A Review of Basic Concepts (Optional)

Chapter **1**

1.1 a. High school GPA is a number usually between 0.0 and 4.0. Therefore, it is quantitative.

b. Country of citizenship: USA, Japan, etc. is qualitative.

c. The scores on the SAT's are numbers between 200 and 800. Therefore, it is quantitative.

d. Gender is either male or female. Therefore, it is qualitative.

e. Parent's income is a number: \$25,000, \$45,000, etc. Therefore, it is quantitative.

f. Age is a number: 17, 18, etc. Therefore, it is quantitative.

1.2 a. The experimental units are the new automobiles. The model name, manufacturer, type of transmission, engine size, number of cylinders, estimated city miles/gallon, and estimated highway miles/gallon are measured on each automobile.

b. Model name, manufacturer, and type of transmission are qualitative. None of these is measured on a numerical scale. Engine size, number of cylinders, estimated city miles/gallon, and estimated highway miles/gallon are all quantitative. Each of these variables is measured on a numerical scale.

1.3 Both the variables current position and type of organization are qualitative. The variable years of experience is quantitative because it is measured on a numerical scale.

1.4 The experimental units are the operational satellites currently in orbit around Earth. The variables country of operator/owner, primary use, and class of orbit are all qualitative because none are measured on a numerical scale. The variables longitudinal position, apogee, launch mass, usable electric power, and expected lifetime are all quantitative variables. All of these variables are measured on a numerical scale.

1.5 a. Species of sea buckthorn is a qualitative variable.

b. Altitude of collection location is a quantitative variable.

c. Total flavonoid content in berries is a quantitative variable.

1.6 Gender and level of education are both qualitative since neither is measured on a numerical scale. Age, income, job satisfaction score, and Machiavellian rating score are all quantitative since they can be measured on a numerical scale.

1.7 a. The population of interest is all decision makers. The sample set is 155 volunteer students. Variables measured were the emotional state and whether to repair a very old car (yes or no).

b. Subjects in the guilty-state group are less likely to repair an old car.

1.8 a. The data would represent the population. These data are all of the data that are of interest to the researchers.

b. If the 80 jamming attacks are actually a sample, then the population would be all jamming attacks by the U.S. military over the past several years.

1.9 a. The experimental units are the participants in the study.

b. The variables of interest are the price of the engagement ring and the level of appreciation. The price of the ring is quantitative, while the level of appreciation is qualitative.

c. The population of interest is average American engaged couples.

d. The sample of 33 respondents is probably not representative of the population. Only engaged couples who used a popular website for engaged couples were used. Those who used this website were probably not representative of all average American engaged couples.

1.10 a. The sample is the set of 505 teenagers selected at random from all U.S. teenagers.

b. The population from which the sample was selected is the set of all teenagers in the U.S.

c. Since the sample was a random sample, it should be representative of the population.

d. The variable of interest is the topics that teenagers most want to discuss with their parents.

e. The inference is expressed as a percent of the population that want to discuss particular topics with their parents.

f. The "margin of error" is the measure of reliability. This margin of error measures the uncertainty of the inference.

1.11 a. The population of interest is all young women who recently participated in a STEM program.

b. The sample is the 159 young women who were recruited to complete an online survey.

c. We would infer that 27% of all young women who recently participated in a STEM program felt that participation in the STEM program increased their interest in science.

1.12 a. The population of interest is the Machiavellian traits in accountants.

b. The sample is 198 accounting alumni of a large southwestern university.

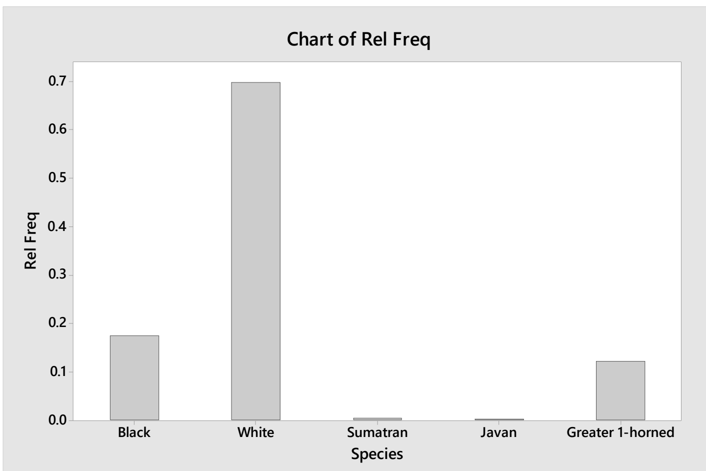
c. The Machiavellian behavior is not necessary to achieve success in the accounting profession.

d. Non-response could bias the results by not including potential other important information that could direct the researcher to a conclusion.

1.13 a. A relative frequency table is:

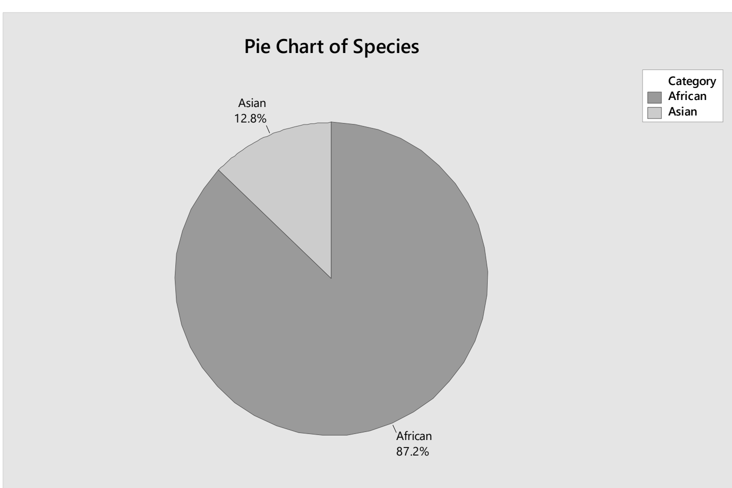
Rhino Species	Frequency	Relative Frequency
African Black	5,000	0.1745
African White	20,000	0.6978
(Asian) Sumatran	100	0.0035
(Asian) Javan	60	0.0021
(Asian) Greater One-Horned	3,500	0.1221
Total	28,660	1.000

b. Using MINITAB, the relative frequency bar graph is:



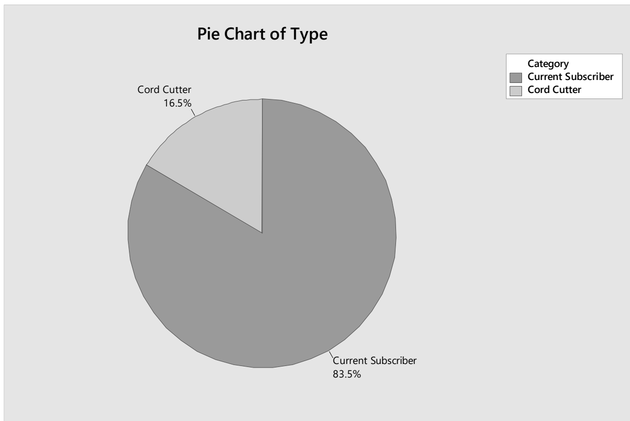
c. The proportion of African rhinos is $0.1745 + 0.6978 = 0.8723$. The proportion of Asian rhinos is $0.0035 + 0.0021 + 0.1221 = 0.1277$.

d. Using MINITAB, the pie chart for these proportions is:

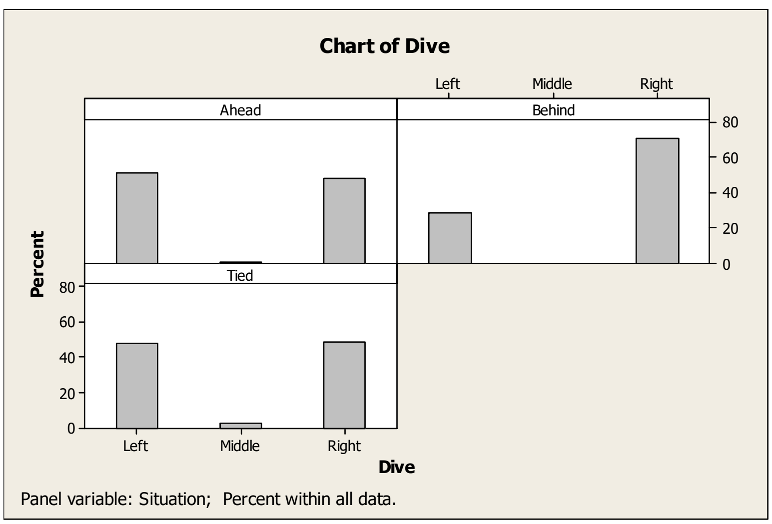


1.14 a. From the pie chart, 76.0% of the sample have a cable/satellite subscription at home. The proportion would be 0.76. This can be found by computing the relative frequency or $1{,}521/2{,}001 = 0.76$.

b. Using MINITAB, the pie chart is:



1.15 Using MINITAB, the side-by-side bar graphs are:



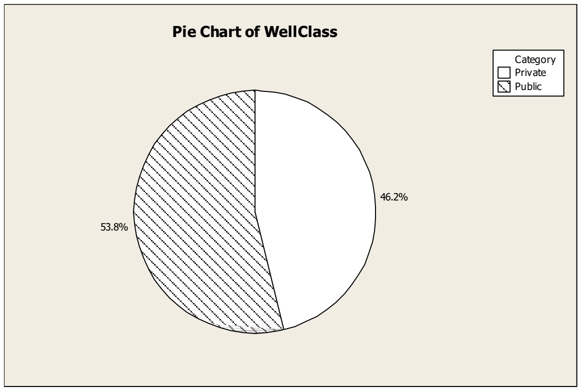
From the graphs, it appears that if the team is either tied or ahead, the goal-keepers tend to dive either right or left with equal probability, with very few diving in the middle. However, if the team is behind, then the majority of goal-keepers tend to dive right (71%).

1.16 a. $\dfrac{196}{504} = 0.3889$ is the proportion of ice melt ponds that had landfast ice.

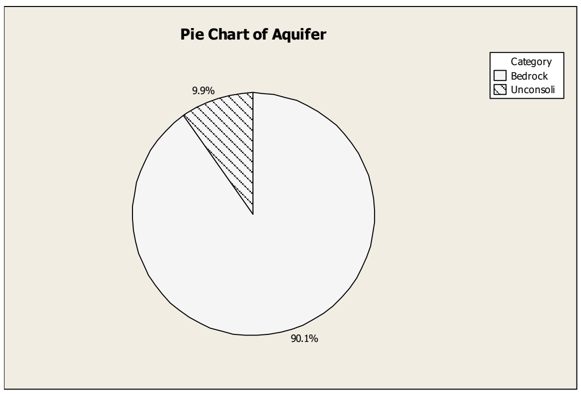
b. Yes, since $\dfrac{88}{504} = 0.1746$ is approximately 17%.

c. The multiyear ice type appears to be significantly different from the first-year ice melt.

1.17 a.


Pie Chart of WellClass
Category
Private
Public
46.2%
53.8%

b.


Pie Chart of Aquifer
Category
Bedrock
Unconsoli
9.9%
90.1%

c.

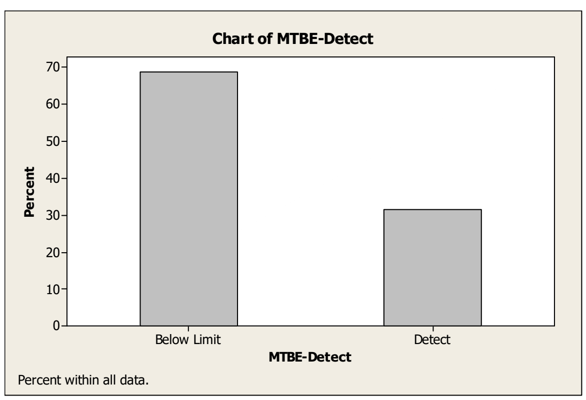
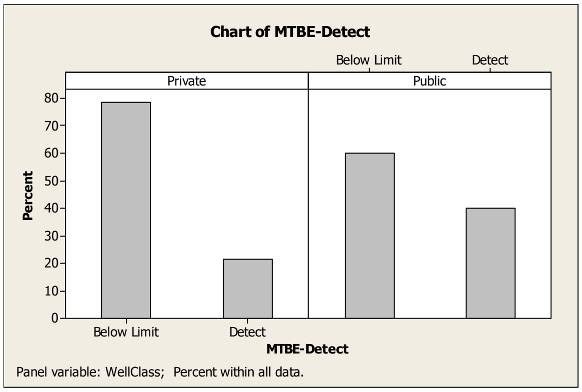


Chart of MTBE-Detect
Percent
70
60
50
40
30
20
10
0
Below Limit
Detect
MTBE-Detect
Percent within all data.

d.



Public wells (40%); Private wells (21%).

1.18 a. The estimated percentage of aftershocks measuring between 1.5 and 2.5 on the Richter scale is approximately 68%.

b. The estimated percentage of aftershocks measuring greater than 3.0 on the Richter scale is approximately 12%.

c. The data are skewed right.

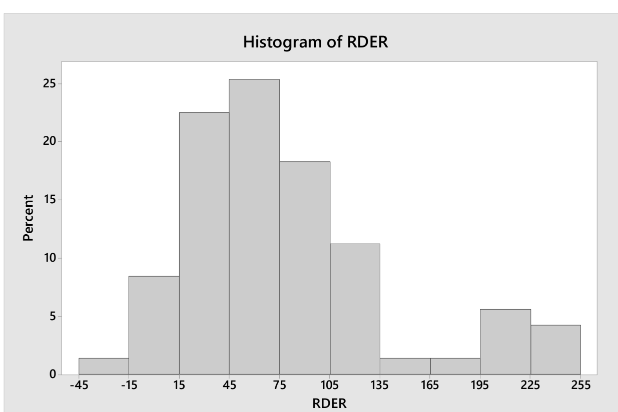
1.19 a. The graph is a frequency histogram.

b. The quantitative variable summarized in the graph is the fup/fumic ratio.

c. The proportion of ratios greater than 1 is $\dfrac{8+5+1}{416}=\dfrac{14}{416}=0.034$.

d. The proportion of ratios less than 0.4 is $\dfrac{181+108}{416}=\dfrac{289}{416}=.695$.

1.20 a. Using MINITAB, the frequency histogram is:



b. From the graph, it appears that about 0.18 of the RDER values are between 75 and 105.

c. From the graph, it appears that about 0.10 of the RDER values are below 15.

1.21 The tem-and-leaf display with the leaves for the honey dosage group bolded.

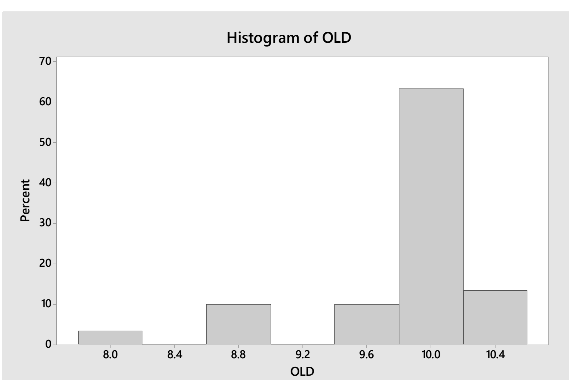
Stem-and-leaf of TotalScore N = 105

1	0	0
4	1	000
4	2	
7	3	000
16	4	000000000
20	5	0000
28	6	00000000
41	7	0000000000000
52	8	00000000000
(13)	9	0000000000000
40	10	0000000000
30	11	000000
24	12	0000000000000
11	13	0000
7	14	0
6	15	00000
1	16	0

Leaf Unit = 0.1

Yes. Most of the scores for the honey dosage tend to be higher than the other treatments.

1.22 a. Using MINITAB, the frequency histogram is:

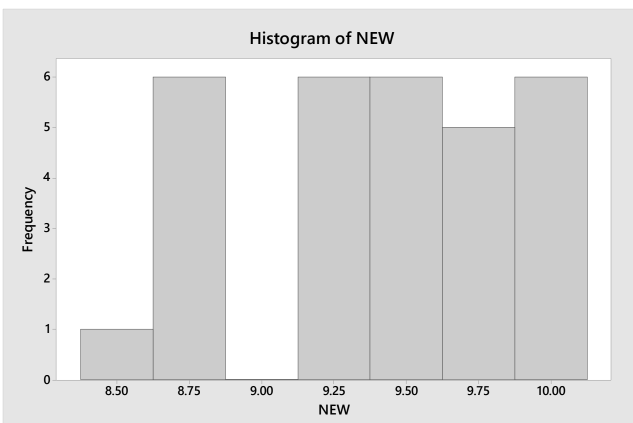


b. Using MINITAB, the stem-and-leaf display is:

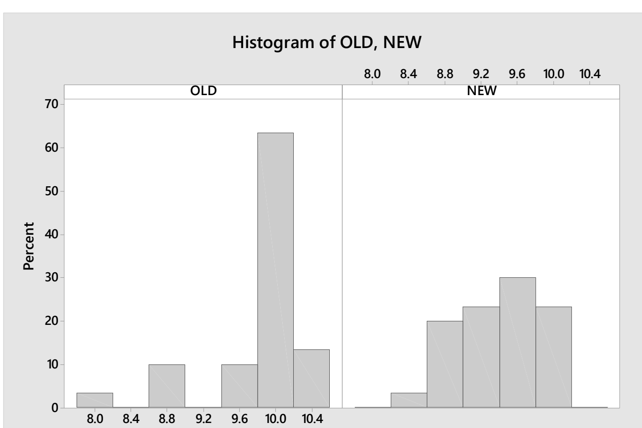
```
Stem-and-leaf of VOLTAGE LOCATION_OLD = 1 N = 30
Leaf Unit = 0.10
1      8  0
1      8
1      8
3      8  77
4      8  8
4      9
4      9
5      9  5
7      9  77
(10)   9  8888889999
13    10  000000111
4     10  222
1     10  5
```

The stem-and-leaf is more informative since the actual values of the old location can be found. The histogram is useful if shape and spread of the data is what is needed, but the actual data points are absorbed in the graph.

c. Using MINITAB, the frequency histogram is:



d. Side-by-side graphs are:



The old process appears to be better than the new process. For the old process, only about 0.13 of the observations are less than 9.2. For the new process, about 0.3 of the observations are below 9.2.

1.23 a. Using MINITAB, the stem-and-leaf display is:

Stem-and-leaf of Score N = 194

1	6	1
1	6	
1	7	
3	7	88
5	8	24
24	8	6667777788889999999
83	9	00000000001111112222222222222223333333333444444444444444444
(84)	9	55555555555555555555666666666666666667777777777777777777777788888888899999999999+
27	10	000000000000000000000000000

Leaf Unit = 1

b. Of the 194 observations, 189 have acceptable standard of sanitation scores. The proportion is $\frac{189}{194} = 0.974$.

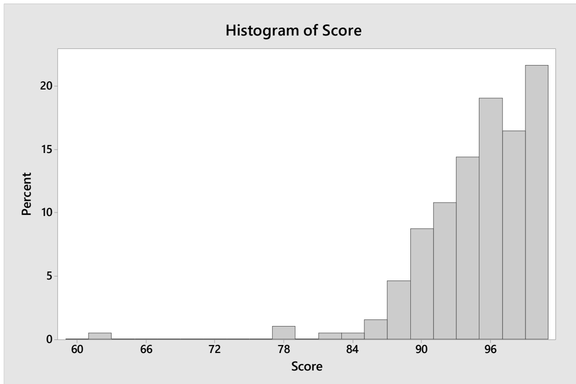
c. The score of 78 is highlighted in bold below.

Stem-and-leaf of Score N = 194

1	6	1
1	6	
1	7	
3	7	**8**8
5	8	24
24	8	6667777788889999999
83	9	00000000001111112222222222222223333333333444444444444444444
(84)	9	55555555555555555555666666666666666667777777777777777777777788888888899999999999+
27	10	000000000000000000000000000

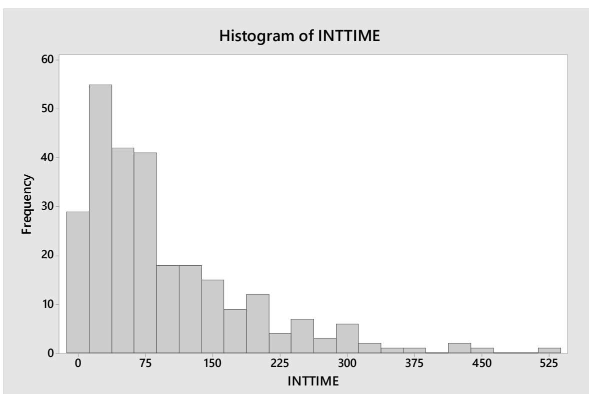
Leaf Unit = 1

d. Using MINITAB, the histogram of the data is:



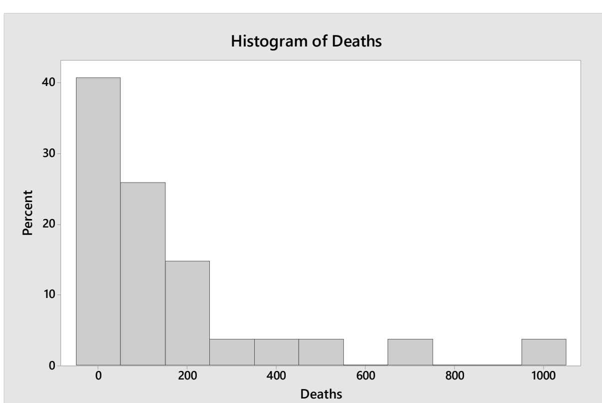
e. The proportion of ships with acceptable sanitation scores is about 0.97.

1.24 Using MINITAB, the histogram is:



The data are skewed right. Answers may vary on whether the phishing attack against the organization was an "inside job."

1.25 a. Using MINITAB, the histograms of the number of deaths is:



b. The interval containing the largest proportion of estimates is 0-50. Almost half of the estimates fall in this interval.

1.26 a. The sample mean is:

$$\bar{y} = \frac{\sum y}{n} = \frac{1+2+3+1+5+6+2+4+1+2+4+2+9}{13} = \frac{42}{13} = 3.231$$

b. The sample variance is:

$$s^2 = \frac{\sum y^2 - \frac{\left(\sum y\right)^2}{n}}{n-1} = \frac{202 - \frac{42^2}{13}}{13-1} = 5.526$$

The standard deviation is: $s = \sqrt{5.526} = 2.351$

c. Using Tchebysheff's Theorem, at least 75% of the observations will fall within 2 standard deviations of the mean. This interval is:
$\bar{y} \pm 2s \Rightarrow 3.231 \pm 2(2.351) \Rightarrow 3.231 \pm 4.702 \Rightarrow (-1.471, 7.933)$
At least 75% of all shaft graves will contain between 0 and 7 sword shafts.

1.27 a. $\bar{y} = 2.12$; The average magnitude for the aftershocks is 2.12.

b. Range = 6.7. The difference between the largest and smallest magnitude is 6.7.

c. $s = 0.66$; About 95% of the magnitudes fall in the interval
$\bar{y} \pm 2s \Rightarrow 2.1197 \pm 2(0.6636) \Rightarrow 2.1197 \pm 1.3272 \Rightarrow (0.79, 3.44)$

d. $\mu =$ mean; $\sigma =$ Standard deviation

1.28 a. The mean RDER score is 78.1885. On average, subjects make 78.19 more error under the irrelevant background speech than under silence.

b. From the histogram in Exercise 1.20, it appears that the data are approximately mound-shaped. By the rule of thumb, approximately 95% of the observations will fall within 2 standard deviations of the mean.

c. We would expect approximately 95% of the observations to fall within the following interval:
$\bar{y} \pm 2s \Rightarrow 78.19 \pm 2(63.24) \Rightarrow 78.19 \pm 126.48 \Rightarrow (-48.29, 204.67)$

1.29 a. Using MINITAB, the descriptive statistics are:

Statistics

Variable	N	Mean	StDev	Minimum	Maximum
Score	194	94.474	4.897	61.000	100.000

$\bar{y} = 94.474$, $s = 4.897$

b. $\bar{y} \pm 2s \Rightarrow 94.474 \pm 2(4.897) \Rightarrow 94.474 \pm 9.794 \Rightarrow (84.680, 104.268)$

c. The percentage of scores that fall in the interval is $\frac{189}{194}(100\%) = 97.42\%$. This number is agrees with the rule of thumb which says that approximately 95% of the observations will fall within 2 standard deviations of the mean.

1.30 a. The average score for Energy Star is 4.44. The average score is close to 5 meaning the average score is close to 'very familiar'.

b. The ecolabel that had the most variation in the numerical responses is Audubon International because it has the largest standard deviation.

c. The interval would be
$\bar{y} \pm 2s \Rightarrow 4.44 \pm 2(0.82) \Rightarrow 4.44 \pm 1.64 \Rightarrow (2.80, 6.08)$

1.31 a. $z = \frac{293 - 353}{30} = -2$ A score of 293 would be 2 standard deviations below the mean.

$z = \frac{413 - 353}{30} = 2$ A score of 413 would be 2 standard deviations above the mean.

Using Tchebysheff's Theorem, at least 3/4 of the observations will be within 2 standard deviations of the mean.

b. For a mound-shaped, symmetric distribution, approximately 95% of the observations will be within 2 standard deviations of the mean, using the rule of thumb.

c. $z = \frac{134 - 184}{25} = -2$ A score of 134 would be 2 standard deviations below the mean.

$z = \frac{234 - 184}{25} = 2$ A score of 234 would be 2 standard deviations above the mean.

Using Tchebysheff's Rule, at least 3/4 of the observations will be within 2 standard deviations of the mean.

d. For a mound-shaped, symmetric distribution, approximately 95% of the observations will be within 2 standard deviations of the mean, using the rule of thumb.

1.32 We will find intervals within 2 standard deviations for each group.
Group T: $\bar{y} \pm 2s \Rightarrow 10.5 \pm 2(7.6) \Rightarrow 10.5 \pm 15.2 \Rightarrow (-4.7, 25.7)$
Group V: $\bar{y} \pm 2s \Rightarrow 3.9 \pm 2(7.5) \Rightarrow 3.9 \pm 15.0 \Rightarrow (-11.1, 18.9)$
Group C: $\bar{y} \pm 2s \Rightarrow 1.4 \pm 2(7.5) \Rightarrow 1.4 \pm 15.0 \Rightarrow (-13.6, 16.4)$
Since the only interval that contains 22.5 is the interval for Group T, the patient is most likely to have come from Group T.

1.33 a. Using Table 1, Appendix D:
$P(-1 \le z \le 1) = P(-1 \le z \le 0) + P(0 \le z \le 1) = 0.3413 + 0.3413 = 0.6826$

b. $P(-1.96 \le z \le 1.96) = P(-1.96 \le z \le 0) + P(0 \le z \le 1.96) = 0.4750 + 0.4750 = 0.9500$

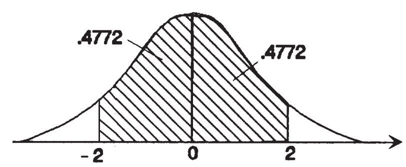
c. Since 1.645 is half way between 1.64 and 1.65 in the table, we will use halfway between the corresponding areas. Halfway between the 2 areas is $\frac{0.4495 + 0.4505}{2} = \frac{0.9}{2} = 0.4500$

$P(-1.645 \le z \le 1.645) = P(-1.645 \le z \le 0) + P(0 \le z \le 1.645) = 0.4500 + 0.4500 = 0.9000$

d. $P(-3 \le z \le 3) = P(-3 \le z \le 0) + P(0 \le z \le 3) = 0.4987 + 0.4987 = 0.9974$

1.34 a. The z-score for $\mu - 2\sigma$ is $z = \frac{(\mu - 2\sigma) - \mu}{\sigma} = -2.$

The z-score for $\mu + 2\sigma$ is $z = \frac{(\mu + 2\sigma) - \mu}{\sigma} = 2.$

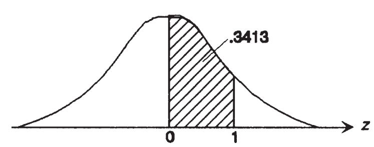


$P(\mu - 2\sigma \le y \le \mu + 2\sigma) = P(-2 \le z \le 2)$
$= P(-2 \le z \le 0) + P(0 \le z \le 2)$

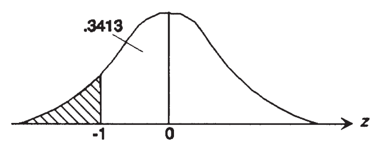
Using Table 1 in Appendix D, $P(-2 \le z \le 0) = 0.4772$ and $P(0 \le z \le 2) = 0\ .4772.$
So $P(\mu - 2\sigma \le y \le \mu + 2\sigma) = 0\ .4772 + 0\ .4772 = 0.9544$

b. The z-score for $y = 108$ is $z = \frac{y - \mu}{\sigma} = \frac{108 - 100}{8} = 1.$

$P(y \ge 108) = P(z \ge 1)$



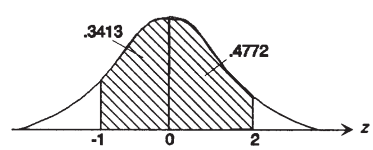
Using Table 1 of Appendix D, we find $P(0 \le z \le 1) = 0.3413.$
so $P(z \ge 1) = 0.5 - 0.3413 = 0.1587$



c. The z-score for $y = 92$ is $z = \frac{y - \mu}{\sigma} = \frac{92 - 100}{8} = -1.$

$P(y \le 92) = P(z \le -1)$

Using Table 1 of Appendix D, we find
$P(-1 \le z \le 0) = 0.3413,$
so $P(z \le -1) = 0.5 - 0.3413 = 0.1587$



d. The z-score for $y = 92$ is $z = \frac{y - \mu}{\sigma} = \frac{92 - 100}{8} = -1.$

The z-score for $y = 116$ is $z = \dfrac{y-\mu}{\sigma} = \dfrac{116-100}{8} = 2.$

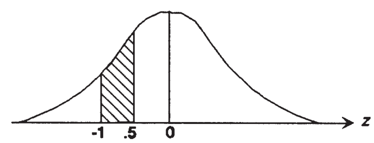
$P(92 \le y \le 116) = P(-1 \le z \le 2)$

Using Table 1 of Appendix D, $P(-1 \le z \le 0) = 0.3413$ and
$P(0 \le z \le 2) = 0.4772.$ So $P(92 \le y \le 116) = P(-1 \le z \le 2)$
$= 0.3413 + 0.4772 = 0.8185.$

e. The z-score for $y = 92$ is $z = \dfrac{y-\mu}{\sigma} = \dfrac{92-100}{8} = -1.$

The z-score for $y = 96$ is $z = \dfrac{y-\mu}{\sigma} = \dfrac{96-100}{8} = -0.5.$

$P(92 \le y \le 96) = P(-1 \le z \le -.5)$

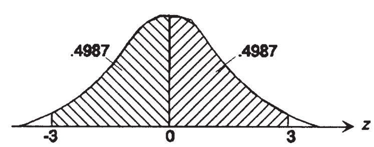


Using Table 1 of Appendix D, $P(-1 \le z \le 0) = 0.3413$ and
$P(-0.5 \le z \le 0) = 0.1915$. So $P(92 \le y \le 96)$
$= P(-1 \le z \le -0.5) = 0.3413 - 0.1915 = 0.1498.$

f. The z-score for $y = 76$ is $z = \dfrac{y-\mu}{\sigma} = \dfrac{76-100}{8} = -3.$

The z-score for $y = 124$ is $z = \dfrac{y-\mu}{\sigma} = \dfrac{124-100}{8} = 3.$

$P(76 \le y \le 124) = P(-3 \le z \le 3)$



Using Table 1 of Appendix D, $P(-3 \le z \le 0) = 0.4987$ and
$P(0 \le z \le 3) = 0.4987.$ So $P(76 \le y \le 124) = P(-3 \le z \le 3) = 0.4987 + 0.4987 = 0.9974.$

1.35 a. Using Table 1, Appendix D:

$P(y > 4) = P\left(z > \dfrac{4-4.44}{0.82}\right) = P(z > -0.54) = P(-0.54 < z < 0) + P(z > 0)$
$= 0.2054 + 0.5 = 0.7054$

b. $P(2 < y < 4) = P\left(\dfrac{2-4.44}{0.82} < z < \dfrac{4-4.44}{0.82}\right) = P(-2.98 < z < -0.54)$
$= P(-2.98 < z < 0) - P(-0.54 < z < 0) = 0.4986 - 0.2054 = 0.2932$

c. $P(y \le 1) = P\left(z \le \dfrac{1-4.44}{0.82}\right) = P(z < -4.20) = P(z < 0) - P(-4.20 < z < 0) \approx 0.5 - 0.5 = 0$

Since the probability of observing a value of 1 or less is so small, it would be extremely unlikely that the ecolabel shown was *Energy Star*.

1.36 a. Using Table 1, Appendix D:

$$P(y<400)=P\left(z<\frac{400-353}{30}\right)=P(z<1.57)=P(0<z<1.57)+P(z<0)$$
$$=0.4418+0.5=.9418$$

b. $$P(y>100)=P\left(z>\frac{100-184}{25}\right)=P(z>-3.36)=P(-3.36<z<0)+P(z>0)$$
$$\approx 0.5+0.5=1.0$$

1.37 a. Using Table 1, Appendix D:

$$P(y\ge 60)=P\left(z\ge\frac{60-59}{5}\right)=P(z\ge 0.2)=P(z\ge 0)-P(0<z<0.2)$$
$$=0.5-0.0793=0.4207$$

b. $$P(y\ge 60)=P\left(z\ge\frac{60-43}{5}\right)=P(z\ge 3.4)=P(z\ge 0)-P(0<z<3.4)\approx 0.5-0.5=0$$

1.38 We want to find y_1 and y_2 such that $P(y_1<y<y_2)=0.90$. First, we must find z_1 and z_2 such that $P(z_1<z<z_2)=0.90$. By symmetry, we know that $P(z_1<z<0)=P(0<z<z_2)=0.90/2=0.4500$. Using Table 1, Appendix D, $z_1=-1.645$ and $z_2=1.645$.

$$z_1=\frac{y_1-\mu}{\sigma}\Rightarrow -1.645=\frac{y_1-64}{2.6}\Rightarrow y_1=64-1.645(2.6)=64-4.277=59.723$$
$$z_2=\frac{y_2-\mu}{\sigma}\Rightarrow 1.645=\frac{y_2-64}{2.6}\Rightarrow y_2=64+1.645(2.6)=64+4.277=68.277$$

Thus, the interval would be $(59.723, 68.277)$.

1.39 For *Tunnel Face*,

$$P(y\le 1)=P\left(z\le\frac{1-1.2}{0.16}\right)=P(z\le -1.25)=0.5-P(-1.25\le z<0)=0.5-0.3944=0.1056$$

For *Tunnel Walls*,

$$P(y\le 1)=P\left(z\le\frac{1-1.4}{0.20}\right)=P(z\le -2.00)=0.5-P(-2.00\le z<0)=0.5-0.4772=0.0228$$

For *Tunnel Crown*,

$$P(y\le 1)=P\left(z\le\frac{1-2.1}{0.70}\right)=P(z\le -1.57)=0.5-P(-1.57\le z<0)=0.5-0.4418=0.0582$$

The probability of failing for *Tunnel Face* is larger than the probabilities of failure for the other two areas. Thus, *Tunnel Face* is more likely to result in failure.

1.40 We have to find the probability of observing $y = 0.7$ or anything more unusual given the two different values of μ.

Without receiving executive coaching: Using Table I, Appendix D with $\mu = 0.75$ and $\sigma = 0.085$,

$$P(y \le 0.7) = P\left(z \le \frac{0.7 - 0.75}{0.085}\right) = P(z \le -0.59) = 0.5 - 0.2224 = 0.2776.$$

After receiving executive coaching: Using Table II, Appendix D with $\mu = 0.52$ and $\sigma = 0.075$,

$$P(y \ge 0.7) = P\left(z \ge \frac{0.7 - 0.52}{0.075}\right) = P(z \ge 2.40) = 0.5 - 0.4918 = 0.0082.$$

Since the probability of observing $y \le 0.7$ for those not receiving executive coaching is much larger than the probability of $y \ge 0.7$ for those receiving executive coaching, it is more likely that the leader did not receive executive coaching.

1.41 a. The relative frequency distribution is:

Value	**Frequency**	**Relative Frequency**
0	26	26/300 = 0.087
1	30	30/300 = 0.100
2	24	0.080
3	29	0.097
4	31	0.103
5	25	0.083
6	42	0.140
7	36	0.120
8	27	0.090
9	30	0.100
	300	1.000

b. $$\bar{y} = \frac{\sum y}{n} = \frac{1404}{300} = 4.68$$

c. $$s^2 = \frac{\sum y^2 - \frac{\left(\sum y\right)^2}{n}}{n-1} = \frac{8942 - \frac{1404^2}{300}}{300-1} = 7.9307$$

d. The 50 sample means are:

4.8333	4.5000	4.5000	5.6667
4.6667	5.0000	4.1667	5.0000
5.1667	4.6667	5.3333	4.1667
4.5000	5.3333	3.8333	2.5000
5.6667	3.8333	4.3333	2.6667
5.0000	4.1667	4.8333	5.5000
7.3333	4.0000	3.5000	2.1667
5.8333	3.3333	3.5000	7.0000
4.0000	4.3333	6.8333	5.8333

6.1667	4.0000	6.8333	2.6667
3.1667	3.8333	5.8333	5.6667
4.8333	5.1667	3.8333	5.5000
5.5000	3.5000		

The frequency distribution for $\overline{y}$ is:

Sample Mean	Frequency	Relative Frequency
2.000 – 2.999	4	4/50 = 0.08
3.000 – 3.999	9	9/50 = 0.18
4.000 – 4.999	16	0.32
5.000 – 5.999	16	0.32
6.000 – 6.999	3	0.06
7.000 – 7.999	2	0.04
	50	1.00

The mean of the sample means is $\overline{\overline{y}} = \frac{\sum \overline{y}}{n} = \frac{234}{50} = 4.68.$

$$s_{\overline{y}}^2 = \frac{\sum \overline{y}^2 - \frac{\left(\sum \overline{y}\right)^2}{n}}{n-1} = \frac{1{,}162.5 - \frac{234^2}{50}}{50-1} = 1.3751, \quad s_{\overline{y}} = \sqrt{1.375} = 1.1726$$

1.42 a. The twenty-five means are:

4.7500	4.0833	6.8333
4.8333	3.8333	4.8333
5.3333	3.9167	5.3333
6.5833	4.3333	3.3333
5.0833	4.3333	4.0833
4.0000	4.5833	4.5833
5.0000	4.5833	4.2500
4.8333	3.5000	5.5833
4.5833		

Using the same intervals as in Exercise 1.41, the frequency distribution for $\overline{y}$ is:

Sample Mean	Frequency	Relative Frequency
2.000 – 2.999	0	0/25 = 0.00
3.000 – 3.999	4	4/25 = 0.16
4.000 – 4.999	14	0.56
5.000 – 5.999	5	0.20
6.000 – 6.999	2	0.08
7.000 – 7.999	0	0.00
	25	1.00

This distribution is not as spread out as that in Exercise 1.41.

b. $\bar{\bar{y}} = \dfrac{\sum \bar{y}}{n} = \dfrac{117}{25} = 4.68$

$$s_{\bar{y}}^2 = \frac{\sum \bar{y}^2 - \frac{\left(\sum \bar{y}\right)^2}{n}}{n-1} = \frac{563.972222 - \frac{117^2}{25}}{25-1} = 0.68384, \quad s_{\bar{y}} = \sqrt{0.68384} = 0.8269$$

The variance for Exercise 1.41 is $s_{\bar{y}} = \sqrt{1.375} = 1.1726$. The standard deviation for this exercise is less than the standard deviation in Exercise 1.41 because the standard deviation in Exercise 1.41 is based on samples of size 6, while the standard deviation in this Exercise is based on samples of size 12. As the sample size increases, the spread of the distribution will decrease.

1.43 a. For $df = n-1 = 10-1 = 9$, $t_0 = 2.262$ yields $P(t > t_0) = 0.025$.

b. For $df = n-1 = 5-1 = 4$, $t_0 = 3.747$ yields $P(t > t_0) = 0.01$.

c. For $df = n-1 = 20-1 = 19$, $t_0 = -2.861$ yields $P(t \le t_0) = 0.005$.

d. For $df = n-1 = 12-1 = 11$, $t_0 = -1.796$ yields $P(t \le t_0) = 0.05$.

1.44 a. For confidence coefficient 0.95, $\alpha = 0.05$ and $\alpha/2 = 0.05/2 = 0.025$. From Table 1, Appendix D, $z_{.025} = 1.96$.

b. We are 95% confident that the true mean HRV for all officers diagnosed with hypertension is between 4.1 and 124.5.

We are 95% confident that the true mean HRV for all officers who are not hypertensive is between 148.0 and 192.6.

c. 95% confidence means that in repeated sampling, 95% of all confidence intervals constructed in the same manner will contain the true mean.

d. To reduce the width of the confidence interval, we would use a smaller confidence coefficient. A smaller confidence coefficient corresponds to a smaller level of confidence. The lower the level of, confidence, the smaller the interval.

1.45 a. $E(\bar{y}) = \mu_{\bar{y}} = \mu = 0.10 \quad Var(\bar{y}) = \dfrac{\sigma^2}{n} = \dfrac{(0.10)^2}{50} \cong 0.0002$

$$\sigma_{\bar{y}} = \frac{s}{\sqrt{n}} = \frac{0.10}{\sqrt{50}} \cong 0.0141$$

b. Since the sample size is greater than 30, the sample distribution of $\bar{y}$ is approximately normal by The Central Limit Theorem.

c. $$P(\bar{y} > 0.13) = P\left(Z > \frac{0.13 - 0.10}{\frac{0.10}{\sqrt{50}}}\right) = P(Z > 2.12) = 0.50 - 0.4830 = 0.0170$$

1.46 a. The parameter of interest for this study is the mean effect size, μ, for all psychological studies of personality and aggressive behavior.

b. It appears to be approximately normal with a few high outliers. Since the sample size is large, the Central Limit Theorem ensures that the data for the average is normally distributed.

c. We can be 95% confident that the interval (0.4786, 0.8167) encloses μ , the true mean effect size.

d. Yes, the researcher can conclude that those who score high on the personality test are more aggressive since 0 is not included in the interval.

1.47 a. $$\bar{y} = \frac{\sum y}{n} = \frac{365}{8} = 45.625$$

b. $$s^2 = \frac{\sum y^2 - \frac{\left(\sum y\right)^2}{n}}{n-1} = \frac{16{,}957 - \frac{\left(\sum 365\right)^2}{8}}{8-1} = 43.410714, \quad s = \sqrt{43.410714} = 6.5887$$

c. For confidence coefficient 0.95, $\alpha = 0.05$ and $\alpha/2 = 0.05/2 = 0.025$. From Table 2, Appendix D, with $df = n - 1 = 8 - 1 = 7$, $t_{0.025} = 2.365$. The 95% confidence interval is:

$$\bar{y} \pm t_{0.025}\frac{s}{\sqrt{n}} \Rightarrow 45.625 \pm 2.365\frac{6.5887}{\sqrt{8}} \Rightarrow 45.625 \pm 5.509 \Rightarrow (40.116, 51.134)$$

d. In order for the interval to be valid, the distribution of all PAI values for music performance anxiety studies must be approximately normal.

e. In repeated sampling, 95% of all intervals constructed will contain the true mean value of μ.

1.48 For confidence coefficient 0.95, $\alpha = 0.05$ and $\alpha/2 = 0.05/2 = 0.025$. From Table 1, Appendix D, $z_{.025} = 1.96$. The 95% confidence interval is:

$$\bar{y} \pm z_{0.025}\frac{\sigma}{\sqrt{n}} \Rightarrow 112 \pm 1.96\frac{560}{\sqrt{2{,}617}} \Rightarrow 112 \pm 21.456 \Rightarrow (90.544, 133.456)$$

We are 95% confident that the true mean tipping point of all daily deal offerings in Korea is between 90.544 and 133.456.

1.49 a. $$E(y) = \mu_{\bar{y}} = \mu = 99.6$$

b. For confidence coefficient 0.95, $\alpha = 0.05$ and $\alpha/2 = 0.05/2 = 0.025$. From Table 1, Appendix D, $z_{0.025} = 1.96$.

$$\bar{y} \pm z_{0.025}\left(s_{\bar{y}}\right) = \bar{y} \pm z_{0.025}\left(\frac{s}{\sqrt{n}}\right) = 99.6 \pm 1.96\left(\frac{12.6}{\sqrt{122}}\right) = 99.6 \pm 2.2 \Rightarrow (97.4, 101.8)$$

c. We are 95% confident that the true mean Mach rating score is between 97.4 and 101.8.

d. Yes, since the value of 85 is not contained in the confidence interval it is unlikely that the true mean Mach rating score could be 85.

1.50 a. For confidence coefficient 0.90, $\alpha = 0.10$ and $\alpha/2 = 0.10/2 = 0.05$. From Table 1, Appendix D, $z_{0.05} = 1.645$. The 90% confidence interval is:

$$\bar{y} \pm z_{0.05}\frac{\sigma}{\sqrt{n}} \Rightarrow 2.42 \pm 1.645\frac{2.84}{\sqrt{86}} \Rightarrow 2.42 \pm 0.504 \Rightarrow (1.916, 2.924)$$

b. We are 90% confident that the true mean intention to comply score is between 1.916 and 2.924.

c. The proportion of all similarly constructed confidence intervals that will contain the true mean is 0.90.

1.51 a. For confidence coefficient 0.90, $\alpha = 0.10$ and $\alpha/2 = 0.10/2 = 0.05$. From Table 2, Appendix D, with $df = n - 1 = 15 - 1 = 14$, $t_{0.05} = 1.761$. The 90% confidence interval is:

$$\bar{y} \pm t_{0.05}\frac{s}{\sqrt{n}} \Rightarrow 18 \pm 1.761\frac{20}{\sqrt{15}} \Rightarrow 18 \pm 9.094 \Rightarrow (8.906, 27.094)$$

b. Yes, the new formula is better than the ASHRAE formula. We are 90% confident that the true mean absolute deviation percentage for the new formula is between 8.906% and 27.094%. This interval is below the 34% for the ASHRAE formula.

1.52 a. The target parameter is μ, the mean difference in error rates for all subjects who perform the memorization tasks.

b. The interval $\bar{y} \pm 2s$ is an interval in which most actual observations will fall. The interval for the population mean is much smaller.

c. For confidence coefficient 0.98, $\alpha = 0.02$ and $\alpha/2 = 0.02/2 = 0.01$. From Table 1, Appendix D, $z_{0.01} = 2.33$. The 98% confidence interval is:

$$\bar{y} \pm z_{0.01}\frac{\sigma}{\sqrt{n}} \Rightarrow 78.1885 \pm 2.33\frac{63.243}{\sqrt{71}} \Rightarrow 78.1885 \pm 17.4880 \Rightarrow (60.7005, 95.6765)$$

We are 98% confident that the true mean difference in error rates for all subjects who perform the memorization tasks is between 60.7005 and 95.6765.

d. 98% confidence means that in repeated sampling, 98% of all intervals constructed in a similar manner will contain the true mean.

e. No. The sample size in the problem is 71, which is greater than 30.

1.53 a. Null Hypothesis $= H_0$

b. Alternative Hypothesis $= H_a$

c. Type I error is when we reject the null hypothesis when the null hypothesis is, in fact, true.

d. Type II error is when we accept the null hypothesis when the null hypothesis is, in fact, not true.

e. Probability of Type I error is α.

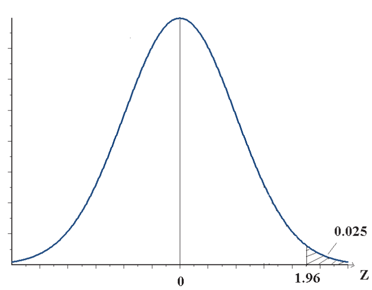
f. Probability of Type II error is β.

g. *p*-value is the observed significance level, which is the probability of observing a value of the test statistics at least as contradictory to the null hypothesis as the observed test statistic value, assuming the null hypothesis is true.

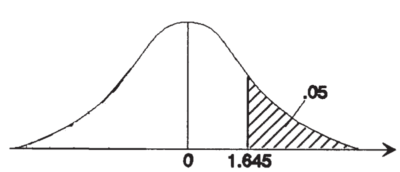
1.54 a. The rejection region is determined by the sampling distribution of the test statistic, the direction of the test $(>, <, \text{or} \neq)$, and the tester's choice of α..

b. No, nothing is proven. When the decision based on sample information is to reject H_0, we run the risk of committing a Type I error. We might have decided in favor of the research hypothesis when, in fact, the null hypothesis was the true statement. The existence of Type I and Type II errors makes it impossible to prove anything using sample information.

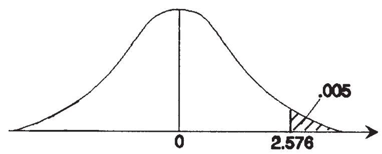
1.55 a. $\alpha = P$ (reject H_0 when H_0 is in fact true) $= P(z > 1.96) = 0.025$



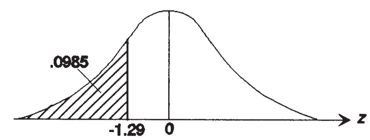
b. $\alpha = P(z > 1.645) = 0.05$



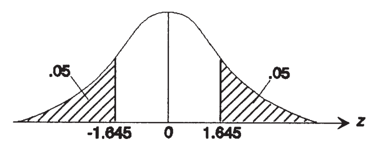
c. $\alpha = P(z > 2.576) = 0.005$



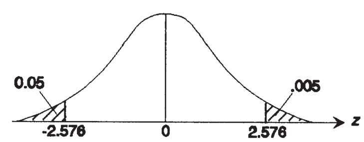
d. $\alpha = P(z < -1.29) = 0.0985$



e. $\alpha = P(z < -1.645) + P(z > 1.645)$
$= 0.05 + 0.05 = 0.10$



f. $\alpha = P(z < -2.576) + P(z > 2.576)$
$= 0.005 + 0.005 = 0.01$



1.56 a. To determine if the average gain in green fees, lessons, or equipment expenditures for participating golf facilities exceed \$2400, we test:

$$H_0: \mu = 2,400$$
$$H_a: \mu > 2,400$$

b. The probability of making a Type I error will be at most 0.05. That is, 5% of the time when repeating this experiment the final conclusion would be that the true mean gain exceeded \$2400 when in fact the true mean was equal to \$2400.

c. The rejection region requires $\alpha = 0.05$ in the upper tail of the z distribution. From Table 1, Appendix D, $z_{.05} = 1.645$. The rejection region is $z > 1.645$.

1.57 To determine if the mean Libor rate of 1.55% is too high, we test:

$$H_0: \mu = 1.55$$
$$H_a: \mu < 1.55$$

1.58 a. The parameter of interest is μ, the mean number of days in the past month that adults walked for the purpose of health or recreation.

b. To determine if the true mean number of days in the past month that adults walked for the purpose of health or recreation is lower than 5.5 days, we test:

$H_0: \mu = 5.5$
$H_a: \mu < 5.5$

c. A Type I error would be concluding the true mean number of days in the past month that adults walked for the purpose of health or recreation is lower than 5.5 days when the true mean is 5.5 days.

d. A Type II error would be concluding the true mean number of days in the past month that adults walked for the purpose of health or recreation is equal to 5.5 days when the true mean is less than 5.5 days.

1.59 a. To determine if the true mean PAI value for similar studies of music performance anxiety exceeds 40, we test:

$H_0: \mu = 40$
$H_a: \mu > 40$

b. The rejection region requires $\alpha = 0.05$ in the upper tail of the t distribution. From Table 2, Appendix D, with $df = n - 1 = 8 - 1 = 7$, $t_{.05} = 1.895$. The rejection region is $t > 1.895$.

c. The test statistic is $t = \frac{\bar{y} - \mu}{s/\sqrt{n}} = \frac{45.63 - 40}{6.59/\sqrt{8}} = 2.416$.

d. Since the observed value of the test statistic falls in the rejection region $(t = 2.416 > 1.895)$, H_0 is rejected. There is sufficient evidence to indicate the true mean PAI value for similar studies of music performance anxiety exceeds 40 at $\alpha = 0.05$.

e. In order for the test to be valid, the distribution of all PAI values for similar studies of music performance anxiety must be approximately normal.

f. From the printout, the p-value is $p = 0.023$. Since the p-value is less than α $(p = 0.023 < 0.05)$, H_0 is rejected.

g. If $\alpha = 0.01$, then the p-value is not less than α $(p = 0.023 \nless 0.01)$. Thus, H_0 is would not be rejected.

1.60 Let μ = true mean heart rate during laughter. To determine if the true mean heart rate during laughter exceeds 71 beats/minutes, we test:

$H_0: \mu = 71$
$H_a: \mu > 71$

The test statistic is $z = \frac{\bar{y} - \mu_0}{s/\sqrt{n}} = \frac{73.5 - 71}{6/\sqrt{90}} = 3.95$.

The rejection region requires $\alpha = 0.05$ in the upper tail of the z distribution. From Table 1, Appendix D, $z_{0.05} = 1.645$. The rejection region is $z > 1.645$.

Since the observed value of the test statistic falls in the rejection region $(z = 3.95 > 1.645)$, H_0 is rejected. There is sufficient evidence to conclude that the true mean heart rate during laughter exceeds 71 beats/minute at $\alpha = 0.05$.

1.61 To determine if the mean difference in error rates for all subjects who perform the memorization tasks exceeds 75%, we test:

$$H_0: \mu = 75$$
$$H_a: \mu > 75$$

The test statistic is $z = \dfrac{\bar{y} - \mu_0}{s/\sqrt{n}} = \dfrac{78.1885 - 75}{63.2429/\sqrt{71}} = 0.42.$

The rejection region requires $\alpha = 0.01$ in the upper tail of the z distribution. From Table 1, Appendix D, $z_{0.01} = 2.33$. The rejection region is $z > 2.33$.

Since the observed value of the test statistic does not fall in the rejection region $(z = 0.42 \ngtr 2.33)$, H_0 is not rejected. There is insufficient evidence to conclude that the true mean difference in error rates for all subjects who perform the memorization tasks exceeds 75% at $\alpha = 0.01$.

1.62 a. To determine if the true mean number of vouchers sold 30 minutes before the tipping point is less than 5, we test:

$$H_0: \mu = 5$$
$$H_a: \mu < 5$$

The test statistic is $z = \dfrac{\bar{y} - \mu_0}{s/\sqrt{n}} = \dfrac{4.73 - 5}{27.58/\sqrt{2,211}} = -0.46.$

The rejection region requires $\alpha = 0.10$ in the lower tail of the z distribution. From Table 1, Appendix D, $z_{0.10} = 1.28$. The rejection region is $z < -1.28$.

Since the observed value of the test statistic does not fall in the rejection region $(z = -0.46 \nless -1.28)$, H_0 is not rejected. There is insufficient evidence to conclude that the true mean number of vouchers sold 30 minutes before the tipping point is less than 5 at $\alpha = 0.10$.

b. To determine if the true mean number of vouchers sold 30 minutes after the tipping point is greater than 10, we test:

$H_0: \mu = 10$
$H_a: \mu > 10$

The test statistic is $z = \dfrac{\bar{y} - \mu_0}{s/\sqrt{n}} = \dfrac{14.26 - 10}{110.71/\sqrt{2,617}} = 1.97.$

The rejection region requires $\alpha = 0.10$ in the upper tail of the z distribution. From Table 1, Appendix D, $z_{0.10} = 1.28$. The rejection region is $z > 1.28$.

Since the observed value of the test statistic falls in the rejection region $(z = 1.97 > 1.28)$, H_0 is rejected. There is sufficient evidence to conclude that the true mean number of vouchers sold 30 minutes after the tipping point is greater than 10 at $\alpha = 0.10$.

1.63 a. To determine whether the mean full-service fee of U.S. funeral homes this year is less than \$8,755, we test:

$H_0: \mu = 8.755$
$H_a: \mu < 8.755$

b. Using MINITAB, the descriptive statistics are:

Statistics

Variable	N	Mean	StDev
NFDA	36	6.819	1.265

The test statistic is $z = \dfrac{\bar{y} - \mu_0}{s/\sqrt{n}} = \dfrac{6.819 - 8.755}{1.265/\sqrt{36}} = -9.18.$

The rejection region requires $\alpha = 0.05$ in the upper tail of the z distribution. From Table 1, Appendix D, $z_{0.05} = 1.645$. The rejection region is $z < -1.645$.

Since the observed value of the test statistic falls in the rejection region $(z = -9.18 < -1.645)$, H_0 is rejected. There is sufficient evidence to conclude that the mean full-service fee of U.S. funeral homes this year is less than \$8,755 at $\alpha = 0.05$.

c. No, we did not have to assume that the data were normally distributed. Since the sample size was larger than 30, the distribution of $\bar{y}$ will be approximately normal.

1.64 a. To determine if the mean heat rate of gas turbines augmented with high pressure inlet fogging exceeds 10,000 kJ/kWh, we test:

$H_0: \mu = 10,000$
$H_a: \mu > 10,000$

The test statistic $z = \dfrac{\bar{y} - \mu_0}{s/\sqrt{n}} = \dfrac{11{,}066.4 - 10{,}000}{1{,}595/\sqrt{67}} = 5.47.$

The p-value is essentially zero and is significantly smaller than the significance level. Thus we can conclude that the true mean heat rate of gas turbines augmented with high pressure inlet fogging is greater than 10,000 kJ/kWh.

b. A Type I error would be if you concluded the true mean is greater than 10,000 kJ/kWh when, in fact, the true mean is equal to 10,000 kJ/kWh . A Type II error would be if you concluded the true mean is equal to 10,000 kJ/kWh when, in fact, the true mean is greater than 10,000 kJ/kWh .

1.65 Using MINITAB, some preliminary calculations are:

Descriptive Statistics

N	Mean	StDev	SE Mean	95% CI for μ
8	0.4225	0.1219	0.0431	(0.3206, 0.5244)

μ: mean of Recall

Test

Null hypothesis H_0: μ = 0.5

Alternative hypothesis H_1: μ ≠ 0.5

T-Value	P-Value
-1.80	0.115

To determine if the mean ratio of repetition for all participants in a similar memory study differs from 0.50, we test:

$$H_0: \mu = 0.5$$
$$H_a: \mu \neq 0.5$$

From the printout, the test statistics is $t = -1.80$.

From the printout, the p-value of the test is $p = 0.115$. No α value was given, so we will use $\alpha = 0.05$. Since the p-value is not less than α $\left(p = 0.115 \not< 0.05\right)$, H_0 is not rejected. There is insufficient evidence to indicate that the mean ratio of repetition for all participants in a similar memory study differs from 0.50 at $\alpha = 0.05$.

1.66 There are three things to describe:

1) Mean: $\mu_{\bar{y}_1 - \bar{y}_2} = \mu_1 - \mu_2$

2) Std Deviation: $\sigma_{\bar{y}_1 - \bar{y}_2} = \sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}$

3) Shape: For sufficiently large samples, the shape of the sampling distribution is approximately normal.

1.67 The two populations must have:
1) relative frequency distributions that are approximately normal, and
2) variances that are equal.
The two samples must both have been randomly and independently chosen.

1.68 a. Let μ_1 = mean leadership value for captains from successful teams, and μ_2 = mean leadership value for captains from unsuccessful teams. To determine whether the mean leadership values for captains from successful and unsuccessful teams differ, we test:

$$H_0: \mu_1 - \mu_2 = 0$$
$$H_a: \mu_1 - \mu_2 \neq 0$$

The p-value is $p = 0.000$. Since the p-value is less than α $(p = 0.000 < 0.05)$, H_0 is rejected. There is sufficient evidence to indicate that the mean leadership values for captains from successful and unsuccessful teams differ at $\alpha = 0.05$.

b. Let μ_1 = mean leadership value for flight attendants from successful teams, and μ_2 = mean leadership value for flight attendants from unsuccessful teams. To determine whether the mean leadership values for flight attendants from successful and unsuccessful teams differ, we test:

$$H_0: \mu_1 - \mu_2 = 0$$
$$H_a: \mu_1 - \mu_2 \neq 0$$

The p-value is $p = 0.907$. Since the p-value is not less than α $(p = 0.907 \nless 0.05)$, H_0 is not rejected. There is insufficient evidence to indicate that the mean leadership values for flight attendants from successful and unsuccessful teams differ at $\alpha = 0.05$.

1.69 For this experiment let μ_1 and μ_2 represent the mean ratings for Group 1 (support favored position) and Group 2 (weaken opposing position), respectively. Then we want to test:

$$H_0: \mu_1 - \mu_2 = 0$$
$$H_a: \mu_1 - \mu_2 \neq 0$$

Calculate the pooled estimate of variance:

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} = \frac{(26-1)(12.5)^2 + (26-1)(12.2)^2}{26 + 26 - 2} = 152.545$$

.

The test statistic is $t = \dfrac{(\bar{y}_1 - \bar{y}_2) - D_0}{\sqrt{s_p^2\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}} = \dfrac{(28.6 - 24.9) - 0}{\sqrt{152.545\left(\dfrac{1}{26} + \dfrac{1}{26}\right)}} = 1.08.$

The rejection region requires $\alpha / 2 = 0.05 / 2 = 0.025$ in each tail of the *t*-distribution. From Table 2, Appendix D, with $df = (n_1 + n_2 - 2) = (26 + 26 - 2) = 50, \ t_{0.025} = 2.009$. The rejection region is $t < -2.009$ or $t > 2.009$.

Since the observed value of the test statistic does not fall in the rejection region $(t = 1.08 \not> 2.009)$, H_0 is not rejected. There is insufficient evidence of a difference between the true mean rating scores for the two groups at $\alpha = 0.05$.

Assumptions: This procedure requires the assumption that the samples of rating scores are randomly and independently selected from normal populations with equal variances.

1.70 a. For none of the five varieties of apricot jelly can we conclude that the mean taste scores of the two protocols differ at $\alpha = 0.05$. None of the *p*-values are less than $\alpha = 0.05$.

b. We can conclude that the mean taste scores for cheese varieties A, C, and D differ for the two protocols at $\alpha = 0.05$ because those varieties of cheese have *p*-values less than $\alpha = 0.05$. We cannot conclude that the mean taste scores for cheese variety B differ for the two protocols at $\alpha = 0.05$ because that variety of cheese has *p*-values greater than $\alpha = 0.05$.

c. The sample sizes in each study were $n_1 = n_2 = 25$. Although these numbers are less than 30, they are fairly close. Therefore, the distributions of $\bar{y}_1$ and $\bar{y}_2$ will be approximately normal.

