

```
> library(faraway) #this command brings in a library of regression functions
> library(psych)
```

Attaching package: 'psych'

The following object is masked from 'package:faraway':

logit

```
> library(stats)
```

```
> library(olsrr)
```

Registered S3 methods overwritten by 'car':

method	from
influence.merMod	lme4
cooks.distance.influence.merMod	lme4
dfbeta.influence.merMod	lme4
dfbetas.influence.merMod	lme4

Attaching package: 'olsrr'

The following object is masked from 'package:faraway':

hsb

The following object is masked from 'package:datasets':

rivers

```
> library(car)
```

Loading required package: carData

Attaching package: 'car'

The following object is masked from 'package:psych':

logit

The following objects are masked from 'package:faraway':

logit, vif

Warning message:

package 'car' was built under R version 3.6.3

```
>
```

```
> #Use the QUASARS dataset from the textbook to examine  $e(i)$  where
```

```
> # $e(i)$  is  $[Y_i - \hat{Y}_{\text{hat}}(i)]$  where  $\hat{Y}_{\text{hat}}(i)$  is  $\hat{Y}_{\text{hat}}$  at  $x_i$  with the  $i$ th observation deleted
```

```
>
```

```

> #read in the data which is in a csv file
> quasars <-
read.csv(file="C:/Users/jmard/Desktop/RegressionMethodsSpring2020/Lecture 03
04FEB2020/QUASAR.csv",header = TRUE)
>
> # n=25 observations on 5 independent variables (covariates) and a response
Y=RFEWIDTH
>
> #Perform a multiple regression using the quasar data with only two predictors:
LINEFLUX and AB1450
> LMOD <- lm(RFEWIDTH ~ LINEFLUX + AB1450,data=quasars)
> summary(LMOD)

```

```

Call:
lm(formula = RFEWIDTH ~ LINEFLUX + AB1450, data = quasars)

```

```

Residuals:
      Min       1Q   Median       3Q      Max
-18.582  -9.881  -1.723   6.753  49.503

```

```

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  1232.778    199.096   6.192 3.12e-06 ***
LINEFLUX      205.424     18.819  10.916 2.39e-10 ***
AB1450        85.739      6.547  13.097 7.28e-12 ***
---

```

```

Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```

```

Residual standard error: 16.32 on 22 degrees of freedom
Multiple R-squared:  0.8913,    Adjusted R-squared:  0.8814
F-statistic: 90.18 on 2 and 22 DF,  p-value: 2.508e-11

```

```

>
> windows(7,7)
> #save graph in pdf
>
pdf(file="C:/Users/jmard/Desktop/RegressionMethodsSpring2020/Output/ei_QUASAR_R_o
ut.pdf")
>
> #Studentized Residuals vs Leverage Plot
> #Graph for detecting outliers without leverage and outliers with leverage
> ols_plot_resid_lev(LMOD)
>
> ##-----##
>
> all_obs <-
data.frame(cbind(quasars$LINEFLUX,quasars$AB1450,quasars$RFEWIDTH,LMOD$fitted.val
ues,LMOD$residuals))
>
> names(all_obs) <- c("LINEFLUX","AB1450","RFEWIDTH","Yhat_all","ei_all")

```

```

> head(all_obs,10L) #note the values for the row corresponding to observation #8
  LINEFLUX AB1450 RFEWIDTH   Yhat_all   ei_all
1   -13.48  19.50      117 135.582396 -18.582396
2   -13.73  19.65       82  97.087342 -15.087342
3   -13.87  18.93       33   6.595665  26.404335
4   -13.27  18.59       92 100.698582  -8.698582
5   -13.56  19.59      114 126.865032 -12.865032
6   -13.95  19.42       50  32.174045  17.825955
7   -13.83  19.18       43  36.247458   6.752542
8   -13.50  20.41      259 209.496738  49.503262
9   -13.66  18.93       58  49.734670   8.265330
10  -13.71  20.00      126 131.204595  -5.204595
> #Now set RFEWIDTH for observation #8 to missing which excludes the observation
from all regression computations.
> all_obs[8,3] <- NA
> head(all_obs,10L)
  LINEFLUX AB1450 RFEWIDTH   Yhat_all   ei_all
1   -13.48  19.50      117 135.582396 -18.582396
2   -13.73  19.65       82  97.087342 -15.087342
3   -13.87  18.93       33   6.595665  26.404335
4   -13.27  18.59       92 100.698582  -8.698582
5   -13.56  19.59      114 126.865032 -12.865032
6   -13.95  19.42       50  32.174045  17.825955
7   -13.83  19.18       43  36.247458   6.752542
8   -13.50  20.41       NA 209.496738  49.503262
9   -13.66  18.93       58  49.734670   8.265330
10  -13.71  20.00      126 131.204595  -5.204595
>
>
> #Fit the regression model with observation #8 missing
> LMODwo8 <- lm(RFEWIDTH ~ LINEFLUX + AB1450,data=all_obs)
> summary(LMODwo8)

```

