Code Library short

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Contents

1 data structure

1.1 access

```
//记得随手啊……亲……down
//时记得优先检查debugup/down/select

inline void rot(int id,int tp)
{
    static int k;
    k=pre[id];
    nxt[k][tp^1]=nxt[id][tp];
    if(nxt[id][tp])
        pre[nxt[id][tp]]=k;
    if(pre[k])
        nxt[pre[k]][k==nxt[pre[k]][1]]=id;
    pre[id]=pre[k];
    nxt[id][tp]=k;
    pre[k]=id;
    up(k);
    up(id);
}

inline void down(int id) //记得随手啊……亲……down
{
    static int i;
    if(rev[id])
    {
        rev[id]=false;
        std::swap(nxt[id][0],nxt[id][1]);
        for(i=0;i<2;++i)
            if(nxt[id][i])
                rev[nxt[id][i]]^=true;
    }
}

int freshen(int id)
{
    int re(id);
    if(pre[id])
        re=freshen(pre[id]);
    down(id);
    return re;
}

inline void splay(int id)//记得随手啊……亲……down
{
    static int rt;
    if(id!=(rt=freshen(id)))
        for(std::swap(fa[id],fa[rt]);pre[id];rot(id,id==nxt[pre[id]][0]));
    /* another faster methond:
    if(id!=rt)
    {
        std::swap(fa[id],fa[rt]);
        do
        {
            rt=pre[id];
            if(pre[rt])
            {
                k=(nxt[pre[rt]][0]==rt);
                if(nxt[rt][k]==id)
                    rot(id,k^1);
                else
                    rot(rt,k);
                rot(id,k);
            }
            else
                rot(id,id==nxt[rt][0]);
        }
        while(pre[id]);
    }
    */
}
```

1.2 Binary Indexed tree

```
inline void update(int pos,const int &val)
{
    while(pos<MAXX)
    {
        tree[pos]+=val;
        pos+=pos&-pos;
    }
}

inline int read(int pos)
{
    int re(0);
    while(pos>0)
    {
        re+=tree[pos];
        pos-=pos&-pos;
    }
    return re;
}
```

1.3 Leftist tree

```
int merge(int a,int b)
{
    if(!a)
        return b;
    if(!b)
        return a;
    if(val[a]<val[b]) // max-heap
        std::swap(a,b);
    if(rand()&1)
        r[a]=merge(r[a],b);
    else
        l[a]=merge(l[a],b);
    fa[l[a]]=fa[r[a]]=a; // set a as father of its sons
    return a;
}
```

1.4 Size Blanced Tree

```
template<class Tp>class sbt
{
    public:
        inline void init()
        {
            rt=cnt=l[0]=r[0]=sz[0]=0;
        }
        inline void ins(const Tp &a)
        {
            ins(rt,a);
        }
        inline void del(const Tp &a)
        {
            del(rt,a);
        }
        inline Tp pred(const Tp &a)
        {
            return pred(rt,a);
        }
        inline Tp succ(const Tp &a)
        {
            return succ(rt,a);
        }
        inline void delsmall(const Tp &a)
        {
            dels(rt,a);
        }
        inline Tp delsel(int a)
        {
            return delsel(rt,a);
        }
    private:
        int cnt,rt,l[MAXX],r[MAXX],sz[MAXX];
        Tp val[MAXX];
        inline void rro(int &pos)
        {
            int k(l[pos]);
            l[pos]=r[k];
            r[k]=pos;
            sz[k]=sz[pos];
            sz[pos]=sz[l[pos]]+sz[r[pos]]+1;
            pos=k;
        }
        inline void lro(int &pos)
        {
            int k(r[pos]);
            r[pos]=l[k];
            l[k]=pos;
            sz[k]=sz[pos];
            sz[pos]=sz[l[pos]]+sz[r[pos]]+1;
            pos=k;
        }
        inline void mt(int &pos,bool flag)
        {
            if(!pos)
                return;
            if(flag)
                if(sz[r[r[pos]]]>sz[l[pos]])
                    lro(pos);
                else
                    if(sz[l[r[pos]]]>sz[l[pos]])
                    {
                        rro(r[pos]);
                        lro(pos);
                    }
                    else
                        return;
            else
                if(sz[l[l[pos]]]>sz[r[pos]])
                    rro(pos);
                else
                    if(sz[r[l[pos]]]>sz[r[pos]])
                    {
                        lro(l[pos]);
                        rro(pos);
                    }
                    else
                        return;
            mt(l[pos],false);
            mt(r[pos],true);
            mt(pos,false);
            mt(pos,true);
        }
        void ins(int &pos,const Tp &a)
        {
            if(pos)
            {
                ++sz[pos];
                if(a<val[pos])
                    ins(l[pos],a);
                else
                    ins(r[pos],a);
                mt(pos,a>=val[pos]);
                return;
            }
            pos=++cnt;
            l[pos]=r[pos]=0;
            val[pos]=a;
            sz[pos]=1;
        }
        Tp del(int &pos,const Tp &a)
        {
            --sz[pos];
            if(val[pos]==a || (a<val[pos] && !l[pos]) || (a>val[pos] && !r[pos]))
            {
                Tp ret(val[pos]);
                if(!l[pos] || !r[pos])
                    pos=l[pos]+r[pos];
                else
                    val[pos]=del(l[pos],val[pos]+1);
                return ret;
            }
            else
                if(a<val[pos])
                    return del(l[pos],a);
                else
                    return del(r[pos],a);
        }
        void dels(int &pos,const Tp &v)
        {
            if(!pos)
                return;
            if(val[pos]<v)
            {
                pos=r[pos];
                dels(pos,v);
                return;
            }
            dels(l[pos],v);
            sz[pos]=1+sz[l[pos]]+sz[r[pos]];
        }
        Tp delsel(int &pos,int k)
```

```
        {
            --sz[pos];
            if(sz[l[pos]]+1==k)
            {
                Tp re(val[pos]);
                if(!l[pos] || !r[pos])
                    pos=l[pos]+r[pos];
                else
                    val[pos]=del(l[pos],val[pos]+1);
                return re;
            }
            if(k>sz[l[pos]])
                return delsel(r[pos],k-1-sz[l[pos]]);
            return delsel(l[pos],k);
        }
};
```

1.5 Sparse Table - rectangle

```
#include<iostream>
#include<cstdio>
#include<algorithm>

#define MAXX 310

int mat[MAXX][MAXX];
int table[9][9][MAXX][MAXX];
int n;
short lg[MAXX];

int main()
{
    for(int i(2);i<MAXX;++i)
        lg[i]=lg[i>>1]+1;
    int T;
    std::cin >> T;
    while (T--)
    {
        std::cin >> n;
        for (int i = 0; i < n; ++i)
            for (int j = 0; j < n; ++j)
            {
                std::cin >> mat[i][j];
                table[0][0][i][j] = mat[i][j];
            }

        // 从小到大计算，保证后来用到的都已经计算过
        for(int i=0;i<=lg[n];++i) // width
        {
            for(int j=0;j<=lg[n];++j) //height
            {
                if(i==0&& j==0)
                    continue;
                for(int ii=0;ii+(1<<j)<=n;++ii)
                    for(int jj=0;jj+(1<<i)<=n;++jj)
                        if(i==0)
                            table[i][j][ii][jj]=std::min(table[i][j-1][ii][jj],table[i][j
                                -1][ii+(1<<(j-1))][jj]);
                        else
                            table[i][j][ii][jj]=std::min(table[i-1][j][ii][jj],table[i-1][
                                j][ii][jj+(1<<(i-1))]);
            }
        }
        long long N;
        std::cin >> N;
        int r1, c1, r2, c2;
        for (int i = 0; i < N; ++i)
        {
            scanf("%d%d%d%d",&r1,&c1,&r2,&c2);
            --r1;
            --c1;
            --r2;
            --c2;
            int w=lg[c2-c1+1];
            int h=lg[r2-r1+1];
            printf("%d\n",std::min(table[w][h][r1][c1],std::min(table[w][h][r1][c2-(1<<w)
                +1],std::min(table[w][h][r2-(1<<h)+1][c1],table[w][h][r2-(1<<h)+1][c2
                -(1<<w)+1]))));
        }
    }
    return 0;
}
```

1.6 Sparse Table - square

```
int num[MAXX][MAXX],max[MAXX][MAXX][10];
short lg[MAXX];

int main()
{
    for(i=2;i<MAXX;++i)
        lg[i]=lg[i>>1]+1;
    scanf("%d%d",&n,&q);
    for(i=0;i<n;++i)
        for(j=0;j<n;++j)
        {
            scanf("%d",num[i]+j);
            max[i][j][0]=num[i][j];
        }
    for(k=1;k<=lg[n];++k)
    {
        l=n+1-(1<<k);
        for(i=0;i<l;++i)
            for(j=0;j<l;++j)
                max[i][j][k]=std::max(std::max(max[i][j][k-1],max[i+(1<<(k-1))][j][k-1]),
                    std::max(max[i][j+(1<<(k-1))][k-1],max[i+(1<<(k-1))][j+(1<<(k-1))][k
                    -1]));
    }
    printf("Case %d:\n",t);
    while(q--)
    {
        scanf("%d%d%d",&i,&j,&l);
        --i;
        --j;
        k=lg[l];
        printf("%d\n",std::max(std::max(max[i][j][k],max[i][j+l-(1<<k)][k]),std::max(max[
            i+l-(1<<k)][j][k],max[i+l-(1<<k)][j+l-(1<<k)][k])));
    }
}
```

1.7 Sparse Table

```
int num[MAXX],min[MAXX][20];
int lg[MAXX];


int main()
{
    for(i=2;i<MAXX;++i)
        lg[i]=lg[i>>1]+1;
    scanf("%d%d",&n,&q);
    for(i=1;i<=n;++i)
    {
        scanf("%d",num+i);
        min[i][0]=num[i];
    }
    for(j=1;j<=lg[n];++j)
    {
        l=n+1-(1<<j);
        j_=j-1;
        j__=(1<<j_);
        for(i=1;i<=l;++i)
            min[i][j]=std::min(min[i][j_],min[i+j__][j_]);
    }
    printf("Case %d:\n",t);
    while(q--)
    {
        scanf("%d%d",&i,&j);
        k=lg[j-i+1];
        printf("%d\n",std::min(min[i][k],min[j-(1<<k)+1][k]));
    }
}
```

1.8 Treap

```
#include<cstdlib>
#include<ctime>
#include<cstring>

struct node
{
    node *ch[2];
    int sz,val,key;
    node(){memset(this,0,sizeof(node));}
    node(int a);
}*null;

node::node(int a):sz(1),val(a),key(rand()-1){ch[0]=ch[1]=null;}

class Treap
{
    inline void up(node *pos)
    {
        pos->sz=pos->ch[0]->sz+pos->ch[1]->sz+1;
    }
    inline void rot(node *&pos,int tp)
    {
        node *k(pos->ch[tp]);
        pos->ch[tp]=k->ch[tp^1];
        k->ch[tp^1]=pos;
        up(pos);
        up(k);
        pos=k;
    }

    void insert(node *&pos,int val)
    {
        if(pos!=null)
        {
            int t(val>=pos->val);
            insert(pos->ch[t],val);
            if(pos->ch[t]->key<pos->key)
                rot(pos,t);
            else
                up(pos);
            return;
        }
        pos=new node(val);
    }
    void rec(node *pos)
    {
        if(pos!=null)
        {
            rec(pos->ch[0]);
            rec(pos->ch[1]);
            delete pos;
        }
    }
    void del(node *&pos,int val)
    {
        if(pos!=null)
        {
            if(pos->val==val)
            {
                int t(pos->ch[1]->key<pos->ch[0]->key);
                if(pos->ch[t]==null)
                {
                    delete pos;
                    pos=null;
                    return;
                }
                rot(pos,t);
                del(pos->ch[t^1],val);
            }
            else
                del(pos->ch[val>pos->val],val);
            up(pos);
        }
    }
    public:
    node *rt;

    Treap():rt(null){}
    inline void insert(int val)
    {
        insert(rt,val);
    }
    inline void reset()
    {
        rec(rt);
        rt=null;
    }
    inline void del(int val)
    {
        del(rt,val);
    }
}treap[MAXX];

init:
```

```
{
    srand(time(0));
    null=new node();
    null->val=0xc0c0c0c0;
    null->sz=0;
    null->key=RAND_MAX;
    null->ch[0]=null->ch[1]=null;
    for(i=0;i<MAXX;++i)
        treap[i].rt=null;
}
```

2 dynamic programming

2.1 knapsack problem

```
multiple-choice knapsack problem:

for 所有的组k
    for v=V..0
        for 所有的属于组ik
            f[v]=max{f[v],f[v-c[i]]+w[i]}
```

2.2 LCIS

```
#include<cstdio>
#include<cstring>
#include<vector>

#define MAXX 1111

int T;
int n,m,p,i,j,k;
std::vector<int>the[2];
int dp[MAXX],path[MAXX];
int ans[MAXX];

int main()
{
    the[0].reserve(MAXX);
    the[1].reserve(MAXX);
    {
        scanf("%d",&n);
        the[0].resize(n);
        for(i=0;i<n;++i)
            scanf("%d",&the[0][i]);
        scanf("%d",&m);
        the[1].resize(m);
        for(i=0;i<m;++i)
            scanf("%d",&the[1][i]);
        memset(dp,0,sizeof dp);
        for(i=0;i<the[0].size();++i)
        {
            n=0;
            p=-1;
            for(j=0;j<the[1].size();++j)
            {
                if(the[0][i]==the[1][j] && n+1>dp[j])
                {
                    dp[j]=n+1;
                    path[j]=p;
                }
                if(the[1][j]<the[0][i] && n<dp[j])
                {
                    n=dp[j];
                    p=j;
                }
            }
        }
        n=0;
        p=-1;
        for(i=0;i<the[1].size();++i)
            if(dp[i]>n)
                n=dp[p=i];
        printf("%d\n",n);
        for(i=n-1;i>=0;--i)
        {
            ans[i]=the[1][p];
            p=path[p];
        }
        for(i=0;i<n;++i)
            printf("%d ",ans[i]);
        puts("");
    }
    return 0;
}
```

2.3 LCS

```
#include<cstdio>
#include<algorithm>
#include<vector>

#define MAXX 111
#define N 128

std::vector<char>the[2];
std::vector<int>dp(MAXX),p[N];

int i,j,k;
char buf[MAXX];
int t;

int main()
{
    the[0].reserve(MAXX);
    the[1].reserve(MAXX);
    while(gets(buf),buf[0]!='#')
    {
        the[0].resize(0);
        for(i=0;buf[i];++i)
            the[0].push_back(buf[i]);
        the[1].resize(0);
        gets(buf);
        for(i=0;buf[i];++i)
            the[1].push_back(buf[i]);
        for(i=0;i<N;++i)
            p[i].resize(0);
        for(i=0;i<the[1].size();++i)
            p[the[1][i]].push_back(i);
        dp.resize(1);
        dp[0]=-1;
        for(i=0;i<the[0].size();++i)
            for(j=p[the[0][i]].size()-1;j>=0;--j)
            {
                k=p[the[0][i]][j];
                if(k>dp.back())
                    dp.push_back(k);
                else
                    *std::lower_bound(dp.begin(),dp.end(),k)=k;
            }
        printf("Case #%d: you can visit at most %d cities.\n",++t,dp.size()-1);
    }
    return 0;
}
```

3 geometry

3.1 3D

```
struct pv
{
  double x,y,z;
  pv() {}
  pv(double xx,double yy,double zz):x(xx),y(yy),z(zz) {}
  pv operator -(const pv& b)const
  {
    return pv(x-b.x,y-b.y,z-b.z);
  }
  pv operator *(const pv& b)const
  {
    return pv(y*b.z-z*b.y,z*b.x-x*b.z,x*b.y-y*b.x);
  }
  double operator &(const pv& b)const
  {
    return x*b.x+y*b.y+z*b.z;
  }
};

//模
double Norm(pv p)
{
  return sqrt(p&p);
}

//绕单位向量 V 旋转 theta 角度
pv Trans(pv pa,pv V,double theta)
{
    double s = sin(theta);
    double c = cos(theta);
    double x,y,z;
    x = V.x;
    y = V.y;
    z = V.z;
    pv pp =
        pv(
                (x*x*(1-c)+c)*pa.x+(x*y*(1-c)-z*s)*pa.y+(x*z*(1-c)+y*s)*pa.z,
                (y*x*(1-c)+z*s)*pa.x+(y*y*(1-c)+c)*pa.y+(y*z*(1-c)-x*s)*pa.z,
                (x*z*(1-c)-y*s)*pa.x+(y*z*(1-c)+x*s)*pa.y+(z*z*(1-c)+c)*pa.z
          );
    return pp;
}

//经纬度转换

x=r*sin()*cos();
y=r*sin()*sin();
z=r*cos();

r=sqrt(x*2+y*2+z*2);//??
r=sqrt(x^2+y^2+z^2);//??

=atan(y/x);
=acos(z/r);

r∞[0,]
[0,2]
[0,]

lat1 [-/2,/2]
lng1 [-,]

pv getpv(double lat,double lng,double r)
{
  lat += pi/2;
  lng += pi;
  return
    pv(r*sin(lat)*cos(lng),r*sin(lat)*sin(lng),r*cos(lat));
}

//经纬度球面距离

#include<cstdio>
#include<cmath>

#define MAXX 1111

char buf[MAXX];
const double r=6875.0/2,pi=acos(-1.0);
double a,b,c,x1,x2,y2,ans;

int main()
{
    double y1;
    while(gets(buf)!=NULL)
    {
        gets(buf);
        gets(buf);

        scanf("%lf^%lf'%lf\"_%s\n",&a,&b,&c,buf);
        x1=a+b/60+c/3600;
        x1=x1*pi/180;
        if(buf[0]=='S')
            x1=-x1;

        scanf("%s",buf);
        scanf("%lf^%lf'%lf\"_%s\n",&a,&b,&c,buf);
        y1=a+b/60+c/3600;
        y1=y1*pi/180;
        if(buf[0]=='W')
            y1=-y1;

        gets(buf);

        scanf("%lf^%lf'%lf\"_%s\n",&a,&b,&c,buf);
        x2=a+b/60+c/3600;
```

```
        x2=x2*pi/180;
        if(buf[0]=='S')
            x2=-x2;

        scanf("%s",buf);
        scanf("%lf°%lf'%lf\" %s\n",&a,&b,&c,buf);
        y2=a+b/60+c/3600;
        y2=y2*pi/180;
        if(buf[0]=='W')
            y2=-y2;

        ans=acos(cos(x1)*cos(x2)*cos(y1-y2)+sin(x1)*sin(x2))*r;
        printf("The distance to the iceberg: %.2lf miles.\n",ans);
        if(ans+0.005<100)
            puts("DANGER!");

        gets(buf);
    }
    return 0;
}

inline bool ZERO(const double &a)
{
    return fabs(a)<eps;
}

//三维向量是否为零
inline bool ZERO(pv p)
{
    return (ZERO(p.x) && ZERO(p.y) && ZERO(p.z));
}

//直线相交
bool LineIntersect(Line3D L1, Line3D L2)
{
    pv s = L1.s-L1.e;
    pv e = L2.s-L2.e;
    pv p  = s*e;
    if (ZERO(p))
        return false;     //是否平行
    p = (L2.s-L1.e)*(L1.s-L1.e);
    return ZERO(p&L2.e);          //是否共面
}

//线段相交
bool inter(pv a,pv b,pv c,pv d)
{
    pv ret = (a-b)*(c-d);
    pv t1 = (b-a)*(c-a);
    pv t2 = (b-a)*(d-a);
    pv t3 = (d-c)*(a-c);
    pv t4 = (d-c)*(b-c);
    return sgn(t1&ret)*sgn(t2&ret) < 0 && sgn(t3&ret)*sgn(t4&ret) < 0;
}

//点在直线上
bool OnLine(pv p, Line3D L)
{
    return ZERO((p-L.s)*(L.e-L.s));
}

//点在线段上
bool OnSeg(pv p, Line3D L)
{
    return (ZERO((L.s-p)*(L.e-p)) && EQ(Norm(p-L.s)+Norm(p-L.e),Norm(L.e-L.s)));
}

//点到直线距离
double Distance(pv p, Line3D L)
{
    return (Norm((p-L.s)*(L.e-L.s))/Norm(L.e-L.s));
}

//线段夹角
//范围值为  之间的弧度[0,]
double Inclination(Line3D L1, Line3D L2)
{
    pv u = L1.e - L1.s;
    pv v = L2.e - L2.s;
    return acos( (u & v) / (Norm(u)*Norm(v)) );
}
```

3.2 3DCH

```
#include<cstdio>
#include<cmath>
#include<vector>
#include<algorithm>

#define MAXX 1111
#define eps 1e-8
#define inf 1e20

struct pv
{
    double x,y,z;
    pv(){}
    pv(const double &xx,const double &yy,const double &zz):x(xx),y(yy),z(zz){}
    inline pv operator-(const pv &i)const
    {
        return pv(x-i.x,y-i.y,z-i.z);
    }
    inline pv operator*(const pv &i)const //叉积
    {
        return pv(y*i.z-z*i.y,z*i.x-x*i.z,x*i.y-y*i.x);
    }
    inline double operator^(const pv &i)const //点积
    {
        return x*i.x+y*i.y+z*i.z;
    }
    inline double len()
    {
        return sqrt(x*x+y*y+z*z);
    }
};

struct pla
{
    short a,b,c;
    bool ok;
    pla(){}
    pla(const short &aa,const short &bb,const short &cc):a(aa),b(bb),c(cc),ok(true){}
    inline void set();
    inline void print()
    {
        printf("%hd %hd %hd\n",a,b,c);
    }
};

inline void pla::set()
{
    to[a][b]=to[b][c]=to[c][a]=fac.size();
}

inline double ptof(const pv &p,const pla &f) //点面距离?
{
    return (pnt[f.b]-pnt[f.a])*(pnt[f.c]-pnt[f.a])^(p-pnt[f.a]);
}

inline double vol(const pv &a,const pv &b,const pv &c,const pv &d)//有向体积，即六面体体积*6
{
    return (b-a)*(c-a)^(d-a);
}

inline double ptof(const pv &p,const short &f) //点到号面的距离pf
{
    return fabs(vol(pnt[fac[f].a],pnt[fac[f].b],pnt[fac[f].c],p)/((pnt[fac[f].b]-pnt[fac[f].a])*(pnt[fac[f].c]-pnt[fac[f].a])).len());
}

void dfs(const short&,const short&);

void deal(const short &p,const short &a,const short &b)
{
    if(fac[to[a][b]].ok)
        if(ptof(pnt[p],fac[to[a][b]])>eps)
            dfs(p,to[a][b]);
        else
        {
            pla add(b,a,p);
            add.set();
            fac.push_back(add);
        }
}

void dfs(const short &p,const short &now)
{
    fac[now].ok=false;
    deal(p,fac[now].b,fac[now].a);
    deal(p,fac[now].c,fac[now].b);
    deal(p,fac[now].a,fac[now].c);
}

inline void make()
{
    fac.resize(0);
    if(n<4)
        return;

    for(i=1;i<n;++i)
        if((pnt[0]-pnt[i]).len()>eps)
        {
            std::swap(pnt[i],pnt[1]);
            break;
        }
    if(i==n)
        return;

    for(i=2;i<n;++i)
        if(((pnt[0]-pnt[1])*(pnt[1]-pnt[i])).len()>eps)
        {
            std::swap(pnt[i],pnt[2]);
            break;
        }
    if(i==n)
        return;

    for(i=3;i<n;++i)
        if(fabs((pnt[0]-pnt[1])*(pnt[1]-pnt[2])^(pnt[2]-pnt[i]))>eps)
        {
            std::swap(pnt[3],pnt[i]);
            break;
        }
    if(i==n)
        return;

    for(i=0;i<4;++i)
    {
        pla add((i+1)%4,(i+2)%4,(i+3)%4);
        if(ptof(pnt[i],add)>0)
            std::swap(add.c,add.b);
        add.set();
        fac.push_back(add);
    }
    for(;i<n;++i)
        for(j=0;j<fac.size();++j)
            if(fac[j].ok && ptof(pnt[i],fac[j])>eps)
            {
                dfs(i,j);
                break;
            }

    short tmp(fac.size());
    fac.resize(0);
    for(i=0;i<tmp;++i)
        if(fac[i].ok)
            fac.push_back(fac[i]);
}

inline pv gc() //重心
{
    pv re(0,0,0),o(0,0,0);
    double all(0),v;
    for(i=0;i<fac.size();++i)
    {
        v=vol(o,pnt[fac[i].a],pnt[fac[i].b],pnt[fac[i].c]);
        re+=(pnt[fac[i].a]+pnt[fac[i].b]+pnt[fac[i].c])*0.25*v;
        all+=v;
    }
    return re*(1/all);
}

inline bool same(const short &s,const short &t) //两面是否相等
{
    pv &a=pnt[fac[s].a],&b=pnt[fac[s].b],&c=pnt[fac[s].c];
    return fabs(vol(a,b,c,pnt[fac[t].a]))<eps && fabs(vol(a,b,c,pnt[fac[t].b]))<eps &&
           fabs(vol(a,b,c,pnt[fac[t].c]))<eps;
}

//表面多边形数目
inline short facetcnt()
{
```

```
    short ans=0;
    for(short i=0;i<fac.size();++i)
    {
        for(j=0;j<i;++j)
            if(same(i,j))
                break;
        if(j==i)
            ++ans;
    }
    return ans;
}

//表面三角形数目
inline short trianglecnt()
{
    return fac.size();
}

//三点构成的三角形面积*2
inline double area(const pv &a,const pv &b,const pv &c)
{
        return (b-a)*(c-a).len();
}

//表面积
inline double area()
{
    double ret(0);
    for(i=0;i<fac.size();++i)
        ret+=area(pnt[fac[i].a],pnt[fac[i].b],pnt[fac[i].c]);
    return ret/2;
}

//体积
inline double volume()
{
    pv o(0,0,0);
    double ret(0);
    for(short i(0);i<fac.size();++i)
        ret+=vol(o,pnt[fac[i].a],pnt[fac[i].b],pnt[fac[i].c]);
    return fabs(ret/6);
}
```

