10.6 a. To compute a moving average using four points, it is important to note that the midpoint of the sum is between the second and third time point. The convention of "dropping it down one line" is used, thereby using the third time point in the series as the starting point. The first moving average in this series which can be calculated in this manner will be from the third quarter in 2008. The general formula for calculating the sum of the four values is:

$$L_t = y_{t-2} + y_{t-1} + y_t + y_{t+1}$$

To calculate the moving average we divide the resulting sum by 4. For example, for the third quarter in 2008,

$$L_{2008,3} = 1322.70 + 1280.00 + 1166.36 + 903.25 = 4672.32$$

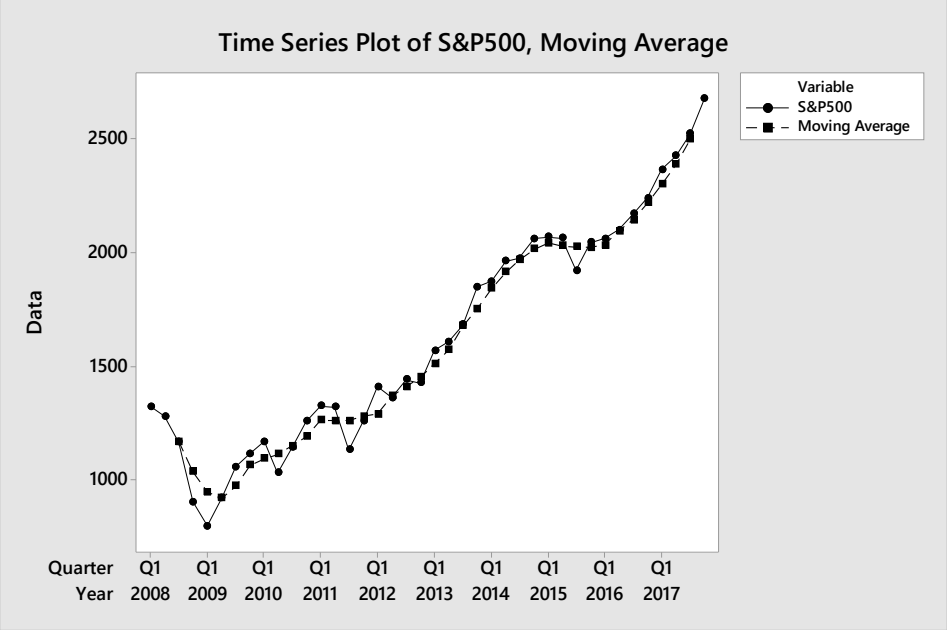
The moving average is $M_{2008,3} = L_{2008,3}/4 = 4672.32/4 = 1168.08$.

The remaining moving averages are calculated in a similar manner and are shown in the table below:

t	Year	Quarter	S&P500	L_t	M_t
1	2008	1	1322.70		
2	2008	2	1280.00		
3	2008	3	1166.36	4672.32	1168.08
4	2008	4	903.25	4147.48	1036.87
5	2009	1	797.87	3786.80	946.70
6	2009	2	919.32	3677.52	919.38
7	2009	3	1057.08	3889.37	972.34
8	2009	4	1115.10	4260.93	1065.23
9	2010	1	1169.43	4372.32	1093.08
10	2010	2	1030.71	4456.44	1114.11
11	2010	3	1141.20	4598.98	1149.74
12	2010	4	1257.64	4755.37	1188.84
13	2011	1	1325.83	5045.30	1261.33
14	2011	2	1320.64	5035.52	1258.88
15	2011	3	1131.42	5035.49	1258.87
16	2011	4	1257.60	5118.13	1279.53
17	2012	1	1408.47	5159.65	1289.91
18	2012	2	1362.16	5468.90	1367.23
19	2012	3	1440.67	5637.48	1409.37
20	2012	4	1426.19	5798.20	1449.55
21	2013	1	1569.19	6042.32	1510.58
22	2013	2	1606.28	6283.20	1570.80
23	2013	3	1681.55	6705.37	1676.34
24	2013	4	1848.36	7008.52	1752.13
25	2014	1	1872.34	7362.47	1840.62
26	2014	2	1960.23	7653.21	1913.30
27	2014	3	1972.29	7863.75	1965.94
28	2014	4	2058.90	8059.31	2014.83
29	2015	1	2067.89	8162.19	2040.55
30	2015	2	2063.11	8109.93	2027.48
31	2015	3	1920.03	8094.97	2023.74
32	2015	4	2043.94	8086.82	2021.70
33	2016	1	2059.74	8122.56	2030.64
34	2016	2	2098.86	8370.81	2092.70
35	2016	3	2168.27	8565.70	2141.42
36	2016	4	2238.83	8868.68	2217.17
37	2017	1	2362.72	9193.23	2298.31