```

Call:
lm(formula = RFEWIDTH ~ LINEFLUX + AB1450, data = all_obs)

```

Residuals:

```

      Min       1Q   Median       3Q      Max
-13.093  -6.310  -1.595   5.062  27.745

```

Coefficients:

```

              Estimate Std. Error t value Pr(>|t|)
(Intercept)   880.501    131.081   6.717 1.20e-06 ***
LINEFLUX      157.423     13.523  11.641 1.27e-10 ***
AB1450         69.848       4.635  15.069 9.85e-13 ***
---

```

```

Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```

```

Residual standard error: 9.747 on 21 degrees of freedom
(1 observation deleted due to missingness)

```

Multiple R-squared: 0.9154, Adjusted R-squared: 0.9073
 F-statistic: 113.6 on 2 and 21 DF, p-value: 5.489e-12

> LMODwo8\$fitted #note the missing fitted value for observation #8. We obtain the fit at observation #8 below.

1	2	3	4	5	6	7	9
120.47795	91.59945	19.26970	89.97504	114.17045	40.90140	43.02861	52.32848
10	11	12	13	14	15	16	17
119.19471	43.98908	118.25542	119.14176	76.20112	93.36154	96.43586	64.00419
18	19	20	21	22	23	24	25
94.27688	121.75841	78.35733	63.46180	53.96348	66.25572	112.09342	56.49818

>
 > x_at_8 <- data.frame(LINEFLUX=-13.5,AB1450=20.41)
 >

> fit_at_8_wo8 <- predict(LMODwo8,newdata = x_at_8)

> fit_at_8_wo8

1
 180.8912

>
 > quasars[8,] #print the 8th row of quasars

	QUASAR	REDSHIFT	LINEFLUX	LUMINOSITY	AB1450	ABSMAG	RFEWIDTH
8	8	2.81	-13.5	45.27	20.41	-25.36	259

>
 > e_at_8_wo8 <- quasars[8,7] - fit_at_8_wo8
 > e_at_8_wo8

1
 78.10882

>
 > check <- 259 - (880.501 + (157.423*-13.5) + (69.848*20.41)) #RFEWIDTH at obs 8
 - Fit at obs 8

> check
 [1] 78.11182

>
 > LMOD\$residuals

1	2	3	4	5	6
-18.5823958	-15.0873417	26.4043354	-8.6985816	-12.8650317	17.8259548
7	8	9	10	11	12
6.7525416	49.5032618	8.2653304	-5.2045950	0.5524107	18.9947144
13	14	15	16	17	18
-4.7025411	-1.7234498	-14.9814419	8.4129569	-7.9700885	-4.0123976
19	20	21	22	23	24
-15.7926644	0.8694368	1.5486756	-0.6238405	-9.8808988	-18.1261402
25					
-0.8782098					

> LMODwo8\$residuals

1	2	3	4	5	6
-3.47794682	-9.59945329	13.73030133	2.02495669	-0.17044522	9.09859984
7	9	10	11	12	13
-0.02861176	5.67151778	6.80528842	-1.98908310	27.74457886	4.85824069
14	15	16	17	18	19

All residuals are shown on the next page.

i	ei	e(8)
1	-18.582	-3.478
2	-15.087	-9.599
3	26.404	13.730
4	-8.699	2.025
5	-12.865	-0.170
6	17.826	9.099
7	6.753	-0.029
8	49.503	78.112
9	8.265	5.672
10	-5.205	6.805
11	0.552	-1.989
12	18.995	27.745
13	-4.703	4.858
14	-1.723	-1.201
15	-14.981	-8.362
16	8.413	12.564
17	-7.970	-9.004
18	-4.012	-3.277
19	-15.793	-5.758
20	0.869	-3.357
21	1.549	-0.462
22	-0.624	-7.963
23	-9.881	-11.256
24	-18.126	-13.093
25	-0.878	-3.498