

JOURNAL OF Econometrics

Journal of Econometrics 105 (2001) 5-26

www.elsevier.com/locate/econbase

Forecasting S&P 100 volatility: the incremental information content of implied volatilities and high-frequency index returns

Bevan J. Blair^a, Ser-Huang Poon^b, Stephen J. Taylor^{c,*}

^aWestLB Asset Management, 33 Gracechurch Street, London EC3V 0AX, UK
^bDepartment of Accounting and Finance, University of Strathclyde, Glasgow G4 0LN, UK
^cDepartment of Accounting and Finance, Lancaster University, Lancaster LA1 4YX, UK

Abstract

The information content of implied volatilities and intraday returns is compared, in the context of forecasting index volatility over horizons from 1 to 20 days. Forecasts of two measures of realised volatility are obtained after estimating ARCH models using daily index returns, daily observations of the VIX index of implied volatility and sums of squares of 5-min index returns. The in-sample estimates show that nearly all relevant information is provided by the VIX index and hence there is not much incremental information in high-frequency index returns. For out-of-sample forecasting, the VIX index provides the most accurate forecasts for all forecast horizons and performance measures considered. The evidence for incremental forecasting information in intraday returns is insignificant. © 2001 Elsevier Science S.A. All rights reserved.

JEL classification: C22; C53; G13; G14

Keywords: ARCH models; Forecasting; High-frequency returns; Implied volatility; Stock index volatility

1. Introduction

The ability of ARCH models to provide good estimates of equity return volatility is well documented. Many studies show that the parameters of a

0304-4076/01/\$- see front matter © 2001 Elsevier Science S.A. All rights reserved. PII: \$0304-4076(01)00068-9

^{*} Corresponding author. Tel.: +44-1524-593624; fax: +44-1524-847321.

E-mail addresses: bevan_blair@westlb.co.uk (B.J. Blair), s.poon@strath.ac.uk (S.-H. Poon), s.taylor@lancaster.ac.uk (S.J. Taylor).

variety of different ARCH models are highly significant in-sample; see Bollerslev (1987), Nelson (1991), Glosten et al. (1993) and surveys by Bollerslev et al. (1992), Bera and Higgins (1993) and Bollerslev et al. (1994).

There is less evidence that ARCH models provide good forecasts of equity return volatility. Some studies (Akgiray, 1989; Heynen and Kat, 1994; Franses and Van Dijk, 1995; Brailsford and Faff, 1996; Figlewski, 1997) examine the out-of-sample predictive ability of ARCH models, with mixed results. All find that a regression of realised volatility on forecast volatility produces a low R^2 statistic (often less than 10%) and hence the predictive power of the forecasts may be questionable. Dimson and Marsh (1990) show that data snooping can produce enhanced in-sample predictive ability that is not transferable to out-of-sample forecasting. Nelson (1992) uses theoretical methods to show that the predictive ability of ARCH models is very good at high frequencies, even when the model is misspecified, but that out-of-sample forecasting of medium- to long-term volatility can be poor. Nelson and Foster (1995) develop conditions for ARCH models to perform favourably at medium- to long-term forecast horizons.

An alternative to ARCH volatility forecasts is to use implied volatilities from options. These are known to covary with realised volatility (Latane and Rendleman, 1976; Chiras and Manaster, 1978) and hence it is of interest to compare the forecasting accuracy of ARCH with implied volatility. To do this many studies use S&P 100 index options which are American. Day and Lewis (1992) find that implied volatilities perform as well but no better than forecasts from ARCH models. Mixtures of the two forecasts outperform both univariate forecasts. Canina and Figlewski (1993) provide contrary evidence to Day and Lewis. They find that implied volatilities are poor forecasts of volatility and that simple historical volatilities outperform implied volatilities. Christensen and Prabhala (1998) show for a much longer period that while implied volatilities are biased forecasts of volatility they outperform historical information models when forecasting volatility.

These papers use implied volatilities that contain measurement errors, because early exercise is ignored and/or dividends are ignored and/or the spot index contains stale prices. The implied volatility series are potentially flawed and this may account for some of the conflicting results. Fleming et al. (1995) describe an implied volatility index (VIX) which eliminates mis-specification problems. We use the same index to obtain new results about the information content of option prices. Fleming (1998) uses a volatility measure similar to VIX to show that implieds outperform historical information.

From the recent studies of Christensen and Prabhala (1998) and Fleming (1998) it can be concluded that the evidence now favours the conclusion that implied volatilities are more informative than daily returns when forecasting equity volatility. The same conclusion has been obtained for foreign exchange volatility from daily data, but with more certainty (Jorion, 1995; Xu and

Taylor, 1995). High-frequency returns, however, have the potential to change these conclusions. Taylor and Xu (1997) show that 5-min FX returns contain volatility information incremental to that provided by options. In this paper we explore the incremental volatility information of high-frequency stock index returns for the first time.

The importance of intraday returns for measuring realised volatility is demonstrated for FX data by Taylor and Xu (1997), Andersen and Bollerslev (1998) and Andersen et al. (2001b), and for equities by Ebens (1999) and Andersen et al. (2001a). Andersen and Bollerslev (1998) prove that regression methods will give low R^2 values when daily squared returns measure realised volatility, even for optimal GARCH forecasts, because squared returns are noisy estimates of volatility. They show that intraday returns can be used to construct a realised volatility series that essentially eliminates the noise in measurements of daily volatility. They find remarkable improvements in the forecasting performance of ARCH models for FX data when they are used to forecast the new realised series, compatible with theoretical analysis. They do not, however, compare ARCH forecasts with implied forecasts.

This paper answers some important empirical questions for the S&P 100 index. Firstly, how does the predictive quality of volatility forecasts from ARCH models, that use daily index returns and/or intraday returns, compare with forecasts from models that use information contained in implied volatilities? Secondly, how important is the selection of the measure of realised volatility in assessing the predictive accuracy of volatility forecasts?

The paper is arranged as follows. Section 2 discusses the various data sets we use and issues surrounding their construction. Methods for estimating and forecasting volatility that use daily index returns, 5-min returns and implied volatilities are presented in Section 3. Section 4 provides results from both in-sample estimation and out-of-sample forecasting of S&P 100 volatility. Section 5 sets out our conclusions.

2. Data

There are three main types of data: daily index returns, daily implied volatilities and 5-min index returns. These are available to us for the 13 years from 1987 to 1999 inclusive. The in-sample period is from 2 January 1987 to 31 December 1992 providing 1519 daily observations, followed by the out-of-sample period from 4 January 1993 to 31 December 1999 providing 1768 daily observations. ¹

¹ An earlier version of this paper used a much shorter out-of-sample period.