1.71 a. Let $\mu_1 =$ mean percentage of board members who are female at firms with nominating committees, and $\mu_2 =$ mean percentage of board members who are female at firms without nominating committees. To determine whether firms with a nominating committee would appoint more female directors than firms without a nominating committee, we test:

$$H_0: \mu_1 - \mu_2 = 0$$
$$H_a: \mu_1 - \mu_2 > 0$$

b. Since the *p*-value is less than α $(p < 0.001 < 0.05)$, H_0 is rejected. There is sufficient evidence to indicate firms with a nominating committee would appoint more female directors than firms without a nominating committee at $\alpha = 0.05$.

c. No, the population percentages for each type of firm do not need to be normally distributed for the inference to be valid because the sample sizes were both greater than 30.

d. $$z = \frac{(\bar{y}_1 - \bar{y}_2) - D_0}{s_{\bar{y}_1 - \bar{y}_2}} = \frac{(\bar{y}_1 - \bar{y}_2) - 0}{s_{\bar{y}_1 - \bar{y}_2}} = 5.51 \Rightarrow \frac{(7.5 - 4.3)}{s_{\bar{y}_1 - \bar{y}_2}} = 5.51 \Rightarrow s_{\bar{y}_1 - \bar{y}_2} = 0.5808$$

For confidence coefficient 0.95, $\alpha = 0.05$ and $\alpha / 2 = 0.05 / 2 = 0.025$. From Table 1, Appendix D, $z_{0.025} = 1.96$. The 95% confidence interval is:

$$(\bar{y}_1 - \bar{y}_2) \pm z_{.025}\left(s_{\bar{y}_1 - \bar{y}_2}\right) \Rightarrow (7.5 - 4.3) \pm 1.96(0.5808) \Rightarrow 3.2 \pm 1.138 \Rightarrow (2.062, 4.338)$$

We are 95% confident that the difference in mean percentage of female board directors at firms with a nominating committee and firms without a nominating committee is between

2.062% and 4.338%. Since both of these numbers are positive, there is evidence that there is a difference in the mean percentages for the two groups.

1.72 a. Let μ_1 = mean voltage reading at the old location, and μ_2 = mean voltage reading at the new location. For confidence coefficient 0.90, $\alpha = 0.10$ and $\alpha/2 = 0.10/2 = 0.05$. From Table 1, Appendix D, $z_{0.05} = 1.645$. The 90% confidence interval is:

$$(\bar{y}_1 - \bar{y}_2) \pm z_{0.05}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \Rightarrow (9.80 - 9.42) \pm 1.645\sqrt{\frac{0.5409^2}{30} + \frac{0.4789^2}{30}}$$

$$\Rightarrow 0.38 \pm 0.217 \Rightarrow (0.163, 0.597)$$

b. No. Since the interval constructed in part (a) contains only positive values, we can conclude that there is evidence that the mean voltage readings are higher at the old location than at the new location.

1.73 a. Using MINITAB, some preliminary calculations are:

Statistics

Variable	N	Mean	StDev
Handshake	5	104.4	24.8
HighFive	5	55.80	13.10
FistBump	5	20.00	6.00

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} = \frac{(5-1)24.8^2 + (5-1)13.1^2}{5 + 5 - 2} = 393.325$$

Let μ_1 = mean percentage of bacteria present relative to the mean of the handshake for the handshake, and μ_2 = mean percentage of bacteria present relative to the mean of the handshake for the high five. For confidence coefficient 0.95, $\alpha = 0.05$ and $\alpha/2 = 0.05/2 = 0.025$. From Table 2, Appendix D, with $df = n_1 + n_2 - 2 = 5 + 5 - 2 = 8$, $t_{0.025} = 2.306$. The 95% confidence interval is:

$$(\bar{y}_1 - \bar{y}_2) \pm t_{0.025}\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} \Rightarrow (104.4 - 55.80) \pm 2.306\sqrt{393.325\left(\frac{1}{5} + \frac{1}{5}\right)}$$

$$\Rightarrow 48.6 \pm 28.92 \Rightarrow (19.68, 77.52)$$

If nearly twice as many bacteria were transferred during a handshake compared with a high five, then the confidence interval for the difference in the mean percentage of bacteria present between the handshake and the fist bump should contain 50%. The above confidence interval does contain 50%, so the confidence interval supports the statement.

b. μ_3 = mean percentage of bacteria present relative to the mean of the handshake for the fist bump.

$$s_p^2 = \frac{(n_2 - 1)s_2^2 + (n_3 - 1)s_3^2}{n_2 + n_3 - 2} = \frac{(5-1)13.1^2 + (5-1)6.0^2}{5 + 5 - 2} = 103.805$$

The 95% confidence interval is:

$$(\bar{y}_2 - \bar{y}_3) \pm t_{0.025}\sqrt{s_p^2\left(\frac{1}{n_2}+\frac{1}{n_3}\right)} \Rightarrow (55.80-20.00) \pm 2.306\sqrt{103.805\left(\frac{1}{5}+\frac{1}{5}\right)}$$

$$\Rightarrow 35.80 \pm 14.86 \Rightarrow (20.94, 50.66)$$

Since this interval contains only positive values, this supports the statement that the fist bump gave a lower transmission of bacteria.

c. Based on the answers to parts a and b, the fist bump is the most hygienic.

1.74 a. If the manipulation was successful, then the positive display group should have a larger mean response than the positive display group. (The smaller the number the higher the agreement.)

b. Using MINITAB, some preliminary calculations are:

Descriptive Statistics: RATING

RULE	N	Mean	StDev	SE Mean
Neutral	67	1.896	0.496	0.061
Positive	78	4.487	0.659	0.075

Let $\mu_1 =$ mean response for the neutral display, and $\mu_2 =$ mean response for the positive display. To determine if the manipulation was successful (mean response for the neutral group is less than that for the positive group), we test:

$$H_0: \mu_1 - \mu_2 = 0$$
$$H_a: \mu_1 - \mu_2 < 0$$

The test statistic is $z = \dfrac{(\bar{y}_1 - \bar{y}_2) - (\mu_1 - \mu_2)}{\sqrt{\left(\dfrac{s_1^2}{n_1}+\dfrac{s_2^2}{n_2}\right)}} = \dfrac{(1.896-4.487)-0}{\sqrt{\left(\dfrac{0.496^2}{67}+\dfrac{0.659^2}{78}\right)}} = -26.96.$

The rejection region requires $\alpha = 0.05$ in the lower tail of the z distribution. Using Table 1, Appendix D, $z_{0.05} = 1.645$. The rejection region is $z < -1.645$.

Since the test statistic is in the rejection region $(z = -26.96 < -1.645)$, H_0 is rejected. There is sufficient evidence to indicate the manipulation was successful at $\alpha = 0.05$.

c. We must assume that the samples are independent.

1.75 a. A paired-samples t-test was used because the samples are not independent. Each subject rated both TV and magazine ads.

b. Since the p-value is so small $(p < 0.001)$, H_0 is rejected at any value of $\alpha > 0.001$. There is sufficient evidence to indicate a difference in mean ratings between TV and magazine ads.

c. For confidence coefficient 0.95, $\alpha = 0.05$ and $\alpha / 2 = 0.05/2 = 0.025$. From Table 1, Appendix D, $z_{0.025} = 1.96$. The 95% confidence interval is:

$$\bar{y}_d \pm z_{0.025}\frac{s_d}{\sqrt{n_d}} \Rightarrow 0.45 \pm 1.96\frac{0.815}{\sqrt{159}} \Rightarrow 0.45 \pm 0.127 \Rightarrow (0.323, 0.577)$$

We are 95% confident that the mean difference between TV and magazine ads is between 0.323 and 0.577. Even though there is a statistical difference between the means of the two types of ads, the difference is quite small compared to the actual responses. Thus, there is no practical difference.

1.76 Let $\mu_d = \mu_1 - \mu_2 =$ before – after. Using MINITAB, some preliminary calculations are:

Descriptive Statistics

Sample	N	Mean	StDev	SE Mean
Before	13	2.513	1.976	0.548
After	13	1.506	1.448	0.402

Test

Null hypothesis	H₀: μ_difference = 0
Alternative hypothesis	H₁: μ_difference > 0

T-Value	P-Value
3.00	0.006

To determine if the photo-red enforcement program was successful in reducing red light-running crash incidents, we test:

$$H_0 : \mu_d = 0$$
$$H_a : \mu_d > 0$$

From the printout, the test statistic is $t = 3.00$ and the p-value is $p = 0.006$. Since the p-value is so small, H_0 is rejected. There is sufficient evidence to indicate the photo-red enforcement program was successful in reducing red light-running crash incidents at any value of $\alpha > 0.006$.

1.77 Let $\mu_d = \mu_1 - \mu_2 =$ North-South – East-West. Using MINITAB, some preliminary calculations are:

Descriptive Statistics

Sample	N	Mean	StDev	SE Mean
NS	5	7719	1548	692
EW	5	7436	1484	663

Estimation for Paired Difference

Mean	StDev	SE Mean	95% CI for μ_difference
283.6	86.4	38.7	(176.3, 390.9)

μ_difference: mean of (NS - EW)

For confidence coefficient 0.95, $\alpha = 0.05$ and $\alpha / 2 = 0.05 / 2 = 0.025$. From Table 2, Appendix D, with $df = n_d - 1 = 5 - 1 = 4$, $t_{0.025} = 2.776$. The 95% confidence interval is:

$$\bar{y}_d \pm z_{0.025} \frac{s_d}{\sqrt{n_d}} \Rightarrow 283.6 \pm 2.776 \frac{86.4}{\sqrt{5}} \Rightarrow 283.6 \pm 107.26 \Rightarrow (176.34, 390.86)$$

We are 95% confident that the difference in mean solar energy amounts generated between north-south oriented roadways and east-west oriented roadways is between 176.34 and 390.86 kilowatt-hours. Since all of the numbers in the interval are positive, this supports the researchers' conclusion.

1.78 From Tables 3, 4, 5, 6 of Appendix D,

a. $F_{0.05} = 3.73$ b. $F_{0.01} = 3.09$ c. $F_{0.025} = 6.52$

d. $F_{0.01} = 3.85$ e. $F_{0.10} = 2.52$ f. $F_{0.05} = 2.94$

1.79 Let σ_1^2 = variance of the number of hippo trails from a water source in the National Reserve and σ_2^2 = variance of the number of hippo trails from a water source in the pastoral ranch. To determine if the variability in the number of hippo trails from a water source in the National Reserve differs from the variability in the number of hippo trails from a water source in the pastoral ranch, we test:

$$H_0: \frac{\sigma_1^2}{\sigma_2^2} = 1$$

$$H_a: \frac{\sigma_1^2}{\sigma_2^2} \neq 1$$

The test statistic is $F = \frac{s_1^2}{s_2^2} = \frac{0.40^2}{0.30^2} = 1.778$.

The rejection region requires $\alpha / 2 = 0.10 / 2 = 0.05$ in the upper tail of the F-distribution. From Table 4, Appendix D, with $\nu_1 = n_1 - 1 = 406 - 1 = 405$ and $\nu_2 = n_2 - 1 = 230 - 1 = 229$, $F_{0.05} \approx 1.00$. The rejection region is $F > 1.00$.

Since the value of the test statistic falls in the rejection region $(F = 1.778 > 1.000)$, H_0 is rejected. There is sufficient evidence to indicate the variability in the number of hippo trails from a water source in the National Reserve differs from the variability in the number of hippo trails from a water source in the pastoral ranch at $\alpha = 0.10$.

1.80 a. We have to assume that the variances of the two groups are the same.

b. Let σ_1^2 = variance of the percentage of bacteria transferred for the handshake and σ_2^2 = variance of the percentage of bacteria transferred for the fist bump. To determine if the variance in percentage of bacteria transferred for the handshake differs from the variance of the percentage of bacteria transferred for the fist bump, we test:

$$H_0: \frac{\sigma_1^2}{\sigma_2^2} = 1$$
$$H_a: \frac{\sigma_1^2}{\sigma_2^2} \neq 1$$

Using the descriptive statistics from Exercise 1.73, the test statistic is
$F = \frac{s_1^2}{s_2^2} = \frac{24.8^2}{6^2} = 17.084.$

The rejection region requires $\alpha / 2 = 0.05 / 2 = 0.025$ in the upper tail of the F-distribution. From Table 5, Appendix D, with $\nu_1 = n_1 - 1 = 4 - 1 = 4$ and $\nu_2 = n_2 - 1 = 5 - 1 = 4$, $F_{0.025} = 9.60$. The rejection region is $F > 9.60$.

Since the value of the test statistic falls in the rejection region $(F = 17.084 > 9.60)$, H_0 is rejected. There is sufficient evidence to indicate the variance in percentage of bacteria transferred for the handshake differs from the variance of the percentage of bacteria transferred for the fist bump at $\alpha = 0.05$.

c. The decisions made in Exercise 1.73 may not be valid. There is evidence that the population variances are not equal, which is a requirement for the test in Exercise 1.73.

1.81 We will test to see if the ratio of the variances differ from 1 or not:

$$H_0: \frac{\sigma_1^2}{\sigma_2^2} = 1$$
$$H_a: \frac{\sigma_1^2}{\sigma_2^2} \neq 1$$

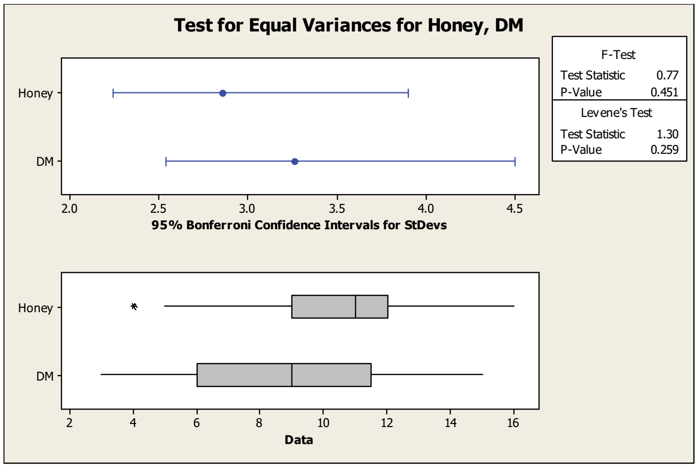
Two assumptions are required for the F test are as follows:
1. The two populations are normally distributed.
2. The samples are randomly and independently selected from their respective populations

The test statistic is $F = \frac{s_1^2}{s_2^2} = \frac{10.604}{8.151} = 1.30.$

The rejection region requires $\alpha / 2 = 0.10 / 2 = 0.05$ in the upper tail of the F distribution. Using Table 4, Appendix D with $\nu_1 = n_1 - 1 = 33 - 1 = 32$ and $\nu_2 = n_2 - 1 = 35 - 1 = 34$, $F_{0.05} = 1.84$. The rejection region is $F > 1.84$.

Since the observed value of the test statistic does not fall in the rejection region $(F = 1.30 \ngtr 1.84)$, H_0 is not rejected. There is insufficient evidence to show that the variances of the two groups differ at $\alpha = 0.10$.

Note: we will always place the larger sample variance in the numerator of the F test.



1.82 Let σ_1^2 = variance for the "support favored position" group and σ_2^2 = variance for the "weaken opposing position" group. To determine if the variances for the two groups differ, we test:

$$H_0: \frac{\sigma_1^2}{\sigma_2^2} = 1$$

$$H_a: \frac{\sigma_1^2}{\sigma_2^2} \neq 1$$

The test statistic is $F = \frac{s_1^2}{s_2^2} = \frac{12.5^2}{12.2^2} = 1.05$.

The rejection region requires $\alpha / 2 = 0.05 / 2 = 0.025$ in the upper tail of the F-distribution. From Table 5, Appendix D, with $v_1 = n_1 - 1 = 26 - 1 = 25$ and $v_2 = n_2 - 1 = 26 - 1 = 25$, $F_{0.025} \approx 2.24$. The rejection region is $F > 2.24$.

Since the value of the test statistic does not fall in the rejection region $(F = 1.05 \not> 2.24)$, H_0 is not rejected. There is insufficient evidence to indicate the variances for the two groups differ at $\alpha = 0.05$.

1.83 Let σ_1^2 = variance of the internal oil content for the vacuum fryer and σ_2^2 = variance of the internal oil content for the two-stage frying process. To determine if the variances for the two groups differ, we test:

$$H_0: \frac{\sigma_1^2}{\sigma_2^2} = 1$$

$$H_a: \frac{\sigma_1^2}{\sigma_2^2} \neq 1$$

The test statistic is $F = \frac{s_1^2}{s_2^2} = \frac{0.011^2}{0.002^2} = 30.25$.

No value of α was given, so we will use $\alpha = 0.10$. The rejection region requires $\alpha/2 = 0.10/2 = 0.05$ in the upper tail of the F-distribution. From Table 4, Appendix D, with $v_1 = n_1 - 1 = 6 - 1 = 5$ and $v_2 = n_2 - 1 = 6 - 1 = 5$, $F_{0.05} = 5.05$. The rejection region is $F > 5.05$.

Since the value of the test statistic falls in the rejection region $(F = 30.25 > 5.05)$, H_0 is rejected. There is sufficient evidence to indicate the variances for the two groups differ at $\alpha = 0.10$. Since there is evidence that the variances of the two groups are different, we would not recommend that the researchers carry out this analysis.

1.84 a. $1 - \left(\frac{1}{K^2}\right) = 1 - \left(\frac{1}{2^2}\right) = 1 - \frac{1}{4} = \frac{3}{4}$

At least ¾ of the measurements will lie within 2 standard deviations of the mean

b. $1 - \left(\frac{1}{K^2}\right) = 1 - \left(\frac{1}{3^2}\right) = 1 - \frac{1}{9} = \frac{8}{9}$

At least 8/9 of the measurements will lie within 3 standard deviations of the mean

c. $1 - \left(\frac{1}{K^2}\right) = 1 - \left(\frac{1}{1.5^2}\right) = 1 - \frac{1}{2.25} = \frac{1.25}{2.25} = \frac{5}{9}$

At least 5/9 of the measurements will lie within 1.5 standard deviations of the mean

1.85 a. $\bar{y} = \frac{\sum_{i=1}^{n} y_i}{n} = \frac{11+2+2+1+9}{5} = \frac{25}{5} = 5$

$$s^2 = \frac{\sum_{i=1}^{n}(y_i - \bar{y})^2}{n-1} = \frac{\sum_{i=1}^{n} y_i^2 - \frac{\left(\sum_{i=1}^{n} y_i\right)^2}{n}}{n-1} = \frac{(11^2 + 2^2 + 2^2 + 1^2 + 9^2) - \frac{(25)^2}{5}}{5-1}$$

$$= \frac{211 - 125}{4} = \frac{86}{4} = 21.5$$

$s = \sqrt{s^2} = \sqrt{21.5} \approx 4.637$

b. $\bar{y} = \frac{\sum_{i=1}^{n} y_i}{n} = \frac{22+9+21+15}{4} = \frac{67}{4} = 16.75$

$$s^2 = \frac{\sum_{i=1}^{n}(y_i - \bar{y})^2}{n-1} = \frac{\sum_{i=1}^{n} y_i^2 - \frac{\left(\sum_{i=1}^{n} y_i\right)^2}{n}}{n-1} = \frac{(22^2 + 9^2 + 21^2 + 15^2) - \frac{(67)^2}{4}}{4-1}$$

$$= \frac{1231 - 1122.25}{3} = \frac{108.75}{3} = 36.25$$

$s = \sqrt{s^2} = \sqrt{36.25} \approx 6.021$

c. $\bar{y} = \frac{\sum_{i=1}^{n} y_i}{n} = \frac{34}{7} = 4.857$

$$s^2 = \frac{\sum_{i=1}^{n} y_i^2 - \frac{\left(\sum_{i=1}^{n} y_i\right)^2}{n}}{n-1} = \frac{344 - \frac{(34)^2}{7}}{7-1} = \frac{178.857}{6} \approx 29.81$$

$$s = \sqrt{s^2} = \sqrt{29.81} \approx 5.460$$

d. $\bar{y} = \frac{\sum_{i=1}^{n} y_i}{n} = \frac{16}{4} = 4$

$$s^2 = \frac{\sum_{i=1}^{n} y_i^2 - \frac{\left(\sum_{i=1}^{n} y_i\right)^2}{n}}{n-1} = \frac{64 - \frac{(16)^2}{4}}{4-1} = \frac{0}{3} \approx 0$$

$$s = \sqrt{s^2} = \sqrt{0} \approx 0$$

1.86 Using Table 1, Appendix D:

a. $P(z \geq 2) = 0.5 - P(0 \leq z \leq 2) = 0.5 - 0.4772 = 0.0228$

b. $P(z \leq -2) = P(z \geq 2) = 0.0228$

c. $P(z \geq -1.96) = 0.5 + P(-1.96 \leq z \leq 0) = 0.5 + 0.4750 = 0.9750$

d. $P(z \geq 0) = 0.5$

e. $P(z \leq -0.5) = 0.5 - P(-0.5 \leq z \leq 0) = 0.5 - 0.1915 = 0.3085$

f. $P(z \leq -1.96) = 0.5 - P(-1.96 \leq z \leq 0) = 0.5 - 0.4750 = 0.0250$

1.87 a. $z = \frac{y - \mu}{\sigma} = \frac{10 - 30}{5} = \frac{-20}{5} = -4$

The sign and magnitude of the z-value indicate that the y-value is 4 standard deviations below the mean.

b. $$z=\frac{y-\mu}{\sigma}=\frac{32.5-30}{5}=\frac{2.5}{5}=0.5$$

The y-value is 0.5 standard deviations above the mean.

c. $$z=\frac{y-\mu}{\sigma}=\frac{30-30}{5}=\frac{0}{5}=0$$

The y-value is equal to the mean of the random variable y.

d. $$z=\frac{y-\mu}{\sigma}=\frac{60-30}{5}=\frac{30}{5}=6$$

The y-value is 6 standard deviations above the mean.

1.88 a. Town where sample collected is qualitative since this variable is not measured on a numerical scale.

b. Type of water supply is qualitative since this variable is not measured on a numerical scale.

c. Acidic level is quantitative since this variable is measured on a numerical scale (pH level 1 to 14).

d. Turbidity level is quantitative since this variable is measured on a numerical scale

e. Temperature quantitative since this variable is measured on a numerical scale.

f. Number of fecal coliforms per 100 millimeters is quantitative since this variable is measured on a numerical scale.

g. Free chlorine-residual (milligrams per liter) is quantitative since this variable is measured on a numerical scale.

h. Presence of hydrogen sulphide (yes or no) is qualitative since this variable is not measured on a numerical scale.

1.89 a. The population is all adults in Tennessee. The sample is 575 study participants.

b. The number of years of education is quantitative since it can be measured on a numerical scale. The insomnia status (normal sleeper or chronic insomnia) is qualitative since it cannot be measured on a numerical scale.

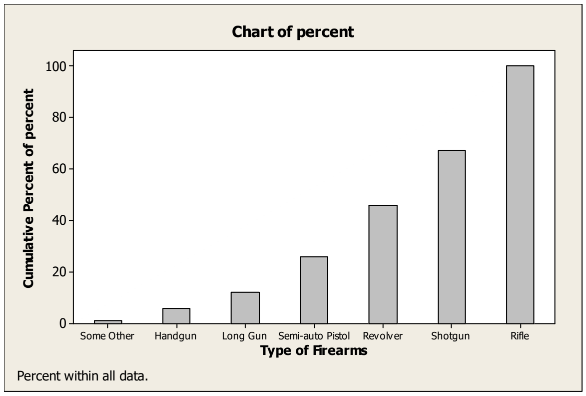
c. Less educated adults are more likely to have chronic insomnia.

1.90 a. Pie chart

b. The type of firearms owned is the qualitative variable.

c. Rifle (33%), shotgun (21%), and revolver (20%) are the most common types of firearm.

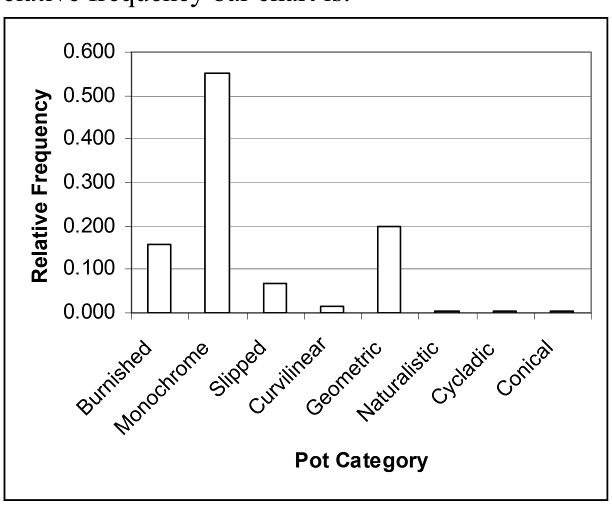
d.



1.91 Suppose we construct a relative frequency bar chart for this data. This will allow the archaeologists to compare the different categories easier. First, we must compute the relative frequencies for the categories. These are found by dividing the frequencies in each category by the total 837. For the burnished category, the relative frequency is 133 / 837 = .159. The rest of the relative frequencies are found in a similar fashion and are listed in the table.

Pot Category	Number Found	Computation	Relative Frequency
Burnished	133	133 / 837	0.159
Monochrome	460	460 / 837	0.550
Slipped	55	55 / 837	0.066
Curvilinear Decoration	14	14 / 837	0.017
Geometric Decoration	165	165 / 837	0.197
Naturalistic Decoration	4	4 / 837	0.005
Cycladic White clay	4	4 / 837	0.005
Cononical cup clay	2	2 / 837	0.002
Total	837		1.001

A relative frequency bar chart is:



The most frequently found type of pot was the Monochrome. Of all the pots found, 55% were Monochrome. The next most frequently found type of pot was the Painted in Geometric Decoration. Of all the pots found, 19.7% were of this type. Very few pots of the types Painted in Naturalistic Decoration, Cycladic White clay, and Conical cup clay were found.

1.92 a. A stem-and-leaf display of the data using MINITAB is:

Stem-and-leaf of FNE N = 25
Leaf Unit = 1.0

2	0	67
3	0	8
6	1	**0**01
10	1	**33**33
12	1	**4**5
(2)	1	**6**6
11	1	8**9**99
7	2	**0**0**11**
3	2	3
2	2	**45**

b. The numbers in bold in the stem-and-leaf display represent the bulimic students. Those numbers tend to be the larger numbers. The larger numbers indicate a greater fear of negative evaluation. Yes, the bulimic students tend to have a greater fear of negative evaluation.

c. A measure of reliability indicates how certain one is that the conclusion drawn is correct. Without a measure of reliability, anyone could just guess at a conclusion.

d. Let $\mu_1 =$ mean FNE score for bulimic students and $\mu_1 =$ mean FNE score for normal students. Using MINITAB, some preliminary calculations are:

Descriptive Statistics: FNESCORE

GROUP	N	Mean	StDev	SE Mean
Bulimic	11	17.82	4.92	1.5
Normal	14	14.14	5.29	1.4

Estimation for Difference

Difference	Pooled StDev	95% CI for Difference
3.68	5.13	(-0.60, 7.95)

For confidence coefficient 0.95, $\alpha = 0.05$ and $\alpha / 2 = 0.05 / 2 = 0.25$. From Table 2, Appendix D, with $df = n_1 + n_2 - 2 = 11 + 14 - 2 = 23$, $t_{0.025} = 2.069$. The 95% confidence interval is:

$$\left(\bar{y}_1 - \bar{y}_2\right) \pm t_{0.025}\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} \Rightarrow \left(17.82 - 14.14\right) \pm 2.069\sqrt{5.13^2\left(\frac{1}{11} + \frac{1}{14}\right)}$$

$$\Rightarrow 3.68 \pm 4.276 \Rightarrow \left(-0.596, 7.956\right)$$

We are 95% confident that the difference in mean FNE scores for bulimic and normal students is between −0.596 and 7.956.

e. We must assume that the distribution of FNE scores for the bulimic students and the distribution of the FNE scores for the normal students are normally distributed. We must also assume that the variances of the two populations are equal. Both sample distributions look somewhat mound-shaped and the sample variances are fairly close in value. Thus, both assumptions appear to be reasonably satisfied.

f. Let σ_{B}^2 = variance of the FNE scores for bulimic students and σ_{N}^2 = variance of the FNE scores for normal students.

To determine if the variances are equal, we test:

$$H_0: \frac{\sigma_B^2}{\sigma_N^2} = 1$$

$$H_a: \frac{\sigma_B^2}{\sigma_N^2} \neq 1$$

The test statistic is $F = \dfrac{\text{Larger sample variance}}{\text{Smaller sample variance}} = \dfrac{s_{\text{N}}^2}{s_{\text{B}}^2} = \dfrac{5.29^2}{4.92^2} = 1.16.$

The rejection region requires $\alpha/2 = 0.05/2 = 0.025$ in the upper tail of the F-distribution with $\nu_{\text{N}} = n_{\text{N}} - 1 = 14 - 1 = 13$ and $\nu_{\text{B}} = n_{\text{B}} - 1 = 11 - 1 = 10$. From Table 5, Appendix D, $F_{0.025} \approx 3.62$. The rejection region is $F > 3.62$.

Since the observed value of the test statistic does not fall in the rejection region $\left(F = 1.16 \not> 3.62\right)$, H_0 is not rejected. It appears that the assumption of equal variances is valid.

1.93 Some preliminary calculations are:

$$\bar{y} = \frac{\sum y}{n} = \frac{110}{5} = 22 \qquad s^2 = \frac{\sum y^2 - \frac{\left(\sum y\right)^2}{n}}{n-1} = \frac{2{,}436 - \frac{110^2}{5}}{5-1} = 4 \qquad s = \sqrt{s^2} = \sqrt{4} = 2$$

To determine if the data collected were fabricated, we test:

$$H_0: \mu = 15$$

$$H_\text{a}: \mu \neq 15$$

The test statistic is $t = \dfrac{\bar{y} - \mu_0}{s/\sqrt{n}} = \dfrac{22 - 15}{2/\sqrt{5}} = 7.83.$

If we want to choose a level of significance to benefit the students, we would choose a small value for α. Suppose we use $\alpha = 0.01$. The rejection region requires $\alpha/2 = 0.01/2 = 0.005$ in each tail of the t distribution with $df = n - 1 = 5 - 1 = 4$. From Table 2, Appendix D, $t_{0.005} = 4.604$. The rejection region is $t < -4.604$ or $t > 4.604$.

Since the observed value of the test statistic falls in the rejection region $\left(t = 7.83 > 4.604\right)$, H_0 is rejected. There is sufficient evidence to indicate the mean data collected were fabricated at $\alpha = 0.01$.

1.94 For the New Location, $\bar{y} = 9.4223$ and $s = 0.4789$.

$\bar{y} \pm s \Rightarrow 9.4223 \pm 0.4789 \Rightarrow (8.9434, 9.9012)$ The Empirical Rule says that approximately 60-80% of the observations should fall in this interval.

$\bar{y} \pm 2s \Rightarrow 9.4223 \pm 2(0.4789) \Rightarrow 9.4223 \pm 0.9578 \Rightarrow (8.4645, 10.3801)$ The Empirical Rule says that approximately 95% of the observations should fall in this interval.

$\bar{y} \pm 3s \Rightarrow 9.4223 \pm 3(0.4789) \Rightarrow 9.4223 \pm 1.4367 \Rightarrow (7.9856, 10.8590)$ The Empirical Rule says that approximately all of the observations should fall in this interval.

For the Old Location, $\bar{y} = 9.8037$ and $s = 0.5409$.

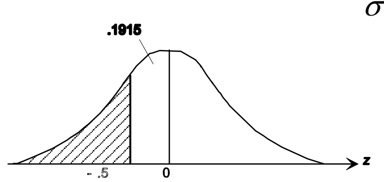
$\bar{y} \pm s \Rightarrow 9.8037 \pm 0.5409 \Rightarrow (9.2628, 10.3446)$ The Empirical Rule says that approximately 60-80% of the observations should fall in this interval.

$\bar{y} \pm 2s \Rightarrow 9.8037 \pm 2(0.5409) \Rightarrow 9.8037 \pm 1.0818 \Rightarrow (8.7219, 10.8855)$ The Empirical Rule says that approximately 95% of the observations should fall in this interval.

$\bar{y} \pm 3s \Rightarrow 9.8037 \pm 3(0.5409) \Rightarrow 9.8037 \pm 1.6227 \Rightarrow (8.1810, 11.4264)$ The Empirical Rule says that approximately all of the observations should fall in this interval.

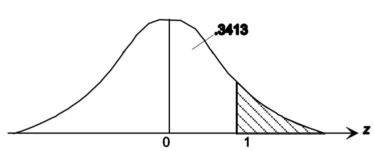
1.95 For each of these questions, we will use Table 1, Appendix D.

a. The z-score for $y = 75$ is $z = \dfrac{y-\mu}{\sigma} = \dfrac{75-80}{10} = -0.50$.



$$\begin{aligned} P(y \le 75) &= P(z \le -0.5) \\ &= 0.5 - P(-0.5 \le z \le 0) \\ &= 0.5 - 0.1915 = 0.3085 \end{aligned}$$

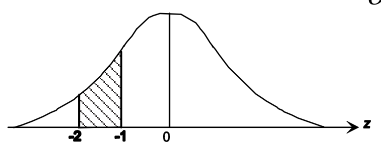
b. The z-score for $y = 90$ is $z = \dfrac{y-\mu}{\sigma} = \dfrac{90-80}{10} = 1.00$.



$$\begin{aligned} P(y \ge 90) &= P(z \ge 1.00) \\ &= 0.5 - P(0 \le z \le 1.00) \\ &= 0.5 - 0.3413 = 0.1587 \end{aligned}$$

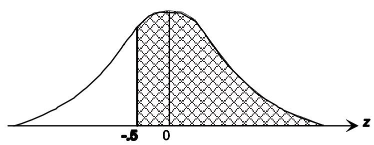
c. The z-score for $y = 60$ is $z = \dfrac{y-\mu}{\sigma} = \dfrac{60-80}{10} = -2$.

The z-score for $y = 70$ is $z = \dfrac{y-\mu}{\sigma} = \dfrac{70-80}{10} = -1$.



$$\begin{aligned} P(60 \le y \le 70) &= P(-2 \le z \le -1) \\ &= P(-2 \le z \le 0) + P(-1 \le z \le 0) \\ &= 0.4772 - 0.3413 = 0.1359 \end{aligned}$$

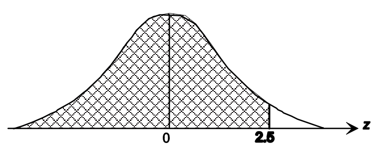
d.



$$\begin{aligned} P(y \ge 75) &= P(z \ge -0.5) \\ &= 0.5 + P(-0.5 \le z \le 0) \\ &= 0.5 + 0.1915 = 0.6915 \end{aligned}$$

e. $P(y = 75) = P(z = -0.5) = 0$

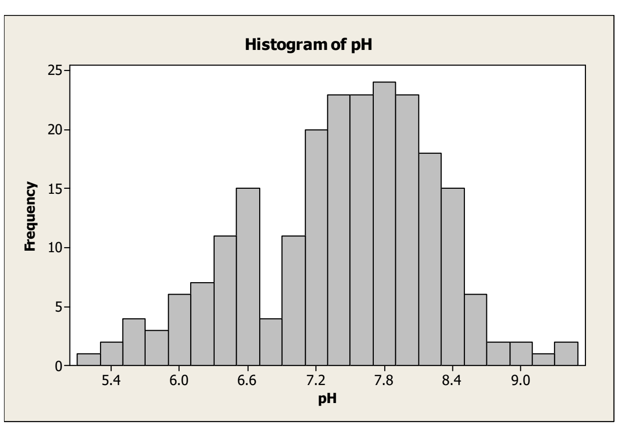
f. The z-score for $y = 105$ is $z = \dfrac{y-\mu}{\sigma} = \dfrac{105-80}{10} = 2.5$.



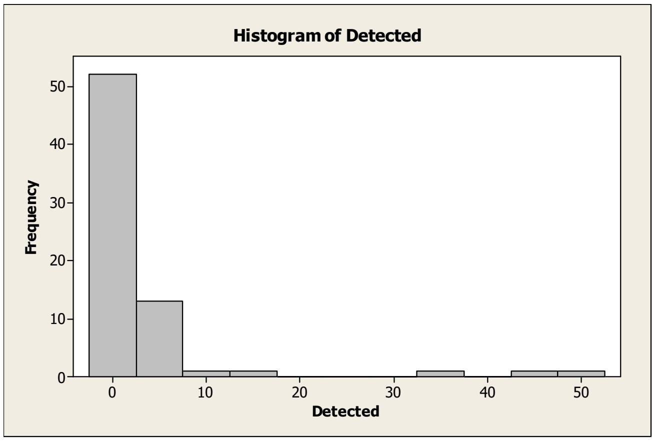
$$\begin{aligned} P(y \le 105) &= P(z \le 2.5) \\ &= 0.5 + P(0 \le z \le 2.5) \\ &= 0.5 + 0.4938 = 0.9938 \end{aligned}$$

1.96 Using Table 1, Appendix D, $P(-1.5 < z < 1.5) = 2(0.4332) = 0.8664$. Approximately 87% of the time *Six Sigma* will meet their goal.

1.97 a. It appears that about 58 out of the 223 or 0.26 of New Hampshire wells have a pH level less than 7.0.



b. It appears that about 6 out of 70 or 0.086 have a value greater than 5 micrograms per liter.



c. $\bar{y} = 7.43,\ \ s = 0.82,\ \ \bar{y} \pm 2s \Rightarrow 7.43 \pm 2(0.82) \Rightarrow (5.79, 9.06);\ 95\%$ (Empirical Rule).

d. $\bar{y} = 1.22,\ \ s = 5.11,\ \ \bar{y} \pm 2s \Rightarrow 1.22 \pm 2(5.11) \Rightarrow (-9.00, 11.44); 75\%$ (Empirical Rule).

1.98 a. $z = \dfrac{y - \mu}{\sigma} = \dfrac{16 - 11}{3.5} = 1.43$

b. Using Table 1, Appendix D,

$$P(10 < y < 15) = P\left(\frac{10 - 11}{3.5} < z < \frac{15 - 11}{3.5}\right) = P(-0.29 < z < 1.14)$$
$$= P(-0.29 < z < 0) + P(0 < z < 1.14) = 0.1141 + 0.3729 = 0.4870$$

c. Using Table 1, Appendix D,

$$P(y > 17) = P\left(z > \frac{17 - 11}{3.5}\right) = P(z > 1.17) = 0.5 - P(0 < z < 1.71) = 0.5 - 0.4564 = 0.0436$$

1.99 For confidence level 0.99, $\alpha = 0.01$ and $\alpha / 2 = 0.01 / 2 = 0.005$. From Table 2, Appendix D, with $df = n - 1 = 13 - 1 = 12,\ \ t_{0.005} = 3.055$. The 99% confidence interval is:

$$\bar{y} \pm t_{0.005}\left(\frac{s}{\sqrt{n}}\right) = 19 \pm 3.055\left(\frac{2.2}{\sqrt{13}}\right) = 19 \pm 1.9 \Rightarrow (17.1, 20.9)$$

We are 99% confident that the true mean quality of the methodology of the Wong scale is between 17.1 and 20.9.

1.100 a. The average daily ammonia concentration is

$$\bar{y} = \frac{\sum y_i}{n} = \frac{1.53 + 1.50 + 1.37 + 1.51 + 1.55 + 1.42 + 1.41 + 1.48}{8} = \frac{11.77}{8} = 1.47 \text{ ppm}$$

b. $$s^2 = \frac{\sum y_i^2 - \frac{\left(\sum y_i\right)^2}{n}}{n-1}$$

$$= \frac{\left(1.53^2 + 1.50^2 + 1.37^2 + 1.51^2 + 1.55^2 + 1.42^2 + 1.41^2 + 1.48^2\right) - \frac{(11.77)^2}{8}}{8-1}$$

$$= \frac{17.3453 - \frac{(11.77)^2}{8}}{8-1} = \frac{0.0287}{7} = 0.0041$$

$$s = \sqrt{s^2} = \sqrt{0.0041} = 0.0640$$

We would expect about most of the daily ammonia levels to fall within $\bar{y} \pm 2s \Rightarrow 1.47 \pm 2(0.0640) \Rightarrow 1.47 \pm 0.128 \Rightarrow (1.34, 1.60)$ ppm.

c. The morning drive-time has more variable ammonia levels as it has the larger standard deviation.

1.101 For this problem, $\mu_{\bar{y}} = \mu = 4.59$ and $\sigma_{\bar{y}} = \frac{\sigma}{\sqrt{n}} = \frac{2.95}{\sqrt{50}} = 0.4172.$

a. $P(\bar{y} \geq 6) = P\left(z \geq \frac{6 - 4.59}{0.4172}\right) = P(z \geq 3.38) \approx 0.5 - 0.5 = 0$

(using Table 1, Appendix D)

Since the probability of observing a sample mean CAHS score of 6 or higher is so small (p is essentially 0), we would not expect to see a sample mean of 6 or higher.

b. μ and/or σ differ from stated values.

1.102 a. For confidence level 0.90, $\alpha = 0.10$ and $\alpha / 2 = 0.10 / 2 = 0.05$. From Table 2, Appendix D, with $df = n - 1 = 29 - 1 = 28$, $t_{0.05} = 1.701$. The 90% confidence interval is:

$$\bar{y} \pm t_{0.05} \frac{s}{\sqrt{n}} \Rightarrow 20.9 \pm 1.701 \frac{3.34}{\sqrt{29}} \Rightarrow 20.9 \pm 1.055 \Rightarrow (19.845, 21.955)$$

We are 90% confident that the true mean number of eggs that a male and female pair of infected spider mites produced is between 19.845 and 21.955.

For confidence level 0.90, $\alpha = 0.10$ and $\alpha / 2 = 0.10 / 2 = 0.05$. From Table 2, Appendix D, with $df = n - 1 = 23 - 1 = 22$, $t_{0.05} = 1.717$. The 90% confidence interval is:

$$\bar{y} \pm t_{0.05} \frac{s}{\sqrt{n}} \Rightarrow 20.3 \pm 1.717 \frac{3.50}{\sqrt{23}} \Rightarrow 20.3 \pm 1.253 \Rightarrow (19.047, 21.553)$$

We are 90% confident that the true mean number of eggs that a treated male infected spider mite produced is between 19.047 and 21.553.

For confidence level 0.90, $\alpha = 0.10$ and $\alpha / 2 = 0.10/2 = 0.05$. From Table 2, Appendix D, with $df = n-1 = 18-1 = 17$, $t_{0.05} = 1.740$. The 90% confidence interval is:

$$\bar{y} \pm t_{0.05}\frac{s}{\sqrt{n}} \Rightarrow 22.9 \pm 1.740\frac{4.37}{\sqrt{18}} \Rightarrow 22.9 \pm 1.792 \Rightarrow (21.108, 24.692)$$

We are 90% confident that the true mean number of eggs that a treated female infected spider mite produced is between 21.108 and 24.692.

For confidence level 0.90, $\alpha = 0.10$ and $\alpha / 2 = 0.10/2 = 0.05$. From Table 2, Appendix D, with $df = n-1 = 21-1 = 20$, $t_{0.05} = 1.725$. The 90% confidence interval is:

$$\bar{y} \pm t_{0.05}\frac{s}{\sqrt{n}} \Rightarrow 18.6 \pm 1.725\frac{2.11}{\sqrt{21}} \Rightarrow 18.6 \pm 0.794 \Rightarrow (17.806, 19.394)$$

We are 90% confident that the true mean number of eggs that a male and female treated pair of infected spider mites produced is between 17.806 and 19.394.

b. It appears that the female treated group produces the highest mean number of eggs.

1.103 a. From the printout, the 95% confidence interval is $(1.6711, 2.1989)$.

b. We are 95% confident that the true mean failure time of used colored display panels is between 1.6711 and 2.1989 years.

c. In repeated sampling, 95% of all confidence intervals constructed will contain the true mean failure time.

1.104 a. For confidence coefficient 0.99, $\alpha = 0.01$ and $\alpha / 2 = 0.01/2 = 0.005$. From Table 1, Appendix D, $z_{0.005} = 2.58$. The 99% confidence interval is:

$$\bar{y} \pm z_{0.005}\frac{s}{\sqrt{n}} \Rightarrow 1.13 \pm 2.58\left(\frac{2.21}{\sqrt{72}}\right) \Rightarrow 1.13 \pm 0.67 \Rightarrow (0.46, 1.80)$$

We are 99% confident that the true mean number of pecks made by chickens pecking at blue string is between 0.458 and 1.802.

b. Yes, there is evidence that chickens are more apt to peck at white string. The mean number of pecks at white string is 7.5. Since 7.5 is not in the 99% confidence interval for the mean number of pecks at blue string, it is not a likely value for the true mean for blue string.

1.105 a. To determine if the mean social interaction score of all Connecticut mental health patients differs from 3, we test:

$H_0: \mu = 3$
$H_a: \mu \neq 3$

The test statistic is $z = \dfrac{\bar{y} - \mu_0}{\sigma_{\bar{y}}} = \dfrac{2.95 - 3}{1.10 / \sqrt{6{,}681}} = -3.72.$

The rejection region requires $\alpha / 2 = 0.01 / 2 = 0.005$ in each tail of the z distribution. From Table 1, Appendix D, $z_{0.005} = 2.58$. The rejection region is $z < -2.58$ or $z > 2.58$.

Since the observed value of the test statistic falls in the rejection region $(z = -3.72 < -2.58)$, H_0 is rejected. There is sufficient evidence to indicate that the mean social interaction score of all Connecticut mental health patients differs from 3 at $\alpha = 0.01$.

b. From the test in part a, we found that the mean social interaction score was statistically different from 3. However, the sample mean score was 2.95. Practically speaking, 2.95 is very similar to 3.0. The very large sample size, $n = 6{,}681$, makes it very easy to find statistical significance, even when no practical significance exists.

c. Because the variable of interest is measured on a 5-point scale, it is very unlikely that the population of the ratings will be normal. However, because the sample size was extremely large, $(n = 6{,}681)$, the Central Limit Theorem will apply. Thus, the distribution of $\bar{y}$ will be normal, regardless of the distribution of y. Thus, the analysis used above is appropriate.

1.106 To determine if the mean alkalinity level of water in the tributary exceeds 50 mpl, we test:

$$H_0: \mu = 50$$
$$H_a: \mu > 50$$

The test statistic is $z = \dfrac{\bar{y} - \mu_0}{s/\sqrt{n}} = \dfrac{67.8 - 50}{14.4/\sqrt{100}} = 12.36.$

The rejection region requires $\alpha = 0.01$ in the upper tail of the z distribution. From Table 1, Appendix D, $z_{0.01} = 2.33$. The rejection region is $z > 2.33$.

Since the observed value of the test statistic falls in the rejection region $(z = 12.36 > 2.33)$, H_0 is rejected. There is sufficient evidence to conclude that the mean alkalinity level of water in the tributary exceeds 50 mpl at $\alpha = 0.01$.

1.107 For this experiment let μ_1 and μ_2 represent the performance level of students in the control group and the rudeness condition group, respectively.

Using MINITAB, some preliminary calculations are:

Statistics

Variable	Condition	N	Mean	StDev
UsesBrick	Control	53	11.81	7.38
	Rude	45	8.511	3.992

To determine if the true mean performance level for students in the rudeness condition is lower than the true mean performance level for students in the control group, we want to test:

$$H_0: \mu_1 - \mu_2 = 0$$
$$H_a: \mu_1 - \mu_2 > 0$$

The test statistic is $z = \dfrac{(\bar{y}_1 - \bar{y}_2) - D_0}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}} = \dfrac{(11.81 - 8.51) - 0}{\sqrt{\dfrac{7.38^2}{53} + \dfrac{3.992^2}{45}}} = 2.81.$

The rejection region requires $\alpha = 0.01$ in the upper tail of the z distribution. From Table 1, Appendix D, $z_{0.01} = 2.33$. The rejection region is $z > 2.33$.
Since the observed value of the test statistic falls in the rejection region $(z = 2.81 > 2.33)$, H_0 is rejected. There is significant evidence to indicate that the true mean performance level for students in the rudeness condition is lower than the true mean performance level for students in the control group at $\alpha = 0.01$.

Assumptions: This procedure requires the assumption that the samples are randomly and independently selected.

1.108 a. $\mu =$ true mean chromatic contrast of crab-spiders on daisies.

b. To determine if the true mean chromatic contrast of crab-spiders on daisies is less than 70, we test:

$$H_0: \mu = 70$$
$$H_a: \mu < 70$$

c. Using MINITAB, some descriptive statistics are:

Statistics

Variable	N	Mean	StDev
CONTRAST	10	57.5	32.6

The test statistic is $t = \dfrac{\bar{y} - \mu_0}{s/\sqrt{n}} = \dfrac{57.5 - 70}{32.6/\sqrt{10}} = -1.21.$

d. The rejection region requires $\alpha = 0.10$ in the lower tail of the t distribution. From Table 2, Appendix D, with $df = n - 1 = 10 - 1 = 9$, $t_{0.10} = 1.383$. The rejection region is $t < -1.383$.

e. Using MINITAB, some calculations are:

Test

Null hypothesis H_0: μ = 70
Alternative hypothesis H_1: μ < 70

T-Value	P-Value
-1.21	0.128

The p-value is $p = 0.128$.

f. Since the test statistics does not fall in the rejection region $\left(t = -1.21 \nless -1.383\right)$, H_0 is not rejected. There is insufficient evidence to indicate the true mean chromatic contrast of crab-spiders on daisies is less than 70 at $\alpha = 0.10$.

1.109 Let $\mu_1 =$ mean height of Australian boys who repeated a grade and $\mu_2 =$ mean height of Australian boys who never repeated a grade.

a. To determine if the average height of Australian boys who repeated a grade is less than the average height of boys who never repeated, we test:

$$H_0: \mu_1 = \mu_2$$
$$H_a: \mu_1 < \mu_2$$

The test statistic is $z = \dfrac{\bar{y}_1 - \bar{y}_2}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}} = \dfrac{-0.04 - 0.30}{\sqrt{\dfrac{1.17^2}{86} + \dfrac{.97^2}{1349}}} = -2.64.$

The rejection region requires $\alpha = 0.05$ in the lower tail of the z distribution. From Table 1, Appendix D, $z_{0.05} = 1.645$. The rejection region is $z < -1.645$.

Since the observed value of the test statistic falls in the rejection region $\left(z = -2.64 < -1.645\right)$, H_0 is rejected. There is sufficient evidence to indicate that the average height of Australian boys who repeated a grade is less than the average height of boys who never repeated at $\alpha = 0.05$.

b. Let $\mu_1 =$ mean height of Australian girls who repeated a grade and $\mu_2 =$ mean height of Australian girls who never repeated a grade.

To determine if the average height of Australian girls who repeated a grade is less than the average height of girls who never repeated, we test:

$$H_0: \mu_1 = \mu_2$$
$$H_a: \mu_1 < \mu_2$$

The test statistic is $z = \dfrac{\bar{y}_1 - \bar{y}_2}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}} = \dfrac{0.26 - 0.22}{\sqrt{\dfrac{0.94^2}{43} + \dfrac{1.04^2}{1366}}} = 0.27.$

The rejection region is $z < -1.645$.

Since the observed value of the test statistic falls in the rejection region $\left(z = 0.27 \nless -1.645\right)$, H_0 is not rejected. There is insufficient evidence to indicate that the average height of Australian girls who repeated a grade is less than the average height of girls who never repeated at $\alpha = 0.05$.

c. From the data, there is evidence to indicate that the average height of Australian boys who repeated a grade is less than the average height of boys who never repeated a grade. However, there is no evidence that the average height of Australian girls who repeated a grade is less than the average height of girls who never repeated.

1.110 To determine if the true mean lacunarity measurement for all grassland pixels differs from 220, we test:

$$H_0 : \mu = 220$$
$$H_a : \mu \neq 220$$

The test statistic is $z = \dfrac{\bar{y} - \mu_0}{s/\sqrt{n}} = \dfrac{225 - 220}{20/\sqrt{100}} = 2.50$.

The rejection region requires $\alpha / 2 = 0.01 / 2 = 0.005$ in each tail of the z distribution. From Table 1, Appendix D, $z_{0.005} = 2.575$. The rejection region is $z < -2.575$ or $z > 2.575$.

Since the observed value of the test statistic does not fall in the rejection region $\left(z = 2.50 \ngtr 2.575\right)$, H_0 is not rejected. There is insufficient evidence to conclude that the true mean lacunarity measurement for all grassland pixels differs from 220 at $\alpha = 0.01$. There is no evidence to indicate the area sampled is not grassland.

1.111 a. Each participant acted as a speaker and an audience member

b. Let $\mu_d = \mu_{\text{speaker}} - \mu_{\text{audience}}$ = true mean number of laugh episodes for speakers minus the true mean number of laugh episodes as an audience member.

c. No, you need sample statistics for differences.

d. When testing the hypothesis: $H_0 : \mu_d = 0$ vs $H_a : \mu_d \neq 0$, the *t*-test revealed that the $p < 0.01$. Thus, we can reject H_0 and conclude that there is a significant difference in the true mean number of laugh episodes for speakers and audience members for any value of $\alpha \geq 0.01$.

1.112 a. The data should be analyzed using a paired-difference analysis because that is how the data were collected. Reaction times were collected twice from each subject, once under the random condition and once under the static condition. Since the two sets of data are not independent, they cannot be analyzed using independent samples analyses.

b. Let μ_1 = mean reaction time under the random condition and μ_2 = mean reaction time under the static condition. Let $\mu_d = \mu_1 - \mu_2$. To determine if there is a difference in mean reaction time between the two conditions, we test:

$$H_0: \mu_d = 0$$
$$H_a: \mu_d \neq 0$$

c. The test statistic is $t = 1.52$ with a p-value of $p = 0.15$. Since the p-value is not small, there is no evidence to reject H_0 for any reasonable value of α. There is insufficient evidence to indicate a difference in the mean reaction times between the two conditions. This supports the researchers' claim that visual search has no memory.

1.113 a. To determine if the dairies in the tri-county market participated in collusive practices, we test:

$$H_0: \mu_1 - \mu_2 = 0$$
$$H_a: \mu_1 - \mu_2 < 0$$

The test statistic is $t = -5.58$ and the p-value is $p = 0.000$. Since the p-value is smaller than $\alpha = 0.01$ $(p = 0.000 < 0.01)$, H_0 is rejected. Therefore, we can conclude that the two mean milk price in the surrounding market is less than the mean milk price in the tri-county market at $\alpha = 0.01$. This indicates that the dairies in the tri-county market participated in collusive practices.

b. To determine if the bid price variance for the surrounding market exceeds the bid price variance for the tri-county market, we test:

$$H_0: \frac{\sigma_1^2}{\sigma_2^2} = 1$$
$$H_a: \frac{\sigma_1^2}{\sigma_2^2} > 1$$

The test statistic is $F = 1.41$ and the p-value is $p = 0.048/2 = 0.024$. If we use $\alpha = 0.05$, then the p-value is less than α $(p = 0.024 < 0.05)$ and H_0 would be rejected. There is sufficient evidence to indicate bid price variance for the surrounding market exceeds the bid price variance for the tri-county market at $\alpha = 0.05$. If we use $\alpha = 0.01$, then the p-value is not less than α $(p = 0.024 \nless 0.01)$ and H_0 would be not rejected. There is insufficient evidence to indicate bid price variance for the surrounding market exceeds the bid price variance for the tri-county market at $\alpha = 0.01$.

1.114 a. Using MINITAB, the output for comparing the mean level of family involvement in science homework assignments of TIPS and ATIPS students is:

Descriptive Statistics: SCIENCE

HWCOND	N	Mean	StDev	SE Mean
ATIPS	98	1.43	1.06	0.11
TIPS	128	2.55	1.27	0.11

Test

Null hypothesis	H_0: $\mu_1 - \mu_2 = 0$
Alternative hypothesis	H_1: $\mu_1 - \mu_2 \neq 0$

T-Value	DF	P-Value
-7.24	222	0.000

Let $\mu_1 =$ mean level of involvement in science homework assignments for ATIPS students and $\mu_2 =$ mean level of involvement in science homework assignments for TIPS students. To determine if the mean level of family involvement in science homework assignments of TIPS and ATIPS students differ, we test:

$$H_0: \mu_1 = \mu_2$$
$$H_a: \mu_1 \neq \mu_2$$

From the printout, the test statistic is $t = -7.24$ and the p-value is $p = 0.000$. Since the p-value is less than $\alpha\left(p = 0.000 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate there is a difference in the mean level of family involvement in science homework assignments between TIPS and ATIPS students at $\alpha = 0.05$.

b. Using MINITAB, the output for comparing the mean level of family involvement in mathematics homework assignments of TIPS and ATIPS students is:

Descriptive Statistics: MATH

HWCOND	N	Mean	StDev	SE Mean
ATIPS	98	1.48	1.22	0.12
TIPS	128	1.56	1.27	0.11

Test

Null hypothesis	H_0: $\mu_1 - \mu_2 = 0$
Alternative hypothesis	H_1: $\mu_1 - \mu_2 \neq 0$

T-Value	DF	P-Value
-0.50	212	0.620

Let $\mu_1 =$ mean level of involvement in mathematics homework assignments for ATIPS students and $\mu_2 =$ mean level of involvement in mathematics homework assignments for TIPS students. To determine if the mean level of family involvement in mathematics homework assignments of TIPS and ATIPS students differ, we test:

$$H_0: \mu_1 = \mu_2$$
$$H_a: \mu_1 \neq \mu_2$$

From the printout, the test statistic is $t = -0.50$ and the p-value is $p = 0.620$. Since the p-value is not less than $\alpha\left(p = 0.620 \not< 0.05\right)$, H_0 is not rejected. There is insufficient evidence to indicate a difference in the mean level of family involvement in mathematics homework assignments between TIPS and ATIPS students at $\alpha = .05$.

c. Using MINITAB, the output for comparing the mean level of family involvement in language arts homework assignments of TIPS and ATIPS students is:

Descriptive Statistics: LANGUAGE

HWCOND	N	Mean	StDev	SE Mean
ATIPS	98	1.01	1.09	0.11
TIPS	128	1.20	1.12	0.099

Test

Null hypothesis $H_0: \mu_1 - \mu_2 = 0$
Alternative hypothesis $H_1: \mu_1 - \mu_2 \neq 0$

T-Value	DF	P-Value
-1.25	211	0.212

Let $\mu_1 =$ mean level of involvement in language arts homework assignments for ATIPS students and $\mu_2 =$ mean level of involvement in language arts homework assignments for TIPS students. To determine if the mean level of family involvement in language arts homework assignments of TIPS and ATIPS students differ, we test:

$$H_0: \mu_1 = \mu_2$$
$$H_a: \mu_1 \neq \mu_2$$

From the printout, the test statistic is $t = -1.25$ and the p-value is $p = 0.212$. Since the p-value is not less than $\alpha \left(p = 0.212 \nless 0.05\right)$, H_0 is not rejected. There is insufficient evidence to indicate a difference in the mean level of family involvement in language arts homework assignments between TIPS and ATIPS students at $\alpha = .05$.

d. Since both sample sizes are greater than 30, the only assumption necessary is:

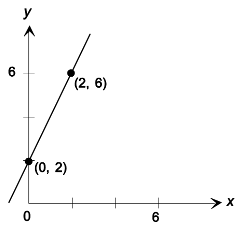
1. The samples are random and independent.

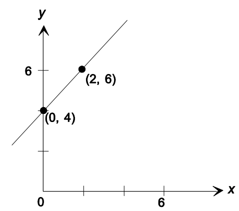
From the information given, there is no reason to dispute this assumption.

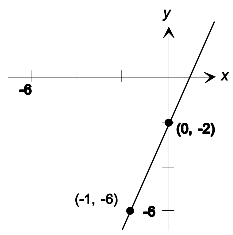
Chapter 3

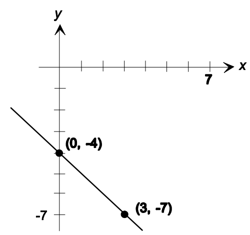
Simple Linear Regression

3.1

a. 

b. 

c. 

d. 

3.2 Since the line passes through the point (0, 1), $1=\beta_0+\beta_1(0)\Rightarrow\beta_0=1$.
Also, since it also passes through the point (2, 3),

$$3=\beta_0+\beta_1(2)\Rightarrow 3=1+2\beta_1\Rightarrow\beta_1=1\Rightarrow y=1+x$$

3.3 a. Using the technique explained in Exercise 3.2:

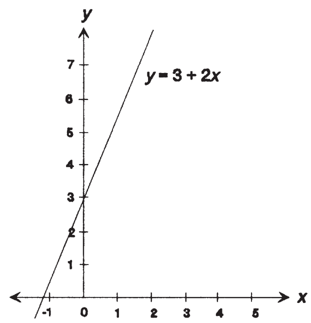
$$\left.\begin{array}{l}2=\beta_0+\beta_1(0)\\6=\beta_0+\beta_1(2)\end{array}\right\}\Rightarrow\left.\begin{array}{l}\beta_0=2\\\beta_1=2\end{array}\right\}\Rightarrow y=2+2x$$

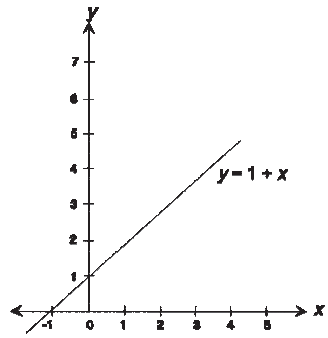
b.
$$\left.\begin{array}{l}4=\beta_0+\beta_1(0)\\6=\beta_0+\beta_1(2)\end{array}\right\}\Rightarrow\left.\begin{array}{l}\beta_0=4\\\beta_1=1\end{array}\right\}\Rightarrow y=4+x$$

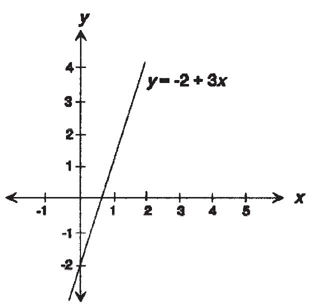
c.
$$\left.\begin{array}{l}-2=\beta_0+\beta_1(0)\\-6=\beta_0+\beta_1(-1)\end{array}\right\}\Rightarrow\left.\begin{array}{l}\beta_0=-2\\\beta_1=4\end{array}\right\}\Rightarrow y=-2+4x$$

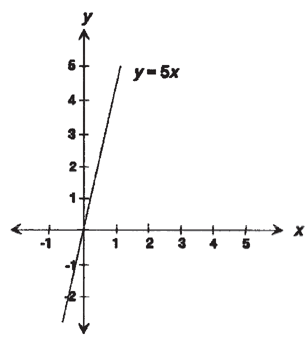
d.
$$\left.\begin{array}{l}-4=\beta_0+\beta_1(0)\\-7=\beta_0+\beta_1(3)\end{array}\right\}\Rightarrow\left.\begin{array}{l}\beta_0=-4\\\beta_1=-1\end{array}\right\}\Rightarrow y=-4-x$$

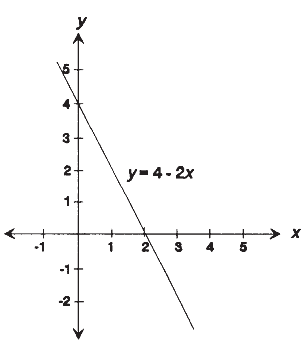
3.4

a. 

b. 

c. 

d. 

e. 

