Bayes’ Rule

Bayesian Data Analysis

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Bayes' Rule

- We've seen that conditional probability of event A given event B is defined via

$$\text{Prob}(A \mid B) = \frac{\text{Prob}(A, B)}{\text{Prob}(B)}.$$

- We also saw that with a little algebraic manipulation, we get

$$\text{Prob}(A, B) = \text{Prob}(A \mid B)\text{Prob}(B).$$

- Since $\text{Prob}(A, B) = \text{Prob}(B, A)$, we can put these together to get

$$\text{Prob}(A \mid B)\text{Prob}(B) = \text{Prob}(A, B) = \text{Prob}(B \mid A)\text{Prob}(A).$$

- *[repeating the last expression]*

$$\mathrm{Prob}(A \mid B)\mathrm{Prob}(B) = \mathrm{Prob}(A, B) = \mathrm{Prob}(B \mid A)\mathrm{Prob}(A).$$

- If we divide both the left and right expressions by $\mathrm{Prob}(B)$, we get **Bayes' Rule**:

$$\mathrm{Prob}(A \mid B) = \frac{\mathrm{Prob}(B \mid A)\mathrm{Prob}(A)}{\mathrm{Prob}(B)}.$$

- Bayes' Rule is also known as Bayes' Law or Bayes' Theorem.

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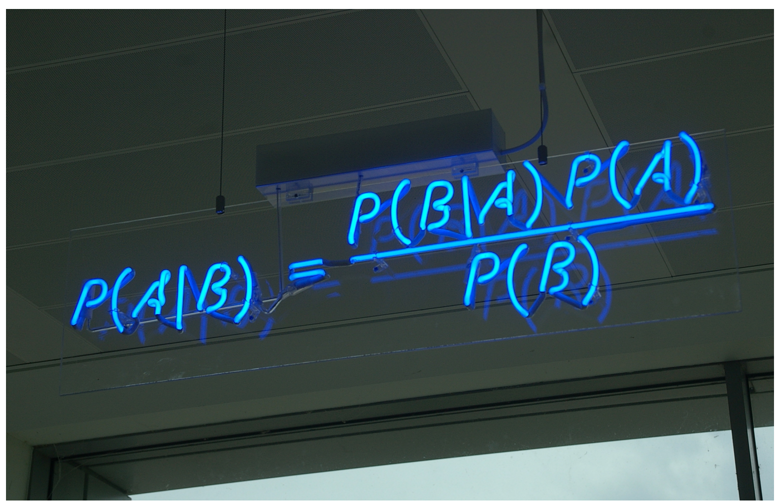

By mattbuck (category) - <span class="int-own-

- On one level, Bayes' Rule is mundane—it only involves the definition of conditional probability plus a small amount of algebra.
- On another level, Bayes' Rule is profound, because it tells us how to convert from one probability that we might know to another probability that we might wish to know.
    - We might know $\text{Prob}(\text{Positive lab result} \mid \text{Dread Disease})$
    - but we *want* to know $\text{Prob}(\text{Dread Disease} \mid \text{Positive lab result})$.

      I corrected a typo on this line.
    - To do so, we also need a bit more information, namely $\text{Prob}(\text{Dread Disease})$ and $\text{Prob}(\text{Positive lab result})$.

- We will go through an example of Bayes' Rule in another segment, but before we do there are three general points to make.

1. It is not always obvious how to calculate $\text{Prob}(B)$.

- - Sometimes it will be easier using the Law of Total Probability,

$$\text{Prob}(B) = \text{Prob}(B \mid A)\text{Prob}(A) + \text{Prob}(B \mid A^c)\text{Prob}(A^c)$$

  - In the language of the previous slide,

$$\text{Prob}(\text{Positive lab result}) =$$

$$\text{Prob}(\text{Positive lab result} \mid \text{Dread Disease})\text{Prob}(\text{Dread Disease})$$

$$+\text{Prob}(\text{Positive lab result} \mid \text{No Dread Disease})\text{Prob}(\text{No Dread Diseas}$$

2.

- - Using Bayes' Rule doesn't make you a Bayesian—it's just a bit of mathematics.
  - However, we will see that we can think of A as a statement of parameters, and B as a statement about evidence. Then

$$\mathrm{Prob}(A \mid B) = \frac{\mathrm{Prob}(B \mid A)\mathrm{Prob}(A)}{\mathrm{Prob}(B)}$$

    is a way of updating our belief about parameters given new evidence.

3. Bayes' Rule has a bit of colorful history.

- - Generally credited to the Reverend Thomas Bayes (1701–1761), it was published 2 years after his death by his friend, Richard Price (1723–1791), who did consider work in preparing the paper (later described as "remarkably opaque"), although the paper does not actually give a statement of Bayes' Rule.
  - It may have been intended as part of an argument for miracles, in opposition to John Hume.
  - Pierre-Simon Laplace (1749–1827) independently derived it and was the first to set out a Bayesian Framework.
  - Everyone uses the picture below of Bayes, but it is very likely not a picture of him.