2.1. Daily index returns

Daily returns from the S&P 100 index are defined in the standard way by the natural logarithm of the ratio of consecutive daily closing levels. Index returns are not adjusted for dividends. Some models in this paper have been estimated using dividend adjusted index returns and the results from volatility estimation and forecasting are not significantly different.

2.2. Implied volatilities

Implied volatilities are considered to be the market's forecast of the volatility of the underlying asset of an option. To calculate an implied volatility an option valuation model is needed as well as inputs for that model (index level, risk free rate, dividends, contractual provisions of the option) and an observed option price. Many of these variables are subject to measurement errors that may induce biases in a series of implied volatilities.

The use of an inappropriate option valuation model will also cause mismeasurements in the implied volatilities. For instance, S&P 100 options are American but if an European model is used to calculate implied volatilities then error will be induced by the omission of the early exercise option. Moreover, when a valuation model is used that includes the early exercise option, if the timing and level of dividends are assumed constant rather than using actual timings and dividends then the early exercise option is not taken up as frequently, pricing errors will occur, and implied volatilities will be mis-measured (see Harvey and Whaley, 1992).

Important biases may also be induced by relatively infrequent trading of the stocks in the index (see Jorion, 1995). Also, if closing prices are used there may be biases due to the different closing times of stock and options markets. Finally, if bid or ask option prices are used the first order autocorrelation of the implied volatility series may be negative and so biases will result from bid/ask effects.

Implied volatilities used in many previous studies (including Day and Lewis, 1992; Canina and Figlewski, 1993; Christensen and Prabhala, 1998) contain relevant measurement errors whose magnitudes are unknown. We follow Fleming et al. (1995) and use an implied volatility index (VIX) that mitigates the above problems. VIX is a weighted index of American implied volatilities calculated from eight near-the-money, near-to-expiry, S&P 100 call and put options and it is constructed in such a way as to eliminate mis-measurement and "smile" effects. This makes it a more accurate measurement of implied market volatility. VIX uses the binomial valuation method with trees that are adjusted to reflect the actual amount and timing of anticipated cash dividends. Each option price is calculated using the midpoint of the most recent bid/ask quote to avoid problems due to bid/ask bounce.

VIX uses both call and put options to increase the amount of information and, most important, to eliminate problems due to mis-measurement of the underlying index and put/call option clientele effects.

VIX is constructed so that it represents a hypothetical option that is at-themoney and has a constant 22 trading days (30 calendar days) to expiry. VIX uses pairs of near-the-money exercise prices, that are just above the current index price and just below it. VIX also uses pairs of times to expiry, one that is nearby (at least eight calendar days to expiration) and one that is second nearby (the following contract month). For a detailed explanation of the construction of the VIX index see Fleming et al. (1995).

Because VIX eliminates most of the problems of mis-measurement we use it as our measure for S&P 100 index implied volatility. Daily values of VIX at the close of option trading are used.² Even though VIX is robust to mis-measurement it is still a biased predictor of subsequent volatility. It is our understanding that bias occurs because of a trading time adjustment that typically multiplies³ conventional implied volatilities by approximately 1.2. The variance of daily index returns from 1987 to 1999 is equivalent to an annualised volatility or standard deviation of 17.6%. The average value of VIX squared is equivalent to an annualised volatility of 21.7% during the same period, which is consistent with an average scaling factor of 1.23. With this in mind, our empirical methods are designed to be robust against this bias.

2.3. High-frequency stock returns

ARCH and implied volatility models provide forecasts of volatility. These forecasts require some measure of the latent volatility process so that their performance can be evaluated. Because the volatility process is not observed researchers have used a variety of methods to compute ex post estimates of volatility, often called realised volatility. The most common method for computing a realised volatility is to square the inter-period returns. For example, if we are forecasting daily volatility the realised measure is the squared daily return. Andersen and Bollerslev (1998) show that this method produces inaccurate forecasts for correctly specified volatility models. Consider a representation of excess returns r_t such that $r_t = \sigma_t z_t$ where σ_t is the time-varying volatility and $z_t \sim \text{i.i.d.}(0,1)$. The realised volatility measure using squared returns is $r_t^2 = \sigma_t^2 z_t^2$ and if σ_t is independent of z_t then $E(r_t^2 | \sigma_t^2) = \sigma_t^2$. However, the realised measure, r_t^2 , will be a very noisy estimate of σ_t^2 because

² VIX data are available at http://www.cboe.com/tools/historical/vix.htm.

³ VIX multiplies the conventional implied by $\sqrt{N_c/N_t}$, with N_c and N_t , respectively, calendar days and trading days until expiry. See Fleming et al. (1995, Eq. (2) and Fig. 3).

not only are ARCH models generally misspecified but poor predictions of r_t^2 will be due to the noisy component z_t^2 .

By sampling more frequently and producing a measure based on intraday data, Andersen and Bollerslev (1998) show that the noisy component is diminished and that in theory the realised volatility is then much closer to the volatility during a day. They find, for foreign exchange data, that the performance of ARCH models improves as the amount of intraperiod data used to measure realised volatility increases. Related results for FX and equity data are presented, respectively, in Andersen et al. (2001b) and Andersen et al. (2001a).

In this paper we use 5-min returns from the S&P 100 index to calculate a measure of realised volatility, here called INTRA. These 5-min returns are constructed from the contemporaneous index levels recorded when S&P 100 options are traded. The original data set contains an index level whenever an option is traded but we have chosen a 5-min frequency as this is the highest and best frequency that Andersen and Bollerslev (1998) use. The latest index level available before a 5-min mark is used in the calculation of 5-min returns. To construct the measure INTRA of daily realised volatility we square and then sum 5-min returns for the period from 08:30 CST until 15:00 CST and then add the square of the previous "overnight" return. Thus, INTRA on a Tuesday, for example, is the squared index return from Monday 15:00 to Tuesday 08:30 plus 78 squared 5-min index returns on Tuesday, commencing with the return from 08:30 to 08:35 and concluding with the return from 14:55 to 15:00. This volatility measure is also used in ARCH models to deduce the information content of intraday returns.

The square root of the average value of *INTRA* from 1987 to 1999 is equivalent to an annualised volatility of 13.4%. This is much less than the equivalent volatility calculated from the variance of daily returns for the same period, namely 17.6%. This is a consequence of substantial positive correlation between consecutive intraday returns, which could be explained by stale prices in the S&P 100 index.