3.5

	Slope (β_1)	***y*-intercept** (β_0)
a.	2	3
b.	1	1
c.	3	−2
d.	5	0
e.	−2	4

3.6 Some preliminary calculations are:

$\sum x = 21 \qquad \sum x^2 = 91 \qquad \bar{x} = \frac{21}{6} = 3.5$

$\sum y = 18 \qquad \sum y^2 = 68 \qquad \bar{y} = \frac{18}{6} = 3 \qquad \sum xy = 78$

a. $SS_{xx} = \sum x^2 - n\bar{x}^2 = 91 - 6(3.5)^2 = 17.5$

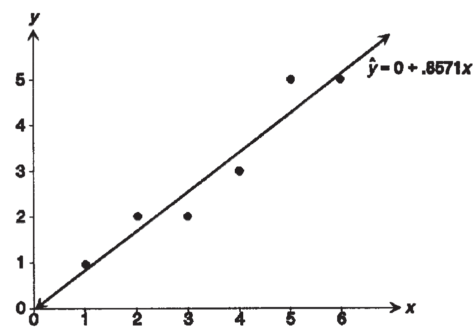
$SS_{xy} = \sum xy - n\bar{x}\bar{y} = 78 - 6(3.5)(3) = 15$

$SS_{yy} = \sum y^2 - n\bar{y}^2 = 68 - 6(3)^2 = 14$

$\hat{\beta}_1 = \frac{SS_{xy}}{SS_{xx}} = \frac{15}{17.5} = 0.8571$

$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1\bar{x} = 3 - (0.8571)(3.5) = 0$

b.



3.7 a. To compute $\hat{\beta}_0$ and $\hat{\beta}_1$, we first construct the following table:

x	y	xy	x^2	y^2
−2	4	−8	4	16
−1	3	−3	1	9
0	3	0	0	9
1	1	1	1	1
2	−1	−2	4	1
$\sum x = 0$	$\sum y = 10$	$\sum xy = -12$	$\sum x^2 = 10$	$\sum y^2 = 36$

Then,

$SS_{xx} = \sum x^2 - \frac{(\sum x)^2}{n} = 10 - \frac{(0)^2}{5} = 10$

$SS_{xy} = \sum xy - \frac{(\sum x)(\sum y)}{n} = -12 - \frac{0(10)}{5} = -12$

$$SS_{yy} = \sum y^2 - \frac{\left(\sum y\right)^2}{n} = 36 - \frac{(10)^2}{5} = 16$$

$$\overline{y} = \frac{\sum y}{n} = \frac{10}{5} = 2 \qquad \overline{x} = \frac{\sum x}{n} = \frac{0}{5} = 0$$

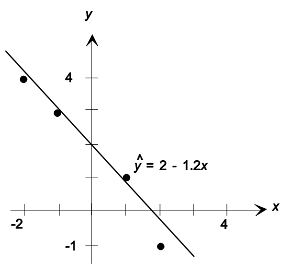
Thus, the least squares estimates of β_0 and β_1 are:

$$\hat{\beta}_1 = \frac{SS_{xy}}{SS_{xx}} = \frac{-12}{10} = -1.2$$

$$\hat{\beta}_0 = \overline{y} - \hat{\beta}_1\overline{x} = 2 - (-1.2)(0) = 2$$

and the equation of the least squares prediction line is $\hat{y} = 2 - 1.2x$.

b.



3.8 a. $y = \beta_0 + \beta_1 x + \varepsilon$

b. Yes, since the data appears to demonstrate a straight-line relationship.

c. $\text{Sales\_Price} = 1.4 + 1.41\,\text{Market\_Val}$

d. $\hat{\beta}_0 = 1.4$, when $x = 0$ (no market value), then the sales price has no practical meaning.

e. Various answers possible. A possible answer for the range on which the slope is $\$100{,}000 < x < \$1{,}000{,}000$.

f. "mean sale price" $= 1.4 + 1.41(\$300{,}000) \approx \$423{,}000$

3.9 a. Yes, there appears to be a positive linear trend. As the height above the horizon increases, the angular size tends to increase.

b & c. A sketch (answers can vary) of the line with lines drawn to the sketch line is:

The estimated deviations and squared deviations are:

ANGLE	HEIGHT	Est Fit	Dev	Sq Dev
321.9	17	322.2	-0.3	0.09
322.3	18	322.3	0.0	0.00
322.4	26	323.0	-0.6	0.36
323.2	32	323.4	-0.2	0.04
323.4	38	323.9	-0.5	0.25
324.4	42	324.2	0.2	0.04
325.0	49	324.8	0.2	0.04
325.7	52	325.0	0.7	0.49
325.8	57	325.4	0.4	0.16
325.0	60	325.7	-0.7	0.49
326.9	63	325.9	1.0	1.00
326.0	67	326.2	-0.2	0.04
325.8	73	326.7	-0.9	0.81
				3.81

The sum of the squared deviations is 3.81.

d. From the sketched line, the y-intercept is about 321 and the slope is about 0.1. These are close to the y-intercept, 320.636, and slope, 0.083, of the regression line.

e. From the printout, the SSE is 3.56465. The sum of squares from the estimated line is 3.81. The SSE from the regression line is smaller.

3.10 a. Using MINITAB, the results are:

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	677.45	677.45	24.41	0.003
VO2Max	1	677.45	677.45	24.41	0.003
Error	6	166.55	27.76		
Lack-of-Fit	5	142.05	28.41	1.16	0.604
Pure Error	1	24.50	24.50		
Total	7	844.00			

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	-27.2	19.8	-1.38	0.217	
VO2Max	0.558	0.113	4.94	0.003	1.00

Regression Equation

HR% = -27.2 + 0.558 VO2Max

The least squares line is $\hat{y} = -27.2 + 0.558x$.

b. Since 0 is not in the range of observed values of VO2Max, the y-intercept does not have a practical interpretation.

c. $\hat{\beta}_1 = 0.558$ For each unit increase in the value of VO2Max, the mean HR% is estimated to increase by 0.558.

3.11 a. No, there does not appear to any trend for cooperation use versus the average payoff.

b. No, there does not appear to any trend for defective use versus the average payoff.

c. Yes, there appears to be somewhat of a linear relationship for average payoff and punishment use.

d. Negative relationship; the more punishment use, the average payoff decreases.

e. Yes, winners tend to punish less than non-winners.

3.12 a. Using MINITAB, some calculations are:

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	6083.84	6083.84	26.35	0.000
Year	1	6083.84	6083.84	26.35	0.000
Error	10	2309.07	230.91		
Lack-of-Fit	9	2301.07	255.67	31.96	0.136
Pure Error	1	8.00	8.00		
Total	11	8392.92			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
15.1956	72.49%	69.74%	54.61%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	-3675	724	-5.08	0.000	
Year	1.870	0.364	5.13	0.000	1.00

Regression Equation

Cost = -3675 + 1.870 Year

The least squares line is $\hat{y} = -3675 + 1.870x$.

b. Since 0 is not in the range of observed values of Year, the y-intercept does not have a practical interpretation.

c. $\hat{\beta}_1 = 1.87$ For each unit increase in cost, the mean cost is estimated to increase by 1.87 million dollars.

3.13 a. Some preliminary calculations are:

$$\sum x = 6167 \qquad \sum y = 135.8 \qquad n = 24$$
$$\sum x^2 = 1{,}641{,}115 \qquad \sum y^2 = 769.72 \qquad \sum xy = 34{,}765$$

$$SS_{xy} = \sum xy - \frac{\left(\sum x\right)\left(\sum y\right)}{n} = 34{,}765 - \frac{(6167)(135.8)}{24} = -129.94167$$

$$SS_{xx} = \sum x^2 - \frac{\left(\sum x\right)^2}{n} = 164{,}115 - \frac{(6167)^2}{24} = 56{,}452.958$$

$$\hat{\beta}_1 = \frac{SS_{xy}}{SS_{xx}} = \frac{-129.94167}{56{,}452.958} = -0.002301769 \cong -0.0023$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1\bar{x} = \frac{135.8}{24} - (-0.002301769)\left(\frac{6167}{24}\right) = 6.249792065 \cong 6.251$$

The least squares line is $\hat{y} = 6.25 - 0.0023x$.

b. $\hat{\beta}_0 = 6.25$ Since $x = 0$ is not in the observed range, $\hat{\beta}_0$ has no interpretation other than being the y-intercept.

$\hat{\beta}_1 = -0.0023$. For each additional increase of 1 part per million of pectin, the mean sweetness index is estimated to decrease by 0.0023.

c. $\hat{y} = 6.25 - 0.0023(300) = 5.56.$

3.14 a. Using MINITAB, some preliminary results are:

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	9.08	9.080	0.18	0.676
CDIFF	1	9.08	9.080	0.18	0.676
Error	22	1116.78	50.763		
Lack-of-Fit	21	1026.06	48.860	0.54	0.813
Pure Error	1	90.72	90.720		
Total	23	1125.86			

Coefficients

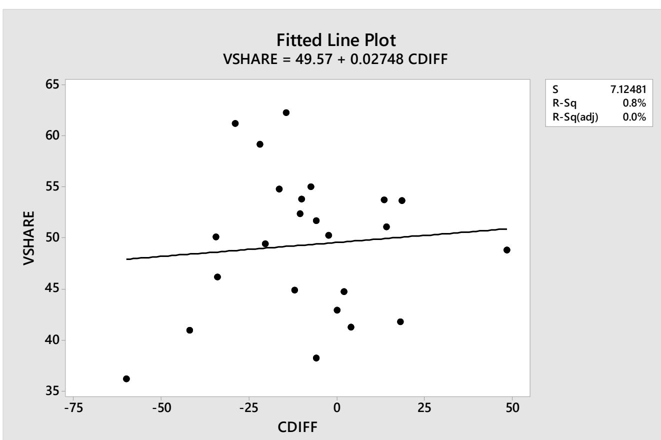
Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	49.57	1.56	31.76	0.000	
CDIFF	0.0275	0.0650	0.42	0.676	1.00

Regression Equation

VSHARE = 49.57 + 0.0275 CDIFF

The least squares line is $\hat{y} = 49.57 + 0.0275x.$

b. Using MINITAB, the scatterplot is:



There does not appear to be much of a linear relationship between Democratic vote share and charisma difference. There might be a slight positive linear trend.

c. $\hat{\beta}_1 = 0.0275$ For each unit increase in charisma difference, the mean Democratic vote share is estimated to increase by 0.0275 points.

3.15 Some preliminary calculations are:

$$\bar{y} = \frac{\sum x}{n} = \frac{103.07}{144} = 0.71576 \qquad \bar{x} = \frac{\sum y}{n} = \frac{792}{144} = 5.5$$

$$SS_{xy} = \sum xy - \frac{\sum x \sum y}{n} = 586.86 - \frac{792(103.07)}{144} = 19.975$$

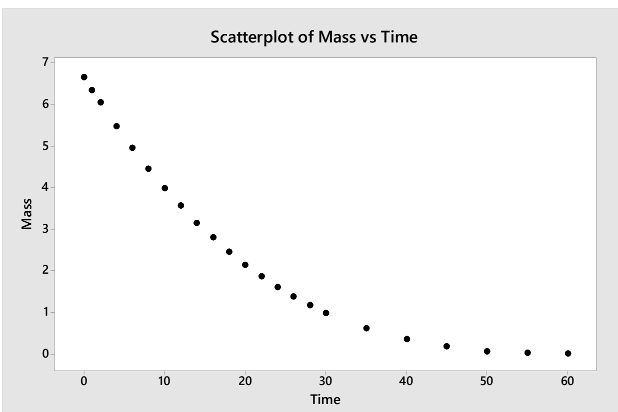
$$SS_{xx} = \sum x^2 - \frac{\left(\sum x\right)^2}{n} = 5{,}112 - \frac{792^2}{144} = 756$$

$$\hat{\beta}_1 = \frac{SS_{xy}}{SS_{xx}} = \frac{19.975}{756} = 0.026421957$$

$$\hat{\beta}_o = \overline{y} - \hat{\beta}_1 \overline{x} = \frac{103.07}{144} - (0.026421957)\left(\frac{792}{144}\right) = 0.570443121$$

The estimated regression line is $\hat{y} = 0.5704 + 0.0264x$. Since $x = 0$ is nonsensical, no practical interpretation of $\hat{\beta}_0 = 0.5704$. For each one-position increase in order, estimated recall proportion increases by $\hat{\beta}_1 = 0.0264$.

3.16 The scatterplot in this problem clearly shows a significantly *nonlinear* trend. Therefore, the linear model is not the best to describe the data in this scatter plot.



Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	89.79	89.7942	122.19	0.000
Time	1	89.79	89.7942	122.19	0.000
Error	21	15.43	0.7349		
Total	22	105.23			

Coefficients

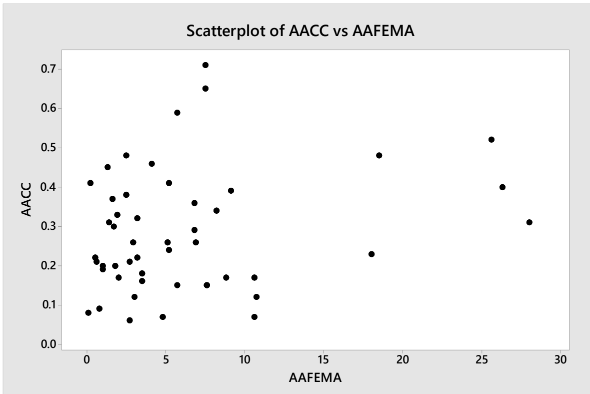
Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	5.221	0.296	17.64	0.000	
Time	-0.1140	0.0103	-11.05	0.000	1.00

Regression Equation

Mass = 5.221 - 0.1140 Time

The fitted regression line is $\hat{y} = 5.221 - 0.1140x$. Since the coefficient of time is negative, there is evidence that the mass of the spill tends to decrease as time increases. For each minute increase in time, the mean mass is estimated to diminish by 5.221 pounds.

3.17 a. Using MINITAB, the scatterplot of the data is:



There does not appear to be any apparent trend in the plot.

b. Using MINITAB, the results are:

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	0.06200	0.06200	2.79	0.102
AAFEMA	1	0.06200	0.06200	2.79	0.102
Error	48	1.06817	0.02225		
Lack-of-Fit	36	0.92617	0.02573	2.17	0.075
Pure Error	12	0.14200	0.01183		
Total	49	1.13016			

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	0.2489	0.0292	8.52	0.000	
AAFEMA	0.00542	0.00324	1.67	0.102	1.00

Regression Equation

AACC = 0.2489 + 0.00542 AAFEMA

The least squares line is $\hat{y} = 0.2489 + 0.00542x$.

The estimated y-intercept is $\hat{\beta}_0 = 0.2489$ and the estimated slope is $\hat{\beta}_1 = 0.00542$.

c. $\hat{\beta}_0 = 0.2489$ Since 0 is not in the observed range of the average annual FEMA relief, the y-intercept has no practical interpretation.
$\hat{\beta}_1 = 0.00542$ For each unit increase in the average annual FEMA relief, the mean average annual number of public corruption convictions is estimated to increase by 0.00542 per 100,000 residents.

3.18 a. $s^2 = \frac{SSE}{n-2} = \frac{0.219}{9-2} = 0.0313$

b. $s = \sqrt{0.0313} = 0.1769$

3.19 a. Using data from Exercise 3.6,

$SSE = SS_{yy} - \hat{\beta}_1 SS_{xy} = 14 - 0.8751(15) = 1.1435$

$s^2 = \frac{SSE}{n-2} = \frac{1.1435}{6-2} = 0.2859 \qquad s = \sqrt{0.2856} = 0.5347$

b. Using data from Exercise 3.7,

$SSE = SS_{yy} - \hat{\beta}_1 SS_{xy} = 16 - (-1.2)(-12) = 1.6$

$s^2 = \frac{SSE}{n-2} = \frac{1.6}{5-2} = 0.5333 \qquad s = \sqrt{0.5333} = 0.7303$

3.20 a. $s^2 = \frac{SSE}{n-2} = \frac{1.04}{28-2} = 0.04 \qquad s = \sqrt{0.04} = 0.2$

b. We would expect most of the observed value to fall within 2s or $2(0.2) = 0.4$ units of the least squares line.

3.21 a. $y = \beta_0 + \beta_1 x + \varepsilon$

b. The least squares line is $\hat{y} = 120 + 0.3456x$.

c. Assumption 1: The mean of the probability distribution of ε is 0.
Assumption 2: The variance of the probability distribution of ε is constant for all settings of the independent variable x.
Assumption 3: The probability distribution of ε is normal.
Assumption 4: The errors associated with any two different observations are independent.

d. $s = 635.187$

e. $\hat{y} \pm 2s \Rightarrow \hat{y} \pm 2(635.187) \Rightarrow \hat{y} \pm 1270.374$

3.22 a. From Exercise 3.12, $s = 15.1956$.

b. We would expect most of the observed values to fall within 2s or $2(15.1956) = 30.3912$ units of the least squares line.

3.23 a. Using calculations from Exercise 3.13,

$$SS_{yy} = \sum y^2 - \frac{\left(\sum y\right)^2}{n} = 769.72 - \frac{(135.8)^2}{24} = 1.318333$$

$$SSE = SS_{yy} - \hat{\beta}_1 SS_{xy} = 1.318333 - (-0.002301769)(-129.94167) = 1.01924$$

$$s^2 = \frac{SSE}{n-2} = \frac{1.01924}{24-2} = 0.0463 \qquad s = \sqrt{0.0463} = 0.2152$$

b. The units of measure for s^2 are square units. It is very difficult to interpret units such as dollars squared, minutes squared, etc.

c. We would expect most of the observed values to fall within 2s or $2(0.2152) = 0.4304$ units of the least squares line.

3.24 a. The estimate of σ^2 is $s^2 = \frac{SSE}{n-2} = \frac{1.06817}{50-2} = 0.02225$.

b. The estimate of σ is $s = \sqrt{0.02225} = 0.1492$.

c. The estimate of σ can be interpreted practically because it is measured in the same units as the data. The units of measure of σ^2 are square units.

d. We would expect most of the observed values to fall within 2s or $2(0.1492) = 0.2984$ units of the least squares line. In this problem, the units of measure is dollars per capita. However, looking at the scatterplot, the data do not fall close to a straight line. The model will not be very accurate in predicting a state's average annual number of public corruption convictions.

3.25 a. The least squares line with the steepest slope is with the pair AB Magnitude Alert and AB Magnitude No-Tone.

b. The least squares line that produces the largest SSE is with the pair AB Magnitude Alert and AB Magnitude No-Tone.

c. The least squares line that produces the smallest estimate of σ is with the pair AB Magnitude Sim and AB Magnitude Alert.

3.26 a. To determine if β_1 differs from 0, we test:

$$H_0: \beta_1 = 0$$
$$H_a: \beta_1 \neq 0$$

The test statistic is $t = \dfrac{\hat{\beta}_1}{s/\sqrt{SS_{xx}}} = \dfrac{0.8571}{0.5345/\sqrt{17.5}} = 6.71$

The rejection region requires $\alpha / 2 = 0.05 / 2 = 0.025$ in each tail of the t distribution. From Table 2, Appendix D, with $df = n - 2 = 6 - 2 = 4$, $t_{0.025} = 2.776$. The rejection region is $t < -2.776$ or $t > 2.776$.

Since the observed value of the test statistic falls in the rejection region $(t = 6.71 > 2.776)$, H_0 is rejected. There is sufficient evidence to indicate that x contributes information for the prediction of y using a linear model at $\alpha = .05$.

b. To determine if β_1 differs from 0, we test:

$$H_0: \beta_1 = 0$$
$$H_a: \beta_1 \neq 0$$

The test statistic is $t = \dfrac{\hat{\beta}_1}{s/\sqrt{SS_{xx}}} = \dfrac{-1.2}{0.7303/\sqrt{10}} = -5.20$

The rejection region requires $\alpha / 2 = 0.05 / 2 = 0.025$ in each tail of the t distribution. From Table 2, Appendix D, with $df = n - 2 = 5 - 2 = 3$, $t_{0.025} = 3.182$. The rejection region is $t < -3.182$ or $t > 3.182$.

Since the observed value of the test statistic falls in the rejection region $(t = -5.20 < -3.182)$, H_0 is rejected. There is sufficient evidence to indicate that x contributes information for the prediction of y using a linear model at $\alpha = .05$.

3.27 a. To determine if there is a positive linear relationship between appraised property value and sale price, we test:

$$H_0: \beta_1 = 0$$
$$H_a: \beta_1 > 0$$

From the printout, the test statistic is $t = 38.132$ and the p-value is $p = 0.000 / 2 = 0.000$. Since the p-value is less than $\alpha\,(p = 0.000 < 0.01)$, H_0 is rejected. There is sufficient evidence to indicate there is a positive linear relationship between appraised property value and sale price at $\alpha = 0.01$.

b. From the printout, the 95% confidence interval is $(1.335, 1.482)$. We are 95% confident that for each \$1000 increase in market value, the mean sale price is estimated to increase by from \$1,335 to \$1,482.

c. In order to obtain a narrower confidence interval, one could lower the confidence level (i.e. to 90%) or increase the sample size.

3.28 Some preliminary calculations are:

$$SS_{yy} = \sum y^2 - \frac{\left(\sum y\right)^2}{n} = 769.72 - \frac{(135.8)^2}{24} = 1.3183333$$

$$SSE = SS_{yy} - \hat{\beta}_1 SS_{xy} = 1.3183333 - (-0.002301796)(-129.94167) = 1.019237592$$

$$s^2 = \frac{SSE}{n-2} = \frac{1.019237592}{22} = 0.046329$$

$$s_{\hat{\beta}_1} = \sqrt{\frac{s^2}{SS_{xx}}} = \sqrt{\frac{0.046329}{56452.958}} = 0.000906$$

For confidence level 0.90, $\alpha = 0.10$ and $\alpha / 2 = 0.10 / 2 = 0.05$. From Table 2, Appendix D with $df = n - 2 = 24 - 2 = 22$, $t_{0.05} = 1.717$.

The confidence interval is:

$$\hat{\beta}_1 \pm t_{0.05} s_{\hat{\beta}_1} \Rightarrow -0.0023 \pm 1.717(0.000906) \Rightarrow (-0.0039, -0.0008)$$

We are 90% confident that the change in the mean sweetness index for each one unit change in the pectin is between −0.0039 and −0.0007.

3.29 a. The equation for the simple linear regression is $y = \beta_0 + \beta_1 x + \varepsilon$.

b. The value of β_0 is probably irrelevant. By definition, β_0 is the mean value of entitlement score for those whose helicopter parent score is 0. We would expect β_1 to be positive. As the helicopter parent score increases, the entitlement score increases.

c. Since the *p*-value is less than $\alpha \left(p = 0.002 < 0.01 \right)$, H_0 is rejected. There is sufficient evidence to indicate there is a positive linear relationship between entitlement scores and helicopter parent score at $\alpha = 0.01$.

3.30 For confidence level 0.95, $\alpha = 0.05$ and $\alpha / 2 = 0.05 / 2 = 0.025$. From Table 2, Appendix D with $df = n - 2 = 50 - 2 = 48$, $t_{0.025} \approx 2.021$. The confidence interval is:

$$\hat{\beta}_1 \pm t_{0.025} s_{\hat{\beta}_1} \Rightarrow 0.00542 \pm 2.021(0.00324) \Rightarrow (-0.0011, 0.0120)$$

We are 95% confident that the increase in the mean state's average annual number of public corruption convictions is between -0.0011 and 0.0120 for each unit increase in the state's average annual FEMA relief.

3.31 a. The equation for the simple linear regression is $y = \beta_0 + \beta_1 x + \varepsilon$.

b. The y-intercept does not have any meaning because 0 cannot be in the range of observed beauty index.

c. For each unit increase in the beauty index, the mean relative success is estimated to increase by 22.91 points.

d. To determine if the slope of the line is positive, we test:

$$H_0 : \beta_1 = 0$$
$$H_a : \beta_1 > 0$$

The test statistic is $t = \dfrac{\hat{\beta}_1}{s_{\hat{\beta}_1}} = \dfrac{22.91}{3.73} = 6.14$.

The rejection region requires $\alpha = 0.01$ in the upper tail of the t distribution. From Table 2, Appendix D, with $df = n - 2 = 641 - 2 = 639$, $t_{0.01} = 2.326$. The rejection region is $t > 2.326$.

Since the observed value of the test statistic falls in the rejection region $(t = 6.14 > 2.326)$, H_0 is rejected. There is sufficient evidence to indicate the slope of the line is positive at $\alpha = 0.01$. There is evidence to indicate that as the beauty index increases, the relative success also increases.

3.32 To determine if the simple linear regression model is useful for predicting Democratic vote share, we test:

$$H_0 : \beta_1 = 0$$
$$H_a : \beta_1 \neq 0$$

The test statistic is $t = \dfrac{\hat{\beta}_1}{s_{\hat{\beta}_1}} = \dfrac{0.0275}{0.0650} = 0.42$ and the p-value is $p = 0.676$. (From Exercise 3.14)

Since the p-value is not less than $\alpha\,(p = 0.676 \nless 0.10)$, H_0 is not rejected. There is insufficient evidence to indicate the simple linear regression model is useful for predicting Democratic vote share at $\alpha = 0.10$.

3.33 Using the calculations from Exercise 3.15 and these calculations:

$$SS_{yy} = \sum y^2 - \frac{\left(\sum y\right)^2}{n} = 83.474 - \frac{103.07^2}{144} = 9.70021597$$

$$SSE = SS_{yy} - \hat{\beta}_1 \left(SS_{xy}\right) = 9.70021597 - (0.026421957)(19.975) = 9.172437366$$
$$s^2 = \frac{SSE}{n-2} = \frac{9.172437366}{144-2} = 0.064594629$$
$$s = \sqrt{s^2} = \sqrt{0.064594629} = 0.254154735$$

To determine if there is a linear trend between the proportion of names recalled and position, we test:

$$H_0 : \beta_1 = 0$$
$$H_a : \beta_1 \neq 0$$

The test statistic is $t = \dfrac{\hat{\beta}_1 - 0}{s_{\hat{\beta}_1}} = \dfrac{\hat{\beta}_1}{s/\sqrt{SS_{xx}}} = \dfrac{0.02642 - 0}{0.25415/\sqrt{756}} = 2.86$

The rejection region requires $\alpha / 2 = 0.01/2 = 0.005$ in each tail of the t distribution. From Table 2, Appendix D, with $df = n - 2 = 144 - 2 = 142$, $t_{0.005} \approx 2.576$. The rejection region is $t < -2.576$ or $t > 2.576$..

Since the observed test statistic falls in the rejection region $(t = 2.86 > 2.576)$, H_0 is rejected. There is sufficient evidence to indicate the proportion of names recalled is linearly related to position at $\alpha = .01$.

3.34 a. To determine if the spill mass tends to diminish linearly as time increases, we test:

$$H_0 : \beta_1 = 0$$
$$H_a : \beta_1 < 0$$

Using information from Exercise 3.16, the test statistic is $t = -11.05$ and the p-value is $p = 0.000/2 = 0.000$. Since the p-value is less than $\alpha\left(p = 0.000 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate the spill mass tends to diminish linearly as time increases at $\alpha = 0.05$.

b. Using MINTAB, the 95% confidence intervals are:

Fits and Diagnostics for All Observations

Obs	Time	Mass	Fit	SE Fit	95% CI
1	0	6.640	5.221	0.296	(4.605, 5.836)
2	1	6.340	5.107	0.288	(4.508, 5.705)
3	2	6.040	4.993	0.280	(4.411, 5.575)
4	4	5.470	4.765	0.264	(4.215, 5.314)
5	6	4.940	4.537	0.249	(4.018, 5.055)
6	8	4.440	4.309	0.236	(3.819, 4.798)
7	10	3.980	4.080	0.223	(3.617, 4.544)
8	12	3.550	3.852	0.211	(3.414, 4.291)
9	14	3.150	3.624	0.201	(3.207, 4.042)
10	16	2.790	3.396	0.192	(2.996, 3.796)
11	18	2.450	3.168	0.186	(2.782, 3.554)

12	20	2.140	2.940	0.181	(2.563, 3.317)
13	22	1.860	2.712	0.179	(2.340, 3.084)
14	24	1.600	2.484	0.179	(2.112, 2.857)
15	26	1.370	2.256	0.182	(1.878, 2.634)
16	28	1.170	2.028	0.186	(1.640, 2.416)
17	30	0.980	1.800	0.193	(1.398, 2.202)
18	35	0.600	1.230	0.218	(0.776, 1.684)
19	40	0.340	0.660	0.251	(0.137, 1.182)
20	45	0.170	0.090	0.290	(-0.513, 0.693)
21	50	0.060	-0.480	0.332	(-1.171, 0.210)
22	55	0.020	-1.051	0.377	(-1.834, -0.267)
23	60	0.000	-1.621	0.423	(-2.500, -0.742)

3.35 a. For each 1% increase in the *ln*(body mass), the mean *ln*(eye mass) is estimated to increase by anywhere from 0.25 to 0.30.

b. For each 1% increase in the *ln*(body mass), the mean *ln*(orbit axis angle) is estimated to decrease by anywhere from 0.14 to 0.50.

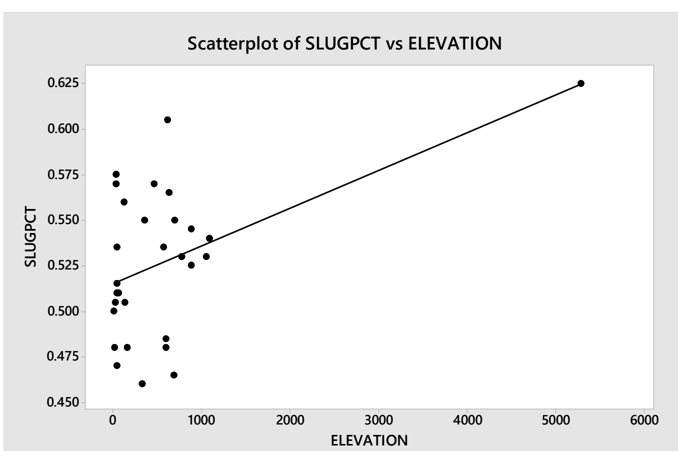
3.36 a. $\hat{\beta}_0 = 0.5151 \qquad \hat{\beta}_1 = 0.000021$

b. To determine if there is a positive linear relationship between elevation and slugging percentage, we test:

$$H_0: \beta_1 = 0$$
$$H_a: \beta_1 > 0$$

From the printout, the test statistic is $t = 2.89$ and the *p*-value is $p = 0.008/2 = 0.004$. Since the *p*-value is less than $\alpha\left(p = 0.004 < 0.01\right)$, H_0 is rejected. There is sufficient evidence to indicate there is a positive linear relationship between elevation and slugging percentage at $\alpha = 0.01$.

c. Using MINITAB, the scatterplot is:



Denver's elevation is much greater than all the others. In addition, if the observation for Denver is deleted, there does not appear to be much of a relationship between elevation and slugging percentage.

d. Using MINITAB, the results are:

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	0.001389	0.001389	0.98	0.332
ELEVATION	1	0.001389	0.001389	0.98	0.332
Error	26	0.036922	0.001420		
Lack-of-Fit	22	0.036685	0.001667	28.08	0.003
Pure Error	4	0.000238	0.000059		
Total	27	0.038311			

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	0.5154	0.0107	48.33	0.000	
ELEVATION	0.000020	0.000020	0.99	0.332	1.00

Regression Equation

SLUGPCT = 0.5154 + 0.000020 ELEVATION

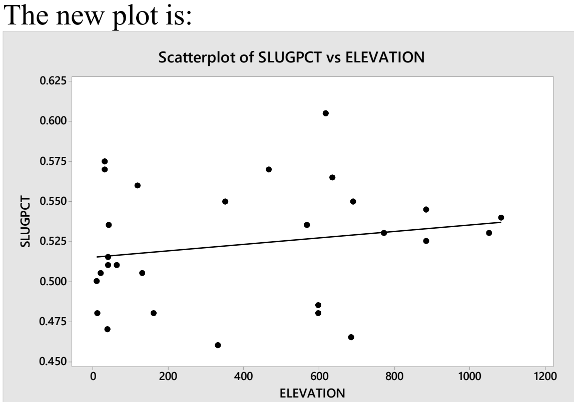
$\hat{\beta}_0 = 0.5154$ $\hat{\beta}_1 = 0.000020$

To determine if there is a positive linear relationship between elevation and slugging percentage, we test:

$$H_0: \beta_1 = 0$$
$$H_a: \beta_1 > 0$$

From the printout, the test statistic is $t = 0.99$ and the *p*-value is $p = 0.332/2 = 0.166$. Since the *p*-value is not less than $\alpha\left(p = 0.166 \not< 0.01\right)$, H_0 is not rejected. There is insufficient evidence to indicate there is a positive linear relationship between elevation and slugging percentage at $\alpha = 0.01$.

The new plot is:



3.37 a. Years of education and yearly income

b. Number of hours playing video games and GPA

3.38 a. If $r = 0.7$, , there is a positive linear relationship between x and y. As x increases, y tends to increase. The slope is positive.

b. If $r = -0.7$, there is a negative linear relationship between x and y. As x increases, y tends to decrease. The slope is negative.

c. If $r = 0$, , there is a 0 slope. There is no linear relationship between x and y.

d. If $r^2 = 0.64$, then r is either 0.8 or -0.8. The linear relationship between x and y could be either positive or negative.

3.39 a. From Exercise 3.6, $SS_{xx} = 17.5$, $SS_{yy} = 14$ and $SS_{xy} = 15$

$$r = \frac{SS_{xy}}{\sqrt{SS_{xx}SS_{yy}}} = \frac{15}{\sqrt{(17.5)(14)}} = 0.9583$$

From Exercise 3.19, $SSE = 1.1435$

$$r^2 = \frac{SS_{yy} - SSE}{SS_{yy}} = \frac{14 - 1.1435}{14} = 0.9183.$$

There is a strong positive correlation between x and y.
We can explain 91.83% of the variation in the sample y's using the linear model with x.

b. In Exercise 3.7, $SS_{xx} = 10$, $SS_{yy} = 16$ and $SS_{xy} = -12$

$$r = \frac{SS_{xy}}{\sqrt{SS_{xx}SS_{yy}}} = \frac{-12}{\sqrt{10(16)}} = -0.9487.$$

In Exercise 3.7, $SSE = SS_{yy} - \hat{\beta}_1 SS_{xy} = 16 - (-1.2)(-12) = 1.6.$

$$r^2 = \frac{SS_{yy} - SSE}{SS_{yy}} = \frac{16 - 1.6}{16} = 0.90.$$

There is a strong positive linear correlation between x and y.
We can explain 90% of the variation in the sample y's using the linear model with x.

3.40 We would expect the crime rate to increase as U.S. population increases. Therefore, we expect a positive correlation between the variables.

3.41 We would expect the GPA of a college student to be correlated to his/her I.Q. As the I.Q. score increases, we would expect the GPA to increase. Thus, the correlation would be positive.

3.42 a. $r = 0.975$. There is a very strong linear relationship between the sale price of a house and the appraised property market value.

b. $r^2 = 0.9516$. 95.16% of the sample home sale prices is explained by the linear relationship between the appraised value of the house and the final market price.

3.43 a. $r^2 = 0.18$. 18% of the sample number of points scored is explained by the linear relationship between the number of points scored and the number of yards from the opposing goal line.

b. $r = -\sqrt{0.18} = -0.424$. The value of r is negative because the coefficient associated with the number of yards from the opposing goal line in the fitted regression line is negative.

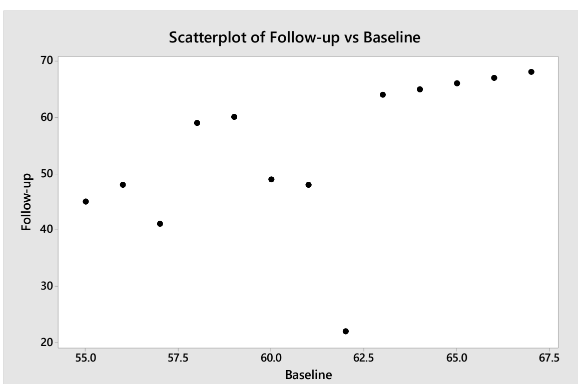
3.44 a. Since the p-value of 0.33 is greater than $\alpha = 0.05$, we cannot conclude that there is a significant linear relationship between cooperation use and average payoff.

b. Since the p-value of 0.66 is greater than $\alpha = 0.05$, we cannot conclude that there is a significant linear relationship between defection use and average payoff.

c. Since the p-value of 0.001 is smaller than $\alpha = 0.05$, we can conclude that there is a significant linear relationship between punishment use and average payoff.

3.45 a. Since the p-value of 0.07 is greater than $\alpha = 0.05$, we cannot conclude that there is a significant linear relationship between baseline and follow-up physical activity for obese young adults; fail to reject $H_0 : \rho = 0$ at $\alpha = .05$.

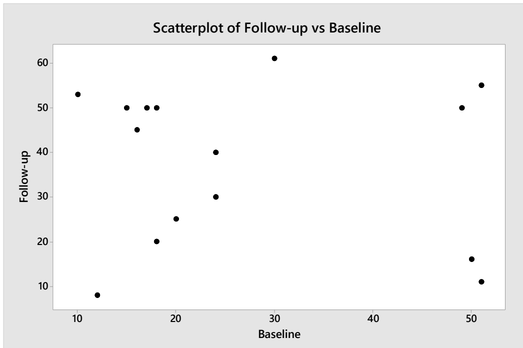
b. A possible scatterplot of the data would be:



c. $r^2 = (.50)^2 = 0.25$, thus 25% of the variability around the sample mean for the total of follow-up number of movements is explained by the linear relationship between the baseline total number of movements for the obese adults and the follow-up total number of movements for the obese adults.

d. Since the correlation value itself is close to zero and the p-value of 0.66 is greater than $\alpha = 0.05$, we cannot conclude that there is a significant linear relationship between baseline and follow-up physical activity for normal weight young adults; fail to reject $H_0 : \rho = 0$ at $\alpha = .05$.

e. A possible scatterplot is:



f. $r^2 = (-.12)^2 = 0.0144$. Thus 1.44% of the variability around the sample mean for the total of follow-up number of movements is explained by the linear relationship between the baseline total number of movements for the normal weight young adults and the total of follow-up number of movements for the normal weight young adults.

3.46 In Exercise 3.13, $SS_{xx} = 56,452.958$ and $SS_{xy} = -129.94167$

$$SS_{yy} = \sum y^2 - \frac{(\sum y)^2}{n} = 769.72 - \frac{135.8^2}{24} = 1.318333$$

$$r = \frac{SS_{xy}}{\sqrt{SS_{xx}SS_{yy}}} = \frac{-129.94167}{\sqrt{56,452.958(1.318333)}} = -0.4763.$$

$$SSE = SS_{yy} - \hat{\beta}_1 SS_{xy} = 1.318333 - (-0.002301769)(-129.94167) = 1.01924.$$

$$r^2 = \frac{SS_{yy} - SSE}{SS_{yy}} = \frac{1.318333 - 1.01924}{1.318333} = 0.2269.$$

22.69% of the variability around the sample mean for the sweetness index can be explained by the linear relationship between the sweetness index and the amount of water-soluble pectin.

3.47 a. There is a rather weak negative linear relationship between the numerical value of a last name and the response time.

b. Since the *p*-value is less than $\alpha\ (p = 0.018 < 0.05)$, H_0 is rejected. There is sufficient evidence to indicate a negative linear relationship between the numerical value of a last name and the response time.

c. Yes, the analysis supports the researchers' *last name effect* theory. Because the correlation coefficient is negative, as the numerical value of the last name increases, the response time tends to decrease.

3.48 Using the values computed in Exercise 3.15:

$$r = \frac{SS_{xy}}{\sqrt{SS_{xx}SS_{yy}}} = \frac{19.975}{\sqrt{756(9.70031597)}} = 0.2333$$

Because *r* is fairly close to 0, there is a very weak positive linear relationship between the proportion of names recalled and position.

$$r^2 = 0.2333^2 = 0.0544$$

5.44% of the sample variance of proportion of names recalled around the sample mean is explained by the linear relationship between proportion of names recalled and position.

3.49 a. To determine if the true population correlation coefficient relating NRMSE and bias is positive, we test:

$$H_0 : \rho = 0$$
$$H_a : \rho > 0$$

The test statistic is $t = \dfrac{r}{\sqrt{\dfrac{1-r^2}{n-2}}} = \dfrac{0.2838}{\sqrt{\dfrac{1-0.2838^2}{3,600-2}}} = 17.753.$

No α value was given, so we will use $\alpha = 0.5$. The rejection region requires $\alpha = 0.5$ in the upper tail of the *t* distribution. From Table 2, Appendix D, with $df = n-2 = 3,600-2 = 3598,\ t_{0.05} = 1.645$. The rejection region is $t > 1.645$.

Since the observed value of the test statistic falls in the rejection region $(t = 17.753 > 1.645)$, H_0 is rejected. There is sufficient evidence to indicate the true population correlation coefficient relating NRMSE and bias is positive at $\alpha = 0.5$.

b. No, we would not recommend using NRMSE as a linear predictor of bias. The estimated correlation coefficient is $r = 0.2838$. This indicates that there is a rather weak positive linear relationship between NRMSE and bias. The sample size was extremely large. The larger the sample size, the easier it is to find statistical significance. In this case, there is statistical significance, but not practical significance.

3.50 a. The sample correlation coefficient between PSI and PHI-F is $r = 0.401$. There is a weak positive linear relationship between the perceived sensory intensity and the perceived hedonic intensity for favorite food.

The sample correlation coefficient between PSI and PHI-L is $r = -0.375$. There is a weak negative linear relationship between the perceived sensory intensity and the perceived hedonic intensity for least favorite food.

b. Yes, we agree that those with the greatest taste intensity tend to experience more extreme food likes and dislikes. As the taste intensity increases, the intensity of favorite foods tends to increase. As the taste intensity increases, the intensity of least favorite foods tends to decrease.

3.51 a. $r^2 = 0.948$. 94.8% of the variability around the mean *ln*(eye mass) is explained by the linear relationship between *ln*(eye mass) and *ln*(body mass).

b. From 3.35a, the relationship between *ln* (eye mass) and *ln* (body mass) is positive. Therefore, $r = \sqrt{0.948} = 0.974$. There is a very strong positive linear relationship between *ln* (eye mass) and *ln* (body mass).

c. $r^2 = .375$. 37.5% of the variability around the mean *ln*(orbit axis angle) is explained by the linear relationship between *ln*(orbit axis angle) and *ln*(body mass).

d. From 3.35b, the relationship between *ln*(orbit axis angle) and *ln*(body mass) is negative. Therefore, $r = -\sqrt{0.375} = -0.612$. There is a moderate negative linear relationship between *ln*(orbit axis angle) and *ln*(body mass).

3.52 a. First, examine the formulas for the confidence interval and the prediction interval. The only difference is that the prediction interval has an extra term (a "1") beneath the radical. Thus, the prediction interval must be wider:

$$\sqrt{\frac{1}{n} + \frac{\left(x_p - \bar{x}\right)^2}{SS_{xx}}} < \sqrt{1 + \frac{1}{n} + \frac{\left(x_p - \bar{x}\right)^2}{SS_{xx}}}$$

The error in estimating the mean value of y, $E(y)$, for a given value of x, say x_p, is the distance between the least squares line, $\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$, and the true line of means, $E(y) = \beta_0 + \beta_1 x$. In contrast, the error in predicting some future of y, $\left(\hat{y} - y_p\right)$ is the sum of two errors: the error of estimating the mean of y, $E(y)$, plus the random error of the actual values of y around its mean. Consequently, the error of predicting a particular value of y will be larger than the error of estimating the mean value of y for a particular value of x.

b. Since the standard error contains the term $\frac{\left(x_p - \bar{x}\right)^2}{SS_{xx}}$, the further x_p is from $\bar{x}$, the larger the standard error. This causes the confidence intervals to be wider for values of x_p further from $\bar{x}$. The implication is our best confidence intervals (narrowest) will be found when $x_p = \bar{x}$.

3.53 a. $\hat{\beta}_1 = \frac{SS_{xy}}{SS_{xx}} = \frac{16.22}{4.77} = 3.400$

$SSE = SS_{yy} - \hat{\beta}_1 SS_{xy} = 59.21 - 3.4(16.22) = 4.062$

$s^2 = \frac{SSE}{n-2} = \frac{4.062}{20-2} = 0.226.$

b. For $x = 2.5$, $\hat{y} = 2.1 + 3.4(2.5) = 10.6$

The form of the 95% confidence interval is $\hat{y} \pm t_{\alpha/2} s \sqrt{\frac{1}{n} + \frac{(x - \bar{x})^2}{SS_{xx}}}$.

For confidence coefficient 0.95, $\alpha = 0.05$ and $\alpha / 2 = 0.05 / 2 = 0.025$. From Table 2, Appendix D, with $df = n - 2 = 20 - 2 = 18$, $t_{0.025} = 2.101$.
The 95% confidence interval is:

$$10.6 \pm 2.101\sqrt{0.226}\sqrt{\frac{1}{20} + \frac{(2.5 - 2.5)^2}{4.77}} \Rightarrow 10.6 \pm 0.223 \Rightarrow (10.377, 10.823)$$

We are 95% confident the mean value of y when $x = 2.5$ is between 10.377 and 10.823.

c. For $x = 2.0, \hat{y} = 2.1 + 3.4(2.0) = 8.9$.

The 95% confidence interval is:

$$8.9 \pm 2.101\sqrt{0.226}\sqrt{\frac{1}{20} + \frac{(2.0 - 2.5)^2}{4.77}} \Rightarrow 8.9 \pm 0.320 \Rightarrow (8.580, 9.220)$$

We are 95% confident the mean value of y when $x = 2.0$ is between 8.580 and 9.220.

d. For $x = 3.0$, $\hat{y} = 2.1 + 3.4(3.0) = 12.3$.

The 95% confidence interval is:

$$12.3 \pm 2.101\sqrt{0.226}\sqrt{\frac{1}{20} + \frac{(3.0 - 2.5)^2}{4.77}} \Rightarrow 12.3 \pm 0.320 \Rightarrow (11.980, 12.620)$$

We are 95% confident the mean value of y when $x = 3.0$ is between 11.980 and 12.620.

e. The width of the interval in (b) is $10.823 - 10.377 = 0.446$.
The width of the interval in (c) is $9.220 - 8.580 = 0.640$.
The width of the interval in (d) is $12.620 - 11.980 = 0.640$.

As the value of x moves away from $\bar{x} = 2.5$, the confidence interval gets wider.

f. The 95% prediction interval is $\hat{y} \pm t_{\alpha/2} s \sqrt{1 + \frac{1}{n} + \frac{(x - \bar{x})^2}{SS_{xx}}}$.

$$12.3 \pm 2.101\sqrt{0.226}\sqrt{1 + \frac{1}{20} + \frac{(3.0 - 2.5)^2}{4.77}} \Rightarrow 12.3 \pm 1.049 \Rightarrow (11.251, 13.349).$$

We are 95% confident that the actual value of y will be between 11.251 and 13.349 when the value of x is 3.

3.54 a. No. We know there is a significant linear relationship between sale price and appraised value. However, the actual sale prices may be scattered quite far from the predicted line.

b. From the printout, the 95% prediction interval for the actual sale price when the appraised value is \$300,000 is $(285.938,\ 561.741)$ or $(\$285{,}938,\ \$561{,}741)$. We are 95% confident that the actual sale price for a home appraised at \$300,000 is between \$285,938 and \$561,741.

c. From the printout, the 95% confidence interval for the mean sale price when the appraised value is \$300,000 is $(408.119,\ 439.560)$ or $(\$408{,}119,\ \$439{,}560)$. We are 95% confident that the mean sale price for a home appraised at \$300,000 is between \$408,119 and \$439,560.

3.55 a. Researchers should use a prediction interval for y with

$$x=10 \Rightarrow \hat{y} \pm t_{\alpha/2} s \sqrt{1+\frac{1}{n}+\frac{\left(x_p-\bar{x}\right)^2}{SS_{xx}}} \Rightarrow \hat{y} \pm t_{\alpha/2} s \sqrt{1+\frac{1}{n}+\frac{\left(10-\bar{x}\right)^2}{SS_{xx}}}.$$

b. Researchers should use a confidence interval for the mean value of y or $E(y)$, with

$$x=10 \Rightarrow \hat{y} \pm t_{\alpha/2} s \sqrt{\frac{1}{n}+\frac{\left(x_p-\bar{x}\right)^2}{SS_{xx}}} \Rightarrow \hat{y} \pm t_{\alpha/2} s \sqrt{\frac{1}{n}+\frac{\left(10-\bar{x}\right)^2}{SS_{xx}}}.$$

3.56 a. We are 95% confident that the actual value of the angular size of the Moon is between 323.502 and 326.108 when the height above the horizon is 50 degrees.

b. We are 95% confident that the mean value of the angular size of the Moon is between 324.448 and 325.163 when the height above the horizon is 50 degrees.

c. No, we would not recommend using the least squares line to predict the angular size of the Moon for a height of 80 degrees because 80 degrees is outside the observed range of data used to construct the least squares line.

3.57 For $x=300$, the confidence interval for $E(y)$ is $(5.45812,\ 5.65964)$. We are 90% confident that the mean sweetness index is between 5.458 and 5.660 when the amount of pectin is 300.

3.58 a. From Exercises 3.15 and 3.33, $\bar{x}=5.5$, $SS_{xx}=756$, $s=0.25415$, and $\hat{y}=0.5704+0.0264x$.
For $x=5$, $\hat{y}=0.5704+0.0264(5)=0.7024$.
For confidence coefficient 0.99, $\alpha=0.01$ and $\alpha/2=0.01/2=0.005$. From Table 2, Appendix D, with $df=n-2=144-2=142$, $t_{0.005}\approx 2.576$. The 99% confidence interval is:

$$\hat{y} \pm t_{\alpha/2} s \sqrt{\frac{1}{n}+\frac{\left(x_p-\bar{x}\right)^2}{SS_{xx}}} \Rightarrow 0.7024 \pm 2.576(0.2542)\sqrt{\frac{1}{144}+\frac{(5-5.5)^2}{756}}$$

$\Rightarrow 0.7024 \pm 0.0559 \Rightarrow (0.6465,\ 0.7583)$

We are 99% confident that the mean recall of all those in the 5th position is between 0.6465 and 0.7583.

b. For confidence coefficient 0.99, $\alpha = 0.01$ and $\alpha/2 = 0.01/2 = 0.005$. From Table 2, Appendix D, with $df = n-2 = 144-2 = 142$, $t_{0.005} \approx 2.576$. The 99% prediction interval is:

$$\hat{y} \pm t_{\alpha/2} s \sqrt{1 + \frac{1}{n} + \frac{(x_p - \bar{x})^2}{SS_{xx}}} \Rightarrow 0.7024 \pm 2.576(0.2542)\sqrt{1 + \frac{1}{144} + \frac{(5-5.5)^2}{756}}$$

$\Rightarrow 0.7024 \pm 0.6572 \Rightarrow (0.0452,\ 1.3596)$

We are 99% confident that the actual recall of a person in the 5th position is between 0.0452 and 1.3596. Since the proportion of names recalled cannot be larger than 1, the actual proportion recalled will be between 0.0452 and 1.000.

c. The prediction interval in part b is wider than the confidence interval in part a. The prediction interval will always be wider than the confidence interval. The confidence interval for the mean is an interval for predicting the mean of all observations for a particular value of *x*. The prediction interval is a confidence interval for the actual value of the dependent variable for a particular value of *x*.

3.59 a. From Exercises 3.16 and 3.34, $\bar{x} = 22.87$, $SS_{xx} = 6906.608$, $s = 0.8573$, and $\hat{y} = 5.22 - 0.114x$.

For $x = 15$, $\hat{y} = 5.22 - 0.114(15) = 3.51$.

For confidence coefficient 0.90, $\alpha = 0.10$ and $\alpha = 0.10/2 = 0.05$. From Table 2, Appendix D, with $df = n-2 = 23-2 = 21$, $t_{0.05} = 1.721$. The 90% confidence interval is:

$$\hat{y} \pm t_{\alpha/2} s \sqrt{\frac{1}{n} + \frac{(x_p - \bar{x})^2}{SS_{xx}}} \Rightarrow 3.51 \pm 1.721(0.8573)\sqrt{\frac{1}{23} + \frac{(15-22.87)^2}{6906.608}} \Rightarrow$$

$3.51 \pm 0.34 \Rightarrow (3.17,\ 3.85)$.

We are 90% confident that the mean mass of all spills with an elapsed time of 15 minutes is between 3.17 and 3.85.

b. For confidence coefficient 0.90, $\alpha = 0.10$ and $\alpha/2 = 0.10/2 = 0.05$. From Table 2, Appendix D, with $df = n-2 = 23-2 = 21$, $t_{0.05} = 1.721$. The 90% prediction interval is:

$$\hat{y} \pm t_{\alpha/2} s \sqrt{1 + \frac{1}{n} + \frac{(x_p - \bar{x})^2}{SS_{xx}}} \Rightarrow 3.51 \pm 1.721(0.8573)\sqrt{1 + \frac{1}{23} + \frac{(15-22.87)^2}{6906.608}} \Rightarrow$$

$3.51 \pm 1.514 \Rightarrow (2.00,\ 5.02)$.

We are 90% confident that the mass of a single spill with an elapsed time of 15 minutes is between 2.00 and 5.02.

3.60 a. To determine if the model is adequate for predicting nitrogen amount, we test:

$$H_0 : \beta_1 = 0$$
$$H_a : \beta_1 \neq 0$$

The test statistic is $t = 32.80$ and the p-value is $p < 0.0001$.

Since the p-value is so small $(p < 0.0001)$, H_0 is rejected for any reasonable value of α. There is sufficient evidence to indicate that the amount of ammonium contributes information for the prediction of the amount of nitrogen removed using a linear model.

b. From the printout, the 95% prediction interval is $(41.8558, 77.8634)$. We are 95% confident that the actual amount of nitrogen removed when the amount of ammonium used is 100mg/l will be between 41.8558 and 77.8634 mg/l.

c. The 95% confidence interval for the mean amount of nitrogen removed when the amount of ammonium used is 100 mg/l will be narrower than the prediction interval. This is because the prediction interval for the actual value contains the variability of locating the mean and the variability of the actual values around the mea. The confidence interval for the mean contains only the variability in locating the mean.

3.61 a. The researchers are interested in the confidence interval for $E(y)$ or the average in-game heart rate of all top-level water polo players who have a maximal oxygen uptake of 150 VO_2max.

b. Using MINITAB, the results are:

Prediction for HR%

Regression Equation

HR% = -27.2 + 0.558 VO2Max

Settings

Variable	Setting
VO2Max	150

Prediction

Fit	SE Fit	95% CI	95% PI
56.4677	3.31256	(48.3622, 64.5733)	(41.2395, 71.6959)

The 95% confidence interval is $(48.3622, 64.5733)$.

c. We are 95% confident that the average in-game heart rate of all top-level polo players with a VO_2max of 150 is between 48.3622 and 64.5733.

3.62 Step 1. We hypothesize a straight-line probabilistic model: $y = \beta_0 + \beta_1 x + \varepsilon$ where y = the monthly price of recycled colored plastic bottles and x = the monthly price of naphtha.

Step 2: Collect the data. The data have been collected.

Step 3: Estimate the unknown parameters in the proposed model. From the exercise, the least squares estimates of β_0 and β_1 are: $\hat{\beta}_0 = -32.35$ $\hat{\beta}_1 = 4.82$
The least squares line is $\hat{y} = -32.35 + 4.82x$.

The least squares estimate of the slope, $\hat{\beta}_1 = 4.82$, implies that the estimated mean monthly price of recycled colored plastic bottles increases by 4.82 for each additional unit increase in the monthly price of naphtha. This interpretation is valid only over the observed values of the monthly price of naphtha. The estimated y-intercept, $\hat{\beta}_0 = -32.35$, has no practical meaning in this example because 0 will not be within the observed range of values for monthly price of naphtha.

Step 4: Specify the probability distribution of the random error component ε. We assume

(1) $E(\varepsilon) = 0$
(2) $\text{Var}(\varepsilon) = \sigma^2$ is constant for all x-values
(3) ε has a normal distribution
(4) ε's are independent

Step 5: To determine if there is a linear relationship between the monthly price of recycled colored plastic bottles and the monthly price of naphtha, we test:

$$H_0 : \beta_1 = 0$$
$$H_a : \beta_1 \neq 0$$

The test statistic is $t = 16.60$.

The rejection region requires $\alpha / 2 = 0.05 / 2 = 0.025$ in each tail of the t distribution. From Table 2, Appendix D, with $df = n - 2 = 120 - 2 = 118$, $t_{0.025} \approx 1.980$. The rejection region is $t < -1.980$ or $t > 1.980$.

Since the observed value of the test statistic falls in the rejection region $(t = 16.60 > 1.980)$, H_0 is rejected. There is sufficient evidence to there is a linear relationship between the monthly price of recycled colored plastic bottles and the monthly price of naphtha at $\alpha = 0.05$.

$r^2 = 0.69$ 69% of the sample variation around the mean monthly price of recycled colored plastic bottles is explained by the linear relationship between the monthly price of recycled colored plastic bottles and the monthly price of naphtha.

3.63 a. Using MINITAB, the results are:

Regression Analysis: Corrupt versus GDP

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	3345.8	3345.76	45.33	0.000
Error	11	811.9	73.81		
Total	12	4157.7			

Model Summary

S	R-sq	R-sq(adj)
8.59141	80.47%	78.70%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	25.89	3.09	8.37	0.000
GDP	0.000985	0.000146	6.73	0.000

Regression Equation

Corrupt = 25.89 + 0.000985 GDP

The fitted regression line is $\hat{y} = 25.89 + 0.000985GPD$.

To determine if GDP per capita is a linear predictor of corruption level, we test:

$$H_0 : \beta_1 = 0$$
$$H_a : \beta_1 \neq 0$$

The test statistic is $t = 6.73$ and the *p*-value is $p = 0.000$. Since the *p*-value is so small, H_0 is rejected. There is sufficient evidence to indicate GDP per capita is a linear predictor of corruption level for any reasonable value of α.

$r^2 = 0.8047$ This indicates that 80.47% of the variability in the corruption values is explained by the linear relationship between the corruption values and the GDP per capita.

b. Using MINITAB, the results are:

Regression Analysis: Corrupt versus PolR

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	2528	2527.6	17.06	0.002
Error	11	1630	148.2		
Total	12	4158			

Model Summary

S	R-sq	R-sq(adj)
12.1732	60.79%	57.23%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	66.06	7.34	9.00	0.000
PolR	-6.25	1.51	-4.13	0.002

Regression Equation

Corrupt = 66.06 - 6.25 PolR

The fitted regression line is $\hat{y} = 66.06 - 6.25PolR$.

To determine if degree of freedom in political rights is a linear predictor of corruption level, we test:

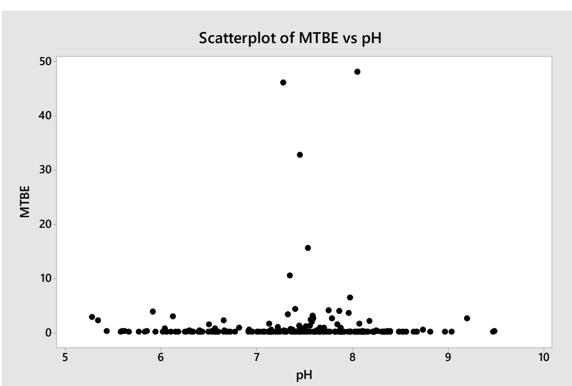
$$H_0 : \beta_1 = 0$$
$$H_a : \beta_1 \neq 0$$

The test statistic is $t = -4.13$ and the p-value is $p = 0.002$. Since the p-value is so small, H_0 is rejected. There is sufficient evidence to indicate GDP per capita is a linear predictor of corruption level for any value of $\alpha > 0.002$.

$r^2 = 0.6079$ This indicates that 60.79% of the variability in the corruption values is explained by the linear relationship between the corruption values and the degree of freedom in political rights.

c. Both variables, GDP per capita and degree of freedom in political rights, are significant predictors of corruption levels. Of the two, GDP per capita is a better predictor because the r^2 value is larger and the p-value for the test is smaller.

3.64 Using MINITAB, a scatterplot of the data is:



From the plot, there does not look like there is a linear relationship between MTBE and pH level.

The proposed linear regression model is $y = \beta_0 + \beta_1 x + \varepsilon$. Using MINITAB, an analysis of the data is:

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	2.01	2.008	0.08	0.782
Error	221	5785.93	26.181		
Total	222	5787.94			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
5.11670	0.03%	0.00%	0.00%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	0.35	3.14	0.11	0.911	
pH	0.116	0.420	0.28	0.782	1.00

The parameter estimates of the least squares line are: $\hat{\beta}_0 = 0.35 \quad \hat{\beta}_1 = 0.116$
The least squares line is $\hat{y} = 0.35 + 0.116x$.

The least squares estimate of the slope, $\hat{\beta}_1 = 0.116,$ implies that the estimated MTBE increases by 0.116 for each additional unit increase in the pH level. This interpretation is valid only over the observed values of the pH level which is from 5.28 to 9.48. The estimated y-intercept, $\hat{\beta}_0 = 0.35$ has no practical meaning in this example because 0 will not be within the observed range of the pH levels.

The estimate of σ is $s = 5.1167$. The value of this estimate is very large compared to most of the values of MTBE.

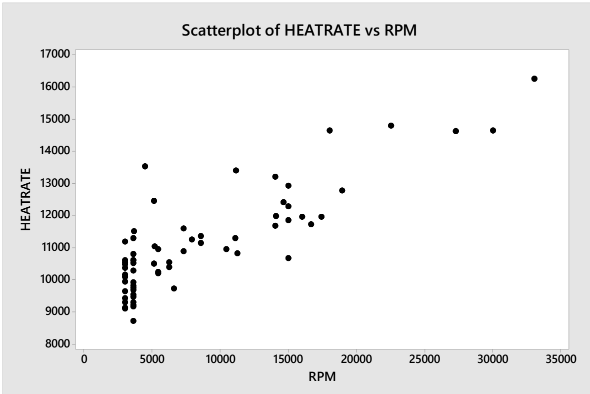
To determine if there is a linear relationship between the MTBE and the pH level, we test:

$$H_0 : \beta_1 = 0$$
$$H_a : \beta_1 \neq 0$$

The test statistic is $t = 0.28$ and the p-value is $p = 0.782$. Since the p-value is so large, H_0 will not be rejected for any reasonable value of α. There is insufficient evidence to indicate there is a linear relationship between the MTBE and the pH level.

$r^2 = 0.00$ This indicates that 0% of the variability in the MTBE values is explained by the linear relationship between the MTBE values and the pH levels. This would indicate that a linear regression model does not explain the relationship between MTBE and pH.

3.65 Using MINITAB, a scatter plot of the data is:



From the plot, there is evidence to indicate a linear relationship between heat rate and speed.

The proposed linear regression model is $y = \beta_0 + \beta_1 x + \varepsilon$. Using MINITAB, an analysis of the data is:

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	119598530	119598530	160.95	0.000
RPM	1	119598530	119598530	160.95	0.000
Error	65	48298678	743057		
Lack-of-Fit	28	28773369	1027620	1.95	0.029
Pure Error	37	19525309	527711		
Total	66	167897208			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
862.007	71.23%	70.79%	69.63%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	9470	164	57.73	0.000	
RPM	0.1917	0.0151	12.69	0.000	1.00

Regression Equation

HEATRATE = 9470 + 0.1917 RPM

The parameter estimates of the least squares line are: $\hat{\beta}_0 = 9470 \quad \hat{\beta}_1 = 0.1917$
The least squares line is $\hat{y} = 9470 + 0.1917x$.

