CS 323 Quiz 2 March 8, 2019

Name:_____

Show all your work

(10) a) Give a Gauss-Seidel method for solving $Ax = b$ where

$$A = \begin{bmatrix} 5 & 2 & 3 \\ 0 & -4 & 2 \\ -1 & 1 & 5 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$$

Solution

$$N = \begin{bmatrix} 5 & 0 & 0 \\ 0 & -4 & 0 \\ -1 & 1 & 5 \end{bmatrix} \quad \text{and} \quad P = \begin{bmatrix} 0 & -2 & -3 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} x_1^{(k+1)} \\ x_2^{(k+1)} \\ x_3^{(k+1)} \end{bmatrix} = \begin{bmatrix} \frac{1}{5} & 0 & 0 \\ 0 & -\frac{1}{4} & 0 \\ \frac{1}{25} & \frac{1}{20} & \frac{1}{5} \end{bmatrix} \left[\begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix} + \begin{bmatrix} 0 & -2 & -3 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1^{(k)} \\ x_2^{(k)} \\ x_3^{(k)} \end{bmatrix} \right]$$

b) Find the iterate matrix M of Gauss Seidel method in part a) and find the norm $\|M\|$.

Solution

$$M = \begin{bmatrix} 0 & -2/5 & -3/5 \\ 0 & 0 & 1/2 \\ 0 & -2/25 & -11/50 \end{bmatrix}$$

$\|M\| = \max\{1, 1/2, 3/10\} = 1.$

c) Does Gauss-Seidel iteration converge?

Solution

The Gauss-Seidel may not converge.