38	2017	2	2423.41	9544.32	2386.08
39	2017	3	2519.36	9979.10	2494.77
40	2017	4	2673.61		

b. The effects of the recession can be seen in the time series plot, with the decline in 2009 by a bull market the rest of the time in the S&P 500. It is difficult to distinguish any seasonal effects as distinct from the overall trends marking the bull and bear market periods.



c. A subjective assessment of the forecast for the 1st quarter of 2018 would need to rely on the continued upward trend exhibited at the end of the time period. An estimate of 2660 would be reasonable.

d. To calculate the exponentially smoothed time series, note that the first value in the series is merely the first value of the time series. The remaining values in the smoothed series are calculated using the formula:

$E_t = wy_t + (1-w)E_{t-1}$, where w is the smoothing constant, and in this case we use $w = 0.3$.

For 2008,1: $E_{2008,1} = y_{2008,1} = 1322.70$

For 2008,2: $E_{2008,2} = wy_{2008,2} + (1-w)E_{2008,1} = 0.3(1280.00) + 0.7(1322.70) = 1309.89$

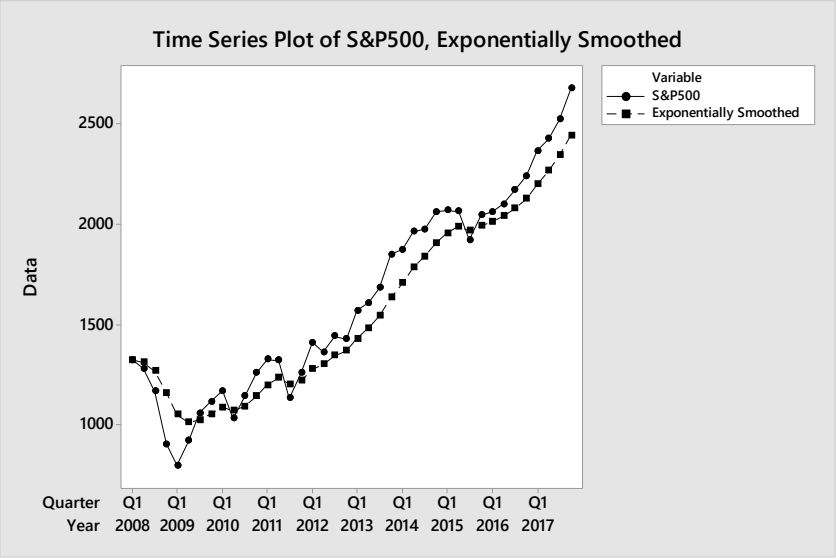
For 2008,3: $E_{2008,3} = wy_{2008,3} + (1-w)E_{2008,2} = 0.3(1166.36) + 0.7(1309.89) = 1266.83$

