generated with MATLAB Release 2013a are somewhat outdated, they nevertheless provide some comparison in terms of the fast development in computer science. To set the point related to the reported timings in Chapter 10, where we have deployed up to 40 logical cores compared to the conducted test at that epoch in parallel on compute nodes with eight logical cores only.

Chapter B : We present a detailed treatment of propositional logic that should allow the reader to grasp crucial logical arguments applied in several proof techniques given in the literature. Propositional logic can be subsumed as a method of formalizing inferences for analyzing reasoning; it is, therefore, at the heart of conducting mathematical proofs. For that purpose, we resume some proof methods and then move along on the basis of the Deduction Theorem (Herbrand Theorem, 1930) to the question under which circumstances a statement is classifiable as non-provable. To answer this question, we need to leave the framework of propositional logic while making some borrowings from predicate logic (first-order logic).

Chapter C : The first section of this appendix reviews crucial aspects of floating-point representation, allowing us to grasp rounding-errors and other kinds of numerical errors caused by machine arithmetic which are captured in greater detail in the second section. These are indispensable concepts for estimating the quality of a computed result.

Chapter D : This appendix reviews specific definitions, concepts, and aspects of group theory. An abstract group allows us to capture the properties of mathematical objects through classification. In this sense, group theory and group algebra are the building blocks for identifying similarities and invariances to classify mathematical objects. Classifications can also be useful in studying games, characterizing solutions and game properties, or performing a stability analysis of solutions. Notably, the study of the pre-kernel in Subsection 4.4.3 pursued this approach by conducting an economic stability analysis by means of the positive general linear groups. We present a concise remainder of group theory that should allow the reader to grasp crucial but abbreviated arguments applied within some proofs of the book.

Chapter E : In this chapter, we will expose the essential ingredients of representation theory and apply them to the analysis of cooperative games with externalities (games in partition function form). Representation theory is the study of the nature and extent to which a given group acts on a vector space. The action of group elements endows, by means of a homomorphism, a structure on the vector space, a representation of the group, from which a classification of the underlying mathematical objects is obtainable. Applying representation techniques to the theory of cooperative games with externalities allows us to decompose the vector space of games in partition function under the group action of the symmetric group into a direct sum of irreducible subspaces from which symmetric linear solutions can be analyzed concisely and elegantly. To make the point, we review some essential results of the work of Sánchez-Pérez (2014, 2017) to discuss a direct sum decomposition of the game space into invariant subspaces. Exemplarily, we present for $n = 4$ and $n = 5$ by means of character theory even a decomposition into the irreducible submodules.

Chapter F : In the course of this appendix, we shall provide some instructions to install the MATLAB toolbox `MatTuGames`. In the initial step, we have to discuss the requirements for running the toolbox properly. Moreover, we also present some add-ons for the toolbox and provide their installation instructions.

The software listed within the requirements section is considered a recommendation. Nothing of the mentioned software tools is really needed to run the toolbox in a basic operating mode. However, to get full functionality out of the toolbox and for accuracy reasons, we strongly recommend its installation. For the impatient user, we provide a quick guide on how to get started and some fundamental operations.

Chapter G: This appendix provides some instructions to install the *Mathematica* package `TuGames`. In particular, we shall focus on a custom installation. Furthermore, we additionally deal with some information on how one can run the package in parallel or run the parallel version of our package and how one can use the **`Cddmathlink`** library to profit from the graphical features of our package. For the users who are even interested in generating some movies to illustrate, for instance, the geometric properties of the (pre-)nucleolus and (pre-)kernel, we also make sure of running the **`Cddmathlink`** library, even in parallel. Furthermore, we also give some pieces of information w.r.t. `MATLink`, which is needed to call our MATLAB toolbox `MatTuGames` within a running *Mathematica* session. For the impatient user, we provide a brief guide to getting started and some basic operations.

Chapter H: In Subsections 2.8.4 and 3.5.8, we shall discuss a MATLAB, respectively, a *Mathematica* program for computing the nucleolus for three-person zero-normalized super-additive games using the formula invented by Leng and Parlar (2010). For comparison reasons, we will present their Maple code for its computation in this appendix.

About the Software

We have selected MATLAB© and *Mathematica*© to conduct computer-based approaches to analyzing cooperative games. We will now give the user some insights into our motives for focusing on these tools. Both software platforms are carefully designed, implemented, maintained, powerful, and reliable. In addition, they have a high-level programming language, powerful graphical capabilities, and relatively fast routines for numerical and symbolic computations. They support parallel computation on multicore processors or general-purpose computation on the graphics processing unit(s) (GPGPU). Furthermore, their functionality is expandable by easily integrating external libraries from `C/C++`, `FORTRAN`, `Python`, or even third-party solvers from different vendors, for example. Besides, they are easy to learn, so one can make quick progress in studying one's own problems. For that reason, they have attracted a great community of users, providing lots of resources, and consequently, it is relatively easy to find advice for arising troubles. All that makes them the prevalent remedy in the academic world and an ideal tool for prototyping to test the preliminaries of ideas. This must be seen in the context that the featuring of creating through their high-level programming language concise codes, enabling, therefore, the users to primarily focus on the concepts of cooperative games, putting the implementation aspects of mathematical tools aside.

MATLAB© Toolbox for Cooperative Game Theory

For the reader who is mainly interested in the computational aspects of TU games using MATLAB©, we offer the software tool `MatTuGames`, which provides the ability to apply enhanced numerical problem-solving for cooperative games with transferable utility. This MATLAB© toolbox is downloadable under the following URL:

http://www.mathworks.com/matlabcentral/fileexchange/35933-mattugames

or alternatively at the URL:

https://github.com/himeinhardt/MatTuGames

As an auxiliary software means to study combinatorial and group theoretical aspects of cooperative games, we have conceived the toolbox `MatRep` under MATLAB© to provide a set of functions for studying representation in symmetric groups.

https://mathworks.com/matlabcentral/fileexchange/62142-matrep-a-matlab-representation-theory-toolbox-symmetric-groups

The designed MATLAB© toolboxes are compatible with MATLAB© R2024a or prior releases.

Mathematica© packages for Cooperative Game Theory

The reader who is more familiar with *Mathematica*© can use the software package `TuGames`. The latest version is **`TUG-3.1.4`**, compatible with *Mathematica*© version 12.0 or higher, and is downloadable from the following URL:

https://github.com/himeinhardt/TuGames

As an additional software means for doing industrial cooperation, we also offer the *Mathematica*© package `TuOlig`, which needs at least **`TUG-3.1.2`**.

https://github.com/himeinhardt/TuOlig

The developed *Mathematica*© packages are compatible with *Mathematica*© 14.1, or prior versions, but not before

version 12.0.
We have used our MATLAB© and *Mathematica*© software tools to create the graphics for the book.

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