The least squares estimate of the slope, $\hat{\beta}_1 = 0.1917$, implies that the estimated heat rate increases by 0.1917 units for each additional unit increase in the speed. This interpretation is valid only over the observed values of the speed level which is from 3,000 to 33,000. The estimated y-intercept, $\hat{\beta}_0 = 9470$ has no practical meaning in this example because 0 will not be within the observed range of the speed levels.

The estimate of σ is $s = 862.007$. We expect most of the observations to fall within $2s = 2(862.007) = 1724.014$ units of their predicted values.

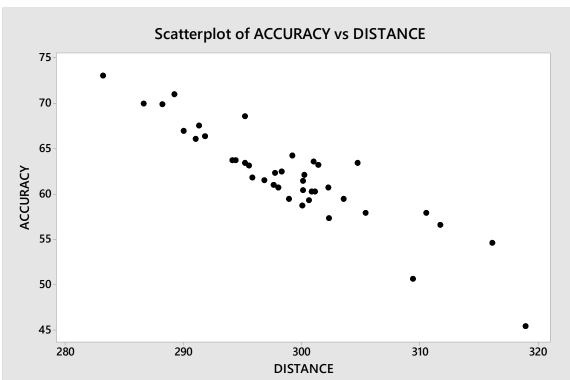
To determine if there is a linear relationship between the heat rate and the speed, we test:

$$H_0 : \beta_1 = 0$$
$$H_a : \beta_1 \neq 0$$

The test statistic is $t = 12.69$ and the p-value is $p = 0.000$. Since the p-value is so small, H_0 will be rejected for any reasonable value of α. There is sufficient evidence to indicate there is a linear relationship between the heat rate and speed.

$r^2 = 0.7173$ This indicates that 71.73% of the variability in the heat rate values is explained by the linear relationship between heat rate and the speed. This indicates that a linear regression line models the relationship between heat rate and speed fairly well.

3.66 Using MINITAB, a scatterplot of the data is:



From the plot, there is evidence to indicate a linear relationship between accuracy and distance.

The proposed linear regression model is $y = \beta_0 + \beta_1 x + \varepsilon$. Using MINITAB, an analysis of the data is:

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	874.99	874.989	174.95	0.000
DISTANCE	1	874.99	874.989	174.95	0.000
Error	38	190.06	5.001		
Lack-of-Fit	36	176.55	4.904	0.73	0.735
Pure Error	2	13.51	6.753		
Total	39	1065.04			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
2.23639	82.16%	81.69%	79.26%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	250.1	14.2	17.58	0.000	
DISTANCE	-0.6294	0.0476	-13.23	0.000	1.00

Regression Equation

ACCURACY = 250.1 - 0.6294 DISTANCE

The parameter estimates of the least squares line are: $\hat{\beta}_0 = 250.1 \quad \hat{\beta}_1 = -0.6294$
The least squares line is $\hat{y} = 250.1 - 0.6294x$.

The least squares estimate of the slope, $\hat{\beta}_1 = -0.6294$, implies that the estimated accuracy decreases by 0.6294 units for each additional yard increase in distance. This interpretation is valid only over the observed values of distance which is from 293.2 to 318.9 yards. The estimated y-intercept, $\hat{\beta}_0 = 250.1$ has no practical meaning in this example because 0 will not be within the observed range of distances.

The estimate of σ is $s = 2.23639$. We expect most of the observations to fall within $2s = 2(2.23639) = 4.473$ units of their predicted values.

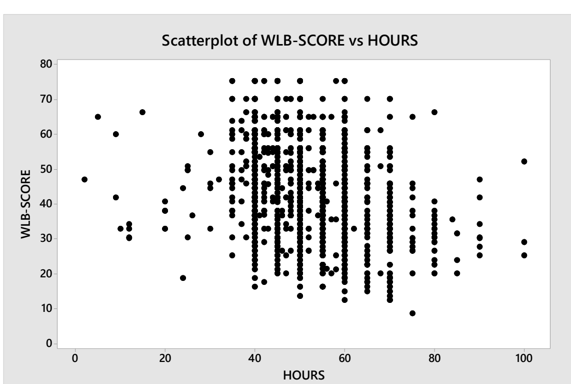
To determine if there is a negative linear relationship between accuracy and distance, we test:

$$H_0: \beta_1 = 0$$
$$H_a: \beta_1 < 0$$

The test statistic is $t = -13.23$ and the p-value is $p = 0.000/2 = 0.000$. Since the p-value is so small, H_0 will be rejected for any reasonable value of α. There is sufficient evidence to indicate there is a negative linear relationship between accuracy and distance.

$r^2 = 0.8216$ This indicates that 82.16% of the variability in the accuracy values is explained by the linear relationship between accuracy and distance. This indicates that a linear regression line models the relationship between accuracy and distance fairly well. The professional golfer has a valid concern.

3.67 Using MINITAB, a scatterplot of the data is:



From the plot, there is evidence to indicate a slight linear relationship between work-life balance scale score and average number of hours worked per week

The proposed linear regression model is $y = \beta_0 + \beta_1 x + \varepsilon$. Using MINITAB, an analysis of the data is:

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	23803	23803.1	157.73	0.000
HOURS	1	23803	23803.1	157.73	0.000
Error	2085	314647	150.9		
Lack-of-Fit	42	11939	284.3	1.92	0.000
Pure Error	2043	302708	148.2		
Total	2086	338451			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
12.2845	7.03%	6.99%	6.84%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	62.50	1.41	44.22	0.000	
HOURS	-0.3467	0.0276	-12.56	0.000	1.00

Regression Equation

WLB-SCORE = 62.50 - 0.3467 HOURS

The parameter estimates of the least squares line are: $\hat{\beta}_0 = 62.50 \quad \hat{\beta}_1 = -0.3467$
The least squares line is $\hat{y} = 62.50 - 0.3467x$.

The least squares estimate of the slope, $\hat{\beta}_1 = -0.3467$, implies that the estimated work-life balance scale score decreases by 0.3467 units for each additional average number of hours worked per week. This interpretation is valid only over the observed values of distance which is from 2 to 100 hours. The estimated y-intercept, $\hat{\beta}_0 = 62.50$ has no practical meaning in this example because 0 will not be within the observed range of hours worked.

The estimate of σ is $s = 12.2845$. We expect most of the observations to fall within $2s = 2(12.2845) = 24.569$ units of their predicted values.

To determine if there is a linear relationship between work-life balance scale score and average number of hours worked per week, we test:

$$H_0: \beta_1 = 0$$
$$H_a: \beta_1 \neq 0$$

The test statistic is $t = -12.56$ and the p-value is $p = 0.000$. Since the p-value is so small, H_0 will be rejected for any reasonable value of α. There is sufficient evidence to indicate there is a linear relationship between work-life balance scale score and average number of hours worked per week.

$r^2 = 0.0703$ This indicates that only 7.03% of the variability in the work-life balance scale scores is explained by the linear relationship between work-life balance scale scores and average number of hours worked per week. This indicates that although there is a significant linear relationship between work-life balance scale score and average number of hours worked per week, the relationship is very weak. Many other factors are influencing work-life balance scale scores.

3.68 Some preliminary calculations are:

$\sum x = 24 \quad \sum y = 77 \quad \sum x^2 = 240 \quad \sum y^2 = 2403 \quad \sum xy = 758$

a. $\hat{\beta}_1 = \dfrac{\sum xy}{\sum x^2} = \dfrac{758}{240} = 3.15833 \approx 3.158$

The fitted model is $\hat{y} = 3.158x$.

b. $SSE = \sum y^2 - \hat{\beta}_1 \sum xy = 2403 - 3.1583333(758) = 8.983359$

$s^2 = \dfrac{SSE}{n-1} = \dfrac{8.983359}{8-1} = 1.283337 \qquad s = \sqrt{1.283337} = 1.1328$

c. To determine if x and y are positively linearly related, we test:

$$H_0: \beta_1 = 0$$
$$H_a: \beta_1 > 0$$

The test statistic is $t = \dfrac{\hat{\beta}_1}{s/\sqrt{\sum x^2}} = \dfrac{3.158333}{1.1328/\sqrt{240}} = 43.193$

The rejection region requires $\alpha = 0.05$ in the upper tail of the t distribution. From Table 2, Appendix D, with $df = n - 1 = 8 - 1 = 7$, $t_{0.05} = 1.895$. The rejection region is $t > 1.895$.

Since the observed value of the test statistic falls in the rejection region $(t = 43.193 > 1.895)$, H_0 is rejected. There is sufficient evidence to indicate that x and y are positively linearly related at $\alpha = 0.05$.

d. The form of the confidence interval for β_1 is $\hat{\beta}_1 \pm t_{0.025}\left(\dfrac{s}{\sqrt{\sum x^2}}\right)$.

For confidence coefficient 0.95, $\alpha = 0.05$ and $\alpha/2 = 0.05/2 = 0.025$. From Table 2, Appendix D, with $df = n-1 = 8-1 = 7$, $t_{0.025} = 2.365$. The 95% confidence interval is:

$$\hat{\beta}_1 \pm t_{0.025}\left(\frac{s}{\sqrt{\sum x^2}}\right) \Rightarrow 3.158 \pm 2.365\left(\frac{1.1328}{\sqrt{240}}\right) \Rightarrow 3.158 \pm 0.173 \Rightarrow (2.985,\ 3.331)$$

e. The point estimate for y when $x = 7$ is $\hat{y} = 3.158(7) = 22.106$. The 95% confidence interval for $E(y)$ is:

$$\hat{y} \pm t_{0.025}s\left(\sqrt{\frac{x_p^2}{\sum x^2}}\right) \Rightarrow 22.106 \pm 2.365(1.1328)\left(\sqrt{\frac{7^2}{240}}\right) \Rightarrow 22.106 \pm 1.211 \Rightarrow (20.895,\ 23.317)$$

f. The 95% prediction interval for y is:

$$\hat{y} \pm t_{0.025}s\left(\sqrt{1+\frac{x_p^2}{\sum x^2}}\right) \Rightarrow 22.106 \pm 2.365(1.1328)\left(\sqrt{1+\frac{7^2}{240}}\right)$$

$$\Rightarrow 22.106 \pm 2.940 \Rightarrow (19.166,\ 25.046)$$

3.69 a. The results of the preliminary calculations are provided below:

$$n = 5,\ \sum x^2 = 30,\ \sum xy = -278,\ \sum y^2 = 2589$$

Substituting into the formula for $\hat{\beta}_1$, we have $\hat{\beta}_1 = \dfrac{\sum xy}{\sum x^2} = \dfrac{-278}{30} = -9.2667$ and the least squares line is $\hat{y} = -9.2667x$.

b. $SSE = \sum y^2 - \hat{\beta}_1 \sum xy = 2589 - (-9.26666677)(-278) = 12.8667$

$$s^2 = \frac{SSE}{n-1} = \frac{12.8667}{5-1} = 3.2167 \quad s = \sqrt{s^2} = \sqrt{3.2167} = 1.7935$$

c. To determine if x and y are negatively linearly related, we test:

$$H_0: \beta_1 = 0$$
$$H_a: \beta_1 < 0$$

Test statistic is $t = \dfrac{\hat{\beta}_1}{\frac{s}{\sqrt{\sum x^2}}} = \dfrac{-9.2667}{\frac{1.7935}{\sqrt{30}}} = -28.30.$

The rejection region requires $\alpha = 0.05$ in the lower tail of the t distribution. From Table 2, Appendix D, with $df = n-1 = 5-1 = 4$, $t_{0.05} = 2.132$. The rejection region is $t < -2.132$.

Since the observed value of the test statistic falls in the rejection region $(t = -28.30 < -2.132)$, H_0 is rejected. There is sufficient evidence to indicate that x and y are negatively linearly related at $\alpha = 0.05$.

d. The form of the confidence interval for β_1 is $\hat{\beta}_1 \pm t_{0.025}\left(\dfrac{s}{\sqrt{\sum x^2}}\right)$.

For confidence coefficient 0.95, $\alpha = 0.05$ and $\alpha/2 = 0.05/2 = 0.025$. From Table 2, Appendix D, with $df = n-1 = 5-1 = 4$, $t_{0.025} = 2.776$. The 95% confidence interval is:

$$\hat{\beta}_1 \pm t_{0.025}\left(\frac{s}{\sqrt{\sum x^2}}\right) \Rightarrow -9.267 \pm 2.776\left(\frac{1.7935}{\sqrt{30}}\right) \Rightarrow -9.267 \pm 0.909 \Rightarrow (-10.176, -8.358).$$

e. The point estimate for y when $x = 1$ is $\hat{y} = \hat{\beta}_1 x = -9.267(1) = -9.267$. The 95% confidence interval for $E(y)$ is:

$$\hat{y} \pm t_{0.025}s\left(\sqrt{\frac{x_p^2}{\sum x^2}}\right) \Rightarrow -9.267 \pm 2.776(1.7935)\left(\sqrt{\frac{1}{30}}\right) \Rightarrow -9.267 \pm 0.909$$

$$\Rightarrow (-10.176, -8.358).$$

f. The 95% prediction interval for y is:

$$\hat{y} \pm t_{0.025}s\left(\sqrt{1+\frac{x_p^2}{\sum x^2}}\right) \Rightarrow -9.267 \pm 2.776(1.7935)\left(\sqrt{1+\frac{1^2}{30}}\right)$$

$$\Rightarrow -9.267 \pm 5.061 \Rightarrow (-14.328, -4.206)$$

3.70 a. The results of the preliminary calculations are provided below:

$$\sum x = 1140 \quad \sum x^2 = 158{,}400 \quad \sum y = 236 \quad \sum xy = 33{,}020 \quad \sum y^2 = 6906$$

Substituting into the formula for $\hat{\beta}_1$, we have $\hat{\beta}_1 = \dfrac{\sum xy}{\sum x^2} = \dfrac{33{,}020}{158{,}400} = 0.208459596 \approx 0.2085$ and the least squares line is $\hat{y} = 0.2085x$.

b. $SSE = \sum y^2 - \hat{\beta}_1 \sum xy = 6906 - (0.208459596)(33{,}020) = 22.664$

$$s^2 = \frac{SSE}{n-1} = \frac{22.664}{10-1} = 2.5182 \quad s = \sqrt{s^2} = \sqrt{2.5182} = 1.5869$$

c. To determine if x and y are positively linearly related, we test:

$$H_0: \beta_1 = 0$$
$$H_a: \beta_1 > 0$$

The test statistic is $t = \dfrac{\hat{\beta}_1}{\dfrac{s}{\sqrt{\sum x^2}}} = \dfrac{0.2085}{\dfrac{1.5869}{\sqrt{158,400}}} = 52.29.$

The rejection region requires $\alpha = 0.05$ in the upper tail of the t distribution. From Table 2, Appendix D, with $df = n - 1 = 10 - 1 = 9$, $t_{0.05} = 1.833$. The rejection region is $t > 1.833$.

Since the observed value of the test statistic falls in the rejection region $(t = 52.29 > 1.833)$, H_0 is rejected. There is sufficient evidence to indicate that x and y are positively linearly related at $\alpha = 0.05$.

d. The form of the confidence interval for β_1 is $\hat{\beta}_1 \pm t_{0.025}\left(\dfrac{s}{\sqrt{\sum x^2}}\right)$.

For confidence coefficient 0.95, $\alpha = 0.05$ and $\alpha/2 = 0.05/2 = 0.025$. From Table 2, Appendix D, with $df = n - 1 = 10 - 1 = 9$, $t_{0.025} = 2.262$. The 95% confidence interval is:

$$\hat{\beta}_1 \pm t_{0.025}\left(\frac{s}{\sqrt{\sum x^2}}\right) \Rightarrow 0.2085 \pm 2.262\left(\frac{1.5869}{\sqrt{158,400}}\right) \Rightarrow 0.2085 \pm 0.0090 \Rightarrow (0.1995,\ 0.2175).$$

e. The point estimate for y when $x = 125$ is $\hat{y} = \hat{\beta}_1 x = 0.2085(125) = 26.06$. The 95% confidence interval for $E(y)$ is:

$$\hat{y} \pm t_{0.025}s\left(\sqrt{\frac{x_p^2}{\sum x^2}}\right) \Rightarrow 26.06 \pm 2.262(1.5869)\left(\sqrt{\frac{125^2}{158,400}}\right) \Rightarrow 26.06 \pm 1.13$$
$$\Rightarrow (24.93,\ 27.19).$$

f. The 95% prediction interval for y is:

$$\hat{y} \pm t_{0.025}s\left(\sqrt{1 + \frac{x_p^2}{\sum x^2}}\right) \Rightarrow 26.06 \pm 2.262(1.5869)\left(\sqrt{1 + \frac{125^2}{158,400}}\right)$$
$$\Rightarrow 26.06 \pm 3.76 \Rightarrow (22.30,\ 29.82)$$

3.71 a. Some preliminary calculations are:

$n = 8 \quad \sum x^2 = 59.75 \qquad \sum xy = 320.5 \qquad \sum y^2 = 1738$

Then, $\hat{\beta}_1 = \frac{\sum xy}{\sum x^2} = \frac{320.5}{59.75} = 5.364016736 \approx 5.364$, and the least squares line is $\hat{y} = 5.364x$.

b. To determine if there is a linear relationship between drug dosage and decrease in pulse rate, we test:

$$H_0 : \beta_1 = 0$$
$$H_a : \beta_1 \neq 0$$

The test statistic is $t = \frac{\hat{\beta}_1}{\frac{s}{\sqrt{\sum x^2}}}$

where $s = \sqrt{s^2} = \sqrt{\frac{SSE}{n-1}} = \sqrt{\frac{\sum y^2 - \hat{\beta}_1 \sum xy}{n-1}} = \sqrt{\frac{1738 - (5.364)(320.5)}{8-1}} = 1.640$

Substituting, we have $t = \frac{5.364}{\frac{1.640}{\sqrt{59.75}}} = 25.28$.

The rejection region requires $\alpha / 2 = 0.10 / 2 = 0.05$ in each tail of the t distribution. From Table 2, Appendix D, with $df = n-1 = 8-1 = 7$, $t_{0.05} = 1.895$. The rejection region is $t < -1.895$ or $t > 1.895$.

Since the observed value of the test statistic falls in the rejection region $(t = 25.28 > 1.895)$, H_0 is rejected. There is sufficient evidence to indicate that drug dosage and decrease in pulse rate are linearly related at $\alpha = 0.10$.

c. We want to predict the decrease in pulse rate y corresponding to a drug dosage of $x_p = 3.5$ cubic centimeters. First, we obtain the point estimate:

$$\hat{y} = \hat{\beta}_1 x = 5.364(3.5) = 18.774$$

For confidence coefficient 0.99, $\alpha = 0.01$ and $\alpha / 2 = 0.01 / 2 = 0.005$. From Table 2, Appendix D, with $df = n-1 = 8-1 = 7$, $t_{0.005} = 3.499$. The 99% confidence interval is:

$$\hat{y} \pm t_{0.005} s \sqrt{1 + \frac{x_p^2}{\sum x^2}} \Rightarrow 18.774 \pm 3.499(1.640)\sqrt{1 + \frac{(3.5)^2}{59.75}} \Rightarrow 18.774 \pm 6.299$$
$$\Rightarrow (12.475, 25.073).$$

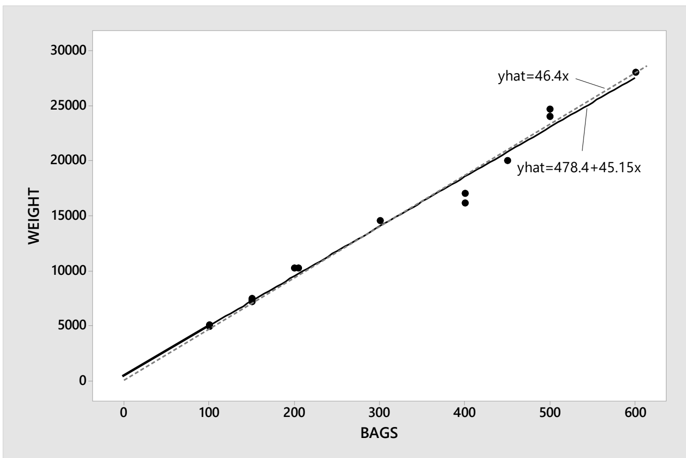
Therefore, we predict the decrease in pulse rate corresponding to a dosage of 3.5cc to fall between 12.475 and 25.073 beats/minute with 99% confidence.

3.72 Some preliminary calculations are:

$\sum x = 4305 \quad \sum x^2 = 1{,}652{,}025 \quad \sum y = 201{,}558 \quad \sum y^2 = 3{,}571{,}211{,}200 \quad \sum xy = 76{,}652{,}695$

a. $\hat{\beta}_1 = \frac{\sum xy}{\sum x^2} = \frac{76{,}652{,}695}{1{,}652{,}025} = 46.39923427 \approx 46.3992$, and the least squares line is $\hat{y} = 46.3992x$.

Using MINITAB, the scatterplot of the data with the fitted line is:



b. $$SSxx = \sum x^2 - \frac{\left(\sum x\right)^2}{n} = 1{,}652{,}025 - \frac{(4305)^2}{15} = 416{,}490$$

$$SS_{xy} = \sum xy - \frac{\left(\sum x\right)\left(\sum y\right)}{n} = 76{,}652{,}695 - \frac{(4305)(201{,}558)}{15} = 18{,}805{,}549$$

$$\hat{\beta}_1 = \frac{SS_{xy}}{SS_{xx}} = \frac{18{,}805{,}549}{416{,}490} = 45.15246224 \approx 45.152$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1\bar{x} = \frac{201{,}558}{15} - 45.1524622\left(\frac{4305}{15}\right) = 478.443$$

The fitted line is $\hat{y} = 478.443 + 45.152x$.

c. Since 0 is not contained in the observed range of values of the number of 50-pound bags in the shipment, $\hat{\beta}_0$ has no practical interpretation. Therefore, a value of $\hat{\beta}_0$ that differs from 0 is not unexpected.

d. First, we need to compute s.

$$SS_{yy} = \sum y^2 - \frac{\left(\sum y\right)^2}{n} = 3{,}571{,}211{,}200 - \frac{(201{,}558)^2}{15} = 862{,}836{,}042$$

$$SSE = SS_{yy} - \hat{\beta}_1 SS_{xy} = 862{,}836{,}042 - 45.15246224(18{,}805{,}549) = 13{,}719{,}200.9$$

$$s^2 = \frac{SSE}{n-2} = \frac{13{,}719{,}200.9}{15-2} = 1{,}055{,}323.146 \qquad s = \sqrt{1{,}055{,}323.146} = 1027.2892$$

To determine if β_0 should be included in the model, we test:

$H_0: \beta_0 = 0$
$H_a: \beta_0 \neq 0$

The test statistic is $t = \dfrac{\hat{\beta}_0}{s\sqrt{\dfrac{1}{n} + \dfrac{\bar{x}^2}{SS_{xx}}}} = \dfrac{478.4}{1027.289\sqrt{\dfrac{1}{15} + \dfrac{287^2}{416{,}490}}} = 0.906.$

The rejection region requires $\alpha/2 = 0.10/2 = 0.05$ in each tail of the t distribution. From Table 2, Appendix D, with $df = n-2 = 15-2 = 13$, $t_{0.05} = 1.771$. The rejection region is $t < -1.771$ or $t > 1.771$.

Since the observed value of the test statistic does not fall in the rejection region $(t = 0.906 \not> 1.771)$, H_0 is not rejected. There is insufficient evidence to indicate that β_0 should be included in the model at $\alpha = 0.10$.

3.73 a. Some preliminary calculations are:

$n = 10 \qquad \sum x^2 = 1{,}933{,}154 \qquad \sum xy = 98{,}946{,}257 \qquad \sum y^2 = 5{,}066{,}358{,}119$

Then, $\hat{\beta}_1 = \dfrac{\sum xy}{\sum x^2} = \dfrac{98{,}946{,}257}{1{,}933{,}154} = 51.18384619 \approx 51.184$, and the least squares prediction equation is $\hat{y} = 51.184x$.

b. To determine if population contributes to the prediction of electricity customers, we test:

$H_0: \beta_1 = 0$
$H_a: \beta_1 \neq 0$

The test statistic is $t = \dfrac{\hat{\beta}_1}{\dfrac{s}{\sqrt{\sum x^2}}}$

where $s = \sqrt{s^2} = \sqrt{\dfrac{SSE}{n-1}} = \sqrt{\dfrac{\left(\sum y^2 - \hat{\beta}_1 \sum xy\right)}{n-1}}$

$$= \sqrt{\frac{5{,}066{,}358{,}119 - 51.18385(98{,}946{,}257)}{10-1}} = 460.4036$$

Substituting, we have $t = \dfrac{51.18}{460.4036 / \sqrt{1,933,154}} = 154.56$

The rejection region requires $\alpha / 2 = 0.01 / 2 = 0.005$ in each tail of the t distribution. From Table 2, Appendix D, with $df = n - 1 = 10 - 1 = 9$, $t_{0.005} = 3.250$. The rejection region is $t < -3.250$ or $t > 3.250$.

Since the observed value of the test statistic falls in the rejection region $(t = 154.56 > 3.250)$, H_0 is rejected. There is sufficient evidence to indicate that population contributes to the prediction of electricity customers at $\alpha = 0.01$.

c. We need the following additional information:

$\sum x = 4286 \quad \sum y = 220,297 \quad SS_{xx} = 96,174.4 \quad SS_{xy} = 4,526,962.8 \quad SS_{yy} = 213,281,298$

$\hat{\beta}_1 = 47.07 \quad \hat{\beta}_0 = 1855.35 \quad SSE = 195,568.4 \quad s^2 = 24,446.05 \quad s = 156.3523$

The least squares prediction equation is $\hat{y} = 1855.35 + 47.07x$.

To determine if population contributes to the prediction of electricity customers, we test:

$$H_0 : \beta_1 = 0$$
$$H_a : \beta_1 \neq 0$$

The test statistic is $t = \dfrac{\hat{\beta}_1}{s / \sqrt{\text{SS}_{xx}}} = \dfrac{47.07}{156.3523 / \sqrt{96,174.4}} = 93.36$

The rejection region requires $\alpha / 2 = 0.01 / 2 = 0.005$ in each tail of the t distribution. From Table 2, Appendix D, with $df = n - 2 = 10 - 2 = 8$, $t_{0.005} = 3.355$. The rejection region is $t < -3.355$ or $t > 3.355$.

Since the observed value of the test statistic falls in the rejection region $(t = 93.36 > 3.355)$, H_0 is rejected. There is sufficient evidence to indicate that population contributes to the prediction of electricity customers at $\alpha = 0.01$.

d. Without running a formal test, we can compare the two models. The value of s for the model $y = \beta_1 x + \varepsilon$ is 460.4036 while the value of s for the model $y = \beta_0 + \beta_1 x + \varepsilon$ is 156.3523. Since the value of s is much smaller for the second model, it appears that the second model should be used.

For a formal test, refer to part (d) of Exercise 3.66.

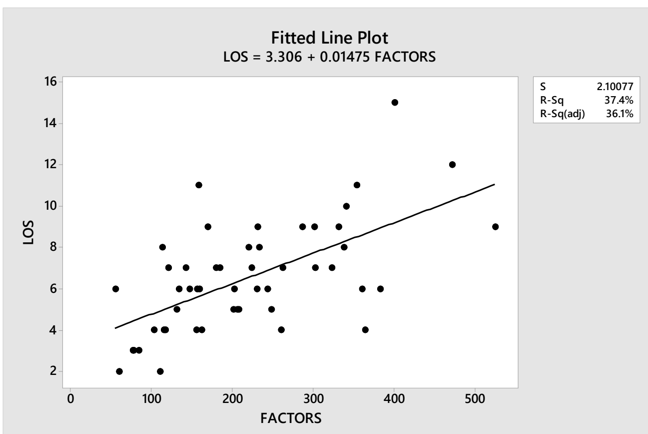
$$H_0 : \beta_0 = 0$$
$$H_a : \beta_0 \neq 0$$

The test statistic is $t = \dfrac{\hat{\beta}_0 - 0}{s\sqrt{\dfrac{1}{n} + \dfrac{\overline{x}^2}{SS_{xx}}}} = \dfrac{1855.35}{156.3523\sqrt{\dfrac{1}{10} + \dfrac{428.6^2}{96{,}174.4}}} = 8.37$

The rejection region requires $\alpha/2 = 0.01/2 = 0.005$ in each tail of the t distribution. From Table 2, Appendix D, with $df = n - 1 = 10 - 1 = 9$, $t_{0.005} = 3.250$. The rejection region is $t < -3.250$ or $t > 3.250$.

Since the observed value of the test statistic falls in the rejection region $(t = 8.37 > 3.250)$, H_0 is rejected. There is sufficient evidence to indicate that β_0 should be included in the model at $\alpha = 0.01$.

3.74 a. Using MINITAB, the scatterplot is:



b. From the printout, the least squares line is $\hat{y} = 3.306 + 0.01475x$.

c. For every one unit increase in the number of factors per patient, we estimate the patient's length of stay to increase 0.01475 days.

d. To determine if the number of factors per patient contributes information for the prediction of the patient's length of stay, we test:

$H_0: \beta_1 = 0$
$H_a: \beta_1 \neq 0$

The test statistic is $t = 5.36$ and the p-value is $p < 0.0001$. Since the p-value is less than $\alpha\,(p < 0.0001 < 0.05)$, H_0 is rejected. There is sufficient evidence to indicate that the number of factors per patient contributes information for the prediction of the patient's length of stay at $\alpha = 0.05$.

e. From the printout, the 95% confidence interval is $(0.00922,\ 0.02029)$. We are 95% confident that for each additional factor per patient, the patient's length of stay will increase between 0.00917 and 0.02033 days.

f. $r = \sqrt{0.3740} = 0.6116$ There appears to be a moderate positive linear relationship between the number of factors and the length of stay.

g. $r^2 = 0.3740$ 37.4% of the variability around the mean length of stay can be explained by the linear relationship between the number of factors and the length of stay.

h. From the printout, the 95% prediction interval is $(2.44798, 10.98081)$.

i. There is a significant linear relationship between length of stay and the number of factors. However, the value of r^2 is only $r^2 = 0.3740$. Thus, only a little over a third of the variability in the lengths of stays is explained by the model. Many other variables could be affecting the lengths of stay other than the number of factors.

3.75 a. $y = \beta_0 + \beta_1 x + \varepsilon$

b. A value of $r = 0.68$ indicates a moderate positive linear relationship between RMP and SET ratings.

c. The slope is positive since the correlation coefficient is positive.

d. Since the *p*-value is so small $(p = 0.001)$, H_0 is rejected for any value of $\alpha > 0.001$. This indicates that there is a significant correlation between RMP and SET ratings.

e. $r^2 = (0.68)^2 = 0.4624$ 46.24% of the variability of the sample SET ratings about their mean can be explained by the linear relationship between the SET ratings and the RMP ratings.

3.76 a. Yes. For the men, as the year increases, the winning time tends to decrease. The straight-line model is $y = \beta_0 + \beta_1 x + \varepsilon$. We would expect the slope to be negative.

b. Yes. For the women, as the year increases, the winning time tends to decrease. The straight-line model is $y = \beta_0 + \beta_1 x + \varepsilon$. We would expect the slope to be negative.

c. Since the slope of the women's line is steeper than that for the men, the slope of the women's line will be greater in absolute value.

d. No. The gathered data is from 1880 to 2000. Using this data to predict the time for the year 2020 would be very risky. We have no idea what the relationship between time and year will be outside the observed range. Thus, we would not recommend using this model.

3.77 Using MINITAB, the analyses are:

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	72.04	72.04	7.11	0.056
DIAMETER	1	72.04	72.04	7.11	0.056
Error	4	40.55	10.14		
Total	5	112.59			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
3.18403	63.98%	54.98%	0.00%

Coefficients

Term	Coef	SE Coef	90% CI	T-Value	P-Value	VIF
Constant	6.35	3.90	(-1.97, 14.68)	1.63	0.179	
DIAMETER	0.950	0.356	(0.190, 1.709)	2.67	0.056	1.00

Regression Equation

POROSITY = 6.35 + 0.950 DIAMETER

Settings

Variable	Setting
DIAMETER	10

Prediction

Fit	SE Fit	90% CI	90% PI
15.8501	1.30529	(13.0674, 18.6327)	(8.51395, 23.1862)

a. The least squares line is $\hat{y} = 6.35 + 0.950x$.

b. $\hat{\beta}_0 = 6.35$ Since 0 is not in the range of observed values for diameter, $\hat{\beta}_0$ has no meaning.

c. From the printout the 90% confidence interval is $(0.190, 1.709)$. We are 90% confident that for each unit increase in diameter, the mean porosity will increase from 0.190 and 1.709 units.

d. From the printout, the 90% prediction interval is $(8.514, 23.186)$.

3.78 Using MINITAB, the analyses are:

Analysis of Variance

Source	DF	Seq SS	Contribution	Adj SS	Adj MS	F-Value	P-Value
Regression	1	0.2330	39.37%	0.2330	0.23300	9.09	0.009
EMPATHY	1	0.2330	39.37%	0.2330	0.23300	9.09	0.009
Error	14	0.3588	60.63%	0.3588	0.02563		
Lack-of-Fit	10	0.2557	43.20%	0.2557	0.02557	0.99	0.552
Pure Error	4	0.1031	17.42%	0.1031	0.02578		
Total	15	0.5918	100.00%				

Model Summary

S	R-sq	R-sq(adj)	PRESS	R-sq(pred)
0.160084	39.37%	35.04%	0.484291	18.16%

Coefficients

Term	Coef	SE Coef	95% CI	T-Value	P-Value	VIF
Constant	-0.392	0.220	(-0.864, 0.079)	-1.79	0.096	
EMPATHY	0.0362	0.0120	(0.0104, 0.0619)	3.02	0.009	1.00

Regression Equation

ACTIVITY = -0.392 + 0.0362 EMPATHY

To determine if people scoring higher in empathy show higher pain-related brain activity, we test:

$$H_0 : \beta_1 = 0$$
$$H_a : \beta_1 > 0$$

The test statistic is $t = 3.02$ and the p-value is $p = 0.009 / 2 = 0.0045$. Since the p-value is very small, H_0 is rejected for any value of $\alpha > 0.0045$. There is sufficient evidence to indicate that people scoring higher in empathy show higher pain-related brain activity at $\alpha > 0.0045$.

3.79 a. Since the p-value for the SG score is $p = 0.739$ and is larger than the significance level of 0.05, then we cannot conclude that ESLR score is linearly related to the SG score.

b. Since the p-value for the SR score is $p = 0.012$ and is smaller than the significance level of 0.05, then we can conclude that ESLR score is linearly related to the SR score.

c. Since the p-value for the ER score is $p = 0.022$ and is smaller than the significance level of 0.05, then we can conclude that ESLR score is linearly related to ER score.

d. $100\left(r^2\right)\%$ of the sample variation in ESLR score can be explained by the linear relationship between ESLR and x (SG, SR, or ER score)

a. 0.2% of the sample variation in ESLR scores around their means can be explained by the linear relationship between ESLR and SG scores.

b. 9.9% of the sample variation in ESLR scores around their means can be explained by the linear relationship between ESLR and SR scores.

c. 7.8% of the sample variation in ESLR scores around their means can be explained by the linear relationship between ESLR and ER scores.

3.80 a. Using MINITAB, the results of the analyses regressing the blood plasma level of 2,3,7,8-TCDD on the fat tissue level of 2,3,7,8-TCDD are:

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	1105.19	1105.19	132.05	0.000
FAT	1	1105.19	1105.19	132.05	0.000
Error	18	150.65	8.37		
Lack-of-Fit	15	137.85	9.19	2.15	0.289
Pure Error	3	12.81	4.27		
Total	19	1255.84			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
2.89303	88.00%	87.34%	80.90%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	-0.150	0.841	-0.18	0.860	
FAT	0.9009	0.0784	11.49	0.000	1.00

Regression Equation

PLASMA = -0.150 + 0.9009 FAT

The fitted prediction equation is $\hat{y} = -0.150 + 0.9009x$.

Using MINITAB, the results of the analyses regressing the fat tissue level of 2,3,7,8-TCDD on the blood plasma level of 2,3,7,8-TCDD are:

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	1198.32	1198.32	132.05	0.000
PLASMA	1	1198.32	1198.32	132.05	0.000
Error	18	163.35	9.07		
Lack-of-Fit	15	154.56	10.30	3.52	0.164
Pure Error	3	8.79	2.93		
Total	19	1361.67			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
3.01245	88.00%	87.34%	80.90%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	0.970	0.846	1.15	0.267	
PLASMA	0.9768	0.0850	11.49	0.000	1.00

Regression Equation

FAT = 0.970 + 0.9768 PLASMA

The fitted prediction equation is $\hat{y} = 0.970 + 0.9768x$.

b. To determine if fat tissue level is a useful predictor of blood plasma level, we test:

$$H_0 : \beta_1 = 0$$
$$H_a : \beta_1 \neq 0$$

The test statistic is $t = 11.49$ and the *p*-value is $p = 0.000$. Since the *p*-value is less than $\alpha\left(p = 0.000 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate fat tissue level is a useful predictor of blood plasma level at $\alpha = 0.05$.

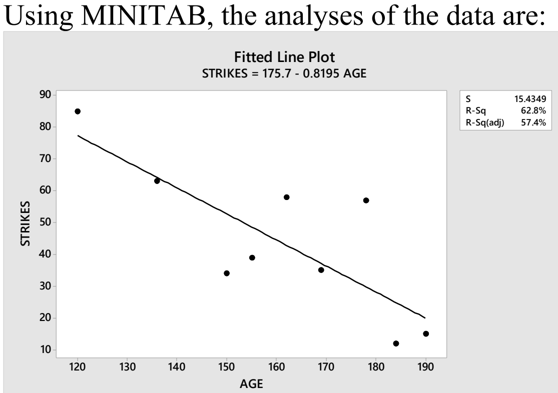
c. To determine if blood plasma level is a useful predictor of fat tissue level, we test:

$$H_0 : \beta_1 = 0$$
$$H_a : \beta_1 \neq 0$$

The test statistic is $t = 11.49$ and the *p*-value is $p = 0.000$. Since the *p*-value is less than $\alpha\left(p = 0.000 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate blood plasma level is a useful predictor of fat tissue level at $\alpha = 0.05$.

d. If we fit a least squares line through the data, the relationship will be the same regardless of which variable is the dependent variable and which variable is the independent variable. The correlation coefficient and the coefficient of determination will be the same regardless of which variable is the dependent variable and which variable is the independent variable.

3.81 Using MINITAB, the analyses of the data are:



Analysis of Variance

Source	DF	Seq SS	Contribution	Adj SS	Adj MS	F-Value	P-Value
Regression	1	2810	62.76%	2810	2809.9	11.79	0.011
AGE	1	2810	62.76%	2810	2809.9	11.79	0.011
Error	7	1668	37.24%	1668	238.2		
Total	8	4478	100.00%				

Model Summary

S	R-sq	R-sq(adj)	PRESS	R-sq(pred)
15.4349	62.76%	57.43%	2582.04	42.33%

Coefficients

Term	Coef	SE Coef	95% CI	T-Value	P-Value	VIF
Constant	175.7	38.6	(84.4, 267.0)	4.55	0.003	
AGE	-0.819	0.239	(-1.384, -0.255)	-3.43	0.011	1.00

Regression Equation

STRIKES = 175.7 - 0.819 AGE

a. The fitted regression line is $\hat{y} = 175.7 - 0.819x$.

b. We see from the plot that there appears to be a moderate negative linear relationship between age and the mean number of strikes.

$\hat{\beta}_0 = 175.7$ Since 0 is not in the observed range of values of age, $\hat{\beta}_0$ has no meaning.

$\hat{\beta}_1 = -0.819$ For each additional day of age for the fish, we estimate that the mean number of strikes will decrease by 0.819 strikes.

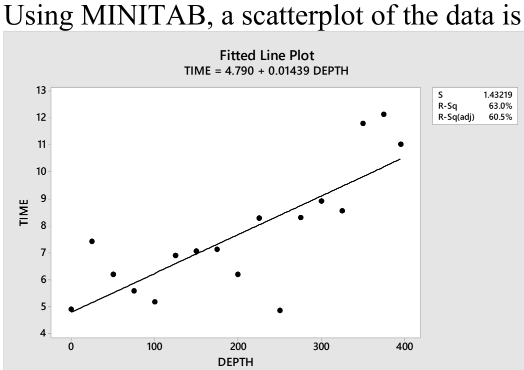
To determine if there is a linear relationship between age of fish and number of strikes, we test:

$$H_0: \beta_1 = 0$$
$$H_a: \beta_1 \neq 0$$

The test statistic is $t = -3.43$ and the p-value is $p = 0.011$. Since the p-value is less than $\alpha\left(p = 0.011 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate there is a linear relationship between age of fish and number of strikes at $\alpha = 0.05$.

$r^2 = 0.6276$ 62.76% of the variability of the mean number of strikes about their mean is explained by the linear relationship between age and number of strikes.

3.82 Using MINITAB, a scatterplot of the data is:



There appears to be a linear relationship between the time to drill 5 feet and the depth at which drilling begins.

Using MINITAB, the analyses of the data are:

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	52.38	52.378	25.54	0.000
DEPTH	1	52.38	52.378	25.54	0.000
Error	15	30.77	2.051		
Total	16	83.15			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
1.43219	63.00%	60.53%	52.23%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	4.790	0.666	7.19	0.000	
DEPTH	0.01439	0.00285	5.05	0.000	1.00

Regression Equation

TIME = 4.790 + 0.01439 DEPTH

The fitted regression line is $\hat{y} = 4.790 + 0.01439x$.

$\hat{\beta}_0 = 4.790$ We estimate the mean time to drill 5 feet when starting at a depth of 0 feet is 4.79 minutes.

$\hat{\beta}_1 = 0.01439$ For each additional foot of depth, we estimate that the mean time to drill 5 feet will increase by 0.0.01439 minutes.

To determine if there is a linear relationship between depth and time, we test:

$$H_0 : \beta_1 = 0$$
$$H_a : \beta_1 \neq 0$$

The test statistic is $t = 5.05$ and the p-value is $p = 0.000$. Since the p-value is less than $\alpha\left(p = 0.000 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate there is a linear relationship between depth and time at $\alpha = 0.05$.

$r^2 = 0.6300$ 63.00% of the variability of the mean time to drill 5 feet about their mean is explained by the linear relationship between time to drill and depth that drilling starts.

3.83 a. To determine if body plus head rotation and active head movement are positively linearly related, we test:

$$H_0 : \beta_1 = 0$$
$$H_a : \beta_1 > 0$$

The test statistic is $t = \dfrac{\hat{\beta}_1}{s_{\hat{\beta}_1}} = \dfrac{0.88 - 0}{0.14} = 6.29.$

The rejection region requires $\alpha = 0.05$ in the upper tail of the t distribution with $df = n - 2 = 39 - 2 = 37$. From Table 2, Appendix D, $t_{0.05} \approx 1.687$. The rejection region is $t > 1.687$.

Since the observed value of the test statistic falls in the rejection region $\left(t = 6.29 > 1.687\right)$, H_0 is rejected. There is sufficient evidence to indicate that the two variables are positively linearly related at $\alpha = 0.05$.

b. For confidence level 0.90, $\alpha = 0.10$ and $\alpha / 2 = 0.10 / 2 = 0.05$. From Table 2, Appendix D, with $df = n - 2 = 39 - 2 = 37$, $t_{0.05} \approx 1.687$. The confidence interval is:

$$\hat{\beta}_1 \pm t_{0.05} s_{\hat{\beta}_1} \Rightarrow 0.88 \pm 1.687\left(0.14\right) \Rightarrow 0.88 \pm 0.24 \Rightarrow \left(0.64, 1.12\right)$$

We are 90% confident that the true value of β_1 is between 0.64 and 1.12.

c. Because the interval in part b contains the value 1, there is no evidence that the true slope of the line differs from 1.

3.84 Using MINITAB, the analyses of the data are:

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	6.096	6.0958	6.74	0.021
RECOVERY	1	6.096	6.0958	6.74	0.021
Error	14	12.654	0.9039		
Lack-of-Fit	7	7.474	1.0677	1.44	0.320
Pure Error	7	5.180	0.7400		
Total	15	18.750			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.950722	32.51%	27.69%	19.69%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	2.970	0.790	3.76	0.002	
RECOVERY	0.1267	0.0488	2.60	0.021	1.00

Regression Equation

LACTATE = 2.970 + 0.1267 RECOVERY

To determine if blood lactate level is linearly related to perceived recovery, we test:

$$H_0 : \beta_1 = 0$$
$$H_a : \beta_1 \neq 0$$

The test statistic is $t = 2.60$ and the p-value is $p = 0.021$. Since the p-value is less than $\alpha\left(p = 0.021 < 0.10\right)$, H_0 is rejected. There is sufficient evidence to indicate blood lactate level is linearly related to perceived recovery at $\alpha = 0.10$.

3.85 a. This relationship will have a negative correlation since the researchers claim an "inverse relationship".

b. Solving $t = \dfrac{r\sqrt{n-2}}{\sqrt{1-r^2}}$ for r using the smallest value of t that leads to a statistically significant result gives: $r^2 = \dfrac{t^2}{t^2+n-2}$. So if $t = 1.645$ leads to a rejection of $H_0 : \rho = 0$, then $r^2 = \dfrac{(1.645)^2}{(1.645)^2 + 337 - 2} = .00801$. Thus, $r = -\sqrt{0.00801} = -0.0895$ since r is negative.

3.86 a. Using MINITAB, the results are:

Analysis of Variance

Source	DF	Seq SS	Contribution	Adj SS	Adj MS	F-Value	P-Value
Regression	1	0.8309	85.38%	0.8309	0.83089	46.73	0.000
TEMP	1	0.8309	85.38%	0.8309	0.83089	46.73	0.000
Error	8	0.1423	14.62%	0.1423	0.01778		
Total	9	0.9731	100.00%				

Model Summary

S	R-sq	R-sq(adj)	PRESS	R-sq(pred)
0.133347	85.38%	83.56%	0.340173	65.04%

Coefficients

Term	Coef	SE Coef	95% CI	T-Value	P-Value	VIF
Constant	-13.49	2.07	(-18.27, -8.71)	-6.51	0.000	
TEMP	-0.05283	0.00773	(-0.07065, -0.03501)	-6.84	0.000	1.00

Regression Equation

PROPPASS = -13.49 - 0.05283 TEMP

The fitted regression line is $\hat{y} = -13.49 - 0.0528x$.

$\hat{\beta}_0 = -13.49$ Since 0 is not within the range of observed value of temperature, $\hat{\beta}_0$ has no meaning.

$\hat{\beta}_1 = -0.0528$ For each degree increase in temperature, the mean proportion of impurity is estimated to decrease by 0.0528.

b. From the printout, the 95% confidence interval for β_1 is $(-0.07065, -0.03501)$. We estimate the mean proportion of impurity will decrease by anywhere from 0.07065 and 0.0351 for each degree increase in temperature. Because 0 is not contained in this interval, there is evidence to indicate that temperature contributes information about the proportions of impurity passing through helium.

c. From the printout, $r^2 = 0.8538$. 85.38% of the variability in the proportion of impurity passing through helium around their means is explained by the linear relationship between the temperature and the proportion of impurity.

d. Using MINITAB, the prediction interval is:

Settings

Variable	Setting
TEMP	-273

Prediction

Fit	SE Fit	95% CI	95% PI
0.931953	0.0557562	(0.803379, 1.06053)	(0.598655, 1.26525)

The 95% prediction interval is $(0.5987, 1.2653)$. We are 95% confident that the actual proportion of impurities will be between 0.5987 and 1.2653 when the temperature is -273 degrees. Since the proportion cannot be greater than 1, the interval really is $(0.5987, 1.0)$.

e. We have no idea what the relationship between temperature and proportion of impurity looks like outside the observed range.

3.87 a. Piano: $r = 0.447$
Because this value is near 0.5, there is a slight positive linear relationship between recognition exposure time and goodness of view for piano.

Bench: $r = -0.057$
Because this value is extremely close to 0, there is an extremely weak negative linear relationship between recognition exposure time and goodness of view for bench.

Motorbike: $r = 0.619$
Because this value is near 0.5, there is a moderate positive linear relationship between recognition exposure time and goodness of view for motorbike.

Armchair: $r = .294$
Because this value is fairly close to 0, there is a weak positive linear relationship between recognition exposure time and goodness of view for armchair.

Teapot: $r = 0.949$
Because this value is very close to 1, there is a strong positive linear relationship between recognition exposure time and goodness of view for teapot.

b. Piano: $r^2 = (0.447)^2 = 0.1998$
19.98% of the total sample variability around the sample mean recognition exposure time is explained by the linear relationship between the recognition exposure time and the goodness of view for piano.

Bench: $r^2 = (-0.057)^2 = 0.0032$
0.32% of the total sample variability around the sample mean recognition exposure time is explained by the linear relationship between the recognition exposure time and the goodness of view for bench.

Motorbike: $r^2 = (0.619)^2 = 0.3832$

38.32% of the total sample variability around the sample mean recognition exposure time is explained by the linear relationship between the recognition exposure time and the goodness of view for motorbike.

Armchair: $r^2 = (0.294)^2 = 0.0864$

8.64% of the total sample variability around the sample mean recognition exposure time is explained by the linear relationship between the recognition exposure time and the goodness of view for armchair.

Teapot: $r^2 = (0.949)^2 = 0.9006$

90.06% of the total sample variability around the sample mean recognition exposure time is explained by the linear relationship between the recognition exposure time and the goodness of view for teapot.

c. The test is:

$$H_0: \beta_1 = 0$$
$$H_a: \beta_1 \neq 0$$

Following are the values of α an $t_{\alpha/2}$ that correspond to $df = n - 2 = 25 - 2 = 23$.

α	**0.20**	**0.10**	**0.05**	**0.02**	**0.01**	**0.002**	**0.001**
$t_{\alpha/2}$	1.319	1.714	2.069	2.500	2.807	3.485	3.767

Piano: $t = 2.40$

$2.069 < 2.40 < 2.500 \Rightarrow p \approx 0.025$

For levels of significance greater than $\alpha = 0.025$, H_0 can be rejected. There is sufficient evidence to indicate that there is a linear relationship between goodness of view and recognition exposure time for piano for $\alpha > 0.025$.

Bench: $t = 0.27$

$0.27 < 1.319 \Rightarrow p > 0.2$

H_0 is not rejected. There is insufficient evidence to indicate that there is a linear relationship between goodness of view and recognition exposure time for bench for $\alpha \leq 0.2$.

Motorbike: $t = 3.78$

$3.78 > 3.767 \Rightarrow p < 0.001$

H_0 can be rejected for $\alpha \geq 0.001$. There is sufficient evidence to indicate that there is a linear relationship between goodness of view and recognition exposure time for motorbike for $\alpha \geq 0.001$.

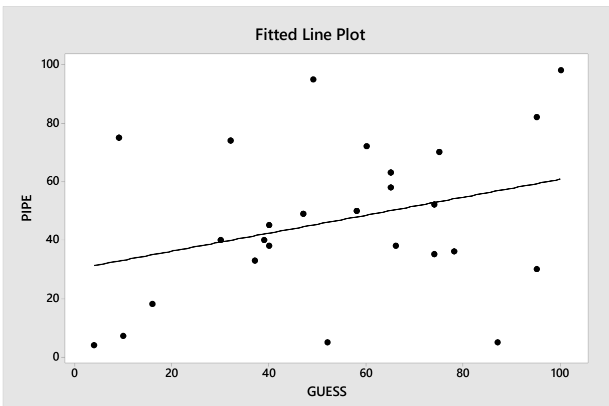
Armchair: $t = 1.47$

$1.319 < 1.47 < 1.717 \Rightarrow p \approx 0.15$

H_0 cannot be rejected for levels of significance $\alpha < 0.15$. There is insufficient evidence to indicate that there is a linear relationship between goodness of view and recognition exposure time for armchair for $\alpha < 0.15$.

Teapot: $t = 14.50$
$14.50 > 3.767 \Rightarrow p < 0.001$
H_0 can be rejected for $\alpha \geq 0.001$. There is sufficient evidence to indicate that there is a linear relationship between goodness of view and recognition exposure time for teapot for $\alpha \geq 0.001$.

3.88 a. Using MINITAB, the scatterplot of the data is:



There is a slight positive linear trend to the data.

b. Using MINITAB, the results are:

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	1779	1778.9	2.63	0.118
GUESS	1	1779	1778.9	2.63	0.118
Error	24	16261	677.6		
Lack-of-Fit	20	14728	736.4	1.92	0.278
Pure Error	4	1534	383.4		
Total	25	18040			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
26.0298	9.86%	6.11%	0.00%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	30.1	11.4	2.63	0.015	
GUESS	0.308	0.190	1.62	0.118	1.00

Regression Equation

PIPE = 30.1 + 0.308 GUESS

The fitted regression line is $\hat{y} = 30.1 + 0.308x$.

$\hat{\beta}_0 = 30.1$ Because 0 is not within the observed values of the dowser's guesses, $\hat{\beta}_0$ has no meaning.

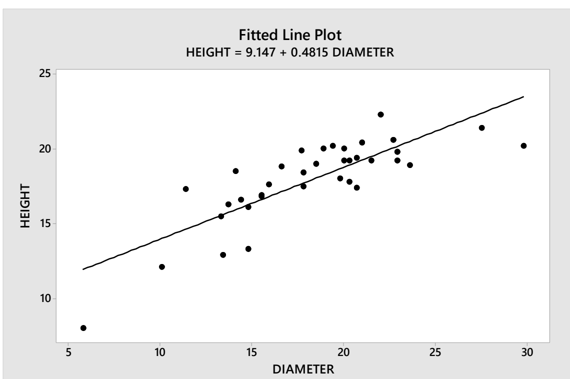
c. To determine if the model is statistically useful for predicting actual pipe location, we test:

$$H_0: \beta_1 = 0$$
$$H_a: \beta_1 \neq 0$$

The test statistic is $t = 1.62$ and the p-value is $p = 0.118$. Since the p-value is not small, H_0 is not rejected. There is insufficient evidence to indicate the model is statistically useful for predicting actual pipe location at $\alpha < 0.118$.

d. Since there is no statistical evidence that there is a linear relationship between the dowsers' guesses and the pipe location, this refutes the conclusion made by the German physicists. In addition, these were the 'best' results of the 'best' dowsers. If there was no relationship between the dowsers' guesses and the pipe location for the 'best' of the 'best', there will not be a relationship between dowsers' guesses and the pipe locations for all of the dowsers.

3.89 a. Using MINITAB, the scatterplot is:



There appears to be a positive linear relationship between breast height diameter and height.

b. Using MINITAB, the results are:

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	183.245	183.245	65.10	0.000
DIAMETER	1	183.245	183.245	65.10	0.000
Error	34	95.703	2.815		
Lack-of-Fit	27	87.893	3.255	2.92	0.073
Pure Error	7	7.810	1.116		
Total	35	278.947			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
1.67773	65.69%	64.68%	57.07%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	9.15	1.12	8.16	0.000	
DIAMETER	0.4815	0.0597	8.07	0.000	1.00

Regression Equation

HEIGHT = 9.15 + 0.4815 DIAMETER

The least squares line is $\hat{y} = 9.15 + 0.4815x$.

$\hat{\beta}_0 = 9.15$

$\hat{\beta}_1 = 0.4815$

c. The least squares line is printed on the scatterplot in part a.

d. To determine if the breast height diameter contributes information for the prediction of tree height, we test:

$$H_0 : \beta_1 = 0$$
$$H_a : \beta_1 \neq 0$$

The test statistic is $t = 8.07$ and the p-value is $p = 0.000$. Since the p-value is less than $\alpha \left(p = 0.000 < 0.05 \right)$, H_0 is rejected. There is sufficient evidence to indicate the breast height diameter contributes information for the prediction of tree height at $\alpha = 0.05$.

e. Using MINITAB, the results are:

Settings

Variable	Setting
DIAMETER	20

Prediction

Fit	SE Fit	90% CI	90% PI
18.7763	0.299602	(18.2697, 19.2829)	(15.8945, 21.6581)

The 90% confidence interval is $(18.2697, 19.2829)$. We are 90% confident that the mean height of trees is between 18.2697m and 19.2829m when the breast height diameter is 20cm.

Chapter

4

Multiple Regression Models

4.1 Associated with each independent variable in a multiple regression model is a $\beta-$coefficient. Since these coefficients are unknown, we estimate them using data from a sample of size *n*. Each estimate eliminates one degree of freedom available for estimating σ^2. Hence, if there are *k* independent variables in the model plus the *y*-intercept, β_0, then there will be $n-(k+1)$ degrees of freedom left to estimate σ^2.

4.2 a. The first order model is $E(y)=\beta_0+\beta_1x_1+\beta_2x_2+\beta_3x_3+\beta_4x_4+\beta_5x_5+\beta_6x_6$.

b. $\hat{\beta}_1$: For each unit increase in air quality, the mean satisfaction score is estimated to increase by 0.122.

$\hat{\beta}_2$: For each unit increase in temperature, the mean satisfaction score is estimated to increase by 0.018.

$\hat{\beta}_3$: For each unit increase in odor/aroma, the mean satisfaction score is estimated to increase by 0.124.

$\hat{\beta}_4$: For each unit increase in music, the mean satisfaction score is estimated to increase by 0.119.

$\hat{\beta}_5$: For each unit increase in noise/sound level, the mean satisfaction score is estimated to increase by 0.101.

$\hat{\beta}_6$: For each unit increase in overall image, the mean satisfaction score is estimated to increase by 0.463.

c. We are 99% confident that mean satisfaction will increase between 0.350 and 0.576 units for each unit increase in overall image.

d. $R^2_{adj}=0.501$. 50.1% of the sample variation in satisfaction scores is explained by the model including the 6 independent variables, adjusted for the number of variables and the sample size.

e. To determine if the overall model is useful for the predicting hotel image, we test:

$H_0: \beta_1=\beta_2=\beta_3=\beta_4=\beta_5=\beta_6=0$
H_a: At least 1 $\beta_i \neq 0$

The test statistic is $F = 71.42$. Using MINITAB with $\nu_1 = k = 6$ and $\nu_2 = n-(k+1) = 422-(6+1) = 415$, the *p*-value is $p = P(F > 71.42) \approx 0$. Since the *p*-value is less than $\alpha\left(p = 0 < 0.01\right)$, H_0 is rejected. There is sufficient evidence to indicate the overall model is useful for predicting hotel image at $\alpha = 0.01$.

4.3 a. From the output, the least squares prediction equation is

$$\hat{y} = -709 + 0.3968x_1 + 0.618x_2 + 0.819x_3 + 0.756x_4 + 1.561x_5 + 0.089x_6 + 1.080x_7 + 0.0175x_8 + 0.0422x_9.$$

b. $\hat{\beta}_1 = 0.3968$: For each additional walk, the mean number of runs scored is estimated to increase by 0.3968 when all other variables are held constant.

$\hat{\beta}_2 = 0.618$: For each additional single, the mean number of runs scored is estimated to increase by 0.618 when all other variables are held constant.

$\hat{\beta}_3 = 0.819$: For each additional double, the mean number of runs scored is estimated to increase by 0.819 when all other variables are held constant.

$\hat{\beta}_4 = 0.756$: For each additional triple, the mean number of runs scored is estimated to increase by 0.756 when all other variables are held constant.

$\hat{\beta}_5 = 1.561$: For each additional home run, the mean number of runs scored is estimated to increase by 1.561 when all other variables are held constant.

$\hat{\beta}_6 = 0.089$: For each additional stolen base, the mean number of runs scored is estimated to increase by 0.089 when all other variables are held constant.

$\hat{\beta}_7 = 1.080$: For each additional times caught stealing, the mean number of runs scored is estimated to increase by 1.080 when all other variables are held constant.

$\hat{\beta}_8 = 0.0175$: For each additional strike out, the mean number of runs scored is estimated to increase by 0.0175 when all other variables are held constant.

$\hat{\beta}_9 = 0.0422$: For each additional ground out, the mean number of runs scored is estimated to increase by 0.0422 when all other variables are held constant.

c. $H_0: \beta_7 = 0$
$H_a: \beta_7 > 0$

The test statistic is $t = 1.27$ and the *p*-value is $p = 0.218/2 = 0.109$. Since the *p*-value is not less than $\alpha\left(p = 0.109 \not< 0.05\right)$, H_0 is not rejected. There is insufficient evidence to indicate that there is a positive linear relationship between the number of runs scored and the number of times caught stealing at $\alpha = .05$.

d. From the printout, the 95% confidence interval for β_5 is $(1.103,\ 2.019)$. We are 95% confident that for each additional home run, the mean number of runs scored will increase by anywhere from 1.103 to 2.019, holding all the other variables constant.

e. Answers will vary.

4.4 a. The model is $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \varepsilon$.

b. To determine if the overall model is useful, we test:

$$H_0: \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$$
$$H_a: \text{At least one } \beta_i \neq 0$$

The test statistic is $F = 4.74$ and the p-value is $p < 0.01$. Since the p-value is less than $\alpha\,(p < 0.01 < 0.05)$, H_0 is rejected. There is sufficient evidence to indicate the model is useful for predicting Machiavellian scores at $\alpha = 0.05$.

c. $R^2 = 0.13$. 13% of the total sample variation in Machiavellian scores is explained by the relationship between the Machiavellian scores and the 4 independent variables.

d. To determine if income is statistically useful predictor of Machiavellian scores, we test:

$$H_0: \beta_4 = 0$$
$$H_a: \beta_4 \neq 0$$

The p-value is $p > 0.10$. Since the p-value is not less than $\alpha\,(p > 0.10 \not< 0.05)$, H_0 is not rejected. There is insufficient evidence to indicate that income is useful for predicting Machiavellian scores adjusted for the other 3 independent variables at $\alpha = 0.05$.

4.5 a. $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$

b. $R^2 = 0.08$ implies that 8% of the sample variation in frequency of marijuana use is explained by the model.

c. The p-value is $p < 0.01$. Since the p-value is so small, H_0 is rejected. There is sufficient evidence to indicate that the model is useful for predicting frequency of marijuana use for any value of $\alpha \geq 0.01$.

d. The p-value is $p < 0.01$. Since the p-value is so small, H_0 is rejected. There is sufficient evidence to indicate that severity of inattention is useful for predicting frequency of marijuana use, adjusted for the other 2 independent variables for any value of $\alpha \geq 0.01$.

e. The p-value is $p > 0.05$. Since the p-value is not small, H_0 is not rejected. There is insufficient evidence to indicate that severity of impulsivity-hyperactivity is useful for predicting frequency of marijuana use, adjusted for the other 2 independent variables for any value of $\alpha \leq 0.05$.

f. The p-value is $p > 0.05$. Since the p-value is not small, H_0 is not rejected. There is insufficient evidence to indicate that level of oppositional-defiant and conduct disorder is useful for predicting frequency of marijuana use, adjusted for the other 2 independent variables for any value of $\alpha \leq 0.05$.