The rest of the values of the exponentially smoothed series are found in a similar manner and are shown in the table below:

t	Year	Quarter	S&P500	E_t
1	2008	1	1322.70	1322.70
2	2008	2	1280.00	1309.89
3	2008	3	1166.36	1266.83
4	2008	4	903.25	1157.76
5	2009	1	797.87	1049.79
6	2009	2	919.32	1010.65
7	2009	3	1057.08	1024.58
8	2009	4	1115.10	1051.73
9	2010	1	1169.43	1087.04
10	2010	2	1030.71	1070.14
11	2010	3	1141.20	1091.46
12	2010	4	1257.64	1141.31
13	2011	1	1325.83	1196.67
14	2011	2	1320.64	1233.86
15	2011	3	1131.42	1203.13
16	2011	4	1257.60	1219.47
17	2012	1	1408.47	1276.17
18	2012	2	1362.16	1301.97
19	2012	3	1440.67	1343.58
20	2012	4	1426.19	1368.36
21	2013	1	1569.19	1428.61
22	2013	2	1606.28	1481.91
23	2013	3	1681.55	1541.80
24	2013	4	1848.36	1633.77
25	2014	1	1872.34	1705.34
26	2014	2	1960.23	1781.81
27	2014	3	1972.29	1838.95
28	2014	4	2058.90	1904.94
29	2015	1	2067.89	1953.82
30	2015	2	2063.11	1986.61
31	2015	3	1920.03	1966.63
32	2015	4	2043.94	1989.83
33	2016	1	2059.74	2010.80
34	2016	2	2098.86	2037.22
35	2016	3	2168.27	2076.53
36	2016	4	2238.83	2125.22
37	2017	1	2362.72	2196.47
38	2017	2	2423.41	2264.55
39	2017	3	2519.36	2340.99
40	2017	4	2673.61	2440.78

E1 in answer is repeated for observations 1 and 2 in R version

A plot of the smoothed series and the S & P 500 values is:



e. As exponential smoothing forecasts are obtained by using the most recent exponentially smoothed value, it follows that $F_{2018,1} = E_{2017,4} =$ 2440.78.

see match_answer in R output

f. There are 40 values in the time series, so $F_{2018,1}$ corresponds with the forty-first time point, and $P = 4$ (i.e. we are using quarterly data). To assess which formula is to be used for the exponentially smoothed component, the trend component, and the seasonal component note that we use:

$E_t = y_t$ when $t = 2$
$E_t = wy_t + (1-w)(E_{t-1} + T_{t-1})$ when $t = 3,4,...,6$ (i.e. up to the value of $P+2=6$)
$E_t = w(y_t / S_{t-P}) + (1-w)(E_{t-1} + T_{t-1})$ when $t > 6$

$T_t = y_2 - y_1$ when $t = 2$
$T_t = v(E_t - E_{t-1}) + (1-v)T_{t-1}$ when $t > 2$

$S_t = y_t / E_t$ when $t = 2,3,...,6$ (i.e. up to the value of $P+2=6$)
$S_t = u(y_t / E_t) + (1-u)S_{t-P}$ when $t > 6$

Therefore, for $t = 2$:
$E_2 = y_2 = 1280.00$
$T_2 = 1280.00 - 1322.70 = -42.70$
$S_2 = y_2 / E_2 = 1280.00 / 1280.00 = 1$

For $t = 3$:
$E_3 = 0.3y_3 + (1-0.3)(E_2 + T_2) = 0.3(1166.36) + 0.7(1280.00 + (-42.70)) = 1216.02$
$T_3 = 0.8(E_3 - E_2) + (1-0.8)T_2 = 0.8(1216.02 - 1280.00) + 0.2(-42.70) = -59.73$
$S_3 = y_3 / E_3 = 1166.36/1216.02 = 0.959$

