CS 323

Homework # 4: due Apr 18

Problem 1 a) Use the Composite Trapezoidal rule with 4 equally spaced subintervals to approximate the following integrals

$$\int_1^2 x \ln x dx.$$

b) Find an up bound of the error of approximation.

Solution: a)

$$T_4 = \frac{1}{8}(1 * \ln 1 + 2 * 1.25 * \ln 1.25 + 2 * 1.5 * \ln 1.5 + 2 * 1.75 * \ln 1.75 + 2 * \ln 2)$$
$$\approx 0.6399$$

b)

$$|\int_1^2 f(x) - T_4| \leq \frac{M_2}{12}(b-a)h^2 = \frac{M_2}{12 * 4^2}.$$

Here, $f'(x) = \ln x + 1$ and $f''(x) = \frac{1}{x}$, so

$$M_2 = \max_{1 \leq x \leq 2} \frac{1}{x} = 1.$$

Finally,

$$|\int_1^2 f(x) - T_4| \leq \frac{M_2}{12}(b-a)h^2 = \frac{1}{12 * 16} \approx 0.0052.$$

Problem 2 a) Use the Composite Simpson rule with 2 equally spaced subintervals to approximate the same integral in problem 1)

b) Find an up bound of the error of approximation.

Solution: a)

$$S_2 = \frac{1}{12}(1 * \ln 1 + 4 * 1.25 * \ln 1.25 + 2 * 1.5 * \ln 1.5 + 4 * 1.75 * \ln 1.75 + 2 * \ln 2)$$
$$\approx 0.63631$$

b)

$$|\int_1^2 f(x) - S_2| \leq \frac{M_4}{180}(1/4)^4.$$

Here, $f''(x) = -\frac{1}{x^2}$ and $f'''(x) = \frac{2}{x^3}$, so

$$M_4 = \max_{1 \le x \le 2} \frac{2}{x^3} = 2.$$

Finally,

$$|\int_1^2 f(x) - S_2| \le \frac{2}{180}(1/4)^4 = \frac{1}{90 * 256} \approx 4.3403 \times 10^{-5}.$$

Problem 3 Use the Gaussian Intergration with two nodes and weights to approximate the integral in problem 1).

Solution We first use a change of variable $x = \frac{1}{2}(3+t)$ to transform the integral

$$\int_1^2 x \ln x dx = \frac{1}{2}\int_{-1}^1 \frac{1}{2}(3+t)\ln(\frac{1}{2}(3+t))dt$$

Use the Gaussian Intergration with two nodes

$$\frac{1}{4}(3-\sqrt{1/3})\ln(\frac{1}{2}(3-\sqrt{1/3})) + \frac{1}{4}(3+\sqrt{1/3})\ln(\frac{1}{2}(3+\sqrt{1/3})) \approx 1.6759$$

Before doing Problems 4, copy the files ***HW4.zip*** *to your home directory. The use of these files will greatly simplify these problems.*

1. `fcn1.m` This file contains the function `fcn1(x)` which takes as input the value x and returns the value fcn1(x) .

2. `fcn2.m` This file contains the function `fcn2(x)` which takes as input the value x and returns the value fcn2(x) .

3. `trap.m` This file contains the function `trap(FunFcn,a,b,n)` which is similar to `mid`, but instead returns the approximation to the integral of Funfcn given by the composite trapezoidal rule on `n` subintervals.

4. `quadtrap.m` Contains function `quadtrap(FunFcn,a,b,tol,ninit,maxn)` taking as inputs the name of a function, the left and right endpoints `a` and `b` of the interval of integration, the absolute error tolerance `tol`, the initial number of subintervals `ninit`, and the maximum number of subintervals allowed `maxn`, and returns a vector whose components are:

the approximation to the integral of `Funfcn` given by the composite trapezoidal rule and an interval doubling strategy, the final number of subintervals used, and the error between the last two approximations.

5. `quadsimp.m` Contains function `quadsimp(FunFcn,a,b,tol,ninit,maxn)` which has the same inputs and outputs as `quadtrap.m`, but uses the composite Simpson's rule instead of the composite trapezoidal rule.

A typical statement in a *Matlab* program which calls one of these functions is:

```
[v, n, err] = quadtrap('fcn1',0,1,.0001,2,100000)
```

Note that the name of the function must be enclosed in quotes and the output of the function is a vector.

Problem 4 Find approximations to the following integrals:

$$\int_0^1 (1-4x(1-x))^{1/3}dx \quad \text{and} \quad \int_0^1 xe^{-x}dx$$

by using the composite trapezoidal rule (`quadtrap.m`) and composite Simpson's rule (`quadsimp.m`). Use the *Matlab* statement `format long` to get extra precision and choose `ninit = 2` and `maxn = 100000`. Run the programs for the choices `tol` $= 10^{-2}$, 10^{-4}, and 10^{-8} and record the error and the number of subintervals used in a table. In addition, for `tol`$= 10^{-16}$ run only the Simpson's rule program on the second function.

Function fcn1.m

tol	Trap error	Trap Sub	Simp error	Simp Sub
10^{-2}	0.00794490220735	16	0.00913042926485	4
10^{-4}	$9.364872715644790e-05$	256	$9.066353416042894e-05$	64
10^{-8}	$9.903734277116882e-09$	65536	$8.784285410179393e-09$	16384

Function fcn2.m

tol	Trap error	Trap Sub	Simp error	Simp Sub
10^{-2}	0.00389476145589	8	$4.551028438593008e-05$	4
10^{-4}	$6.103234470516972e-05$	64	$4.551028438593008e-05$	4
10^{-8}	$3.725290242950763e-09$	8192	$7.028795323549275e-10$	64
10^{-16}			$5.551115123125783e-17$	4096