3.3 circle&ploy's area

```
bool InCircle(Point a,double r)
{
  return cmp(a.x*a.x+a.y*a.y,r*r) <= 0;
  //这里判断的时候 EPS 一定不要太小!!
}

double CalcArea(Point a,Point b,double r)
{
  Point p[4];
  int tot = 0;
  p[tot++] = a;

  Point tv = Point(a,b);
  Line tmp = Line(Point(0,0),Point(tv.y,-tv.x));
  Point near = LineToLine(Line(a,b),tmp);
  if (cmp(near.x*near.x+near.y*near.y,r*r) <= 0)
  {
    double A,B,C;
    A = near.x*near.x+near.y*near.y;
    C = r;
    B = C*C-A;
    double tvl = tv.x*tv.x+tv.y*tv.y;
    double tmp = sqrt(B/tvl); //这样做只用一次开根
    p[tot] = Point(near.x+tmp*tv.x,near.y+tmp*tv.y);
    if (OnSeg(Line(a,b),p[tot]) == true)  tot++;
    p[tot] = Point(near.x-tmp*tv.x,near.y-tmp*tv.y);
    if (OnSeg(Line(a,b),p[tot]) == true)  tot++;
  }
  if (tot == 3)
  {
    if (cmp(Point(p[0],p[1]).Length(),Point(p[0],p[2]).Length()) > 0)
      swap(p[1],p[2]);
  }
  p[tot++] = b;

  double res = 0.0,theta,a0,a1,sgn;
  for (int i = 0;i < tot-1;i++)
  {
    if (InCircle(p[i],r) == true && InCircle(p[i+1],r) == true)
    {
      res += 0.5*xmult(p[i],p[i+1]);
    }
    else
    {
      a0 = atan2(p[i+1].y,p[i+1].x);
      a1 = atan2(p[i].y,p[i].x);
      if (a0 < a1)  a0 += 2*pi;
      theta = a0-a1;
      if (cmp(theta,pi) >= 0) theta = 2*pi-theta;
      sgn = xmult(p[i],p[i+1])/2.0;
      if (cmp(sgn,0) < 0) theta = -theta;
      res += 0.5*r*r*theta;
    }
  }
  return res;
}

//调用

area2 = 0.0;
for (int i = 0;i < resn;i++) //遍历每条边，按照逆时针
    area2 += CalcArea(p[i],p[(i+1)%resn],r);
```

3.4 circle's area

```
//去重
{
    for (int i = 0; i < n; i++)
    {
        scanf("%lf%lf%lf",&c[i].c.x,&c[i].c.y,&c[i].r);
        del[i] = false;
    }
    for (int i = 0; i < n; i++)
        if (del[i] == false)
        {
            if (c[i].r == 0.0)
                del[i] = true;
            for (int j = 0; j < n; j++)
                if (i != j)
                    if (del[j] == false)
                        if (cmp(Point(c[i].c,c[j].c).Len()+c[i].r,c[j].r) <= 0)
                            del[i] = true;
        }
    tn = n;
    n = 0;
    for (int i = 0; i < tn; i++)
        if (del[i] == false)
            c[n++] = c[i];
}

//ans[i表示被覆盖]次的面积i
const double pi = acos(-1.0);
const double eps = 1e-8;
struct Point
{
    double x,y;
    Point(){}
    Point(double _x,double _y)
    {
        x = _x;
        y = _y;
    }
    double Length()
    {
        return sqrt(x*x+y*y);
    }
};
struct Circle
{
    Point c;
    double r;
};
struct Event
{
    double tim;
    int typ;
    Event(){}
    Event(double _tim,int _typ)
    {
        tim = _tim;
        typ = _typ;
    }
};

int cmp(const double& a,const double& b)
{
    if (fabs(a-b) < eps)     return 0;
    if (a < b)  return -1;
    return 1;
}

bool Eventcmp(const Event& a,const Event& b)
{
    return cmp(a.tim,b.tim) < 0;
}

double Area(double theta,double r)
{
    return 0.5*r*r*(theta-sin(theta));
}

double xmult(Point a,Point b)
{
    return a.x*b.y-a.y*b.x;
}

int n,cur,tote;
Circle c[1000];
double ans[1001],pre[1001],AB,AC,BC,theta,fai,a0,a1;
Event e[4000];
Point lab;

int main()
{
    while (scanf("%d",&n) != EOF)
    {
        for (int i = 0;i < n;i++)
            scanf("%lf%lf%lf",&c[i].c.x,&c[i].c.y,&c[i].r);
        for (int i = 1;i <= n;i++)
            ans[i] = 0.0;
        for (int i = 0;i < n;i++)
        {
            tote = 0;
            e[tote++] = Event(-pi,1);
            e[tote++] = Event(pi,-1);
            for (int j = 0;j < n;j++)
                if (j != i)
                {
                    lab = Point(c[j].c.x-c[i].c.x,c[j].c.y-c[i].c.y);
                    AB = lab.Length();
                    AC = c[i].r;
                    BC = c[j].r;
                    if (cmp(AB+AC,BC) <= 0)
                    {
                        e[tote++] = Event(-pi,1);
                        e[tote++] = Event(pi,-1);
                        continue;
                    }
                    if (cmp(AB+BC,AC) <= 0) continue;
                    if (cmp(AB,AC+BC) > 0)  continue;
                    theta = atan2(lab.y,lab.x);
                    fai = acos((AC*AC+AB*AB-BC*BC)/(2.0*AC*AB));
                    a0 = theta-fai;
                    if (cmp(a0,-pi) < 0)     a0 += 2*pi;
                    a1 = theta+fai;
                    if (cmp(a1,pi) > 0)  a1 -= 2*pi;
                    if (cmp(a0,a1) > 0)
                    {
                        e[tote++] = Event(a0,1);
                        e[tote++] = Event(pi,-1);
                        e[tote++] = Event(-pi,1);
                        e[tote++] = Event(a1,-1);
                    }
                    else
                    {
                        e[tote++] = Event(a0,1);
                        e[tote++] = Event(a1,-1);
                    }
                }
            sort(e,e+tote,Eventcmp);
            cur = 0;
            for (int j = 0;j < tote;j++)
            {
                if (cur != 0 && cmp(e[j].tim,pre[cur]) != 0)
                {
                    ans[cur] += Area(e[j].tim-pre[cur],c[i].r);
                    ans[cur] += xmult(Point(c[i].c.x+c[i].r*cos(pre[cur]),c[i].c.y+c[i].r*
                        sin(pre[cur])),
                        Point(c[i].c.x+c[i].r*cos(e[j].tim),c[i].c.y+c[i].r*sin(e[j].
```

```
                                tim))))/2.0;
                }
                cur += e[j].typ;
                pre[cur] = e[j].tim;
            }
        }
        for (int i = 1;i < n;i++)
            ans[i] -= ans[i+1];
        for (int i = 1;i <= n;i++)
            printf("[%d] = %.3f\n",i,ans[i]);
    }
    return 0;
}
```

3.5 circle

```
//单位圆覆盖
#include<cstdio>
#include<cmath>
#include<vector>
#include<algorithm>

#define MAXX 333
#define  eps 1e-8

struct pv
{
    double x,y;
    pv(){}
    pv(const double &xx,const double &yy):x(xx),y(yy){}
    inline pv operator-(const pv &i)const
    {
        return pv(x-i.x,y-i.y);
    }
    inline double cross(const pv &i)const
    {
        return x*i.y-y*i.x;
    }
    inline void print()
    {
        printf("%lf %lf\n",x,y);
    }
    inline double len()
    {
        return sqrt(x*x+y*y);
    }
}pnt[MAXX];

struct node
{
    double k;
    bool flag;
    node(){}
    node(const double &kk,const bool &ff):k(kk),flag(ff){}
    inline bool operator<(const node &i)const
    {
        return k<i.k;
    }
};

std::vector<node>alpha;

short n,i,j,k,l;
short ans,sum;
double R=2;
double theta,phi,d;
const double pi(acos(-1.0));

int main()
{
    alpha.reserve(MAXX<<1);
    while(scanf("%hd",&n),n)
    {
        for(i=0;i<n;++i)
            scanf("%lf %lf",&pnt[i].x,&pnt[i].y);
        ans=0;
        for(i=0;i<n;++i)
        {
            alpha.resize(0);
            for(j=0;j<n;++j)
                if(i!=j)
                {
                    if((d=(pnt[i]-pnt[j]).len())>R)
                        continue;
                    if((theta=atan2(pnt[j].y-pnt[i].y,pnt[j].x-pnt[i].x))<0)
                        theta+=2*pi;
                    phi=acos(d/R);
                    alpha.push_back(node(theta-phi,true));
                    alpha.push_back(node(theta+phi,false));
                }
            std::sort(alpha.begin(),alpha.end());
            for(j=0;j<alpha.size();++j)
            {
                if(alpha[j].flag)
                    ++sum;
                else
                    --sum;
                ans=std::max(ans,sum);
            }
        }
        printf("%hd\n",ans+1);
    }
    return 0;
}

//最小覆盖圆

#include<cstdio>
#include<cmath>

#define MAXX 511
#define eps 1e-8

struct pv
{
    double x,y;
    pv(){}
    pv(const double &xx,const double &yy):x(xx),y(yy){}
    inline pv operator-(const pv &i)const
    {
        return pv(x-i.x,y-i.y);
    }
    inline pv operator+(const pv &i)const
    {
        return pv(x+i.x,y+i.y);
    }
    inline double cross(const pv &i)const
    {
        return x*i.y-y*i.x;
    }
    inline pv operator*(const double &a)const
    {
        return pv(x*a,y*a);
    }
    inline pv operator/(const double &a)const
    {
        return pv(x/a,y/a);
    }
    inline double len()
    {
        return sqrt(x*x+y*y);
    }
}pnt[MAXX],o,t1,lt,aa,bb,cc,dd;

short n,i,j,k,l;
double r,u;

inline pv ins(const pv &a1,const pv &a2,const pv &b1,const pv &b2)
{
    t1=a2-a1;
    lt=b2-b1;
    u=(b1-a1).cross(lt)/(t1).cross(lt);
    return a1+t1*u;
}

inline pv get(const pv &a,const pv &b,const pv &c)
{
    aa=(a+b)/2;
    bb.x=aa.x-a.y+b.y;
    bb.y=aa.y+a.x-b.x;
    cc=(a+c)/2;
    dd.x=cc.x-a.y+c.y;
    dd.y=cc.y+a.x-c.x;
    return ins(aa,bb,cc,dd);
}

int main()
{
    while(scanf("%hd",&n),n)
    {
        for(i=0;i<n;++i)
            scanf("%lf %lf",&pnt[i].x,&pnt[i].y);
        o=pnt[0];
        r=0;
        for(i=1;i<n;++i)
            if((pnt[i]-o).len()>r+eps)
            {
                o=pnt[i];
                r=0;
                for(j=0;j<i;++j)
                    if((pnt[j]-o).len()>r+eps)
                    {
                        o=(pnt[i]+pnt[j])/2;
                        r=(o-pnt[j]).len();
                        for(k=0;k<j;++k)
                            if((o-pnt[k]).len()>r+eps)
                            {
                                o=get(pnt[i],pnt[j],pnt[k]);
                                r=(o-pnt[i]).len();
                            }
                    }
            }
        printf("%.2lf %.2lf %.2lf\n",o.x,o.y,r);
    }
    return 0;
}

//两原面积交
double dis(int x,int y)
{
    return sqrt((double)(x*x+y*y));
}

double area(int x1,int y1,int x2,int y2,double r1,double r2)
{
    double s=dis(x2-x1,y2-y1);
    if(r1+r2<s) return 0;
    else if(r2-r1>s) return PI*r1*r1;
    else if(r1-r2>s) return PI*r2*r2;
    double q1=acos((r1*r1+s*s-r2*r2)/(2*r1*s));
    double q2=acos((r2*r2+s*s-r1*r1)/(2*r2*s));
    return (r1*r1*q1+r2*r2*q2-r1*s*sin(q1));
}

//三角形外接圆
{
    for (int i = 0; i < 3; i++)
        scanf("%lf%lf",&p[i].x,&p[i].y);
    tp = pv((p[0].x+p[1].x)/2,(p[0].y+p[1].y)/2);
    l[0] = Line(tp,pv(tp.x-(p[1].y-p[0].y),tp.y+(p[1].x-p[0].x)));
    tp = pv((p[0].x+p[2].x)/2,(p[0].y+p[2].y)/2);
    l[1] = Line(tp,pv(tp.x-(p[2].y-p[0].y),tp.y+(p[2].x-p[0].x)));
    tp = LineToLine(l[0],l[1]);
    r = pv(tp,p[0]).Length();
    printf("(%.6f,%.6f,%.6f)\n",tp.x,tp.y,r);
}

//三角形内切圆
{
    for (int i = 0; i < 3; i++)
        scanf("%lf%lf",&p[i].x,&p[i].y);
    if (xmult(pv(p[0],p[1]),pv(p[0],p[2])) < 0)
        swap(p[1],p[2]);
    for (int i = 0; i < 3; i++)
        len[i] = pv(p[i],p[(i+1)%3]).Length();
    tr = (len[0]+len[1]+len[2])/2;
    r = sqrt((tr-len[0])*(tr-len[1])*(tr-len[2])/tr);
    for (int i = 0; i < 2; i++)
    {
        v = pv(p[i],p[i+1]);
        tv = pv(-v.y,v.x);
        tr = tv.Length();
        tv = pv(tv.x*r/tr,tv.y*r/tr);
        tp = pv(p[i].x+tv.x,p[i].y+tv.y);
        l[i].s = tp;
        tp = pv(p[i+1].x+tv.x,p[i+1].y+tv.y);
        l[i].e = tp;
    }
    tp = LineToLine(l[0],l[1]);
    printf("(%.6f,%.6f,%.6f)\n",tp.x,tp.y,r);
}
```

3.6 closest point pair

```
//演算法笔记1

struct Point {double x, y;} p[10], t[10];
bool cmpx(const Point& i, const Point& j) {return i.x < j.x;}
bool cmpy(const Point& i, const Point& j) {return i.y < j.y;}

double DnC(int L, int R)
{
    if (L >= R) return 1e9; // 沒有點、只有一個點。

    /* : 把所有點分成左右兩側，點數盡量一樣多。Divide */

    int M = (L + R) / 2;

    /* : 左側、右側分別遞迴求解。Conquer */

    double d = min(DnC(L,M), DnC(M+1,R));
    //  if (d == 0.0) return d; // 提早結束

    /* : 尋找靠近中線的點，並依座標排序。MergeYO(NlogN)。 */

    int N = 0;  // 靠近中線的點數目
    for (int i=M;    i>=L && p[M].x - p[i].x < d; --i) t[N++] = p[i];
    for (int i=M+1; i<=R && p[i].x - p[M].x < d; ++i) t[N++] = p[i];
    sort(t, t+N, cmpy); // Quicksort O(NlogN)

    /* : 尋找橫跨兩側的最近點對。MergeO(N)。 */

    for (int i=0; i<N-1; ++i)
        for (int j=1; j<=2 && i+j<N; ++j)
            d = min(d, distance(t[i], t[i+j]));

    return d;
}

double closest_pair()
{
    sort(p, p+10, cmpx);
    return DnC(0, N-1);
}


//演算法笔记2

struct Point {double x, y;} p[10], t[10];
bool cmpx(const Point& i, const Point& j) {return i.x < j.x;}
bool cmpy(const Point& i, const Point& j) {return i.y < j.y;}

double DnC(int L, int R)
{
    if (L >= R) return 1e9; // 沒有點、只有一個點。

    /* : 把所有點分成左右兩側，點數盡量一樣多。Divide */

    int M = (L + R) / 2;

    // 先把中線的座標記起來，因為待會重新排序之後會跑掉。X
    double x = p[M].x;

    /* : 左側、右側分別遞迴求解。Conquer */

    // 遞迴求解，並且依照座標重新排序。Y
    double d = min(DnC(L,M), DnC(M+1,R));
    //  if (d == 0.0) return d; // 提早結束

    /* : 尋找靠近中線的點，並依座標排序。MergeYO(N)。 */

    // 尋找靠近中線的點，先找左側。各點已照座標排序了。Y
    int N = 0;  // 靠近中線的點數目
    for (int i=0; i<=M; ++i)
        if (x - p[i].x < d)
            t[N++] = p[i];

    // 尋找靠近中線的點，再找右側。各點已照座標排序了。Y
    int P = N;  // 為分隔位置P
    for (int i=M+1; i<=R; ++i)
        if (p[i].x - x < d)
            t[N++] = p[i];

    // 以座標排序。使用YMerge 方式，合併已排序的兩陣列。Sort
    inplace_merge(t, t+P, t+N, cmpy);

    /* : 尋找橫跨兩側的最近點對。MergeO(N)。 */

    for (int i=0; i<N; ++i)
        for (int j=1; j<=2 && i+j<N; ++j)
            d = min(d, distance(t[i], t[i+j]));

    /* : 重新以座標排序所有點。MergeYO(N)。 */

    // 如此一來，更大的子問題就可以直接使用Merge 。Sort
    inplace_merge(p+L, p+M+1, p+R+1, cmpy);

    return d;
}

double closest_pair()
{
    sort(p, p+10, cmpx);
    return DnC(0, N-1);
}

//mzry
//分治
double calc_dis(Point &a ,Point &b) {
  return sqrt((a.x-b.x)*(a.x-b.x) + (a.y-b.y)*(a.y-b.y));
}
//別忘了排序
bool operator<(const Point &a ,const Point &b) {
  if(a.y != b.y) return a.x < b.x;
  return a.x < b.x;
}
double Gao(int l ,int r ,Point pnts[]) {
  double ret = inf;
  if(l == r) return ret;
  if(l+1 ==r) {
    ret = min(calc_dis(pnts[l],pnts[l+1]) ,ret);
    return ret;
  }
  if(l+2 ==r) {
    ret = min(calc_dis(pnts[l],pnts[l+1]) ,ret);
    ret = min(calc_dis(pnts[l],pnts[l+2]) ,ret);
    ret = min(calc_dis(pnts[l+1],pnts[l+2]) ,ret);
    return ret;
  }

  int mid = l+r>>1;
  ret = min (ret ,Gao(l ,mid,pnts));
  ret = min (ret , Gao(mid+1, r,pnts));

  for(int c = l ; c<=r; c++)
    for(int d = c+1; d<=c+7 && d<=r; d++) {
      ret = min(ret , calc_dis(pnts[c],pnts[d]));
    }
  return ret;
}

//增量
#include <iostream>
#include <cstdio>
#include <cstring>
#include <map>
#include <vector>
#include <cmath>
#include <algorithm>
#define Point pair<double,double>
using namespace std;

const int step[9][2] = {{-1,-1},{-1,0},{-1,1},{0,-1},{0,0},{0,1},{1,-1},{1,0},{1,1}};
int n,x,y,nx,ny;
map<pair<int,int>,vector<Point > > g;
vector<Point > tmp;
Point p[20000];
double tx,ty,ans,nowans;
vector<Point >::iterator it,op,ed;
pair<int,int> gird;
bool flag;

double Dis(Point p0,Point p1)
{
  return sqrt((p0.first-p1.first)*(p0.first-p1.first)+
         (p0.second-p1.second)*(p0.second-p1.second));
}

double CalcDis(Point p0,Point p1,Point p2)
{
  return Dis(p0,p1)+Dis(p0,p2)+Dis(p1,p2);
}

void build(int n,double w)
{
  g.clear();
  for (int i = 0;i < n;i++)
    g[make_pair((int)floor(p[i].first/w),(int)floor(p[i].second/w))].push_back(p[i]);
}

int main()
{
  int t;
  scanf("%d",&t);
  for (int ft = 1;ft <= t;ft++)
  {
    scanf("%d",&n);
    for (int i = 0;i < n;i++)
    {
      scanf("%lf%lf",&tx,&ty);
      p[i] = make_pair(tx,ty);
    }
    random_shuffle(p,p+n);
    ans = CalcDis(p[0],p[1],p[2]);
    build(3,ans/2.0);
    for (int i = 3;i < n;i++)
    {
      x = (int)floor(2.0*p[i].first/ans);
      y = (int)floor(2.0*p[i].second/ans);
      tmp.clear();
      for (int k = 0;k < 9;k++)
      {
        nx = x+step[k][0];
        ny = y+step[k][1];
        gird = make_pair(nx,ny);
        if (g.find(gird) != g.end())
        {
          op = g[gird].begin();
          ed = g[gird].end();
          for (it = op;it != ed;it++)
            tmp.push_back(*it);
        }
      }
      flag = false;
      for (int j = 0;j < tmp.size();j++)
        for (int k = j+1;k < tmp.size();k++)
        {
          nowans = CalcDis(p[i],tmp[j],tmp[k]);
          if (nowans < ans)
          {
            ans = nowans;
            flag = true;
          }
        }
      if (flag == true)
        build(i+1,ans/2.0);
      else
        g[make_pair((int)floor(2.0*p[i].first/ans),(int)floor(2.0*p[i].second/ans))].
              push_back(p[i]);
    }
    printf("%.3f\n",ans);
  }
}
```

3.7 ellipse

```
sq(x-h)/sq(q) + sq(y-k)/sq(b) = 1

x=h+a*cos(t);
y=k+b*sin(t);

area: pi*a*b;
distance from center to focus: f=sqrt(sq(a)-sq(b));
eccentricity: e=sqrt(a-sq(b/a))=f/a;
focal parameter: sq(b)/sqrt(sq(a)-sq(b))=sq(b)/f;

double circumference(double a,double b) // accuracy: pow(0.5,53);
{
    double x=a;
    double y=b;
    if(x<y)
        std::swap(x,y);
    double digits=53,tol=sqrt(pow(0.5,digits));
    if(digits*y<tol*x)
        return 4*x;
    double s=0,m=1;
    while(x>(tol+1)*y)
    {
        double tx=x;
        double ty=y;
```

```
        x=0.5f*(tx+ty);
        y=sqrt(tx*ty);
        m*=2;
        s+=m*pow(x-y,2);
    }
    return pi*(pow(a+b,2)-s)/(x+y);
}
```

3.8 Graham's scan

```
pv pnt[MAXX];

inline bool com(const pv &a,const pv &b)
{
    if(fabs(t=(a-pnt[0]).cross(b-pnt[0]))>eps)
        return t>0;
    return (a-pnt[0]).len()<(b-pnt[0]).len();
}

inline void graham(std::vector<pv> &ch,const int n)
{
    std::nth_element(pnt,pnt,pnt+n);
    std::sort(pnt+1,pnt+n,com);
    ch.resize(0);
    ch.push_back(pnt[0]);
    ch.push_back(pnt[1]);
    static int i;
    for(i=2;i<n;++i)
        if(fabs((pnt[i]-ch[0]).cross(ch[1]-ch[0]))>eps)
        {
            ch.push_back(pnt[i++]);
            break;
        }
        else
            ch.back()=pnt[i];
    for(;i<n;++i)
    {
        while((ch.back()-ch[ch.size()-2]).cross(pnt[i]-ch[ch.size()-2])<eps)
            ch.pop_back();
        ch.push_back(pnt[i]);
    }
}
```

3.9 half-plane intersection

```
//解析几何方式abc
inline pv ins(const pv &p1,const pv &p2)
{
    u=fabs(a*p1.x+b*p1.y+c);
    v=fabs(a*p2.x+b*p2.y+c);
    return pv((p1.x*v+p2.x*u)/(u+v),(p1.y*v+p2.y*u)/(u+v));
}

inline void get(const pv& p1,const pv& p2,double & a,double & b,double & c)
{
    a=p2.y-p1.y;
    b=p1.x-p2.x;
    c=p2.x*p1.y-p2.y*p1.x;
}

inline pv ins(const pv &x,const pv &y)
{
    get(x,y,d,e,f);
    return pv((b*f-c*e)/(a*e-b*d),(a*f-c*d)/(b*d-a*e));
}

std::vector<pv>p[2];
inline bool go()
{
    k=0;
    p[k].resize(0);
    p[k].push_back(pv(-inf,inf));
    p[k].push_back(pv(-inf,-inf));
    p[k].push_back(pv(inf,-inf));
    p[k].push_back(pv(inf,inf));
    for(i=0;i<n;++i)
    {
        get(pnt[i],pnt[(i+1)%n],a,b,c);
        c+=the*sqrt(a*a+b*b);
        p[!k].resize(0);
        for(l=0;l<p[k].size();++l)
            if(a*p[k][l].x+b*p[k][l].y+c<eps)
                p[!k].push_back(p[k][l]);
            else
            {
                m=(l+p[k].size()-1)%p[k].size();
                if(a*p[k][m].x+b*p[k][m].y+c<-eps)
                    p[!k].push_back(ins(p[k][m],p[k][l]));
                m=(l+1)%p[k].size();
                if(a*p[k][m].x+b*p[k][m].y+c<-eps)
                    p[!k].push_back(ins(p[k][m],p[k][l]));
            }
        k=!k;
        if(p[k].empty())
            break;
    }
    //结果在p[k中]
    return p[k].empty();
}

//计算几何方式
//本例求多边形核

inline pv ins(const pv &a,const pv &b)
{
    u=fabs(ln.cross(a-pnt[i]));
    v=fabs(ln.cross(b-pnt[i]))+u;
    tl=b-a;
    return pv(u*tl.x/v+a.x,u*tl.y/v+a.y);
}

int main()
{
    j=0;
    for(i=0;i<n;++i)
    {
        ln=pnt[(i+1)%n]-pnt[i];
        p[!j].resize(0);
        for(k=0;k<p[j].size();++k)
            if(ln.cross(p[j][k]-pnt[i])<=0)
                p[!j].push_back(p[j][k]);
            else
            {
                l=(k-1+p[j].size())%p[j].size();
                if(ln.cross(p[j][l]-pnt[i])<0)
                    p[!j].push_back(ins(p[j][k],p[j][l]));
                l=(k+1)%p[j].size();
                if(ln.cross(p[j][l]-pnt[i])<0)
                    p[!j].push_back(ins(p[j][k],p[j][l]));
            }
        j=!j;
    }
    //结果在p[j中]
}

//mrzy

bool HPIcmp(Line a, Line b)
{
    if (fabs(a.k - b.k) > eps)
        return a.k < b.k;
    return ((a.s - b.s) * (b.e-b.s)) < 0;
}

Line Q[100];

void HPI(Line line[], int n, Point res[], int &resn)
{
    int tot = n;
    std::sort(line, line + n, HPIcmp);
    tot = 1;
    for (int i = 1; i < n; i++)
        if (fabs(line[i].k - line[i - 1].k) > eps)
            line[tot++] = line[i];
    int head = 0, tail = 1;
    Q[0] = line[0];
    Q[1] = line[1];
    resn = 0;
    for (int i = 2; i < tot; i++)
    {
        if (fabs((Q[tail].e-Q[tail].s)*(Q[tail - 1].e-Q[tail - 1].s)) < eps || fabs((Q[
            head].e-Q[head].s)*(Q[head + 1].e-Q[head + 1].s)) < eps)
            return;
        while (head < tail && (((Q[tail]&Q[tail - 1]) - line[i].s) * (line[i].e-line[i].s)
            ) > eps)
            --tail;
        while (head < tail && (((Q[head]&Q[head + 1]) - line[i].s) * (line[i].e-line[i].s)
            ) > eps)
            ++head;
        Q[++tail] = line[i];
    }
    while (head < tail && (((Q[tail]&Q[tail - 1]) - Q[head].s) * (Q[head].e-Q[head].s)) >
        eps)
        tail--;
    while (head < tail && (((Q[head]&Q[head + 1]) - Q[tail].s) * (Q[tail].e-Q[tail].s)) >
        eps)
        head++;
    if (tail <= head + 1)
        return;
    for (int i = head; i < tail; i++)
        res[resn++] = Q[i] & Q[i + 1];
    if (head < tail + 1)
        res[resn++] = Q[head] & Q[tail];
}
```