The remaining values are shown in the table below:

t	Year	Quarter	S&P500	E_t	T_t	S_t
1	2008	1	1322.70			
2	2008	2	1280.00	1280.00	-42.70	1.000
3	2008	3	1166.36	1216.02	-59.73	0.959
4	2008	4	903.25	1080.38	-120.46	0.836
5	2009	1	797.87	911.31	-159.35	0.876
6	2009	2	919.32	802.16	-119.18	1.146
7	2009	3	1057.08	808.71	-18.60	1.133
8	2009	4	1115.10	953.21	111.88	1.003
9	2010	1	1169.43	1146.27	176.83	0.948
10	2010	2	1030.71	1195.98	75.13	1.004
11	2010	3	1141.20	1191.91	11.77	1.045
12	2010	4	1257.64	1218.76	23.83	1.017
13	2011	1	1325.83	1289.44	61.31	0.988
14	2011	2	1320.64	1340.17	52.84	0.995
15	2011	3	1131.42	1299.83	-21.71	0.958
16	2011	4	1257.60	1265.51	-31.80	1.006
17	2012	1	1408.47	1291.25	14.24	1.039
18	2012	2	1362.16	1324.68	29.59	1.011
19	2012	3	1440.67	1399.19	65.53	0.994
20	2012	4	1426.19	1450.79	54.38	0.994
21	2013	1	1569.19	1506.52	55.47	1.041
22	2013	2	1606.28	1569.80	61.72	1.017
23	2013	3	1681.55	1649.70	76.26	1.007
24	2013	4	1848.36	1765.85	108.17	1.021
25	2014	1	1872.34	1851.65	90.28	1.026
26	2014	2	1960.23	1937.38	86.64	1.015
27	2014	3	1972.29	2004.66	71.15	0.995
28	2014	4	2058.90	2058.32	57.16	1.010
29	2015	1	2067.89	2085.58	33.24	1.009
30	2015	2	2063.11	2093.21	12.75	1.000
31	2015	3	1920.03	2052.97	-29.64	0.965
32	2015	4	2043.94	2023.20	-29.75	1.010
33	2016	1	2059.74	2008.02	-18.09	1.017
34	2016	2	2098.86	2022.55	8.00	1.019
35	2016	3	2168.27	2095.31	59.81	1.000
36	2016	4	2238.83	2173.36	74.41	1.020
37	2017	1	2362.72	2270.26	92.40	1.029
38	2017	2	2423.41	2367.39	96.18	1.021
39	2017	3	2519.36	2480.29	109.56	1.008
40	2017	4	2673.61	2599.08	116.94	1.024

see Et (level)
Tt (trend)
St (season)
in R output

The forecast for quarter 1 of 2018, where $t = 41\ (n = 40)$ is given by:

$$F_t = (E_n + T_n)S_{n+1-P} = (E_{40} + T_{40})S_{40+1-4} = (E_{40} + T_{40})S_{37}$$
$$= (2599.08 + 116.94)1.029 = 2794.78$$

10.7 a. To compute a moving average using three points, the midpoint of the sum is the second time point. The first moving average in this series which can be calculated in this manner will be from 2001.

The general formula for calculating the sum of the three values is:

$$L_t = y_{t-1} + y_t + y_{t+1}$$

To calculate the moving average we divide the resulting sum by 3. For example, for the 2001 moving average point

$$L_{2001} = y_{2000} + y_{2001} + y_{2002} = 279 + 271 + 310 = 860$$

The moving average, $M_{2001} = L_{2001}/3 = 860/3 = 286.67$. The remaining moving averages may be calculated in a similar manner, and are shown in the table below:

Year	Price	L_t	M_t
2000	279		
2001	271	860	286.67
2002	310	944	314.67
2003	363	1083	361.00
2004	410	1218	406.00
2005	445	1458	486.00
2006	603	1743	581.00
2007	695	2170	723.33
2008	872	2539	846.33
2009	972	3069	1023.00
2010	1225	3769	1256.33
2011	1572	4466	1488.67
2012	1669	4652	1550.67
2013	1411	4346	1448.67
2014	1266		
2015	1160		
2016	1251		
2017	1257		

The plot of the price and moving average series is: