Marginal, Joint, and Conditional Probability

Bayesian Data Analysis

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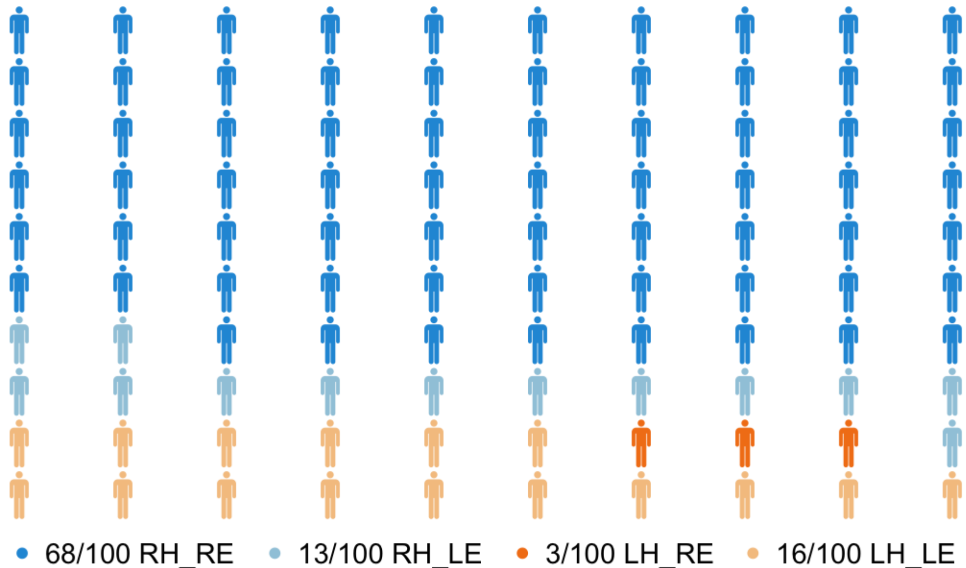
Some data to get us started

In last year's class, I asked the students to report their dominant hand and their dominant eye. To simplify things, the table below shows numbers scaled up to a class of 100.

	left-eyed	right-eyed	Total
left-handed	16	3	19
right-handed	13	68	81
Total	29	71	100

We will use this table to distinguish among the concepts of *marginal probability*, *joint probability*, and *conditional probability*.

- Here's another way of visualizing the same data.
- The figure has a separate icon for each person in the class.



- Let us write RH for the event of a randomly selected student from the class being right handed and
- RE for the event of a randomly selected student being right eyed.
- Notice that RH is an event defined in terms of the handedness variable, while RE is an event defined in terms of the “eyedness” variable.
- The *size* of RH, written $|\mathrm{RH}|$, is the number of elements in the event, namely 81.
- The size of the RH^c (the complement of RH), written $|\mathrm{RH}^c|$, is the number of elements in the universe but not in the event, namely 19.
- The size of the universe in this case is $|\mathrm{U}| = 100$.

Marginal Probability

- What is the probability of RH, Prob(RH)?
- Simply

$$\text{Prob(RH)} = \frac{|\text{RH}|}{|\text{U}|} = \frac{81}{100}.$$

- Similarly,

$$\text{Prob}(\text{RH}^c) = |\text{RH}^c|/|\text{U}| = 19/100$$

- and

$$\text{Prob(RE)} = |\text{RE}|/|\text{U}| = 71/100$$

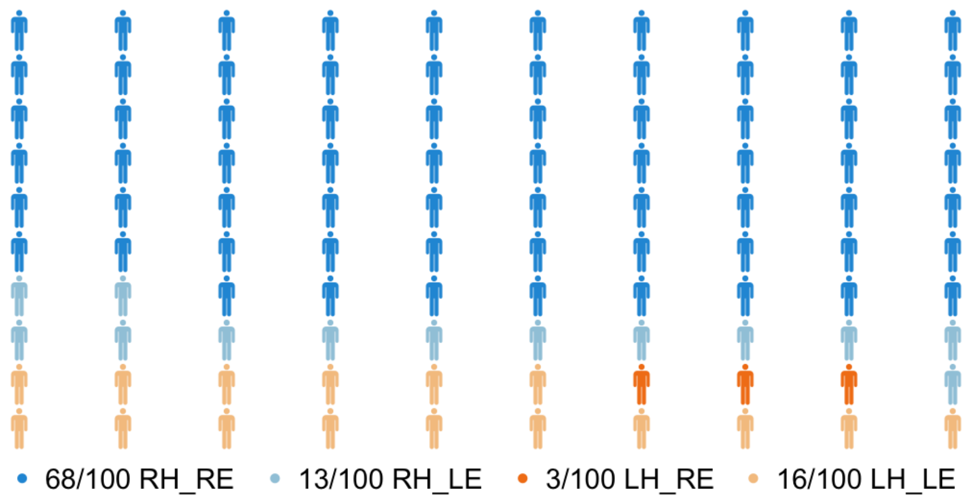
- $$\text{Prob}(\text{RE}^c) = |\text{RE}^c|/|\text{U}| = 29/100.$$

Marginal Probability cont.

- By *marginal probability*, we mean the probabilities involved in one variable while just ignoring any other variables.
- For example, the probabilities of events having to do with handedness, while ignoring left- or right-eyedness.
- Or the probabilities of events having to do with eyedness, while ignoring handedness.
- You can think of these as having to do with the *margins* of the table—just the row totals or just the column totals.

Here’s another way to think of it.

- The marginal distribution for handedness is based just considering blue or orange.
- The marginal distribution for eyeness is based on just considering light or dark.

