https://textbooks.math.gatech.edu/ila/eigenvectors.html

Eigenvalues and Eigenvectors

Here is an important definition regarding matrices.

Let A be an k×k matrix.

1. An ***eigenvector*** of A is a *nonzero* vector v in Rk such that Av=λv, for some scalar λ.
2. An ***eigenvalue*** of A is a scalar λ such that the equation Av=λv has a *nontrivial* solution.

If Av=λv for v not equal to 0, we say that λ is the ***eigenvalue for*** v, and that v is an ***eigenvector for*** λ.

The German prefix "eigen" roughly translates to "self" or "own". An eigenvector of A is a vector that is taken to a multiple of itself by the matrix transformation T(x)=Ax, which perhaps explains the terminology. On the other hand, "eigen" is often translated as "characteristic"; we may think of an eigenvector as describing an intrinsic, or characteristic, property of A.

Note: Eigenvalues and eigenvectors are only for square matrices.

```
> #properties of eigenvalues/eigenvectors
>
> A <- matrix(c(1.34,-0.16,0.19,-0.16, 0.62, -0.13,0.19,-0.13,1.49), 3, 3,
byrow=TRUE)
> A
      [,1]  [,2]  [,3]
[1,]  1.34 -0.16  0.19
[2,] -0.16  0.62 -0.13
[3,]  0.19 -0.13  1.49
> eval <- eigen(A)  #extract principal components
> eval$values
[1] 1.6571563 1.2157875 0.5770563
>
> (evec <- eval$vectors)  #show eigenvectors
           [,1]        [,2]      [,3]
[1,]  0.5729000  0.79965881 0.1798093
[2,] -0.1883644 -0.08505274 0.9784094
[3,]  0.7976869 -0.59440039 0.1019005
>
> #Show some properties of eigenvalues/eigenvectors
> library(matlib)
>
> #1. Orthogonality
> crossprod(evec)
              [,1]          [,2]          [,3]
[1,]  1.000000e+00 -5.551115e-17  1.526557e-16
[2,] -5.551115e-17  1.000000e+00 -3.469447e-17
[3,]  1.526557e-16 -3.469447e-17  1.000000e+00
> (t(evec)%*%evec)  #matrix(eigenvectors)'matrix(eigenvectors)
              [,1]          [,2]          [,3]
[1,]  1.000000e+00 -5.551115e-17  1.526557e-16
[2,] -5.551115e-17  1.000000e+00 -3.469447e-17
[3,]  1.526557e-16 -3.469447e-17  1.000000e+00
> #2. trace(A) = sum of diagonal elements of A = sum of eigenvalues
> sum(eval$values)
[1] 3.45
> tr(A)
[1] 3.45
>
> #3. determinant (A) = product of eigenvalues (A)
> prod(eval$values)
[1] 1.162624
> det(A)
[1] 1.162624
>
> #4. rank (A) = number of non-zero eigenvalues(A)
> sum(eval$values != 0)
[1] 3
> R(A)
[1] 3
```