3.10 kdtree

```
#include <iostream>
#include <cstdio>
#include <cstdlib>
#include <algorithm>
#include <stack>
#include <algorithm>
using namespace std;
#define MAXN 100010
typedef long long ll;
struct Point{
    ll x,y;
    void operator =(const Point &p){
        x=p.x; y=p.y;
    }
    ll dis(const Point &a){
        return (x-a.x)*(x-a.x)+(y-a.y)*(y-a.y);
    }
}point[MAXN],pp[MAXN];

struct Node{
    int split;//{0,1} 表示垂直于 0轴的超平面，表示垂直于轴的超平面x1y
    Point p;//点
}tree[MAXN*4];

bool cmpx(const Point &a,const Point &b)
{
    return a.x<b.x;
}

bool cmpy(const Point &a,const Point &b)
{
    return a.y<b.y;
}

void initTree(int x,int y,int split,int pos)
{
    if(y<x) return ;
    int mid=(x+y)>>1;
    random_shuffle(point+x,point+y);
    if(split==0) nth_element(point+x,point+mid,point+y+1,cmpx);
    else nth_element(point+x,point+mid,point+y+1,cmpy);
    tree[pos].split=split;
    tree[pos].p=point[mid];
    initTree(x,mid-1,(split^1),2*pos);
    initTree(mid+1,y,(split^1),2*pos+1);
}

ll ans;
void insert(int x,int y,Point &p,int pos)
{
    if(y<x) return ;
    int mid=(x+y)>>1;
    ll temp=p.dis(tree[pos].p);
    if(temp!=0) ans=min(ans,temp);
    if(tree[pos].split==0){
        if(p.x<=tree[pos].p.x){
            insert(x,mid-1,p,2*pos);
            if(ans>=(p.x-tree[pos].p.x)*(p.x-tree[pos].p.x))
                insert(mid+1,y,p,2*pos+1);
        }
        else{
            insert(mid+1,y,p,2*pos+1);
            if(ans>=(p.x-tree[pos].p.x)*(p.x-tree[pos].p.x))
                insert(x,mid-1,p,2*pos);
        }
```

```
    }
    else
    {
        if(p.y<=tree[pos].p.y){
            insert(x,mid-1,p,2*pos);
            if(ans>=(p.y-tree[pos].p.y)*(p.y-tree[pos].p.y))
                insert(mid+1,y,p,2*pos+1);
        }
        else{
            insert(mid+1,y,p,2*pos+1);
            if(ans>=(p.y-tree[pos].p.y)*(p.y-tree[pos].p.y))
                insert(x,mid-1,p,2*pos);
        }
    }
}

int main()
{
    int cases,n;
    scanf("%d",&cases);
    while(cases--)
    {
        scanf("%d",&n);
        for(int i=1;i<=n;i++){
            scanf("%I64d%I64d",&pp[i].x,&pp[i].y);
            point[i]=pp[i];
        }
        initTree(1,n,0,1);
        for(int i=1;i<=n;i++){
            ans=1LL<<62;
            insert(1,n,pp[i],1);
            printf("%I64d\n",ans);
        }
    }
    return 0;
}
```

3.11 others

```
eps如果

sqrt(a), asin(a), acos(a) 中的是你自己算出来并传进来的，那就得小心了。如果本来应该是的，由于浮点误差，可能实际是一个绝对值很小的负数（比如aa0-1e），这样-12sqrt(a)应得的，直接因0不在定义域而出错。类似地，如果本来应该是 ±aa1则,asin(a)、acos(a)也有可能出错。因此，对于此种函数，必需事先对进行校正。a现在考虑一种情况，题目要求输出保留两位小数。有个的正确答案的精确值是

case0按理应该输出，但你的结果可能是恭喜:005,0:010:005000000001()，也有可能是悲剧0:004999999999()，如果按照printf("%.2lf", a)输出，那你的遭遇将和括号里的字相同。如果为正，则输出
aa + eps，否则输出a - 。eps不要输出

-0.000注意的数据范围

double

a==b   fabs(a-b)<eps
a!=b   fabs(a-b)>eps
a<b    a+eps<b
a<=b   a<b+eps
a>b    a>b+eps
a>=b   a+eps>b三角函数



cos/sin/tan 输入弧度
acos 输入，输出 [-1,+1][0,]
asin 输入，输出 [-1,+1][-/2,+/2]
atan 输出 [-/2,+/2]
atan2 输入(y,x)注意顺序()返回,tan(y/x) 。,[-,+]都是零的时候会发生除零错误xy

other

log 自然对数(ln)
log10 你猜……
ceil 向上
floor 向下

round

cpp: 四舍六入五留双
java: add 0.5,then floor
cpp:（一）当尾数小于或等于时，直接将尾数舍去。
4（二）当尾数大于或等于时，将尾数舍去并向前一位进位。
6（三）当尾数为，而尾数后面的数字均为时，应看尾数 “” 的前一位：若前一位数字此时为奇数，就应向前进一位；若前一位数字此时为偶数，则应将尾数舍去。数字 “” 在此时应被视为偶数。
5050（四）当尾数为，而尾数 “” 的后面还有任何不是的数字时，无论前一位在此时为奇数还是偶数，也无论 “” 后面不为的数字在哪一位上，都应向前进一位。
55050

rotate mat:
[ cos(theta)  -sin(theta) ]
[ sin(theta)   cos(theta) ]
```

3.12 Pick's theorem

```
给定顶点座标均是整点（或正方形格点）的简单多边形

A面积：
i内部格点数目：
b边上格点数目：
A = i + b/2 - 。1取格点的组成图形的面积为一单位。在平行四边形格点，皮克定理依然成立。套用于任意三角形格点，皮克定理则是


A = 2i + b - 。2
```

3.13 PointInPoly

```
/*射线法
, 多边形可以是凸的或凹的的顶点数目要大于等于
poly3返回值为：

0  -- 点在内poly
1  -- 点在边界上poly
2  -- 点在外poly
*/

int inPoly(pv p,pv poly[], int n)
{
  int i, count;
  Line ray, side;

  count = 0;
  ray.s = p;
  ray.e.y  = p.y;
  ray.e.x  = -1;  //-, 注意取值防止越界! INF

  for (i = 0; i < n; i++)
  {
    side.s = poly[i];
    side.e = poly[(i+1)%n];

    if(OnSeg(p, side))
      return 1;

    // 如果平行轴则不作考虑sidex
    if (side.s.y == side.e.y)
      continue;

        if (OnSeg(side.s, ray))
        {
            if (side.s.y > side.e.y)
                count++;
        }
        else
            if (OnSeg(side.e, ray))
            {
                if (side.e.y > side.s.y)
                    count++;
            }
            else
                if (inter(ray, side))
                    count++;
  }
  return ((count % 2 == 1) ? 0 : 2);
}
```

3.14 rotating caliper

```
//最远点对

inline double go()
{
    l=ans=0;
    for(i=0;i<n;++i)
    {
        t1=pnt[(i+1)%n]-pnt[i];
        while(abs(t1.cross(pnt[(l+1)%n]-pnt[i]))>=abs(t1.cross(pnt[l]-pnt[i])))
            l=(l+1)%n;
        ans=std::max(ans,std::max(dist(pnt[l],pnt[i]),dist(pnt[l],pnt[(i+1)%n])));
    }
    return ans;
}

//两凸包最近距离
double go()
{
    sq=sp=0;
    for(i=1;i<ch[1].size();++i)
        if(ch[1][sq]<ch[1][i])
            sq=i;
    tp=sp;
    tq=sq;
    ans=(ch[0][sp]-ch[1][sq]).len();
    do
    {
        a1=ch[0][sp];
        a2=ch[0][(sp+1)%ch[0].size()];
        b1=ch[1][sq];
        b2=ch[1][(sq+1)%ch[1].size()];
        tpv=b1-(b2-a1);
        tpv.x = b1.x - (b2.x - a1.x);
        tpv.y = b1.y - (b2.y - a1.y);
        len=(tpv-a1).cross(a2-a1);
        if(fabs(len)<eps)
        {
            ans=std::min(ans,p2l(a1,b1,b2));
            ans=std::min(ans,p2l(a2,b1,b2));
            ans=std::min(ans,p2l(b1,a1,a2));
            ans=std::min(ans,p2l(b2,a1,a2));
            sp=(sp+1)%ch[0].size();
            sq=(sq+1)%ch[1].size();
        }
        else
            if(len<-eps)
            {
                ans=std::min(ans,p2l(b1,a1,a2));
                sp=(sp+1)%ch[0].size();
            }
            else
            {
                ans=std::min(ans,p2l(a1,b1,b2));
                sq=(sq+1)%ch[1].size();
            }
    }while(tp!=sp || tq!=sq);
    return ans;
}

//外接矩形 by mzry
inline void solve()
{
    resa = resb = 1e100;
    double dis1,dis2;
    Point xp[4];
    Line l[4];
    int a,b,c,d;
    int sa,sb,sc,sd;
    a = b = c = d = 0;
    sa = sb = sc = sd = 0;
    Point va,vb,vc,vd;
    for (a = 0; a < n; a++)
    {
        va = Point(p[a],p[(a+1)%n]);
        vc = Point(-va.x,-va.y);
        vb = Point(-va.y,va.x);
        vd = Point(-vb.x,-vb.y);
        if (sb < sa)
        {
            b = a;
            sb = sa;
        }
        while (xmult(vb,Point(p[b],p[(b+1)%n])) < 0)
        {
            b = (b+1)%n;
            sb++;
        }
        if (sc < sb)
        {
            c = b;
            sc = sb;
```

```
        }
        while (xmult(vc,Point(p[c],p[(c+1)%n])) < 0)
        {
            c = (c+1)%n;
            sc++;
        }
        if (sd < sc)
        {
            d = c;
            sd = sc;
        }
        while (xmult(vd,Point(p[d],p[(d+1)%n])) < 0)
        {
            d = (d+1)%n;
            sd++;
        }

        //卡在 p[a],p[b],p[c],p[d] 上
        sa++;
    }
}

//合并凸包给定凸多边形
 P = { p(1) , ... , p(m) } 和 Q = { q(1) , ... , q(n) ，一个点对} (p(i), q(j)) 形成 P 和 Q 之
    间的桥当且仅当:

(p(i), q(j)) 形成一个并踵点对。
p(i-1), p(i+1), q(j-1), q(j+1) 都位于由 (p(i), q(j)) 组成的线的同一侧。假设多边形以标准形式给出并且
    顶点是以顺时针序排列，算法如下：、分别计算



1 P 和 Q 拥有最大 y 坐标的顶点。如果存在不止一个这样的点，取  x 坐标最大的。、构造这些点的逐平切线，
2 以多边形处于其右侧为正方向（因此他们指向 x 轴正方向）。、同时顺时针旋转两条切线直到其中一条与边相交。
3 得到一个新的并踵点对 (p(i), q(j)) 。对于平行边的情况，得到三个并踵点对。、对于所有有效的并踵点对
4 (p(i), q(j)): 判定 p(i-1), p(i+1), q(j-1), q(j+1) 是否都位于连接点 (p(i), q(j)) 形成的线的同
    一侧。如果是，这个并踵点对就形成了一个桥，并标记他。、重复执行步骤和步骤直到切线回到他们原来的位置
534、所有可能的桥此时都已经确定了。
6 通过连续连接桥间对应的凸包链来构造合并凸包。上述的结论确定了算法的正确性。运行时间受步骤，，约束。


 156 他们都为 O(N) 运行时间（N 是顶点总数）。因此算法拥有现行的时间复杂度。一个凸多边形间的桥实际上确定了另
    一个有用的概念：多边形间公切线。同时，桥也是计算凸多边形交的算法核心。



//临界切线、计算
1 P 上 y 坐标值最小的顶点（称为 yminP ）和  Q 上 y 坐标值最大的顶点（称为）。 ymaxQ、为多边形在
2 yminP 和 ymaxQ 处构造两条切线 LP 和 LQ 使得他们对应的多边形位于他们的右侧。此时  LP 和 LQ 拥有不同的
    方向，并且  yminP 和 ymaxQ 成为了多边形间的一个对踵点对。、令
3 p(i)= , yminP q(j)= 。ymaxQ (p(i), q(j)) 构成了多边形间的一个对踵点对。检测是否
    有  p(i-1),p(i+1) 在线 (p(i), q(j)) 的一侧，并且  q(j-1),q(j+1) 在另一侧。如果成立，
    (p(i), q(j)) 确定了一条线。CS、旋转这两条线，
4 直到其中一条和其对应的多边形的边重合。、一个新的对踵点对确定了。
5 如果两条线都与边重合，总共三对对踵点对（原先的顶点和新的顶点的组合）需要考虑。对于所有的对踵点对，执行上面
    的测试。、重复执行步骤和步骤，
645 直到新的点对为(yminP,ymaxQ)。、输出
7线。CS

//最小最大周长面积外接矩形//、计算全部四个多边形的端点，
1 称之为， xminP ，xmaxP ，yminP 。ymaxP、通过四个点构造
2 P 的四条切线。他们确定了两个“卡壳”集合。、如果一条（或两条）线与一条边重合，
3 那么计算由四条线决定的矩形的面积，并且保存为当前最小值。否则将当前最小值定义为无穷大。、顺时针旋转线直到其
    中一条和多边形的一条边重合。
4、计算新矩形的周长面积，
5/ 并且和当前最小值比较。如果小于当前最小值则更新，并保存确定最小值的矩形信息。、重复步骤和步骤，
645 直到线旋转过的角度大于度。90、输出外接矩形的最小周长。
7
```

3.15 shit

```
struct pv
{
    double x,y;
    pv():x(0),y(0){}
    pv(double xx,double yy):x(xx),y(yy){}
    inline pv operator+(const pv &i)const
    {
        return pv(x+i.x,y+i.y);
    }
    inline pv operator-(const pv &i)const
    {
        return pv(x-i.x,y-i.y);
    }
    inline bool operator ==(const pv &i)const
    {
        return fabs(x-i.x)<eps && fabs(y-i.y)<eps;
    }
    inline bool operator<(const pv &i)const
    {
        return y==i.y?x<i.x:y<i.y;
    }
    inline double cross(const pv &i)const
    {
        return x*i.y-y*i.x;
    }
    inline double dot(const pv &i)const
    {
        return x*i.x+y*i.y;
    }
    inline double len()
    {
        return sqrt(x*x+y*y);
    }
    inline pv rotate(pv p,double theta)
    {
        static pv v;
        v=*this-p;
        static double c,s;
        c=cos(theta);
        s=sin(theta);
        return pv(p.x+v.x*c-v.y*s,p.y+v.x*s+v.y*c);
    }
};

inline int dblcmp(double d)
{
    if(fabs(d)<eps)
        return 0;
    return d>eps?1:-1;
}

inline int cross(pv *a,pv *b) // 不相交0 不规范1 规范2
{
    int d1=dblcmp((a[1]-a[0]).cross(b[0]-a[0]));
    int d2=dblcmp((a[1]-a[0]).cross(b[1]-a[0]));
    int d3=dblcmp((b[1]-b[0]).cross(a[0]-b[0]));
    int d4=dblcmp((b[1]-b[0]).cross(a[1]-b[0]));
    if((d1^d2)==-2 && (d3^d4)==-2)
        return 2;
    return ((d1==0 && dblcmp((b[0]-a[0]).dot(b[0]-a[1]))<=0 ) ||
            (d2==0 && dblcmp((b[1]-a[0]).dot(b[1]-a[1]))<=0 ) ||
            (d3==0 && dblcmp((a[0]-b[0]).dot(a[0]-b[1]))<=0 ) ||
            (d4==0 && dblcmp((a[1]-b[0]).dot(a[1]-b[1]))<=0));
}

inline bool pntonseg(const pv &p,const pv *a)
{
    return fabs((p-a[0]).cross(p-a[1]))<eps && (p-a[0]).dot(p-a[1])<eps;
}

pv rotate(pv v,pv p,double theta,double sc=1) // rotate vector v, theta    [0,2]
{
    static pv re;
    re=p;
    v=v-p;
    p.x=sc*cos(theta);
    p.y=sc*sin(theta);
    re.x+=v.x*p.x-v.y*p.y;
    re.y+=v.x*p.y+v.y*p.x;
    return re;
}

struct line
{
    pv pnt[2];
    line(double a,double b,double c) // a*x + b*y + c = 0
    {
#define maxl 1e2 //preciseness should not be too high ( compare with eps )
        if(fabs(b)>eps)
        {
            pnt[0]=pv(maxl,(c+a*maxl)/(-b));
            pnt[1]=pv(-maxl,(c-a*maxl)/(-b));
        }
        else
        {
            pnt[0]=pv(-c/a,maxl);
            pnt[1]=pv(-c/a,-maxl);
        }
#undef maxl
    }
    pv cross(const line &v)const
    {
        double a=(v.pnt[1]-v.pnt[0]).cross(pnt[0]-v.pnt[0]);
        double b=(v.pnt[1]-v.pnt[0]).cross(pnt[1]-v.pnt[0]);
        return pv((pnt[0].x*b-pnt[1].x*a)/(b-a),(pnt[0].y*b-pnt[1].y*a)/(b-a));
    }
};

inline std::pair<pv,double> getcircle(const pv &a,const pv &b,const pv &c)
{
    static pv ct;
    ct=line(2*(b.x-a.x),2*(b.y-a.y),a.len()-b.len()).cross(line(2*(c.x-b.x),2*(c.y-b.y),b.
        len()-c.len()));
    return std::make_pair(ct,sqrt((ct-a).len()));
}
```

3.16 sort - polar angle

```
inline bool cmp(const Point& a,const Point& b)
{
    if (a.y*b.y<= 0)
    {
        if (a.y > 0 || b.y > 0)
            return a.y < b.y;
        if (a.y == 0 && b.y == 0)
            return a.x < b.x;
    }
    return a.cross(b) > 0;
}
```

3.17 triangle

```
Area:
p=(a+b+c)/2
area=sqrt(p*(p-a)*(p-b)*(p-c));
area=a*b*sin(C)/2;
area=sq(a)*sin(B)*sin(C)/2/sin(B+C);
area=sq(a)/2/(cot(B)+cot(C));

centroid:
    center of mass
    intersection of triangle's three triangle medians

Trigonometric conditions:
tan(A/2)*tan(B/2)+tan(B/2)*tan(C/2)+tan(A/2)*tan(C/2)==1
sq(sin(A/2))+sq(sin(B/2))+sq(sin(C/2))+2*sin(A/2)*sin(B/2)*sin(C/2)==1

Circumscribed circle:
diameter=a*b*c/(2*area);
diameter=sqrt(2*area/sin(A)/sin(B)/sin(c));
diameter=a/sin(A)=b/sin(B)=c/sin(C);

Incircle:
inradius=2*area/(a+b+c);
coordinates(x,y)=a*{xa,ya}/(a+b+c)+b*{xb,yb}/(a+b+c)+c*{xc,yc}/(a+b+c);

Excircles:
radius[a]=2*area/(b+c-a);
radius[b]=2*area/(a+c-b);
radius[c]=2*area/(a+b-c);

Steiner circumellipse (least area circumscribed ellipse)
    area= area * 4*pi/3/sqrt(3);
    center is the triangle's centroid.

Steiner inellipse ( maximum area inellipse )
    area= area * pi/3/sqrt(3);
    center is the triangle's centroid.

Fermat Point:当有一个内角不小于 ° 时，费马点为此角对应顶点。
120当三角形的内角都小于 ° 时

120以三角形的每一边为底边，向外做三个正三角形

ABC, 'BCA, 'CAB, '连接
CC, 'BB, 'AA, 则三条线段的交点就是所求的点。'
```

4 geometry/tmp

4.1 circle

```
struct circle
{
    point p;
    double r;
    circle(){}
    circle(point _p,double _r):
    p(_p),r(_r){};
    circle(double x,double y,double _r):
    p(point(x,y)),r(_r){};
    circle(point a,point b,point c)//三角形的外接圆
    {
      p=line(a.add(b).div(2),a.add(b).div(2).add(b.sub(a).rotleft())).crosspoint(line(c.
          add(b).div(2),c.add(b).div(2).add(b.sub(c).rotleft())));
      r=p.distance(a);
    }
    circle(point a,point b,point c,bool t)//三角形的内切圆
    {
      line u,v;
      double m=atan2(b.y-a.y,b.x-a.x),n=atan2(c.y-a.y,c.x-a.x);
      u.a=a;
    u.b=u.a.add(point(cos((n+m)/2),sin((n+m)/2)));
      v.a=b;
     m=atan2(a.y-b.y,a.x-b.x),n=atan2(c.y-b.y,c.x-b.x);
      v.b=v.a.add(point(cos((n+m)/2),sin((n+m)/2)));
      p=u.crosspoint(v);
      r=line(a,b).dispointtoseg(p);
    }
    void input()
    {
        p.input();
        scanf("%lf",&r);
    }
    void output()
    {
        printf("%.2lf %.2lf %.2lf\n",p.x,p.y,r);
    }
    bool operator==(circle v)
    {
      return ((p==v.p)&&dblcmp(r-v.r)==0);
    }
    bool operator<(circle v)const
    {
      return ((p<v.p)||(p==v.p)&&dblcmp(r-v.r)<0);
    }
    double area()
    {
      return pi*sqr(r);
    }
    double circumference()
    {
      return 2*pi*r;
    }
    //0 圆外
    //1 圆上
    //2 圆内
    int relation(point b)
    {
        double dst=b.distance(p);
        if (dblcmp(dst-r)<0)return 2;
        if (dblcmp(dst-r)==0)return 1;
        return 0;
    }
    int relationseg(line v)
    {
        double dst=v.dispointtoseg(p);
        if (dblcmp(dst-r)<0)return 2;
        if (dblcmp(dst-r)==0)return 1;
        return 0;
    }
    int relationline(line v)
    {
        double dst=v.dispointtoline(p);
        if (dblcmp(dst-r)<0)return 2;
        if (dblcmp(dst-r)==0)return 1;
        return 0;
    }
    //过a 两点b 半径的两个圆r
    int getcircle(point a,point b,double r,circle&c1,circle&c2)
    {
        circle x(a,r),y(b,r);
        int t=x.pointcrosscircle(y,c1.p,c2.p);
    if (!t)return 0;
        c1.r=c2.r=r;
        return t;
    }
    //与直线相切u 过点q 半径的圆r1
    int getcircle(line u,point q,double r1,circle &c1,circle &c2)
    {
      double dis=u.dispointtoline(q);
      if (dblcmp(dis-r1*2)>0)return 0;
      if (dblcmp(dis)==0)
      {
        c1.p=q.add(u.b.sub(u.a).rotleft().trunc(r1));
        c2.p=q.add(u.b.sub(u.a).rotright().trunc(r1));
        c1.r=c2.r=r1;
        return 2;
      }
      line u1=line(u.a.add(u.b.sub(u.a).rotleft().trunc(r1)),u.b.add(u.b.sub(u.a).rotleft
          ().trunc(r1)));
      line u2=line(u.a.add(u.b.sub(u.a).rotright().trunc(r1)),u.b.add(u.b.sub(u.a).
          rotright().trunc(r1)));
    circle cc=circle(q,r1);
      point p1,p2;
      if (!cc.pointcrossline(u1,p1,p2))cc.pointcrossline(u2,p1,p2);
      c1=circle(p1,r1);
      if (p1==p2)
    {
      c2=c1;return 1;
      }
      c2=circle(p2,r1);
      return 2;
  }
  //同时与直线u,相切v 半径的圆r1
    int getcircle(line u,line v,double r1,circle &c1,circle &c2,circle &c3,circle &c4)
    {
      if (u.parallel(v))return 0;
      line u1=line(u.a.add(u.b.sub(u.a).rotleft().trunc(r1)),u.b.add(u.b.sub(u.a).rotleft
          ().trunc(r1)));
      line u2=line(u.a.add(u.b.sub(u.a).rotright().trunc(r1)),u.b.add(u.b.sub(u.a).
          rotright().trunc(r1)));
      line v1=line(v.a.add(v.b.sub(v.a).rotleft().trunc(r1)),v.b.add(v.b.sub(v.a).rotleft
          ().trunc(r1)));
      line v2=line(v.a.add(v.b.sub(v.a).rotright().trunc(r1)),v.b.add(v.b.sub(v.a).
          rotright().trunc(r1)));
      c1.r=c2.r=c3.r=c4.r=r1;
      c1.p=u1.crosspoint(v1);
      c2.p=u1.crosspoint(v2);
      c3.p=u2.crosspoint(v1);
      c4.p=u2.crosspoint(v2);
      return 4;
    }
    //同时与不相交圆cx,相切cy 半径为的圆r1
    int getcircle(circle cx,circle cy,double r1,circle&c1,circle&c2)
      {
          circle x(cx.p,r1+cx.r),y(cy.p,r1+cy.r);
          int t=x.pointcrosscircle(y,c1.p,c2.p);
      if (!t)return 0;
          c1.r=c2.r=r1;
          return t;
      }
      int pointcrossline(line v,point &p1,point &p2)//求与线段交要先判断relationseg
      {
          if (!(*this).relationline(v))return 0;
          point a=v.lineprog(p);
          double d=v.dispointtoline(p);
          d=sqrt(r*r-d*d);
          if (dblcmp(d)==0)
          {
              p1=a;
              p2=a;
              return 1;
          }
          p1=a.sub(v.b.sub(v.a).trunc(d));
          p2=a.add(v.b.sub(v.a).trunc(d));
          return 2;
      }
      //5 相离
    //4 外切
    //3 相交
    //2 内切
    //1 内含
      int relationcircle(circle v)
      {
        double d=p.distance(v.p);
        if (dblcmp(d-r-v.r)>0)return 5;
        if (dblcmp(d-r-v.r)==0)return 4;
        double l=fabs(r-v.r);
        if (dblcmp(d-r-v.r)<0&&dblcmp(d-l)>0)return 3;
        if (dblcmp(d-l)==0)return 2;
        if (dblcmp(d-l)<0)return 1;
      }
      int pointcrosscircle(circle v,point &p1,point &p2)
      {
        int rel=relationcircle(v);
        if (rel==1||rel==5)return 0;
        double d=p.distance(v.p);
        double l=(d+(sqr(r)-sqr(v.r))/d)/2;
        double h=sqrt(sqr(r)-sqr(l));
        p1=p.add(v.p.sub(p).trunc(l).add(v.p.sub(p).rotleft().trunc(h)));
        p2=p.add(v.p.sub(p).trunc(l).add(v.p.sub(p).rotright().trunc(h)));
        if (rel==2||rel==4)
        {
          return 1;
        }
      return 2;
      }
      //过一点做圆的切线 先判断点和圆关系()
      int tangentline(point q,line &u,line &v)
      {
        int x=relation(q);
        if (x==2)return 0;
        if (x==1)
        {
          u=line(q,q.add(q.sub(p).rotleft()));
          v=u;
          return 1;
        }
        double d=p.distance(q);
        double l=sqr(r)/d;
        double h=sqrt(sqr(r)-sqr(l));
        u=line(q,p.add(q.sub(p).trunc(l).add(q.sub(p).rotleft().trunc(h))));
        v=line(q,p.add(q.sub(p).trunc(l).add(q.sub(p).rotright().trunc(h))));
        return 2;
    }
      double areacircle(circle v)
      {
        int rel=relationcircle(v);
        if (rel>=4)return 0.0;
          if (rel<=2)return min(area(),v.area());
          double d=p.distance(v.p);
          double hf=(r+v.r+d)/2.0;
          double ss=2*sqrt(hf*(hf-r)*(hf-v.r)*(hf-d));
          double a1=acos((r*r+d*d-v.r*v.r)/(2.0*r*d));
          a1=a1*r*r;
          double a2=acos((v.r*v.r+d*d-r*r)/(2.0*v.r*d));
          a2=a2*v.r*v.r;
          return a1+a2-ss;
      }
      double areatriangle(point a,point b)
      {
          if (dblcmp(p.sub(a).det(p.sub(b))==0))return 0.0;
          point q[5];
          int len=0;
          q[len++]=a;
          line l(a,b);
          point p1,p2;
          if (pointcrossline(l,q[1],q[2])==2)
          {
              if (dblcmp(a.sub(q[1]).dot(b.sub(q[1])))<0)q[len++]=q[1];
              if (dblcmp(a.sub(q[2]).dot(b.sub(q[2])))<0)q[len++]=q[2];
          }
          q[len++]=b;
          if (len==4&&dblcmp(q[0].sub(q[1]).dot(q[2].sub(q[1])))>0))swap(q[1],q[2]);
          double res=0;
          int i;
          for (i=0;i<len-1;i++)
          {
              if (relation(q[i])==0||relation(q[i+1])==0)
              {
                  double arg=p.rad(q[i],q[i+1]);
                  res+=r*r*arg/2.0;
              }
              else
              {
                  res+=fabs(q[i].sub(p).det(q[i+1].sub(p))/2.0);
              }
          }
          return res;
      }
};
```