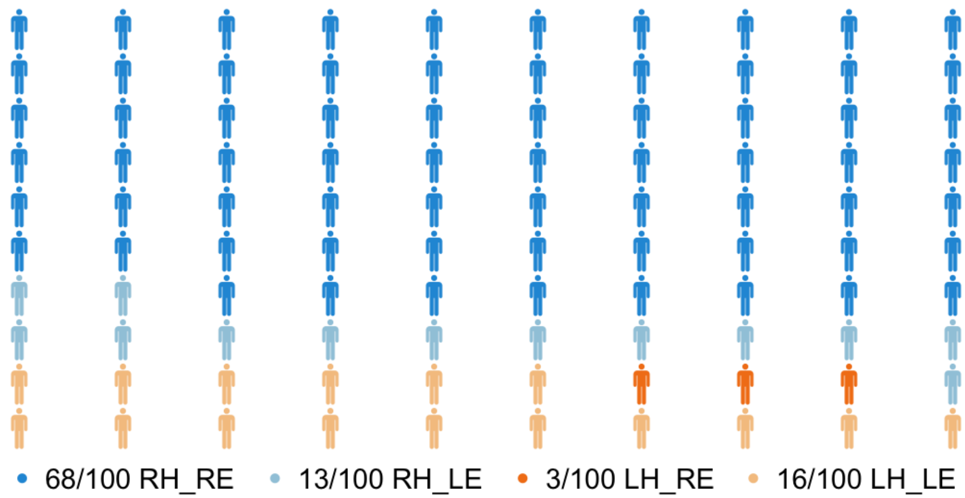


Joint Probability

- What is the probability of a student being right-handed **and** right-eyed?
- We can write that as $\text{Prob}(\text{RH and RE})$ and calculate it as $\text{Prob}(\text{RH and RE}) = 68/100$.
- In the figure, it corresponds to the proportion of dark blue individuals.
- The *joint probability* is the probability of an event defined in terms of both variables, namely both handedness and eyedness.
- By the way, statisticians get tired of writing the and, and so will just separate the events with a comma when they mean to imply an and. That is, $\text{Prob}(\text{RH}, \text{RE}) = \text{Prob}(\text{RH and RE})$.

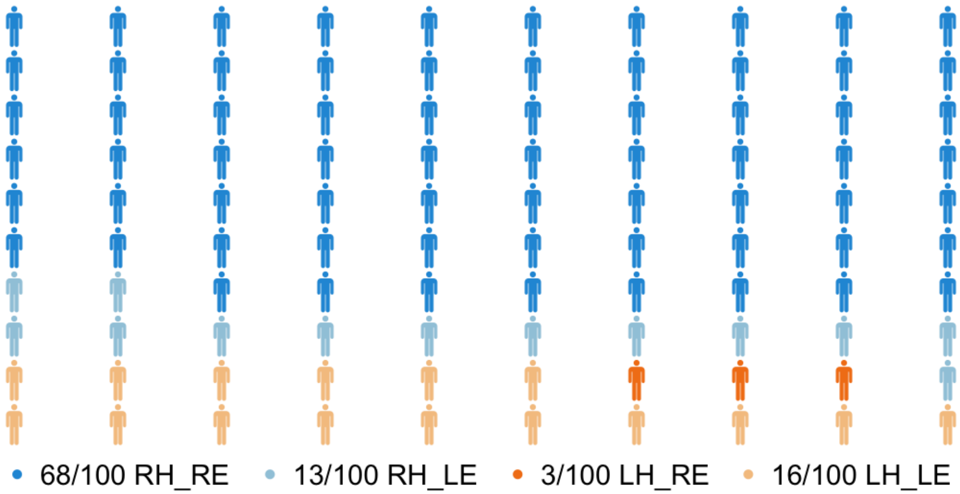
Joint Probability cont.

- The joint probability of RH, and RE, namely $\mathrm{Prob}(\mathrm{RH}, \mathrm{RE})$ is the probability of picking a dark blue student.
- Since $|\mathrm{RH}\text{ and }\mathrm{RE}| = 68$ and $|U|$ = 100, we have $\mathrm{Prob}(\mathrm{RH}, \mathrm{RE}) = 68/100$.



Conditional Probability

- What about the probability that a person is left-eyed, *given* that they are right-handed?
- This is like saying that we restrict the universe to right-handed students and then asking the probability that such a student is left-eyed?
- In terms of the figure, among all blue icons, what is the probability of picking one that is light blue?



Conditional Probability cont.

- We just need to divide the size of right-handed, left-eyed students, $|RH, RE^c| = 13$, by the size of the restricted universe, $|RH| = 68 + 13 = 81$. *Corrected*
- The notation for the result is

$$\text{Prob(Left-eyed|right-handed)} = \text{Prob}(\text{RE}^c|\text{RH}),$$

  where the "|" means "conditional on".
- Thus

$$\text{Prob}(\text{RE}^c|\text{RH}) = \frac{|RE^c, RH|}{|RH|} = \frac{|RE^c, RH|/|U|}{|RH|/|U|}$$

$$= \frac{\text{Prob}(\text{RE}^c, \text{RH})}{\text{Prob(RH)}} = 13/81.$$

Conditional Probability Definition.

- We can actually use this as the definition of conditional probability.
- For events A and B, we define

$$\text{Prob}(A|B) = \frac{\text{Prob}(A, B)}{\text{Prob}(B)}.$$

Where to find out more

- For this course, we will not go further into the idea of conditional probability.
- If you would like to read more, however, I suggest the chapter in *Probability and Bayesian Modeling*: https://bayesball.github.io/BOOK/conditional-probability.html
    - The chapter uses some R code, but you can just skip over that.