The Bayes' Box and the Grid Method

Bayesian Data Analysis

Steve Buyske

Bayes' Box

- Let's return to Bayes' Rule for a moment to discuss a method called " Bayes' Box".
- When we have a discrete parameter, this is a way to calculate the denominator of Bayes' Rule and from there the posterior.

- We will do it via an example.
- Suppose that we have a coin with probability θ of heads, and suppose we toss it twice and observe 2 heads.
- Suppose also that there are only three possible values of θ, namely $1/4$, $1/2$, and $3/4$, and we judge them equally likely.
- On the next slide we'll construct the Bayes' box.

Bayes' Box p1

θ	Prob(θ)	Prob(HH $\mid$ θ)	Prob(HH $\mid$ θ)Prob(θ)	Prob(θ $\mid$ HH)
1/4	1/3			
1/2	1/3			
3/4	1/3			

Bayes' Box p2

θ	Prob(θ)	Prob(HH | θ)	Prob(HH | θ)Prob(θ)	Prob(θ | HH)
1/4	1/3	1/4 * 1/4 = 1/16		
1/2	1/3	1/2 * 1/2 = 1/4		
3/4	1/3	3/4 * 3/4 = 9/16		

Bayes' Box p3

θ	$\text{Prob}(\theta)$	$\text{Prob}(\text{HH} \mid \theta)$	$\text{Prob}(\text{HH} \mid \theta)\text{Prob}(\theta)$	$\text{Prob}(\theta \mid \text{HH})$
1/4	1/3	1/4 * 1/4 = 1/16	1/3 * 1/16 = 1/48	
1/2	1/3	1/2 * 1/2 = 1/4	1/3 * 1/4 = 1/12 = 4/48	
3/4	1/3	3/4 * 3/4 = 9/16	1/3 * 9/16 = 9/48	

Bayes' Box p4

θ	Prob(θ)	Prob(HH | θ)	Prob(HH | θ)Prob(θ)	Prob(θ | HH)
1/4	1/3	1/4 * 1/4 = 1/16	1/3 * 1/16 = 1/48	
1/2	1/3	1/2 * 1/2 = 1/4	1/3 * 1/4 = 1/12 = 4/48	
3/4	1/3	3/4 * 3/4 = 9/16	1/3 * 9/16 = 9/48	

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1/48 + 4/48 + 9/48 = 14/48
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Bayes' Box p4

θ	Prob(θ)	Prob(HH | θ)	Prob(HH | θ)Prob(θ)	Prob(θ | HH)
1/4	1/3	1/4 * 1/4 = 1/16	1/3 * 1/16 = 1/48	(1/48)/(14/48) = 1/14
1/2	1/3	1/2 * 1/2 = 1/4	1/3 * 1/4 = 1/12 = 4/48	(4/48)/(14/48) = 4/14
3/4	1/3	3/4 * 3/4 = 9/16	1/3 * 9/16 = 9/48	(9/48)/(14/48) = 9/14

`1/48 + 4/48 + 9/48 = 14/48`

Bayes' Box and the Grid Method

- With a discrete parameter, we could program the Bayes' Box method to find the posterior even if there were many possible values of θ.
- For a continuous parameter, we could get a good approximation by chopping up the continuous parameter into a grid.
- Notice that to do so, we just need to be able to evaluate the prior and the likelihood—the integral in the denominator is replaced by a sum.
- The finer the grid, the more accurate the approximation.
- There may be a problem if we don't know a bound for θ.
- There will definitely be a problem if there are more than just a few parameters, because the number of grid points will become very large.