4.2 circles

```
const int maxn=500;
struct circles
{
  circle c[maxn];
  double ans[maxn];//ans[i表示被覆盖了]次的面积i
  double pre[maxn];
  int n;
  circles(){}
  void add(circle cc)
  {
    c[n++]=cc;
  }
  bool inner(circle x,circle y)
  {
    if (x.relationcircle(y)!=1)return 0;
    return dblcmp(x.r-y.r)<=0?1:0;
  }
  void init_or()//圆的面积并去掉内含的圆
  {
    int i,j,k=0;
    bool mark[maxn]={0};
    for (i=0;i<n;i++)
    {
      for (j=0;j<n;j++)if (i!=j&&!mark[j])
      {
        if ((c[i]==c[j])||inner(c[i],c[j]))break;
      }
      if (j<n)mark[i]=1;
    }
    for (i=0;i<n;i++)if (!mark[i])c[k++]=c[i];
    n=k;
  }
  void init_and()//圆的面积交去掉内含的圆
  {
    int i,j,k=0;
    bool mark[maxn]={0};
    for (i=0;i<n;i++)
    {
      for (j=0;j<n;j++)if (i!=j&&!mark[j])
      {
        if ((c[i]==c[j])||inner(c[j],c[i]))break;
      }
      if (j<n)mark[i]=1;
    }
    for (i=0;i<n;i++)if (!mark[i])c[k++]=c[i];
    n=k;
  }
  double areaarc(double th,double r)
    {
      return 0.5*sqr(r)*(th-sin(th));
    }
  void getarea()
  {
    int i,j,k;
    memset(ans,0,sizeof(ans));
    vector<pair<double,int> >v;
    for (i=0;i<n;i++)
    {
      v.clear();
      v.push_back(make_pair(-pi,1));
      v.push_back(make_pair(pi,-1));
      for (j=0;j<n;j++)if (i!=j)
      {
        point q=c[j].p.sub(c[i].p);
        double ab=q.len(),ac=c[i].r,bc=c[j].r;
        if (dblcmp(ab+ac-bc)<=0)
          {
              v.push_back(make_pair(-pi,1));
          v.push_back(make_pair(pi,-1));
                continue;
          }
        if (dblcmp(ab+bc-ac)<=0)continue;
        if (dblcmp(ab-ac-bc)>0) continue;
        double th=atan2(q.y,q.x),fai=acos((ac*ac+ab*ab-bc*bc)/(2.0*ac*ab));
        double a0=th-fai;
        if (dblcmp(a0+pi)<0)a0+=2*pi;
        double a1=th+fai;
        if (dblcmp(a1-pi)>0)a1-=2*pi;
        if (dblcmp(a0-a1)>0)
        {
          v.push_back(make_pair(a0,1));
          v.push_back(make_pair(pi,-1));
          v.push_back(make_pair(-pi,1));
          v.push_back(make_pair(a1,-1));
        }
        else
        {
          v.push_back(make_pair(a0,1));
          v.push_back(make_pair(a1,-1));
        }
      }
      sort(v.begin(),v.end());
      int cur=0;
      for (j=0;j<v.size();j++)
      {
        if (cur&&dblcmp(v[j].first-pre[cur]))
        {
          ans[cur]+=areaarc(v[j].first-pre[cur],c[i].r);
          ans[cur]+=0.5*point(c[i].p.x+c[i].r*cos(pre[cur]),c[i].p.y+c[i].r*sin(pre[cur]))
                .det(point(c[i].p.x+c[i].r*cos(v[j].first),c[i].p.y+c[i].r*sin(v[j].first)
                ));
        }
        cur+=v[j].second;
        pre[cur]=v[j].first;
      }
    }
    for (i=1;i<=n;i++)
    {
      ans[i]-=ans[i+1];
    }
  }
};
```

4.3 halfplane

```
struct halfplane:public line
{
  double angle;
  halfplane(){}
  //表示向量 a->逆时针b左侧()的半平面
  halfplane(point _a,point _b)
  {
    a=_a;
    b=_b;
  }
  halfplane(line v)
  {
    a=v.a;
    b=v.b;
  }
  void calcangle()
  {
    angle=atan2(b.y-a.y,b.x-a.x);
  }
  bool operator<(const halfplane &b)const
  {
    return angle<b.angle;
  }
};
struct halfplanes
{
  int n;
  halfplane hp[maxp];
  point p[maxp];
  int que[maxp];
  int st,ed;
  void push(halfplane tmp)
  {
    hp[n++]=tmp;
  }
  void unique()
  {
    int m=1,i;
    for (i=1;i<n;i++)
    {
      if (dblcmp(hp[i].angle-hp[i-1].angle))hp[m++]=hp[i];
      else if (dblcmp(hp[m-1].b.sub(hp[m-1].a).det(hp[i].a.sub(hp[m-1].a))>0))hp[m-1]=hp[i
            ];
    }
    n=m;
  }
  bool halfplaneinsert()
  {
    int i;
    for (i=0;i<n;i++)hp[i].calcangle();
    sort(hp,hp+n);
    unique();
    que[st=0]=0;
    que[ed=1]=1;
    p[1]=hp[0].crosspoint(hp[1]);
    for (i=2;i<n;i++)
    {
      while (st<ed&&dblcmp((hp[i].b.sub(hp[i].a).det(p[ed].sub(hp[i].a))))<0)ed--;
      while (st<ed&&dblcmp((hp[i].b.sub(hp[i].a).det(p[st+1].sub(hp[i].a))))<0)st++;
      que[++ed]=i;
      if (hp[i].parallel(hp[que[ed-1]]))return false;
      p[ed]=hp[i].crosspoint(hp[que[ed-1]]);
    }
    while (st<ed&&dblcmp(hp[que[st]].b.sub(hp[que[st]].a).det(p[ed].sub(hp[que[st]].a)))
          <0)ed--;
    while (st<ed&&dblcmp(hp[que[ed]].b.sub(hp[que[ed]].a).det(p[st+1].sub(hp[que[ed]].a)))
          <0)st++;
    if (st+1>=ed)return false;
    return true;
  }
  void getconvex(polygon &con)
  {
    p[st]=hp[que[st]].crosspoint(hp[que[ed]]);
    con.n=ed-st+1;
    int j=st,i=0;
    for (;j<=ed;i++,j++)
    {
      con.p[i]=p[j];
    }
  }
};
```

4.4 line

```
struct line
{
    point a,b;
    line(){}
    line(point _a,point _b)
    {
        a=_a;
        b=_b;
    }
    bool operator==(line v)
    {
      return (a==v.a)&&(b==v.b);
    }
    //倾斜角angle
    line(point p,double angle)
    {
      a=p;
      if (dblcmp(angle-pi/2)==0)
      {
        b=a.add(point(0,1));
      }
      else
      {
        b=a.add(point(1,tan(angle)));
      }
    }
    //ax+by+c=0
    line(double _a,double _b,double _c)
    {
      if (dblcmp(_a)==0)
      {
        a=point(0,-_c/_b);
        b=point(1,-_c/_b);
      }
      else if (dblcmp(_b)==0)
      {
        a=point(-_c/_a,0);
        b=point(-_c/_a,1);
      }
      else
      {
        a=point(0,-_c/_b);
        b=point(1,(-_c-_a)/_b);
      }
    }
    void input()
    {
        a.input();
        b.input();
    }
    void adjust()
    {
```

```
    if (b<a)swap(a,b);
  }
  double length()
  {
    return a.distance(b);
  }
  double angle()//直线倾斜角 0<=angle<180
 {
  double k=atan2(b.y-a.y,b.x-a.x);
  if (dblcmp(k)<0)k+=pi;
  if (dblcmp(k-pi)==0)k=pi;
  return k;
 }
  //点和线段关系
  //1 在逆时针
  //2 在顺时针
  //3 平行
  int relation(point p)
  {
    int c=dblcmp(p.sub(a).det(b.sub(a)));
    if (c<0)return 1;
    if (c>0)return 2;
    return 3;
  }
  bool pointonseg(point p)
  {
    return dblcmp(p.sub(a).det(b.sub(a)))==0&&dblcmp(p.sub(a).dot(p.sub(b)))<=0;
  }
  bool parallel(line v)
  {
    return dblcmp(b.sub(a).det(v.b.sub(v.a)))==0;
  }
  //2 规范相交
  //1 非规范相交
  //0 不相交
  int segcrossseg(line v)
  {
    int d1=dblcmp(b.sub(a).det(v.a.sub(a)));
    int d2=dblcmp(b.sub(a).det(v.b.sub(a)));
    int d3=dblcmp(v.b.sub(v.a).det(a.sub(v.a)));
    int d4=dblcmp(v.b.sub(v.a).det(b.sub(v.a)));
    if ((d1^d2)==-2&&(d3^d4)==-2)return 2;
    return (d1==0&&dblcmp(v.a.sub(a).dot(v.a.sub(b)))<=0||
            d2==0&&dblcmp(v.b.sub(a).dot(v.b.sub(b)))<=0||
            d3==0&&dblcmp(a.sub(v.a).dot(a.sub(v.b)))<=0||
            d4==0&&dblcmp(b.sub(v.a).dot(b.sub(v.b)))<=0);
  }
  int linecrossseg(line v)//*this seg v line
  {
    int d1=dblcmp(b.sub(a).det(v.a.sub(a)));
    int d2=dblcmp(b.sub(a).det(v.b.sub(a)));
    if ((d1^d2)==-2)return 2;
    return (d1==0||d2==0);
  }
  //0 平行
 //1 重合
 //2 相交
  int linecrossline(line v)
  {
    if ((*this).parallel(v))
    {
      return v.relation(a)==3;
    }
    return 2;
  }
  point crosspoint(line v)
  {
    double a1=v.b.sub(v.a).det(a.sub(v.a));
    double a2=v.b.sub(v.a).det(b.sub(v.a));
    return point((a.x*a2-b.x*a1)/(a2-a1),(a.y*a2-b.y*a1)/(a2-a1));
  }
  double dispointtoline(point p)
  {
    return fabs(p.sub(a).det(b.sub(a)))/length();
  }
  double dispointtoseg(point p)
  {
    if (dblcmp(p.sub(b).dot(a.sub(b)))<0||dblcmp(p.sub(a).dot(b.sub(a)))<0)
    {
      return min(p.distance(a),p.distance(b));
    }
    return dispointtoline(p);
  }
  point lineprog(point p)
  {
    return a.add(b.sub(a).mul(b.sub(a).dot(p.sub(a))/b.sub(a).len2()));
  }
  point symmetrypoint(point p)
  {
    point q=lineprog(p);
    return point(2*q.x-p.x,2*q.y-p.y);
  }
};
```

4.5 line3d

```
struct line3
{
  point3 a,b;
  line3(){}
  line3(point3 _a,point3 _b)
  {
    a=_a;
    b=_b;
  }
  bool operator==(line3 v)
  {
    return (a==v.a)&&(b==v.b);
  }
  void input()
  {
    a.input();
    b.input();
  }
  double length()
  {
    return a.distance(b);
  }
  bool pointonseg(point3 p)
  {
    return dblcmp(p.sub(a).det(p.sub(b)).len())==0&&dblcmp(a.sub(p).dot(b.sub(p)))<=0;
  }
  double dispointtoline(point3 p)
  {
    return b.sub(a).det(p.sub(a)).len()/a.distance(b);
  }
  double dispointtoseg(point3 p)
  {
    if (dblcmp(p.sub(b).dot(a.sub(b)))<0||dblcmp(p.sub(a).dot(b.sub(a)))<0)
    {
      return min(p.distance(a),p.distance(b));
    }
    return dispointtoline(p);
  }
  point3 lineprog(point3 p)
  {
    return a.add(b.sub(a).trunc(b.sub(a).dot(p.sub(a))/b.distance(a)));
  }
  point3 rotate(point3 p,double ang)//绕此向量逆时针角度parg
 {
  if (dblcmp((p.sub(a).det(p.sub(b)).len()))==0)return p;
  point3 f1=b.sub(a).det(p.sub(a));
  point3 f2=b.sub(a).det(f1);
  double len=fabs(a.sub(p).det(b.sub(p)).len()/a.distance(b));
  f1=f1.trunc(len);f2=f2.trunc(len);
  point3 h=p.add(f2);
  point3 pp=h.add(f1);
  return h.add((p.sub(h)).mul(cos(ang*1.0))).add((pp.sub(h)).mul(sin(ang*1.0)));
 }
};
```

4.6 plane

```
struct plane
{
  point3 a,b,c,o;
  plane(){}
  plane(point3 _a,point3 _b,point3 _c)
  {
    a=_a;
    b=_b;
    c=_c;
    o=pvec();
  }
  plane(double _a,double _b,double _c,double _d)
  {
    //ax+by+cz+d=0
    o=point3(_a,_b,_c);
  if (dblcmp(_a)!=0)
  {
    a=point3((-_d-_c-_b)/_a,1,1);
  }
  else if (dblcmp(_b)!=0)
  {
    a=point3(1,(-_d-_c-_a)/_b,1);
  }
  else if (dblcmp(_c)!=0)
  {
    a=point3(1,1,(-_d-_a-_b)/_c);
  }
  }
  void input()
  {
    a.input();
    b.input();
    c.input();
    o=pvec();
  }
  point3 pvec()
  {
    return b.sub(a).det(c.sub(a));
  }
 bool pointonplane(point3 p)//点是否在平面上
  {
    return dblcmp(p.sub(a).dot(o))==0;
  }
  //0 不在
 //1 在边界上
 //2 在内部
  int pointontriangle(point3 p)//点是否在空间三角形上abc
  {
    if (!pointonplane(p))return 0;
    double s=a.sub(b).det(c.sub(b)).len();
    double s1=p.sub(a).det(p.sub(b)).len();
    double s2=p.sub(a).det(p.sub(c)).len();
    double s3=p.sub(b).det(p.sub(c)).len();
    if (dblcmp(s-s1-s2-s3))return 0;
    if (dblcmp(s1)&&dblcmp(s2)&&dblcmp(s3))return 2;
    return 1;
  }
  //判断两平面关系
  //0 相交
  //1 平行但不重合
  //2 重合
  bool relationplane(plane f)
  {
    if (dblcmp(o.det(f.o).len()))return 0;
    if (pointonplane(f.a))return 2;
    return 1;
  }
  double angleplane(plane f)//两平面夹角
  {
    return acos(o.dot(f.o)/(o.len()*f.o.len()));
  }
  double dispoint(point3 p)//点到平面距离
 {
  return fabs(p.sub(a).dot(o)/o.len());
 }
  point3 pttoplane(point3 p)//点到平面最近点
 {
  line3 u=line3(p,p.add(o));
  crossline(u,p);
  return p;
 }
  int crossline(line3 u,point3 &p)//平面和直线的交点
  {
    double x=o.dot(u.b.sub(a));
    double y=o.dot(u.a.sub(a));
    double d=x-y;
    if (dblcmp(fabs(d))==0)return 0;
    p=u.a.mul(x).sub(u.b.mul(y)).div(d);
    return 1;
  }
  int crossplane(plane f,line3 &u)//平面和平面的交线
  {
    point3 oo=o.det(f.o);
    point3 v=o.det(oo);
    double d=fabs(f.o.dot(v));
    if (dblcmp(d)==0)return 0;
    point3 q=a.add(v.mul(f.o.dot(f.a.sub(a))/d));
    u=line3(q,q.add(oo));
    return 1;
  }
};
```

4.7 point

```
using namespace std;

#define mp make_pair
#define pb push_back

const double eps=1e-8;
const double pi=acos(-1.0);
const double inf=1e20;
const int maxp=8;

int dblcmp(double d)
{
    if (fabs(d)<eps)return 0;
    return d>eps?1:-1;
}

inline double sqr(double x)
{
    return x*x;
}

struct point
{
    double x,y;
    point(){}
    point(double _x,double _y):
    x(_x),y(_y){};
    void input()
    {
        scanf("%lf%lf",&x,&y);
    }
    void output()
    {
        printf("%.2f %.2f\n",x,y);
    }
    bool operator==(point a)const
    {
        return dblcmp(a.x-x)==0&&dblcmp(a.y-y)==0;
    }
    bool operator<(point a)const
    {
        return dblcmp(a.x-x)==0?dblcmp(y-a.y)<0:x<a.x;
    }
    double len()
    {
        return hypot(x,y);
    }
    double len2()
    {
        return x*x+y*y;
    }
    double distance(point p)
    {
        return hypot(x-p.x,y-p.y);
    }
    point add(point p)
    {
        return point(x+p.x,y+p.y);
    }
    point sub(point p)
    {
        return point(x-p.x,y-p.y);
    }
    point mul(double b)
    {
        return point(x*b,y*b);
    }
    point div(double b)
    {
        return point(x/b,y/b);
    }
    double dot(point p)
    {
        return x*p.x+y*p.y;
    }
    double det(point p)
    {
        return x*p.y-y*p.x;
    }
    double rad(point a,point b)
    {
      point p=*this;
      return fabs(atan2(fabs(a.sub(p).det(b.sub(p))),a.sub(p).dot(b.sub(p))));
  }
    point trunc(double r)
  {
    double l=len();
    if (!dblcmp(l))return *this;
    r/=l;
    return point(x*r,y*r);
  }
    point rotleft()
    {
        return point(-y,x);
    }
    point rotright()
    {
        return point(y,-x);
    }
    point rotate(point p,double angle)//绕点逆时针旋转角度pangle
    {
        point v=this->sub(p);
        double c=cos(angle),s=sin(angle);
        return point(p.x+v.x*c-v.y*s,p.y+v.x*s+v.y*c);
    }
};
```

4.8 point3d

```
struct point3
{
  double x,y,z;
  point3(){}
  point3(double _x,double _y,double _z):
  x(_x),y(_y),z(_z){};
  void input()
  {
    scanf("%lf%lf%lf",&x,&y,&z);
  }
  void output()
  {
    printf("%.2lf %.2lf %.2lf\n",x,y,z);
  }
  bool operator==(point3 a)
  {
      return dblcmp(a.x-x)==0&&dblcmp(a.y-y)==0&&dblcmp(a.z-z)==0;
  }
  bool operator<(point3 a)const
  {
      return dblcmp(a.x-x)==0?dblcmp(y-a.y)==0?dblcmp(z-a.z)<0:y<a.y:x<a.x;
  }
  double len()
  {
      return sqrt(len2());
  }
  double len2()
  {
      return x*x+y*y+z*z;
  }
  double distance(point3 p)
  {
      return sqrt((p.x-x)*(p.x-x)+(p.y-y)*(p.y-y)+(p.z-z)*(p.z-z));
  }
  point3 add(point3 p)
  {
      return point3(x+p.x,y+p.y,z+p.z);
  }
  point3 sub(point3 p)
  {
      return point3(x-p.x,y-p.y,z-p.z);
  }
 point3 mul(double d)
 {
   return point3(x*d,y*d,z*d);
 }
 point3 div(double d)
 {
   return point3(x/d,y/d,z/d);
 }
 double dot(point3 p)
  {
      return x*p.x+y*p.y+z*p.z;
  }
  point3 det(point3 p)
  {
      return point3(y*p.z-p.y*z,p.x*z-x*p.z,x*p.y-p.x*y);
  }
  double rad(point3 a,point3 b)
  {
    point3 p=(*this);
    return acos(a.sub(p).dot(b.sub(p))/(a.distance(p)*b.distance(p)));
  }
  point3 trunc(double r)
  {
      r/=len();
      return point3(x*r,y*r,z*r);
  }
  point3 rotate(point3 o,double r) // building?
  {
  }
};
```

4.9 polygon

```
struct polygon
{
    int n;
    point p[maxp];
    line l[maxp];
    void input()
    {
      n=4;
      p[0].input();
      p[2].input();
      double dis=p[0].distance(p[2]);
      p[1]=p[2].rotate(p[0],pi/4);
      p[1]=p[0].add((p[1].sub(p[0])).trunc(dis/sqrt(2.0)));
      p[3]=p[2].rotate(p[0],2*pi-pi/4);
      p[3]=p[0].add((p[3].sub(p[0])).trunc(dis/sqrt(2.0)));
    }
    void add(point q)
    {
      p[n++]=q;
    }
    void getline()
    {
        for (int i=0;i<n;i++)
        {
            l[i]=line(p[i],p[(i+1)%n]);
        }
    }
    struct cmp
  {
      point p;
      cmp(const point &p0){p=p0;}
      bool operator()(const point &aa,const point &bb)
      {
          point a=aa,b=bb;
          int d=dblcmp(a.sub(p).det(b.sub(p)));
          if (d==0)
          {
              return dblcmp(a.distance(p)-b.distance(p))<0;
          }
          return d>0;
      }
  };
    void norm()
    {
        point mi=p[0];
        for (int i=1;i<n;i++)mi=min(mi,p[i]);
        sort(p,p+n,cmp(mi));
    }
    void getconvex(polygon &convex)
    {
        int i,j,k;
        sort(p,p+n);
        convex.n=n;
        for (i=0;i<min(n,2);i++)
        {
            convex.p[i]=p[i];
        }
        if (n<=2)return;
        int &top=convex.n;
        top=1;
        for (i=2;i<n;i++)
        {
            while (top&&convex.p[top].sub(p[i]).det(convex.p[top-1].sub(p[i]))<=0)
                top--;
            convex.p[++top]=p[i];
        }
        int temp=top;
```

```
        convex.p[++top]=p[n-2];
        for (i=n-3;i>=0;i--)
        {
            while (top!=temp&&convex.p[top].sub(p[i]).det(convex.p[top-1].sub(p[i]))<=0)
                top--;
            convex.p[++top]=p[i];
        }
    }
    bool isconvex()
    {
      bool s[3];
      memset(s,0,sizeof(s));
      int i,j,k;
      for (i=0;i<n;i++)
      {
        j=(i+1)%n;
        k=(j+1)%n;
        s[dblcmp(p[j].sub(p[i]).det(p[k].sub(p[i])))+1]=1;
        if (s[0]&&s[2])return 0;
      }
      return 1;
    }
     //3 点上
   //2 边上
   //1 内部
   //0 外部
     int relationpoint(point q)
     {
       int i,j;
       for (i=0;i<n;i++)
       {
         if (p[i]==q)return 3;
       }
       getline();
       for (i=0;i<n;i++)
       {
         if (l[i].pointonseg(q))return 2;
       }
       int cnt=0;
       for (i=0;i<n;i++)
     {
       j=(i+1)%n;
       int k=dblcmp(q.sub(p[j]).det(p[i].sub(p[j])));
       int u=dblcmp(p[i].y-q.y);
       int v=dblcmp(p[j].y-q.y);
       if (k>0&&u<0&&v>=0)cnt++;
       if (k<0&&v<0&&u>=0)cnt--;
     }
     return cnt!=0;
     }
     //1 在多边形内长度为正
     //2 相交或与边平行
   //0 无任何交点
     int relationline(line u)
     {
       int i,j,k=0;
       getline();
       for (i=0;i<n;i++)
       {
         if (l[i].segcrossseg(u)==2)return 1;
         if (l[i].segcrossseg(u)==1)k=1;
       }
       if (!k)return 0;
       vector<point>vp;
       for (i=0;i<n;i++)
       {
         if (l[i].segcrossseg(u))
         {
           if (l[i].parallel(u))
           {
             vp.pb(u.a);
             vp.pb(u.b);
             vp.pb(l[i].a);
             vp.pb(l[i].b);
             continue;
           }
           vp.pb(l[i].crosspoint(u));
         }
       }
       sort(vp.begin(),vp.end());
       int sz=vp.size();
       for (i=0;i<sz-1;i++)
       {
         point mid=vp[i].add(vp[i+1]).div(2);
         if (relationpoint(mid)==1)return 1;
       }
       return 2;
     }
     //直线切割凸多边形左侧u
     //注意直线方向
     void convexcut(line u,polygon &po)
     {
         int i,j,k;
         int &top=po.n;
         top=0;
         for (i=0;i<n;i++)
         {
             int d1=dblcmp(p[i].sub(u.a).det(u.b.sub(u.a)));
             int d2=dblcmp(p[(i+1)%n].sub(u.a).det(u.b.sub(u.a)));
             if (d1>=0)po.p[top++]=p[i];
             if (d1*d2<0)po.p[top++]=u.crosspoint(line(p[i],p[(i+1)%n]));
         }
     }
     double getcircumference()
     {
         double sum=0;
         int i;
         for (i=0;i<n;i++)
         {
             sum+=p[i].distance(p[(i+1)%n]);
         }
         return sum;
     }
     double getarea()
     {
         double sum=0;
         int i;
         for (i=0;i<n;i++)
         {
             sum+=p[i].det(p[(i+1)%n]);
         }
         return fabs(sum)/2;
     }
     bool getdir()//代表逆时针1 代表顺时针0
     {
         double sum=0;
         int i;
         for (i=0;i<n;i++)
         {
             sum+=p[i].det(p[(i+1)%n]);
         }
         if (dblcmp(sum)>0)return 1;
         return 0;
     }
     point getbarycentre() // centroid
     {
         point ret(0,0);
         double area=0;
         int i;
         for (i=1;i<n-1;i++)
         {
             double tmp=p[i].sub(p[0]).det(p[i+1].sub(p[0]));
             if (dblcmp(tmp)==0)continue;
             area+=tmp;
             ret.x+=(p[0].x+p[i].x+p[i+1].x)/3*tmp;
             ret.y+=(p[0].y+p[i].y+p[i+1].y)/3*tmp;
         }
         if (dblcmp(area))ret=ret.div(area);
         return ret;
     }
     double areaintersection(polygon po) // refer: HPI
     {
     }
     double areaunion(polygon po)
     {
       return getarea()+po.getarea()-areaintersection(po);
     }
     double areacircle(circle c)
     {
     int i,j,k,l,m;
     double ans=0;
     for (i=0;i<n;i++)
     {
       int j=(i+1)%n;
       if (dblcmp(p[j].sub(c.p).det(p[i].sub(c.p)))>=0)
       {
         ans+=c.areatriangle(p[i],p[j]);
       }
       else
       {
         ans-=c.areatriangle(p[i],p[j]);
       }
     }
     return fabs(ans);
     }
     //多边形和圆关系
   //0 一部分在圆外
   //1 与圆某条边相切
   //2 完全在圆内
     int relationcircle(circle c)
     {
       getline();
       int i,x=2;
       if (relationpoint(c.p)!=1)return 0;
       for (i=0;i<n;i++)
       {
         if (c.relationseg(l[i])==2)return 0;
         if (c.relationseg(l[i])==1)x=1;
       }
       return x;
     }
     void find(int st,point tri[],circle &c)
     {
       if (!st)
       {
         c=circle(point(0,0),-2);
       }
       if (st==1)
       {
         c=circle(tri[0],0);
       }
       if (st==2)
       {
         c=circle(tri[0].add(tri[1]).div(2),tri[0].distance(tri[1])/2.0);
       }
       if (st==3)
       {
         c=circle(tri[0],tri[1],tri[2]);
       }
     }
     void solve(int cur,int st,point tri[],circle &c)
     {
       find(st,tri,c);
       if (st==3)return;
       int i;
       for (i=0;i<cur;i++)
       {
         if (dblcmp(p[i].distance(c.p)-c.r)>0)
         {
           tri[st]=p[i];
           solve(i,st+1,tri,c);
         }
       }
     }
     circle mincircle()//点集最小圆覆盖
   {
     random_shuffle(p,p+n);
     point tri[4];
     circle c;
     solve(n,0,tri,c);
     return c;
   }
   int circlecover(double r)//单位圆覆盖
   {
     int ans=0,i,j;
     vector<pair<double,int> >v;
     for (i=0;i<n;i++)
     {
       v.clear();
       for (j=0;j<n;j++)if (i!=j)
       {
         point q=p[i].sub(p[j]);
         double d=q.len();
         if (dblcmp(d-2*r)<=0)
         {
           double arg=atan2(q.y,q.x);
           if (dblcmp(arg)<0)arg+=2*pi;
           double t=acos(d/(2*r));
           v.push_back(make_pair(arg-t+2*pi,-1));
           v.push_back(make_pair(arg+t+2*pi,1));
         }
       }
       sort(v.begin(),v.end());
       int cur=0;
       for (j=0;j<v.size();j++)
       {
         if (v[j].second==-1)ans=max(ans,++cur);
         else --cur;
```

```
        ans=max(ans,cur);
      }
    }
    return ans+1;
  }
  int pointinpolygon(point q)//点在凸多边形内部的判定
  {
    if (getdir())reverse(p,p+n);
    if (dblcmp(q.sub(p[0]).det(p[n-1].sub(p[0])))==0)
    {
      if (line(p[n-1],p[0]).pointonseg(q))return n-1;
      return -1;
    }
    int low=1,high=n-2,mid;
    while (low<=high)
    {
      mid=(low+high)>>1;
      if (dblcmp(q.sub(p[0]).det(p[mid].sub(p[0])))>=0&&dblcmp(q.sub(p[0]).det(p[mid+1].
          sub(p[0])))<0)
      {
        polygon c;
        c.p[0]=p[mid];
        c.p[1]=p[mid+1];
        c.p[2]=p[0];
        c.n=3;
        if (c.relationpoint(q))return mid;
        return -1;
      }
      if (dblcmp(q.sub(p[0]).det(p[mid].sub(p[0])))>0)
      {
        low=mid+1;
      }
      else
      {
        high=mid-1;
      }
    }
    return -1;
  }
};
```

