

Bonferroni Joint Confidence Intervals $(1-\alpha) \times 100\%$ CIs

Start with Joint CIs on B_1, B_2 .

Let A = event that CI on B_1 does not cover B_1 . $P(A) = \alpha$

Let B = event that CI on B_2 does not cover B_2 . $P(B) = \alpha$

$P(\text{CI on } B_1 \text{ covers } B_1 \text{ and CI on } B_2 \text{ covers } B_2) = P(\bar{A} \cap \bar{B})$.

However, $P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B})$ by De Morgan's law.

$$\begin{aligned} P(\overline{A \cup B}) &= 1 - P(A \cup B) = 1 - [P(A) + P(B) - P(A \cap B)] \\ &= 1 - P(A) - P(B) + P(A \cap B) \geq 1 - P(A) - P(B) = 1 - 2\alpha \end{aligned}$$

∴ Compute $(1 - \frac{\alpha}{2}) \times 100\%$ CI on B_1 and
 $(1 - \frac{\alpha}{2}) \times 100\%$ CI on B_2 .

The confidence associated with both CIs covering
 their parameters $\geq 1 - 2(\frac{\alpha}{2}) = \underline{\underline{1 - \alpha}}$

Bonferroni Joint CIs are known to be conservative.

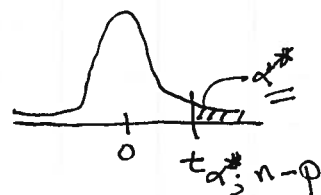
This method works for m joint CIs.

Compute $(1 - \frac{\alpha}{m}) \times 100\%$ CIs on each of m parameters.

Family wise/Experimentwise confidence $\geq 1 - \frac{\alpha}{m} (m) = \boxed{1 - \alpha}$

∴ for $m=2$

Compute
$$\begin{cases} \hat{B}_1 \pm t_{\frac{\alpha}{4}; n-p} s(\hat{B}_1) \\ \hat{B}_2 \pm t_{\frac{\alpha}{4}; n-p} s(\hat{B}_2) \end{cases}$$



$\alpha^* = \alpha / (2xm)$