4.6 a. The sum of the errors (SE) equals zero and the sum of squares of the errors (SSE) is minimized.

b. For every one unit increase in the betweenness centrality (number of shortest paths between peers), the estimate of the mean lead-user rating increases by 0.42.

c. The p-value is $p = 0.002$. Since the p-value is less than $\alpha \left(p = 0.002 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate that betweenness centrality is useful for predicting lead-user rating adjusted for the other 3 independent variables at $\alpha = 0.05$.

4.7 a. To determine if the overall model is adequate, we test:

$$H_0: \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$$
$$H_a: \text{At least one } \beta_i \neq 0$$

The test statistic is $F = 4.38$ and the p-value is $p = 0.0907$. Since the p-value is less than $\alpha \left(p = 0.0907 < 0.10\right)$, H_0 is rejected. There is sufficient evidence to indicate the model is useful for predicting grafting efficiency at $\alpha = 0.10$.

b. $R_a^2 = 0.6286$. 62.86% of the total variation in grafting efficiency values is explained by the model including the 4 independent variables.

c. $s = 11.22062$. Most of the observed values of grafting efficiency will fall within $2s = 2\left(11.22062\right) = 22.44$ units of their predicted values.

d. The 90% confidence interval for β_3 is $\left(-0.21804, 1.08404\right)$. We are 90% confident that for each degree increase in reaction temperature, the mean value of grafting efficiency will change anywhere from -0.218 and 1.084.

e. To determine if reaction time is statistically useful predictor of grafting efficiency, we test:

$$H_0: \beta_4 = 0$$
$$H_a: \beta_4 \neq 0$$

The test statistic is $t = -0.74$ and the p-value is $p = 0.5027$. Since the p-value is not small, H_0 is not rejected. There is insufficient evidence to indicate that reaction time is statistically useful predictor of grafting efficiency adjusted for the other 3 independent variables for any reasonable value of α.

4.8 a. To determine if the overall model is useful for predicting the percentage of silver in the alloy, we test:

$H_0: \beta_1 = \beta_2 = \beta_3 = 0$
H_a : At least one $\beta_i \neq 0$

The p-value is $p = 0.049$. Since the p-value is less than $\alpha\left(p = 0.049 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate the overall model is useful for predicting the percentage of silver in the alloy at $\alpha = 0.05$.

$R^2 = 0.075$. 7.5% of the total sample variation of the percentages of silver in the alloy can be explained by the model containing the 3 independent variables.

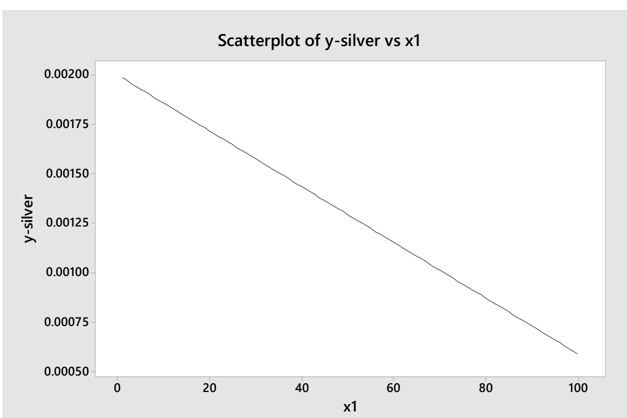
b. To determine if the overall model is useful for predicting the percentage of iron in the alloy, we test:

$H_0: \beta_1 = \beta_2 = \beta_3 = 0$
H_a : At least one $\beta_i \neq 0$

The p-value is $p < 0.001$. Since the p-value is less than $\alpha\left(p < 0.001 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate the overall model is useful for predicting the percentage of iron in the alloy at $\alpha = 0.05$.

$R^2 = 0.783$. 78.3% of the total sample variation of the percentages of iron in the alloy can be explained by the model containing the 3 independent variables.

c. Using MINITAB, the relationship between percentage of silver and proportion of aluminum scraps from cans could look something like:

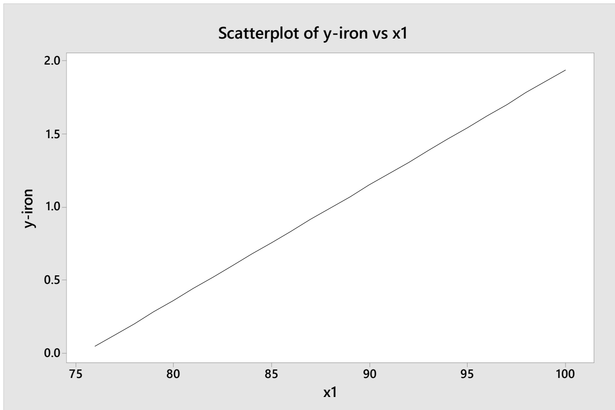


To determine if the relationship between percentage of silver and the proportion of aluminum scraps from cans is significant, we test:

$H_0: \beta_1 = 0$
$H_a: \beta_1 \neq 0$

The p-value is $p = 0.015$. Since the p-value is less than $\alpha\left(p = 0.015 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate the relationship between percentage of silver and the proportion of aluminum scraps from cans is significant at $\alpha = 0.05$.

d. Using MINITAB, the relationship between percentage of iron and proportion of aluminum scraps from cans could look something like:



To determine if the relationship between percentage of iron and the proportion of aluminum scraps from cans is significant, we test:

$$H_0 : \beta_1 = 0$$
$$H_a : \beta_1 \neq 0$$

The p-value is $p < 0.001$. Since the p-value is less than $\alpha\left(p < 0.001 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate the relationship between percentage of iron and the proportion of aluminum scraps from cans is significant at $\alpha = 0.05$.

4.9 a. $\hat{\beta}_1 = 2.006$. For every unit increase in the proportion of block with low density, the mean population density is estimated to increase by 2.006 units, holding high density constant. $\hat{\beta}_2 = 5.006$. For every unit increase in proportion of block with high density, the mean population density is estimated to increase by 5.006 units, holding low density constant.

b. $R^2 = 0.686$. 68.6% of the sample variation in population density is explained by the model containing the 2 independent variables.

c. $H_0 : \beta_1 = \beta_2 = 0$
H_a : At least one $\beta_i \neq 0$

d. $$F = \frac{R^2 / k}{\left(1 - R^2\right) / \left[n - \left(k + 1\right)\right]} = \frac{0.686 / 2}{\left(1 - 0.686\right) / \left[125 - \left(2 + 1\right)\right]} = 133.27$$

e. The rejection region requires $\alpha = 0.01$ in the upper tail of the F distribution with $\nu_1 = k = 2$ and $\nu_2 = n - \left(k + 1\right) = 125 - \left(2 + 1\right) = 122$. Using Table 6, Appendix D, $F = 4.79$. The rejection region is $F > 4.79$.

Since the observed value of the test statistic falls in the rejection region $(F = 133.27 > F = 4.79)$, H_0 is rejected. There is sufficient evidence to conclude that the model containing 2 independent variables is adequate for predicting population density at $\alpha = 0.01$.

4.10 a. The model is $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \varepsilon$.

b. The p-value is $p = 0.005$. Since the p-value is less than $\alpha\,(p = 0.005 < 0.01)$, H_0 is rejected. There is sufficient evidence to indicate there is a negative linear relationship between the number of years played golf and change from routine adjusted for the other independent variables at $\alpha = 0.01$.

c. Yes, since the estimate of β_3 was negative implying that the greater the number of years playing golf the smaller the change of routine score.

d. $H_0: \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$
H_a : At least one $\beta_i \neq 0$

e. The rejection region requires $\alpha = 0.01$ in the upper tail of the F distribution with $\nu_1 = k = 4$ and $\nu_2 = n - (k+1) = 393 - (4+1) = 388$. Using MINITAB, $F = 3.37$. The rejection region is $F > 3.37$.

f. **Thrill**: The test statistic is $F = 5.56$. Since the observed value of the test statistic falls in the rejection region $(F = 5.56 > F = 3.37)$, H_0 is rejected. There is sufficient evidence to conclude that the model containing 4 independent variables is adequate for predicting thrill at $\alpha = 0.01$.

Boredom-alleviation: The test statistic is $F = 3.02$. Since the observed value of the test statistic does not fall in the rejection region $(F = 3.02 \not> F = 3.37)$, H_0 is not rejected. There is insufficient evidence to conclude that the model containing 4 independent variables is adequate for predicting boredom-alleviation at $\alpha = 0.01$.

Surprise: The test statistic is $F = 3.33$. Since the observed value of the test statistic does not fall in the rejection region $(F = 3.33 \not> F = 3.37)$, H_0 is not rejected. There is insufficient evidence to conclude that the model containing 4 independent variables is adequate for predicting surprise at $\alpha = 0.01$.

g. **Thrill**: The p-value is $p < 0.001$. Since the p-value is less than $\alpha\,(p < 0.001 < 0.01)$, H_0 is rejected.

Boredom-alleviation: The p-value is $p = 0.018$. Since the p-value is not less than $\alpha\,(p = 0.018 \not< 0.01)$, H_0 is not rejected.

Surprise: The p-value is $p = 0.011$. Since the p-value is not less than $\alpha\,(p = 0.011 \not< 0.01)$, H_0 is not rejected.

These all agree with the conclusions in part f.

h. **Thrill**: $R^2 = 0.055$. 5.5% of the total variation in the thrill scores is explained by the model containing the 4 independent variables.

Boredom-alleviation: $R^2 = 0.030$. 3.0% of the total variation in the boredom-alleviation scores is explained by the model containing the 4 independent variables.

Surprise: $R^2 = 0.023$. 2.3% of the total variation in the surprise scores is explained by the model containing the 4 independent variables.

4.11 a. $\hat{y} = 1.81231 + 0.10875x_1 + 0.00017x_2$

b. $\hat{\beta}_1 = 0.10875$. For every 1-mile increase in road length, the mean number of crashes is estimated to increase 0.109, holding AADT constant.

$\hat{\beta}_2 = 0.00017$. For every one-vehicle increase in AADT, the mean number of crashes is estimated to increase by 0.00017, holding length constant.

c. For confidence level 0.99, $\alpha = 0.01$ and $\alpha / 2 = 0.01/2 = 0.005$. Using MINITAB with $df = n-(k+1) = 100-(2+1) = 97$, $t_{0.005} = 2.627$. The 99% confidence interval is

$$\hat{\beta}_1 \pm t_{0.005} s_{\hat{\beta}_1} \Rightarrow 0.109 \pm 2.627(0.03166) \Rightarrow 0.109 \pm 0.083 \Rightarrow (0.026,\ 0.192)$$

We are 99% confident that the true increase in mean number of crashes when the length increases by 1 mile on the interstate highways is between 0.026 miles and 0.192 miles, holding AADT constant.

d. For confidence level 0.99, $\alpha = 0.01$ and $\alpha / 2 = 0.01/2 = 0.005$. Using MINITAB with $df = n-(k+1) = 100-(2+1) = 97$, $t_{0.005} = 2.627$. The 99% confidence interval is

$$\hat{\beta}_2 \pm t_{0.005} s_{\hat{\beta}_2} \Rightarrow 0.00017 \pm 2.627(0.00003) \Rightarrow 0.00017 \pm 0.00008 \Rightarrow (0.00009,\ 0.00025)$$ W

e are 99% confident that the true increase in mean number of crashes when AADT increases by 1 unit on the interstate highways is between 0.00009 miles and 0.00025 miles, holding length constant.

e. $\hat{y} = 1.20785 + 0.06343x_1 + 0.00056x_2$

$\hat{\beta}_1 = 0.06343$. For every 1-mile increase in road length, the mean number of crashes is estimated to increase 0.063, holding AADT constant.

$\hat{\beta}_2 = 0.00056$. For every one-vehicle increase in AADT, the mean number of crashes is estimated to increase by 0.00056, holding length constant.

$$\hat{\beta}_1 \pm t_{0.005} s_{\hat{\beta}_1} \Rightarrow 0.063 \pm 2.627(0.01809) \Rightarrow 0.063 \pm 0.048 \Rightarrow (0.015,\ 0.111)$$

We are 99% confident that the true increase in mean number of crashes when the length increases by 1 mile on the non-interstate highways is between 0.015 miles and 0.111 miles, holding AADT constant.

$$\hat{\beta}_2 \pm t_{0.005} s_{\hat{\beta}_2} \Rightarrow 0.00056 \pm 2.627(0.00012) \Rightarrow 0.00056 \pm 0.00032 \Rightarrow (0.00024, 0.00088)$$

We are 99% confident that the true increase in mean number of crashes when AADT increases by 1 unit on the non-interstate highways is between 0.00024 miles and 0.00088 miles, holding length constant.

4.12 a. Using MINITAB, the results of fitting the first-order model are:

Analysis of Variance

Source	DF	Seq SS	Contribution	Adj SS	Adj MS	F-Value	P-Value
Regression	4	827.83	49.76%	827.833	206.958	40.85	0.000
GENDER	1	674.64	40.55%	52.904	52.904	10.44	0.001
SELFESTM	1	57.66	3.47%	8.650	8.650	1.71	0.193
BODYSAT	1	19.62	1.18%	25.569	25.569	5.05	0.026
IMPREAL	1	75.91	4.56%	75.907	75.907	14.98	0.000
Error	165	835.95	50.24%	835.955	5.066		
Lack-of-Fit	144	620.62	37.30%	620.622	4.310	0.42	0.999
Pure Error	21	215.33	12.94%	215.333	10.254		
Total	169	1663.79	100.00%				

Model Summary

S	R-sq	R-sq(adj)	PRESS	R-sq(pred)
2.25087	49.76%	48.54%	885.289	46.79%

Coefficients

Term	Coef	SE Coef	95% CI	T-Value	P-Value	VIF
Constant	14.011	0.775	(12.480, 15.542)	18.07	0.000	
GENDER	-2.186	0.677	(-3.522, -0.851)	-3.23	0.001	3.56
SELFESTM	-0.0479	0.0367	(-0.1204, 0.0245)	-1.31	0.193	2.62
BODYSAT	-0.322	0.143	(-0.606, -0.039)	-2.25	0.026	6.16
IMPREAL	0.493	0.127	(0.242, 0.745)	3.87	0.000	1.03

Regression Equation

DESIRE = 14.011 - 2.186 GENDER - 0.0479 SELFESTM - 0.322 BODYSAT + 0.493 IMPREAL

The least squares prediction equation is
$\hat{y} = 14.011 - 2.186x_1 - 0.0479x_2 - 0.322x_3 + 0.493x_4$.

b. $\hat{\beta}_0 = 14.011$. This has no meaning other than the y-intercept.

$\hat{\beta}_1 = -2.186$. The mean value of desire to have cosmetic surgery is estimated to be 2.186 units lower for males than females, holding all other variables constant.

$\hat{\beta}_2 = -0.0479$. For each unit increase in self-esteem, the mean value of desire to have cosmetic surgery is estimated to decrease by 0.0479 units, holding all other variables constant.

$\hat{\beta}_3 = -0.322$. For each unit increase in body satisfaction, then mean value of desire to have cosmetic surgery is estimated to decrease by 0.322 units, holding all other variables constant.

$\hat{\beta}_4 = 0.493$. For each unit increase in impression of reality TV, the mean value of desire to have cosmetic surgery is estimated to increase by 0.493 units, holding all other variables constant.

c. To determine if the overall model is useful for predicting desire to have cosmetic surgery, we test:

$H_0: \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$
$H_a:$ At least one $\beta_i \neq 0$

From the printout, the test statistic is $F = 40.85$ and the p-value is $p = 0.000$.
Since the p-value is less than $\alpha\left(p = 0.000 < .01\right)$, H_0 is rejected. There is sufficient evidence to indicate the overall model is useful for predicting desire to have cosmetic surgery at $\alpha = 0.01$.

d. R_a^2 is the preferred measure of model fit. From the printout, $R_a^2 = 0.4854$. This indicates that 48.54% of the total sample variation in desire values is explained by the model containing gender, self-esteem, body satisfaction and impression of reality TV, adjusting for the sample size and the number of variables in the model.

e. To determine if the desire to have cosmetic surgery decreases linearly as level of body satisfaction increases, we test:

$H_0: \beta_3 = 0$
$H_a: \beta_3 < 0$

From the printout, the test statistic is $t = -2.25$ and the p-value is $p = 0.026/2 = 0.013$.

Since the p-value is less than $\alpha\left(p = 0.013 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate the desire to have cosmetic surgery decreases linearly as level of body satisfaction increases, holding all other variables constant at $\alpha = 0.05$.

f. From the printout, the 95% confidence interval is $\left(0.242,\ 0.745\right)$.

We are 95% confident that the increase in mean desire for cosmetic surgery is between 0.242 and 0.745 for each unit increase in impression of reality TV, holding all other variables constant.

4.13 a. Using MINITAB, the results are:

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	2	0.11914	0.05957	1.00	0.391
MassFlux	1	0.10832	0.10832	1.82	0.198
HeatFlux	1	0.01082	0.01082	0.18	0.676
Error	15	0.89350	0.05957		
Total	17	1.01264			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.244063	11.77%	0.00%	0.00%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	1.088	0.184	5.92	0.000	
MassFlux	-0.000234	0.000174	-1.35	0.198	1.00
HeatFlux	-0.080	0.188	-0.43	0.676	1.00

Regression Equation

Diameter = 1.088 - 0.000234 MassFlux - 0.080 HeatFlux

The fitted model is $\hat{y} = 1.088 - 0.000234x_1 - 0.080x_2$.

To determine if the overall model is adequate, we test:

$$H_0: \beta_1 = \beta_2 = 0$$
$$H_a: \text{At least one } \beta_i \neq 0$$

The test statistic is $F = 1.00$ and the p-value is $p = 0.391$. Since the p-value is not small, H_0 is not rejected. There is insufficient evidence to indicate the overall model is adequate for any $\alpha < 0.391$.

b. Using MINITAB, the results are:

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	2	1.92985E+11	96492253030	121.31	0.000
MassFlux	1	6614715359	6614715359	8.32	0.011
HeatFlux	1	1.86370E+11	1.86370E+11	234.31	0.000
Error	15	11931113453	795407564		
Total	17	2.04916E+11			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
28203.0	94.18%	93.40%	91.27%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	-1030	21237	-0.05	0.962	
MassFlux	-57.9	20.1	-2.88	0.011	1.00
HeatFlux	332037	21692	15.31	0.000	1.00

Regression Equation

Density = -1030 - 57.9 MassFlux + 332037 HeatFlux

The fitted model is $\hat{y} = -1,030 - 57.9x_1 + 332,037x_2$. .

To determine if the overall model is adequate, we test:

$$H_0 : \beta_1 = \beta_2 = 0$$
$$H_a : \text{At least one } \beta_i \neq 0$$

The test statistic is $F = 121.31$ and the p-value is $p = 0.000$. Since the p-value is so small, H_0 is rejected. There is sufficient evidence to indicate the overall model is adequate for any reasonable value of α.

c. The density is better predicted by mass flux and heat flux. The model predicting diameter is not adequate and should not be used, while the model predicting density is adequate.

4.14 a. $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$ where x_1 = latitude, x_2 = longitude, and x_3 = depth.

b. Using MINITAB, the results of the regression analysis are:

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	3	505770	168590	15.80	0.000
LATITUDE	1	189284	189284	17.74	0.000
LONGITUDE	1	182342	182342	17.09	0.000
DEPTH-FT	1	53179	53179	4.98	0.026
Error	323	3446791	10671		
Total	326	3952562			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
103.301	12.80%	11.99%	10.88%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	-86868	31224	-2.78	0.006	
LATITUDE	-2219	527	-4.21	0.000	1.20
LONGITUDE	1542	373	4.13	0.000	1.31
DEPTH-FT	-0.350	0.157	-2.23	0.026	1.22

Regression Equation

ARSENIC = -86868 - 2219 LATITUDE + 1542 LONGITUDE - 0.350 DEPTH-FT

The fitted regression line is $\hat{y} = -86,868 - 2,219x_1 + 1,542x_2 - 0.350x_3$.

c. $\hat{\beta}_0 = -86,868$. The only interpretation for $\hat{\beta}_0$ is that it is the y-intercept.

$\hat{\beta}_1 = -2,219$. We estimate that the mean arsenic level will decrease by 2,219 μg / liter for each additional degree increase in latitude.

$\hat{\beta}_2 = 1,542$. We estimate that the mean arsenic level will increase by 1,542 μg / liter for each additional degree increase in longitude.

$\hat{\beta}_3 = -0.350$. We estimate that the mean arsenic level will decrease by 0.350 μg / liter for each additional foot increase in depth.

d. $s = 103.301$. We would expect that most observed values of arsenic levels to fall within $2s = 2(103.301) = 206.602$ units of their predicted values.

e. $R^2 = 0.1280$. 12.80% of the total variation in the arsenic levels is explained by the regression model containing latitude, longitude, and depth.

$R^2_{adj} = 0.1199$. 11.99% of the total variation in the arsenic levels is explained by the regression model containing latitude, longitude, and depth, adjusted for the number of independent variables in the model and the sample size.

f. To determine if the overall model is adequate, we test:

$$H_0: \beta_1 = \beta_2 = \beta_3 = 0$$
$$H_a: \text{At least one } \beta_i \neq 0$$

The test statistic is $F = 15.80$ and the p-value is $p = 0.000$. Since the p-value is less than $\alpha\ (p = 0.000 < 0.05)$, H_0 is rejected. There is sufficient evidence to indicate the overall model is adequate for predicting arsenic levels at $\alpha = 0.05$.

g. This model is questionable. Even though the model is adequate for predicting arsenic levels, the R^2 level is quite low. Only 12.8% of the variation in the arsenic levels is explained by the model. In addition, the standard deviation is quite large.

4.15 a. $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5$

b. Using MINITAB, the results of the regression analysis are:

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	5	155055273	31011055	147.30	0.000
RPM	1	8574188	8574188	40.73	0.000
INLET-TEMP	1	7929432	7929432	37.67	0.000
EXH-TEMP	1	3641364	3641364	17.30	0.000
CPRATIO	1	30	30	0.00	0.991
AIRFLOW	1	774427	774427	3.68	0.060
Error	61	12841935	210524		
Total	66	167897208			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
458.828	92.35%	91.72%	90.35%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	13614	870	15.65	0.000	
RPM	0.0888	0.0139	6.38	0.000	2.99
INLET-TEMP	-9.20	1.50	-6.14	0.000	13.31
EXH-TEMP	14.39	3.46	4.16	0.000	7.32
CPRATIO	0.4	29.6	0.01	0.991	4.93
AIRFLOW	-0.848	0.442	-1.92	0.060	3.15

Regression Equation

HEATRATE = 13614 + 0.0888 RPM - 9.20 INLET-TEMP + 14.39 EXH-TEMP + 0.4 CPRATIO - 0.848 AIRFLOW

The fitted regression model is $\hat{y} = 13,614 + 0.0888x_1 - 9.20x_2 + 14.39x_3 + 0.4x_4 - 0.848x_5$.

c. $\hat{\beta}_0 = 13,614$. This is simply the estimate of the y-intercept.

$\hat{\beta}_1 = 0.0888$. We estimate that the mean heat rate will increase by 0.0888 kJ/kW-hr for each additional increase in revolutions per minute, holding all other variables constant.

$\hat{\beta}_2 = -9.20$. We estimate that the mean heat rate will decrease by 9.20 kJ/kW-hr for each additional degree increase in inlet-temperature, holding all other variables constant.

$\hat{\beta}_3 = 14.39$. We estimate that the mean heat rate will increase by 14.39 kJ/kW-hr for each additional degree increase in exhaust-temperature, holding all other variables constant.

$\hat{\beta}_4 = -0.848$. We estimate that the mean heat rate will decrease by 0.848 kJ/kW-hr with every one kg/sec increase in mass air flow, holding all other variables constant.

d. $s = 458.828$. Almost all of the observed heat rates will fall within $2s = 2(458.828) = 917.656$ kJ/kW-hr of the model predicted values.

e. $R_a^2 = 0.9172$. 91.72% of the total variation in heat rates is explained by the model containing the 5 independent variables, adjusted for the number of independent variables in the model and the sample size.

f. To determine if the model is adequate for predicting heat rates, we test:

$$H_0: \beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = 0$$
$$H_a: \text{ At least one } \beta_i \neq 0$$

The test statistic is $F = 147.30$ and the p-value is $p = 0.000$. Since the p-value is less than $\alpha\left(p = 0.000 < 0.01\right)$, H_0 is rejected. There is sufficient evidence to indicate that the model is useful in predicting the heat rate at $\alpha = 0.01$.

4.16 a. The least squares prediction equation is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_6$

b. To determine the overall model adequacy, we test:

$$H_0: \beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = \beta_6 = 0$$
$$H_a: \text{ At least one } \beta_i \neq 0$$

c. The test statistic is $F = 32.47$ and the p-value is $p < 0.001$. Since the p-value is less than $\alpha\left(p < 0.001 < 0.01\right)$, H_0 is rejected. There is sufficient evidence to indicate that the model is adequate for predicting the urban/rural rating at $\alpha = 0.01$.

d. $R^2 = 0.44$. 44% of the total variation in the urban/rural ratings is explained by the regression model containing the 6 independent variables.

$R_{adj}^2 = 0.43$. 43% of the total variation in the urban/rural ratings is explained by the regression model containing the 6 independent variables, adjusted for the number of independent variables in the model and the sample size.

e. The null hypothesis would be $H_0: \beta_4 = 0$.

f. To determine population growth contributes to the model, we test:

$$H_0: \beta_4 = 0$$
$$H_a: \beta_4 \neq 0$$

The p-value is $p = 0.860$. Since the p-value is not less than $\alpha\left(p = 0.860 \nless 0.01\right)$, H_0 is not rejected. There is insufficient evidence to indicate that population growth contributes to the model at $\alpha = 0.01$.

4.17 a. To determine if the model is important for predicting IQ, we test:

$H_0: \beta_1 = \beta_2 = \ldots = \beta_{18} = 0$
H_a: At least one $\beta_i \neq 0$

The test statistic is $F = \dfrac{R^2 / k}{(1-R^2)/[n-(k+1)]} = \dfrac{0.95/18}{(1-0.95)/\left[20-(18+1)\right]} = 1.056.$

The rejection region requires $\alpha = 0.05$ in the upper tail if the F distribution. Using MINITAB, with $v_1 = k = 18$ and $v_2 = n-(k+1) = 20-(18+1) = 1$, $F = 247.32$. The rejection region is $F > 247.32$.

Since the observed value of the test statistic does not fall in the rejection region $(F = 1.056 \not> 247.32)$, H_0 is not rejected. There is insufficient evidence to indicate that at least one variable is important for predicting IQ at $\alpha = 0.05$. What happened? The inclusion of 18 independent variables in the model reduces the number of degrees of freedom in the denominator to 1. The result is an inflation of the critical value, which makes it difficult to reject H_0. In order to have more degrees of freedom available for estimating σ^2, the researcher should either collect more data or include fewer independent variables in the model.

b. $R_a^2 = 1 - \dfrac{(n-1)}{n-(k+1)}(1-R^2) = 1 - \dfrac{20-1}{20-(18+1)}(1-0.95) = 0.05$

After adjusting for the sample size and the number of parameters in the model, approximately 5% of the sample variation in IQ is "explained" by the model.

4.18 a. For $x_1 = 10$, $x_2 = 0$, and $x_3 = 1$, $\hat{y} = 52{,}484 + 2{,}941(10) + 16{,}880(0) + 11{,}108(1) = 93{,}002$..

b. For $x_1 = 10$, $x_2 = 1$, and $x_3 = 0$, $\hat{y} = 52{,}484 + 2{,}941(10) + 16{,}880(1) + 11{,}108(0) = 98{,}774$.

c. A 95% prediction interval is preferred over point estimates because point estimates do not take into account variability. The prediction intervals take into account the variability in the salaries of individuals with the same qualifications.

4.19 From Exercise 4.3, the prediction equation is

$$\hat{y} = -709 + 0.3968x_1 + 0.618x_2 + 0.819x_3 + 0.756x_4 + 1.561x_5 + 0.089x_6 + 1.080x_7 + 0.0175x_8 + 0.0422x_9.$$

Answers will vary.

4.20 a. For $x_1 = 1$, $x_2 = 10$, $x_3 = 5$, and $x_4 = 2$,
$\hat{y} = 3.58 + 0.01(1) - 0.06(10) - 0.01(5) + 0.42(2) = 3.78.$

b. For $x_1 = 0$, $x_2 = 8$, $x_3 = 10$, and $x_4 = 4$,
$\hat{y} = 3.58 + 0.01(0) - 0.06(8) - 0.01(10) + 0.42(4) = 4.68.$

4.21 The 90% prediction interval for the 6th observation is $(8.3999, 65.8979)$. We are 90% confident that the actual grafting efficiency of chemical run with initial concentration of 2, cardanol concentration of 15, reaction temperature of 35, and reaction time of 8 is between 8.3999 and 65.8979. It appears that the maximum grafting efficiency is predicted when initial concentration of 1, cardanol concentration of 5, reaction temperature of 35, and reaction time of 6 because the lower and upper limits of the prediction interval for these settings are the highest of all prediction intervals.

4.22 a. The confidence interval is $(13.42, 14.31)$. We are 95% confident that the mean desire to have cosmetic surgery is between 13.42 and 14.31 for females with a self-esteem of 24, body satisfaction of 3, and impression of reality TV of 4.

b. The confidence interval is $(8.79, 10.89)$. We are 95% confident that the mean desire to have cosmetic surgery is between 8.79 and 10.89 for males with a self-esteem of 22, body satisfaction of 9, and impression of reality TV of 4.

4.23 According to the analysis on the MINITAB printout below we can predict the arsenic level for the lowest latitude (23.7547), the highest longitude (90.6617), and the lowest depth (25.0).

Prediction for ARSENIC

Regression Equation

ARSENIC = -86868 - 2219 LATITUDE + 1542 LONGITUDE - 0.350 DEPTH-FT

Settings

Variable	Setting
LATITUDE	23.7547
LONGITUDE	90.6617
DEPTH-FT	25

Prediction

Fit	SE Fit	95% CI	95% PI
232.535	23.2692	(186.757, 278.314)	(24.2148, 440.856)

With 95% confidence we can predict that the arsenic level is between 24.2148 and 440.856 when the lowest latitude is 23.7547, the highest longitude is 90.6617, and the least depth is 25.0.

4.24 a. With 95% confidence we can predict that the heat rate level is between 11,599.6 and 13,665.5 for RPM = 7500, INLET-TEMP = 1000, EXH-TEMP = 525, CPRATIO = 13.5, and AIRFLOW = 10.0.

b. With 95% confidence we can say that the mean heat rate level is between 12,157.9 and 13,107.1 for RPM = 7500, INLET-TEMP = 1000, EXH-TEMP = 525, CPRATIO = 13.5, and AIRFLOW = 10.0.

c. The confidence interval for $E(y)$ will always be narrower than the corresponding prediction interval for a single point. The variance for a single point includes the variation for locating the mean plus the variation of the y's once the mean has been located. The variance for $E(y)$ only includes the variation for locating the mean.

4.25 a. The first-order model would be $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$.

b. Using MINITAB, the results are:

Regression Analysis: Project versus IntraPers, StressMan, Mood
Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	3	70.29	23.429	2.66	0.077
IntraPers	1	44.25	44.251	5.03	0.037
StressMan	1	34.19	34.186	3.89	0.063
Mood	1	10.48	10.482	1.19	0.289
Error	19	167.08	8.794		
Total	22	237.37			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
2.96544	29.61%	18.50%	3.79%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	86.90	3.20	27.17	0.000	
IntraPers	-0.2099	0.0936	-2.24	0.037	1.06
StressMan	0.1515	0.0769	1.97	0.063	1.06
Mood	0.0733	0.0671	1.09	0.289	1.09

Regression Equation

Project = 86.90 - 0.2099 IntraPers + 0.1515 StressMan + 0.0733 Mood

The fitted regression model is $\hat{y} = 86.90 - 0.2099x_1 + 0.1515x_2 + 0.0733x_3$.

c. To determine if the overall model is useful for prediction y, we test:

$$H_0 : \beta_1 = \beta_2 = \beta_3 = 0$$
$$H_a : \text{At least one } \beta_i \neq 0$$

The test statistic is $F = 2.66$ and the p-value is $p = 0.077.$. Since the p-value is less than $\alpha\left(p = 0.077 < 0.10\right)$, H_0 is rejected. There is sufficinet evidence to indicate the overall model is useful for predicting y at $\alpha = 0.10$.

d. $R^2_{adj} = 0.1850$. 18.5% of the sample variation in the project scores is explained by the model containing the 3 variables, adjusted for the simple size and the number of parameters in the model.

$s = 2.96544$ and $2s = 2\left(2.96544\right) = 5.93$. Approximately 95% of the observed values of project scores will fall within 5.93 units of their predicted values.

e. Using MINITAB, the resutls are:

Prediction for Project
Regression Equation

Project = 86.90 - 0.2099 IntraPers + 0.1515 StressMan + 0.0733 Mood

Settings

Variable	Setting
IntraPers	20
StressMan	30
Mood	25

Prediction

Fit	SE Fit	95% CI	95% PI
89.0837	0.892843	(87.2149, 90.9524)	(82.6017, 95.5656)

The 95% prediction interval is $(82.60, 95.57)$. We are 95% confident that the actual project score will be between 82.60 and 95.57 when the range of interpersonal scores is 20, the range of management scores is 30, and the range of mood scores is 25.

4.26 a. The model would be $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2$.

b. The model would be $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2$.

c. Based on the graph the better model would be $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2$. The graph indicates that the effect of size of the party on the size of the tip depends on whether a compliment was given.

4.27 a. To determine the overall adequacy of the model, we test:

$$H_0: \beta_1 = \beta_2 = \beta_3 = 0$$
$$H_a: \text{At least one } \beta_i \neq 0$$

The test statistic is $F = 31.98$ and the *p*-value is $p < 0.001$. Since the *p*-value is less than $\alpha \left(p < 0.001 < 0.01\right)$, H_0 is rejected. There is sufficient evidence to indicate the overall model is adequate for predicting the severity of tilting at $\alpha = 0.01$.

b. To determine if the rate of change of severity of tilting with perceived effect of experience on tilting depends on poker experience, we test:

$$H_0: \beta_3 = 0$$
$$H_a: \beta_3 \neq 0$$

The test statistic is $t = 5.61$ and the *p*-value is $p < 0.001$. Since the *p*-value is less than $\alpha \left(p < 0.001 < 0.01\right)$, H_0 is rejected. There is sufficient evidence to indicate the rate of change of severity of tilting with perceived effect of experience on tilting depends on poker experience at $\alpha = 0.01$.

4.28 a. To determine the overall adequacy of the model, we test:

$H_0: \beta_1 = \beta_2 = \beta_3 = 0$
H_a : At least one $\beta_i \neq 0$

The test statistic is $F = 226.35$ and the p-value is $p < 0.001$. Since the p-value is less than $\alpha\left(p < 0.001 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate the overall model is adequate for predicting y at $\alpha = 0.05$.

b. To determine if interaction exists, we test:

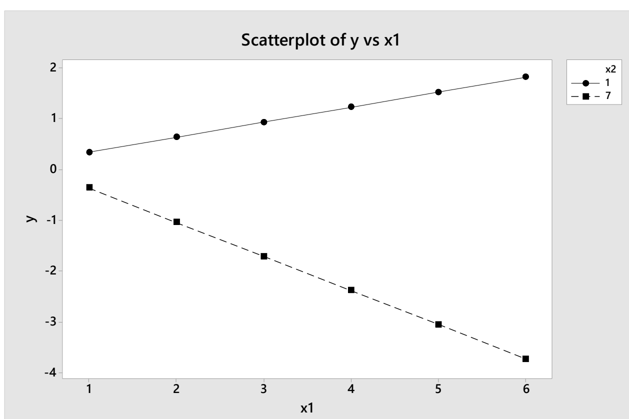
$H_0: \beta_3 = 0$
$H_a: \beta_3 \neq 0$

The test statistic is $t = -3.09$ and the p-value is $p < 0.01$. Since the p-value is less than $\alpha\left(p < 0.01 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate interaction is present at $\alpha = 0.05$.

c. The prediction line when $x_2 = 1$ is
$\hat{y} = \hat{\beta}_0 + 0.426x_1 + 0.044(1) - 0.157x_1(1) = \hat{\beta}_0 + 0.044 + 0.269x_1$.

d. The prediction line when $x_2 = 7$ is
$\hat{y} = \hat{\beta}_0 + 0.426x_1 + 0.044(7) - 0.157x_1(7) = \hat{\beta}_0 + 0.308 - 0.673x_1$.

e. A possible graph is:



4.29 a. $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2$

b. If the 2 variables interact, then the linear relationship between negative feelings score and number ahead in line depends on number behind in line.

c. The p-value is $p > 0.25$. Since the p-value is not small, H_0 is not rejected. There is insufficient evidence to indicate that the number ahead in line and the number behind in line interact to affect negative feelings score for any reasonable level of α.

d. $\beta_1 > 0$ should be positive and $\beta_2 < 0$ should be negative.

4.30 a. The least squares prediction equation is $\hat{y} = 11.779 - 1.972x_1 + 0.585x_4 - 0.553x_1x_4$.

b. For $x_1 = 1$ and $x_4 = 5$, $\hat{y} = 11.779 - 1.972(1) + 0.585(5) - 0.553(1)(5) = 9.967$.

c. To determine if the model is adequate, we test:

$H_0 : \beta_1 = \beta_2 = \beta_3 = 0$
H_a : At least $1 \beta_i \neq 0$

The test statistic is $F = 45.086$ and the p-value is $p = 0.000$.

Since the p-value is less than $\alpha(p = 0.000 < 0.10)$, H_0 is rejected. There is sufficient evidence to indicate the model is adequate in predicting desire to have cosmetic surgery at $\alpha = 0.10$.

d. $R_a^2 = 0.439$. 43.9% of the sample variation in the desire to have cosmetic surgery around its mean is explained by the model containing gender, impression of reality TV and the interaction of the two variables, adjusted for the number of terms in the model and the sample size.

e. $s = 2.350$. Most of the observed values of desire will fall within $2s = 2(2.350) = 4.70$ units of their predicted values.

f. To determine if gender and impression of reality TV interact, we test:

$H_0 : \beta_3 = 0$
$H_a : \beta_3 \neq 0$

The test statistic is $t = -2.004$ and the p-value is $p = 0.047$. .

Since the p-value is less than $\alpha(p = 0.047 < 0.10)$, H_0 is rejected. There is sufficient evidence to indicate gender and impression of reality TV interact to affect desire to have cosmetic surgery at $\alpha = 0.10$.

4.31 a. The first-order model would be $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4$.

b. The β -coefficient that measures the effect of flexibility on relationship quality independently of the other independent variables is β_1.

c. The β -coefficient that measures the effect of reputation on relationship quality independently of the other independent variables is β_2.

The β -coefficient that measures the effect of empathy on relationship quality independently of the other independent variables is β_3.

The β -coefficient that measures the effect of task alignment on relationship quality independently of the other independent variables is β_4.

d. The interaction model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_1 x_4 + \beta_6 x_2 x_4 + \beta_7 x_3 x_4$.

e. The null hypothesis to determine if the effect of flexibility (x_1) on relationship quality (y) depends on task alignment (x_4) would be $H_0 : \beta_5 = 0$.

f. The null hypothesis to determine if the effect of reputation (x_2) on relationship quality (y) depends on task alignment (x_4) would be $H_0 : \beta_6 = 0$.

The null hypothesis to determine if the effect of empathy (x_3) on relationship quality (y) depends on task alignment (x_4) would be $H_0 : \beta_7 = 0$.

g. Yes. Since none of the interaction terms are significant, there is no evidence to indicate the impact of each $x\,(x_1,\, x_2,\, \text{or } x_3)$ on y depends on x_4.

4.32 a. To determine if the overall model is statistically useful for predicting the day of the onset of the flight season, we test:

$$H_0 : \beta_1 = \beta_2 = \beta_3 = 0$$
$$H_a : \text{At least } 1\, \beta_i \neq 0$$

The p-value of the F-test is $p = 0.0032$.

Since the p-value is less than $\alpha\,(p = 0.0032 < 0.01)$, H_0 is rejected. There is sufficient evidence to indicate the overall model is statistically useful for predicting the day of the onset of the flight season at $\alpha = 0.01$.

b. $R^2 = 0.73$. 73% of the sample variation in the day of the onset of the flight season around its mean is explained by the model containing the first date when less than 10 cm of snow was measured, average July temperature and the interaction of the two variables.

c. Yes. Since H_0 is rejected, there is evidence that the interaction term is significant.

d. For $x_2 = 30$, the fitted model is
$\hat{y} = -144.2 + 2.05x_1 + 35.4(30) - 0.21x_1(30) = 917.8 - 4.25x_1$.
The estimate of the change in the mean day of the onset of the flight season for each 1-day increase in the timing of the snowmelt is -4.25 days when $x_2 = 30$.

4.33 a. The interaction model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_1 x_3 + \beta_5 x_2 x_3$.

b. Using MINITAB, the regression results are:

Regression Analysis: ARSENIC versus LATITUDE, ... PTH, LONG-DEPTH

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	5	542303	108461	10.21	0.000
LATITUDE	1	15689	15689	1.48	0.225
LONGITUDE	1	757	757	0.07	0.790
DEPTH-FT	1	26250	26250	2.47	0.117
LAT-DEPTH	1	9128	9128	0.86	0.355
LONG-DEPTH	1	33777	33777	3.18	0.076
Error	321	3410258	10624		
Total	326	3952562			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
103.072	13.72%	12.38%	10.43%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	10845	67720	0.16	0.873	
LATITUDE	-1280	1053	-1.22	0.225	4.82
LONGITUDE	217	815	0.27	0.790	6.29
DEPTH-FT	-1549	986	-1.57	0.117	48397303.04
LAT-DEPTH	-11.0	11.9	-0.93	0.355	3967014.08
LONG-DEPTH	20.0	11.2	1.78	0.076	51350321.12

Regression Equation

ARSENIC = 10845 - 1280 LATITUDE + 217 LONGITUDE - 1549 DEPTH-FT - 11.0 LAT-DEPTH
+ 20.0 LONG-DEPTH

The least squares prediction equation is
$\hat{y} = 10,845 - 1280x_1 + 217x_2 - 1549x_3 - 11.0x_1x_3 + 20.0x_2x_3$.

c. To determine if latitude and depth interact to affect arsenic levels, we test:

$$H_0: \beta_4 = 0$$
$$H_a: \beta_4 \neq 0$$

The test statistic is $t = -0.93$ and the p-value is $p = 0.355$. Since the p-value is not less than $\alpha\left(p = 0.355 \nless 0.05\right)$, H_0 is not rejected. There is insufficient evidence to indicate latitude and depth interact to affect arsenic levels at $\alpha = 0.05$.

d. To determine if longitude and depth interact to affect arsenic levels, we test:

$$H_0: \beta_5 = 0$$
$$H_a: \beta_5 \neq 0$$

The test statistic is $t = 1.78$ and the p-value is $p = 0.076$. Since the p-value is not less than $\alpha\left(p = 0.076 \not< 0.05\right)$, H_0 is not rejected. There is insufficient evidence to indicate longitude and depth interact to affect arsenic levels at $\alpha = 0.05$.

e. Since neither of the interactions is significant, then the effect of latitude on the arsenic levels does not depend on the depth and the effect of longitude on the arsenic levels does not depend on depth.

4.34 a. The model would be $E\left(y\right) = \beta_0 + \beta_2 x_2 + \beta_3 x_3 + \beta_5 x_5 + \beta_6 x_2 x_5 + \beta_7 x_3 x_5$.

b. Using MINITAB, the results of the regression are:

Regression Analysis: HEATRATE versus INLET-TEMP, ... OW, EXH_FLOW

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	5	156875371	31375074	173.64	0.000
INLET-TEMP	1	68495630	68495630	379.09	0.000
EXH-TEMP	1	28318845	28318845	156.73	0.000
AIRFLOW	1	6527	6527	0.04	0.850
INLET_FLOW	1	10415228	10415228	57.64	0.000
EXH_FLOW	1	4806495	4806495	26.60	0.000
Error	61	11021838	180686		
Total	66	167897208			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
425.072	93.44%	92.90%	91.88%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	13945	1044	13.35	0.000	
INLET-TEMP	-15.138	0.777	-19.47	0.000	4.17
EXH-TEMP	28.84	2.30	12.52	0.000	3.78
AIRFLOW	-0.69	3.63	-0.19	0.850	247.07
INLET_FLOW	0.02277	0.00300	7.59	0.000	305.16
EXH_FLOW	-0.0543	0.0105	-5.16	0.000	708.39

Regression Equation

HEATRATE = 13945 - 15.138 INLET-TEMP + 28.84 EXH-TEMP - 0.69 AIRFLOW + 0.02277 INLET_FLOW - 0.0543 EXH_FLOW

The least squares prediction equation is
$\hat{y} = 13{,}945 - 15.138x_2 + 28.84x_3 - 0.69x_5 + 0.02277x_2x_5 - 0.0543x_3x_5$.

c. To determine if the inlet temperature and air flow rate interact to affect heat rate, we test:

$$H_0: \beta_6 = 0$$
$$H_a: \beta_6 \neq 0$$

The test statistic is $t = 7.59$ and the p-value is $p = 0.000$. Since the p-value is less than $\alpha\left(p = 0.000 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate the inlet temperature and air flow rate interact to affect heat rate at $\alpha = 0.05$.

d. To determine if the exhaust temperature and air flow rate interact to affect heat rate, we test:

$$H_0: \beta_7 = 0$$
$$H_a: \beta_7 \neq 0$$

The test statistic is $t = -5.16$ and the p-value is $p = 0.000$. Since the p-value is less than $\alpha\left(p = 0.000 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate the exhaust temperature and air flow rate interact to affect heat rate at $\alpha = 0.05$.

e. The tests indicate that the effect of inlet temperature on heat rate depends on the air flow rate and the effect of exhaust temperature on the heat rate depends on the air flow rate.

4.35 a. To determine if the overall model is adequate, we test:

$$H_0: \beta_1 = \beta_2 = 0$$
$$H_a: \text{ At least one } \beta_i \neq 0$$

The test statistic is $F = 48.15$ and the p-value is $p = 0.000$. Since the p-value is less than $\alpha\left(p = 0.000 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate that the overall model is adequate at $\alpha = 0.05$.

b. To determine if the rate of increase of angle size with height above the horizon is slower for greater heights, we test:

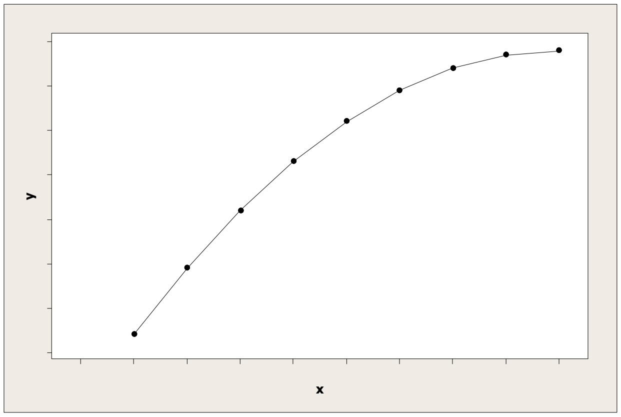
$$H_0: \beta_2 = 0$$
$$H_a: \beta_2 < 0$$

The test statistic is $t = -1.33$ and the p-value is $p = 0.214 / 2 = 0.107$. Since the p-value is not less than $\alpha\left(p = 0.107 \nless 0.05\right)$, H_0 is not rejected. There is insufficient evidence to indicate that the rate of increase of angle size with height above the horizon is slower for greater heights at $\alpha = 0.05$.

4.36 a. The correlation coefficient, r, only measures the linear relationship between two variables. If the relationship between two variables is curvilinear, then r should not be used to measure the relationship.

b. The model is $E\left(y\right) = \beta_0 + \beta_1 x + \beta_2 x^2$.

c. A possible graph is:



d. If the theory is supported, then the expected sign of β_2 is negative.

e. To determine if task performance will increase as a level of conscientiousness increase, but at a decreasing rate, we test:

$$H_0: \beta_2 = 0$$
$$H_a: \beta_2 < 0$$

The p-value is $(p < 0.05)$. Since the p-value is less than $\alpha\,(p < 0.05)$, H_0 is rejected. There is sufficient evidence to indicate that task performance will increase as the level of conscientiousness increases, but at a decreasing rate at $\alpha = 0.05$.

4.37 a. To determine if the quadratic model is useful, we test:

$$H_0: \beta_1 = \beta_2 = 0$$
$$H_a: \text{ At least one } \beta_i \neq 0$$

The test statistic is $F = \dfrac{R^2 / k}{(1-R^2)/[n-(k+1)]} = \dfrac{0.12/2}{(1-0.12)/\left[388-(2+1)\right]} = 26.25.$

The rejection region requires $\alpha = 0.05$ in the upper tail of the F distribution. From Table 4, Appendix D, with $\nu_1 = k = 2$ and $\nu_2 = n-(k+1) = 388-(2+1) = 385$, $F \approx 3.00$. The rejection region is $F > 3.00$.

Since the observed value of the test statistic falls in the rejection region $(F = 26.25 > 3.00)$, H_0 is rejected. There is sufficient evidence to indicate that the quadratic model is useful at $\alpha = 0.05$.

b. To determine if the leadership ability will increase at a decreasing rate with assertiveness, we test:

$$H_0: \beta_2 = 0$$
$$H_a: \beta_2 < 0$$

c. The test statistic is $t = -3.97$ and the p-value is $p < 0.01/2 = 0.005$. Since the p-value is less than $\alpha\left(p < 0.005 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate that the leadership ability will increase at a decreasing rate with assertiveness at $\alpha = 0.05$.

4.38 a. The quadratic model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_1^2$.

b. β_2 allows for a curvilinear relationship between y and x_1.

c. The term will negative. If the rate of increase decreases as the independent variable increases, then the coefficient corresponding to the quadratic term is negative.

4.39 a. $\hat{\beta}_0 = 6.13$. Since 0 is not in the observed range (one cannot have the ball on the goal line), this has no meaning other than the y-intercept.

$\hat{\beta}_1 = .141$. Since the quadratic term is present in the model, this is no longer the slope of the line. It is simply a location parameter.

$\hat{\beta}_2 = -0.0009$. Since this term is negative, it indicates that the shape of the relationship is mound-shaped, or concave downward. As the distance from the goal line increases, the predicted number of points scored will increase to some point and then start decreasing.

b. $R^2 = 0.226$. 22.6% of the sample variation in the number of points scored around their mean is explained by the quadratic relationship between the number of points scored and the number of yards from the opposing goal line.

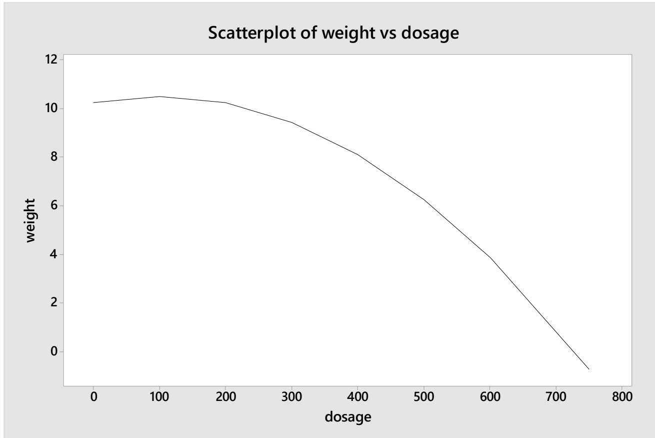
c. No. Even though the value of R^2 has increased, we do not know if the increase is statistically significant.

d. To determine if the quadratic model is a better fit, we would test:

$$H_0: \beta_2 = 0$$
$$H_a: \beta_2 \neq 0$$

4.40 Because the graph is curved, we would hypothesize that the model should be $E\left(y\right) = \beta_0 + \beta_1 x + \beta_2 x^2$. Since the curve opens down, β_2 will be negative.

4.41 a. Using MINITAB, a possible graph is:



b. For $x = 500$, $\hat{y} = 10.25 + 0.0053(500) - 0.0000266(500)^2 = 6.25$.

c. For $x = 0$, $\hat{y} = 10.25 + 0.0053(0) - 0.0000266(0)^2 = 10.25$.

d. Notice in the table below the smallest positive difference in the estimated weight change and the control group is from dosage level 200.

dosage	weight	10.25-weight
0	10.250	0.000
100	10.514	-0.264
200	10.246	0.004
500	6.250	4.000
750	-0.737	10.988

4.42 a. To test the researcher's theory, we need to test to see if the relationship is curved and opens down. Thus, we would test:

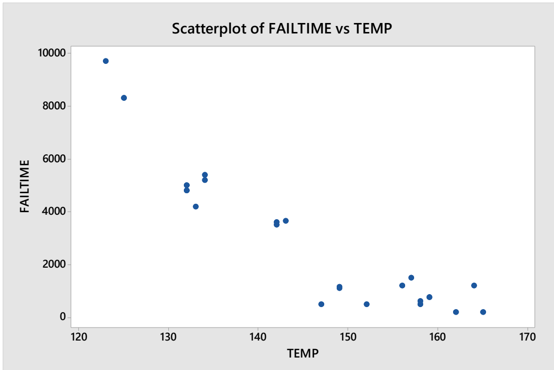
$$H_0 : \beta_2 = 0$$
$$H_a : \beta_2 < 0$$

b. The test statistic is $t = \frac{\hat{\beta}_2}{s_{\hat{\beta}_2}} = \frac{-0.279}{0.039} = -7.15$.

The rejection region requires $\alpha = 0.01$ in the lower tail of the *t*-distribution with $df = n - (k+1) = 419{,}225 - (2+1) = 419{,}222$, . From Table 2, Appendix D, $t_{0.01} = 2.326$. The rejection region is $t < -2.326$.

Since the observed value of the test statistic falls in the rejection region $(t = -7.15 < -2.326)$, H_0 is rejected. There is sufficient evidence to support the researchers' claim at $\alpha = 0.01$.

4.43 a. Using MINITAB, the scatterplot of the data is:



It appears that the relationship between failure time and solder temperature is curvilinear.

b. Using MINITAB, the results of the regression analysis is:

Regression Analysis: FAILTIME versus TEMP, TEMPSQ

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	2	144830280	72415140	152.93	0.000
TEMP	1	18711461	18711461	39.51	0.000
TEMPSQ	1	15166293	15166293	32.03	0.000
Error	19	8997107	473532		
Lack-of-Fit	14	8945857	638990	62.34	0.000
Pure Error	5	51250	10250		
Total	21	153827386			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
688.137	94.15%	93.54%	92.22%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	154243	21868	7.05	0.000	
TEMP	-1909	304	-6.29	0.000	688.04
TEMPSQ	5.93	1.05	5.66	0.000	688.04

Regression Equation

FAILTIME = 154243 - 1909 TEMP + 5.93 TEMPSQ

The least squares prediction equation is $\hat{y} = 154,243 - 1909x + 5.93x^2$.

c. To determine if there is an upward curvature in the relationship between failure time and solder temperature, we test:

$$H_0: \beta_2 = 0$$
$$H_a: \beta_2 > 0$$

The test statistic is $t = 5.66$ and the p-value is $p = 0.000/2 = 0.000$. Since the p-value is less than $\alpha\left(p = 0.000 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate there is an upward curvature in the relationship between failure time and solder temperature at $\alpha = 0.05$.

4.44 a. Using MINITAB, the results of the regression analysis are:

Regression Analysis: YEARS versus AGE, AGE-SQ

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	2	1450.95	725.47	13.15	0.000
AGE	1	89.73	89.73	1.63	0.211
AGE-SQ	1	14.20	14.20	0.26	0.615
Error	35	1930.45	55.16		
Lack-of-Fit	19	1194.61	62.87	1.37	0.266
Pure Error	16	735.83	45.99		
Total	37	3381.39			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
7.42668	42.91%	39.65%	32.40%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	-8.9	10.2	-0.87	0.389	
AGE	0.704	0.552	1.28	0.211	43.48
AGE-SQ	-0.00341	0.00673	-0.51	0.615	43.48

Regression Equation

YEARS = -8.9 + 0.704 AGE - 0.00341 AGE-SQ

The fitted regression line is $\hat{y} = -8.9 + 0.704x - 0.00341x^2$.

b. To determine the overall adequacy, we test:

$$H_0: \beta_1 = \beta_2 = 0$$
$$H_a: \text{ At least one } \beta_i \neq 0$$

The test statistic is $F = 13.15$ and the p-value is $p = 0.000 = 0.000$. Since the p-value is less than $\alpha\left(p = 0.000 < 0.01\right)$, H_0 is rejected. There is sufficient evidence to indicate the overall model is adequate at $\alpha = 0.01$.

c. To determine if the relationship between age and the number of years shopping on Black Friday is quadratic, we test:

$$H_0: \beta_2 = 0$$
$$H_a: \beta_2 \neq 0$$

The test statistic is $t = -0.51$ and the p-value is $p = 0.615$. Since the p-value is not less than $\alpha\left(p = 0.615 \not< 0.01\right)$, H_0 is not rejected. There is insufficient evidence to indicate the relationship between age and the number of years shopping on Black Friday is quadratic at $\alpha = 0.01$.

4.45 a. Using MINITAB, the results of the regression analysis are:

Regression Analysis: RATE versus EST, EST_SQ

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	2	18138955261	9069477631	8.91	0.002
EST	1	148551356	148551356	0.15	0.706
EST_SQ	1	132549926	132549926	0.13	0.722
Error	21	21371254395	1017678781		
Lack-of-Fit	15	21307470620	1420498041	133.62	0.000
Pure Error	6	63783775	10630629		
Total	23	39510209656			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
31901.1	45.91%	40.76%	0.00%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	-288	8049	-0.04	0.972	
EST	1.39	3.65	0.38	0.706	32.54
EST_SQ	0.000035	0.000097	0.36	0.722	32.54

Regression Equation

RATE = -288 + 1.39 EST + 0.000035 EST_SQ

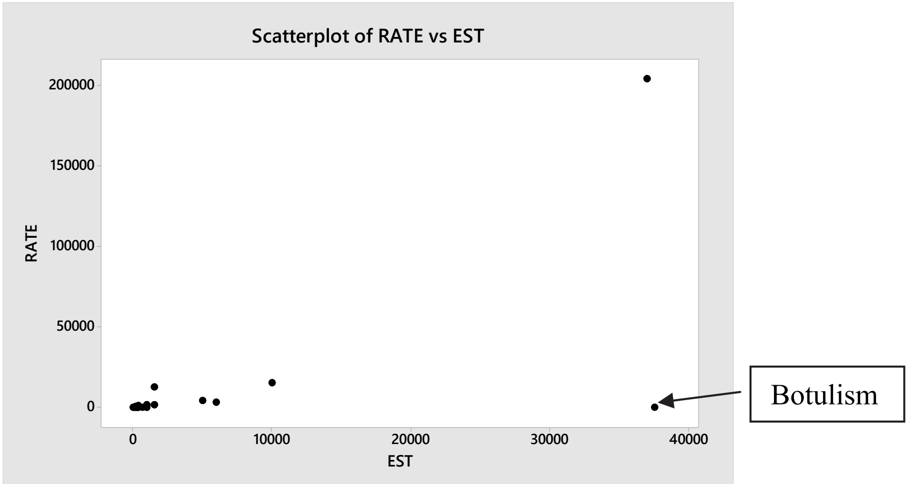
The fitted regression model is $\hat{y} = -288 + 1.39x + 0.000035x^2$.

To determine if the incidence rate is curvilinearly related to estimated rate, we test:

$$H_0: \beta_2 = 0$$
$$H_a: \beta_2 \neq 0$$

The test statistic is $t = 0.36$ and the p-value is $p = 0.722$. Since the p-value is not less than $\alpha\left(p = 0.722 \not< 0.05\right)$, H_0 is not rejected. There is insufficient evidence to indicate the incidence rate is curvilinearly related to estimated rate at $\alpha = 0.05$.

b. Using MINITAB, the scatterplot of the data is:



The value of rate corresponding to Botulism is much lower than we would estimate with the other observations.

c. Using MINITAB, the results of the regression analysis are:

Regression Analysis: RATE versus EST, EST_SQ

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	2	39251490541	19625745270	2582.35	0.000
EST	1	496483	496483	0.07	0.801
EST_SQ	1	2284006914	2284006914	300.53	0.000
Error	20	151998825	7599941		
Lack-of-Fit	14	88215049	6301075	0.59	0.803
Pure Error	6	63783775	10630629		
Total	22	39403489366			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
2756.80	99.61%	99.58%	83.53%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	735	696	1.06	0.303	
EST	-0.081	0.317	-0.26	0.801	17.69
EST_SQ	0.000151	0.000009	17.34	0.000	17.69

Regression Equation

RATE = 735 - 0.081 EST + 0.000151 EST_SQ

The fitted regression model is $\hat{y} = 735 - 0.081x + 0.000151x^2$.

To determine if the incidence rate is curvilinearly related to estimated rate, we test:

$$H_0: \beta_2 = 0$$
$$H_a: \beta_2 \neq 0$$

The test statistic is $t = 17.34$ and the p-value is $p = 0.000$. Since the p-value is less than $\alpha\left(p = 0.000 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate the incidence rate is curvilinearly related to estimated rate at $\alpha = 0.05$. Yes, the fit of the model has improved. The R^2 value increased from 0.4591 to 0.9961.

4.46 a. Let x_1 and x_2 represent the two quantitative independent variables. A first-order linear model that includes only the first-order terms for the variables is $E\left(y\right) = \beta_0 + \beta_1 x_1 + \beta_2 x_2$.

b. Let x_1, x_2, x_3, and x_4 represent the four quantitative independent variables. A first-order linear model that includes only the first-order terms for the variables is $E\left(y\right) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4$.

4.47 a. Let x_1 and x_2 represent the two quantitative independent variables. A complete second-order linear model that includes the variables is $E\left(y\right) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1^2 + \beta_4 x_2^2 + \beta_5 x_1 x_2$.

b. Let x_1, x_2 and x_3 represent the three quantitative independent variables. A complete second-order linear model that includes the variables is $E\left(y\right) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_1^2 + \beta_5 x_2^2 + \beta_6 x_3^2 + \beta_7 x_1 x_2 + \beta_8 x_1 x_3 + \beta_9 x_2 x_3$.

4.48 a. The model is $E\left(y\right) = \beta_0 + \beta_1 x$, where $x = \begin{cases} 1 \text{ if A} \\ 0 \text{ if B} \end{cases}$

$\beta_0 =$ mean of y for the $x = B$ level $= \mu_{\text{B}}$
$\beta_1 =$ difference in the mean levels of y for the $x = \text{A}$ and $x = \text{B}$ levels $= \mu_{\text{A}} - \mu_{\text{B}}$

b. For a qualitative independent variable with four levels (A, B, C, and D), the model requires three dummy variables as shown below. (We have arbitrarily selected level D as the "base" level.)

$$x_1 = \begin{cases} 1 \text{ if level A} \\ 0 \text{ if not} \end{cases} \qquad x_2 = \begin{cases} 1 \text{ if level B} \\ 0 \text{ if not} \end{cases} \qquad x_3 = \begin{cases} 1 \text{ if level C} \\ 0 \text{ if not} \end{cases}$$

Then the model is written $E\left(y\right) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$.
The following table shows the values of the dummy variables and the mean response $E\left(y\right)$ for each of the four levels.