4.10 polygons

```
struct polygons
{
  vector<polygon>p;
  polygons()
  {
    p.clear();
  }
  void clear()
  {
    p.clear();
  }
  void push(polygon q)
  {
    if (dblcmp(q.getarea()))p.pb(q);
  }
  vector<pair<double,int> >e;
  void ins(point s,point t,point X,int i)
  {
    double r=fabs(t.x-s.x)>eps?(X.x-s.x)/(t.x-s.x):(X.y-s.y)/(t.y-s.y);
    r=min(r,1.0);r=max(r,0.0);
    e.pb(mp(r,i));
  }
  double polyareaunion()
  {
    double ans=0.0;
    int c0,c1,c2,i,j,k,w;
    for (i=0;i<p.size();i++)
    {
      if (p[i].getdir()==0)reverse(p[i].p,p[i].p+p[i].n);
    }
    for (i=0;i<p.size();i++)
    {
      for (k=0;k<p[i].n;k++)
      {
        point &s=p[i].p[k],&t=p[i].p[(k+1)%p[i].n];
        if (!dblcmp(s.det(t)))continue;
        e.clear();
        e.pb(mp(0.0,1));
        e.pb(mp(1.0,-1));
        for (j=0;j<p.size();j++)if (i!=j)
        {
          for (w=0;w<p[j].n;w++)
          {
            point a=p[j].p[w],b=p[j].p[(w+1)%p[j].n],c=p[j].p[(w+1+p[j].n)%p[j].n];
            c0=dblcmp(t.sub(s).det(c.sub(s)));
            c1=dblcmp(t.sub(s).det(a.sub(s)));
            c2=dblcmp(t.sub(s).det(b.sub(s)));
            if (c1*c2<0)ins(s,t,line(s,t).crosspoint(line(a,b)),-c2);
            else if (!c1&&c0*c2<0)ins(s,t,a,-c2);
            else if (!c1&&!c2)
            {
              int c3=dblcmp(t.sub(s).det(p[j].p[(w+2)%p[j].n].sub(s)));
              int dp=dblcmp(t.sub(s).dot(b.sub(a)));
              if (dp&&c0)ins(s,t,a,dp>0?c0*((j>i)^(c0<0)):-(c0<0));
              if (dp&&c3)ins(s,t,b,dp>0?-c3*((j>i)^(c3<0)):c3<0);
            }
          }
        }
        sort(e.begin(),e.end());
        int ct=0;
        double tot=0.0,last;
        for (j=0;j<e.size();j++)
        {
          if (ct==p.size())tot+=e[j].first-last;
          ct+=e[j].second;
          last=e[j].first;
        }
        ans+=s.det(t)*tot;
      }
    }
    return fabs(ans)*0.5;
  }
};
```

5 graph

5.1 Articulation

```
void dfs(int now,int fa)  // 从开始now1
{
    int p(0);
    dfn[now]=low[now]=cnt++;
    for(std::list<int>::const_iterator it(edge[now].begin());it!=edge[now].end();++it)
        if(dfn[*it]==-1)
        {
            dfs(*it,now);
            ++p;
            low[now]=std::min(low[now],low[*it]);
            if((now==1&& p>1) || (now!=1 && low[*it]>=dfn[now])) // 如果从出发点出发的子节点不能由兄弟节点到达，那么出发点为割点。如果现节点不是出发点，但是其子孙节点不能达到祖先节点，那么该节点为割点
                ans.insert(now);
        }
        else
            if(*it!=fa)
                low[now]=std::min(low[now],dfn[*it]);
}
```

5.2 Biconnected Component - Edge

```
// hdu 4612
#include<cstdio>
#include<algorithm>
#include<set>
#include<cstring>
#include<stack>
#include<queue>

#define MAXX 200111
#define MAXE (1000111*2)
#pragma comment(linker, "/STACK:16777216")

int edge[MAXX],to[MAXE],nxt[MAXE],cnt;
#define v to[i]
inline void add(int a,int b)
{
    nxt[++cnt]=edge[a];
    edge[a]=cnt;
    to[cnt]=b;
}

int dfn[MAXX],low[MAXX],col[MAXX],belong[MAXX];
int idx,bcnt;
std::stack<int>st;

void tarjan(int now,int last)
{
    col[now]=1;
    st.push(now);
    dfn[now]=low[now]=++idx;
    bool flag(false);
    for(int i(edge[now]);i;i=nxt[i])
    {
        if(v==last && !flag)
        {
            flag=true;
            continue;
        }
        if(!col[v])
        {
            tarjan(v,now);
            low[now]=std::min(low[now],low[v]);
            /*
            if(low[v]>dfn[now])
            then this is a bridge
            */
        }
        else
            if(col[v]==1)
                low[now]=std::min(low[now],dfn[v]);
    }
    col[now]=2;
    if(dfn[now]==low[now])
    {
        ++bcnt;
        static int x;
        do
        {
            x=st.top();
            st.pop();
            belong[x]=bcnt;
        }while(x!=now);
    }
}

std::set<int>set[MAXX];

int dist[MAXX];
std::queue<int>q;
int n,m,i,j,k;

inline int go(int s)
{
    static std::set<int>::const_iterator it;
    memset(dist,0x3f,sizeof dist);
    dist[s]=0;
    q.push(s);
    while(!q.empty())
    {
        s=q.front();
        q.pop();
        for(it=set[s].begin();it!=set[s].end();++it)
            if(dist[*it]>dist[s]+1)
            {
                dist[*it]=dist[s]+1;
                q.push(*it);
            }
    }
    return std::max_element(dist+1,dist+1+bcnt)-dist;
}

int main()
{
    while(scanf("%d %d",&n,&m),(n||m))
    {
        cnt=0;
        memset(edge,0,sizeof edge);
        while(m--)
        {
            scanf("%d %d",&i,&j);
            add(i,j);
            add(j,i);
        }

        memset(dfn,0,sizeof dfn);
        memset(belong,0,sizeof belong);
```

```
        memset(low,0,sizeof low);
        memset(col,0,sizeof col);
        bcnt=idx=0;
        while(!st.empty())
            st.pop();

        tarjan(1,-1);
        for(i=1;i<=bcnt;++i)
            set[i].clear();
        for(i=1;i<=n;++i)
            for(j=edge[i];j;j=nxt[j])
                set[belong[i]].insert(belong[to[j]]);
        for(i=1;i<=bcnt;++i)
            set[i].erase(i);
        /*
        printf("%d\n",dist[go(go(1))]);
        for(i=1;i<=bcnt;++i)
            printf("%d\n",dist[i]);
        puts("");
        */
        printf("%d\n",bcnt-1-dist[go(go(1))]);
    }
    return 0;
}
```

5.3 Blossom algorithm

```
#include<cstdio>
#include<vector>
#include<cstring>
#include<algorithm>

#define MAXX 233

bool map[MAXX][MAXX];
std::vector<int>p[MAXX];
int m[MAXX];
int vis[MAXX];
int q[MAXX],*qf,*qb;

int n;

inline void label(int x,int y,int b)
{
    static int i,z;
    for(i=b+1;i<p[x].size();++i)
        if(vis[z=p[x][i]]==1)
        {
            p[z]=p[y];
            p[z].insert(p[z].end(),p[x].rbegin(),p[x].rend()-i);
            vis[z]=0;
            *qb++=z;
        }
}

inline bool bfs(int now)
{
    static int i,x,y,z,b;
    for(i=0;i<n;++i)
        p[i].resize(0);
    p[now].push_back(now);
    memset(vis,-1,sizeof vis);
    vis[now]=0;
    qf=qb=q;
    *qb++=now;

    while(qf<qb)
        for(x=*qf++,y=0;y<n;++y)
            if(map[x][y] && m[y]!=y && vis[y]!=1)
            {
                if(vis[y]==-1)
                    if(m[y]==-1)
                    {
                        for(i=0;i+1<p[x].size();i+=2)
                        {
                            m[p[x][i]]=p[x][i+1];
                            m[p[x][i+1]]=p[x][i];
                        }
                        m[x]=y;
                        m[y]=x;
                        return true;
                    }
                    else
                    {
                        p[z=m[y]]=p[x];
                        p[z].push_back(y);
                        p[z].push_back(z);
                        vis[y]=1;
                        vis[z]=0;
                        *qb++=z;
                    }
                else
                {
                    for(b=0;b<p[x].size() && b<p[y].size() && p[x][b]==p[y][b];++b);
                    --b;
                    label(x,y,b);
                    label(y,x,b);
                }
            }
    return false;
}

int i,j,k;
int ans;

int main()
{
    scanf("%d",&n);
    for(i=0;i<n;++i)
        p[i].reserve(n);
    while(scanf("%d %d",&i,&j)!=EOF)
    {
        --i;
        --j;
        map[i][j]=map[j][i]=true;
    }
    memset(m,-1,sizeof m);
    for(i=0;i<n;++i)
        if(m[i]==-1)
        {
            if(bfs(i))
                ++ans;
            else
                m[i]=i;
        }
    printf("%d\n",ans<<1);
    for(i=0;i<n;++i)
        if(i<m[i])
            printf("%d %d\n",i+1,m[i]+1);
    return 0;
}
```

5.4 chu-liu algorithm

```
#include<cstdio>
#include<cstring>
#include<algorithm>

const int inf = 0x5fffffff;

int n,m,u,v,cost,dis[1001][1001],L;
int pre[1001],id[1001],visit[1001],in[1001];

void init(int n)
{
    L = 0;
    for (int i = 0; i < n; i++)
        for (int j = 0; j < n; j++)
            dis[i][j] = inf;
}

struct Edge
{
    int u,v,cost;
};

Edge e[1001*1001];


int zhuliu(int root,int n,int m,Edge e[])
{
    int res = 0,u,v;
    while (true)
    {
        for (int i = 0; i < n; i++)
            in[i] = inf;
        for (int i = 0; i < m; i++)
            if (e[i].u != e[i].v && e[i].cost < in[e[i].v])
            {
                pre[e[i].v] = e[i].u;
                in[e[i].v] = e[i].cost;
            }
        for (int i = 0; i < n; i++)
            if (i != root)
                if (in[i] == inf)
                    return -1;
        int tn = 0;
        memset(id,-1,sizeof(id));
        memset(visit,-1,sizeof(visit));
        in[root] = 0;
        for (int i = 0; i < n; i++)
        {
            res += in[i];
            v = i;
            while (visit[v] != i && id[v] == -1 && v != root)
            {
                visit[v] = i;
                v = pre[v];
            }
            if(v != root && id[v] == -1)
            {
                for(int u = pre[v] ; u != v ; u = pre[u])
                    id[u] = tn;
                id[v] = tn++;
            }
        }
        if(tn == 0) break;
        for (int i = 0; i < n; i++)
            if (id[i] == -1)
                id[i] = tn++;
        for (int i = 0; i < m;)
        {
            int v = e[i].v;
            e[i].u = id[e[i].u];
            e[i].v = id[e[i].v];
            if (e[i].u != e[i].v)
                e[i++].cost -= in[v];
            else
                std::swap(e[i],e[--m]);
        }
        n = tn;
        root = id[root];
    }
    return res;
}

int main()
{
    while (scanf("%d%d",&n,&m) != EOF)
    {
        init(n);
        for (int i = 0; i < m; i++)
        {
            scanf("%d%d%d",&u,&v,&cost);
            if (u == v) continue;
            dis[u][v] = std::min(dis[u][v],cost);
        }
        L = 0;
        for (int i = 0; i < n; i++)
            for (int j = 0; j < n; j++)
                if (dis[i][j] != inf)
                {
                    e[L].u = i;
                    e[L].v = j;
                    e[L++].cost = dis[i][j];
                }
        printf("%d\n",zhuliu(0,n,L,e));
    }
    return 0;
}
```

5.5 Covering problems

最大团以及相关知识独立集：独立集是指图的顶点集的一个子集该子集的导出子图的点互不相邻如果一个独立集不是任何一个独立集的子集

,.，那么称这个独立集是一个极大独立集一个图中包含顶点数目最多的独立集称为最大独立集。最大独立集一定是极大独立集，但是极大独立集不一定是最大的独立集。.支配集：与独立集相对应的就是支配集，支配集也是图顶点集的一个子集，设是图的一个支配集，则对于图中的任意一个顶点，要么属于集合

```
 SGus, 要么与中的顶点相邻。在中除去任何元素后不再是支配集，则支配集是极小支配集。称的所有支配集中顶点个数最少的支配集为最小支配集，最小支配集中的顶点个数成为支配数。ssssG最大团：图的顶点的子集，设是最大团，则中任意两点相邻。若，是最大团，则

 GDDuvu,有边相连，其补图vu,没有边相连，所以图的最大团vG其补图的最大独立集。给定无向图=G = (V;E)，如果属于，并且对于任意UVu,包含于vU 有< u; v 包含于>，则称是的完全子图，的完全子图是的团，当且仅当不包含在的更大的完全子图中，的最大团是指中所含顶点数目最多的团。如果属于，并且对于任意EUGGUGUGGGUVu; 包含于有vU< u; v 不包含于>，则称是的空子图，的空子图是的独立集，当且仅当不包含在的更大的独立集，的最大团是指中所含顶点数目最多的独立集。EUGGUGUGGG性质：最大独立集最小覆盖集



+= V最大团补图的最大独立集
=最小覆盖集最大匹配
=

minimum cover:
vertex cover vertex bipartite graph = maximum cardinality bipartite matching找完最大二分匹配後，有三種情況要分別處理：甲、

 X 側未匹配點的交錯樹們。乙、
 Y 側未匹配點的交錯樹們。丙、層層疊疊的交錯環們（包含單獨的匹配邊）。這三個情況互不干涉。用

 Graph Traversal 建立甲、乙的交錯樹們，剩下部分就是丙。要找點覆蓋，甲、乙是取盡奇數距離的點，丙是取盡偶數距離的點、或者是取盡奇數距離的點，每塊連通分量可以各自為政。另外，小心處理的話，是可以印出字典順序最小的點覆蓋的。已經有最大匹配時，求點覆蓋的時間複雜度等同於一次

 Graph Traversal 的時間。

vertex cover edge

edge cover vertex首先在圖上求得一個
 Maximum Matching 之後，對於那些單身的點，都由匹配點連過去。如此便形成了 Minimum Edge Cover 。

edge cover edge

path cover vertex
general graph: NP-H
tree: DP
DAG: 将每个节点拆分为入点和出点,ans节点数匹配数=-

path cover edge
minimize the count of euler path ( greedy is ok? )

cycle cover vertex
general: NP-H
weighted: do like path cover vertex, with KM algorithm

cycle cover edge
NP-H
```

5.6 Difference constraints

```
for a - b<= c
    add(b,a,c);最短路得最远解最长路得最近解根据情况反转边反转方向及边权



//?()全点得普通解

0
```

5.7 Flow network

```
Maximum weighted closure of a graph:所有由这个子图中的点出发的边都指向这个子图，那么这个子图为原图的一个（闭合子图）

closure每个节点向其所有依赖节点连边，容量

inf源点向所有正权值节点连边，容量为该权值所有负权值节点向汇点连边，容量为该权值绝对值以上均为有向边最大权为



sum正权值新图的最小割{}-{}残量图中所有由源点可达的点即为所选子图




Eulerian circuit:计入度和出度之差无向边任意定向出入度之差为奇数则无解然后构图



:原图有向边不变，容量
1 // 好像需要在新图中忽略有向边？无向边按之前认定方向，容量
1源点向所有度数为正的点连边，容量
abs度数(/2)所有度数为负的点向汇点连边，容量
abs度数(/2)两侧均满流则有解相当于规约为可行流问题注意连通性的


trick终点到起点加一条有向边即可将问题转为问题

pathcircuit



Feasible flow problem:
refer Feasible flow problem.cpp由超级源点出发的边全部满流则有解有源汇时，由汇点向源点连边，下界上界

0即可转化为无源无汇上下界流inf对于每条边

<a->b cap{u,d, 建边}><ss->b cap(u)、><a->st cap(u)、><a->b cap(d-u)>

Maximum flow: 好像也可以二分//将流量还原至原图后，在残量网络上继续完成最大流
//直接把和设为原来的，此时输出的最大流即是答案
sourcesinkst不需要删除或者调整
t->弧s
Minimum flow: 好像也可以二分//建图时先不连汇点到源点的边，新图中完成最大流之后再连原汇至原源的边完成第二次最大流，此时
t->这条弧的流量即为最小流s判断可行流存在还是必须连原汇原源的边之后查看满流
->所以可以使用跑流加
 ->弧ts跑流，-> 最后检查超级源点满流情况来一步搞定
tips:合并流量、减少边数来加速




Minimum cost feasible flow problem:
TODO看起来像是在上面那样跑费用流就行了……




Minimum weighted vertex cover edge for bipartite graph:
for all vertex in X:
edge < s->x cap(weight(x)) >
for all vertex in Y:
edge < y->t cap(weight(y)) >
for original edges
edge < x->y cap(inf) >

ans={maximum flow}={minimum cut}残量网络中的所有简单割
( 源点可达( && 汇点不可达) || 源点不可达( && 汇点可达) )对应着解



Maximum weighted vertex independent set for bipartite graph:
ans=Sum点权{}-value{Minimum weighted vertex cover edge}解应该就是最小覆盖集的补图吧……方格取数




: // refer: hdu 3820 golden eggs取方格获得收益当取了相邻方格时付出边的代价必取的方格到源汇的边的容量



/inf相邻方格之间的边的容量为代价
*2
ans=sum方格收益最大流{}-{}最小割的唯一性



: // refer关键边。有向边起点为:集，终点为集st从源和汇分别能够到的点集是所有点时，最小割唯一也就是每一条增广路径都仅有一条边满流注意查看的是实际的网络，不是残量网络具体来说






void rr(int now)
{
    done[now]=true;
    ++cnt;
    for(int i(edge[now]);i!=-1;i=nxt[i])
        if(cap[i] && !done[v])
            rr(v);
}

void dfs(int now)
{
    done[now]=true;
    ++cnt;
    for(int i(edge[now]);i!=-1;i=nxt[i])
        if(cap[i^1] && !done[v])
            dfs(v);
}

memset(done,0,sizeof done);
cnt=0;
rr(source);
dfs(sink);
puts(cnt==n?"UNIQUE":"AMBIGUOUS");



Tips:两点间可以不止有一种边，也可以不止有一条边，无论有向无向
;两点间容量则可以设法化简为一个点
inf;点权始终要转化为边权
;不参与决策的边权设为来排除掉
inf;贪心一个初始不合法情况，然后通过可行流调整
; // refer: 混合图欧拉回路存在性、有向无向图中国邮差问题遍历所有边至少一次后回到原点/()按时间拆点时间层……?
();
```

5.8 Hamiltonian circuit

```
//if every point connect with not less than [(N+1)/2] points
#include<cstdio>
#include<algorithm>
#include<cstring>

#define MAXX 177
#define MAX (MAXX*MAXX)

int edge[MAXX],nxt[MAX],to[MAX],cnt;

inline void add(int a,int b)
{
    nxt[++cnt]=edge[a];
    edge[a]=cnt;
    to[cnt]=b;
}

bool done[MAXX];
int n,m,i,j,k;

inline int find(int a)
{
    static int i;
    for(i=edge[a];i;i=nxt[i])
        if(!done[to[i]])
        {
            edge[a]=nxt[i];
            return to[i];
        }
    return 0;
}

int a,b;
int next[MAXX],pre[MAXX];
bool mat[MAXX][MAXX];

int main()
{
    while(scanf("%d_%d",&n,&m)!=EOF)
    {
        for(i=1;i<=n;++i)
            next[i]=done[i]=edge[i]=0;
        memset(mat,0,sizeof mat);
        cnt=0;
        while(m--)
        {
            scanf("%d_%d",&i,&j);
            add(i,j);
            add(j,i);
            mat[i][j]=mat[j][i]=true;
        }
        a=1;
        b=to[edge[a]];
        cnt=2;
        done[a]=done[b]=true;
        next[a]=b;
        while(cnt<n)
```

```
        {
            while(i=find(a))
            {
                next[i]=a;
                done[a=i]=true;
                ++cnt;
            }
            while(i=find(b))
            {
                next[b]=i;
                done[b=i]=true;
                ++cnt;
            }
            if(!mat[a][b])
                for(i=next[a];next[i]!=b;i=next[i])
                    if(mat[a][next[i]] && mat[i][b])
                    {
                        for(j=next[i];j!=b;j=next[j])
                            pre[next[j]]=j;
                        for(j=b;j!=next[i];j=pre[j])
                            next[j]=pre[j];
                        std::swap(next[i],b);
                        break;
                    }
            next[b]=a;
            for(i=a;i!=b;i=next[i])
                if(find(i))
                {
                    a=next[b=i];
                    break;
                }
        }
        while(a!=b)
        {
            printf("%d ",a);
            a=next[a];
        }
        printf("%d\n",b);
    }
    return 0;
}
```

5.9 Hopcroft-Karp algorithm

```
#include<cstdio>
#include<cstring>

#define MAXX 50111
#define MAX 150111

int nx,p;
int i,j,k;
int x,y;
int ans;
bool flag;

int edge[MAXX],nxt[MAX],to[MAX],cnt;

int cx[MAXX],cy[MAXX];
int px[MAXX],py[MAXX];

int q[MAXX],*qf,*qb;

bool ag(int i)
{
    int j,k;
    for(k=edge[i];k;k=nxt[k])
        if(py[j=to[k]]==px[i]+1)
        {
            py[j]=0;
            if(cy[j]==-1 || ag(cy[j]))
            {
                cx[i]=j;
                cy[j]=i;
                return true;
            }
        }
    return false;
}

int main()
{
    scanf("%d %*d %d",&nx,&p);
    while(p--)
    {
        scanf("%d %d",&i,&j);
        nxt[++cnt]=edge[i];
        edge[i]=cnt;
        to[cnt]=j;
    }
    memset(cx,-1,sizeof cx);
    memset(cy,-1,sizeof cy);
    while(true)
    {
        memset(px,0,sizeof(px));
        memset(py,0,sizeof(py));
        qf=qb=q;
        flag=false;

        for(i=1;i<=nx;++i)
            if(cx[i]==-1)
                *qb++=i;
        while(qf!=qb)
            for(k=edge[i=*qf++];k;k=nxt[k])
                if(!py[j=to[k]])
                {
                    py[j]=px[i]+1;
                    if(cy[j]==-1)
                        flag=true;
                    else
                    {
                        px[cy[j]]=py[j]+1;
                        *qb++=cy[j];
                    }
                }
        if(!flag)
            break;
        for(i=1;i<=nx;++i)
            if(cx[i]==-1 && ag(i))
                ++ans;
    }
    printf("%d\n",ans);
    return 0;
}
```

