The Law of Total Probability

Bayesian Data Analysis

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The Law of Total Probability

- The Law of Total Probability sounds rather grand, but in its simplest form it simply states, for two events A and B, that

$$\text{Prob}(A) = \text{Prob}(A, B) + \text{Prob}(A, B^c).$$

- Now we know from the definition of conditional probability that

$$\text{Prob}(A|B) = \text{Prob}(A, B)/\text{Prob}(\text{B}),$$

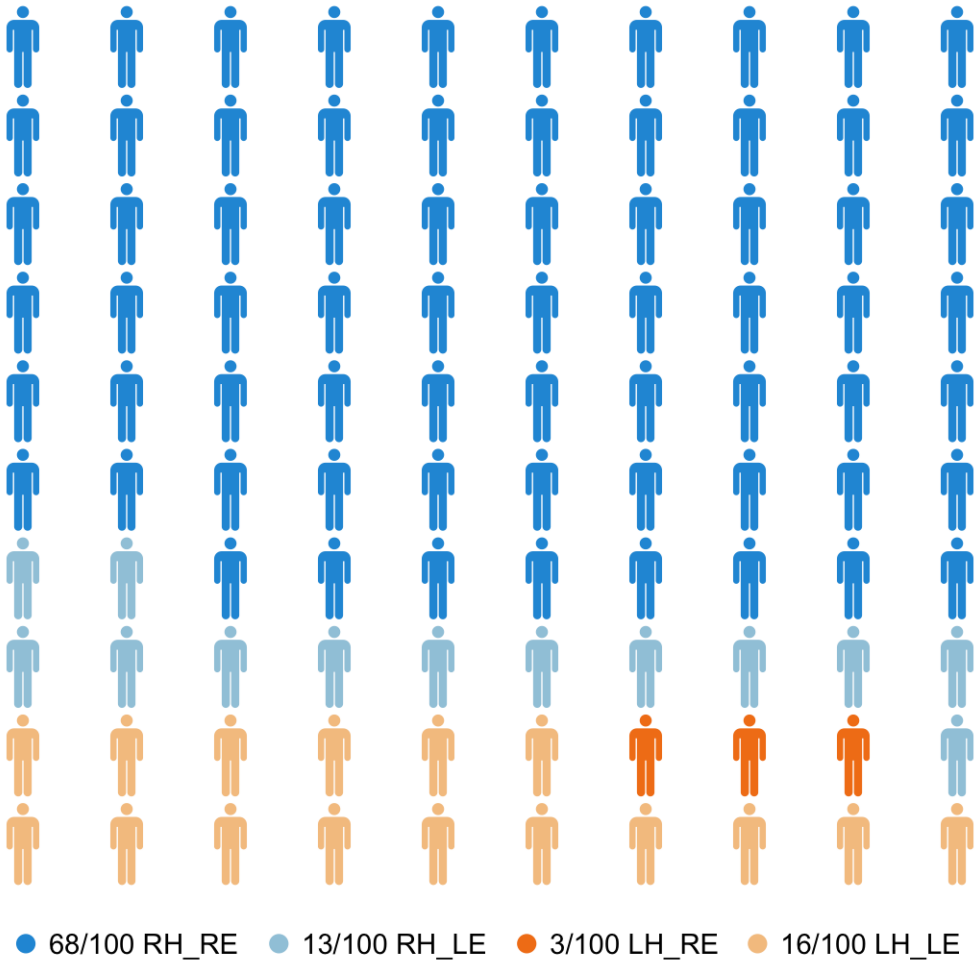
  which we can rewrite as

$$\text{Prob}(A, B) = \text{Prob}(A|B)\text{Prob}(\text{B}).$$

- Combine that with the first bullet point, and we get

$$\text{Prob}(A) = \text{Prob}(A|B)\text{Prob}(B) + \text{Prob}(A|B^c)\text{Prob}(B^c).$$

- Let’s say that A = “right-eyed” and B = “right-handed”



A more general version

- We've conditioned on B and B^c. Together, they make up the universe.
- More generally, suppose $B_1, \ldots, B_k$ are a set of mutually exclusive events that make up the universe.
- The Law of Total Probability can then be written as

$$\text{Prob}(A) = \text{Prob}(A|B_1)\text{Prob}(B_1) + \cdots + \text{Prob}(A|B_k)\text{Prob}(B_k).$$