Level	x_1	x_2	x_3	$E\left(y\right)$
A	1	0	0	$\beta_0 + \beta_1 = \mu_{\text{A}}$
B	0	1	0	$\beta_0 + \beta_2 = \mu_B$
C	0	0	1	$\beta_0 + \beta_3 = \mu_{\text{C}}$
D	0	0	0	$\beta_0 = \mu_{\text{D}}$

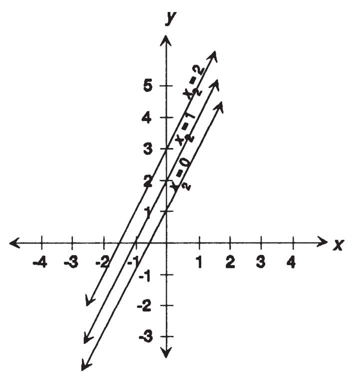
From the table, we obtain the interpretations of the β's.

$\beta_0 = \mu_D$, $\beta_1 = \mu_A - \mu_D$, $\beta_2 = \mu_B - \mu_D$, and $\beta_3 = \mu_C - \mu_D$.

4.49 a. For $x_2 = 0$, $E(y) = 1 + 2x_1 + 0 = 1 + 2x_1$

For $x_2 = 1$, $E(y) = 1 + 2x_1 + 1 = 2 + 2x_1$

For $x_2 = 2$, $E(y) = 1 + 2x_1 + 2 = 3 + 2x_1$



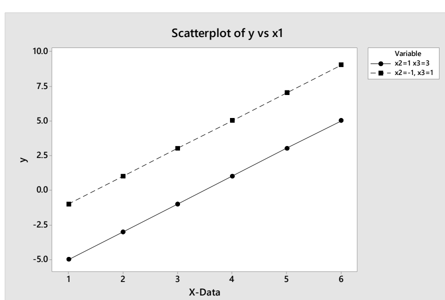
b. The graphed curves are first-order.

c. The three lines are parallel. Each line has a slope of 2.

d. When $E(y)$ is graphed as a function of one of the independent variables for various values of the other independent variable, the lines will be parallel.

4.50 a. For $x_2 = 1$ and $x_3 = 3$, $y = 1 + 2x_1 + (1) - 3(3) \Rightarrow y = -7 + 2x_1$.

Using MINITAB, the graph might look like:



b. For $x_2 = -1$ and $x_3 = 1$, $y = 1 + 2x_1 + (-1) - 3(1) \Rightarrow y = -3 + 2x_1$. (See graph in part a.)

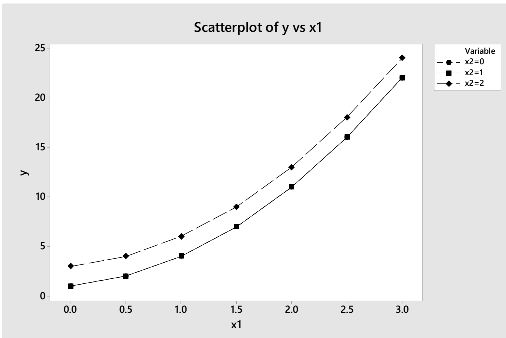
c. The geometric relationship in a first-order model for $E(y)$ as a function of one independent variable for various combinations of values of the other independent variables is parallel lines. The y-intercept is determined by the combination of values of the other independent variables. The slope is the coefficient of **the one** independent variable.

4.51 a. For $x_2 = 0$, $y = 1 + x_1 - (0) + 2x_1^2 + (0)^2 \Rightarrow y = 1 + x_1 + 2x_1^2$.

For $x_2 = 1$, $y = 1 + x_1 - (1) + 2x_1^2 + (1)^2 \Rightarrow y = 1 + x_1 + 2x_1^2$.

For $x_2 = 2$, $y = 1 + x_1 - (2) + 2x_1^2 + (2)^2 \Rightarrow y = 3 + x_1 + 2x_1^2$.

Using MINITAB, the graph might look like:



Note, the lines are exactly the same for $x_2 = 0$ and $x_2 = 1$.

b. The graphed curves are second-order because they have the term x_1^2.

c. The curves have the same shape - they are just shifted up or down.

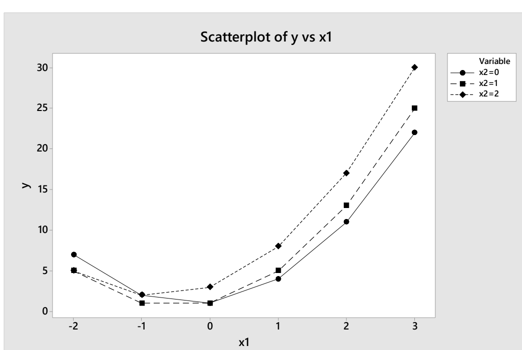
d. No, the variables do not interact. The relationship between y and x_1 is not affected by the value of x_2.

4.52 a. For $x_2 = 0$, $y = 1 + x_1 - (0) + x_1(0) + 2x_1^2 + (0)^2 \Rightarrow y = 1 + x_1 + 2x_1^2$.

For $x_2 = 1$, $y = 1 + x_1 - (1) + x_1(1) + 2x_1^2 + (1)^2 \Rightarrow y = 1 + 2x_1 + 2x_1^2$.

For $x_2 = 2$, $y = 1 + x_1 - (2) + x_1(2) + 2x_1^2 + (2)^2 \Rightarrow y = 3 + 3x_1 + 2x_1^2$.

Using MINITAB, a possible graph is:



b. The curves are second-order because they contain a squared term x_1^2.

c. Because the coefficient in front of the x_1^2 is the same regardless of the value of x_2, the shape of the curves are all the same. However, the coefficient in front of the x_1 term changes with the value of x_2, which shifts the curve left and right. The value of the constant term changes with different values of x_2. These different values shifts the curves up and down.

d. Yes, the independent variables x_1 and x_2 interact. Although the shape of the x_1 curve does not change for different x_2 values, the effect of x_1 on y does change for different values of x_2. The effect of x_1 on y changes for different values of x_2 because the curve is shifted to the right or left.

e. The term x_1x_2 allows the curve to be shifted to the right or left along the x_1 axis.

4.53 a. First, we define the dummy variables, using radio show as the base level:

$$x_1 = \begin{cases} 1 \text{ if continuous verbal} \\ 0 \text{ if not} \end{cases} \qquad x_2 = \begin{cases} 1 \text{ if late verbal} \\ 0 \text{ if not} \end{cases} \qquad x_3 = \begin{cases} 1 \text{ if no verbal} \\ 0 \text{ if not} \end{cases}$$

Then the model is written $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$.

b. $\hat{\beta}_0 = \hat{\mu}_4 = 49.00$. This is the estimated mean percentage of billboards recalled by the student drivers listening to the radio.

$\hat{\beta}_1 = \hat{\mu}_1 - \hat{\mu}_4 = 34.30 - 49.00 = -14.70$. This is the estimated difference between the mean percentage of billboards recalled by the student drivers in the continuous verbal condition and the student drivers listening to the radio.

$\hat{\beta}_2 = \hat{\mu}_2 - \hat{\mu}_4 = 63.40 - 49.00 = 14.40$. This is the estimated difference between the mean percentage of billboards recalled by the student drivers in the late verbal condition and the student drivers listening to the radio.

$\hat{\beta}_3 = \hat{\mu}_3 - \hat{\mu}_4 = 63.90 - 49.00 = 14.90$. This is the estimated difference between the mean percentage of billboards recalled by the student drivers in the no verbal condition and the student drivers listening to the radio.

c. Using MINITAB, the results are:

Regression Analysis: RECALL versus x1, x2, x3

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	3	5922	1973.9	5.39	0.004
x1	1	1080	1080.5	2.95	0.095
x2	1	1037	1036.8	2.83	0.101
x3	1	1110	1110.1	3.03	0.090
Error	36	13189	366.4		
Total	39	19111			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
19.1409	30.99%	25.23%	14.80%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	49.00	6.05	8.10	0.000	
x1	-14.70	8.56	-1.72	0.095	1.50
x2	14.40	8.56	1.68	0.101	1.50
x3	14.90	8.56	1.74	0.090	1.50

Regression Equation

RECALL = 49.00 - 14.70 x1 + 14.40 x2 + 14.90 x3

From the printout, the fitted regression line is $\hat{y} = 49 - 14.70x_1 + 14.40x_2 + 14.90x_3$. This agrees with part b.

d. To determine if the mean recall percentage differs for students in the four groups, we test:

$$H_0: \beta_1 = \beta_2 = \beta_3 = 0$$
$$H_a: \text{ At least one } \beta_i \neq 0$$

The test statistic is $F = 5.39$ and the p-value is $p = 0.004$. Since the p-value is less than $\alpha\left(p = 0.004 < 0.01\right)$, H_0 is rejected. There is sufficient evidence to indicate the mean recall percentage differs for students in the four groups at $\alpha = 0.01$.

4.54 a. The model is $E\left(y\right) = \beta_0 + \beta_1 x$ where $x = \begin{cases} 1 & \text{if election was effected by a World War} \\ 0 & \text{otherwise} \end{cases}$

b. An expression for the mean Democratic vote share during all years when there is no World War is β_0.

c. An expression for the mean Democratic vote share during all years when there is a World War is $\beta_0 + \beta_1$.

d. Using MINITAB, the results are:

Regression Analysis: VSHARE versus WAR

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	12.51	12.51	0.25	0.624
WAR	1	12.51	12.51	0.25	0.624
Error	22	1113.36	50.61		
Total	23	1125.86			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
7.11387	1.11%	0.00%	0.00%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	49.60	1.55	31.95	0.000	
WAR	-2.18	4.39	-0.50	0.624	1.00

Regression Equation

VSHARE = 49.60 - 2.18 WAR

To determine if the mean Democratic vote share during all years when there is a World War differs from the mean when there is no World War, we test:

$$H_0: \beta_1 = 0$$
$$H_a: \beta_1 \neq 0$$

The test statistic is $t = -0.50$ and the p-value is $p = 0.624$. Since the p-value is not less than $\alpha\left(p = 0.624 \nless 0.10\right)$, H_0 is not rejected. There is insufficient evidence to indicate the mean Democratic vote share during all years when there is a World War differs from the mean when there is no World War at $\alpha = 0.10$.

e. The model would be $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2$.

$$\text{where } x_1 = \begin{cases} 1 & \text{if Democrat incumbent} \\ 0 & \text{otherwise} \end{cases} \qquad x_2 = \begin{cases} 1 & \text{if Republican incumbent} \\ 0 & \text{otherwise} \end{cases}$$

f. An expression for the mean Democratic vote share during all years when there is no incumbent running is β_0.

g. An expression for the mean Democratic vote share during all years when a Republican incumbent is running is $\beta_0 + \beta_2$.

h. An expression for the difference between the mean Democratic vote share for all years when a Democratic incumbent is running and when there is no incumbent running is β_1.

i. Using MINITAB, the results are:

Regression Analysis: VSHARE versus INCUMB1, INCUMB2

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	2	318.62	159.309	4.14	0.030
INCUMB1	1	239.09	239.093	6.22	0.021
INCUMB2	1	1.02	1.017	0.03	0.872
Error	21	807.25	38.440		
Total	23	1125.86			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
6.20003	28.30%	21.47%	5.51%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	46.47	2.34	19.83	0.000	
INCUMB1	8.00	3.21	2.49	0.021	1.43
INCUMB2	0.51	3.12	0.16	0.872	1.43

Regression Equation

VSHARE = 46.47 + 8.00 INCUMB1 + 0.51 INCUMB2

To determine if the mean Democratic vote share differs depending on the incumbent running, we test:

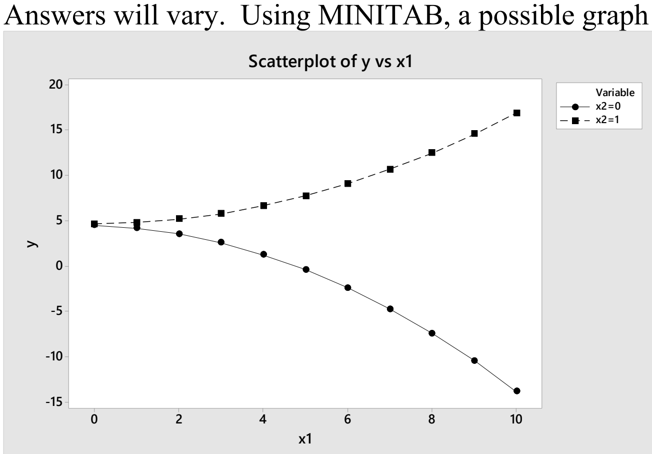
$$H_0: \beta_1 = \beta_2 = 0$$
$$H_a: \text{At least } 1\ \beta_i \neq 0$$

The test statistic is $F = 4.14$ and the p-value is $p = 0.030$. Since the p-value is less than $\alpha\left(p = 0.030 < 0.10\right)$, H_0 is rejected. There is sufficient evidence to indicate the mean Democratic vote share differs depending on the incumbent running at $\alpha = 0.10$.

4.55 a. When $x_2 = 0$, $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_1^2 + \beta_3(0) + \beta_4 x_1(0) + \beta_5 x_1^2(0) = \beta_0 + \beta_1 x_1 + \beta_2 x_1^2$.

b. When $x_2 = 1$,
$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_1^2 + \beta_3(1) + \beta_4 x_1(1) + \beta_5 x_1^2(1) = (\beta_0 + \beta_3) + (\beta_1 + \beta_4)x_1 + (\beta_2 + \beta_5)x_1^2$.

c. Answers will vary. Using MINITAB, a possible graph is:



The graph of y when $x_2 = 0$ is an example of plotting the line when the team leader is not effective. This has a downward curvature. The graph of y when $x_2 = 1$ is an example of plotting the line when the team leader is effective. This has an upward curvature.

4.56 a. $x_1 = \begin{cases} 1 \text{ if manual} \\ 0 \text{ if not} \end{cases}$ $x_2 = \begin{cases} 1 \text{ if clay} \\ 0 \text{ if not} \end{cases}$ $x_3 = \begin{cases} 1 \text{ if gravel} \\ 0 \text{ if not} \end{cases}$ $x_4 = \begin{cases} 1 \text{ if East} \\ 0 \text{ if not} \end{cases}$ $x_5 = \begin{cases} 1 \text{ if South} \\ 0 \text{ if not} \end{cases}$

$$x_6 = \begin{cases} 1 \text{ if West} \\ 0 \text{ if not} \end{cases} \quad x_7 = \begin{cases} 1 \text{ if Southeast} \\ 0 \text{ if not} \end{cases}$$

b. The model for wine quality as a function of grape-picking method is $E(y) = \beta_0 + \beta_1 x_1$.
$\beta_0 =$ mean wine quality for automated grape-picking method
$\beta_1 =$ difference in mean wine quality between manual and automated grape-picking method

c. The model for wine quality as a function of soil type is $E(y) = \beta_0 + \beta_2 x_2 + \beta_3 x_3$.
$\beta_0 =$ mean wine quality for soil type sand
$\beta_2 =$ difference in mean wine quality between clay and sand soil type
$\beta_3 =$ difference in mean wine quality between gravel and sand soil type

d. The model for wine quality as a function of slope orientation is
$E(y) = \beta_0 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_6 + \beta_7 x_7$.
$\beta_0 =$ mean wine quality for slope orientation southwest
$\beta_4 =$ difference in mean wine quality between east and southwest slope orientation
$\beta_5 =$ difference in mean wine quality between south and southwest slope orientation
$\beta_6 =$ difference in mean wine quality between west and southwest slope orientation
$\beta_7 =$ difference in mean wine quality between southeast and southwest slope orientation

4.57 a. There are a total of 4 different gene marker combinations: AA, AB, BA, and BB.

b. Using BB as the base level, the dummy variables are:

$$x_1 = \begin{cases} 1 \text{ if AA} \\ 0 \text{ if not} \end{cases} \quad x_2 = \begin{cases} 1 \text{ if AB} \\ 0 \text{ if not} \end{cases} \quad x_3 = \begin{cases} 1 \text{ if BA} \\ 0 \text{ if not} \end{cases}$$

The model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$.

c. $\beta_0 =$ mean extent of the disease for gene marker BB
$\beta_1 =$ difference in mean extent of the disease between AA and BB gene markers
$\beta_2 =$ difference in mean extent of the disease between AB and BB gene markers
$\beta_3 =$ difference in mean extent of the disease between BA and BB gene markers

d. To determine if the overall model is statistically useful, the null hypothesis is

$$H_0: \beta_1 = \beta_2 = \beta_3 = 0$$

4.58 a. Suppose we use the no help level as the base level. The dummy variables will be:

$$x_1 = \begin{cases} 1 \text{ if completed solution} \\ 0 \text{ if not} \end{cases} \quad x_2 = \begin{cases} 1 \text{ if check figures} \\ 0 \text{ if not} \end{cases}$$

The model for knowledge gain as a function of homework assistant group is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2$.

b. In terms of the model, the difference between the mean knowledge gain between students in the complete solution group and the no help group is β_1.

c. Using MINITAB, the regression analysis is:

Regression Analysis: IMPROVE versus x1, x2

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	2	6.643	3.322	0.45	0.637
x1	1	2.803	2.803	0.38	0.538
x2	1	1.121	1.121	0.15	0.697
Error	72	527.357	7.324		
Total	74	534.000			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
2.70636	1.24%	0.00%	0.00%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	2.433	0.494	4.92	0.000	
x1	-0.483	0.781	-0.62	0.538	1.22
x2	0.287	0.733	0.39	0.697	1.22

Regression Equation

IMPROVE = 2.433 - 0.483 x1 + 0.287 x2

The regression model is $\hat{y} = 2.433 - 0.483x_1 + 0.287x_2$.

d. To determine if the model is useful, we test:

$$H_0: \beta_1 = \beta_2 = 0$$
$$H_a: \text{At least } 1\, \beta_i \neq 0$$

The test statistic is $F = 0.45$ and the p-value is $p = 0.637$. Since the p-value is not less than $\alpha\left(p = 0.637 < 0.05\right)$, H_0 is not rejected. There is insufficient evidence to indicate the model is useful at $\alpha = 0.05$.

4.59 a. The complete second-order model is

$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_1^2 + \beta_6 x_2^2 + \beta_7 x_3^2 + \beta_8 x_4^2 + \beta_9 x_1 x_2 + \beta_{10} x_1 x_3 + \beta_{11} x_1 x_4 + \beta_{12} x_2 x_3 + \beta_{13} x_2 x_4 + \beta_{14} x_3 x_4$$

b. The terms in the model that allow for curvilinear relationships are $\beta_5 x_1^2$, $\beta_6 x_2^2$, $\beta_7 x_3^2$, and $\beta_8 x_4^2$.

4.60 a. To determine if the overall model is adequate, we test:

$$H_0: \beta_1 = \beta_2 = \cdots = \beta_{12} = 0$$
$$H_a: \text{At least } 1\, \beta_i \neq 0$$

The test statistic is $F = 26.9$.

The rejection region requires $\alpha = 0.05$ in the upper tail of the F distribution with $\nu_1 = 12$ and $\nu_2 = n - (k+1) = 148 - (12+1) = 135$. Using MINITAB, $F_{0.05} = 1.82$. The rejection region is $F > 1.82$.

Since the observed value of the test statistic falls in the rejection region $(F = 26.9 > 1.82)$, H_0 is rejected. There is sufficient evidence to indicate the overall model is adequate in predicting the card price at $\alpha = 0.05$.

b. To determine if race has an impact on the value of professional football players' rookie cards, we test:

$$H_0: \beta_1 = 0$$
$$H_a: \beta_1 \neq 0$$

The test statistic is $t = -1.014$ and the p-value is $p = 0.312$. Since the p-value is not small, H_0 is not rejected. There is insufficient evidence to indicate race has an impact on the value of professional football players' rookie cards for any value of $\alpha < 0.312$.

c. To determine if card vintage has an impact on the value of professional football players' rookie cards, we test:

$$H_0: \beta_3 = 0$$
$$H_a: \beta_3 \neq 0$$

The test statistic is $t = -10.92$ and the p-value is $p = 0.000$. Since the p-value is less than $\alpha\,(p = 0.000 < 0.05)$, H_0 is rejected. There is sufficient evidence to indicate card vintage has an impact on the value of professional football players' rookie cards at $\alpha = 0.05$.

d. The model is
$E[\ln(y)] = \beta_0 + \beta_1 x_4 + \beta_2 x_5 + \beta_3 x_6 + \ldots + \beta_9 x_{12} + \beta_{10} x_4 x_5 + \beta_{11} x_4 x_6 + \ldots + \beta_{17} x_4 x_{12}$.

4.61 a. The model is $E(y) = \beta_0 + \beta_1 x_1$.

b. $\beta_0 =$ mean relative optimism if the analyst worked for a sell-side firm.

c. Yes, since $\beta_1 =$ difference in mean relative optimism between buy-side and sell-side analysts.

d. Yes. We would expect the value of β_1 to be negative is the buy-side firms are less optimistic than sell-side firms.

4.62 a. We define the dummy variables as:

$$x_1 = \begin{cases} 1 \text{ if ban/others} \\ 0 \text{ if not} \end{cases} \quad x_2 = \begin{cases} 1 \text{ if ban/no others} \\ 0 \text{ if not} \end{cases} \quad x_3 = \begin{cases} 1 \text{ if no ban/others} \\ 0 \text{ if not} \end{cases}$$

The model would be $E(\text{MVL}) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$.

b. Using MINITAB, the regression results are:

Regression Analysis: MVL versus X1, X2, X3

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	3	189.391	63.130	105.37	0.000
X1	1	125.640	125.640	209.70	0.000
X2	1	107.559	107.559	179.52	0.000
X3	1	1.184	1.184	1.98	0.162
Error	173	103.651	0.599		
Total	176	293.042			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.774039	64.63%	64.02%	63.09%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	6.376	0.104	61.09	0.000	
X1	2.137	0.148	14.48	0.000	1.38
X2	2.171	0.162	13.40	0.000	1.33
X3	0.253	0.180	1.41	0.162	1.27

Regression Equation

MVL = 6.376 + 2.137 X1 + 2.171 X2 + 0.253 X3

The least squares prediction equation is $\hat{y} = 6.376 + 2.137x_1 + 2.171x_2 + 0.253x_3$.

c. $\hat{\beta}_0 = 6.376$. We estimate the mean MVL is 6.376 for the no ban/no others market.

$\hat{\beta}_1 = 2.137$. We estimate the difference in mean MVL between ban/others and no ban/no other markets is 2.137.

$\hat{\beta}_2 = 2.171$. We estimate the difference in mean MVL between ban/no others and no ban/no other markets is 2.171.

$\hat{\beta}_3 = 0.253$. We estimate the difference in mean MVL between no ban/others and no ban/no other markets is 0.253.

d. To determine if the overall model is useful, we test:

$H_0: \beta_1 = \beta_2 = \beta_3 = 0$
H_a: At least $1 \beta_i \neq 0$

The test statistic is $F = 105.37$ and the p-value is $p = 0.000$. Since the p-value is less than $\alpha\left(p = 0.000 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate the overall model is useful at $\alpha = 0.05$. This indicates that the overall mean MVL values differ among the 4 markets.

e. From the printout, $\hat{\beta}_0 = 6.376 = \hat{\mu}_4$. This is the mean for the no ban/no others market.
$\hat{\mu}_1 = \hat{\beta}_1 + \hat{\beta}_0 = 2.137 + 6.376 = 8.513.$
$\hat{\mu}_2 = \hat{\beta}_2 + \hat{\beta}_0 = 2.171 + 6.376 = 8.547.$
$\hat{\mu}_3 = \hat{\beta}_3 + \hat{\beta}_0 = 0.253 + 6.376 = 6.629.$

4.63 a. We are 95% confident that for each 1% increase in body mass, the estimated percentage increase in eye mass is between $\left(e^{0.25} - 1\right)100\% = 28.4\%$ and $\left(e^{0.30} - 1\right)100\% = 35.0\%$.

b. We are 95% confident that for each 1% increase in body mass, the estimated percentage decrease in orbit axis angle is between $\left(e^{-0.05} - 1\right)100\% = 4.9\%$ and $\left(e^{-0.14} - 1\right)100\% = 13.1\%$.

4.64 a. $\hat{y} = 80.22 + 156.5x_1 - 42.3x_1^2 + 272.84x_2 + 760.1x_1x_2 + 46.95x_1^2x_2$

b. To determine if the overall model is useful for predicting transcript copy number, we test:

$H_0: \beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = 0$
H_a: At least $1 \beta_i \neq 0$

The test statistic is $F = 417.05$ and the p-value is $p < 0.0001$. Since the p-value is less than $\alpha\left(p < 0.0001 < 0.01\right)$, H_0 is rejected. There is significant evidence to indicate that the prediction equation is useful for predicting transcript copy number at $\alpha = 0.01$.

c. To determine if the transcript number is curvilinearly related to proportion of RNA, we test:

$H_0: \beta_2 = 0$
$H_a: \beta_2 \neq 0$

The test statistic is $t = -0.34$ and the p-value is $p = 0.7340$. Since the p-value is not less than $\alpha\left(p = 0.7340 \nless 0.01\right)$, H_0 is not rejected. There is insufficient evidence that transcript copy number is curvilinearly related to the proportion of RNA at $\alpha = 0.01$.

We also need to test:

$H_0 : \beta_5 = 0$
$H_a : \beta_5 \neq 0$

The test statistic is $t = 0.27$ and the p-value is $p = 0.7898$. Since the p-value is not less than $\alpha \left(p = 0.7898 \nless 0.01\right)$, H_0 is not rejected. There is insufficient evidence that transcript copy number is curvilinearly related to the proportion of RNA at $\alpha = 0.01$.

4.65 Models (a) and (b) are nested. The complete model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2$ and the reduced model is $E(y) = \beta_0 + \beta_1 x_1$.
Models (a) and (d) are nested. The complete model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2$ and the reduced model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2$.
Models (a) and (e) are nested. The complete model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 + \beta_4 x_1^2 + \beta_5 x_2^2$ and the reduced model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2$.
Models (b) and (c) are nested. The complete model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_1^2$ and the reduced model is $E(y) = \beta_0 + \beta_1 x_1$.
Models (b) and (d) are nested. The complete model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2$ and the reduced model is $E(y) = \beta_0 + \beta_1 x_1$.

Models (b) and (e) are nested. The complete model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 + \beta_4 x_1^2 + \beta_5 x_2^2$ and the reduced model is $E(y) = \beta_0 + \beta_1 x_1$.
Models (c) and (e) are nested. The complete model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 + \beta_4 x_1^2 + \beta_5 x_2^2$ and the reduced model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_1^2$.
Models (d) and (e) are nested. The complete model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 + \beta_4 x_1^2 + \beta_5 x_2^2$ and the reduced model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2$.

4.66 a. The parameters involved in testing that no curvature exists are β_7, β_8, and β_9.

b. The hypotheses are:

$H_0 : \beta_7 = \beta_8 = \beta_9 = 0$
H_a : At least 1 $\beta_i \neq 0$

c. To determine if x_3 is useful for the prediction of $E(y)$, we test:

$H_0 : \beta_3 = \beta_5 = \beta_6 = \beta_9 = 0$
H_a : At least 1 $\beta_i \neq 0$

4.67 a. The complete first-order model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_6$.

b. The reduced model would be $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4$. To determine if the leadership score of either the purser or head flight attendant (or both) is useful for predicting team goal attainment, we test:

$$H_0 : \beta_5 = \beta_6 = 0$$
$$H_a : \text{At least } 1\, \beta_i \neq 0$$

c. Because the R^2 value increased from 0.02 to 0.25, we would think that the null hypothesis would be rejected. We would conclude that the leadership score of either the purser or head flight attendant (or both) is useful for predicting team goal attainment for successful cabin crews. However, the complete model is still not that useful. Only 25% of the variation in team goal attainment scores can be explained by the complete model.

d. To determine if the leadership score of either the purser or head flight attendant (or both) is useful for predicting team goal attainment, we test:

$$H_0 : \beta_5 = \beta_6 = 0$$
$$H_a : \text{At least } 1\, \beta_i \neq 0$$

The p-value is $p < 0.05$. Since the p-value is less than $\alpha\,(p < 0.05)$, H_0 is rejected. There is sufficient evidence to indicate that the leadership score of either the purser or head flight attendant (or both) is useful for predicting team goal attainment for successful cabin crews at $\alpha = 0.05$.

e. Because the R^2 value only increased from 0.14 to 0.15, we would think that the null hypothesis would not be rejected. We would conclude that there is no evidence to indicate leadership score of either the purser or head flight attendant (or both) is useful for predicting team goal attainment for unsuccessful cabin crews.

f. To determine if the leadership score of either the purser or head flight attendant (or both) is useful for predicting team goal attainment, we test:

$$H_0 : \beta_5 = \beta_6 = 0$$
$$H_a : \text{At least } 1\, \beta_i \neq 0$$

The p-value is $p > 0.10$. Since the p-value is not less than $\alpha\,(p > 0.10 \nless 0.05)$, H_0 is not rejected. There is insufficient evidence to indicate that the leadership score of either the purser or head flight attendant (or both) is useful for predicting team goal attainment for unsuccessful cabin crews at $\alpha = 0.05$.

4.68 a. The model would be $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_1 x_4 + \beta_6 x_2 x_4 + \beta_7 x_3 x_4$.

b. Using MINITAB, the results are:

Regression Analysis: DESIRE versus GENDER, SELFESTM, ... BODY_IMP

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	7	853.89	121.984	24.40	0.000
GENDER	1	4.15	4.152	0.83	0.363
SELFESTM	1	2.98	2.979	0.60	0.441
BODYSAT	1	0.40	0.404	0.08	0.777
IMPREAL	1	11.50	11.502	2.30	0.131
GEN_IMP	1	0.08	0.080	0.02	0.899
EST_IMP	1	0.61	0.605	0.12	0.728
BODY_IMP	1	4.67	4.671	0.93	0.335
Error	162	809.90	4.999		
Lack-of-Fit	141	594.56	4.217	0.41	0.999
Pure Error	21	215.33	10.254		
Total	169	1663.79			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
2.23593	51.32%	49.22%	46.80%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	13.09	2.01	6.50	0.000	
GENDER	-1.89	2.07	-0.91	0.363	33.88
SELFESTM	-0.091	0.118	-0.77	0.441	27.24
BODYSAT	0.135	0.475	0.28	0.777	68.43
IMPREAL	0.746	0.492	1.52	0.131	15.63
GEN_IMP	-0.065	0.511	-0.13	0.899	40.56
EST_IMP	0.0098	0.0281	0.35	0.728	68.28
BODY_IMP	-0.112	0.116	-0.97	0.335	98.28

Regression Equation

DESIRE = 13.09 - 1.89 GENDER - 0.091 SELFESTM + 0.135 BODYSAT + 0.746 IMPREAL - 0.065 GEN_IMP + 0.0098 EST_IMP - 0.112 BODY_IMP

The fitted regression equation is
$\hat{y} = 13.09 - 1.89x_1 - 0.091x_2 + 0.135x_3 + 0.746x_4 - 0.065x_1x_4 + 0.0098x_2x_4 - 0.112x_3x_4$.

To determine the overall utility of the model, we test:

$$H_0: \beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = \beta_6 = \beta_7 = 0$$
$$H_a: \text{At least } 1\ \beta_i \neq 0$$

The test statistic is $F = 24.40$ and the p-value is $p = 0.000$.. Since the p-value is so small, H_0 will be rejected for any reasonable value of α.. There is sufficient evidence to indicate the model is useful for predicting desire to have cosmetic surgery.

c. To determine if impression of reality TV interacts with each of the other independent variables, the null hypothesis is:

$$H_0: \beta_5 = \beta_6 = \beta_7 = 0$$

d. The reduced model is $E(y)=\beta_0+\beta_1x_1+\beta_2x_2+\beta_3x_3+\beta_4x_4$. This model was fit in Exercise 4.12 with $SSE_R=835.95$.

The test statistic is $F=\dfrac{(SSE_R-SSE_C)/(k-g)}{SSE_C/[n-(k+1)]}=\dfrac{(835.95-809.90)/(7-4)}{809.90/[170-(7+1)]}=1.74$.

Since no α was given, we will use $\alpha=0.05$. The rejection region requires $\alpha=0.05$ in the upper tail of the F-distribution with $\nu_1=k-g=7-4=3$ and $\nu_2=n-(k+1)=170-(7+1)=162$. Using MINITAB, $F_{0.05}=2.66$. The rejection region is $F>2.66$.

Since the observed value of the test statistic does not fall in the rejection region $(F=1.74 \not> 2.66)$, H_0 is not rejected. There is insufficient evidence to indicate impression of reality TV interacts with at least one of the other independent variables at $\alpha=0.05$.

4.69 a. To determine whether the rate of increase of team performance with time pressure depends on effectiveness of the team leader, we test:

$H_0: \beta_4=\beta_5=0$
$H_a:$ At least one $\beta_i \neq 0$

b. For fixed time pressure, to determine whether the mean team performance differs for teams with effective and non-effective team leaders, we test:

$H_0: \beta_3=\beta_4=\beta_5=0$
$H_a:$ At least one $\beta_i \neq 0$

4.70 a. $R^2=0.101$. 10.1% of the sample variation in aggression score is explained by Model 1.
$R^2=0.555$. 55.5% of the sample variation in aggression score is explained by Model 2.

b. The hypotheses to compare the 2 models are:

$H_0: \beta_5=\beta_6=\beta_7=\beta_8=0$
$H_a:$ At least one $\beta_i \neq 0$

c. Yes, the two models are nested since all of the terms of the first model and four additional terms are included in the second.

d. Since the p-value is less than $\alpha\,(p<0.001<0.05)$, H_0 is rejected. There is sufficient evidence to conclude that the additional terms contribute to the prediction of y at $\alpha=0.05$.

e. The new model would be

$$E(y)=\beta_0+\beta_1x_1+\beta_2x_2+\beta_3x_3+\beta_4x_4+\beta_5x_5+\beta_6x_6+\beta_7x_7+\beta_8x_8$$
$$\beta_9x_5x_6+\beta_{10}x_5x_7+\beta_{11}x_5x_8+\beta_{12}x_6x_7+\beta_{13}x_6x_8+\beta_{14}x_7x_8.$$

f. The hypotheses to compare the 2 models are:

$$H_0 : \beta_9 = \beta_{10} = \cdots = \beta_{14} = 0$$
$$H_a : \text{At least one } \beta_i \neq 0$$

Since the p-value is not less than $\alpha\left(p > 0.10 \not< 0.05\right)$, H_0 is not rejected. There is insufficient evidence to conclude that the additional terms in model 3contribute to the prediction of y at $\alpha = 0.05$.

4.71 a. The complete second-order model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 + \beta_4 x_1^2 + \beta_5 x_2^2$.

b. To determine if the curvature terms are useful for predicting heat rate, we test:

$$H_0 : \beta_4 = \beta_5 = 0$$
$$H_a : \text{At least one } \beta_i \neq 0$$

c. The complete model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 + \beta_4 x_1^2 + \beta_5 x_2^2$ and the reduced model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2$.

d. $SSE_R = 25{,}310{,}639$, $SSE_C = 19{,}370{,}350$, $MSE_C = 317{,}547$

e. The test statistic is

$$F = \frac{\left(SSE_R - SSE_C\right)/(k-g)}{SSE_C/\left[n-(k+1)\right]} = \frac{(25{,}310{,}639 - 19{,}370{,}350)/(5-3)}{19{,}370{,}350/\left[67-(5+1)\right]} = 9.35.$$

f. The rejection region requires $\alpha = 0.10$ in the upper tail of the F-distribution with $\nu_1 = k - g = 5 - 3 = 2$ and $\nu_2 = n - (k+1) = 67 - (5+1) = 61$. Using MINITAB, $F_{0.10} = 2.39$. The rejection region is $F > 2.39$.

g. Since the observed value of the test statistic falls in the rejection region $\left(F = 9.35 > 2.39\right)$, H_0 is rejected. There is sufficient evidence to indicate the curvature terms are useful for predicting heat rate at $\alpha = 0.10$.

4.72 a. Multiple t-tests result in an increased Type I error rate.

b. To determine if the transcript number is curvilinearly related to proportion of RNA, we test:

$$H_0 : \beta_2 = \beta_5 = 0$$
$$H_a : \text{At least one } \beta_i \neq 0$$

c. The test statistic is $F = 0.060$ and the p-value is $p = 0.942$. Since the p-value is not small, H_0 is not rejected. There is insufficient evidence to conclude that transcript number is curvilinearly related to proportion of RNA at any reasonable value of α.

d. $$F = \frac{(SSE_R - SSE_C)/(k-g)}{MSE_C} = \frac{(89{,}171.145 - 88{,}818.679)/(5-3)}{2960.623} = 0.06$$

4.73 a. To determine if the quadratic terms in the model are useful, we test:

$H_0: \beta_4 = \beta_5 = 0$
H_a: At least one $\beta_i \neq 0$

b. The complete model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 + \beta_4 x_2^2 + \beta_5 x_1 x_2^2$ and the reduced model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2$.

c. To determine if the interaction terms in the model are useful, we test:

$H_0: \beta_3 = \beta_5 = 0$
H_a: At least one $\beta_i \neq 0$

d. The complete model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 + \beta_4 x_2^2 + \beta_5 x_1 x_2^2$ and the reduced model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_4 x_2^2$.

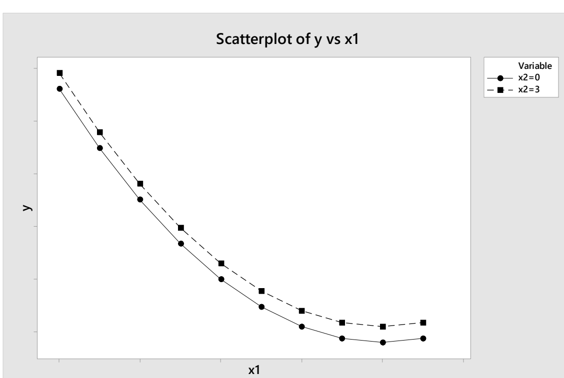
e. To determine if the dummy terms in the model are useful, we test:

$H_0: \beta_1 = \beta_3 = \beta_5 = 0$
H_a: At least one $\beta_i \neq 0$

f. The complete model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 + \beta_4 x_2^2 + \beta_5 x_1 x_2^2$ and the reduced model is $E(y) = \beta_0 + \beta_2 x_2 + \beta_4 x_2^2$.

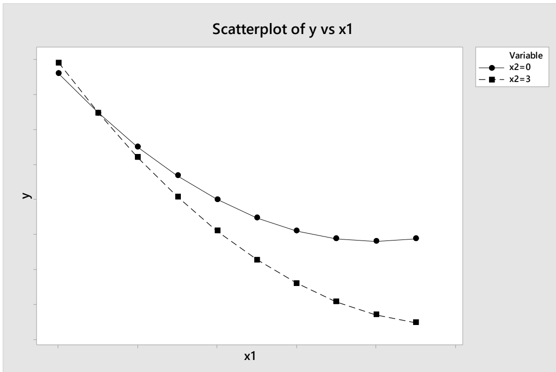
4.74 a. Since the researchers hypothesize that the income will decrease at a decreasing rate, the sign of β_2 will be positive.

b. Answers will vary. A possible graph is:



c. A complete second-order model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_1^2 + \beta_3 x_2 + \beta_4 x_1 x_2 + \beta_5 x_1^2 x_2$.

d. Answers will vary. A possible graph is:



e. To compare the two models, we test:

$$H_0 : \beta_4 = \beta_5 = 0$$
$$H_a : \text{At least one } \beta_i \neq 0$$

f. Using MINITAB, the results of fitting the reduced model in part a are:

Regression Analysis: Income versus Agree, Agree_Sq, Gender
Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	3	18708663846	6236221282	104.17	0.000
Error	96	5747214158	59866814		
Total	99	24455878004			

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	-21657	31780	-0.68	0.497
Agree	37155	19257	1.93	0.057
Agree_Sq	-7056	2903	-2.43	0.017
Gender	25482	1552	16.42	0.000

Using MINITAB, the results of fitting the complete model in part c are:

Regression Analysis: Income versus Agree, Agree_Sq, ... A, G_Agree_Sq
Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	5	18808157832	3761631566	62.61	0.000
Error	94	5647720172	60082129		
Total	99	24455878004			

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	-9847	45303	-0.22	0.828
Agree	27248	28743	0.95	0.346
Agree_Sq	-5169	4520	-1.14	0.256
Gender	42549	71654	0.59	0.554
G_A	-4765	43177	-0.11	0.912
G_Agree_Sq	-128	6474	-0.02	0.984

The test statistic is

$$F = \frac{(SSE_R - SSE_C)/(k-g)}{MSE_C} = \frac{(5,747,214,158 - 5,647,720,172)/(5-3)}{60,082,129} = 0.83.$$

The rejection region requires $\alpha = 0.10$ in the upper tail of the F distribution with $\nu_1 = k - g = 5 - 3 = 2$ and $\nu_2 = n - (k+1) = 100 - (5+1) = 94$. Using MINITAB, $F_{0.10} = 2.36$. The rejection region is $F > 2.36$.

Since the observed value of the test statistic does not fall in the rejection region $(F = 0.83 \not> 2.36)$, H_0 is not rejected. There is insufficient evidence to conclude that gender and agreeableness score interact to affect income at $\alpha = 0.10$.

4.75 a. The interaction model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_1 x_2 + \beta_5 x_1 x_3 + \beta_6 x_2 x_3$.

b. When $x_1 = 1000$ and $x_2 = 50$,

$$E(y) = \beta_0 + \beta_1(1000) + \beta_2(50) + \beta_3 x_3 + \beta_4(1000)(50) + \beta_5(1000)x_3 + \beta_6(50)x_3$$
$$= \beta_0 + 1000\beta_1 + 50\beta_2 + 50,000\beta_4 + (\beta_3 + 1000\beta_5 + 50\beta_6)x_3.$$

Thus, the slope is $(\beta_3 + 1000\beta_5 + 50\beta_6)$.

c. When $x_1 = 1000$ and $x_2 = 150$,

$$E(y) = \beta_0 + \beta_1(1000) + \beta_2(150) + \beta_3 x_3 + \beta_4(1000)(150) + \beta_5(1000)x_3 + \beta_6(150)x_3$$
$$= \beta_0 + 1000\beta_1 + 150\beta_2 + 150,000\beta_4 + (\beta_3 + 1000\beta_5 + 150\beta_6)x_3.$$

Thus, the slope is $(\beta_3 + 1000\beta_5 + 150\beta_6)$.

d. Using MINITAB, the results of the regression analyses are:

Regression Analysis: FORCE versus SPEED, RATE, PCTWT, ... , RATE_WT

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	6	216288	36047.9	378.90	0.000
Error	9	856	95.1		
Total	15	217144			

Model Summary

S	R-sq	R-sq(adj)
9.75392	99.61%	99.34%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	343.1	19.0	18.02	0.000
SPEED	0.01812	0.00732	2.48	0.035
RATE	1.988	0.146	13.58	0.000
PCTWT	9.38	1.46	6.41	0.000
SP_RATE	-0.000012	0.000049	-0.26	0.803
SP_WT	-0.000375	0.000488	-0.77	0.462
RATE_WT	0.01250	0.00975	1.28	0.232

Regression Equation

FORCE = 343.1 + 0.01812 SPEED + 1.988 RATE + 9.38 PCTWT - 0.000012 SP_RATE - 0.000375 SP_WT + 0.01250 RATE_WT

The least squares prediction equation is
$\hat{y} = 343.1 + 0.01812x_1 + 1.988x_2 + 9.38x_3 - 0.000012x_1x_2 - 0.000375x_1x_3 + 0.01250x_2x_3$.

e. To determine if any of the interaction terms contribute to the usefulness of the model, we must first fit the reduced model $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$.

Using MINITAB, the results are:

Regression Analysis: FORCE versus SPEED, RATE, PCTWT

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	3	216069	72022.9	803.98	0.000
Error	12	1075	89.6		
Total	15	217144			

Model Summary

S	R-sq	R-sq(adj)
9.46485	99.50%	99.38%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	340.62	8.53	39.93	0.000
SPEED	0.01313	0.00237	5.55	0.000
RATE	2.0875	0.0473	44.11	0.000
PCTWT	9.875	0.473	20.87	0.000

Regression Equation

FORCE = 340.62 + 0.01313 SPEED + 2.0875 RATE + 9.875 PCTWT

To determine if any of the interaction terms contribute to the usefulness of the model, we test:

$H_0: \beta_4 = \beta_5 = \beta_6 = 0$

H_a : At least one $\beta_i \neq 0$

The test statistic is $F = \dfrac{(SSE_R - SSE_C)/(k-g)}{MSE_C} = \dfrac{(1075-856)/(6-3)}{95.1} = 0.77$.

The rejection region requires $\alpha = 0.05$ in the upper tail of the F distribution with $\nu_1 = k - g = 6 - 3 = 3$ and $\nu_2 = n - (k+1) = 16 - (6+1) = 9$. Using Table4, Appendix D, $F_{0.05} = 3.86$. The rejection region is $F > 3.86$.

Since the observed value of the test statistic does not fall in the rejection region $(F = 0.77 \not> 3.86)$, H_0 is not rejected. There is insufficient evidence to conclude that any of the interaction terms contribute to the usefulness of the model at $\alpha = 0.05$.

4.76 Since the variation of the sampling distribution for estimating the mean value of y has a smaller standard deviation than the sampling distribution for estimating a particular value of y $\left(\sqrt{\frac{1}{n} + \frac{(x-\bar{x})^2}{SS_{xx}}}\ vs\ \sqrt{1 + \frac{1}{n} + \frac{(x-\bar{x})^2}{SS_{xx}}}\right)$, the confidence interval will always be narrower.

4.77 a. First, we define the dummy variables:

$$x_1 = \begin{cases} 1 \text{ if groundwater} \\ 0 \text{ if not} \end{cases} \qquad x_2 = \begin{cases} 1 \text{ if subsurface flow} \\ 0 \text{ if not} \end{cases}$$

The model would be $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2$.

b. $\beta_0 =$ mean nitrate concentration when the water source is overground flow $= \mu_{\text{Overground}}$

$\beta_1 =$ difference in mean nitrate concentration between water sources groundwater and overground flow $= \mu_{\text{Groundwatrer}} - \mu_{\text{Overground}}$

$\beta_2 =$ difference in mean nitrate concentration between water sources subsurface flow and overground flow $= \mu_{\text{Subsurface flow}} - \mu_{\text{Overground}}$

4.78 a. $R^2_{adj} = 0.76$ 76% of the sample variability in the SAT-Math scores about their means is explained by the linear relationship between the SAT-Math scores and the independent variables score on PSAT and whether the student was coached or not.

b. Foe confidence coefficient 0.95, $\alpha = 0.05$ and $\alpha/2 = 0.05/2 = 0.025$. From Table 1, Appendix D, with $df = n - (k+1) = 3492 - (2+1) = 3489$, $t_{0.025} = 1.960$. The 95% confidence interval is:

$$\hat{\beta}_2 \pm t_{0.025} s_{\hat{\beta}_2} \Rightarrow 19 \pm 1.960(3) \Rightarrow 19 \pm 5.88 \Rightarrow (13.12,\ 24.88)$$

We are 95% confident that the difference in the mean SAT-Math scores between students who were coached and those who were not is between 13.12 and 24.88 points, holding PSAT scores constant.

c. Since the interval in part b does not contain negative numbers, there is evidence that students who were coached scored better than students who were not coached.

d. The hypothesized model would be
$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_6 + \beta_7 x_7 + \beta_8 x_8 + \beta_9 x_9 + \beta_{10} x_{10}$.

e. To compare the two models, the null hypothesis would be $H_0: \beta_3 = \beta_4 = \beta_5 = \cdots = \beta_{10} = 0$

f. The test was statistically significant. Thus, H_0 was rejected. There is sufficient evidence to indicate that at least one of the "control" variables contributes to the prediction of SAT-Math scores.

g . $R^2_{adj} = 0.79$. 79% of the sample variability of the SAT-Math scores around their means is explained by the proposed model relating SAT-Math scores to the 10 independent variables, adjusting for the sample size and the number of β parameters in the model.

h. For confidence coefficient 0.95, $\alpha = 0.05$ and $\alpha / 2 = 0.05/2 = 0.025$. From Table 2, Appendix D, with $df = n - (k+1) = 3492 - (10+1) = 3481$, $t_{0.025} = 1.96$. The 95% confidence interval is:

$$\hat{\beta}_2 \pm t_{0.025} s_{\hat{\beta}_2} \Rightarrow 14 \pm 1.96(3) \Rightarrow 14 \pm 5.88 \Rightarrow (8.12, 19.88)$$

We are 95% confident that the difference in the mean SAT-Math scores between students who were coached and those who were not is between 8.12 and 19.88 points, holding all the other variables constant.

i. Yes. From part b, the confidence interval for β_2 was $(13.12,\ 24.88)$. In part h, the confidence interval for β_2 was $(8.12,\ 19.88)$. Even though coaching is significant in both models, the change in the mean SAT-Math scores is not as great if the control variables are added to the model. Notice that each point estimate is within the confidence interval of the other.

j. The complete model would be:

$$\begin{aligned} E(y) = {} & \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_6 + \beta_7 x_7 + \beta_8 x_8 + \beta_9 x_9 + \beta_{10} x_{10} \\ & + \beta_{11} x_2 x_1 + \beta_{12} x_2 x_3 + \beta_{13} x_2 x_4 + \beta_{14} x_2 x_5 + \beta_{15} x_2 x_6 + \beta_{16} x_2 x_7 + \beta_{17} x_2 x_8 \\ & + \beta_{18} x_2 x_9 + \beta_{19} x_2 x_{10} \end{aligned}$$

k. The null hypothesis would be $H_0: \beta_{11} = \beta_{12} = \beta_{13} = \cdots = \beta_{19} = 0$. The test would be comparing nested models using an F-test.

4.79 a. To determine if the model is useful for predicting the number of hours needed, we test:

$$H_0: \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$$
$$H_a: \text{At least one } \beta_i \neq 0$$

The test statistic is $F = 72.12$ and the p-value is $p = 0.000$. Since the p-value is less than $\alpha \left(p = 0.000 < 0.01 \right)$, H_0 is rejected. There is sufficient evidence to indicate the model is useful for predicting the number of hours needed at $\alpha = 0.01$.

b. The 95% confidence interval is $\left(1448.65, 2424.14\right)$. We are 95% confident that the true mean number of hours for all boilers with characteristics $x_1 = 150{,}000$, $x_2 = 500$, $x_3 = 1$, and $x_4 = 0$ will fall between 1448.65 and 2424.14.

The 95% prediction interval is $\left(47.8402, 3824.95\right)$. We are 95% confident that the actual number of hours for all boilers with characteristics $x_1 = 150{,}000$, $x_2 = 500$, $x_3 = 1$, and $x_4 = 0$ will fall between 47.8402 and 3824.95.

c. We would use the 95% confidence interval to estimate the average hours required for all industrial mud boilers with a capacity of 150,000 lb/hr and a design pressure of 500 psi.

4.80 a. The first order model for $E\left(y\right)$ as a function of the first five independent variables is:

$$E\left(y\right) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5$$

b. To test the utility of the model, we test:

$$H_0 : \beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = 0$$
$$H_a : \text{ At least one } \beta_i \neq 0$$

The test statistic is $F = 34.47$ and the p-value is $p < 0.001$. Since the p-value is so small, H_0 is rejected. There is sufficient evidence to indicate the model is useful for predicting GSI at $\alpha > 0.001$.

$R^2 = 0.469$. 46.9% of the variability in the GSI scores around their means is explained by the model including the first five independent variables.

c. The first order model for $E\left(y\right)$ as a function of the first seven independent variables is:

$$E\left(y\right) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_6 + \beta_7 x_7$$

d. $R^2 = 0.603$. 60.3% of the variability in the GSI scores around their means is explained by the model including the first seven independent variables.

e. Since the p-values associated with the variables DES and PDEQ-SR are both less than 0.001, there is evidence that both variables contribute to the prediction of GSI, adjusted for all the other variables already in the model for $\alpha > 0.001$.

4.81 a. $\hat{\beta}_0 = 325{,}790$. This means that the mean percentage of motor vehicles without catalytic converters would be 325,790% when a given year is 0. Since the years range only from 1984 to 1999, 0 is not in the observed range. Thus $\hat{\beta}_0$ has no practical interpretation.

b. $\hat{\beta}_1 = -321.67$. When the quadratic term is present in the model $\left(x^2\right)$, the coefficient associated with the linear term is no longer the slope of the line. Rather, it is a location parameter and has no practical interpretation.

c. Since the sign of $\hat{\beta}_2$ is positive, the curve opens upward.

d. Since we have no idea what the relationship between y and x will be outside the observed range, we should not use the least squares prediction equation to predict the value of y for values of x outside the observed range.

4.82 a. The p-value associated with testing the significance of β_3 is $p > 0.10$. Since the p-value is so large, the interaction between implicit and explicit self-esteem does not have a significant effect on the dependent variable. The p-values associated with testing the significance of β_1 and with testing the significance of β_2 are both less than 0.001. Since the p-values are so small, the main effects of both implicit and explicit self-esteem are both significant for predicting accuracy.

b. The p-value associated with testing the significance of β_3 is $p < 0.001$. Since the p-value is so small, there is evidence to indicate the effect of implicit self-esteem on the dependent variable depends on the value of explicit self-esteem.

4.83 a. The first order model is $E\left(y\right) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5$.

b. The null hypothesis for testing the contribution of strength of client-therapist relationship is:

$$H_0 : \beta_4 = 0$$

c. The test statistic is $t = 4.408$ and the p-value is $p = 0.001$. Since the p-value is so small, H_0 is rejected. There is sufficient evidence to indicate that the strength of client-therapist relationship contributes to the prediction of a client's reaction to the report.

d. Yes. Since the estimate of the β coefficient for the interaction term is positive, males will have a higher estimated β coefficient for the therapist's age. With all other variables held constant, as age increases, males will have a higher estimated Client reaction to the report.

e. $R^2 = 0.2946$. 29.46% of the variability in the client's reaction scores can be explained by the model containing the 5 independent variables.

4.84 a. Since as the level of x_2 goes from 0 to 9, we expect the level of funniness to increase, then level off, then decrease, we are expecting to see negative or downward curvature. The sign we expect to see on β_2 is negative.

b. To determine if the quadratic model relating pain to funniness rating is useful, we test:

$H_0: \beta_1 = \beta_2 = 0$
H_a: At least one $\beta_i \neq 0$

The test statistic is $F = 1.60$. (given)

The rejection region requires $\alpha = 0.05$ in the upper tail of the F distribution with numerator $\nu_1 = k = 2$ and denominator $\nu_2 = n - (k+1) = 32 - (2+1) = 29$. From Table 4, Appendix D, $F_{0.05} = 3.33$. The rejection region is $F > 3.33$.

Since our observed test statistic did not fall in the rejection region ($F = 1.60 \not> 3.33$), H_0 is not rejected. There is insufficient evidence to indicate the quadratic model relating pain to funniness rating is useful at $\alpha = 0.05$.

c. To determine if the quadratic model relating aggression/hostility to funniness rating is useful, we test:

$H_0: \beta_1 = \beta_2 = 0$
H_a: At least one $\beta_i \neq 0$

The test statistic is $F = 1.61$. (given).

The rejection region requires $\alpha = 0.05$ in the upper tail of the F distribution with numerator $\nu_1 = k = 2$ and denominator $\nu_2 = n - (k+1) = 32 - (2+1) = 29$. From Table 4, Appendix D, $F_{0.05} = 3.33$. The rejection region is $F > 3.33$.

Since our observed test statistic did not fall in the rejection region $(F = 1.61 \not> 3.33)$, H_0 is not rejected. There is insufficient evidence to indicate the quadratic model relating aggression/hostility to funniness rating is useful at $\alpha = 0.05$.

4.85 a. Since there are only 2 levels of Type, only 1 dummy variable is needed. Let

$$x_1 = \begin{cases} 1 & \text{if aerosol/spray} \\ 0 & \text{otherwise} \end{cases}$$

The model is $E(y) = \beta_0 + \beta_1 x_1$.

b. Using MINITAB, the results of the regression are:

Regression Analysis: Cost versus x1
Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	0.04086	0.04086	0.06	0.814
Error	12	8.49728	0.70811		
Total	13	8.53814			

Model Summary

S	R-sq	R-sq(adj)
0.841491	0.48%	0.00%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	0.887	0.344	2.58	0.024
x1	-0.109	0.454	-0.24	0.814

Regression Equation

Cost = 0.887 - 0.109 x1

The fitted regression line is $\hat{y} = 0.887 - 0.109x_1$.

c. To determine if repellent type is a useful predictor of cost-per-use, we test: $H_0: \beta_1 = 0$

d. The test statistic is $t = -0.24$ and the p-value is $p = 0.814$. Since the p-value is not less than $\alpha\left(p = 0.814 \nless 0.10\right)$, H_0 is not rejected. There is insufficient evidence to indicate that repellant type is a useful predictor of cost-per-use at $\alpha = 0.10$.

e. The model is the same as that in part a.

Using MINITAB, the results of the regression are:

Regression Analysis: Hours versus x1
Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	9.287	9.287	0.21	0.653
Error	12	525.427	43.786		
Total	13	534.714			

Model Summary

S	R-sq	R-sq(adj)
6.61707	1.74%	0.00%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	5.92	2.70	2.19	0.049
x1	1.65	3.57	0.46	0.653

Regression Equation

Hours = 5.92 + 1.65 x1

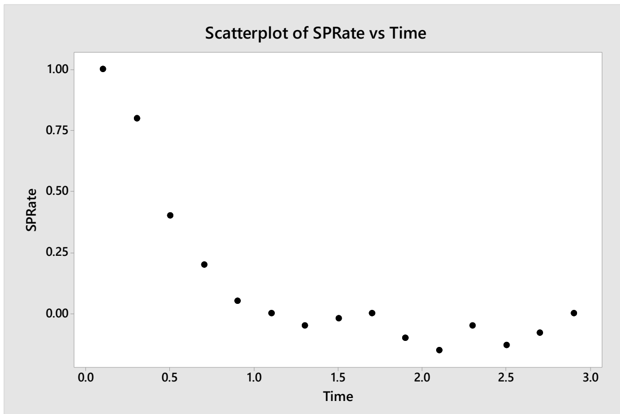
The fitted regression line is $\hat{y} = 5.92 + 1.65x_1$.

To determine if repellent type is a useful predictor of maximum number of hours of protection, we test:

$$H_0: \beta_1 = 0$$
$$H_a: \beta_1 \neq 0$$

The test statistic is $t = 0.46$ and the p-value is $p = 0.653$. Since the p-value is not less than α $(p = 0.653 \not< 0.10)$, H_0 is not rejected. There is insufficient evidence to indicate that repellant type is a useful predictor of maximum number of hours of protection at $\alpha = 0.10$.

4.86 a. Using MINITAB, the scatterplot is:



From the graph, it appears that there is a curvilinear relationship.

b. Using MINITAB, the regression results are:

Regression Analysis: SPRate versus Time

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	2	1.5478	0.77391	75.65	0.000
Error	12	0.1228	0.01023		
Total	14	1.6706			

Model Summary

S	R-sq	R-sq(adj)
0.101142	92.65%	91.43%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	1.0070	0.0790	12.75	0.000
Time	-1.167	0.122	-9.57	0.000
Time_sq	0.2898	0.0394	7.36	0.000

Regression Equation

SPRate = 1.0070 – 1.167 Time + 0.2898 Time_sq

The least squares prediction line is $\hat{y} = 1.0070 - 1.167x + 0.2898x^2$.

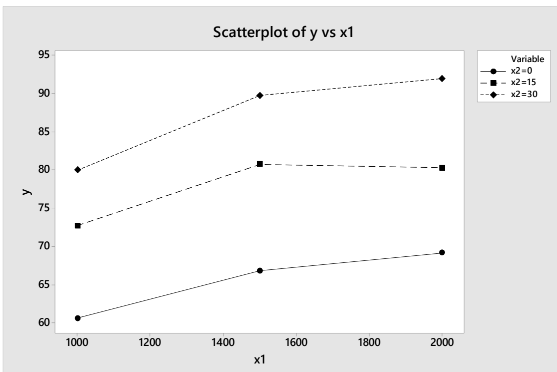
c. To determine if there is an upward curvature in the relationship between surface production rate and time after turnoff, we test:

$H_0: \beta_2 = 0$
$H_a: \beta_2 > 0$

The test statistic is $t = 7.36$ and the p-value is $p = 0.000./2 = 0.000$. Since the p-value is less than $\alpha\left(p = 0.000 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate an upward curvature in the relationship between surface production rate and time after turnoff at $\alpha = 0.05$.

4.87 a. The complete second-order model is $E\left(y\right) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1^2 + \beta_4 x_2^2 + \beta_5 x_1 x_2$.

b. Using MINITAB, the plot is:



Based on the plot, the complete second-order mode is not necessary. The shape of the plots for each level of dipping angle are similar.

c. Using MINITAB, the regression results are:

Regression Analysis: OILREC versus PRESSURE, DIPANGLE, PRESS_ANG

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	3	843.19	281.064	44.67	0.000
Error	5	31.46	6.292		
Total	8	874.65			

Model Summary

S	R-sq	R-sq(adj)
2.50838	96.40%	94.25%

Coefficients

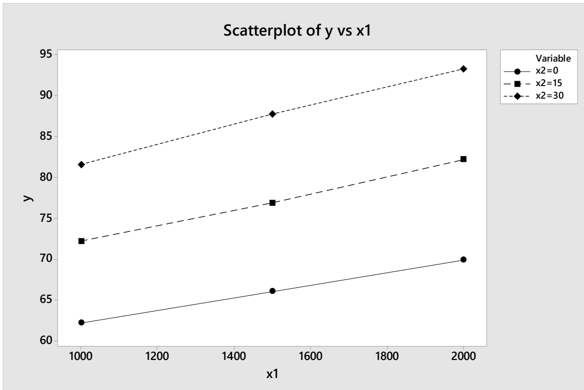
Term	Coef	SE Coef	T-Value	P-Value
Constant	54.50	5.03	10.83	0.000
PRESSURE	0.00770	0.00324	2.38	0.063
DIPANGLE	0.554	0.260	2.13	0.086
PRESS_ANG	0.000113	0.000167	0.68	0.528

Regression Equation

OILREC = 54.50 + 0.00770 PRESSURE + 0.554 DIPANGLE + 0.000113 PRESS_ANG

The prediction equation is $\hat{y} = 54.50 + 0.00770x_1 + 0.554x_2 + 0.000113x_1x_2$.

d. Using MINITAB, the plot is:



It appears that the model will provide an adequate fit.

e. To determine if the model is adequate, we test:

$$H_0: \beta_1 = \beta_2 = \beta_3 = 0$$
$$H_a: \text{At least one } \beta_i \neq 0$$

The test statistic is $F = 44.67$ and the p-value is $p = 0.000$. Since the p-value is less than $\alpha\left(p = 0.000 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate the model is adequate at $\alpha = 0.05$.

f. To determine if there is interaction between pressure and dipping angle, we test:

$$H_0: \beta_3 = 0$$
$$H_a: \beta_3 \neq 0$$

The test statistic is $t = 0.68$ and the p-value is $p = 0.528$. Since the p-value is not less than $\alpha\left(p = 0.528 \not< 0.05\right)$, H_0 is not rejected. There is insufficient evidence to indicate there is interaction between pressure and dipping angle at $\alpha = 0.05$.