3. Methodology for forecasting volatility

3.1. In-sample models

To compare the in-sample performance of several models that use information from stock index returns and implied volatilities, ARCH models are estimated for daily index returns r_t from 2 January 1987 to 31 December 1992, with a dummy term d_t for the Black Monday crash in 1987. The most

general specification is as follows:

$$r_t = \mu + \psi_1 d_t + \varepsilon_t,\tag{1}$$

$$\varepsilon_t = h_t^{1/2} z_t, \quad z_t \sim \text{i.i.d.}(0, 1),$$
 (2)

$$h_{t} = \frac{\alpha_{0} + \alpha_{1} \varepsilon_{t-1}^{2} + \alpha_{2} s_{t-1} \varepsilon_{t-1}^{2} + \psi_{2} d_{t-1}}{1 - \beta L} + \frac{\gamma INTRA_{t-1}}{1 - \beta_{1}L} + \frac{\delta VIX_{t-1}^{2}}{1 - \beta_{V}L}.$$
 (3)

Here L is the lag operator, h_t is the conditional variance of the return in period t, s_{t-1} is 1 when $\varepsilon_{t-1} < 0$ and otherwise it is zero, d_t is 1 when t refers to 19 October 1987 and is 0 otherwise, VIX_{t-1} is the daily implied index volatility computed from an annualised volatility as $VIX/\sqrt{252}$ and $INTRA_{t-1}$ is the measure of volatility based on 5-min returns from the S&P 100 index. The parameters ψ_1 and ψ_2 , respectively, represent the excess return on the crash day (-24%) and an exceptional, transient effect in variance immediately following the crash.

By placing restrictions on certain parameters six different volatility models are obtained using different daily information sets:

- 1. The GJR(1,1) model of Glosten et al. (1993) with a crash dummy in the variance equation, also estimated by Blair et al. (2001a, b): $\gamma = \delta = \beta_I = \beta_V = 0$.
- 2. A volatility model that uses intraday returns information alone: $\alpha_1 = \alpha_2 = \delta = \beta_V = 0$.
- 3. A specification that uses the historic information in both daily returns and *INTRA* but ignores option information: $\delta = \beta_V = 0$.
- 4. A volatility model that uses the latest information in the *VIX* series alone: $\alpha_1 = \alpha_2 = \gamma = \beta_1 = 0$.
- 5. A low-frequency specification that only uses the information provided by daily measurements of *VIX* and index returns: $\gamma = \beta_I = 0$.
- 6. A volatility model that uses information in both *INTRA* and *VIX* without explicit use of daily returns: $\alpha_1 = \alpha_2 = 0$.
 - These are all special cases of:
- 7. The unrestricted model using all the available information in daily returns, *INTRA* and *VIX*.

The parameters are estimated by the usual quasi-likelihood methodology, 4 so that the likelihood function is defined by assuming standardised returns, z_t , have Normal distributions even though this assumption is known to be false. Inferences are made using robust p-values, based on Bollerslev and Wooldridge (1992) standard errors, that are robust against mis-specification of the shape of the conditional distributions. It is then possible to attempt to

⁴ The general parameter vector is $(\mu, \alpha_0, \alpha_1, \alpha_2, \beta, \psi_1, \psi_2, \gamma, \beta_1, \delta, \beta_V)$. The log-likelihood function was maximised using GAUSS with the constraints $\alpha_1, \alpha_1 + \alpha_2 \ge 0$.

decide which types of information are required to obtain a satisfactory description of the conditional distributions of daily returns. To assess predictive power, ε_t^2 is regressed on the conditional variance h_t . Higher values of the correlation R indicate more accurate in-sample forecasts of ε_t^2 .

3.2. Forecasting methods

Time series of forecasts are obtained by estimating rolling ARCH models. Each model is estimated initially over the final 1000 trading days of the in-sample period, from 20 January 1989 to 31 December 1992, and forecasts of realised volatility are made for the next day, say day T+1, using the in-sample parameter estimates. The model and data are then rolled forward one day, deleting the observation(s) at time T-999 and adding on the observation(s) at time T+1, re-estimated and a forecast is made for time T+2. This rolling method is repeated until the end of the out-of-sample forecast period. The one-step-ahead forecasts provide predictions for 4 January 1993 to 31 December 1999 inclusive and define time series of length 1768. On each day, forecasts are also made for 5, 10 and 20 day volatility.

Two measures of realised volatility are predicted. The first measure is squared excess returns, so that forecasts are made at time T of

$$(r_{T+1} - \mu)^2$$
 and $\sum_{j=1}^{N} (r_{T+j} - \mu)^2$, $N = 5$, 10, 20.

These quantities are predicted because a correctly specified ARCH model provides optimal forecasts of $(r_{T+1} - \mu)^2$. We assume the conditional expected return μ is constant and our results are not sensitive to the choice of μ ; results are reported for an assumed annual expected return of 10%. The second measure of realised volatility is *INTRA* that uses 5-min and "overnight" returns. Forecasts are produced at time T for

$$INTRA_{T+1}$$
 and $\sum_{i=1}^{N} INTRA_{T+j}$, $N = 5, 10, 20.$

These quantities can be expected to provide a more predictable measure of realised volatility than squared excess daily returns, although on average $INTRA_{T+1}$ is less than $(r_{T+1} - \mu)^2$ as already remarked in Section 2.3. Four different sets of forecasts are estimated for each measure of realised volatility to compare predictive power. These forecasts are first presented for the realised measure defined by squared excess returns.

3.2.1. Historic volatility

A simple estimate of volatility is evaluated to see what simple methods can achieve in comparison with more sophisticated methodologies. The simple one-step-ahead forecast is here called historic volatility (HV) and it equals the sample variance of daily returns over the period from time T-99 to time T inclusive, given by

$$h_{T+1} = \frac{1}{100} \sum_{j=0}^{99} (r_{T-j} - \bar{r}_T)^2, \quad \bar{r}_T = \frac{1}{100} \sum_{j=0}^{99} r_{T-j}.$$
 (4)

To produce 5, 10 and 20 day volatility forecasts the one-step-ahead forecast is, respectively, multiplied by 5, 10 and 20.

3.2.2. GJR(1,1) forecasts

The one-step-ahead forecast, h_{T+1} , of $(r_{T+1} - \mu)^2$ is defined by the recursive formula,

$$h_{t+1} = \alpha_0 + \alpha_1 r_t^2 + \alpha_2 s_t r_t^2 + \beta h_t, \tag{5}$$

where s_t equals 1 when $r_t < 0$ and otherwise equals 0. The parameters α_0 , α_1 , α_2 , β are estimated from the information $I_T^{(1)} = \{r_{T-i}, 0 \le i \le 999\}$. Forecasts for 5, 10 and 20 day volatility are produced by aggregating expectations

$$E(h_{T+j}|I_T^{(1)}) = \alpha_0 + p_{gir}E(h_{T+j-1}|I_T^{(1)}), \quad j > 1,$$
(6)

with $p_{\rm gjr}$ the persistence equal to $\alpha_1 + \frac{1}{2}\alpha_2 + \beta$ for the GJR(1,1) model, here assuming that returns have symmetric distributions. Then $\sum_{j=1}^{N} (r_{T+j} - \mu)^2$ is forecast to be $\sum_{j=1}^{N} E(h_{T+j}|I_T^{(1)})$.

3.2.3. Forecasts of volatility using VIX

These forecasts use information contained in VIX alone to forecast volatility. The one-step-ahead forecast, h_{T+1} , is obtained by estimating an ARCH model with conditional variances defined by

$$h_t = \alpha_0 + \frac{\delta V I X_{t-1}^2}{1 - \beta_V L}.\tag{7}$$

Parameters α_0 , δ , β_V are estimated from $I_T^{(2)} = \{r_{T-i}, VIX_{T-i}, 0 \le i \le 999\}$. To produce 5, 10 and 20 day volatility forecasts it is assumed that $\mathrm{E}(h_{T+j}|I_T^{(2)}) = \mathrm{E}(h_{T+j-1}|I_T^{(2)})$ for j>1 so that the one-step-ahead forecast is, respectively, multiplied by 5, 10 and 20. This simple multiplicative method may handicap VIX forecasts, however, information about the term structure of implied volatility is not available from VIX.

3.2.4. Forecasts of volatility using intraday returns

These forecasts use the information in intraday returns alone to forecast volatility. Now h_{T+1} is obtained from conditional variances defined by

$$h_t = \alpha_0 + \frac{\gamma INTRA_{t-1}}{1 - \beta_1 L} \tag{8}$$

and the parameters α_0 , γ , β_I are estimated ⁵ from $I_T^{(3)} = \{r_{T-i}, INTRA_{T-i}, 0 \le i \le 999\}$. To produce 5, 10 and 20 day volatility forecasts, we aggregate expectations defined by

$$E(h_{T+j}|I_T^{(3)}) = (1 - \beta_1)\alpha_0 + \beta_1 E(h_{T+j-1}|I_T^{(3)}) + \gamma E(INTRA_{T+j-1}|I_T^{(3)}), \quad j > 1.$$

To make progress, it is necessary to assume that there is a proportional relationship between expectations of squared excess returns and *INTRA*, so that

$$E(INTRA_{T+j}|I_T^{(k)}) = c_T E((r_{T+j} - \mu)^2 | I_T^{(k)}) = c_T E(h_{T+j} | I_T^{(k)})$$
(9)

for some constant c_T , that is the same for all positive j and information sets indexed by k = 1, 2, 3. Positive dependence among intraday returns causes c_T to be less than one as noted in Section 2.3. Making the assumption stated by Eq. (9), expectations are calculated from

$$E(h_{T+j}|I_T^{(3)}) = (1 - \beta_I)\alpha_0 + (\beta_I + c_T\gamma)\beta_I E(h_{T+j-1}|I_T^{(3)}), \quad j > 1$$
(10)

and $\sum_{j=1}^{N} (r_{T+j} - \mu)^2$ is forecast to be $\sum_{j=1}^{N} \mathrm{E}(h_{T+j}|I_T^{(3)})$. To calculate the forecasts at time T, the constant c_T is estimated by the ratio

$$\hat{c}_T = \sum_{t=T-999}^{T} INTRA_t / \sum_{t=T-999}^{T} (r_t - \mu)^2.$$
 (11)

3.2.5. Forecasts of INTRA

To predict future values of *INTRA* we again make the plausible assumption stated in Eq. (9). Then if some method (*HV*, *GJR*, *VIX*, or *INTRA*) produces a forecast $f_{T,N}$ of $\sum_{j=1}^{N} (r_{T+j} - \mu)^2$ then that method's forecast of $\sum_{j=1}^{N} INTRA_{T+j}$ equals $\hat{c}_T f_{T,N}$.

3.3. Forecast evaluation

Mean square forecast errors are compared to assess the relative predictive accuracy of the four forecasting methods. Given forecasts $x_{T,N}$ made at times T = s, ..., n - N of quantities $y_{T,N}$ known at time T + N, we report the proportion of variance explained by the forecasts:

$$P = 1 - \frac{\sum_{T=s}^{n-N} (y_{T,N} - x_{T,N})^2}{\sum_{T=s}^{n-N} (y_{T,N} - \bar{y})^2}.$$
 (12)

We also report the squared correlation, R^2 , from the regression

$$y_{T,N} = \alpha + \beta x_{T,N} + u_{T,N},$$

⁵ Specifications (5), (7) and (8) are estimated with the constraints α_1 , $\alpha_1 + \alpha_2$, β , β_I , $\beta_V \ge 0$.

which provides the proportion of variance explained by the ex post best linear combination $\alpha + \beta x_{T,N}$. Hence P, which measures the accuracy of forecasts, is at most equal to the popular R^2 statistic that is often interpreted as a measure of information content.

The above regression only measures the predictive power of a single method in isolation. Day and Lewis (1992) found that mixtures of forecasts from ARCH models and from implied volatilities produced higher adjusted R^2 statistics than univariate forecasts. Consequently, multiple regressions are also estimated. Multiple R^2 statistics are used to assess the information content of the mixtures. No model that uses a mixture of the VIX, GJR and intraday information sets is used to produce forecasts directly. Rather bivariate and trivariate mixtures of forecasts are evaluated to see if the implied volatility forecasts subsume the information in GJR and intraday return forecasts.

4. Results

4.1. In-sample ARCH results

Table 1 presents parameter estimates, robust t-ratios, log-likelihoods and squared correlations R^2 for the seven ARCH models defined in Section 3.1. The results are obtained from 6 years of data, from 2 January 1987 to 31 December 1992. The log-likelihoods increase monotonically across the columns of Table 1. The excess log-likelihood for a model is defined as its maximum log-likelihood minus the corresponding figure for the first model.

The first model is the standard GJR model, that uses previous index returns to describe the conditional variance, augmented by a transient volatility increase following the crash on 19 October 1987. The multiplier for squared negative returns $(\alpha_1 + \alpha_2)$ is 3 times that for squared positive returns (α_1) , indicating a substantial asymmetric effect, although the estimates of α_1 and α_2 are not significantly different from zero at the 10% level reflecting the relatively short sample period. The persistence estimate is $\alpha_1 + \frac{1}{2}\alpha_2 + \beta = 0.9693$.

The second model uses only the measure INTRA based on 5-min returns to describe conditional variances. It has a substantially higher log-likelihood than the GJR model. The INTRA series is filtered by the lag function $\gamma/(1-\beta_1L)$ with β_I approximately 0.25. This suggests that most of the volatility information in 5-min returns can be obtained from the most recent day although long memory effects in INTRA, beyond the scope of this paper, might be anticipated from the results of Ebens (1999) and Andersen et al. (2001a). The low value of β_I contrasts with the high value of β for daily returns (0.94 for Model 1).

When all the information in the history of daily and intraday index returns is used in the third model, the log-likelihood is not significantly higher than

Table 1 Models for the S&P 100 index, daily returns from January 1987 to December 1992a

Daily index returns r_t are modelled by the ARCH specification:

$$r_t = \mu + \psi_1 d_t + \varepsilon_t,$$

$$\varepsilon_t = h_t^{1/2} z_t, \quad z_t \sim \text{i.i.d.}(0,1),$$

$$\begin{aligned} & r_t - \mu + \psi_1 u_t + \varepsilon_t, \\ & \varepsilon_t = h_t^{1/2} z_t, \quad z_t \sim \text{i.i.d.}(0,1), \\ & h_t = \frac{\alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 s_{t-1} \varepsilon_{t-1}^2 + \psi_2 d_{t-1}}{1 - \beta L} + \frac{\gamma INTRA_{t-1}}{1 - \beta_1 L} + \frac{\delta VIX_{t-1}^2}{1 - \beta_V L}, \\ & d_t \text{ is 1 for 19 October 1987 and otherwise is 0.} \end{aligned}$$

 d_t is 1 for 19 October 1987 and otherwise is 0,

 s_t is 1 if ε_t is negative, otherwise s_t is zero,

INTRA is a sum of squared intraday returns, VIX is a measure of implied volatility.

Parameter	Model						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$\alpha_0 \times 10^6$	2.6891	44.797	0.7298	9.6881	12.5372	6.3594	0.3110
	(1.53)	(3.85)	(1.48)	(0.74)	(0.72)	(0.55)	(0.70)
α_1	0.0136		0.0085		0.0741		0.0029
	(1.53)		(1.56)		(1.34)		(0.47)
α_2	0.0280		-0.0078		-0.0485		-0.0029
	(1.19)		(-0.79)		(-1.20)		(-0.47)
β	0.9417	0.9793	0.9773	0.5209	-0.3039	0.5954	0.9695
•	(33.67)	(118.7)	(130.8)	(1.53)	(-1.93)	(3.58)	(95.89)
$\psi_2 \times 10^3$	1.563	0.590	0.432	2.104	0.800	1.618	0.259
,	(2.13)	(1.67)	(1.26)	(1.40)	(0.81)	(0.68)	(0.67)
γ		0.6396	0.5661			0.3718	0.3742
•		(3.64)	(3.32)			(1.86)	(1.86)
$\beta_{ m I}$		0.2523	0.2350			0.0360	
, -		(1.51)	(1.53)			(0.23)	(0.39)
δ				0.4313	0.4101	0.3283	0.2816
				(6.31)	(6.06)	(4.47)	(3.27)
$\beta_{ m V}$				0.1509	0.1553	0.1943	0.1778
•				(2.73)	(2.59)	(3.40)	(3.49)
log-L	4833.66	4845.80	4848.35	4851.97	4858.48	4859.70	4860.79
Excess log-I	_	12.14	14.69	18.31	24.82	26.04	27.13
R^2	0.197	0.318	0.307	0.377	0.367	0.376	0.358

a Notes—Parameters are estimated by maximising the quasi-log-likelihood function, defined by assuming conditional normal densities. Robust t-ratios, calculated using GAUSS, are shown in parentheses. Excess log-likelihood is relative to Model (1). Estimates of μ range from 0.00039 to 0.00047. ψ_1 is -0.237. The constraints α_1 , $\alpha_1 + \alpha_2 \ge 0$ are applied.

for Model 2 using the non-robust likelihood ratio test and a 5% significance level. Furthermore, the parameters for the variable INTRA are similar for Models 2 and 3 whilst the parameters for squared daily returns are much smaller for Model 3 than for Model 1.

The only information used in the fourth model to describe conditional variances is contained in the VIX series. The significance of this information is clear from the high robust t-ratio for the parameter δ , equal to 6.31. The fourth model has a higher likelihood than the second model that uses daily and 5-min returns. Model 4 has the same number of parameters as Model 2 and the difference in their log-likelihoods is 6.17. Thus implied volatilities are more informative than 5-min returns, at least in-sample. The VIX series is filtered by the function $\delta/(1-\beta_V L)$ with β_V approximately 0.15. Although β_V is near zero, it is significantly different at the 5% level with a robust t-ratio of 2.73. This result suggests that the options market is not informationally efficient, since the latest option prices do not contain all relevant information about the next day's volatility. Although VIX quadrupled on the crash day, from 36% on Friday 16th to 150% on Monday 19th, the estimate of the crash parameter ψ_2 is positive.

Models 5–7 are compared with Model 4 to try to decide if *VIX* contains all the relevant information about the next day's volatility. Any incremental information in the history of 5-min and daily returns is probably to be found in the 5-min returns, firstly because Model 6 has a higher log-likelihood than Model 5 and secondly because Model 7 that uses all the information improves only slightly on Model 6 that does not use daily returns. Consequently, we focus on comparing Model 4 that uses *VIX* information with Model 6 that uses both *VIX* and *INTRA* information.

The log-likelihood for Model 6 is 7.73 more than for Model 4. The former model has an additional two parameters and hence a non-robust likelihood-ratio test would conclusively prefer the former model. However, the robust t-ratio for the INTRA parameter γ equals 1.86 so that γ has a robust one-tailed p-value that is low (3.1%) but not very close to zero. It may be concluded that whilst VIX is more informative than INTRA there is probably incremental information in 5-min returns.

As VIX squared and INTRA do not have comparable magnitudes, for reasons given in Sections 2.2 and 2.3, the fact that the VIX parameter δ is less than the INTRA parameter γ is not particularly relevant when comparing information content. The average level of VIX squared is approximately 2.6 times the average level of INTRA. In the equation for the conditional variance h_t the ratio of the impact of VIX information to the impact of INTRA information is therefore approximately

2.6
$$\frac{\delta/(1-\beta_{\rm V})}{\gamma/(1-\beta_{\rm I})} = 2.8$$

using the parameter estimates for Model 6. This ratio emphasises that *VIX* is much more informative than *INTRA* when applying an in-sample methodology.

Fig. 1 provides a comparison of the in-sample conditional variances for three ARCH specifications and the squared returns that they forecast. The specifications are Models 1, 2 and 4 that, respectively, use the information

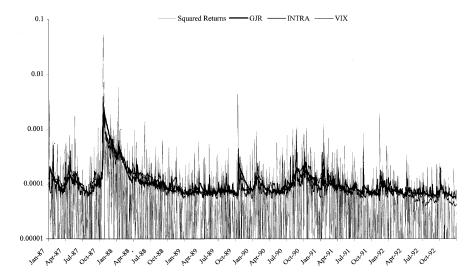


Fig. 1. In-sample comparison of conditional variance forecasts of squared returns (logarithmic scale) 2 January 1987–31 December 1992.

in daily returns (labelled GJR in Fig. 1), intraday returns (labelled INTRA) and implied volatilities (labelled VIX). The values of the four time series are plotted on a logarithmic scale truncated at 10^{-5} . It can be seen that the squared returns are a very noisy series and that the three conditional variances move closely together.

4.2. Out-of-sample forecasting

The out-of-sample accuracy of volatility forecasts is compared from 4 January 1993 to 31 December 1999. Fig. 2(a) provides similar information to Fig. 1 and shows the one-step ahead *GJR*, *INTRA* and *VIX* forecasts compared with the realised volatility defined by squared returns. It is now possible to see periods when the three forecasts are some way apart and other periods when they are very similar. Fig. 2(b) provides comparisons when there is much less noise in the series of realised volatility. The quantity predicted is now the sum of *INTRA* values over 20 days. It is seen that volatility was low for the first 4 years on the plot (1993–1996), higher in 1997 and then much higher in 1998 and 1999. The *GJR* forecasts are the most sensitive to large movements in the index and occasional spikes in the forecasts can be seen. The *VIX* forecasts can be seen to track the quantity forecasted more closely than the other forecasts during some periods, for example throughout 1993 at the left-hand side of the figure. Fig. 3 plots the sums of *INTRA* values over 20 days against the *VIX* forecasts, for which R^2 equals 0.57.

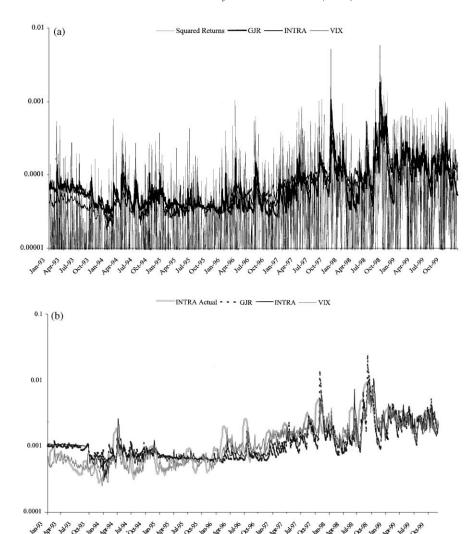


Fig. 2. (a) Out-of-sample comparison of 1-day ahead forecasts of squared returns (logarithmic scale) 4 January 1993–31 December 1999. (b) Actual values and forecasts of realised volatility over 20 days, measured using intraday returns (logarithmic scale) 4 January 1993–31 December 1999.

4.2.1. Relative accuracy of the forecasts

Tables 2 and 3 summarise the out-of-sample accuracy of the volatility forecasts. Each table provides information for forecasts 1, 5, 10 and 20 days ahead of the two measures of realised volatility. Both measures are sums of squares, summing squared daily excess returns for one measure and squared 5-min and overnight returns for the other measure. Table 2 reports the

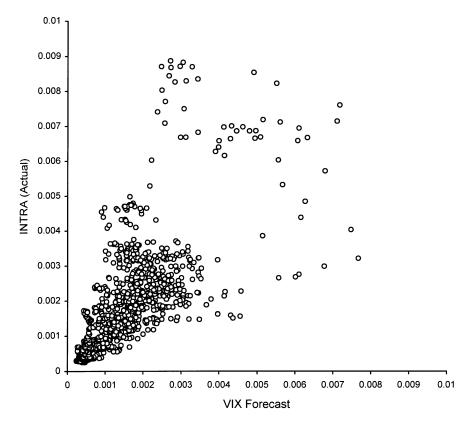


Fig. 3. Scatter plot of actual values and VIX forecasts of realised volatility over 20 days, measured using intraday returns 4 January 1993–31 December 1999.

accuracy measure P defined by Eq. (12). This proportion of explained variability is a linear function of the mean square error of forecasts, with higher values corresponding to more accurate forecasts. Table 3 reports the (multiple) squared correlation R^2 from regressions of realised volatility on one or more forecasts. These R^2 statistics provide useful information about the information content of the various forecasts.

Tables 2 and 3 unequivocally support the conclusion that the implied volatility index VIX provides more accurate forecasts of realised volatility than the historic volatility HV, the low-frequency ARCH forecast GJR given by the model of Glosten et al. (1993) and the intraday volatility INTRA calculated from 5-min and overnight returns. The VIX forecasts are more accurate than the other univariate forecasts for all 16 combinations of realised volatility measure (sums of squared excess returns or INTRA), forecast horizon (1, 5, 10 or 20) and summary statistic $(P \text{ or } R^2)$.

Table 2
The relative accuracy of volatility forecasts from January 1993 to December 1999

Realised volatility $y_{T,N}$ is defined by either $\sum_{j=1}^{N} (r_{T+j} - \mu)^2$ or $\sum_{j=1}^{N} INTRA_{T+j}$. Forecasts $x_{T,N}$ made at time T are obtained from historic volatility estimates HV, the GJR model, the INTRA series or the VIX series. These forecasts are defined in Section 3.2 and make use of Eqs. (4) (HV), (5) (GJR), (7) (VIX) and (8) (INTRA). The accuracy of forecasts is measured by the proportion of explained variability,

$$P = 1 - \frac{\sum_{T=s}^{n-N} (y_{T,N} - x_{T,N})^2}{\sum_{T=s}^{n-N} (y_{T,N} - \bar{y})^2},$$

which is a linear function of the mean square error of the forecasts. Values of P are calculated from 1769 - N forecasts.

A. Values of P for forecasts of sums of squared excess returns								
Forecast	N = 1	N=5	N = 10	N = 20				
HV	0.037	0.089	0.112	0.128				
GJR	0.106	0.085	0.016	0.013				
INTRA	0.099	0.204	0.214	0.250				
VIX	0.115	0.239	0.297	0.348				
B. Values of P	for forecasts of sum	s of the intraday vo	latility measure INTI	RA				
HV	0.167	0.243	0.255	0.255				
GJR	0.375	0.289	0.169	0.181				
INTRA	0.383	0.494	0.455	0.465				
VIX	0.401	0.533	0.534	0.545				

The proportions of explained variability P given in Table 2 can be summarised by five remarks. First, VIX always has the highest value. ⁶ Second, GJR and INTRA have similar values when forecasting for the next day. Third, INTRA ranks second to VIX when forecasting five or more days into the future. Fourth, the values of P generally increase as the horizon N increases, with the exception of the GJR forecasts. Fifth, the values of P are much higher for predictions of INTRA than for predictions of squared daily excess returns, as anticipated from the work of Andersen and Bollerslev (1998). Changing the target to be forecast from squared daily excess returns to sums of squared intraday returns approximately quadruples the proportions P for one-day ahead forecasts and more than doubles them for 5-day ahead forecasts.

The rows of Table 3 are arranged in the order that produces columns of ranked values of R^2 for N = 5, 10, 20 and almost a ranked column when N is 1. It is seen first that the variance of the 100 previous daily returns is less informative than all other forecasts. Second, INTRA has a higher statistic

 $^{^6}$ However, there are few differences in the values of P that are statistically significant at conventional levels.

Table 3
Correlations and multiple correlations for regressions of realised volatility on volatility forecasts from January 1993 to December 1999

Realised volatility is defined by either $\sum_{j=1}^{N} (r_{T+j} - \mu)^2$ or $\sum_{j=1}^{N} INTRA_{T+j}$. Forecasts made at time T are obtained from historic volatility estimates HV, the GJR model, the INTRA series or the VIX series. These forecasts are defined in Section 3.2 and make use of Eqs. (4) (HV), (5) (GJR), (7) (VIX) and (8) (INTRA). The table documents the squared correlation R^2 from regressions of realised volatility on one or more forecasts. Values of R^2 are calculated from 1769 - N forecasts.

A. Values of R ² for forec Explanatory variables		of squared excess re $N = 5$	turns $N = 10$	N = 20				
HV	0.043	0.111	0.151	0.197				
GJR	0.118	0.181	0.189	0.223				
INTRA	0.099	0.212	0.238	0.285				
GJR & INTRA	0.119	0.217	0.240	0.287				
VIX	0.129	0.249	0.304	0.356				
VIX & INTRA	0.129	0.250	0.304	0.356				
VIX & GJR	0.139	0.253	0.304	0.356				
VIX, GJR & INTRA	0.144	0.253	0.304	0.356				
B. Values of R^2 for forecasts of sums of the intraday volatility measure INTRA								
HV	0.185	0.282	0.309	0.335				
GJR	0.423	0.449	0.395	0.403				
INTRA	0.385	0.506	0.482	0.499				
GJR & INTRA	0.443	0.525	0.490	0.504				
VIX	0.445	0.567	0.559	0.569				
VIX & INTRA	0.448	0.575	0.563	0.576				
VIX & GJR	0.491	0.586	0.564	0.576				
VIX, GJR & INTRA	0.495	0.586	0.565	0.577				

than GJR, except when N is 1, as also occurs in Table 2. Third, linear combinations of GJR and INTRA are less informative than VIX. Fourth, combinations of VIX and other forecasts are apparently only more informative than VIX alone when forecasting 1 or 5 days ahead using VIX and GJR. The lack of incremental information is striking for forecasts 10 and 20 days ahead; the value of R^2 increases by less than 0.001 when the first realised volatility measure is regressed on VIX, GJR and INTRA instead of VIX alone, whilst the increase is from 0.559 to 0.565 for the second measure when N equals 10 and from 0.569 to 0.577 when N equals 20. The superior performance of VIX as the forecast horizon increases is consistent with its design as a measure of market expectations over 22 trading days. Fifth, the values of R^2 generally increase as N increases and the values are higher, often much higher, when realised volatility is measured using intraday returns.

The differences between the two performance measures, $R^2 - P$, are generally small. In particular, they are always less than 0.05 for the VIX forecasts

thus indicating that a linear function of these forecasts, optimised ex post, is not much more accurate than the ex ante forecasts. However, the differences $R^2 - P$ are substantial for the GJR forecasts and this is a consequence of the low values of P when forecasting 10 or 20 days ahead.

4.2.2. Properties of univariate forecast errors

Tests for bias in the forecasts of squared excess returns find no evidence of bias, for all horizons N that have been considered. There is, however, statistically significant evidence that the forecasts of future INTRA values are downward biased for the out-of-sample period. For example, the average value of the INTRA forecast obtained from VIX information is 86% of the average value of INTRA, for each of the four forecasting horizons; t-tests for a zero mean applied to non-overlapping forecast errors range from 3.02 to 4.09. The observed level of bias is consistent with an overall increase in estimates \hat{c}_T of the terms c_T defined by Eq. (9), from 0.6 in 1993 to 0.8 in 1999, that is consistent with a decrease in the dependence between consecutive intraday returns. As the estimates \hat{c}_T use information from the previous 4 years they are, with hindsight, less than the parameters c_T that they estimate and hence the forecasts are on average less than the intraday volatility that they predict.

The values of R^2 in Table 3 imply that there is not much incremental predictive information in the GJR and INTRA forecasts, and that any such information is to be found at the shortest forecasting horizons. The increases in R^2 when GJR and INTRA are both added to the regressions that only employ VIX are 5.0%, 1.9%, 0.6% and 0.8% when predicting realised volatility measured by INTRA at horizons 1, 5, 10 and 20, respectively; the increases are much less when predicting squared excess returns and their sums. Consequently, we only discuss the incremental predictive information beyond VIX for one-step ahead forecasts. Let GJRf, VIXf and INTRAf refer to the forecasts of the next squared excess return defined by Eqs. (5), (7) and (8), respectively. A heteroscedasticity-adjusted regression of the forecast errors from VIX on the three sets of forecasts then gives

$$(r_{T+1} - \mu)^2 - VIXf_T = a + 0.16VIXf_T + 0.27GJRf_T$$
$$-0.23INTRAf_T + \text{residual}$$

with standard errors 0.16, 0.13 and 0.15 for the three forecasts and a negative intercept a. The R^2 of this regression is only 0.4% with an F-statistic equal to 2.37 and a p-value of 7%. The standard errors ignore the sampling error in the parameters that are used to calculate the forecasts and hence they underestimate the correct standard errors, see West (2001). Nevertheless, it can be concluded that the evidence for incremental forecasting information

in daily and intraday returns is insignificant when predicting squared daily returns.

The forecasts of $INTRA_{T+1}$ are \hat{c}_T multiplied by the forecasts of the squared excess return. They provide the estimated regression

$$INTRA_{T+1} - \hat{c}_T VIX f_T = a - 0.02 \hat{c}_T VIX f_T + 0.42 \hat{c}_T GJR f_T$$

$$-0.12 \hat{c}_T INTRA f_T + \text{residual}.$$

The forecast errors now vary significantly with the forecasts. The R^2 of the regression is 3.1%, the F-statistic equals 18.85 and the constant a has a t-ratio of -5.45 reflecting the bias in the forecasts discussed above. The standard errors of the three forecasts are 0.08, 0.06 and 0.07 with t-ratios of -0.27 for VIXf, 6.54 for GJRf and -1.67 for INTRAf. These t-ratios must be interpreted cautiously because the standard errors may be misleading. The high t-ratio for GJRf compared with that for INTRAf is counterintuitive. The evidence for some incremental information is more intuitive and can be attributed to the mismatch between the 1-day ahead forecast horizon and the 1-month horizon that defines VIX.

5. Conclusions

Previous studies of low-frequency (daily or weekly) index returns and implied volatilities have produced conflicting conclusions about the informational efficiency of the S&P 100 options market. Our in-sample analysis of low-frequency data using ARCH models finds no evidence for incremental information in daily index returns beyond that provided by the *VIX* index of implied volatilities. This conclusion is in agreement with the recent evidence of Christensen and Prabhala (1998) and Fleming (1998). The extension here of the historic information set to include high-frequency (5-min) returns shows, furthermore, that there appears to be only minor incremental information in high-frequency returns; this information is almost subsumed by implied volatilities.

Out-of-sample comparisons of volatility forecasts show that *VIX* provides more accurate forecasts than either low- or high-frequency index returns, regardless of the definition of realised volatility and the horizon of the forecasts. A mixture of the *VIX* forecasts and forecasts from index returns shows that there is probably some incremental forecasting information in daily returns when forecasting 1-day ahead. For the longer forecast horizon of 20 trading days, that closely matches the life of the hypothetical option that defines *VIX*, daily observations of *VIX* contain all the relevant forecasting information.

Our results for equity volatility confirm the conclusion of Andersen and Bollerslev (1998), obtained for foreign exchange volatility, that intraday

returns provide much more accurate measures of realised volatility than daily returns as well as providing more accurate forecasts. Although high-frequency returns are highly informative about future volatility, we have found that implied volatilities were more informative throughout our sample period from 1987 to 1999.

Acknowledgements

We are particularly grateful to two anonymous referees, Francis Diebold, Kenneth West, Dan Galai, Nagpurnanand Prabhala and Jan Jakobsen for their constructive comments on earlier versions of this paper. We have also benefited from the comments of participants at meetings held by the Bolsa de Derivados do Porto (Oporto, 1999), the London School of Economics (London, 1999), the American Finance Association (Boston, 2000), the Financial Management Association (Edinburgh, 2000) and the Bachelier Finance Society (Paris, 2000). We thank Robert Whaley for *VIX* data.

References

- Akgiray, V., 1989. Conditional heteroskedasticity in time series of stock returns: evidence and forecasts. Journal of Business 62, 55–80.
- Andersen, T.G., Bollerslev, T., 1998. Answering the skeptics: yes standard volatility models do provide accurate forecasts. International Economic Review 39, 885–905.
- Andersen, T.G., Bollerslev, T., Diebold, F.X., Ebens, H., 2001a. The distribution of realized stock return volatility. Journal of Financial Economics 61, 43–76.
- Andersen, T.G., Bollerslev, T., Diebold, F.X., Labys, P., 2001b. The distribution of exchange rate volatility. Journal of the American Statistical Association 96, 42–55.
- Bera, A.K., Higgins, M.L., 1993. ARCH models: properties, estimation and testing. Journal of Economic Surveys 7, 305–362.
- Blair, B.J., Poon, S.H., Taylor, S.J., 2001a. Asymmetric and crash effects in stock volatility. Applied Financial Economics, forthcoming.
- Blair, B.J., Poon, S.H., Taylor, S.J., 2001b. Modelling S&P 100 volatility: the information content of stock returns. Journal of Banking and Finance 25, 1665–1679.
- Bollerslev, T., 1987. A conditional heteroskedastic time series model for speculative prices and rates of returns. Review of Economics and Statistics 69, 542–547.
- Bollerslev, T., Chou, R.Y., Kroner, K.P., 1992. ARCH modeling in finance: a review of theory and empirical evidence. Journal of Econometrics 52, 5–59.
- Bollerslev, T., Engle, R.F., Nelson, D.B., 1994. ARCH models. In: Handbook of Econometrics, Vol. IV. North-Holland Amsterdam, pp. 2959–3037.
- Bollerslev, T., Wooldridge, J.M., 1992. Quasi-maximum likelihood estimation and inference in dynamic models with time-varying covariances. Econometric Reviews 11, 143–179.
- Brailsford, T.J., Faff, R.W., 1996. An evaluation of volatility forecasting techniques. Journal of Banking and Finance 20, 419–438.
- Canina, L., Figlewski, S., 1993. The informational content of implied volatility. Review of Financial Studies 6, 659–681.

- Chiras, D.P., Manaster, S., 1978. The information content of option prices and a test for market efficiency. Journal of Financial Economics 6, 213–234.
- Christensen, B.J., Prabhala, N.R., 1998. The relation between implied and realized volatility. Journal of Financial Economics 50, 125–150.
- Day, T.E., Lewis, C.M., 1992. Stock market volatility and the informational content of stock index options. Journal of Econometrics 52, 267–287.
- Dimson, E., Marsh, P., 1990. Volatility forecasting without data-snooping. Journal of Banking and Finance 14, 399-421.
- Ebens, H., 1999. Realized stock volatility. Working Paper, Department of Economics, John Hopkins University.
- Figlewski, S., 1997. Forecasting volatility. Financial Markets. Institutions and Instruments 6, 1–88.
- Fleming, J., 1998. The quality of market volatility forecasts implied by S&P 100 index option prices. Journal of Empirical Finance 5, 317–345.
- Fleming, J., Ostdiek, B., Whaley, R.E., 1995. Predicting stock market volatility: a new measure. Journal of Futures Markets 15, 265–302.
- Franses, P.H., Van Dijk, D., 1995. Forecasting stock market volatility using (non-linear) GARCH models. Journal of Forecasting 15, 229–235.
- Glosten, L.R., Jagannathan, R., Runkle, D.E., 1993. On the relation between the expected value and the volatility of the nominal excess return on stocks. Journal of Finance 48, 1779–1801.
- Harvey, C.R., Whaley, R.E., 1992. Dividends and S&P 100 index options. Journal of Futures Markets 12, 123–137.
- Heynen, R.C., Kat, H.M., 1994. Volatility prediction: a comparison of stochastic volatility, GARCH(1,1) and EGARCH(1,1) models. Journal of Derivatives 2 (2), 50–65.
- Jorion, P., 1995. Predicting volatility in the foreign exchange market. Journal of Finance 50, 507–528.
- Latane, H.A., Rendleman, R.J., 1976. Standard deviations of stock price ratios implied in option prices. Journal of Finance 31, 369–381.
- Nelson, D.B., 1991. Conditional heteroskedasticity in asset returns: a new approach. Econometrica 59, 347–370.
- Nelson, D.B., 1992. Filtering and forecasting with misspecified ARCH models I: getting the right variance with the wrong model. Journal of Econometrics 52, 61–90.
- Nelson, D.B., Foster, D.P., 1995. Filtering and forecasting with misspecified ARCH models II: making the right forecast with the wrong model. Journal of Econometrics 67, 303–335.
- Taylor, S.J., Xu, X., 1997. The incremental volatility information in one million foreign exchange quotations. Journal of Empirical Finance 4, 317–340.
- West, K.D., 2001. Encompassing tests when no model is encompassing. Journal of Econometrics, this issue.
- Xu, X., Taylor, S.J., 1995. Conditional volatility and the informational efficiency of the PHLX currency options markets. Journal of Banking and Finance 19, 803–821.