

Name: \_\_\_\_\_

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(10) **Problem** Given the data points  $(1, 2), (2, 0), (3, 1), (4, -1)$ ,

a) Find the cubic interpolation polynomial in the Lagrange form. DO NOT SIMPLIFY.

**Solution**

$$p(x) = 2 \frac{(x-2)(x-3)(x-4)}{(1-2)(1-3)(1-4)} + \frac{(x-1)(x-2)(x-4)}{(3-1)(3-2)(3-4)} - \frac{(x-1)(x-2)(x-3)}{(4-1)(4-2)(4-3)}.$$

b) Find the cubic interpolation polynomial in the Newton form. DO NOT SIMPLIFY.

**Solution** Calculate the divided difference using the divided difference table,

$x_i$	$y_i$	$f[\cdot, \cdot]$	$f[\cdot, \cdot, \cdot]$	$f[\cdot, \cdot, \cdot, \cdot]$
1	2	$\frac{2-0}{1-2} = -2$	$\frac{-2-1}{1-3} = 1.5$	$\frac{1.5-(-1.5)}{1-4} = -1$
2	0	$\frac{0-1}{2-3} = 1$	$\frac{1-(-2)}{2-4} = -1.5$	—
3	1	$\frac{2-0}{1-2} = -2$	—	—
4	-1	—	—	—

The interpolation polynomial in the Newton's form is

$$p(x) = 2 - 2(x-1) + 1.5(x-1)(x-2) - (x-1)(x-2)(x-3).$$