4.88 a. First, we define the dummy variables:

$$x_1 = \begin{cases} 1 & \text{if none} \\ 0 & \text{otherwise} \end{cases} \quad x_2 = \begin{cases} 1 & \text{if low} \\ 0 & \text{otherwise} \end{cases} \quad x_3 = \begin{cases} 1 & \text{if medium} \\ 0 & \text{otherwise} \end{cases}$$

The model would be $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$.

b. $\beta_0 =$ the mean proportion of times the subject's eyes fixated on the speaker's mouth for high noise.

$\beta_1 =$ the difference in the mean proportion of times the subject's eyes fixated on the speaker's mouth between none and high noise.
$\beta_2 =$ the difference in the mean proportion of times the subject's eyes fixated on the speaker's mouth between low and high noise.
$\beta_3 =$ the difference in the mean proportion of times the subject's eyes fixated on the speaker's mouth between medium and high noise.

c. To determine if there is a difference in the mean proportion of times the subject's eyes fixated on the speaker's mouth among the 4 noise groups, we test:

$$H_0: \beta_1 = \beta_2 = \beta_3 = 0$$
$$H_a: \text{At least one } \beta_i \neq 0$$

4.89 a. First, we define the dummy variables:

$$x_1 = \begin{cases} 1 & \text{if youngest third} \\ 0 & \text{otherwise} \end{cases} \qquad x_2 = \begin{cases} 1 & \text{if middle third} \\ 0 & \text{otherwise} \end{cases}$$

The model would be $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2$.

b. From the table,

$$\hat{\beta}_0 = \mu_{\text{oldest third}} = 0.16$$
$$\hat{\beta}_1 = \mu_{\text{youngest third}} - \mu_{\text{oldest third}} = 0.33 - 0.16 = 0.17$$
$$\hat{\beta}_2 = \mu_{\text{middle third}} - \mu_{\text{oldest third}} = 0.33 - 0.16 = 0.17$$

c. The model for the girls is the same as for the boys.
From the table,

$$\hat{\beta}_0 = \mu_{\text{oldest third}} = 0.21$$
$$\hat{\beta}_1 = \mu_{\text{youngest third}} - \mu_{\text{oldest third}} = 0.27 - 0.21 = 0.06$$
$$\hat{\beta}_2 = \mu_{\text{middle third}} - \mu_{\text{oldest third}} = 0.18 - 0.21 = -0.03$$

4.90 a. The complete model is

$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_6 + \beta_7 x_7 + \beta_8 x_8 + \beta_9 x_9 + \beta_{10} x_{10} + \beta_{11} x_{11} + \beta_{12} x_{12} + \beta_{13} x_{13} + \beta_{14} x_{14} + \beta_{15} x_{15} + \beta_{16} x_{16} + \beta_{17} x_{17} + \beta_{18} x_{18} + \beta_{19} x_{19} + \beta_{20} x_{20} + \beta_{21} x_{21}$$

where variables $x_1 - x_4$ are the demographic variables, $x_5 - x_{11}$ are the diagnostic variables, $x_{12} - x_{15}$ are the treatment variables, and $x_{16} - x_{21}$ are the community variables.

b. To determine if the 7 diagnostic variables contribute information for the prediction of community adjustment, we test:

$$H_0: \beta_5 = \beta_6 = \cdots = \beta_{11} = 0$$
$$H_a: \text{At least one } \beta_i \neq 0$$

c. The reduced model would be

$$E(y)=\beta_0+\beta_1x_1+\beta_2x_2+\beta_3x_3+\beta_4x_4+\beta_{12}x_{12}+\beta_{13}x_{13}+\beta_{14}x_{14}+\beta_{15}x_{15}$$
$$+\beta_{16}x_{16}+\beta_{17}x_{17}+\beta_{18}x_{18}+\beta_{19}x_{19}+\beta_{20}x_{20}+\beta_{21}x_{21}$$

d. The test statistic is $F=59.3$ and the p-value is $p<=0.0001$. Since the p-value is so small, H_0 is rejected. There is sufficient evidence to indicate at least one of the diagnostic variables contributes to the prediction of community adjustment for any reasonable value of α.

4.91 a. To determine whether the rate of increase of emotional distress with experience is different for the two groups, we test:

$$H_0:\beta_4=\beta_5=0$$
$$H_a:\text{At least one } \beta_i\neq 0$$

b. To determine whether there are differences in mean emotional distress levels that are attributable to exposure group, we test:

$$H_0:\beta_3=\beta_4=\beta_5=0$$
$$H_a:\text{At least one } \beta_i\neq 0$$

c. The test statistic is $F=\dfrac{(SSE_R-SSE_C)/(k-g)}{SSE_C\big/\left[n-(k+1)\right]}=\dfrac{(795.23-783.90)/(5-2)}{783.90\big/\left[200-(5+1)\right]}=0.93$

The rejection region requires $\alpha=0.05$ in the upper tail of the F distribution with $\nu_1=k-g=5-2=3$ and $\nu_2=n-(k+1)=200-(5+1)=194$. Using MINITAB, $F_{0.05}=2.65$. The rejection region is $F>2.65$.

Since the observed value of the test statistic does not fall in the rejection region $(F=0.93\not>2.65)$, H_0 is not rejected. There is insufficient evidence to conclude there are differences in mean emotional distress levels that are attributable to exposure group at $\alpha=0.05$.

4.92 a. Since temperature is measured on a numerical scale, it is quantitative.

b. Since relative humidity is measured on a numerical scale, it is quantitative.

c. Since organic compound is not measured on a numerical scale, it is qualitative.

d. Let $x_1=\begin{cases}1 \text{ if benzene}\\0 \text{ if not}\end{cases}$ $x_2=\begin{cases}1 \text{ if toulene}\\0 \text{ if not}\end{cases}$

$x_3=\begin{cases}1 \text{ if chloroform}\\0 \text{ if not}\end{cases}$ $x_4=\begin{cases}1 \text{ if methanol}\\0 \text{ if not}\end{cases}$

The model would then be $E(y)=\beta_0+\beta_1x_1+\beta_2x_2+\beta_3x_3+\beta_4x_4$.

e. $\beta_0 = \mu_A$; $\beta_1 = \mu_B - \mu_A; \beta_2 = \mu_T - \mu_A; \beta_3 = \mu_C - \mu_A; \beta_4 = \mu_M - \mu_A$.

f. To see if the mean sorption rates of the five organic compounds differ, we test:

$$H_0: \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$$
$$H_a: \text{At least one } \beta_i \neq 0$$

The test statistic would be an F.

4.93 a. The first-order model would be $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2$.

b. Using MINITAB, the results of the regression analysis are:

Regression Analysis: Earnings versus Age, Hours
Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	2	5018232	2509116	8.36	0.005
Error	12	3600196	300016		
Total	14	8618428			

Model Summary

S	R-sq	R-sq(adj)
547.737	58.23%	51.26%

Coefficients

Term	Coef	SE Coef	95% CI	T-Value	P-Value	VIF
Constant	-20	653	(-1443, 1402)	-0.03	0.976	
Age	13.35	7.67	(-3.36, 30.07)	1.74	0.107	1.01
Hours	243.7	63.5	(105.3, 382.1)	3.84	0.002	1.01

Regression Equation

Earnings = -20 + 13.35 Age + 243.7 Hours

The least squares line is $\hat{y} = -20 + 13.35x_1 + 243.7x_2$.

c. $\hat{\beta}_0 = -20$. This has no meaning because $x_1 = 0$ and $x_2 = 0$ are not in the observed range.

$\hat{\beta}_1 = 13.35$. The mean annual earnings is estimated to increase by \$13.35 for each additional year of age, holding hours worked per day constant.

$\hat{\beta}_2 = 243.7$. The mean annual earnings is estimated to increase by \$243.70 for each additional hour worked per day, holding year of age constant.

d. To determine if the overall model is useful for predicting annual earnings, we test:

$$H_0: \beta_1 = \beta_2 = 0$$
$$H_a: \text{At least one } \beta_i \neq 0$$

The test statistic is $F = 8.36$ and the *p*-value is $p = 0.005$. Since the *p*-value is less than $\alpha\left(p = 0.005 < 0.01\right)$, H_0 is rejected. There is sufficient evidence to indicate the overall model is useful for predicting annual earnings at $\alpha = 0.01$.

e. $R^2_{adj} = 0.5126$. 51.26% of the variation in annual earnings is explained by the model containing age and the number of hours worked per day, adjusted for the number of variables in the model and the sample size.

f. $s = 547.737$. We would expect almost all values of annual earnings to fall within $2s = 2(547.737) = 1095.474$ units of their least squares predicted values.

g. To determine if age is a useful predictor of earnings, we test:

$$H_0: \beta_1 = 0$$
$$H_a: \beta_1 \neq 0$$

The test statistic is $t = 1.74$ and the *p*-value is $p = 0.107$. Since the *p*-value is not less than $\alpha\left(p = 0.107 \not< 0.01\right)$, H_0 is not rejected. There is insufficient evidence to indicate age is a useful predictor of annual earnings at $\alpha = 0.01$.

h. From the printout, the 95% confidence interval for β_2 is (105.3, 382.1). We are 95% confident that for every additional hour worked per day, the mean annual earnings will increase from \$105.30 to \$382.1, holding age constant.

i. Using MINITAB, the results are:

Prediction for Earnings
Regression Equation

Earnings = -20 + 13.35 Age + 243.7 Hours

Settings

Variable	Setting
Age	45
Hours	10

Prediction

Fit	SE Fit	95% CI	95% PI
3017.56	182.352	(2620.25, 3414.87)	(1759.75, 4275.38)

The 95% prediction interval for annual earnings when age is 45 and hours worked is 10 is (1759.75, 4275.38). We are 95% confident that a man aged 45 who works 10 hours per day will earn somewhere between \$1759.75 and \$4275.38 per year.

The 95% confidence interval for annual earnings when age is 45 and hours worked is 10 is (2620.25, 3414.87). We are 95% confident that the mean earnings of all men aged 45 who work 10 hours per day will be somewhere between \$2620.25 and \$3414.87 per year.

j. Using MINITAB, the results of fitting the interaction model are:

Regression Analysis: Earnings versus Age, Hours, Age_Hours

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	3	5287427	1762476	5.82	0.012
Error	11	3331000	302818		
Total	14	8618428			

Model Summary

	R-sq	R-sq(adj)
550.289	61.35%	50.81%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	1042	1304	0.80	0.441
Age	-13.2	29.2	-0.45	0.659
Hours	103	162	0.64	0.537
Age_Hours	3.62	3.84	0.94	0.366

Regression Equation

Earnings = 1042 - 13.2 Age + 103 Hours + 3.62 Age_Hours

The least squares prediction equation is $\hat{y} = 1042 - 13.2x_1 + 103x_2 + 3.62x_1x_2$.

k. When $x_2 = 10$, $= \hat{y} = 1042 - 13.2x_1 + 103(10) + 3.62x_1(10) = 2072 + 23x_1$. The slope relating annual earnings to age when hours worked is 10 is 23. We estimate that for each additional year of age, the annual earnings will increase by $23 when the hours worked per day is 10.

l. When $x_1 = 40$, $= \hat{y} = 1042 - 13.2(40) + 103x_2 + 3.62(40)x_2 = 514 + 247.8x_2$. The slope relating annual earnings to hours worked when age is 40 is 247.8. We estimate that for each additional hour worked, the annual earnings will increase by $247.80 when the worker is 40 years old.

m. To determine if age and hours worked interact, the null hypothesis is:

$$H_0: \beta_3 = 0$$

n. The p-value is $p = 0.366$.

o. Since the p-value is not small, H_0 is not rejected. There is insufficient evidence to indicate that age and hours worked interact to affect annual earnings.

4.94 a. To determine if any of the supervisory behavior variables affect the intrinsic job satisfaction, we test:

$$H_0: \beta_5 = \beta_6 = \beta_7 = \beta_8 = 0$$
$$H_a: \text{At least one } \beta_i \neq 0$$

For males, the rejection region requires $\alpha = 0.05$ in the upper tail of the F distribution with $\nu_1 = k - g = 8 - 4 = 4$ and $\nu_2 = n - (k+1) = 244 - (8+1) = 235$. Using MINITAB, $F_{0.05} = 2.41$. The rejection region is $F > 2.41$.

For females, the rejection region requires $\alpha = 0.05$ in the upper tail of the F distribution with $\nu_1 = k - g = 8 - 4 = 4$ and $\nu_2 = n - (k+1) = 153 - (8+1) = 144$. Using MINITAB, $F_{0.05} = 2.43$. The rejection region is $F > 2.43$.

b. For males, reduced model: $R^2 = 0.218$. This implies that 21.8% of the sample variability of the intrinsic job satisfaction scores is explained by the model containing age, education level, firm experience, and sales experience.

For males, complete model: $R^2 = 0.408$. This implies that 40.8% of the sample variability of the intrinsic job satisfaction scores is explained by the model containing age, education level, firm experience, sales experience, contingent reward behavior, noncontingent reward behavior, contingent punishment behavior, and noncontingent punishment behavior.

Since the R^2 value increased from 0.218 to 0.408 after the 4 supervisory behavior variables were added to the model, it appears that they did have an impact on intrinsic job satisfaction for the males.

For females, reduced model: $R^2 = 0.268$. This implies that 26.8% of the sample variability of the intrinsic job satisfaction scores is explained by the model containing age, education level, firm experience, and sales experience.

For females, complete model: $R^2 = 0.496$. This implies that 49.6% of the sample variability of the intrinsic job satisfaction scores is explained by the model containing age, education level, firm experience, sales experience, contingent reward behavior, noncontingent reward behavior, contingent punishment behavior, and noncontingent punishment behavior.

Since the R^2 value increased from 0.268 to 0.496 after the 4 supervisory behavior variables were added to the model, it appears that they did have an impact on intrinsic job satisfaction for the females.

c. For Males: The test statistic is $F_{\text{males}} = 13.00$. From part a, the rejection region is $F > 2.41$. Since the observed value of the test statistic falls in the rejection region $(F = 13.00 > 2.41)$, H_0 is rejected. There is sufficient evidence to indicate that at least one of the supervisory behavior variables contribute to the prediction of intrinsic job satisfaction at $\alpha = 0.05$.

For Females: The test statistic is $F_{\text{females}} = 9.05$. From part a, the rejection region is $F > 2.43$. Since the observed value of the test statistic falls in the rejection region $(F = 9.05 > 2.43)$, H_0 is rejected. There is sufficient evidence to indicate that at least one of the supervisory behavior variables contribute to the prediction of intrinsic job satisfaction at $\alpha = 0.05$.

4.95 a. First we define the dummy variable:

$$x_9 = \begin{cases} 1 & \text{if male} \\ 0 & \text{otherwise} \end{cases}$$

The complete model is

$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_6 + \beta_7 x_7 + \beta_8 x_8 + \beta_9 x_9$$
$$+ \beta_{10} x_1 x_9 + \beta_{11} x_2 x_9 + \beta_{12} x_3 x_9 + \beta_{13} x_4 x_9 + \beta_{14} x_5 x_9 + \beta_{15} x_6 x_9 + \beta_{16} x_7 x_9 + \beta_{17} x_8 x_9$$

b. The null hypothesis for testing whether gender has an effect is:

$$H_0 : \beta_9 = \beta_{10} = \cdots = \beta_{17} = 0$$

c. To conduct this test, both a complete and reduced model that contains only $x_1, x_2, \ldots, x_8$ should be fit. The SSE's of these two models are then compared in the partial F test to determine if the gender variable has an effect on job satisfaction.

4.96 a. The model would be $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4$.

b. Using MINITAB, the results are:

Regression Analysis: RFEWIDTH versus REDSHIFT, ... INOSITY, AB1450

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	4	49163	12290.7	51.72	0.000
Error	20	4753	237.6		
Total	24	53915			

Model Summary

S	R-sq	R-sq(adj)
15.4155	91.18%	89.42%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	21088	18553	1.14	0.269
REDSHIFT	108.5	88.7	1.22	0.236
LINEFLUX	558	316	1.77	0.093
LUMINOSITY	-340	321	-1.06	0.302
AB1450	85.68	6.27	13.66	0.000

Regression Equation

RFEWIDTH = 21088 + 108.5 REDSHIFT + 558 LINEFLUX - 340 LUMINOSITY + 85.68 AB1450

The fitted regression line is $\hat{y} = 21088 + 108.5x_1 + 558x_2 - 340x_3 + 85.68x_4$

c. $\hat{\beta}_0 = 21088$. This is simply the estimate of the y-intercept because none of the independent variables have 0 in their observed ranges.

$\hat{\beta}_1 = 108.5$. We estimate that the mean equivalent width will increase by 108.5 for each additional increase of 1 unit of redshift, all other variables held constant.

$\hat{\beta}_2 = 558$. We estimate that the mean equivalent width will increase by 558 for each additional increase of 1 unit of lineflux, all other variables held constant.

$\hat{\beta}_3 = -340$. We estimate that the mean equivalent width will decrease by 340 for each additional increase of 1 unit of line luminosity, all other variables held constant.

$\hat{\beta}_4 = 85.68$. We estimate the mean equivalent width will increase by 85.68 for each additional increase of 1 unit of AB_{1450}, all other variables held constant.

d. To determine if redshift is a useful linear predictor of equivalent width, we test:

$$H_0: \beta_1 = 0$$
$$H_a: \beta_1 \neq 0$$

The test statistic is $t = 1.22$ and the p-value is $p = .236$. Since the p-value is not less than $\alpha\left(p = 0.238 \not< 0.05\right)$, H_0 is not rejected. There is insufficient evidence to indicate that redshift is a useful linear predictor of equivalent width, adjusted for the other variables in the model at $\alpha = 0.05$.

e. $R^2 = 0.9118$. 91.18% of the variability in the equivalent width scores is explained by the model with the 4 independent variables.

$R_a^2 = 0.8942$. 89.42% of the variability in the equivalent width scores is explained by the model with the 4 independent variables, adjusted for the number of parameters in the model and the sample size.

R_a^2 is preferred because it takes into account the number of parameters in the model and the sample size.

f. To determine if the model is adequate, we test:

$$H_0: \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$$
$$H_a: \text{At least one } \beta_i \neq 0$$

The test statistic is $F = 51.72$ and the p-value was $p = 0.000$. Since the p-value is so small, H_0 is rejected. There is sufficient evidence to indicate the model is adequate for any reasonable value of α.

4.97 a. The model would be $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2$.

b. If turntable speed and blade position interact, then the linear relationship between number of defects and turntable speed depends on blade position.

c. This would imply that the value of $\beta_3 < 0$. As the value of x_2 gets larger, the slope of the line will get smaller if $\beta_3 < 0$.

Chapter **5**

Principles of Model Building

5.1 Ball speed, x_1, is quantitative and whether the dropped ball buckled at impact, x_2, is qualitative.

5.2 All of the independent variables, US long-term bond yield (x_1), foreign country's spot exchange rate (x_2), and US spot exchange rate (x_3) are all quantitative variables.

5.3 a. Company's annual percentage return on assets is a quantitative variable. Answers will vary. Possible values could be from 20% to 100%.

b. Size of the company is a quantitative variable. Answers will vary. Possible values could be from 5 to several thousand.

c. CEO age is a quantitative variable. Answers will vary. Possible values could be from 35 to 75.

d. CEO salary is a quantitative variable. Answers will vary. Possible values could be from $100,000 to $10,000,000.

e. CEO social class origin is a qualitative variable. Answers will vary. Possible values could be lower class, middle class, and upper class.

f. CEO gender is a qualitative variable. Possible values are male and female.

g. Whether or not the CEO attended an elite university is a qualitative variable. Possible values are yes and no.

h. Whether or not the CEO founded the company is a qualitative variable. Possible values are yes and no.

5.4 Each of the variables, waist circumference, cardiorespiratory fitness, and body mass index are all quantitative variables.

5.5 a. This variable would be a qualitative variable as there are only two qualitative values.

b. The number of days between forecast and fiscal year-end is a quantitative variable since it can be measured on a numerical scale.

c. The number of quarters the analyst had worked with the firm is a quantitative variable since it can be measured on a numerical scale.

5.6 The level of bullying is a quantitative scale as it can be measured on a numerical scale and the perceived organizational support is a qualitative variable since it has only three non-numerical values ("low", "neutral", or "high").

5.7 Both independent variables are qualitative. Juror gender has only two values (male, female) and whether or not expert testimony was given only has two values (yes, no).

5.8 Water source is a qualitative variable since it contains only three possible values "groundwater, subsurface flow, or overground flow". Silica concentration is a quantitative variable since it can be measured on a numerical scale.

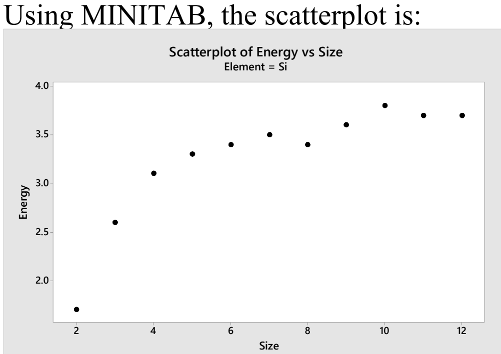
5.9 The assumption of a normal distribution prohibits the use of a qualitative dependent variable.

5.10 a. i. First-order
ii. Third-order
iii. First-order
iv. Second-order

b. i. $E(y) = \beta_0 + \beta_1 x$
ii. $E(y) = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3$
iii. $E(y) = \beta_0 + \beta_1 x$
iv. $E(y) = \beta_0 + \beta_1 x + \beta_2 x^2$

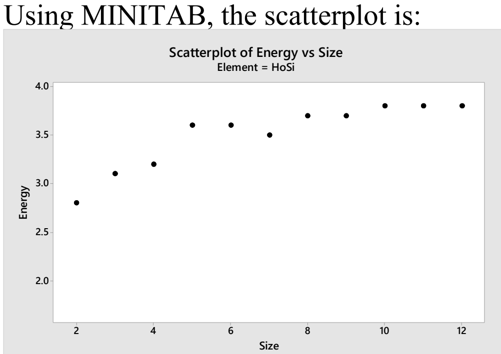
c. i. $\beta_1 > 0$ since the slope of the line is positive.
ii. $\beta_3 > 0$ refer to Figure 5.4.
iii. $\beta_1 < 0$ since the slope of the line is negative.
iv. $\beta_2 < 0$ since the parabola opens downward.

5.11 a. Using MINITAB, the scatterplot is:



The second-order model is suggested because the data form a curved line.

b. Using MINITAB, the scatterplot is:



The second-order model is suggested because the data form a curved line.

c. Using MINITAB, the results are:

Regression Analysis: Energy-S versus Size, Size-Sq

Model Summary

S	R-sq	R-sq(adj)
0.211896	90.52%	88.14%

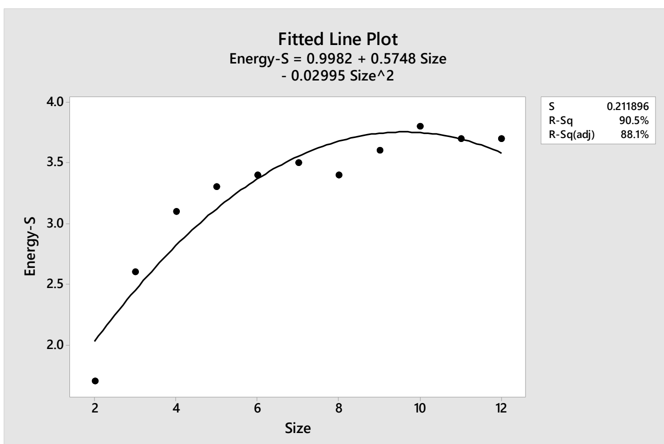
Analysis of Variance

Source	DF	SS	MS	F	P
Regression	2	3.42807	1.71404	38.17	0.000
Error	8	0.35920	0.04490		
Total	10	3.78727			

Sequential Analysis of Variance

Source	DF	SS	F	P
Linear	1	2.65827	21.19	0.001
Quadratic	1	0.76980	17.14	0.003

The fitted line plot is:



The R-squared value is $R^2 = 0.9052$. Thus, 90.52% of the variability in the binding energy for the pure silicon coating is explained by the 2nd –order model with cluster size.

d. Using MINITAB, the results are:

Regression Analysis: Energy-H versus Size, Size-Sq

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	2	1.03460	0.517301	55.56	0.000
Error	8	0.07449	0.009311		
Total	10	1.10909			

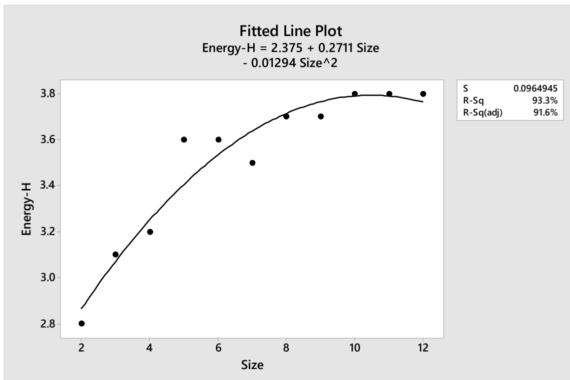
Model Summary

S	R-sq	R-sq(adj)
0.0964945	93.28%	91.60%

Coefficients

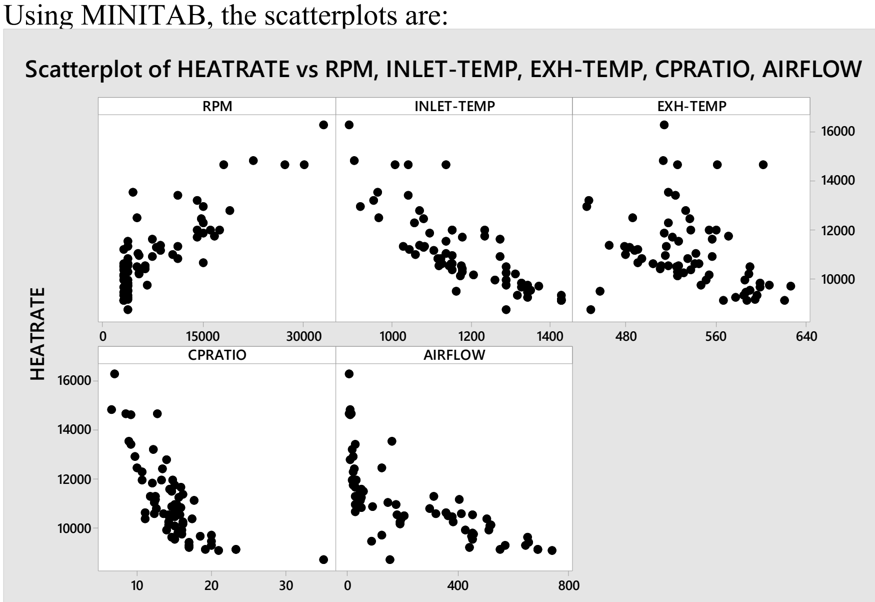
Term	Coef	SE Coef	T-Value	P-Value
Constant	2.375	0.147	16.19	0.000
Size	0.2711	0.0470	5.76	0.000
Size-Sq	-0.01294	0.00329	-3.93	0.004

The fitted line plot is:



The R-squared value is $R^2 = 0.9328$. Thus, 93.28% of the variability in the binding energy for the holmium-silicon mix coating is explained by the 2nd –order model with cluster size.

5.12 Using MINITAB, the scatterplots are:

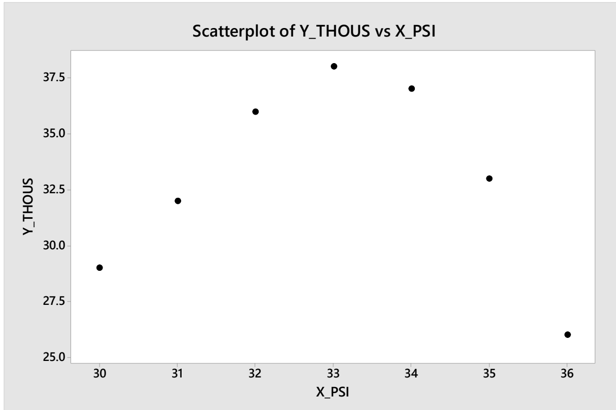


The relationship between heat rate and all of the variables looks rather linear except for CPRATIO. This plot looks curved. Thus a model might be:

$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_4^2$$

5.13 Since both graphs looked curved, we would propose second-order models for both males and females. The models would be $E(y) = \beta_0 + \beta_1 x + \beta_2 x^2$.

5.14 a. Using MINITAB, the scatterplot is:



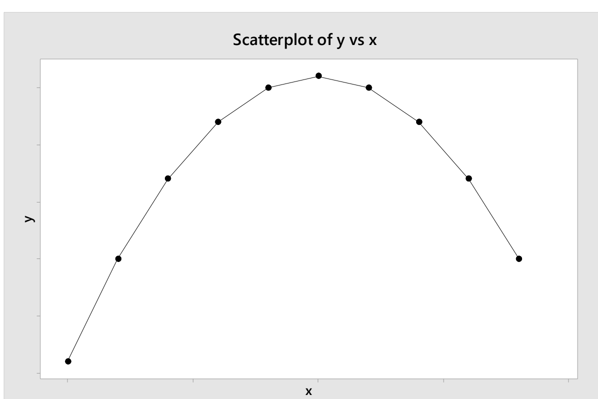
b. If we only use the values 30, 31, 32, and 33 for x, we would fit a linear model, $E(y) = \beta_0 + \beta_1 x$. If we only use the values 33, 34, 35, and 36 for x, we would fit a linear model, $E(y) = \beta_0 + \beta_1 x$. If we use all the data, we would fit a second-order model because there is definitely curvature to the data, $E(y) = \beta_0 + \beta_1 x + \beta_2 x^2$.

5.15 To determine if the rate of decrease of distance with increasing passenger volume levels off for more crowded platforms, we test:

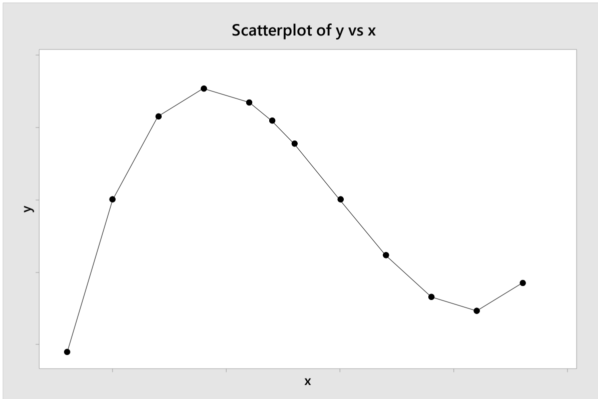
$$H_0: \beta_2 = 0$$
$$H_a: \beta_2 > 0$$

The test statistic is $t = 5.59$ and the p-value is $p = 0.000 / 2 = 0.000$. Since the p-value is less than $\alpha\left(p = 0.000 < 0.01\right)$, H_0 is rejected. There is sufficient evidence to indicate the rate of decrease of distance with increasing passenger volume levels off for more crowded platforms at $\alpha = 0.01$.

5.16 a. We would propose a second-order model: $E(y) = \beta_0 + \beta_1 x + \beta_2 x^2$. A possible graph is:



b. We would propose a third-order model: $E(y) = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3$. A possible graph is:

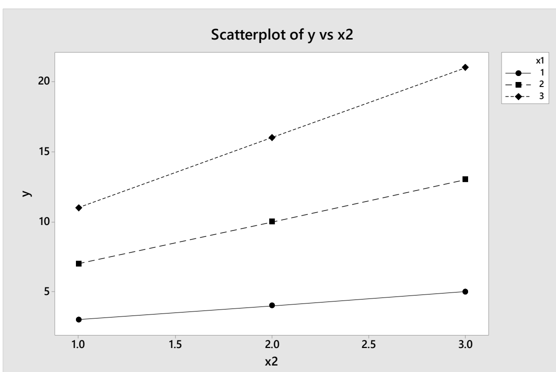


c. To determine which of the 2 models is better, one would compare the 2 models using a partial F-test. These 2 models are nested. We would compare the complete model to the reduced model to determine if the cubic term is necessary.

5.17 a. Both frequency and amplitude are quantitative variables.

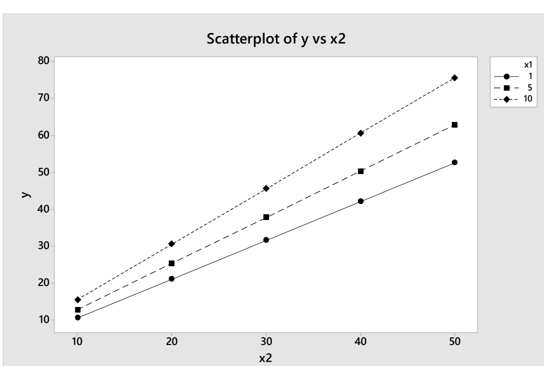
b. The first order model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2$.

c. The interaction model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2$. A possible sketch is:



d. The complete second-order model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 + \beta_4 x_1^2 + \beta_5 x_2^2$.

e. A possible graph is:



This graph is similar to the graph in part c.

f. The first-order model with all 2-way interactions is:

$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_1 x_2 + \beta_5 x_1 x_3 + \beta_6 x_2 x_3.$$

g. The complete second-order model is:

$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_1 x_2 + \beta_5 x_1 x_3 + \beta_6 x_2 x_3 + \beta_7 x_1^2 + \beta_8 x_2^2 + \beta_9 x_3^2.$$

5.18 a. The complete second-order model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 + \beta_4 x_1^2 + \beta_5 x_2^2$.

b. The fitted model is $\hat{y} = 1 + 0.17x_1 + 0.34x_2 + 0.32x_1 x_2 - 0.30x_1^2 + 0.07x_2^2$.

c. To determine if the rate of increase of work engagement with leader's perception is slower for high leader perception values, we test:

$$H_0 : \beta_4 = 0$$
$$H_a : \beta_4 < 0$$

The test statistic is $t = -1.88$ and the *p*-value is $p < 0.10 / 2 = 0.05$. Since the *p*-value is less than $\alpha\left(p < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate the rate of increase of work engagement with leader's perception is slower for high leader perception values, adjusted for the other variables in the model at $\alpha = 0.05$.

d. To determine if the relationship between a subordinate's work engagement and the subordinate's perception of the LMX relationship is linearly increasing one, we first test to see if the relationship is curvilinear:

$$H_0 : \beta_5 = 0$$
$$H_a : \beta_5 \neq 0$$

The test statistic is $t = 1.17$ and the *p*-value is $p > 0.10$. Since the *p*-value is not less than $\alpha\left(p > 0.10 \nless 0.05\right)$, H_0 is not rejected. There is insufficient evidence to indicate the relationship between a subordinate's work engagement and the subordinate's perception of the LMX relationship is quadratic, adjusted for the other variables in the model at $\alpha = 0.05$.

Now we will test to see if the relationship between a subordinate's work engagement and the subordinate's perception of the LMX relationship is linearly increasing one, we test:

$$H_0 : \beta_2 = 0$$
$$H_a : \beta_2 > 0$$

The test statistic is $t = 4.86$ and the *p*-value is $p < 0.01$. Since the *p*-value is less than $\alpha\left(p < 0.01 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate the relationship between a subordinate's work engagement and the subordinate's perception of the LMX relationship is linearly increasing one, adjusted for the other variables in the model at $\alpha = 0.05$.

5.19 a. The complete second-order model is:
$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 + \beta_4 x_1^2 + \beta_5 x_2^2$.

b. The model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2$.

c. The model with interaction is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2$.

d. For fixed x_2, the slope becomes $\beta_1 + \beta_3 x_2$.

e. For fixed x_1, the slope becomes $\beta_2 + \beta_3 x_1$.

5.20 a. The complete second-order model is
$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 + \beta_4 x_1^2 + \beta_5 x_2^2$.

b. $R^2 = 0.14$. 14% of the variability in the goal of improving efficiency is explained by the second-order model containing level of CEO leadership and level o congruence between the CEO and the VP.

c. This could show that the relationship between the congruence between CEO and VP has a negative effect on the efficiency of the company; i.e. if the CEO an VP do not "get along" in the eye business eye, the efficiency of the company could suffer.

d. The p-value is $p = 0.02$. Since the p-value is less than $\alpha\ (p = 0.02 < 0.05)$, H_0 is rejected. There is sufficient evidence to imply that the interaction between the CEO leadership and the CEO and VP congruence has a significant impact on helping to predict the variability in the goal of improving efficiency, adjusted for other variables in the model at $\alpha = 0.05$.

5.21 a. The complete second-order model is
$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 + \beta_4 x_1^2 + \beta_5 x_2^2$.

b. Using MINITAB, the results of the regression analysis are:

Regression Analysis: HEATRATE versus RPM, CPRATIO, ... _SQ, CPR_SQ

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	5	148526859	29705372	93.55	0.000
Error	61	19370350	317547		
Total	66	167897208			

Model Summary

S	R-sq	R-sq(adj)
563.513	88.46%	87.52%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	15583	1143	13.63	0.000
RPM	0.078	0.110	0.71	0.481
CPRATIO	-523	103	-5.06	0.000
RPM_CPR	0.00445	0.00558	0.80	0.428
RPM_SQ	-0.000000	0.000002	-0.09	0.927
CPR_SQ	8.84	2.16	4.09	0.000

Regression Equation

HEATRATE = 15583 + 0.078 RPM - 523 CPRATIO + 0.00445 RPM_CPR - 0.000000 RPM_SQ + 8.84 CPR_SQ

The regression equation is

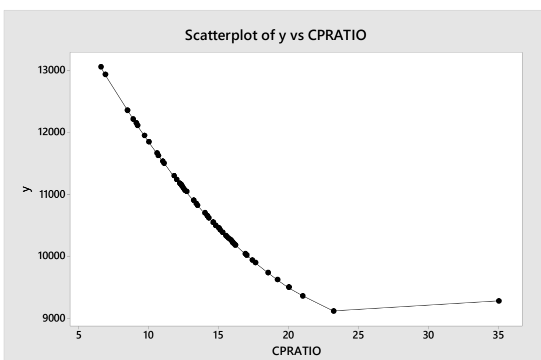
$\hat{y} = 15{,}583 + 0.078x_1 - 523x_2 + 0.0044x_1x_2 - 0.000000x_1^2 + 8.84x_2^2$

c. To determine if the model is useful for predicting y, we test:

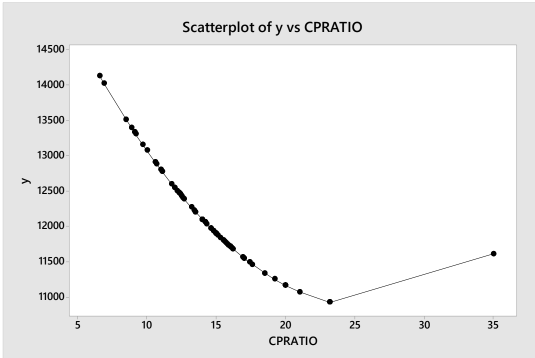
$$H_0: \beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = 0$$
$$H_a: \text{At least one } \beta_i \neq 0$$

The test statistic is $F = 93.55$ and the p-value is $p = 0.000$. Since the p-value is so small, H_0 is rejected. There is sufficient evidence to indicate the model is useful for predicting y at any reasonable level of α.

d. When cycle speed is 5000 rpm, the graph is:



e. When cycle speed is 15,000 rpm, the graph is:



f. Graphs have similar shape. The graph with cycle speed at 15,000 rpm is shifted above the graph when cycle speed is 5000 rpm.

5.22 a. A complete second-order model would be
$$E(y)=\beta_0+\beta_1x_1+\beta_2x_2+\beta_3x_3+\beta_4x_1x_2+\beta_5x_1x_3+\beta_6x_2x_3+\beta_7x_1^2+\beta_8x_2^2+\beta_9x_3^2.$$

b. Using MINITAB, the results of the regression analysis are:

Regression Analysis: Project versus IntraPers, StressMan, ... , Mood_Sq

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	9	74.43	8.270	0.66	0.731
Error	13	162.94	12.534		
Total	22	237.37			

Model Summary

S	R-sq	R-sq(adj)
3.54028	31.36%	0.00%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	88.0	19.7	4.46	0.001
IntraPers	-0.141	0.745	-0.19	0.853
StressMan	-0.084	0.560	-0.15	0.882
Mood	0.14	1.01	0.13	0.896
Pers_Stres	-0.0024	0.0172	-0.14	0.890
Pers_Mood	-0.0010	0.0247	-0.04	0.969
Stres_Mood	-0.0006	0.0129	-0.05	0.961
Pers_Sq	0.0003	0.0158	0.02	0.983
Stres_Sq	0.0061	0.0109	0.56	0.584
Mood_Sq	-0.00058	0.00961	-0.06	0.953

Regression Equation

Project = 88.0 - 0.141 IntraPers - 0.084 StressMan + 0.14 Mood - 0.0024 Pers_Stres
- 0.0010 Pers_Mood - 0.0006 Stres_Mood + 0.0003 Pers_Sq + 0.0061 Stres_Sq
- 0.00058 Mood_Sq

The fitted model is

$$\hat{y} = 88.0 - 0.141x_1 - 0.084x_2 + 0.14x_3 - 0.0024x_1x_2 - 0.0010x_1x_3 - 0.0006x_2x_3 + 0.0003x_1^2 + 0.0061x_2^2 - 0.00058x_3^2.$$

c. To determine if the overall model is adequate, we test:

$H_0: \beta_1 = \beta_2 = \cdots = \beta_9 = 0$
$H_a:$ At least one $\beta_i \neq 0$

The test statistic is $F = 0.66$ and the p-value is $p = 0.731$. Since the p-value is not small, H_0 is not rejected. There is insufficient evidence to indicate the model is adequate for predicting y at any reasonable level of α.

d. To determine if the range of intrapersonal scores is curvilinearly related to average project score, we test:

$H_0: \beta_7 = 0$
$H_a: \beta_7 \neq 0$

The test statistic is $t = 0.02$ and the p-value is $p = 0.983$. Since the p-value is not less than $\alpha\left(p = 0.983 \nless 0.01\right)$, H_0 is not rejected. There is insufficient evidence to indicate the range of intrapersonal scores is curvilinearly related to average project score, adjusted for all the other variables in the model at $\alpha = 0.01$.

e. To determine if the range of stress management scores is curvilinearly related to average project score, we test:

$H_0: \beta_8 = 0$
$H_a: \beta_8 \neq 0$

The test statistic is $t = 0.56$ and the p-value is $p = 0.584$. Since the p-value is not less than $\alpha\left(p = 0.584 \nless 0.01\right)$, H_0 is not rejected. There is insufficient evidence to indicate the range of stress management scores is curvilinearly related to average project score, adjusted for all the other variables in the model at $\alpha = 0.01$.

f. To determine if the range of mood scores is curvilinearly related to average project score, we test:

$H_0: \beta_9 = 0$
$H_a: \beta_9 \neq 0$

The test statistic is $t = -0.06$ and the p-value is $p = 0.953$. Since the p-value is not less than $\alpha\left(p = 0.953 \nless 0.01\right)$, H_0 is not rejected. There is insufficient evidence to indicate the range of mood scores is curvilinearly related to average project score, adjusted for all the other variables in the model at $\alpha = 0.01$.

5.23 a. The complete second-order model is

$$E(y)=\beta_0+\beta_1x_1+\beta_2x_2+\beta_3x_5+\beta_4x_6+\beta_5x_1x_2+\beta_6x_1x_5+\beta_7x_1x_6+\beta_8x_2x_5$$
$$+\beta_9x_2x_6+\beta_{10}x_5x_6+\beta_{11}x_1^2+\beta_{12}x_2^2+\beta_{13}x_5^2+\beta_{14}x_6^2.$$

b. We would need to compare the complete model in part a to the reduced model

$$E(y)=\beta_0+\beta_1x_1+\beta_2x_2+\beta_3x_5+\beta_4x_6+\beta_5x_1x_2+\beta_6x_1x_5+\beta_7x_1x_6+\beta_8x_2x_5$$
$$+\beta_9x_2x_6+\beta_{10}x_5x_6.$$

We would test:

$H_0: \beta_{11}=\beta_{12}=\beta_{13}=\beta_{14}=0$

H_a :At least one $\beta_i \neq 0$

5.24 a. The equation is $u=\dfrac{x-\bar{x}}{s_x}$.

b. Some preliminary calculations are:

$n=7 \quad \sum x=231 \quad \sum x^2=7651$

$$\bar{x}=\frac{\sum x}{n}=\frac{231}{7}=33 \qquad s_x=\sqrt{\frac{\sum x^2-\frac{\left(\sum x\right)^2}{n}}{n-1}}=\sqrt{\frac{7651-\frac{(231)^2}{7}}{7-1}}=\sqrt{4.6667}=2.16$$

Then, using $u=\dfrac{x-\bar{x}}{s_x}=\dfrac{x-33}{2.16}$:

x	30	31	32	33	34	35	36
u	-1.389	-0.926	-0.463	0	0.463	0.926	1.389

c. Let $x_1=x$ and $x_2=x^2$. Some preliminary calculations are:

$\sum x_1=231 \quad \sum x_1^2=7651 \quad \sum x_2=7651 \quad \sum x_2^2=8{,}484{,}595 \quad \sum x_1x_2=254{,}331$

$$SS_{x_1x_1}=\sum x_1^2-\frac{\left(\sum x_1\right)^2}{n}=7651-\frac{231^2}{7}=28$$

$$SS_{x_2x_2}=\sum x_2^2-\frac{\left(\sum x_2\right)^2}{n}=8{,}484{,}595-\frac{7651^2}{7}=122{,}052$$

$$SS_{x_1x_2}=\sum x_1x_2-\frac{\left(\sum x_1\right)\left(\sum x_2\right)}{n}=254{,}331-\frac{231(7651)}{7}=1848$$

$$r=\frac{SS_{x_1x_2}}{\sqrt{SS_{x_1x_1}SS_{x_2x_2}}}=\frac{1848}{\sqrt{28(122{,}052)}}=0.9997$$

d. Let $u_1 = u$ and $u_2 = u^2$. Some preliminary calculations are:

$$\sum u_1 = 0 \quad \sum u_1^2 = 6.002 \quad \sum u_2 = 6.002 \quad \sum u_2^2 = 9.007 \quad \sum u_1 u_2 = 0$$

$$SS_{u_1u_1} = \sum u_1^2 - \frac{\left(\sum u_1\right)^2}{n} = 6.002 - \frac{0^2}{7} = 6.002$$

$$SS_{u_2u_2} = \sum u_2^2 - \frac{\left(\sum u_2\right)^2}{n} = 9.007 - \frac{6.002^2}{7} = 3.861$$

$$SS_{u_1u_2} = \sum u_1 u_2 - \frac{\left(\sum u_1\right)\left(\sum u_2\right)}{n} = 0 - \frac{0(6.002)}{7} = 0$$

$$r = \frac{SS_{u_1u_2}}{\sqrt{SS_{u_1u_1} SS_{u_2u_2}}} = \frac{0}{\sqrt{6.002(3.861)}} = 0$$

e. Using MINITAB, the results of the regression analysis are:

Regression Analysis: Y_THOUS versus u, u_sq

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	2	111.000	55.500	44.40	0.002
Error	4	5.000	1.250		
Total	6	116.000			

Model Summary

S	R-sq	R-sq(adj)
1.11803	95.69%	93.53%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	37.571	0.645	58.21	0.000
u	-0.463	0.456	-1.01	0.368
u_sq	-5.332	0.569	-9.37	0.001

Regression Equation

Y_THOUS = 37.571 - 0.463 u - 5.332 u_sq

The fitted regression line is $\hat{y} = 37.571 - 0.463u - 5.332u^2$.

If we test for curvature, the test statistic is $t = -9.37$ and the p-value is $p = 0.001$. Since the p-value is so small, there is evidence that the second-order model is significant.

5.25 a. Let $x_1 = x$ and $x_2 = x^2$. Some preliminary calculations are:

$$\sum x_1 = 302 \quad \sum x_1^2 = 5818 \quad \sum x_2 = 5818 \quad \sum x_2^2 = 2{,}876{,}122 \quad \sum x_1 x_2 = 125{,}438$$

$$SS_{x_1x_1} = \sum x_1^2 - \frac{\left(\sum x_1\right)^2}{n} = 5818 - \frac{302^2}{20} = 1257.8$$

$$SS_{x_2x_2} = \sum x_2^2 - \frac{\left(\sum x_2\right)^2}{n} = 2{,}876{,}122 - \frac{5818^2}{20} = 1{,}183{,}665.8$$

$$SS_{x_1x_2} = \sum x_1x_2 - \frac{\left(\sum x_1\right)\left(\sum x_2\right)}{n} = 125{,}438 - \frac{302(5818)}{20} = 37{,}586.2$$

$$r = \frac{SS_{x_1x_2}}{\sqrt{SS_{x_1x_1}SS_{x_2x_2}}} = \frac{37{,}586.2}{\sqrt{1257.8(1{,}183{,}665.8)}} = 0.974$$

Since the correlation is so close to 1, this implies that x and x^2 are highly correlated with each other. Thus, the *t*-tests are not independent and there is evidence of multicollinearity. Coding is recommended.

b. The coding equation is $u = \dfrac{x - \bar{x}}{s_x}$.

$$\bar{x} = \frac{\sum x}{n} = \frac{302}{20} = 15.1 \qquad s_x = \sqrt{\frac{\sum x^2 - \frac{\left(\sum x\right)^2}{n}}{n-1}} = \sqrt{\frac{5818 - \frac{(302)^2}{20}}{20-1}} = \sqrt{66.2} = 8.1363$$

The equation is $u = \dfrac{x - 15.1}{8.1363}$. The values are found in the following table:

x	u	x	u
21	0.725	11	-0.504
2	-1.61	20	0.602
4	-1.364	17	0.234
25	1.217	18	0.356
16	0.111	19	0.479
26	1.34	7	-0.996
6	-1.118	29	1.708
22	0.848	5	-1.241
23	0.971	14	-0.135
9	-0.75	8	-0.873

c. Let $u_1 = u$ and $u_2 = u^2$. Some preliminary calculations are:

$$\sum u_1 = 0 \quad \sum u_1^2 = 18.999 \quad \sum u_2 = 18.999 \quad \sum u_2^2 = 31.8746 \quad \sum u_1u_2 = -0.74038$$

$$SS_{u_1u_1} = \sum u_1^2 - \frac{\left(\sum u_1\right)^2}{n} = 18.999 - \frac{0^2}{20} = 18.999$$

$$SS_{u_2u_2} = \sum u_2^2 - \frac{\left(\sum u_2\right)^2}{n} = 31.8746 - \frac{18.999^2}{20} = 13.8265$$

$$SS_{u_1u_2} = \sum u_1u_2 - \frac{\left(\sum u_1\right)\left(\sum u_2\right)}{n} = -0.74038 - \frac{0(18.999)}{20} = -0.74038$$

$$r = \frac{SS_{u_1u_2}}{\sqrt{SS_{u_1u_1}SS_{u_2u_2}}} = \frac{-0.74038}{\sqrt{18.999(13.8265)}} = -0.0457$$

Since the correlation between u and u^2 is very small, the problem of multicollinearity has been greatly diminished.

d. Using MINITAB, the results of the regression analysis are:

Regression Analysis: DISTANCE versus u, u_sq

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	2	0.70785	0.353924	65.39	0.000
Error	17	0.09201	0.005412		
Total	19	0.79986			

Model Summary

S	R-sq	R-sq(adj)
0.0735672	88.50%	87.14%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	0.0983	0.0250	3.93	0.001
u	-0.1640	0.0169	-9.71	0.000
u_sq	0.1107	0.0198	5.59	0.000

Regression Equation

DISTANCE = 0.0983 - 0.1640 u + 0.1107 u_sq

The fitted regression line is $\hat{y} = 0.0983 - 0.1640u + 0.1107u^2$.

If we test for curvature, the test statistic is $t = 5.59$ and the p-value is $p = 0.000$. Since the p-value is so small, there is evidence that the second-order model is significant.

5.26 Using MINITAB, the correlation between temperature and temperature squared is:

Correlation: TEMP, TEMP_SQ
Correlations
Pearson correlation 0.999

Since the correlation is so high, we can use a coding system to help reduce round-off errors. Using the coding system for observational data leads to

$$u = \frac{x - \bar{x}}{s_x} = \frac{x - 146.182}{12.9712}.$$

Using MINITAB, the correlation between u and u^2 is:

Correlation: U, U_SQ
Correlations
Pearson correlation -0.239

Using MINITAB, the results of the regression analysis are:

Regression Analysis: FAILTIME versus U, U_SQ
Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	2	144830280	72415140	152.93	0.000
Error	19	8997107	473532		
Total	21	153827386			

Model Summary

S	R-sq	R-sq(adj)
688.137	94.15%	93.54%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	1900	223	8.51	0.000
U	-2276	155	-14.72	0.000
U_SQ	998	176	5.66	0.000

Regression Equation
FAILTIME = 1900 - 2276 U + 998 U_SQ

The fitted regression line is $\hat{y} = 1900 - 2276u + 998u^2$.

5.27 Using MINITAB, the correlation between Height and height-squared is:

Correlation: HEIGHT, HT_SQ
Correlations
Pearson correlation 0.985

Since the correlation is so high, we can use a coding system to help reduce round-off errors. Using the coding system for observational data leads to

$$u = \frac{x - \bar{x}}{s_x} = \frac{x - 45.6932}{18.5310}.$$

Using MINITAB, the correlation between u and u^2 is:

Correlation: U, U_SQ

Correlations

Pearson correlation -0.237

Using MINITAB, the results of the regression analysis are:

Regression Analysis: ANGLE versus U, U_SQ

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	2	29.182	14.5911	48.15	0.000
Error	10	3.030	0.3030		
Total	12	32.212			

Model Summary

S	R-sq	R-sq(adj)
0.550468	90.59%	88.71%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	324.678	0.232	1400.79	0.000
U	1.494	0.164	9.13	0.000
U_SQ	-0.251	0.189	-1.33	0.214

Regression Equation

ANGLE = 324.678 + 1.494 U - 0.251 U_SQ

The fitted regression line is $\hat{y} = 324.678 + 1.494u - 0.251u^2$.

If we test for curvature, the test statistic is $t = -1.33$ and the p-value is $p = 0.214$. Since the p-value is not small, there is no evidence that the second-order model is significant. Thus, we should fit a first-order model.

5.28 a. First, we define the dummy variables:

$$x_1 = \begin{cases} 1 & \text{if Lower} \\ 0 & \text{otherwise} \end{cases} \qquad x_2 = \begin{cases} 1 & \text{if Middle} \\ 0 & \text{otherwise} \end{cases}$$

The model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2$.

b. $\beta_0 =$ mean strategic risk taking for those in the upper class
$\beta_1 =$ difference in mean strategic risk taking between those in the lower class and those in the upper class
$\beta_2 =$ difference in mean strategic risk taking between those in the middle class and those in the upper class

c. To test for differences among the social class levels, a test for model adequacy is run.

5.29 a. First, define the dummy variables:

$$x_1 = \begin{cases} 1 & \text{if male} \\ 0 & \text{otherwise} \end{cases} \qquad x_2 = \begin{cases} 1 & \text{if expert testifies} \\ 0 & \text{otherwise} \end{cases}$$

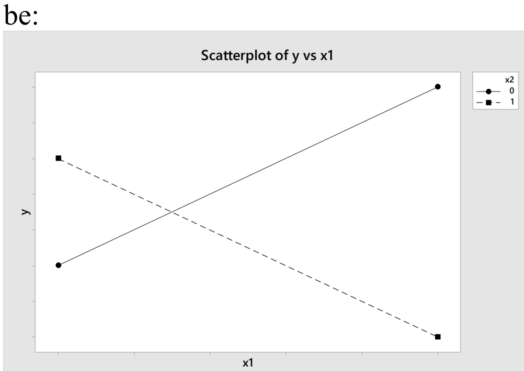
The model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2$.

$\beta_0 =$ mean likelihood of changing verdict for females and no expert testimony.
$\beta_1 =$ difference in mean likelihood of changing a verdict between males and females.
$\beta_2 =$ difference in mean likelihood of changing a verdict between expert testimony and no expert testimony.

b. The model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2$.

$\beta_0 = \mu_{f,n}$
$\beta_1 = \mu_{m,n} - \mu_{f,n}$
$\beta_2 = \mu_{f,y} - \mu_{f,n}$
$\beta_3 = \mu_{m,y} - \mu_{m,n} - \mu_{f,y} + \mu_{f,n} = \left(\mu_{m,y} - \mu_{m,n}\right) - \left(\mu_{f,y} - \mu_{f,n}\right)$

c. The interaction model in part b would represent the hypotheses. A possible graph would be:



5.30 a. Group is the qualitative independent variable. It must be coded into two dummy variables since it has three levels.

b. First, define the dummy variables:

$$x_1 = \begin{cases} 1 & \text{if Group 2} \\ 0 & \text{otherwise} \end{cases} \qquad x_2 = \begin{cases} 1 & \text{if Group 3} \\ 0 & \text{otherwise} \end{cases}$$

The model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2$.

c. $\beta_0 = \mu_1 =$ mean milk production for Group 1 (man-made shade structure).

$\beta_1 = \mu_2 - \mu_1 =$ difference in mean milk production between Group 2 and Group 1 (tree shade minus man-made shade structure).
$\beta_2 = \mu_3 - \mu_1 =$ difference in mean milk production between Group 3 and Group 1 (no shade minus man-made shade structure).

5.31 a. $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_1 x_2 + \beta_5 x_1 x_3$

where $x_1 = \begin{cases} 1 \text{ if manual} \\ 0 \text{ if automated} \end{cases}$ $x_2 = \begin{cases} 1 \text{ if clay} \\ 0 \text{ if not} \end{cases}$ $x_3 = \begin{cases} 1 \text{ if gravel} \\ 0 \text{ if not} \end{cases}$

b. $\beta_0 = \mu_{\text{automated/Sand}}$

c. $\beta_0 + \beta_1 + \beta_2 + \beta_4 = \mu_{\text{manual/clay}}$

d. $\beta_1 = \mu_{\text{manual/sand}} - \mu_{\text{automated/sand}}$

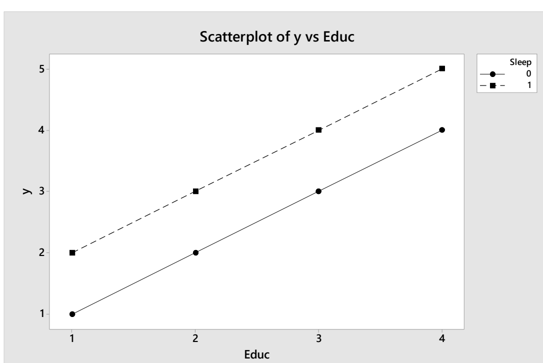
5.32 a. First define the dummy variables:

$x_1 = \begin{cases} 1 & \text{if chronic insomnia} \\ 0 & \text{otherwise} \end{cases}$ $x_2 = \begin{cases} 1 & \text{if some college} \\ 0 & \text{otherwise} \end{cases}$ $x_3 = \begin{cases} 1 & \text{if high school graduate} \\ 0 & \text{otherwise} \end{cases}$

$x_4 = \begin{cases} 1 & \text{if high school dropout} \\ 0 & \text{otherwise} \end{cases}$

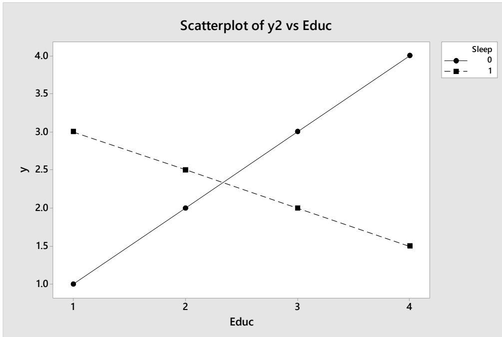
The main effects model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4$.

A possible graph is:



b. The interaction model is
$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_1 x_2 + \beta_6 x_1 x_3 + \beta_7 x_1 x_4$.

A possible graph is:



c. Since there is no interaction present, the main effects model should be used: $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4$.

5.33 a. The interaction model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2$.

where $x_1 = \begin{cases} 1 \text{ if ambiguous} \\ 0 \text{ if common} \end{cases}$ $\quad x_2 = \begin{cases} 1 \text{ if high} \\ 0 \text{ if low} \end{cases}$

b. $\hat{\beta}_0 = \mu_{\text{com/low}} = 7.8$

$\hat{\beta}_1 = \mu_{\text{amb/low}} - \mu_{\text{com/low}} = 18.0 - 7.8 = 10.2$

$\hat{\beta}_2 = \mu_{\text{com/high}} - \mu_{\text{com/low}} = 6.3 - 7.8 = -1.5$

$\hat{\beta}_3 = \mu_{\text{amb/high}} - \mu_{\text{com/high}} - \mu_{\text{amb/low}} + \mu_{\text{com/low}} = 6.1 - 6.3 - 18.0 + 7.8 = -10.4$

c. We would use a *t*-test to test:

$$H_0: \beta_3 = 0$$
$$H_a: \beta_3 \neq 0$$

d. Using MINITAB, the results of the regression analysis are:

Regression Analysis: NUMBER versus NAME_Ambig, ... h, LOAD_NAME

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	3	2423.1	807.690	84.86	0.000
Error	96	913.7	9.517		
Total	99	3336.7			

Model Summary

S	R-sq	R-sq(adj)
3.08504	72.62%	71.76%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	7.800	0.617	12.64	0.000
LOAD_High	-1.520	0.873	-1.74	0.085
NAME_Ambig	10.200	0.873	11.69	0.000
LOAD_NAME	-10.36	1.23	-8.40	0.000

Regression Equation

```
NUMBER = 7.800 - 1.520 LOAD_High + 10.200 NAME_Ambig - 10.36 LOAD_NAME
```

The fitted regression line is $\hat{y} = 7.8 + 10.2x_1 - 1.52x_2 - 10.36x_1x_2$.
Except for rounding errors, the estimates are the same.

5.34 The proposed model would not reach the administration's objective. It would force the difference in the mean salaries of lectures and assistant professors to be the same as the difference in the mean salaries of associate professors and full professors.

To meet their objective, the administration should fit the following model:

$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$$

where $x_1 = \begin{cases} 1 & \text{if lecturer} \\ 0 & \text{otherwise} \end{cases}$ $\quad x_2 = \begin{cases} 1 & \text{if assistant professor} \\ 0 & \text{otherwise} \end{cases}$

$$x_2 = \begin{cases} 1 & \text{if associate professor} \\ 0 & \text{otherwise} \end{cases}$$

5.35 a. Define the dummy variables:

$$x_1 = \begin{cases} 1 & \text{if large/private} \\ 0 & \text{otherwise} \end{cases} \quad x_2 = \begin{cases} 1 & \text{if small/public} \\ 0 & \text{otherwise} \end{cases} \quad x_3 = \begin{cases} 1 & \text{if small/private} \\ 0 & \text{otherwise} \end{cases}$$

b. The model would be $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$.

$$\beta_0 = \mu_{\text{large/public}}$$
$$\beta_1 = \mu_{\text{large/private}} - \mu_{\text{large/public}}$$
$$\beta_2 = \mu_{\text{small/public}} - \mu_{\text{large/public}}$$
$$\beta_3 = \mu_{\text{small/private}} - \mu_{\text{large/public}}$$

c. Since the p-value is so small $(p < 0.001)$, H_0 is rejected. There is sufficient evidence to indicate there are differences in the mean likelihood of reporting sustainability policies.

d. The dummy variables would be

$$x_1 = \begin{cases} 1 & \text{if small} \\ 0 & \text{otherwise} \end{cases} \quad x_2 = \begin{cases} 1 & \text{if private} \\ 0 & \text{otherwise} \end{cases}$$

e. The model would be $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2$.

f. Large/public: $E(y) = \beta_0 + \beta_1(0) + \beta_2(0) = \beta_0$.
Large/private: $E(y) = \beta_0 + \beta_1(0) + \beta_2(1) = \beta_0 + \beta_2$.