5.10 Improved Shortest Augmenting Path Algorithm

```
int source,sink;

inline long long go(const int N=sink)
{
    static int now,N,i;
    static long long min,mf;
    memset(gap,0,sizeof gap);
    memset(h,0,sizeof h);
    memcpy(w,edge,sizeof w);
    gap[0]=N;
    mf=0;

    pre[now=source]=-1;
    while(h[source]<N)
    {
rep:
        if(now==sink)
        {
            min=inf;
            for(i=pre[sink];i!=-1;i=pre[to[i^1]])
                if(min>=cap[i])
                {
                    min=cap[i];
                    now=to[i^1];
                }
            for(i=pre[sink];i!=-1;i=pre[to[i^1]])
            {
                cap[i]-=min;
                cap[i^1]+=min;
            }
            mf+=min;
        }
        for(int &i(w[now]);i!=-1;i=nxt[i])
            if(cap[i] && h[v]+1==h[now])
            {
                pre[now=v]=i;
                goto rep;
            }
        if(!--gap[h[now]])
            return mf;
        min=N;
        for(i=w[now]=edge[now];i!=-1;i=nxt[i])
            if(cap[i])
                min=std::min(min,(long long)h[v]);
        ++gap[h[now]=min+1];
        if(now!=source)
            now=to[pre[now]^1];
    }
    return mf;
}
```

5.11 k Shortest Path

```
#include<cstdio>
#include<cstring>
#include<queue>
#include<vector>

int K;

class states
{
    public:
        int cost,id;
};

int dist[1000];

class cmp
{
    public:
        bool operator ()(const states &i,const states &j)
        {
            return i.cost>j.cost;
        }
};

class cmp2
{
    public:
        bool operator ()(const states &i,const states &j)
        {
            return i.cost+dist[i.id]>j.cost+dist[j.id];
        }
};

struct edges
{
    int to,next,cost;
} edger[100000],edge[100000];

int headr[1000],head[1000],Lr,L;

void dijkstra(int s)
{
    states u;
    u.id=s;
    u.cost=0;
    dist[s]=0;
    std::priority_queue<states,std::vector<states>,cmp> q;
    q.push(u);
    while (!q.empty())
    {
        u=q.top();
        q.pop();
        if (u.cost!=dist[u.id])
            continue;
        for (int i=headr[u.id]; i!=-1; i=edger[i].next)
        {
            states v=u;
            v.id=edger[i].to;
            if (dist[v.id]>dist[u.id]+edger[i].cost)
            {
                v.cost=dist[v.id]=dist[u.id]+edger[i].cost;
                q.push(v);
            }
        }
    }
}

int num[1000];
```

```
inline void init(int n)
{
    Lr=L=0;
    memset(head,-1,4*n);
    memset(headr,-1,4*n);
    memset(dist,63,4*n);
    memset(num,0,4*n);
}

void add_edge(int u,int v,int x)
{
    edge[L].to=v;
    edge[L].cost=x;
    edge[L].next=head[u];
    head[u]=L++;
    edger[Lr].to=u;
    edger[Lr].cost=x;
    edger[Lr].next=headr[v];
    headr[v]=Lr++;
}

inline int a_star(int s,int t)
{
    if (dist[s]==0x3f3f3f3f)
        return -1;
    std::priority_queue<states,std::vector<states>,cmp2> q;
    states tmp;
    tmp.id=s;
    tmp.cost=0;
    q.push(tmp);
    while (!q.empty())
    {
        states u=q.top();
        q.pop();
        num[u.id]++;
        if (num[t]==K)
            return u.cost;
        for (int i=head[u.id]; i!=-1; i=edge[i].next)
        {
            int v=edge[i].to;
            tmp.id=v;
            tmp.cost=u.cost+edge[i].cost;
            q.push(tmp);
        }
    }
    return -1;
}

int main()
{
    int n,m;
    scanf("%d%d",&n,&m);
    init(n);
    for (int i=0; i<m; i++)
    {
        int u,v,x;
        scanf("%d%d%d",&u,&v,&x);
        add_edge(u-1,v-1,x);
    }
    int s,t;
    scanf("%d%d%d",&s,&t,&K);
    if (s==t)
        ++K;
    dijkstra(t-1);
    printf("%d\n",a_star(s-1,t-1));
}
```

5.12 Kariv-Hakimi Algorithm

```
//Absolute Center of a graph, not only a tree
#include<cstdio>
#include<algorithm>
#include<vector>
#include<cstring>
#include<set>

#define MAXX 211
#define inf 0x3f3f3f3f

int e[MAXX][MAXX],dist[MAXX][MAXX];
double dp[MAXX],ta;
int ans,d;
int n,m,a,b;
int i,j,k;
typedef std::pair<int,int> pii;
std::vector<pii>vt[2];
bool done[MAXX];
typedef std::pair<double,int> pdi;
std::multiset<pdi>q;
int pre[MAXX];

int main()
{
    vt[0].reserve(MAXX);
    vt[1].reserve(MAXX);
    scanf("%d%d",&n,&m);
    memset(e,0x3f,sizeof(e));
    while(m--)
    {
        scanf("%d%d%d",&i,&j,&k);
        e[i][j]=e[j][i]=std::min(e[i][j],k);
    }
    for(i=1;i<=n;++i)
        e[i][i]=0;
    memcpy(dist,e,sizeof(dist));
    for(k=1;k<=n;++k)
        for(i=1;i<=n;++i)
            for(j=1;j<=n;++j)
                dist[i][j]=std::min(dist[i][j],dist[i][k]+dist[k][j]);
    ans=inf;
    for(i=1;i<=n;++i)
        for(j=i;j<=n;++j)
            if(e[i][j]!=inf)
            {
                vt[0].resize(0);
                vt[1].resize(0);
                static int i;
                for(i=1;i<=n;++i)
                    vt[0].push_back(pii(dist[i][i],dist[j][i]));
                std::sort(vt[0].begin(),vt[0].end());
                for(i=0;i<vt[0].size();++i)
                {
                    while(!vt[1].empty() && vt[1].back().second<=vt[0][i].second)
                        vt[1].pop_back();
                    vt[1].push_back(vt[0][i]);
                }
                d=inf;
                if(vt[1].size()==1)
                    if(vt[1][0].first<vt[1][0].second)
                    {
                        ta=0;
                        d=(vt[1][0].first<<1);
                    }
                    else
                    {
                        ta=e[i][j];
                        d=(vt[1][0].second<<1);
                    }
                else
                    for(i=1;i<vt[1].size();++i)
                        if(d>e[i][j]+vt[1][i-1].first+vt[1][i].second)
                        {
                            ta=(e[i][j]+vt[1][i].second-vt[1][i-1].first)/(double)2.0f;
                            d=e[i][j]+vt[1][i-1].first+vt[1][i].second;
                        }
                if(d<ans)
                {
                    ans=d;
                    a=i;
                    b=j;
                    dp[i]=ta;
                    dp[j]=e[i][j]-ta;
                }
            }
    printf("%d\n",ans);
    for(i=1;i<=n;++i)
        if(i!=a && i!=b)
            dp[i]=1e20;
    q.insert(pdi(dp[a],a));
    if(a!=b)
        q.insert(pdi(dp[b],b));
    if(a!=b)
        pre[b]=a;
    while(!q.empty())
    {
        k=q.begin()->second;
        q.erase(q.begin());
        if(done[k])
            continue;
        done[k]=true;
        for(i=1;i<=n;++i)
            if(e[k][i]!=inf && dp[k]+e[k][i]<dp[i])
            {
                dp[i]=dp[k]+e[k][i];
                q.insert(pdi(dp[i],i));
                pre[i]=k;
            }
    }
    vt[0].resize(0);
    for(i=1;i<=n;++i)
        if(pre[i])
            if(i<pre[i])
                printf("%d %d\n",i,pre[i]);
            else
                printf("%d %d\n",pre[i],i);
    return 0;
}
```

5.13 Kuhn-Munkres algorithm

```
bool match(int u)
{
    vx[u]=true;
    for(int i=1;i<=n;++i)
        if(lx[u]+ly[i]==g[u][i]&&!vy[i])
        {
            vy[i]=true;
            if(!d[i]||match(d[i]))
            {
                d[i]=u;
                return true;
            }
        }
    return false;
}

inline void update()
{
    int i,j;
    int a=1<<30;
    for(i=1;i<=n;++i)if(vx[i])
        for(j=1;j<=n;++j)if(!vy[j])
            a=min(a,lx[i]+ly[j]-g[i][j]);
    for(i=1;i<=n;++i)
    {
        if(vx[i])lx[i]-=a;
        if(vy[i])ly[i]+=a;
    }
}

void km()
{
    int i,j;
    for(i=1;i<=n;++i)
    {
        lx[i]=ly[i]=d[i]=0;
        for(j=1;j<=n;++j)
            lx[i]=max(lx[i],g[i][j]);
    }
    for(i=1;i<=n;++i)
    {
        while(true)
        {
            memset(vx,0,sizeof(vx));
            memset(vy,0,sizeof(vy));
            if(match(i))
                break;
            update();
        }
    }
    int ans=0;
    for(i=1;i<=n;++i)
        if(d[i]!=0)
            ans+=g[d[i]][i];
    printf("%d\n",ans);
}

int main()
{
    while(scanf("%d",&n)!=EOF)
    {
        for(int i=1;i<=n;++i)gets(s[i]);
        memset(g,0,sizeof(g));
        for(int i=1;i<=n;++i)
```

```
            for(int j=1;j<=n;++j)
                if(i!=j)
                    g[i][j]=cal(s[i],s[j]);
        km();
    }
    return 0;
}
```

5.14 Manhattan minimum spanning tree

```
#include <cstdio>
#include <algorithm>
#include <cstring>
#include <iostream>

using namespace std;

const int maxn = 60000;

struct node {int x, y, k[2];} b[maxn];
struct bian {int a, b, c;} g[maxn * 8];
struct point{int k[2];} d[maxn * 8];
long long s[maxn], ans;
int i, n, m, a[maxn], lim, h, mid, bh[maxn * 2], f[maxn], num, e[maxn * 2], next[maxn * 2], first[maxn], tot;
int comx(int p, int q) {return b[p].x < b[q].x;}
int comy(int p, int q) {return b[p].y < b[q].y;}
int comc(const bian &p, const bian &q) {return p.c < q.c;}
int dist(int p, int q) {return abs(b[p].x - b[q].x) + abs(b[p].y - b[q].y);}
int maxbh(int p, int q, int k) {return b[p].k[k] > b[q].k[k] ? p : q;}
int minbh(int p, int q, int k) {return b[p].k[k] < b[q].k[k] ? p : q;}
int getfa(int x) {if (f[x] != x) f[x] = getfa(f[x]); return f[x];}
long long gcd(long long p, long long q) {return (!p || !q) ? p + q : gcd(q, p % q);}
void link(int u, int v)
{
    e[++num] = v, next[num] = first[u], first[u] = num;
    e[++num] = u, next[num] = first[v], first[v] = num;
}
void add(int x, int k)
{
    int y = h + b[x].k[1]; d[y].k[0] = minbh(d[y].k[0], x, 0);
    for (y >>= 1; y; y >>= 1) d[y].k[0] = minbh(d[y << 1].k[0], d[y << 1 ^ 1].k[0], 0);
    y = h + b[x].k[0];
    d[y].k[1] = k ? maxbh(x, d[y].k[1], 1) : minbh(d[y].k[1], x, 1);
    for (y >>= 1; y; y >>= 1)
        d[y].k[1] = k ? maxbh(x, d[y << 1].k[1], 1) : minbh(d[y << 1 ^ 1].k[1], x, 1);
}
int ask(int l, int r, int k, int boss)
{
    for (mid = 0, l += h - 1, r += h + 1; (l ^ r) != 1; l >>= 1, r >>= 1)
    {
        if (!(l & 1)) mid = boss ? maxbh(mid, d[l + 1].k[k], k) : minbh(mid, d[r - 1].k[k], k);
        if (r & 1) mid = boss ? maxbh(mid, d[r - 1].k[k], k) : minbh(mid, d[r - 1].k[k], k);
    } return mid;
}
void manhattan()
{
    sort(bh + 1, bh + m + 1, comx);
    b[0].k[0] = maxn * 3, b[0].k[1] = -1;
    for (add(bh[m], 1), i = m - 1; i; add(bh[i], 1), --i)
    {
        g[++tot].a = bh[i], g[tot].b = ask(b[bh[i]].k[1], lim, 0, 0);
        g[tot].c = dist(g[tot].a, g[tot].b);
        if (g[tot].b == 0) --tot;
        g[++tot].a = bh[i], g[tot].b = ask(1, b[bh[i]].k[0], 1, 1);
        g[tot].c = dist(g[tot].a, g[tot].b);
        if (g[tot].b == 0) --tot;
    }
    b[0].k[1] = b[0].k[0];
    memset(d, 0, sizeof(d));
    sort(bh + 1, bh + m + 1, comy);
    for (add(bh[m], 0), i = m - 1; i; add(bh[i], 0), --i)
    {
        g[++tot].a = bh[i], g[tot].b = ask(1, b[bh[i]].k[1], 0, 0);
        g[tot].c = dist(g[tot].a, g[tot].b);
        if (g[tot].b == 0) --tot;
        g[++tot].a = bh[i], g[tot].b = ask(1, b[bh[i]].k[0], 1, 0);
        g[tot].c = dist(g[tot].a, g[tot].b);
        if (g[tot].b == 0) --tot;
    }
}
void kruskal()
{
    sort(g + 1, g + tot + 1, comc);
    for (i = 1; i <= tot; ++i)
    {
        int f1 = getfa(g[i].a), f2 = getfa(g[i].b);
        if (f1 != f2) link(g[i].a, g[i].b), f[f1] = f2;
    } tot = 0; memset(f, 0, sizeof(f));
}
void dfs(int x, int fa)
{
    bh[++tot] = x;
    for (int p = first[x]; p; p = next[p])
        if (e[p] != fa) dfs(e[p], x), bh[++tot] = x;
}
void del(int l, int r)
{
    if (l > r) return ;
    for (int j = l; j <= r; ++j)
        ans -= 1LL * f[a[j]] * f[a[j]], ans += 1LL * (--f[a[j]]) * f[a[j]];
}
void ins(int l, int r)
{
    if (l > r) return ;
    for (int j = l; j <= r; ++j)
        ans -= 1LL * f[a[j]] * f[a[j]], ans += 1LL * (++f[a[j]]) * f[a[j]];
}
int main()
{
    freopen("hose.in", "r", stdin);
    freopen("hose.out", "w", stdout);
    scanf("%d%d", &n, &m);
    for (i = 1; i <= n; ++i)
        scanf("%d", a+i);
    for (i = 1; i <= m; ++i)
    {
        scanf("%d%d", &b[bh[i] = f[i] = i].x, &b[i].y);
        b[i].k[0] = b[i].x + b[i].y;
        b[i].k[1] = b[i].y - b[i].x + maxn;
        lim = max(lim, max(b[i].k[0], b[i].k[1]));
    }
    for (h = 1; h <= lim; h <<= 1);
    manhattan();
    kruskal();
    dfs(1, 0);
    ins(b[bh[1]].x, b[bh[1]].y);
    for (s[1] = ans, i = 2; i <= tot; s[bh[i]] = ans, ++i)
    {
        ins(b[bh[i]].x, b[bh[i - 1]].x - 1);
        ins(b[bh[i - 1]].y + 1, b[bh[i]].y);
        del(b[bh[i - 1]].x, b[bh[i]].x - 1);
        del(b[bh[i]].y + 1, b[bh[i - 1]].y);
    }
    for (i = 1; i <= m; ++i)
    {
        long long fz = s[i] - b[i].k[1] - 1 + maxn, fm = 1LL * (b[i].k[1] + 1 - maxn) * (b[i].k[1] - maxn);
        long long gys = gcd(fz, fm);
        printf("%lld/%lld\n", fz/gys, fm/gys);
    }
    return 0;
}
```

```
#include<iostream>
#include<cstdio>
#include<algorithm>
#include<cmath>
#include<cstring>
#define maxn 55000
#define inf 2147483647
using namespace std;
struct query
{
    int l,r,s,w;
}a[maxn];
int c[maxn];
long long col[maxn],size[maxn],ans[maxn];
int n,m,cnt,len;

long long gcd(long long x,long long  y)
{
    return (!x)?y:gcd(y%x,x);
}

bool cmp(query a,query b)
{
    return (a.w==b.w)?a.r<b.r:a.w<b.w;
}

int main()
{
    //freopen("hose.in","r",stdin);
    scanf("%d%d",&n,&m);
    for (int i=1;i<=n;i++) scanf("%d",&c[i]);
    len=(int)sqrt(m);
    cnt=(len*len==m)?len:len+1;
    for (int i=1;i<=m;i++)
    {
        scanf("%d%d",&a[i].l,&a[i].r);
        if (a[i].l>a[i].r) swap(a[i].l,a[i].r);
        size[i]=a[i].r-a[i].l+1;
        a[i].w=a[i].l/len+1;
        a[i].s=i;
    }
    sort(a+1,a+m+1,cmp);
    int i=1;
    while (i<=m)
    {
        int now=a[i].w;
        memset(col,0,sizeof(col));
        for (int j=a[i].l;j<=a[i].r;j++) ans[a[i].s]+=2*(col[c[j]]++);
        i++;
        for (;a[i].w==now;i++)
        {
            ans[a[i].s]=ans[a[i-1].s];
            for (int j=a[i-1].r+1;j<=a[i].r;j++)
                ans[a[i].s]+=2*(col[c[j]]++);
            if (a[i-1].l<a[i].l)
                for (int j=a[i-1].l;j<a[i].l;j++)
                    ans[a[i].s]-=2*(--col[c[j]]);
            else
                for (int j=a[i].l;j<a[i-1].l;j++)
                    ans[a[i].s]+=2*(col[c[j]]++);
        }
    }
    long long all,num;
    for (int i=1;i<=m;i++)
    {
        if (size[i]==1) all=1; else all=size[i]*(size[i]-1);
        num=gcd(ans[i],all);
        printf("%lld/%lld\n",ans[i]/num,all/num);
    }
    return 0;
}
```

5.15 Minimum Ratio Spanning Tree

```
#include<cstdio>
#include<cstring>
#include<cmath>

#define MAXX 1111

struct
{
    int x,y;
    double z;
} node[MAXX];

struct
{
    double l,c;
} map[MAXX][MAXX];

int n,l,f[MAXX],pre[MAXX];
double dis[MAXX];

double mst(double x)
{
    int i,j,tmp;
    double min,s=0,t=0;
    memset(f,0,sizeof(f));
    f[1]=1;
    for (i=2; i<=n; i++)
    {
```

```
        dis[i]=map[1][i].c-map[1][i].l*x;
        pre[i]=1;
    }
    for (i=1; i<n; i++)
    {
        min=1e10;
        for (j=1; j<=n; j++)
            if (!f[j] && min>dis[j])
            {
                min=dis[j];
                tmp=j;
            }
        f[tmp]=1;
        t+=map[pre[tmp]][tmp].l;
        s+=map[pre[tmp]][tmp].c;
        for (j=1; j<=n; j++)
            if (!f[j] && map[tmp][j].c-map[tmp][j].l*x<dis[j])
            {
                dis[j]=map[tmp][j].c-map[tmp][j].l*x;
                pre[j]=tmp;
            }
    }
    return s/t;
}

int main()
{
    int i,j;
    double a,b;
    while (scanf("%d",&n),n);
    {
        for (i=1; i<=n; i++)
            scanf("%f%f%lf",&node[i].x,&node[i].y,&node[i].z);
        for (i=1; i<=n; i++)
            for (j=i+1; j<=n; j++)
            {
                map[j][i].l=map[i][j].l=sqrt(1.0*(node[i].x-node[j].x)*(node[i].x-node[j].x)+(node[i].y-node[j].y)*(node[i].y-node[j].y));
                map[j][i].c=map[i][j].c=fabs(node[i].z-node[j].z);
            }
        a=0,b=mst(a);
        while (fabs(b-a)>1e-8)
        {
            a=b;
            b=mst(a);
        }
        printf("%.3lf\n",b);
    }
    return 0;

}
```

5.16 Minimum-cost flow problem

```
// like Edmonds-Karp Algorithm
inline void add(int a,int b,int c,int d)
{ adde(a,b,c,d);adde(b,a,0,-d);}

int dist[MAXX],pre[MAXX];
int source,sink;
std::queue<int>q;
bool in[MAXX];

inline bool go()
{
    static int now,i;
    memset(dist,0x3f,sizeof dist);
    dist[source]=0;
    pre[source]=-1;
    q.push(source);
    in[source]=true;
    while(!q.empty())
    {
        in[now=q.front()]=false;
        q.pop();
        for(i=edge[now];i!=-1;i=nxt[i])
            if(cap[i] && dist[v]>dist[now]+cst[i])
            {
                dist[v]=dist[now]+cst[i];
                pre[v]=i;
                if(!in[v])
                {
                    q.push(v);
                    in[v]=true;
                }
            }
    }
    return dist[sink]!=inf;
}

inline int mcmf(int &flow)
{
    static int ans,i;
    flow=ans=0;
    while(go())
    {
        static int min;
        min=inf;
        for(i=pre[sink];i!=-1;i=pre[to[i^1]])
            min=std::min(min,cap[i]);
        flow+=min;
        ans+=min*dist[sink];
        for(i=pre[sink];i!=-1;i=pre[to[i^1]])
        {
            cap[i]-=min;
            cap[i^1]+=min;
        }
    }
    return ans;
}
```

5.17 Spanning tree

```
Minimum Bottleneck Spanning Tree && All-pairs vertexes' Minimum Bottleneck Path
Minimum Diameter Spanning Tree : Kariv-Hakimi Algorithm
Directed Minimum Spanning Tree : -ChuLiu/Edmonds' algorithm
Second-best MST
Degree-constrained MST
Minimum Ratio Spanning Tree
Manhattan MST

Enumerate All MST
Count Spanning Trees
Minimum Steiner Tree
k-MST
```

5.18 Stable Marriage

```
//对于每个预备队列中的对象，及被匹配对象，先按照喜好程度排列匹配对象

while(!g.empty())  // 预备匹配队列
{
    if(dfn[edge[g.front()].front()]==-1)
        dfn[edge[g.front()].front()]=g.front(); // 如果目前还没尝试匹配过的对象没有被任何别的对象占据
    else
    {
        for(it=edge[edge[g.front()].front()].begin();it!=edge[edge[g.front()].front()].end();++it)
            if(*it==dfn[edge[g.front()].front()] || *it==g.front()) //如果被匹配对象更喜欢正在被匹配的人或现在准备匹配的对象
                break;
        if(*it==g.front()) //如果更喜欢新的
        {
            g.push_back(dfn[edge[g.front()].front()]);
            dfn[edge[g.front()].front()]=g.front();
        }
        else
            g.push_back(g.front()); //否则放到队尾，重新等待匹配
    }
    edge[g.front()].pop_front(); //每组匹配最多只考虑一次
    g.pop_front();
}
```

5.19 Stoer-Wagner Algorithm

```
#include<cstdio>
#include<cstring>

const int maxn=510;

int map[maxn][maxn];
int n;

void contract(int x,int y)//合并两个点
{
    int i,j;
    for (i=0; i<n; i++)
        if (i!=x)
        {
            map[x][i]+=map[y][i];
            map[i][x]+=map[i][y];
        }
    for (i=y+1; i<n; i++)
        for (j=0; j<n; j++)
        {
            map[i-1][j]=map[i][j];
            map[j][i-1]=map[j][i];
        }
    n--;
}

int w[maxn],c[maxn];
int sx,tx;

int mincut() //求最大生成树，计算最后一个点的割，并保存最后一条边的两个顶点
{
    static int i,j,k,t;
    memset(c,0,sizeof(c));
    c[0]=1;
    for (i=0; i<n; i++)
        w[i]=map[0][i];
    for (i=1; i+1<n; i++)
    {
        t=k=-1;
        for (j=0; j<n; j++)
            if (c[j]==0&&w[j]>k)
                k=w[t=j];
        c[sx=t]=1;
        for (j=0; j<n; j++)
            w[j]+=map[t][j];
    }
    for (i=0; i<n; i++)
        if (c[i]==0)
            return w[tx=i];
}
int main()
{
    int i,j,k,m;
    while (scanf("%d%d",&n,&m)!=EOF)
    {
        memset(map,0,sizeof(map));
        while (m--)
        {
            scanf("%d%d%d",&i,&j,&k);
            map[i][j]+=k;
            map[j][i]+=k;
        }
        int mint=999999999;
        while (n>1)
        {
            k=mincut();
            if (k<mint) mint=k;
            contract(sx,tx);
        }
        printf("%d\n",mint);
    }
    return 0;
}
```

5.20 Strongly Connected Component

```
//缩点后注意自环
void dfs(const short &now)
{
    dfn[now]=low[now]=cnt++;
    st.push(now);
    for(std::list<short>::const_iterator it(edge[now].begin());it!=edge[now].end();++it)
        if(dfn[*it]==-1)
        {
            dfs(*it);
            low[now]=std::min(low[now],low[*it]);
        }
        else
            if(sc[*it]==-1)
                low[now]=std::min(low[now],dfn[*it]);
    if(dfn[now]==low[now])
    {
        while(sc[now]==-1)
```

```
        {
            sc[st.top()]=p;
            st.pop();
        }
        ++p;
    }
}
```

6 math

6.1 cantor

```
const int PermSize = 12;
int fac[PermSize] = {1, 1, 2, 6, 24, 120, 720, 5040, 40320, 362880, 3628800, 39916800};

inline int Cantor(int a[])
{
    int i, j, cnt;
    int res = 0;
    for (i = 0; i < PermSize; ++i)
    {
        cnt = 0;
        for (j = i + 1; j < PermSize; ++j)
            if (a[i] > a[j])
                ++cnt;
        res = res + cnt * fac[PermSize - i - 1];
    }
    return res;
}

bool h[13];

inline void UnCantor(int x, int res[])
{
    int i,j,l,t;
    for (i = 1;i <= 12;i++)
        h[i] = false;
    for (i = 1; i <= 12; i++)
    {
        t = x / fac[12 - i];
        x -= t * fac[12 - i];
        for (j = 1, l = 0; l <= t; j++)
            if (!h[j])
                l++;
        j--;
        h[j] = true;
        res[i - 1] = j;
    }
}
```

6.2 Discrete logarithms - BSGS

```
//The running time of BSGS and the space complexity is O(\sqrt{n})
//Pollard's rho algorithm for logarithms' running time is approximately O(\sqrt{p}) where p is n's largest prime factor.
#include<cstdio>
#include<cmath>
#include<cstring>

struct Hash // std::map is bad. clear()时会付出巨大的代价
{
    static const int mod=100003; // prime is good
    static const int MAXX=47111; // bigger than sqrt(c)
    int hd[mod],nxt[MAXX],cnt;
    long long v[MAXX],k[MAXX]; // a^k  v (mod c)
    inline void init()
    {
        memset(hd,0,sizeof hd);
        cnt=0;
    }
    inline long long find(long long v)
    {
        static int now;
        for(now=hd[v%mod];now;now=nxt[now])
            if(this->v[now]==v)
                return k[now];
        return -1ll;
    }
    inline void insert(long long k,long long v)
    {
        if(find(v)!=-1ll)
            return;
        nxt[++cnt]=hd[v%mod];
        hd[v%mod]=cnt;
        this->v[cnt]=v;
        this->k[cnt]=k;
    }
}hash;

long long gcd(long long a,long long b)
{
    return b?gcd(b,a%b):a;
}

long long exgcd(long long a,long long b,long long &x,long long &y)
{
    if(b)
    {
        long long re(exgcd(b,a%b,x,y)),tmp(x);
        x=y;
        y=tmp-(a/b)*y;
        return re;
    }
    x=1ll;
    y=0ll;
    return a;
}

inline long long bsgs(long long a,long long b,long long c) // a^x  b (mod c)
{
    static long long x,y,d,g,m,am,k;
    static int i,cnt;
    a%=c;
    b%=c;
    x=1ll%c; // if c==1....
    for(i=0;i<100;++i)
    {
        if(x==b)
            return i;
        x=(x*a)%c;
    }
    d=1ll%c;
    cnt=0;
    while((g=gcd(a,c))!=1ll)
    {
        if(b%g)
            return -1ll;
        ++cnt;
        c/=g;
        b/=g;
        d=a/g*d%c;
    }
    hash.init();
    m=sqrt((double)c); // maybe need a ceil
    am=1ll%c;
    hash.insert(0,am);
    for(i=1;i<=m;++i)
    {
        am=am*a%c;
        hash.insert(i,am);
    }
    for(i=0;i<=m;++i)
    {
        g=exgcd(d,c,x,y);
        x=(x*b/g%c+c)%c;
        k=hash.find(x);
        if(k!=-1ll)
            return i*m+k+cnt;
        d=d*am%c;
    }
    return -1ll;
}

long long k,p,n;

int main()
{
    while(scanf("%lld %lld %lld",&k,&p,&n)!=EOF)
    {
        if(n>p || (k=bsgs(k,n,p))==-1ll)
            puts("Orz, I' cant find D!");
        else
            printf("%lld\n",k);
    }
    return 0;
}
```

6.3 Divisor function

```
sum of positive divisors function
(n)=(pow(p[0],a[0]+1)-1)/(p[0]-1)* (pow(p[1],a[1]+1)-1)/(p[1]-1)* ... (pow(p[n-1],a[n-1]+1)-1);
```

6.4 Gaussian elimination

```
#define N

inline int ge(int a[N][N],int n) // 返回系数矩阵的秩
{
    static int i,j,k,l;
    for(j=i=0;j<n;++j) //第行i第,列j
    {
        for(k=i;k<n;++k)
            if(a[k][j])
                break;
        if(k==n)
            continue;
        for(l=0;l<=n;++l)
            std::swap(a[i][l],a[k][l]);
        for(l=0;l<=n;++l)
            if(l!=i && a[l][j])
                for(k=0;k<=n;++k)
                    a[l][k]^=a[i][k];
        ++i;
    }
    for(j=i;j<n;++j)
        if(a[j][n])
            return -1; //无解
    return i;
}
/*
 */

void dfs(int v)
{
    if(v==n)
    {
        static int x[MAXX],ta[MAXX][MAXX];
        static int tmp;
        memcpy(x,ans,sizeof(x));
        memcpy(ta,a,sizeof(ta));
        for(i=l-1;i>=0;--i)
        {
            for(j=i+1;j<n;++j)
                ta[i][n]^=(x[j]&&ta[i][j]); //迭代消元求解
            x[i]=ta[i][n];
        }
        for(tmp=i=0;i<n;++i)
            if(x[i])
                ++tmp;
        cnt=std::min(cnt,tmp);
        return;
    }
    ans[v]=0;
    dfs(v+1);
    ans[v]=1;
    dfs(v+1);
}

inline int ge(int a[N][N],int n)
{
    static int i,j,k,l;
    for(i=j=0;j<n;++j)
    {
        for(k=i;k<n;++k)
            if(a[k][i])
                break;
        if(k<n)
        {
            for(l=0;l<=n;++l)
                std::swap(a[i][l],a[k][l]);
            for(k=0;k<n;++k)
                if(k!=i && a[k][i])
                    for(l=0;l<=n;++l)
                        a[k][l]^=a[i][l];
            ++i;
        }
```

```
        else //将不定元交换到后面去
        {
            l=n-1-j+i;
            for(k=0;k<r;++k)
                std::swap(a[k][l],a[k][i]);
        }
    }
    if(i==n)
    {
        for(i=cnt=0;i<r;++i)
            if(a[i][n])
                ++cnt;
        printf("%d\n",cnt);
        continue;
    }
    for(j=i;j<r;++j)
        if(a[j][n])
            break;
    if(j<n)
        puts("impossible");
    else
    {
        memset(ans,0,sizeof(ans));
        cnt=111;
        dfs(l=i);
        printf("%d\n",cnt);
    }
}

/*
 */

inline void ge(int a[N][N],int m,int n) // m*n
{
    static int i,j,k,l,b,c;
    for(i=j=0;i<m && j<r;++j)
    {
        for(k=i;k<m;++k)
            if(a[k][j])
                break;
        if(k==m)
            continue;
        for(l=0;l<=r;++l)
            std::swap(a[i][l],a[k][l]);
        for(k=0;k<m;++k)
            if(k!=i && a[k][j])
            {
                b=a[k][j];
                c=a[i][j];
                for(l=0;l<=r;++l)
                    a[k][l]=((a[k][l]*c-a[i][l]*b)%7+7)%7;
            }
        ++i;
    }
    for(j=i;j<m;++j)
        if(a[j][n])
            break;
    if(j<m)
    {
        puts("Inconsistent data.");
        return;
    }
    if(i<n)
        puts("Multiple solutions.");
    else
    {
        memset(ans,0,sizeof(ans));
        for(i=n-1;i>=0;--i)
        {
            k=a[i][n];
            for(j=i+1;j<r;++j)
                k=((k-a[i][j]*ans[j])%7+7)%7;
            while(k%a[i][i])
                k+=7;
            ans[i]=(k/a[i][i])%7;
        }
        for(i=0;i<r;++i)
            printf("%d%c",ans[i],i+1==n?'\n':' ');
    }
}
```

6.5 inverse element

```
inline void getInv2(int x,int mod)
{
    inv[1]=1;
    for (int i=2; i<=x; i++)
        inv[i]=(mod-(mod/i)*inv[mod%i]%mod)%mod;
}

long long power(long long x,long long y,int mod)
{
    long long ret=1;
    for (long long a=x%mod; y; y>>=1,a=a*a%mod)
        if (y&1)
            ret=ret*a%mod;
    return ret;
}

inline int getInv(int x,int mod)//为素数mod
{
    return power(x,mod-2);
}
```

6.6 Linear programming

```
#include<cstdio>
#include<cstring>
#include<cmath>
#include<algorithm>

#define MAXN 33
#define MAXM 33
#define eps 1e-8

double a[MAXN][MAXM],b[MAXN],c[MAXM];
double x[MAXM],d[MAXN][MAXM];
int ix[MAXN+MAXM];
double ans;
int n,m;
int i,j,k,r,s;
double D;

inline bool simplex()
{
    r=n;
    s=m++;
    for(i=0;i<n+m++i)
        ix[i]=i;
    memset(d,0,sizeof d);
    for(i=0;i<r;++i)
    {
        for(j=0;j+k<m++j)
            d[i][j]=-a[i][j];
        d[i][m-1]=1;
        d[i][m]=b[i];
        if(d[r][m]>d[i][m])
            r=i;
    }
    for(j=0;j+k<m++j)
        d[n][j]=c[j];
    d[n+1][m-1]=-1;
    while(true)
    {
        if(r<n)
        {
            std::swap(ix[s],ix[r+m]);
            d[r][s]=1./d[r][s];
            for(j=0;j<=m++j)
                if(j!=s)
                    d[r][j]*=-d[r][s];
            for(i=0;i<=n+1;++i)
                if(i!=r)
                {
                    for(j=0;j<=m++j)
                        if(j!=s)
                            d[i][j]+=d[r][j]*d[i][s];
                    d[i][s]*=d[r][s];
                }
        }
        r=-1;
        s=-1;
        for(j=0;j<m++j)
            if((s<0 || ix[s]>ix[j]) && (d[n+1][j]>eps || (d[n+1][j]>-eps && d[n][j]>eps)))
                s=j;
        if(s<0)
            break;
        for(i=0;i<r;++i)
            if(d[i][s]<-eps && (r<0 || (D=(d[r][m]/d[r][s]-d[i][m]/d[i][s]))<-eps || (D<
                eps && ix[r+m]>ix[i+m])))
                r=i;
        if(r<0)
            return false;
    }
    if(d[n+1][m]<-eps)
        return false;
    for(i=m;i<n+m++i)
        if(ix[i]+1<m)
            x[ix[i]]=d[i-m][m]; // answer
    ans=d[n][m]; // maxium value
    return true;
}

int main()
{
    while(scanf("%d %d",&m,&n)!=EOF)
    {
        for(i=0;i<m++i)
            scanf("%lf",c+i);  // max{ sum{c[i]*x[i]} }
        for(i=0;i<r;++i)
        {
            for(j=0;j<m++j)
                scanf("%lf",a[i]+j); // sum{ a[i]*x[i] }<= b
            scanf("%lf",b+i);
            b[i]*=n;
        }
        simplex();
        printf("Nasa can spend %.01f taka.\n",ceil(ans));
    }
    return 0;
}
```

6.7 Lucas' theorem

```
#include<cstdio>
#include<cstring>
#include<iostream>

int mod;
long long num[100000];
int ni[100],mi[100];
int len;

void init(int p)
{
    mod=p;
    num[0]=1;
    for (int i=1; i<p; i++)
        num[i]=i*num[i-1]%p;
}

void get(int n,int ni[],int p)
{
    for (int i = 0; i < 100; i++)
        ni[i] = 0;
    int tlen = 0;
    while (n != 0)
    {
        ni[tlen++] = n%p;
        n /= p;
    }
    len = tlen;
}

long long power(long long x,long long y)
{
    long long ret=1;
    for (long long a=x%mod; y; y>>=1,a=a*a%mod)
        if (y&1)
            ret=ret*a%mod;
    return ret;
}

long long getInv(long long x)//mod 为素数
{
    return power(x,mod-2);
}

long long calc(int n,int m,int p)//C(n,m)%p
{
    init(p);
    long long ans=1;
```

```
    for (; n&&m&&ans; n/=p,m/=p)
    {
        if (n%p>=m%p)
            ans = ans*num[n%p]%p *getInv(num[m%p]%p)%p *getInv(num[n%p-m%p])%p;
        else
            ans=0;
    }
    return ans;
}

int main()
{
    int t;
    scanf("%d",&t);
    while (t--)
    {
        int n,m,p;
        scanf("%d%d%d",&n,&m,&p);
        printf("%lld\n",calc(n+m,m,p));
    }
    return 0;
}
```

6.8 Matrix

```
struct Matrix
{
    const int N(52);
    int a[N][N];
    inline Matrix operator*(const Matrix &b)const
    {
        static Matrixres;
        static int i,j,k;
        for(i=0;i<N;++i)
            for(j=0;j<N;++j)
            {
                res.a[i][j]=0;
                for(k=0;k<N;++k)
                    res.a[i][j]+=a[i][k]*b.a[k][j];
            }
        return res;
    }
    inline Matrix operator^(int y)const
    {
        static Matrix res,x;
        static int i,j;
        for(i=0;i<N;++i)
        {
            for(j=0;j<N;++j)
            {
                res.a[i][j]=0;
                x.a[i][j]=a[i][j];
            }
            res.a[i][i]=1;
        }
        for(;y;y>>=1,x=x*x)
            if(y&1)
                res=res*x;
        return res;
    }
};

Fibonacci Matrix
[1 1]
[1 0]
```

6.9 Multiset

```
Permutation:
MultiSet S={1 m,4 s,4 i,2 p}
P(S)=(1+4+4+2)!/1!/4!/4!/2!

Combination:
MultiSet S∞={ a1∞, a2,... ∞ ak}
C(S,r)=(r+k-1)!/r!/(k-1)!=C(r,r+k-1)

if(r>min{count(element[i])})
    you have to resolve this problem with inclusion-exclusion principle.

MS T={3 a,4 b,5 c}
MS T∞*={ a∞, b∞, c}
A1={C(T*,10)|count(a)>3} // C(6,8)
A2={C(T*,10)|count(b)>4} // C(5,7)
A3={C(T*,10)|count(c)>5} // C(4,6)

C(T,10)=C(T*,10)-(|A1|+|A2|+|A3|)+(|A1  A2|+|A1  A3|+|A2  A3|)-|A1  A2  A3|
        C(10,12)                    C(1,3)    C(0,2)    0          0
ans=6
```

6.10 Pell's equation

```
/*
find the (x,y)pair that x^2-n*y^2=1
these is not solution if and only if n is a square number.

solution:
simply brute-force search the integer y, get (x1,y1). ( toooo slow in some situation )
or we can enumerate the continued fraction of sqrt(n), as (x/y), it will be much more
       faster

other solution pairs' matrix:
[ x1 n*y1 ]
[ y1 x1   ]
k-th solution is pow({matrix},k)
*/

import java.util.*;
import java.math.*;

public class Main
{
    static BigInteger p,q,p1,p2,p3,q1,q2,q3,a1,a2,a0,h1,h2,g1,g2,n0;
    static int n,t;
    static void solve()
    {
        p2=BigInteger.ONE;
        p1=BigInteger.ZERO;
        q2=BigInteger.ZERO;
        q1=BigInteger.ONE;
        a0=a1=BigInteger.valueOf((long)Math.sqrt(n));
        g1=BigInteger.ZERO;
        h1=BigInteger.ONE;
        n0=BigInteger.valueOf(n);
        while(true)
        {
            g2=a1.multiply(h1).subtract(g1);
            h2=(n0.subtract(g2.multiply(g2))).divide(h1);
            a2=(g2.add(a0)).divide(h2);
            p=p2.multiply(a1).add(p1);
            q=q2.multiply(a1).add(q1);
            if(p.multiply(p).subtract(n0.multiply(q.multiply(q))).equals(BigInteger.ONE))
                return ;
            a1=a2;
            g1=g2;
            h1=h2;
            p1=p2;
            p2=p;
            q1=q2;
            q2=q;
        }
    }
    public static void main(String[] args)
    {
        Scanner in=new Scanner(System.in);
        t=in.nextInt();
        for(int i=0;i<t;++i)
        {
            n=in.nextInt();
            solve();
            System.out.println(p+" "+q);
        }
    }
}
```

6.11 Pollard's rho algorithm

```
#include<cstdio>
#include<cstdlib>
#include<list>

short T;
unsigned long long a;
std::list<unsigned long long>fac;

inline unsigned long long multi_mod(const unsigned long long &a,unsigned long long b,const
       unsigned long long &n)
{
    unsigned long long exp(a%n),tmp(0);
    while(b)
    {
        if(b&1)
        {
            tmp+=exp;
            if(tmp>n)
                tmp-=n;
        }
        exp<<=1;
        if(exp>n)
            exp-=n;
        b>>=1;
    }
    return tmp;
}

inline unsigned long long exp_mod(unsigned long long a,unsigned long long b,const unsigned
       long long &c)
{
    unsigned long long tmp(1);
    while(b)
    {
        if(b&1)
            tmp=multi_mod(tmp,a,c);
        a=multi_mod(a,a,c);
        b>>=1;
    }
    return tmp;
}

inline bool miller_rabbin(const unsigned long long &n,short T)
{
    if(n==2)
        return true;
    if(n<2 || !(n&1))
        return false;
    unsigned long long a,u(n-1),x,y;
    short t(0),i;
    while(!(u&1))
    {
        ++t;
        u>>=1;
    }
    while(T--)
    {
        a=rand()%(n-1)+1;
        x=exp_mod(a,u,n);
        for(i=0;i<t;++i)
        {
            y=multi_mod(x,x,n);
            if(y==1 && x!=1 && x!=n-1)
                return false;
            x=y;
        }
        if(y!=1)
            return false;
    }
    return true;
}

unsigned long long gcd(const unsigned long long &a,const unsigned long long &b)
{
    return b?gcd(b,a%b):a;
}

inline unsigned long long pollar_rho(const unsigned long long n,const unsigned long long &
       c)
{
    unsigned long long x(rand()%(n-1)+1),y,d,i(1),k(2);
    y=x;
    while(true)
    {
        ++i;
        x=(multi_mod(x,x,n)+c)%n;
        d=gcd((x-y+n)%n,n);
        if(d>1 && d<n)
            return d;
        if(x==y)
            return n;
        if(i==k)
        {
```

```
            k<<=1;
            y=x;
        }
    }
}

void find(const unsigned long long &n,short c)
{
    if(n==1)
        return;
    if(miller_rabbin(n,6))
    {
        fac.push_back(n);
        return;
    }
    unsigned long long p(n);
    short k(c);
    while(p>=n)
        p=pollar_rho(p,c--);
    find(p,k);
    find(n/p,k);
}

int main()
{
    scanf("%d",&T);
    while(T--)
    {
        scanf("%llu",&a);
        fac.clear();
        find(a,120);
        if(fac.size()==1)
            puts("Prime");
        else
        {
            fac.sort();
            printf("%llu\n",fac.front());
        }
    }
    return 0;
}
```

6.12 Reduced Residue System

```
Euler's totient function:对正整数，欧拉函数  是少于或等于的数中与互质的数的数目，也就是对的简化剩余系的大小。

nnnn
(1)（唯一和互质的数就是本身）。=111若
m,互质， n(mn) =(m) (n)。对于来说，所有这样的数的和为
nn *(n)。/2

inline long long phi(int n)
{
    static int i;
    static int re;
    re=n;
    for(i=0;prm[i]*prm[i]<=n;++i)
        if(n%prm[i]==0)
        {
            re-=re/prm[i];
            do
                n/=prm[i];
            while(n%prm[i]==0);
        }
    if(n!=1)
        re-=re/n;
    return re;
}

inline void Euler()
{
    static int i,j;
    phi[1]=1;
    for(i=2;i<MAXX;++i)
        if(!phi[i])
            for(j=i;j<MAXX;j+=i)
            {
                if(!phi[j])
                    phi[j]=j;
                phi[j]=phi[j]/i*(i-1);
            }
}

Multiplicative order:

the multiplicative order of a modulo n is the smallest positive integer k with
a^k 1(mod n).对的简化剩余系中的所有

mx,ord(x)都一定是 (m)的一个约数(aka. Euler's totient theorem)求

:
method 、根据定义，对 1(m)分解素因子之后暴力枚举所有 (m)的约数，找到最小的一个，满足d pow(x,d,m)==1
method 、2
inline long long ord(long long x,long long m)
{
    static long long ans;
    static int i,j;
    ans=phi(m);
    for(i=0;i<fac.size();++i)
        for(j=0;j<fac[i].second && pow(x,ans/fac[i].first,m)==1;++j)
            ans/=fac[i].first;
    return ans;
}


Primitive root:若

ord(x)==(m)，则为的一个原根m因此只需检查所有
 pow(x,d) {为 d(m)的约数} 找到使 pow(x,d)%m==1 的所有，当且仅当这样的只有一个，并且为 dd(m)的时候，是
        的一个原根m当且仅当

m= 1,2,4,pow(p,n),2*pow(p,n) {为奇质数p,为正整数n} 时，存在原根n // 应该是指存在对于完全剩余系的原根
        ……？当存在原根时，原根数目为

m((m))求：枚举每一个简化剩余系中的数，若对于的每一个质因子


iip[j],pow(i ,(m)/p[j])%都不为，那么为的一个原根。也就是说，m1imord(i)==(m)。最小原根通常极小。


Carmichael function:

(n) is defined as the smallest positive integer m such that
    pow(a,m)%n==1 { for a!=1 && gcd(a,n)==1 }也就是简化剩余系完全剩余系中存在乘法群中无法得到的数
(1)中所有的x lcm{ord(x)}

if n=pow(p[0],a[0])*pow(p[1],a[1])*...*pow(p[m-1],a[m-1])
    then (n)=lcm ((pow(p[0],a[0])) ,(pow(p[1],a[1])) ,... ,(pow(p[m-1],a[m-1])));

if n=pow(2,a)*pow(p[0],a[0])*pow(p[1],a[1])*...*pow(p[m-1],a[m-1])
    then (n)=lcm(pow(2,c) ,(pow(p[0],a[0])) ,(pow(p[1],a[1])) ,... ,(pow(p[m-1],a[m-1])));
    { c=0 if a<2; c=1 if a==2; c=a-2 if a>3; }


Carmichael's theorem:
if gcd(a,n)==1
    then pow(a ,(n))%n==1
```

6.13 System of linear congruences

```
// minimal val that for all (m,a) , val%m == a
#include<cstdio>

#define MAXX 11

int T,t;
int m[MAXX],a[MAXX];
int n,i,j,k;
int x,y,c,d;
int lcm;

int exgcd(int a,int b,int &x,int &y)
{
    if(b)
    {
        int re(exgcd(b,a%b,x,y)),tmp(x);
        x=y;
        y=tmp-(a/b)*y;
        return re;
    }
    x=1;
    y=0;
    return a;
}

int main()
{
    scanf("%d",&T);
    for(t=1;t<=T;++t)
    {
        scanf("%d",&n);
        lcm=1;
        for(i=0;i<n;++i)
        {
            scanf("%d",m+i);
            lcm*=m[i]/exgcd(lcm,m[i],x,y);
        }
        for(i=0;i<n;++i)
            scanf("%d",a+i);
        for(i=1;i<n;++i)
        {
            c=a[i]-a[0];
            d=exgcd(m[0],m[i],x,y);
            if(c%d)
                break;
            y=m[i]/d;
            c/=d;
            x=(x*c%y+y)%y;
            a[0]+=m[0]*x;
            m[0]*=y;
        }
        printf("Case %d: %d\n",t,i<n?-1:(a[0]?a[0]:lcm));
    }
    return 0;
}
```

7 other

7.1 bigint

```
// header files
#include <cstdio>
#include <string>
#include <algorithm>
#include <iostream>

struct Bigint
{
    // representations and structures
    std::string a; // to store the digits
    int sign; // sign = -1 for negative numbers, sign = 1 otherwise
    // constructors
    Bigint() {} // default constructor
    Bigint( std::string b ) { (*this) = b; } // constructor for std::string
    // some helpful methods
    int size() // returns number of digits
    {
        return a.size();
    }
    Bigint inverseSign() // changes the sign
    {
        sign *= -1;
        return (*this);
    }
    Bigint normalize( int newSign ) // removes leading 0, fixes sign
    {
        for( int i = a.size() - 1; i > 0 && a[i] == '0'; i-- )
            a.erase(a.begin() + i);
        sign = ( a.size() == 1 && a[0] == '0' ) ? 1 : newSign;
        return (*this);
    }
    // assignment operator
    void operator = ( std::string b ) // assigns a std::string to Bigint
    {
        a = b[0] == '-' ? b.substr(1) : b;
        reverse( a.begin(), a.end() );
        this->normalize( b[0] == '-' ? -1 : 1 );
    }
    // conditional operators
    bool operator < ( const Bigint &b ) const // less than operator
    {
        if( sign != b.sign )
            return sign < b.sign;
        if( a.size() != b.a.size() )
            return sign == 1 ? a.size() < b.a.size() : a.size() > b.a.size();
        for( int i = a.size() - 1; i >= 0; i-- )
            if( a[i] != b.a[i] )
                return sign == 1 ? a[i] < b.a[i] : a[i] > b.a[i];
```

```
        return false;
    }
    bool operator == ( const Bigint &b ) const // operator for equality
    {
        return a == b.a && sign == b.sign;
    }

    // mathematical operators
    Bigint operator + ( Bigint b ) // addition operator overloading
    {
        if( sign != b.sign )
            return (*this) - b.inverseSign();
        Bigint c;
        for(int i = 0, carry = 0; i<a.size() || i<b.size() || carry; i++ )
        {
            carry+=(i<a.size() ? a[i]-48 : 0)+(i<b.a.size() ? b.a[i]-48 : 0);
            c.a += (carry % 10 + 48);
            carry /= 10;
        }
        return c.normalize(sign);
    }

    Bigint operator - ( Bigint b ) // subtraction operator overloading
    {
        if( sign != b.sign )
            return (*this) + b.inverseSign();
        int s = sign; sign = b.sign = 1;
        if( (*this) < b )
            return ((b - (*this)).inverseSign()).normalize(-s);
        Bigint c;
        for( int i = 0, borrow = 0; i < a.size(); i++ )
        {
            borrow = a[i] - borrow - (i < b.size() ? b.a[i] : 48);
            c.a += borrow >= 0 ? borrow + 48 : borrow + 58;
            borrow = borrow >= 0 ? 0 : 1;
        }
        return c.normalize(s);
    }
    Bigint operator * ( Bigint b ) // multiplication operator overloading
    {
        Bigint c("0");
        for( int i = 0, k = a[i] - 48; i < a.size(); i++, k = a[i] - 48 )
        {
            while(k--)
                c = c + b; // ith digit is k, so, we add k times
            b.a.insert(b.a.begin(), '0'); // multiplied by 10
        }
        return c.normalize(sign * b.sign);
    }
    Bigint operator / ( Bigint b ) // division operator overloading
    {
        if( b.size() == 1 && b.a[0] == '0' )
            b.a[0] /= ( b.a[0] - 48 );
        Bigint c("0"), d;
        for( int j = 0; j < a.size(); j++ )
            d.a += "0";
        int dSign = sign * b.sign;
        b.sign = 1;
        for( int i = a.size() - 1; i >= 0; i-- )
        {
            c.a.insert( c.a.begin(), '0');
            c = c + a.substr( i, 1 );
            while( !( c < b ) )
            {
                c = c - b;
                d.a[i]++;
            }
        }
        return d.normalize(dSign);
    }
    Bigint operator % ( Bigint b ) // modulo operator overloading
    {
        if( b.size() == 1 && b.a[0] == '0' )
            b.a[0] /= ( b.a[0] - 48 );
        Bigint c("0");
        b.sign = 1;
        for( int i = a.size() - 1; i >= 0; i-- )
        {
            c.a.insert( c.a.begin(), '0');
            c = c + a.substr( i, 1 );
            while( !( c < b ) )
                c = c - b;
        }
        return c.normalize(sign);
    }

    // output method
    void print()
    {
        if( sign == -1 )
            putchar('-');
        for( int i = a.size() - 1; i >= 0; i-- )
            putchar(a[i]);
    }
};



int main()
{
    Bigint a, b, c; // declared some Bigint variables
    ////////////////////////
    // taking Bigint input //
    ////////////////////////

    std::string input; // std::string to take input
    std::cin >> input; // take the Big integer as std::string
    a = input; // assign the std::string to Bigint a

    std::cin >> input; // take the Big integer as std::string
    b = input; // assign the std::string to Bigint b

    ////////////////////////////////
    // Using mathematical operators //
    ////////////////////////////////

    c = a + b; // adding a and b
    c.print(); // printing the Bigint
    puts(""); // newline

    c = a - b; // subtracting b from a
    c.print(); // printing the Bigint
    puts(""); // newline

    c = a * b; // multiplying a and b
    c.print(); // printing the Bigint
    puts(""); // newline

    c = a / b; // dividing a by b
    c.print(); // printing the Bigint
    puts(""); // newline

    c = a % b; // a modulo b
    c.print(); // printing the Bigint
    puts(""); // newline

    ////////////////////////////////
    // Using conditional operators //
    ////////////////////////////////

    if( a == b )
        puts("equal"); // checking equality
    else
        puts("not equal");

    if( a < b )
        puts("a is smaller than b"); // checking less than operator

    return 0;
}
```

