Credible Intervals

Bayesian Data Analysis

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Summarizing a Posterior

- The posterior tells us everything we know about a parameter, but we would like to have ways of summarizing it.
- We could use give a measure of location, such as
  - Mean
  - Median
  - Mode (MAP)
- We could also give a measure of spread, such as
  - Standard deviation
  - Interquartile range

Credible intervals

- A popular summary is to give a *credible interval*.
  - For concreteness, let's talk about the 95% credible.
- Credible intervals are the Bayesian analog of confidence intervals.
  - They incorporate both location and spread of the posterior.
  - The interpretation is that there is a 95% probability that the parameter is inside the 95% credible interval.
  - Credible intervals are often given as 90% instead of 95%, in part because the boundaries of a 90% credible interval are more stable than the boundaries of a 95% credible interval.
  - You'll also see 50% credible intervals.

How to Construct a Credible Interval

- Credible intervals are not unique.
- There are two main approaches to constructing a credible interval.
- The first is the *central posterior credible interval*, also known as the *equal tails credible interval*.
    - For a 90% central posterior credible interval, sinmply find the points corresponding to the 5th and 95th percentiles of the posterior.
    - This is straightforward to calculate, and for a reasonably symmetric posterior it works well.
    - Consider, though, if we constructed a credible interval this way for the probability of a success after getting three failures in three trials.

Highest Density Interval

- The other main approach is the *highest density credible interval*, sometimes called the *most plausible Bayesian credible interval*, and often abbeviated as *HDI*.
- The definition is a little tricky, but once you are used to it the resulting interval will seem quite reasonable.
- The definition is, for a 90% HDI, all values of θ such that the probability density of θ is greater than or equal to w, where w is defined such that

$$\int_{\theta \text{ such that } p(\theta) \geq w} p(\theta)\, d\theta = 0.90.$$

- Every point in the HDI has a higher probability than every point outside the HDI.
- The HDI is harder to calculate than the central posterior credible interval, but
  - we will use R functions that do the work for us.
  - It is important to understand the concept, just not the calculation.
- For a multi-modal distribution, the HDI may consist of separated regions.