First Simple Example

Bayesian Data Analysis

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A first example

This example is more conceptual than an illustration of how we will actually do our calculations

Basic idea:

- Count all the ways that the data can occur according to our various assumptions and our model.
- Assumptions with more ways that are consistent with the data, and with our prior beliefs, are more plausible.

A first example cont.

- For this toy example, let’s say that we have a bag with four marbles, each of which is either red or white.
- The manufacturer says that the number of red marbles is random.
- Let us suppose that the manufacturer also gives us the overall probability of each configuration of the number of red marbles in a bag—we’ll call that the “prior probability.”

Table of prior probabilities

Prior Probability	Red marbles	White marbles
0	0	4
3/6	1	3
2/6	2	2
1/6	3	1
0	4	0

- Notice that the prior probabilities add up to 1.0
- If you want to think in terms of a parameter, you can think of the parameter as the number of red marbles.
  - That parameter has five discrete possible values: 0, 1, …, 4.

New data

Now suppose we pull out a marble, which turns out to be red. We have 5 different sets of assumptions about the bag (the rows in the table). For each set of assumptions, how many ways could we get data such as we observed?

Prior Probability	Red marbles	White marbles	How many ways to pull one red marble in one draw?
0	0	4	0
3/6	1	3	1
2/6	2	2	2
1/6	3	1	3
0	4	0	4

- Later, we will say that this new column is proportional to the “likelihood.”
- The likelihood is measure of how “likely” the data is given the model and the particular value of the parameter.

Constructing the posterior probability

Now, let's multiply the prior with the new column

Prior Probability	Red marbles	White marbles	How many ways to pull one red marble in one draw?	Prior × count
0	0	4	0	0
3/6	1	3	1	3/6
2/6	2	2	2	4/6
1/6	3	1	3	3/6
0	4	0	4	0

Constructing the posterior probability cont.

The last column is not a probability, since it does not add up to 1.0, but if we divided each value by (0 + 3/6 + 4/6 + 3/6 + 0) = 10/6, then it would be. We will later call the new values the “posterior probability.”

Prior Probability	Red marbles	White marbles	How many ways to pull one red marble in one draw?	Prior $\times$ count	Posterior Probability
0	0	4	0	0	0
3/6	1	3	1	3/6	3/10
2/6	2	2	2	4/6	4/10
1/6	3	1	3	3/6	3/10
0	4	0	4	0	0

What we can say so far

- Right now, we have very little data,
- But—in part because of the fairly strong statement of prior probabilities—
- We can already say that we think that we think that the most probable state of the bag is 2 red and 2 white marbles.

Adding another experiment

- Now suppose that—after setting the red marble that we drew aside—we decide to perform another experiment, namely to draw another marble.
- The posterior probability from the old experiment—in the last table—will now serve as an updated prior probability for the new experiment.
- Otherwise, the calculations are pretty similar.

Updating the posterior probability

Original Prior	New Prior (= old posterior)	Red marbles	White marbles	How many ways to pull one red marble in one draw conditional on having already drawn a red marble?	Prior × count	New Posterior Probability
0	0	0	4	0	0	0
3/6	3/10	1	3	0	0	0
2/6	4/10	2	2	1	4/10	4/10
1/6	3/10	3	1	2	6/10	6/10
0	0	4	0	3	0	0

The Bayesian approach naturally incorporates going from experiment to experiment

- Starting with an original set of prior probabilities, after getting data from an experiment (drawing a marble) we found the posterior probabilities.
- Those posterior probabilities become the new prior for the next experiment, which we combine with new data to get new prior probabilities.

Vocabulary lesson

So what do we mean when we write posterior probability?

- The **posterior probability** is the probability of an event after all background information and evidence is taken into account.
- The **prior probability** is a way of operationalizing the background information into a quantitative form.
- The new evidence is turned, with the help of a statistical model, into a quantitative form that we will call the **likelihood**.
- The posterior probability combines the prior probability with the likelihood.
- We use the terms posterior probability and prior probability so often we'll often just call them the posterior and the prior.