7.2 java

```
//Scanner

Scanner in=new Scanner(new FileReader("asdf"));
PrintWriter pw=new PrintWriter(new Filewriter("out"));
boolean         in.hasNext();
String          in.next();
BigDecimal      in.nextBigDecimal();
BigInteger      in.nextBigInteger();
BigInteger      in.nextBigInteger(int radix);
double          in.nextDouble();
int             in.nextInt();
int             in.nextInt(int radix);
String          in.nextLine();
long            in.nextLong();
long            in.nextLong(int radix);
short           in.nextShort();
short           in.nextShort(int radix);
int             in.radix(); //Returns this scanner's default radix.
Scanner         in.useRadix(int radix);// Sets this scanner's default radix to the specified
        radix.
void            in.close();//Closes this scanner.

//String

char            str.charAt(int index);
int             str.compareTo(String anotherString); // <0 if less. ==0 if equal. >0 if
       greater.
int             str.compareToIgnoreCase(String str);
String          str.concat(String str);
boolean         str.contains(CharSequence s);
boolean         str.endsWith(String suffix);
boolean         str.startsWith(String preffix);
boolean         str.startsWith(String preffix,int toffset);
int             str.hashCode();
int             str.indexOf(int ch);
int             str.indexOf(int ch,int fromIndex);
int             str.indexOf(String str);
int             str.indexOf(String str,int fromIndex);
int             str.lastIndexOf(int ch);
int             str.lastIndexOf(int ch,int fromIndex);
//(ry
int             str.length();
String          str.substring(int beginIndex);
String          str.substring(int beginIndex,int endIndex);
String          str.toLowerCase();
String          str.toUpperCase();
String          str.trim();// Returns a copy of the string, with leading and trailing
        whitespace omitted.

//StringBuilder
StringBuilder str.insert(int offset,...);
StringBuilder str.reverse();
void            str.setCharAt(int index,int ch);

//BigInteger
compareTo(); equals(); doubleValue(); longValue(); hashCode(); toString(); toString(int
       radix); max(); min(); mod(); modPow(BigInteger exp,BigInteger m); nextProbablePrime
       (); pow();
andNot(); and(); xor(); not(); or(); getLowestSetBit(); bitCount(); bitLength(); setBig(
       int n); shiftLeft(int n); shiftRight(int n);
add(); divide(); divideAndRemainder(); remainder(); multiply(); subtract(); gcd(); abs();
       signum(); negate();

//BigDecimal
movePointLeft(); movePointRight(); precision(); stripTrailingZeros(); toBigInteger();
       toPlainString();


//sort
class pii implements Comparable
{
    public int a,b;
    public int compareTo(Object i)
    {
        pii c=(pii)i;
        return a==c.a?c.b-b:c.a-a;
    }
}

class Main
{
    public static void main(String[] args)
    {
        pii[] the=new pii[2];
        the[0]=new pii();
        the[1]=new pii();
        the[0].a=1;
        the[0].b=1;
        the[1].a=1;
        the[1].b=2;
        Arrays.sort(the);
        for(int i=0;i<2;++i)
            System.out.printf("%d %d\n",the[i].a,the[i].b);
    }
}
```

7.3 others

```
god damn it windows:
#pragma comment(linker, "/STACK:16777216")
#pragma comment(linker,"/STACK:102400000,102400000")
```

```

chmod +x [filename]

while true; do
./gen > input
./sol < input > output.sol
./bf < input > output.bf

diff output.sol output.bf
if[ $? -ne 0];then break fi
done、状态状态状态状态状态状态状态状态状态状态状态

1、
2calm_down();calm_down();calm_down();、读完题目读完题目读完题目
3、不盲目跟版
4、考虑换题换想法
5/、对数离线
6//hash观察问题本身点　区间互转//、对数调整精度
6.1 or 将乘法转换成加法、点化区间，区间化点
6.2、数组大小……
7
```

8 search

8.1 dlx

```
精确覆盖：给定一个矩阵，现在要选择一些行，使得每一列有且仅有一个。
011每次选定一个元素个数最少的列，从该列中选择一行加入答案，删除该行所有的列以及与该行冲突的行。重复覆盖：给定一个矩阵，现在要选择一些行，使得每一列至少有一个。

011每次选定一个元素个数最少的列，从该列中选择一行加入答案，删除该行所有的列。与该行冲突的行可能满足重复覆盖。
```

8.2 dlx - exact cover

```
#include<cstdio>
#include<cstring>
#include<algorithm>
#include<vector>

#define N 256
#define MAXN N*22
#define MAXM N*5
#define inf 0x3f3f3f3f
const int MAXX(MAXN*MAXM);

bool mat[MAXN][MAXM];

int u[MAXX],d[MAXX],l[MAXX],r[MAXX],ch[MAXX],rh[MAXX];
int sz[MAXM];
std::vector<int>ans(MAXX);
int hd,cnt;

inline int node(int up,int down,int left,int right)
{
    u[cnt]=up;
    d[cnt]=down;
    l[cnt]=left;
    r[cnt]=right;
    u[down]=d[up]=l[right]=r[left]=cnt;
    return cnt++;
}

inline void init(int n,int m)
{
    cnt=0;
    hd=node(0,0,0,0);
    static int i,j,k,r;
    for(j=1;j<=m++j)
    {
        ch[j]=node(cnt,cnt,l[hd],hd);
        sz[j]=0;
    }
    for(i=1;i<=n;++i)
    {
        r=-1;
        for(j=1;j<=m++j)
            if(mat[i][j])
            {
                if(r==-1)
                {
                    r=node(u[ch[j]],ch[j],cnt,cnt);
                    rh[r]=i;
                    ch[r]=ch[j];
                }
                else
                {
                    k=node(u[ch[j]],ch[j],l[r],r);
                    rh[k]=i;
                    ch[k]=ch[j];
                }
                ++sz[j];
            }
    }
}

inline void rm(int c)
{
    l[r[c]]=l[c];
    r[l[c]]=r[c];
    static int i,j;
    for(i=d[c];i!=c;i=d[i])
        for(j=r[i];j!=i;j=r[j])
        {
            u[d[j]]=u[j];
            d[u[j]]=d[j];
            --sz[ch[j]];
        }
}

inline void add(int c)
{
    static int i,j;
    for(i=u[c];i!=c;i=u[i])
        for(j=l[i];j!=i;j=l[j])
        {
            ++sz[ch[j]];
            u[d[j]]=d[u[j]]=j;
        }
    l[r[c]]=r[l[c]]=c;
}

bool dlx(int k)
{
    if(hd==r[hd])
    {
        ans.resize(k);
        return true;
    }
    int s=inf,c;
    int i,j;
    for(i=r[hd];i!=hd;i=r[i])
        if(sz[i]<s)
        {
            s=sz[i];
            c=i;
        }
    rm(c);
    for(i=d[c];i!=c;i=d[i])
    {
        ans[k]=rh[i];
        for(j=r[i];j!=i;j=r[j])
            rm(ch[j]);
        if(dlx(k+1))
            return true;
        for(j=l[i];j!=i;j=l[j])
            add(ch[j]);
    }
    add(c);
    return false;
}

#include <cstdio>
#include <cstring>

#define N 1024
#define M 1024*110
using namespace std;

int l[M], r[M], d[M], u[M], col[M], row[M], h[M], res[N], cntcol[N];
int dcnt = 0;
inline void addnode(int &x)
{
    ++x;
    r[x] = l[x] = u[x] = d[x] = x;
}
inline void insert_row(int rowx, int x)
{
    r[l[rowx]] = x;
    l[x] = l[rowx];
    r[x] = rowx;
    l[rowx] = x;
}
inline void insert_col(int colx, int x)
{
    d[u[colx]] = x;
    u[x] = u[colx];
    d[x] = colx;
    u[colx] = x;
}
inline void dlx_init(int cols)
{
    memset(h, -1, sizeof(h));
    memset(cntcol, 0, sizeof(cntcol));
    dcnt = -1;
    addnode(dcnt);
    for (int i = 1; i <= cols; ++i)
    {
        addnode(dcnt);
        insert_row(0, dcnt);
    }
}
inline void remove(int c)
{
    l[r[c]] = l[c];
    r[l[c]] = r[c];
    for (int i = d[c]; i != c; i = d[i])
        for (int j = r[i]; j != i; j = r[j])
        {
            u[d[j]] = u[j];
            d[u[j]] = d[j];
            cntcol[col[j]]--;
        }
}
inline void resume(int c)
{
    for (int i = u[c]; i != c; i = u[i])
        for (int j = l[i]; j != i; j = l[j])
        {
            u[d[j]] = j;
            d[u[j]] = j;
            cntcol[col[j]]++;
        }
    l[r[c]] = c;
    r[l[c]] = c;
}
bool DLX(int deep)
{
    if (r[0] == 0)
    {
//Do anything you want to do here
        printf("%d", deep);
        for (int i = 0; i < deep; ++i) printf(" %d", res[i]);
        puts("");
        return true;
    }
    int min = INT_MAX, tempc;
    for (int i = r[0]; i != 0; i = r[i])
        if (cntcol[i] < min)
        {
            min = cntcol[i];
            tempc = i;
        }
    remove(tempc);
    for (int i = d[tempc]; i != tempc; i = d[i])
    {
        res[deep] = row[i];
        for (int j = r[i]; j != i; j = r[j]) remove(col[j]);
        if (DLX(deep + 1)) return true;
        for (int j = l[i]; j != i; j = l[j]) resume(col[j]);
    }
    resume(tempc);
    return false;
}
//插入矩阵中的节点"1"
inline void insert_node(int x, int y)
{
    cntcol[y]++;
    addnode(dcnt);
```

```
    row[dcnt] = x;
    col[dcnt] = y;
    insert_col(y, dcnt);
    if (h[x] == -1) h[x] = dcnt;
    else insert_row(h[x], dcnt);
}
int main()
{
    int n, m;
    while (~scanf("%d%d", &n, &m))
    {
        dlx_init(m);
        for (int i = 1; i <= n; ++i)
        {
            int k, x;
            scanf("%d", &k);
            while (k--)
            {
                scanf("%d", &x);
                insert_node(i, x);
            }
        }
        if (!DLX(0))
            puts("NO");
    }
    return 0;
}
```

8.3 dlx - repeat cover

```
#include<cstdio>
#include<cstring>
#include<algorithm>

#define MAXN 110
#define MAXM 1000000
#define INF 0x7FFFFFFF

using namespace std;

int G[MAXN][MAXN];
int L[MAXM], R[MAXM], U[MAXM], D[MAXM];
int size, ans, S[MAXM], H[MAXM], C[MAXM];
bool vis[MAXN * 100];
void Link(int r, int c)
{
    U[size] = c;
    D[size] = D[c];
    U[D[c]] = size;
    D[c] = size;
    if (H[r] < 0)
        H[r] = L[size] = R[size] = size;
    else
    {
        L[size] = H[r];
        R[size] = R[H[r]];
        L[R[H[r]]] = size;
        R[H[r]] = size;
    }
    S[c]++;
    C[size++] = c;
}
void Remove(int c)
{
    int i;
    for (i = D[c]; i != c; i = D[i])
    {
        L[R[i]] = L[i];
        R[L[i]] = R[i];
    }
}
void Resume(int c)
{
    int i;
    for (i = D[c]; i != c; i = D[i])
        L[R[i]] = R[L[i]] = i;
}
int A()
{
    int i, j, k, res;
    memset(vis, false, sizeof(vis));
    for (res = 0, i = R[0]; i; i = R[i])
    {
        if (!vis[i])
        {
            res++;
            for (j = D[i]; j != i; j = D[j])
            {
                for (k = R[j]; k != j; k = R[k])
                    vis[C[k]] = true;
            }
        }
    }
    return res;
}
void Dance(int now)
{
    if (R[0] == 0)
        ans = min(ans, now);
    else if (now + A() < ans)
    {
        int i, j, temp, c;
        for (temp = INF, i = R[0]; i; i = R[i])
        {
            if (temp > S[i])
            {
                temp = S[i];
                c = i;
            }
        }
        for (i = D[c]; i != c; i = D[i])
        {
            Remove(i);
            for (j = R[i]; j != i; j = R[j])
                Remove(j);
            Dance(now + 1);
            for (j = L[i]; j != i; j = L[j])
                Resume(j);
            Resume(i);
        }
    }
}
void Init(int m)
{
    int i;
    for (i = 0; i <= m; i++)
    {
        R[i] = i + 1;
        L[i + 1] = i;
        U[i] = D[i] = i;
        S[i] = 0;
    }
    R[m] = 0;
    size = m + 1;
}
```

8.4 fibonacci knapsack

```
#include<stdio.h>
#include<stdlib.h>
#include<algorithm>

#define MAXX 71

struct mono
{
    long long weig,cost;
}goods[MAXX];

short n,T,t,i;
long long carry,sumw,sumc;
long long ans,las[MAXX];

int com(const void *n,const void *m)
{
    struct mono *a=(struct mono *)n,*b=(struct mono *)m;
    if(a->weig!=b->weig)
        return a->weig-b->weig;
    else
        return b->cost-a->cost;
}

bool comp(const struct mono a,const struct mono b)
{
    if(a.weig!=b.weig)
        return a.weig<b.weig;
    else
        return b.cost<a.cost;
}

void dfs(short i,long long cost_n,long long carry_n,short last)
{
    if(ans<cost_n)
        ans=cost_n;
    if(i==n || goods[i].weig>carry_n || cost_n+las[i]<=ans)
        return;
    if(last || (goods[i].weig!=goods[i-1].weig && goods[i].cost>goods[i-1].cost))
        dfs(i+1,cost_n+goods[i].cost,carry_n-goods[i].weig,1);
    dfs(i+1,cost_n,carry_n,0);
}

int main()
{
    //    freopen("asdf","r",stdin);
    scanf("%hd",&T);
    for(t=1;t<=T;++t)
    {
        scanf("%hd%lld",&n,&carry);
        sumw=0;
        sumc=0;
        ans=0;
        for(i=0;i<n;++i)
        {
            scanf("%lld%lld",&goods[i].weig,&goods[i].cost);
            sumw+=goods[i].weig;
            sumc+=goods[i].cost;
        }
        if(sumw<=carry)
        {
            printf("Case %hd: %lld\n",t,sumc);
            continue;
        }
//        qsort(goods,n,sizeof(struct mono),com);
        std::sort(goods,goods+n,comp);
        for(i=0;i<n;++i)
        {
//            printf("%lld %lld\n",goods[i].weig,goods[i].cost);
            las[i]=sumc;
            sumc-=goods[i].cost;
        }
        dfs(0,0,carry,1);
        printf("Case %hd: %lld\n",t,ans);
    }
    return 0;
}
```

9 string

9.1 Aho-Corasick Algorithm

```
//trie graph
#include<cstring>
#include<queue>

#define MAX 1000111
#define N 26

int nxt[MAX][N],fal[MAX],cnt;
bool ed[MAX];
char buf[MAX];

inline void init(int a)
{
    memset(nxt[a],0,sizeof(nxt[0]));
    fal[a]=0;
    ed[a]=false;
}

inline void insert()
{
    static int i,p;
    for(i=p=0;buf[i];++i)
    {
        if(!nxt[p][map[buf[i]]])
            init(nxt[p][map[buf[i]]]=++cnt);
        p=nxt[p][map[buf[i]]];
    }
    ed[p]=true;
}
```

```

inline void make()
{
    static std::queue<int>q;
    int i,now,p;
    q.push(0);
    while(!q.empty())
    {
        now=q.front();
        q.pop();
        for(i=0;i<N++i)
            if(nxt[now][i])
            {
                q.push(p=nxt[now][i]);
                if(now)
                    fal[p]=nxt[fal[now]][i];
                ed[p]|=ed[fal[p]];
            }
            else
                nxt[now][i]=nxt[fal[now]][i]; // 使用本身的存串的时候注意已被重载trienxt
    }
}

// normal version

#define N 128

char buf[MAXX];
int cnt[1111];

struct node
{
    node *fal,*nxt[N];
    int idx;
    node() { memset(this,0,sizeof node); }
}*rt;
std::queue<node*>Q;

void free(node *p)
{
    for(int i(0);i<N++i)
        if(p->nxt[i])
            free(p->nxt[i]);
    delete p;
}

inline void add(char *s,int idx)
{
    static node *p;
    for(p=rt;*s;++s)
    {
        if(!p->nxt[*s])
            p->nxt[*s]=new node();
        p=p->nxt[*s];
    }
    p->idx=idx;
}

inline void make()
{
    Q.push(rt);
    static node *p,*q;
    static int i;
    while(!Q.empty())
    {
        p=Q.front();
        Q.pop();
        for(i=0;i<N++i)
            if(p->nxt[i])
            {
                q=p->fal;
                while(q)
                {
                    if(q->nxt[i])
                    {
                        p->nxt[i]->fal=q->nxt[i];
                        break;
                    }
                    q=q->fal;
                }
                if(!q)
                    p->nxt[i]->fal=rt;
                Q.push(p->nxt[i]);
            }
    }
}

inline void match(const char *s)
{
    static node *p,*q;
    for(p=rt;*s;++s)
    {
        while(p!=rt && !p->nxt[*s])
            p=p->fal;
        p=p->nxt[*s];
        if(!p)
            p=rt;
        for(q=p;q!=rt && q->idx;q=q->fal) // why q->idx ? looks like not necessary at all, I delete it in an other solution
            ++cnt[q->idx];
    }
}
//可以考虑一下，拉直指针来跳过无效的匹配dfsfal
//在线调整关键字存在性的时候，可以考虑欧拉序压扁之后使用或者线段树进行区间修改BIT
```

9.2 Gusfield's Z Algorithm

```

inline void make(int *z,char *buf)
{
    int i,j,l,r;
    l=0;
    r=1;
    z[0]=strlen(buf);
    for(i=1;i<z[0];++i)
        if(r<=i || z[i-l]>=r-i)
        {
            j=std::max(i,r);
            while(j<z[0] && buf[j]==buf[j-i])
                ++j;
            z[i]=j-i;
            if(i<j)
            {
                l=i;
                r=j;
            }
        }
        else
            z[i]=z[i-l];
}

for(i=1;i<len && i+z[i]<len;++i); //i可能最小循环节长度=
```

9.3 Manacher's Algorithm

```

inline int match(const int a,const int b,const std::vector<int> &str)
{
    static int i;
    i=0;
    while(a-i>=0&& b+i<str.size() && str[a-i]==str[b+i])
        ++i;
    return i;
}

inline void go(int *z,const std::vector<int> &str)
{
    static int c,l,r,i,ii,n;
    z[0]=1;
    c=l=r=0;
    for(i=1;i<str.size();++i)
    {
        ii=(l<<1)-i;
        n=r+1-i;

        if(i>r)
        {
            z[i]=match(i,i,str);
            l=i;
            r=i+z[i]-1;
        }
        else
            if(z[ii]==n)
            {
                z[i]=n+match(i-n,i+n,str);
                l=i;
                r=i+z[i]-1;
            }
            else
                z[i]=std::min(z[ii],n);
        if(z[i]>z[c])
            c=i;
    }
}

inline bool check(int *z,int a,int b) //检查子串[a,b是否回文]
{
    a=a*2-1;
    b=b*2-1;
    int m=(a+b)/2;
    return z[m]>=b-m+1;
}
```

9.4 Morris-Pratt Algorithm

```

inline void make(char *buf,int *fal)
{
    static int i,j;
    fal[0]=-1;
    for(i=1,j=-1;buf[i];++i)
    {
        while(j>=0&& buf[j+1]!=buf[i])
            j=fal[j];
        if(buf[j+1]==buf[i])
            ++j;
        fal[i]=j;
    }

}

inline int match(char *p,char *t,int* fal)
{
    static int i,j,re;
    re=0;
    for(i=0,j=-1;t[i];++i)
    {
        while(j>=0&& p[j+1]!=t[i])
            j=fal[j];
        if(p[j+1]==t[i])
            ++j;
        if(!p[j+1])
        {
            ++re;
            j=fal[j];
        }
    }
    return re;
}
```

9.5 smallest representation

```
int min(char a[],int len)
{
    int i = 0,j = 1,k = 0;
    while (i < len && j < len && k < len)
    {
        int cmp = a[(j+k)%len]-a[(i+k)%len];
        if (cmp == 0)
            k++;
        else
        {
            if (cmp > 0)
                j += k+1;
            else
                i += k+1;
            if (i == j) j++;
            k = 0;
        }
    }
    return std::min(i,j);
}
```

9.6 Suffix Array - DC3 Algorithm

```
#include<cstdio>
#include<cstring>
#include<algorithm>

```

9.7 Suffix Array - Prefix-doubling Algorithm

```
#define MAXX 1111
#define F(x) ((x)/3+((x)%3==1?0:tb))
#define G(x) ((x)<tb?(x)*3+1:((x)-tb)*3+2)

int wa[MAXX],wb[MAXX],wv[MAXX],ws[MAXX];

inline bool c0(const int *str,const int &a,const int &b)
{
    return str[a]==str[b] && str[a+1]==str[b+1] && str[a+2]==str[b+2];
}

inline bool c12(const int *str,const int &k,const int &a,const int &b)
{
    if(k==2)
        return str[a]<str[b] || str[a]==str[b] && c12(str,1,a+1,b+1);
    else
        return str[a]<str[b] || str[a]==str[b] && wv[a+1]<wv[b+1];
}

inline void sort(int *str,int *a,int *b,const int &n,const int &m)
{
    memset(ws,0,sizeof(ws));
    int i;
    for(i=0;i<n;++i)
        ++ws[wv[i]=str[a[i]]];
    for(i=1;i<m;++i)
        ws[i]+=ws[i-1];
    for(i=n-1;i>=0;--i)
        b[--ws[wv[i]]]=a[i];
}

inline void dc3(int *str,int *sa,const int &n,const int &m)
{
    int *strn(str+n);
    int *san(sa+n),tb((n+1)/3),ta(0),tbc(0),i,j,k;
    str[n]=str[n+1]=0;
    for(i=0;i<n;++i)
        if(i%3)
            wa[tbc++]=i;
    sort(str+2,wa,wb,tbc,m);
    sort(str+1,wb,wa,tbc,m);
    sort(str,wa,wb,tbc,m);
    for(i=j=1,strn[F(wb[0])]=0;i<tbc;++i)
        strn[F(wb[i])]=c0(str,wb[i-1],wb[i])?j-1:j++;
    if(j<tbc)
        dc3(strn,san,tbc,j);
    else
        for(i=0;i<tbc;++i)
            san[strn[i]]=i;
    for(i=0;i<tbc;++i)
        if(san[i]<tb)
            wb[ta++]=san[i]*3;
    if(n%3==1)
        wb[ta++]=n-1;
    sort(str,wb,wa,ta,m);
    for(i=0;i<tbc;++i)
        wv[wb[i]=G(san[i])]=i;
    for(i=j=k=0;i<ta && j<tbc;)
        sa[k++]=c12(str,wb[j]%3,wa[i],wb[j])?wa[i++]:wb[j++];
    while(i<ta)
        sa[k++]=wa[i++];
    while(j<tbc)
        sa[k++]=wb[j++];
}

int rk[MAXX],lcpa[MAXX],sa[MAXX*3];
int str[MAXX*3]; //必须int

int main()
{
    scanf("%d%d",&n,&j);
    for(i=0;i<n;++i)
    {
        scanf("%d",&k);
        num[i]=k-j+100;
        j=k;
    }
    num[n]=0;

    dc3(num,sa,n+1,191); //191: 中取值范围，桶排序str

    for(i=1;i<=n;++i) // 数组rank
        rk[sa[i]]=i;
    for(i=k=0;i<n;++i) // 数组lcp
        if(!rk[i])
            lcpa[0]=0;
        else
        {
            j=sa[rk[i]-1];
            if(k>0)
                --k;
            while(num[i+k]==num[j+k])
                ++k;
            lcpa[rk[i]]=k;
        }


    for(i=1;i<=n;++i)
        sptb[0][i]=i;
    for(i=1;i<=lg[n];++i)  //sparse table RMQ
    {
        k=n+1-(1<<i);
        for(j=1;j<=k;++j)
        {
            a=sptb[i-1][j];
            b=sptb[i-1][j+(1<<(i-1))];
            sptb[i][j]=lcpa[a]<lcpa[b]?a:b;
        }
    }
}

inline int ask(int l,int r)
{
    a=lg[r-l+1];
    r-=(1<<a)-1;
    l=sptb[a][l];
    r=sptb[a][r];
    return lcpa[l]<lcpa[r]?l:r;
}

inline int lcp(int l,int r) // 字符串上[l,r区间的]rmq
{
    l=rk[l];
    r=rk[r];
    if(l>r)
        std::swap(l,r);
    return lcpa[ask(l+1,r)];
}
```

```
int wx[maxn],wy[maxn],*x,*y,wss[maxn],wv[maxn];

inline bool cmp(int *r,int n,int a,int b,int l)
{
    return a+l<n && b+l<n && r[a]==r[b] && r[a+l]==r[b+l];
}
void da(int str[],int sa[],int n,int m)
{
    int *s = str;
    int *x=wx,*y=wy,*t,p;
    int i,j;
    for(i=0; i<m; i++)
        wss[i]=0;
    for(i=0; i<n; i++)
        wss[x[i]=s[i]]++;
    for(i=1; i<m; i++)
        wss[i]+=wss[i-1];
    for(i=n-1; i>=0; i--)
        sa[--wss[x[i]]]=i;
    for(j=1,p=1; p<n && j<n; j*=2,m=p)
    {
        for(i=n-j,p=0; i<n; i++)
            y[p++]=i;
        for(i=0; i<n; i++)
            if(sa[i]-j>=0)
                y[p++]=sa[i]-j;
        for(i=0; i<n; i++)
            wv[i]=x[y[i]];
        for(i=0; i<m; i++)
            wss[i]=0;
        for(i=0; i<n; i++)
            wss[wv[i]]++;
        for(i=1; i<m; i++)
            wss[i]+=wss[i-1];
        for(i=n-1; i>=0; i--)
            sa[--wss[wv[i]]]=y[i];
        for(t=x,x=y,y=t,p=1,i=1,x[sa[0]]=0; i<n; i++)
            x[sa[i]]=cmp(y,n,sa[i-1],sa[i],j)?p-1:p++;
    }
}
```