Small/public: $E(y) = \beta_0 + \beta_1(1) + \beta_2(0) = \beta_0 + \beta_1$.
Small/private: $E(y) = \beta_0 + \beta_1(1) + \beta_2(1) = \beta_0 + \beta_1 + \beta_2$.

g. The difference between large and small firms for public firms is: $(\beta_0 + \beta_1) - (\beta_0) = \beta_1$
The difference between large and small firms for private firms is:
$(\beta_0 + \beta_1 + \beta_2) - (\beta_0 + \beta_2) = \beta_1$

h. The model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2$.

i. Large/public: $E(y) = \beta_0 + \beta_1(0) + \beta_2(0) + \beta_3(0)(0) = \beta_0$.
Large/private: $E(y) = \beta_0 + \beta_1(0) + \beta_2(1) + \beta_3(0)(1) = \beta_0 + \beta_2$.
Small/public: $E(y) = \beta_0 + \beta_1(1) + \beta_2(0) + \beta_3(1)(0) = \beta_0 + \beta_1$.
Small/private: $E(y) = \beta_0 + \beta_1(1) + \beta_2(1) + \beta_3(1)(1) = \beta_0 + \beta_1 + \beta_2 + \beta_3$.

j. The difference between large and small firms for public firms is: $(\beta_0 + \beta_1) - (\beta_0) = \beta_1$
The difference between large and small firms for private firms is:
$(\beta_0 + \beta_1 + \beta_2 + \beta_3) - (\beta_0 + \beta_2) = \beta_1 + \beta_3$

5.36 a. First, define the dummy variable:

$$x_2 = \begin{cases} 1 & \text{if server complimented} \\ 0 & \text{otherwise} \end{cases}$$

The model corresponding to Theory 1 is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2$.

b. The model corresponding to Theory 2 is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2$.

c. Because the effect of party size on the tip size depends on whether the server complimented or not, the model corresponding to Theory 2 will have a better fit.

5.37 a. $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_1^2 + \beta_3 x_2 + \beta_4 x_3 + \beta_5 x_1 x_2 + \beta_6 x_1 x_3 + \beta_7 x_1^2 x_2 + \beta_8 x_1^2 x_3$,

where $x_1 =$ level of bullying, $x_2 = \begin{cases} 1 & \text{if low} \\ 0 & \text{otherwise} \end{cases}$, $x_3 = \begin{cases} 1 & \text{if neutral} \\ 0 & \text{otherwise} \end{cases}$

b.
$$\begin{aligned} E(y) &= \beta_0 + \beta_1(25) + \beta_2(25)^2 + \beta_3(1) + \beta_4(0) + \beta_5(25)(1) \\ &\quad + \beta_6(25)(0) + \beta_7(25)^2(1) + \beta_8(25)(0) \\ &= \beta_0 + 25\beta_1 + 625\beta_2 + \beta_3 + 25\beta_5 + 625\beta_7 \end{aligned}$$

c. We would use a nested F-test of $H_0: \beta_2 = \beta_7 = \beta_8 = 0$

d. $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_1 x_2 + \beta_5 x_1 x_3$

e. For low, the equation is

$$E(y)=\beta_0+\beta_1x_1+\beta_2(1)+\beta_3(0)+\beta_4x_1(1)+\beta_5x_1(0)=\beta_0+\beta_2+(\beta_1+\beta_4)x_1$$

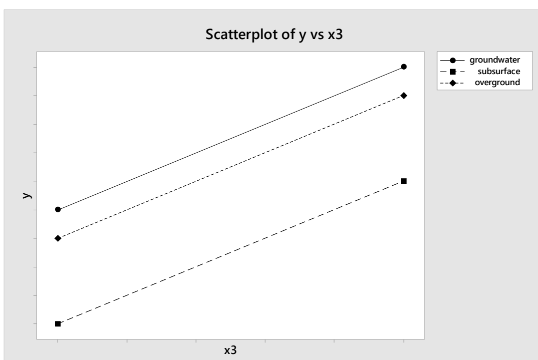
The slope of the line is $(\beta_1+\beta_4)$.
For neutral, the equation is
$$E(y)=\beta_0+\beta_1x_1+\beta_2(0)+\beta_3(1)+\beta_4x_1(0)+\beta_5x_1(1)=\beta_0+\beta_3+(\beta_1+\beta_5)x_1$$
The slope of the line is $(\beta_1+\beta_5)$.
For high, the equation is
$$E(y)=\beta_0+\beta_1x_1+\beta_2(0)+\beta_3(0)+\beta_4x_1(0)+\beta_5x_1(0)=\beta_0+\beta_1x_1$$
The slope of the line is (β_1).

5.38 a. First, define the independent variables:

$$x_1=\begin{cases}1 & \text{if subsurface flow}\\ 0 & \text{otherwise}\end{cases} \qquad x_2=\begin{cases}1 & \text{if overground flow}\\ 0 & \text{otherwise}\end{cases} \qquad x_3=\text{silica concentration}$$

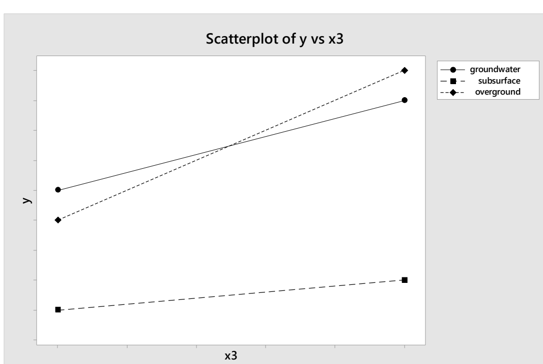
The first-order model is $E(y)=\beta_0+\beta_1x_1+\beta_2x_2+\beta_3x_3$.

Using MINITAB, a possible graph is:



b. The new model would be $E(y)=\beta_0+\beta_1x_1+\beta_2x_2+\beta_3x_3+\beta_4x_1x_3+\beta_5x_2x_3$.

Using MINITAB, a possible graph is:



5.39 a. $E(y)=\beta_0+\beta_1x_1+\beta_2x_2+\beta_3x_3+\beta_4x_4+\beta_5x_5+\beta_6x_6+\beta_7x_7+\beta_8x_1x_2+\beta_9x_1x_3$
$+\beta_{10}x_1x_4+\beta_{11}x_1x_5+\beta_{12}x_1x_6+\beta_{13}x_1x_7+\beta_{14}x_2x_4+\beta_{15}x_2x_5+\beta_{16}x_2x_6$
$+\beta_{17}x_2x_7+\beta_{18}x_3x_4+\beta_{19}x_3x_5+\beta_{20}x_3x_6+\beta_{21}x_3x_7+\beta_{22}x_1x_2x_4+\beta_{23}x_1x_2x_5$
$\beta_{24}x_1x_2x_6+\beta_{25}x_1x_2x_7+\beta_{26}x_1x_3x_4+\beta_{27}x_1x_3x_5+\beta_{28}x_1x_3x_6+\beta_{29}x_1x_3x_7$

where $x_1=\begin{cases}1 \text{ if manual}\\ 0 \text{ if automated}\end{cases}$ $x_2=\begin{cases}1 \text{ if clay}\\ 0 \text{ if not}\end{cases}$ $x_3=\begin{cases}1 \text{ if gravel}\\ 0 \text{ if not}\end{cases}$

$x_4=\begin{cases}1 \text{ if East}\\ 0 \text{ if otherwise}\end{cases}$ $x_5=\begin{cases}1 \text{ if South}\\ 0 \text{ if otherwise}\end{cases}$ $x_6=\begin{cases}1 \text{ if West}\\ 0 \text{ if otherwise}\end{cases}$

$x_7=\begin{cases}1 \text{ if Southeast}\\ 0 \text{ if otherwise}\end{cases}$

b. $\beta_0=\mu_{\text{Automated/Sand/SW}}$

c. For grapes picked manually, from clay soil, with east orientation, the mean is $\beta_0+\beta_1+\beta_2+\beta_4+\beta_8+\beta_{10}+\beta_{14}+\beta_{22}$

d. For grapes picked manually with soil type sand and orientation southwest, the mean is $\beta_0+\beta_1$. For grapes automatically picked with soil type sand and orientation southwest, the mean is β_0. The difference is β_1.

e. $\beta_8=\beta_9=\beta_{22}=\beta_{23}=\beta_{24}=\beta_{25}=\beta_{26}=\beta_{27}=\beta_{28}=\beta_{29}=0$

5.40 a. First, define the dummy variables:

$x_3=\begin{cases}1 \text{ if advanced}\\ 0 \text{ if not}\end{cases}$ $x_4=\begin{cases}1 \text{ if aeroderivative}\\ 0 \text{ if not}\end{cases}$

The complete second-order model is:

$$E(y)=\beta_0+\beta_1x_1+\beta_2x_2+\beta_3x_3+\beta_4x_4+\beta_5x_1^2+\beta_6x_2^2+\beta_7x_1x_3+\beta_8x_1x_4$$
$$+\beta_9x_2x_3+\beta_{10}x_2x_4+\beta_{11}x_1^2x_3+\beta_{12}x_1^2x_4+\beta_{13}x_2^2x_3+\beta_{14}x_2^2x_4+\beta_{15}x_1x_2$$
$$+\beta_{16}x_1x_2x_3+\beta_{17}x_1x_2x_4$$

b. If the model type is traditional, then $x_3=0$ and $x_4=0$. The model would be $E(y)=\beta_0+\beta_1x_1+\beta_2x_2+\beta_5x_1^2+\beta_6x_2^2+\beta_{15}x_1x_2$.

If the model type is advanced, then $x_3=1$ and $x_4=0$. The model would be

$$E(y)=\beta_0+\beta_1x_1+\beta_2x_2+\beta_3+\beta_5x_1^2+\beta_6x_2^2+\beta_7x_1+\beta_9x_2+\beta_{11}x_1^2$$
$$+\beta_{13}x_2^2+\beta_{15}x_1x_2+\beta_{16}x_1x_2$$
$$=(\beta_0+\beta_3)+(\beta_1+\beta_7)x_1+(\beta_2+\beta_9)x_2+(\beta_5+\beta_{11})x_1^2$$
$$+(\beta_6+\beta_{13})x_2^2+(\beta_{15}+\beta_{16})x_1x_2$$

If the model type is aeroderivative, then $x_3 = 0$ and $x_4 = 1$. The model would be

$$\begin{aligned}E(y) &= \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_4 + \beta_5 x_1^2 + \beta_6 x_2^2 + \beta_8 x_1 \\ &\quad + \beta_{10} x_2 + \beta_{12} x_1^2 + \beta_{14} x_2^2 + \beta_{15} x_1 x_2 + \beta_{17} x_1 x_2 \\ &= (\beta_0 + \beta_4) + (\beta_1 + \beta_8) x_1 + (\beta_2 + \beta_{10}) x_2 + (\beta_5 + \beta_{12}) x_1^2 \\ &\quad + (\beta_6 + \beta_{14}) x_2^2 + (\beta_{15} + \beta_{17}) x_1 x_2\end{aligned}$$

All three of these equations are complete second-order models in x_1 and x_2.

c. Using MINITAB, the results are:

Regression Analysis: HEATRATE versus RPM, CPRATIO, ... RATIO_SQ_x4

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	17	153410877	9024169	30.52	0.000
Error	49	14486331	295639		
Total	66	167897208			

Model Summary

S	R-sq	R-sq(adj)
543.727	91.37%	88.38%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	19343	3125	6.19	0.000
RPM	0.018	0.148	0.12	0.906
CPRATIO	-1073	421	-2.55	0.014
x3	-7395	8419	-0.88	0.384
x4	-207132	118784	-1.74	0.087
RPM-SQ	0.000000	0.000003	0.17	0.863
CPRATIO_SQ	29.7	14.6	2.04	0.047
RPM_CPRATIO	0.00685	0.00802	0.85	0.397
RPM_x3	0.97	1.21	0.80	0.426
RPM_x4	13.04	7.77	1.68	0.100
CPRATIO_x3	651	917	0.71	0.481
CPRATIO_x4	14880	8327	1.79	0.080
RPM_CPRATIO_x3	-0.0431	0.0405	-1.06	0.293
RPM_CPRATIO_x4	-0.503	0.300	-1.68	0.100
RPM_SQ_x3	-0.000011	0.000090	-0.12	0.903
RPM_SQ_x4	-0.000183	0.000111	-1.66	0.104
CPRATIO_SQ_x3	-17.4	25.0	-0.70	0.488
CPRATIO_SQ_x4	-249	132	-1.88	0.066

Regression Equation

HEATRATE = 19343 + 0.018 RPM - 1073 CPRATIO - 7395 x3 - 207132 x4 + 0.000000 RPM-SQ
+ 29.7 CPRATIO_SQ + 0.00685 RPM_CPRATIO + 0.97 RPM_x3 + 13.04 RPM_x4
+ 651 CPRATIO_x3 + 14880 CPRATIO_x4 - 0.0431 RPM_CPRATIO_x3 -
0.503 RPM_CPRATIO_x4
- 0.000011 RPM_SQ_x3 - 0.000183 RPM_SQ_x4 - 17.4 CPRATIO_SQ_x3 -
249 CPRATIO_SQ_x4

The fitted regression line is

$$\hat{y} = 19343 + 0.018x_1 - 1073x_2 + 0.000000x_1^2 + 29.7x_2^2 + 0.00685x_1x_2 - 7395x_3$$
$$- 207132x_4 + 0.97x_1x_3 + 13.04x_1x_4 + 651x_2x_3 + 14880x_2x_4 - 0.000011x_1^2x_3$$
$$- 0.000183x_1^2x_4 - 17.4x_2^2x_3 - 249x_2^2x_4 - 0.0431x_1x_2x_3 - 0.503x_1x_2x_4$$

d. To determine if the overall model is adequate, we test:

$$H_0: \beta_1 = \beta_2 = \cdots = \beta_{17} = 0$$
$$H_a: \text{ At least one } \beta_i \neq 0$$

The test statistic is $F = 30.52$ and the p-value is $p = 0.000$. Since the p-value is so small, H_0 is rejected. There is sufficient evidence to indicate the model is adequate for any reasonable value of α.

e. We must first fit the model without the terms for the engine type or without x_3 and x_4. Using MINITAB, the results are:

Regression Analysis: HEATRATE versus RPM, CPRATIO, ... SQ, RPM_CPR

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	5	148526859	29705372	93.55	0.000
Error	61	19370350	317547		
Total	66	167897208			

Model Summary

S	R-sq	R-sq(adj)
563.513	88.46%	87.52%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	15583	1143	13.63	0.000
RPM	0.078	0.110	0.71	0.481
CPRATIO	-523	103	-5.06	0.000
RPM_SQ	-0.000000	0.000002	-0.09	0.927
CPR_SQ	8.84	2.16	4.09	0.000
RPM_CPR	0.00445	0.00558	0.80	0.428

Regression Equation

HEATRATE = 15583 + 0.078 RPM - 523 CPRATIO - 0.000000 RPM_SQ + 8.84 CPR_SQ + 0.00445 RPM_CPR

To determine if the second-order response surface is identical for each level of engine type, we test:

$$H_0: \beta_3 = \beta_4 = \cdots = \beta_{14} = \beta_{16} = \beta_{17} = 0$$
$$H_a: \text{ At least one } \beta_i \neq 0$$

The test statistic is

$$F = \frac{(SSE_R - SSE_C)/(k-g)}{MSE_c} = \frac{(19{,}370{,}350 - 14{,}486{,}331)/(17-5)}{295{,}639} = 1.38$$

The rejection region requires $\alpha = 0.05$ in the upper tail of the F distribution with $\nu_1 = k - g = 17 - 5 = 12$ and $\nu_2 = n - (k+1) = 49$. Using MINITAB, $F = 1.96$. The rejection region is $F > 1.96$.

Since the observed value of the test statistic does not fall in the rejection region $(F = 1.38 \not> 1.96)$, H_0 is not rejected. There is insufficient evidence to indicate the second-order response surface is different for the different levels of engine type at $\alpha = 0.05$.

5.41 a. Let $x_1 =$ elevation and $x_2 = \begin{cases} 1 \text{ if east} \\ 0 \text{ otherwise} \end{cases}$

The first-order model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2$.

b. The graphs will vary. However, for each value of slope face, the slopes of the straight-line relationship between mean lead level and elevation will vary. The lines will not be parallel. The line for slope face West will have a slope equal to β_1. The line for slope face East will have a slope equal to $\beta_1 + \beta_3$.

c. For slope face East, the slope of the line is $\beta_1 + \beta_3$. Thus, for each foot increase in elevation, the mean lead level will increase by $\beta_1 + \beta_3$.

d. Using MINITAB, the results of the analysis are:

Regression Analysis: LEAD versus ELEVATION, SLOPE, ELE_SLOPE

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	3	20.24	6.746	0.26	0.857
Error	66	1738.08	26.335		
Total	69	1758.32			

Model Summary

S	R-sq	R-sq(adj)
5.13172	1.15%	0.00%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	2.38	5.39	0.44	0.660
ELEVATION	0.00181	0.00214	0.84	0.401
SLOPE	3.20	7.67	0.42	0.678
ELE_SLOPE	-0.00133	0.00303	-0.44	0.663

Regression Equation

LEAD = 2.38 + 0.00181 ELEVATION + 3.20 SLOPE - 0.00133 ELE_SLOPE

To determine if the overall model is useful for predicting lead level, we test:

$H_0: \beta_1 = \beta_2 = \beta_3 = 0$
$H_a:$ At least one $\beta_i \neq 0$

From the printout, the test statistic is $F = 0.26$ and the p-value is $p = 0.857$. Since the p-value is not less than $\alpha\left(p = 0.857 \not< 0.10\right)$, H_0 is not rejected. There is insufficient evidence to indicate that the overall model is useful for predicting lead level at $\alpha = 0.10$.

e. The complete second-order model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_1^2 + \beta_3 x_2 + \beta_4 x_1 x_2 + \beta_5 x_1^2 x_2$.

5.42 a. The model would be $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2$.

b. The model when affected by war would be $E(y) = \beta_0 + \beta_1 x_1 + \beta_2(1) + \beta_3 x_1(1) = \beta_0 + \beta_2 + (\beta_1 + \beta_3)x_1$. The slope would be $\beta_1 + \beta_3$.

c. The model when not affected by war would be $E(y) = \beta_0 + \beta_1 x_1 + \beta_2(0) + \beta_3 x_1(0) = \beta_0 + \beta_1 x_1$. The slope would be β_1.

d. The model when there is no charisma difference would be $E(y) = \beta_0 + \beta_1(0) + \beta_2 x_2 + \beta_3(0)x_2 = \beta_0 + \beta_2 x_2$. The effect of a world war would be β_2.

e. The model when the charisma difference is 50 would be $E(y) = \beta_0 + \beta_1(50) + \beta_2 x_2 + \beta_3(50)x_2 = \beta_0 + 50\beta_1 + (\beta_2 + 50\beta_3)x_2$. The effect of a world war would be $\beta_2 + 50\beta_3$.

f. Using MINITAB, the results are:

Regression Analysis: VSHARE versus CDIFF, WAR, CDIFF_WAR

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	3	216.3	72.11	1.59	0.224
Error	20	909.5	45.48		
Total	23	1125.9			

Model Summary

S	R-sq	R-sq(adj)
6.74364	19.21%	7.10%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	49.37	1.53	32.21	0.000
CDIFF	-0.0378	0.0701	-0.54	0.596
WAR	7.17	6.11	1.17	0.255
CDIFF_WAR	0.378	0.180	2.10	0.049

Regression Equation
VSHARE = 49.37 - 0.0378 CDIFF + 7.17 WAR + 0.378 CDIFF_WAR

To determine if the linear effect of charisma difference on the mean Democratic vote share depends on world war status, we test:

$$H_0: \beta_3 = 0$$
$$H_a: \beta_3 \neq 0$$

The test statistic is $t = 2.10$ and the *p*-value is $p = 0.049$. Since the *p*-value is less than $\alpha\left(p = 0.049 < 0.10\right)$, H_0 is rejected. There is sufficient evidence to indicate that the linear effect of charisma difference on the mean Democratic vote share depends on world war status at $\alpha = 0.10$.

5.43 a. The parallel line model is $E\left(y\right) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$. (no interaction)

b. The interaction model would be $E\left(y\right) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_1 x_2 + \beta_5 x_1 x_3$.

c. The slope of the line for TDS-3A waste is $\beta_1 + \beta_4$.
The slope of the line for FE waste is $\beta_1 + \beta_5$.
The slope of the line for AL waste is β_1.

d. To test for the presence of temperature-waste type interaction, use a nested *F*-test

$$H_0: \beta_4 = \beta_5 = 0$$
$$H_a: \text{ At least one } \beta_i \neq 0$$

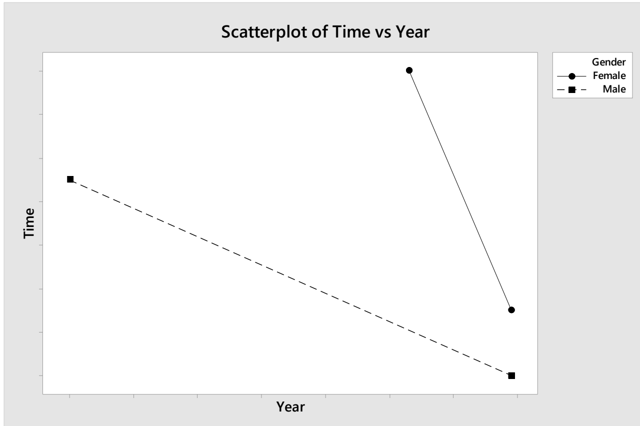
To perform this test, we would first fit the complete model (model given in part b) and then fit the reduced model (model given in part a). By comparing the two models, we could decide whether to reject H_0 or not.

5.44 Winning time is a quantitative variable as it can be measured on a numerical scale. Year of race is a quantitative variable as it can be measured on a numerical scale. Gender is a qualitative variable as the responses (male and female) fall into one of two categories.

5.45 a. The first order model is $E\left(y\right) = \beta_0 + \beta_1 x_1 + \beta_2 x_2$.

b. $\beta_0 =$ mean winning time for females in 1880.
$\beta_1 =$ change in mean winning time for each year since 1880, holding gender constant
$\beta_2 =$ difference in mean winning time between males and female, holding year constant

c. The interaction model is $E\left(y\right) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2$. A possible sketch is:



d. The complete second-order model is
$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_3 + \beta_3 x_1 x_3 + \beta_4 x_1^2 + \beta_5 x_3^2$.

e. The model with the main effect of gender is
$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_3 + \beta_3 x_1 x_3 + \beta_4 x_1^2 + \beta_5 x_3^2 + \beta_6 x_2$.

f. The model allowing for interaction terms is
$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_3 + \beta_3 x_1 x_3 + \beta_4 x_1^2 + \beta_5 x_3^2 + \beta_6 x_2 + \beta_7 x_1 x_2 + \beta_8 x_3 x_2 + \beta_9 x_1 x_3 x_2 + \beta_{10} x_1^2 x_2 + \beta_{11} x_3^2 x_2.$$

g. This will happen when β_8, β_9 and β_{11} are all 0.

h. This will happen when β_5, β_8, β_9 and β_{11} are all 0.

i. This will happen when $\beta_6 - \beta_{11}$ are all 0.

5.46 a. Number of preincident psychological symptoms is a quantitative variable ranging from zero to a high value of maybe 10 or 15.

b. Years of experience is a quantitative variable ranging from 0 to around 40.

c. Cigarette smoking behavior is a qualitative variable. Possible values for this variable is smokes or does not smoke.

d. Level of social support is a qualitative variable. Possible values of the variable are low, medium, or high.

e. Marital status is a qualitative variable. Possible values of the variable are single, married, divorced, widowed, or separated.

f. Age is a quantitative variable with values ranging from a low of 18 to a high of about 65.

g. Ethnic status is a qualitative variable. Possible values of this variable are Caucasian, Afro-American, American Indian, Asian, etc.

h. Exposure to a chemical fire is a qualitative variable. Possible values of this variable are exposed or not exposed.

i. Educational level is a qualitative variable. Possible values of this variable are below high school, high school diploma, some college, bachelor's degree, master's degree, or doctorate degree.

j. Distance lived from site of incident is a quantitative variable, ranging from 0 miles to maybe 40 miles.

k. Gender is a qualitative variable with values male or female.

5.47 a. The model would be $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$.

where $x_1 = \begin{cases} 1 \text{ if low} \\ 0 \text{ if not} \end{cases}$ $\quad x_2 = \begin{cases} 1 \text{ if moderate} \\ 0 \text{ if not} \end{cases}$ $\quad x_3 = \begin{cases} 1 \text{ if high} \\ 0 \text{ if not} \end{cases}$

b. $\beta_0 = \mu_{\text{None}}$
$\beta_1 = \mu_{\text{Low}} - \mu_{\text{None}}$
$\beta_2 = \mu_{\text{Mod}} - \mu_{\text{None}}$
$\beta_3 = \mu_{\text{High}} - \mu_{\text{None}}$

c. To test for differences among emotional stress means for the 4 social support levels, we test:

$$H_0: \beta_1 = \beta_2 = \beta_3 = 0$$
$$H_a: \text{ At least one } \beta_i \neq 0$$

The test statistic would be an *F*.

5.48 a. Since the variable state of birth is measured on a nonnumerical scale, it is qualitative. Examples of state of birth are Florida, Ohio, etc.

b. Since the variable age is measured using a numerical scale, it is quantitative. Examples of ages are 15 years, 25 years, etc.

c. Since the variable education level is measured on a nonnumerical scale, it is qualitative. Examples of education level are high school graduate, college graduate, etc.

d. Since the variable tenure with firm is measured using a numerical scale, it is quantitative. We assume that tenure with a firm is the number of years of service. Examples of tenure with a firm are 15 years, 25 years, etc.

e. Since the variable total compensation is measured using a numerical scale, it is quantitative. Examples of total compensation are $45,000, $57,843, etc.

f. Since the variable area of expertise is measured on a nonnumerical scale, it is qualitative. Examples of areas of expertise are statistics, accounting, etc.

5.49 a. Age represents a quantitative variable ranging from a low value of perhaps 16 to a high of 70.

b. Years in therapy can range between say 0 and 80. This is a quantitative independent variable.

c. Highest educational degree would probably be broken down into five levels: none, high school diploma, bachelor's degree, master's degree, or doctorate. These categories are indicative of a qualitative variable.

d. Job classification is categorical in nature and therefore a qualitative independent variable. Some possible "levels" are laborer, foreman, secretary, administrative assistant, manager, vice president, etc.

e. Religious preference is also qualitative. Four possible levels are: Protestant, Catholic, Jewish, other.

f. It is impossible to assign numerical values to classifications such as: single, married, divorced, widowed, separated, other. Thus, marital status is a qualitative variable.

g. IQ is considered a quantitative variable with a wide range of values from 0 to 175 or more.

h. Gender is a qualitative variable with the two levels, male and female.

5.50 A possible model is $E(y) = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3$.

5.51 a. The MINITAB printout below shows the correlation between the three parameters:

Correlation: x, x_sq, x_cu

Correlations

	x	x_sq
x_sq	0.975	
x_cu	0.928	0.987

Cell Contents
Pearson correlation

b. Let $u = \dfrac{x - \overline{x}}{s_x} = \dfrac{x - 5.5}{3.03}$

The MINITAB printout below shows the correlation between the coded variables:

Correlation: u, u_sq, u_cu

Correlations

	u	u_sq
u_sq	0.000	
u_cu	0.923	0.000

Cell Contents
Pearson correlation

The level is reduced in two of the three pairs.

5.52 a. The complete second-order model is
$E(y)=\beta_0+\beta_1x_1+\beta_2x_1^2+\beta_3x_2+\beta_4x_1x_2+\beta_5x_1^2x_2$.

b. For noncoached students, $x_2=0$. The equation would be
$E(y)=\beta_0+\beta_1x_1+\beta_2x_1^2+\beta_3(0)+\beta_4x_1(0)+\beta_5x_1^2(0)=\beta_0+\beta_1x_1+\beta_2x_1^2$.
The y-intercept is β_0, the shift parameter is β_1, and the rate of curvature is β_2.

c. For coached students, $x_2=1$. The equation would be

$$E(y)=\beta_0+\beta_1x_1+\beta_2x_1^2+\beta_3(1)+\beta_4x_1(1)+\beta_5x_1^2(1)$$
$$=(\beta_0+\beta_3)+(\beta_1+\beta_4)x_1+(\beta_2+\beta_5)x_1^2.$$

The y-intercept is $(\beta_0+\beta_3)$, the shift parameter is $(\beta_1+\beta_4)$, and the rate of curvature is $(\beta_2+\beta_5)$.

d. To determine if coaching has an effect on SAT-Math, we test:

$H_0: \beta_3=\beta_4=\beta_5=0$
H_a: At least one $\beta_i \neq 0$

The test statistic would be an F.

5.53 a. For males $(x_5=0)$ and the model is $E(y)=\beta_0+\beta_1x_1+\beta_2x_2+\beta_3x_3+\beta_4x_4$.

b. $\beta_1=$ difference in mean starting salaries between male graduating seniors in Business Administration and Nursing, $\mu_{BA}-\mu_N$.

c. $\beta_2=$ difference in mean starting salaries between male graduating seniors in Engineering and Nursing, $\mu_E-\mu_N$.

d. $\beta_3=$ difference in mean starting salaries between male graduating seniors in Liberal Arts and Sciences and Nursing, $\mu_{LAS}-\mu_N$.

e. $\beta_4=$ difference in mean starting salaries between male graduating seniors in Journalism and Nursing, $\mu_J-\mu_N$.

f. For females $(x_5=1)$ and the model is $E(y)=(\beta_0+\beta_5)+\beta_1x_1+\beta_2x_2+\beta_3x_3+\beta_4x_4$.

g. $\beta_1=$ difference in mean starting salaries between female graduating seniors in Business Administration and Nursing, $\mu_{BA}-\mu_N$. This is the same as in part (b).

h. $\beta_2=$ difference in mean starting salaries between female graduating seniors in Engineering and Nursing, $\mu_E-\mu_N$.. This is the same as in part (c).

i. $\beta_3 =$ difference in mean starting salaries between female graduating seniors in Liberal Arts and Sciences and Nursing, $\mu_{\text{LAS}} - \mu_{\text{N}}$. This is the same as in part (d).

j. $\beta_4 =$ difference in mean starting salaries between female graduating seniors in Journalism and Nursing, $\mu_{\text{J}} - \mu_{\text{N}}$. This is the same as in part (e).

k. For a given college, β_1 is the difference in mean starting salary between females and males, $\mu_{\text{F}} - \mu_{\text{M}}$.

l. To determine if gender has an effect on average starting salary, we test:

$$H_0: \beta_5 = 0$$
$$H_a: \beta_5 \neq 0$$

The test statistic is $t = -2.72$ and the p-value is $p = 0.0066$. Since the p-value is less than $\alpha\left(p = 0.0066 < 0.01\right)$, H_0 is rejected. There is sufficient evidence to indicate that gender has an effect on average starting salary at $\alpha = 0.01$.

5.54 a. The interaction model is
$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_1 x_5 + \beta_7 x_2 x_5 + \beta_8 x_3 x_5 + \beta_9 x_4 x_5$.

b. $\beta_1 =$ difference in mean starting salaries between male graduating seniors in Business Administration and Nursing, $\mu_{\text{M,BA}} - \mu_{\text{M,N}}$.

c. $\beta_2 =$ difference in mean starting salaries between male graduating seniors in Engineering and Nursing, $\mu_{\text{M,E}} - \mu_{\text{M,N}}$.

d. $\beta_3 =$ difference in mean starting salaries between male graduating seniors in Liberal Arts and Sciences and Nursing, $\mu_{\text{M,LAS}} - \mu_{\text{M,N}}$.

e. $\beta_4 =$ difference in mean starting salaries between male graduating seniors in Journalism and Nursing, $\mu_{\text{M,J}} - \mu_{\text{M,N}}$.

f. $\beta_5 =$ difference in mean starting salaries between female and male graduating seniors in Nursing, $\mu_{\text{F,N}} - \mu_{\text{M,N}}$.

g. To determine if interaction exists, we test:

$$H_0: \beta_6 = \beta_7 = \beta_8 = \beta_9 = 0$$
$$H_a: \text{At least one } \beta_i \neq 0$$

We would fit the model in Exercise 5.53 (reduced model) and the model in part a (complete model) and run a partial F test.

5.55 a. The model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$

where $x_1 = \begin{cases} 1 & \text{if boy} \\ 0 & \text{if girl} \end{cases}$ $x_2 = \begin{cases} 1 & \text{if youngest third} \\ 0 & \text{if not} \end{cases}$

$x_3 = \begin{cases} 1 & \text{if middle Third} \\ 0 & \text{if not} \end{cases}$

b. $\beta_0 = \mu_{\text{Girls/Oldest}}$, $\beta_1 = \mu_{\text{Boys}} - \mu_{\text{Girls}}$, $\beta_2 = \mu_{\text{Youngest}} - \mu_{\text{Oldest}}$,
$\beta_3 = \mu_{\text{Middle}} - \mu_{\text{Oldest}}$

c. The interaction model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_1 x_2 + \beta_5 x_1 x_3$.

d. $\hat{\beta}_0 = \hat{\mu}_{\text{Girls/Oldest}} = 0.21$

$\hat{\beta}_1 = \hat{\mu}_{\text{Boys/Oldest}} - \hat{\mu}_{\text{Girls/Oldest}} = 0.16 - 0.21 = -0.05$

$\hat{\beta}_2 = \hat{\mu}_{\text{Girls/Youngest}} - \hat{\mu}_{\text{Girls/Oldest}} = 0.27 - 0.21 = 0.06$

$\hat{\beta}_3 = \hat{\mu}_{\text{Girls/Middle}} - \hat{\mu}_{\text{Girls/Oldest}} = 0.18 - 0.21 = -0.03$

$\hat{\beta}_4 = \hat{\mu}_{\text{Boys/Youngest}} - \hat{\mu}_{\text{Boys/Oldest}} - \hat{\mu}_{\text{Girls/Youngest}} + \hat{\mu}_{\text{Girls/Oldest}}$
$= 0.33 - 0.16 - 0.27 + 0.21 = 0.11$

$\hat{\beta}_5 = \hat{\mu}_{\text{Boys/Middle}} - \hat{\mu}_{\text{Boys/Oldest}} - \hat{\mu}_{\text{Girls/Middle}} + \hat{\mu}_{\text{Girls/Oldest}}$
$= 0.33 - 0.16 - 0.18 + 0.21 = 0.20$

e. To determine if the difference between the mean standardized heights of boys and girls is the same across all three age tertiles, we use a nested *F*-test:

$H_0: \beta_4 = \beta_5 = 0$
H_a: At least one $\beta_i \neq 0$

5.56 a. The quadratic model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_1^2$.

b. Since there are only there levels (12, 24, and 36) of the independent variable, a cubic model could not be fit.

c. The complete second-order model is
$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_1^2 + \beta_3 x_2 + \beta_4 x_3 + \beta_5 x_1 x_2 + \beta_6 x_1 x_3 + \beta_7 x_1^2 x_2 + \beta_8 x_1^2 x_3$.

where $x_2 = \begin{cases} 1 & \text{if paint type A} \\ 0 & \text{if not} \end{cases}$ $x_3 = \begin{cases} 1 & \text{if paint type B} \\ 0 & \text{if not} \end{cases}$

5.57 a. The two independent variables, packaging and location, are qualitative. The two types of packaging generate one term in the model, while the four locations induce three terms. A main effect model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4$.

where $x_1 = \begin{cases} 1 & \text{if } P_1 \\ 0 & \text{if } P_2 \end{cases}$ $\quad x_3 = \begin{cases} 1 & \text{if } L_2 \\ 0 & \text{if not} \end{cases}$

$x_2 = \begin{cases} 1 & \text{if } L_1 \\ 0 & \text{if not} \end{cases}$ $\quad x_4 = \begin{cases} 1 & \text{if } L_3 \\ 0 & \text{if not} \end{cases}$

This model assumes that Packaging and Location affect total sales independently.

b. If the previous assumption is incorrect and Packaging and Location do interact, then a more appropriate model is:

$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_1 x_2 + \beta_6 x_1 x_3 + \beta_7 x_1 x_4$$

There are eight parameters in this model. There are also a total of eight Packaging-Location combinations, which permits an estimate of $E(y)$ for each combination.

c. To determine if packaging and location interact to affect weekly sales, we test:

$H_0: \beta_5 = \beta_6 = \beta_7 = 0$
H_a: At least one $\beta_i \neq 0$

The test statistic is $F = \dfrac{(SSE_R - SSE_C)/(k-g)}{SSE_C / [n-(k+1)]} = \dfrac{(422.36 - 346.65)/(7-4)}{346.65/[40-(7+1)]} = 2.33$.

The rejection region requires $\alpha = 0.05$ in the upper tail of the F distribution with $\nu_1 = k - g = 7 - 4 = 3$ and $\nu_2 = n-(k+1) = 40-(7+1)$=32. Using Table 4, Appendix D, $F_{0.05} \approx 2.92$. The rejection region is $F >$ 2.92.

Since the observed value of the test statistic does not fall in the rejection region $(F = 2.33 \not> 2.92)$, H_0 in not rejected. There is insufficient evidence to indicate that the interaction between location and packaging is important in estimating mean weekly sales at $\alpha = 0.05$.

This implies that the company can choose the location and packaging type independently of each other. The choice of packaging type does not depend on location.

5.58 a. The complete second-order model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 + \beta_4 x_1^2 + \beta_5 x_2^2$.

b. Using MINITAB, the results are:

Regression Analysis: Earnings versus Age, Hours, A_H, ... _sq, Hours_sq

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	5	6520245	1304049	5.59	0.013
Error	9	2098183	233131		
Total	14	8618428			

Model Summary

S	R-sq	R-sq(adj)
482.837	75.65%	62.13%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	606	2331	0.26	0.801
Age	119.7	64.6	1.85	0.097
Hours	-140	492	-0.28	0.783
A_H	2.66	3.42	0.78	0.456
Age_sq	-1.571	0.691	-2.27	0.049
Hours_sq	8.1	26.7	0.30	0.769

Regression Equation

Earnings = 606 + 119.7 Age - 140 Hours + 2.66 A_H - 1.571 Age_sq + 8.1 Hours_sq

The least squares prediction equation is
$\hat{y} = 606 + 119.7x_1 - 140x_2 + 2.66x_1x_2 - 1.571x_1^2 + 8.1x_2^2$.

c. To determine if the model is useful, we test:

$$H_0: \beta_1 = \beta_2 = \cdots = \beta_5 = 0$$
$$H_a: \text{ At least one } \beta_i \neq 0$$

The test statistic is $F = 5.59$ and the p-value is $p = 0.013$. Since the p-value is less than $\alpha\left(p = 0.013 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate the model is useful for predicting annual earnings at $\alpha = 0.05$.

d. To determine if the 2nd-order terms are necessary for predicting annual earnings, we test:

$$H_0: \beta_3 = \beta_4 = \beta_5 = 0$$
$$H_a: \text{ At least one } \beta_i \neq 0$$

We would then fit the reduced model, $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2$, and compute the test statistic by comparing the complete model to the reduced model.

e. Using MINITAB, the results of fitting the reduced model are:

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	2	5018232	2509116	8.36	0.005
Error	12	3600196	300016		
Total	14	8618428			

The test statistic is
$$F = \frac{(SSE_R - SSE_C)/(k-g)}{SSE_C/[n-(k+1)]} = \frac{(3{,}600{,}196 - 2{,}098{,}183)/(5-2)}{2{,}098{,}183/[15-(5+1)]} = 2.15.$$

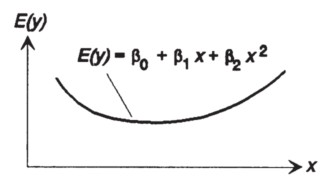
The rejection region requires $\alpha = 0.05$ in the upper tail of the F distribution with $\nu_1 = k - g = 5 - 2 = 3$ and $\nu_2 = n - (k+1) = 15 - (5+1) = 9$. From Table 4, Appendix D, $F_{0.05} = 3.86$. The rejection region is $F > 3.86$.

Since the observed value of the test statistic does not fall in the rejection region $(F = 2.15 \not> 3.86)$, H_0 is not rejected. There is insufficient evidence to indicate the 2nd-order terms are necessary for predicting annual earnings at $\alpha = 0.05$.

5.59 The management is advocating a second-order model relating y, assembly time, to x, time since lunch:

$$E(y) = \beta_0 + \beta_1 x + \beta_2 x^2$$

Furthermore, the average assembly time is expected to decrease for small values of x and then gradually increase as x gets larger. We expect β_2 to be positive and the general shape of the model to look like this:



5.60 a. Let x_1 = NDF and x_2 = AMAP. The first-order model would be $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2$.

b. To determine if the model is adequate, we test:

$$H_0: \beta_1 = \beta_2 = 0$$
$$H_a: \text{ At least one } \beta_i \neq 0$$

The p-value is $p = 0.001$. Since the p-value is so small, H_0 is rejected. The model is adequate for predicting TME for any reasonable value of α.

$R^2 = 0.988$. 98.8% of the sample variation in the TME scores is explained by the first-order model with NDF and AMAP.

c. The complete 2nd-order model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 + \beta_4 x_1^2 + \beta_5 x_2^2$.

d. To determine if the 2nd-order terms are useful, we test:

$$H_0: \beta_3 = \beta_4 = \beta_5 = 0$$
$$H_a: \text{ At least one } \beta_i \neq 0$$

We now fit a complete model and compare it to the reduced model to determine if the 2^{nd}-order terms contribute to the model.

5.61 a. The independent variables, foot mass, leg length, velocity, pressure, and impulse are all quantitative variables. The independent variable, foot type, is qualitative.

b. Let $x_1 =$ pressure and $x_2 =$ leg length. The model would be $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2$.

c. When $x_1 = 40$,
$E(y) = \beta_0 + \beta_1(40) + \beta_2 x_2 + \beta_3(40)x_2 = \beta_0 + 40\beta_1 + (\beta_2 + 40\beta_3)x_2$.
For every increase of 1 meter in leg length, the heel depth will increase by $\beta_2 + 40\beta_3$ when pressure is 40.

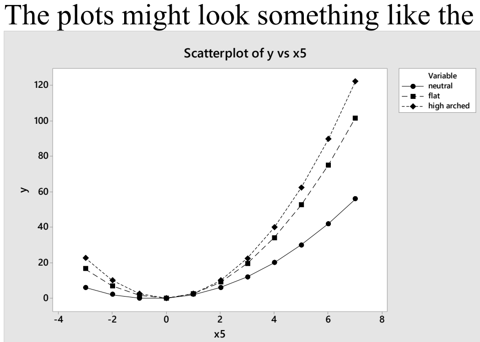
d. Let $x_3 = \begin{cases} 1 & \text{if neutral} \\ 0 & \text{otherwise} \end{cases}$ and $x_4 = \begin{cases} 1 & \text{if flat} \\ 0 & \text{otherwise} \end{cases}$

The model would be $E(y) = \beta_0 + \beta_1 x_3 + \beta_2 x_4$.

$\beta_0 =$ mean heel depth for high arch type.
$\beta_1 =$ difference in mean heel depth between neutral and high arch types.
$\beta_2 =$ difference in mean heel depth between flat and high arch types.

e. Let $x_5 =$ velocity. The complete 2^{nd}-order model would be
$E(y) = \beta_0 + \beta_1 x_3 + \beta_2 x_4 + \beta_3 x_5 + \beta_4 x_3 x_5 + \beta_5 x_4 x_5 + \beta_6 x_5^2 + \beta_7 x_3 x_5^2 + \beta_8 x_4 x_5^2$.

f. The plots might look something like the following:



5.62 a. Let $x_1 = \begin{cases} 1 & \text{if VHA} \\ 0 & \text{if no VHA} \end{cases}$

The model would be $E(y) = \beta_0 + \beta_1 x_1$.

Using MINITAB, the results are:

Regression Analysis: CH4 versus x1
Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	42280	42280	5.03	0.034
Error	25	210230	8409		
Total	26	252510			

Model Summary

S	R-sq	R-sq(adj)
91.7016	16.74%	13.41%

Coefficients

Term	Coef	SE Coef	95% CI	T-Value	P-Value	VIF
Constant	99.2	25.4	(46.8, 151.6)	3.90	0.001	
x1	79.2	35.3	(6.5, 151.9)	2.24	0.034	1.00

Regression Equation

CH4 = 99.2 + 79.2 x1

To determine if the mean amount of methane gas emitted differs for the 2 types of sludge, we test:

$$H_0: \beta_1 = 0$$
$$H_a: \beta_1 \neq 0$$

The test statistic is $F = 5.03$ and the p-value is $p = 0.034$. Since the p-value is less than $\alpha\left(p = 0.034 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate the mean amount of methane gas emitted differs for the 2 types of sludge at $\alpha = 0.05$.

$\beta_1 =$ difference in mean amount of methane gas emitted between sludge with VHA and sludge without VHA. From the printout, $\hat{\beta}_1 = 79.2$ and the 95% confidence interval for β_1 is $(6.5, 151.9)$. We are 95% confident that the difference in mean amount of methane gas emitted between sludge with VHA and sludge without VHA is between 6.5 and 151.9.

b. We would fit the model $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2$. Using MINITAB, the results are:

Regression Analysis: CH4 versus x1, x2, x1x2
Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	3	238658	79552.7	132.10	0.000
Error	23	13851	602.2		
Total	26	252510			

Model Summary

S	R-sq	R-sq(adj)
24.5405	94.51%	93.80%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	-137.0	24.1	-5.69	0.000
x1	-27.5	34.0	-0.81	0.426
x2	6.719	0.656	10.24	0.000
x1x2	3.038	0.928	3.27	0.003

Regression Equation

CH4 = -137.0 - 27.5 x1 + 6.719 x2 + 3.038 x1x2

The fitted regression line is $\hat{y} = -137.0 - 27.5x_1 + 6.719x_2 + 3.038x_1x_2$.

To determine if the emission rates differ for the two types of sludge, we test:

$$H_0: \beta_3 = 0$$
$$H_a: \beta_3 \neq 0$$

The test statistic is $t = 3.27$ and the p-value is $p = 0.003$. Since the p-value is less than $\alpha\left(p = 0.003 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate the emission rates differ for the two types of sludge at $\alpha = 0.05$.

For sludge with no VHA, the fitted model is $\hat{y} = -137.0 - 27.5(0) + 6.719x_2 + 3.038(0)x_2 = -137.0 + 6.719x_2$. The emission rate is 6.719.

For sludge with VHA, the fitted model is $\hat{y} = -137.0 - 27.5(1) + 6.719x_2 + 3.038(1)x_2 = -164.5 + 9.757x_2$. The emission rate is 9.757.

c. The complete 2nd-order model is $E(y) = \beta_0 + \beta_1x_1 + \beta_2x_2 + \beta_3x_1x_2 + \beta_4x_2^2 + \beta_5x_1x_2^2$.

d. To determine whether curvature exists in the relationship between methane gas emission and treatment time, we test:

$$H_0: \beta_4 = \beta_5 = 0$$
$$H_a: \text{At least } \beta_i \neq 0$$

e. The complete model is $E(y) = \beta_0 + \beta_1x_1 + \beta_2x_2 + \beta_3x_1x_2 + \beta_4x_2^2 + \beta_5x_1x_2^2$. The reduced model is $E(y) = \beta_0 + \beta_1x_1 + \beta_2x_2 + \beta_3x_1x_2$.

f. Using MINITAB, the results from the complete model are:

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	5	244222	48844.4	123.76	0.000
Error	21	8288	394.7		
Total	26	252510			

The test statistic is $F = \frac{(SSE_R - SSE_C)/(k-g)}{SSE_C/[n-(k+1)]} = \frac{(13{,}851-8288)/(5-3)}{8288/[27-(5+1)]} = 7.05.$

The rejection region requires $\alpha = 0.05$ in the upper tail of the F distribution with $\nu_1 = k - g = 5 - 3 = 2$ and $\nu_2 = n-(k+1) = 27-(5+1) = 21$. From Table 4, Appendix D, $F_{0.05} = 3.47$. The rejection region is $F > 3.47$.

Since the observed value of the test statistic falls in the rejection region $(F = 7.05 > 3.47)$, H_0 is rejected. There is sufficient evidence to indicate curvature exists in the relationship between methane gas emission and treatment time at $\alpha = 0.05$.

5.63 Using MINITAB, the regression analysis is:

Regression Analysis: DEMAND versus p, p_sq

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	2	159298	79649.0	382.26	0.000
Error	5	1042	208.4		
Total	7	160340			

Model Summary

S	R-sq	R-sq(adj)
14.4348	99.35%	99.09%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	8383	1246	6.73	0.001
p	-3898	746	-5.22	0.003
p_sq	491	111	4.41	0.007

Regression Equation

DEMAND = 8383 - 3898 p + 491 p_sq

Using MINITAB, the correlation between p and p_sq is

Correlation: p, p_sq

Correlations

Pearson correlation 1.000

Next, we code the data using $u = \frac{p - \bar{p}}{s_p} = \frac{p - 3.35}{0.244949}$. The regression results with the coded data are:

Regression Analysis: DEMAND versus u, u_sq

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	2	159298	79649.0	382.26	0.000
Error	5	1042	208.4		
Total	7	160340			

Model Summary

S	R-sq	R-sq(adj)
14.4348	99.35%	99.09%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	835.59	7.76	107.67	0.000
u	-148.92	5.46	-27.30	0.000
u_sq	29.46	6.68	4.41	0.007

Regression Equation

DEMAND = 835.59 - 148.92 u + 29.46 u_sq

Using MINITAB, the correlation between *p* and *p_sq* is

Correlation: u, u_sq
Correlations

Pearson correlation 0.000

The correlation between *p* and *p_sq* is $r = 1.000$, while the correlation between *u* and *u_sq* is $r = 0.000$. By using the coded data, the problem with multicollinearity has essentially been removed.

5.64 a. Let $x_2 = \begin{cases} 1 & \text{if method} = G \\ 0 & \text{otherwise} \end{cases}$ $\quad x_3 = \begin{cases} 1 & \text{if method} = R_1 \\ 0 & \text{otherwise} \end{cases}$

The first-order, main effects model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$.

b. $\beta_0 =$ y-intercept for method R_2

$\beta_1 =$ change in mean shelf life for each unit change in drug potency, with method held constant

$\beta_2 =$ difference in mean shelf life between methods G and R_2.

$\beta_3 =$ difference in mean shelf life between methods R_1 and R_2.

c. The model allowing the slopes to differ is
$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_1 x_2 + \beta_5 x_1 x_3$.

d. For method G, the slope is $\beta_1 + \beta_4$.
For method R_1, the slope is $\beta_1 + \beta_5$.
For method R_2, the slope is β_1.

5.65 a. To compute the coded data, we use:
$u_1 = \dfrac{x_1 - \bar{x}_1}{s_{x_1}} = \dfrac{x_1 - 90}{8.3205}$ and $u_2 = \dfrac{x_2 - \bar{x}_2}{s_{x_2}} = \dfrac{x_2 - 55}{4.16025}$

The coded data are:

u_1	u_2	y	u_1	u_2	y	u_1	u_2	y
-1.20	-1.20	50.8	0	-1.20	63.4	1.20	-1.20	46.6
-1.20	-1.20	50.7	0	-1.20	61.6	1.20	-1.20	49.1
-1.20	-1.20	49.4	0	-1.20	63.4	1.20	-1.20	46.4
-1.20	0	93.7	0	0	93.8	1.20	0	69.8
-1.20	0	90.9	0	0	92.1	1.20	0	72.5
-1.20	0	90.9	0	0	97.4	1.20	0	73.2
-1.20	1.20	74.5	0	1.20	70.9	1.20	1.20	38.7
-1.20	1.20	73	0	1.20	68.8	1.20	1.20	42.5
-1.20	1.20	71.2	0	1.20	71.3	1.20	1.20	41.4

b. Using MINITAB, the correlation coefficients are:

Correlation: x1, x1_sq
Correlations
Pearson correlation 0.999

Correlation: u1, u1_sq
Correlations
Pearson correlation 0.000

c. Using MINITAB, the correlation coefficients are:

Correlation: x2, x2_sq
Correlations
Pearson correlation 1.000

Correlation: u2, u2_sq
Correlations
Pearson correlation 0.000

d. Using MINITAB, the results are:

Regression Analysis: y versus u1, u2, u1u2, u1_sq, u2_sq
Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	5	8402.26	1680.45	596.32	0.000
Error	21	59.18	2.82		
Total	26	8461.44			

Model Summary

S	R-sq	R-sq(adj)
1.67870	99.30%	99.13%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	94.926	0.722	131.40	0.000
u1	-7.623	0.329	-23.15	0.000
u2	3.277	0.329	9.95	0.000
u1u2	-5.037	0.335	-15.01	0.000
u1_sq	-9.235	0.474	-19.46	0.000
u2_sq	-19.804	0.474	-41.74	0.000

Regression Equation

y = 94.926 - 7.623 u1 + 3.277 u2 - 5.037 u1u2 - 9.235 u1_sq - 19.804 u2_sq

The fitted equation is $\hat{y} = 94.926 - 7.623u_1 + 3.277u_2 - 5.037u_1u_2 - 9.235u_1^2 - 19.804u_2^2$.

Chapter **6**

Variable Screening Methods

6.1 a. The hypothesized equation of the final stepwise regression would be $E(y) = \beta_0 + \beta_1 x_5 + \beta_2 x_2$.

b. $R^2 = 0.771$. 77.1% of the sample variation in heel depth is explained by the model containing pressure and leg length.

c. To determine if the final stepwise model is useful, we test:

$$H_0: \beta_1 = \beta_2 = 0$$
$$H_a: \text{At least one } \beta_i \neq 0$$

The p-value is $p < 0.001$. Since the p-value is so small, H_0 is rejected. There is sufficient evidence to indicate the final stepwise model is useful at any reasonable value of α.

d. There were 6 t-tests run in the first step and $(6-1) = 5$ t-tests run in the second step. The minimum number of t-tests run is $6 + 5 = 11$.

e. The probability of making one Type I error is α. The probability of making at least one Type I error during the stepwise analysis is
$P(\text{At least one Type I error}) = 1 - P(\text{No Type I errors})$

$$= 1 - \binom{n}{0}\alpha^0 (1-\alpha)^{n-0} = 1 - \binom{11}{0}\alpha^0 (1-\alpha)^{11}$$

6.2 a. The form of the model fit in step 1 is $E(y) = \beta_0 + \beta_1 x_i$. There will be 8 models fit. The best independent variable picked in this step is the one which produces the largest (absolute) t-value.

b. Suppose that x_3 was selected in step 1. The form of the model fit in step 2 is $E(y) = \beta_0 + \beta_1 x_3 + \beta_2 x_i$. There will be 7 models fit in this step. The best independent variable picked in this step is the one which produces the largest (absolute) t-value, adjusted for x_3 being in the model.

c. Suppose that x_5 was selected in step 2. The form of the model fit in step 3 is $E(y) = \beta_0 + \beta_1 x_3 + \beta_2 x_5 + \beta_3 x_i$. There will be 6 models fit in this step. The best independent variable picked in this step is the one which produces the largest (absolute) t-value, adjusted for x_3 and x_5 being in the model.

d. One would next want to check for 2nd-order terms and interaction terms among the 3 variables.

6.3 a. There will be 5 models fit in the first step.

b. There will be 4 models fit in the second step.

c. There will be 3 models fit in the third step.

d. There will be 2 models fit in the fourth step.

e. The total number of t-tests would be $5+4+3+2=14$. The probability of making one Type I error is $\alpha=0.05$. The probability of making at least one Type I error during the stepwise analysis is

$P(\text{At least one Type I error}) = 1 - P(\text{No Type I errors})$

$$= 1-\binom{n}{0}\alpha^0(1-\alpha)^{n-0} = 1-\binom{14}{0}0.05^0(1-0.05)^{14} = 0.5123$$

6.4 a. There are eight different one-variable models that can be fit to the data.

b. $t=\dfrac{\hat{\beta}_1}{s_{\hat{\beta}_1}}$ is larger than any of the other t-values.

c. There would be an additional seven two-variable combinations that could be fitted to the model once x_1 is fitted.

d. $\hat{\beta}_1=-0.28$. The difference in the mean relative error in estimating effort between developers and project leaders is estimated to be -0.28.
$\hat{\beta}_2=0.27$. The difference in the mean relative error in estimating effort between previous accuracy of more than 20% and previous accuracy less than 20% is estimated to be 0.27.

e. One should be wary of using the model in part d as the final model because it contains only indicator variables and only first-order terms.

6.5 a. There would be 11 models fit for first step in the stepwise regression.

b. There would be 10 possible two variable models.

c. There would only be one model possible at step 11.

d. $E(y)=\beta_0+\beta_1x_{11}+\beta_2x_4+\beta_3x_2+\beta_4x_7+\beta_5x_{10}+\beta_6x_1+\beta_7x_9+\beta_8x_3$

e. $R^2=0.677$. 67.7% of sample variation in overall satisfaction is explained by the model containing variables x_{11}, x_4, x_2, x_7, x_{10}, x_1, x_9, and x_3.

f. No interaction or higher-order terms were tested.

6.6 a. If there is enough data, use stepwise regression since there are so many variables possible to predict the tensile yield strength.

b. There were 9 variables selected. Therefore, 10 steps were performed. There were $11+10+9+8+7+6+5+4+3+2=65$ first order models that were fit in the stepwise routine.

c. No. There is a high probability of at least one Type I error in the stepwise procedure. They should use these variables to build a model, possibly with higher-order terms.

6.7 a. No. The best set of independent variables may not include x_4, x_5, and x_6. Once these variables are in the model, the best set may not be discovered. Also, there could be significant interaction or second-order effects.

b. The model would be $E(y)=\beta_0+\beta_1x_4+\beta_2x_5+\beta_3x_6+\beta_4x_4x_5+\beta_5x_4x_6+\beta_6x_5x_6$.

c. Use a nested F-test of $H_0:\beta_4=\beta_5=\beta_6=0$

d. The biologist could consider interaction and higher-order terms, and investigate whether there are additional variables that could affect the amount of marine life. In addition, the biologist could look at all possible models rather than using stepwise regression.

6.8 a. Using MINITAB, with alpha-to-enter 0.05 and alpha-to-remove 0.05, the results are:

Regression Analysis: Y versus X1, X2, X3, X4, X5, X6, X7

Stepwise Selection of Terms

Candidate terms: X1, X2, X3, X4, X5, X6, X7

	-----Step 1-----		-----Step 2-----		-----Step 3-----	
	Coef	P	Coef	P	Coef	P
Constant	88.19		75.97		77.73	
X5	0.04914	0.000	0.04954	0.000	0.05827	0.000
X2			0.1318	0.007	0.1363	0.004
X4					-0.0347	0.049
S		12.7007		11.9018		11.5428
R-sq		34.49%		43.63%		48.06%
R-sq(adj)		33.18%		41.33%		44.81%
R-sq(pred)		30.01%		37.75%		41.39%
Mallows' Cp		18.78		11.47		8.95

α to enter = 0.05, α to remove = 0.05

The least squares prediction equation is: $\hat{y}=77.73+0.05827x_5+0.1363x_2-0.0347x_4$.

b. $\hat{\beta}_0=77.73$. This is simply the y-intercept 0 is not in the observed ranges for x_5, x_2, and x_4.

$\hat{\beta}_1=0.05827$. We estimate the mean number of hours worked per day by the clerical staff to increase by 0.058 hours for every one additional check cashed, holding the number of money orders or gift certificates sold and the number of change order transactions processed constant.

$\hat{\beta}_2 = 0.1363$. We estimate the number of hours worked per day by the clerical staff to increase by 0.136 hours for every one additional money order or gift certificate sold, holding the number of change order transactions processed and the number of checks cashed constant.

$\hat{\beta}3 = -0.035$. We estimate the number of hours worked per day by the clerical staff to decrease by 0.035 hours for every one additional change order transaction processed, holding the number of money orders or gift certificates sold and the number of checks cashed constant.

c. This model was selected as the best model after many different models were fit to the data. There are two main dangers associated with using and interpreting this model:
1. The overall probability with making at least one Type I error is extremely high.
2. This model only includes linear components. Before using this model, the statistician should consider both quadratic and interaction relationships in the model.

6.9 a. There are (i) $\binom{4}{1} = 4$ models with 1 variable

(ii) $\binom{4}{2} = 6$ models with 2 variables

(iii) $\binom{4}{3} = 4$ models with 3 variables

and (iv) $\binom{4}{4} = 1$ model with 4 variables

b. Using MINITAB, the results are:

Best Subsets Regression: Y versus X1, X2, X3, X4

Response is Y

Vars	R-Sq	R-Sq (adj)	PRESS	R-Sq (pred)	Mallows Cp	S	X 1	X 2	X 3	X 4
1	21.3	19.7	10507.4	14.7	2.5	13.921			X	
1	8.6	6.7	12053.3	2.1	10.7	15.005		X		
1	0.7	0.0	13341.0	0.0	15.8	15.636				X
1	0.0	0.0	13141.7	0.0	16.2	15.692	X			
2	24.7	21.7	10494.2	14.8	2.3	13.752		X	X	
2	23.7	20.6	10461.0	15.0	3.0	13.848			X	X
2	21.4	18.2	10759.8	12.6	4.5	14.053	X		X	
2	9.1	5.4	12503.7	0.0	12.4	15.111		X		X
3	26.6	22.0	10489.3	14.8	3.1	13.719		X	X	X
3	24.8	20.1	10759.0	12.6	4.3	13.885	X	X	X	
3	23.9	19.1	10688.6	13.2	4.9	13.972	X		X	X
3	9.1	3.4	12950.5	0.0	14.4	15.267	X	X		X
4	26.8	20.6	10710.4	13.0	5.0	13.846	X	X	X	X

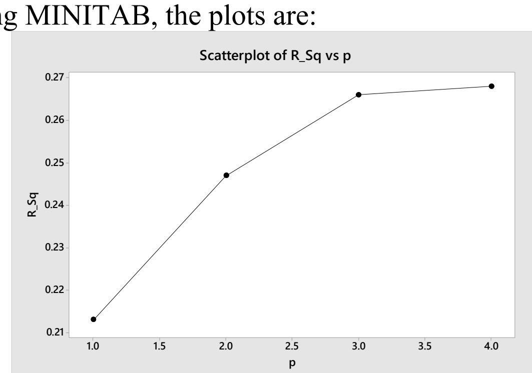
(i) for the 1 variable model, the maximum $R^2 = 0.213$, minimum MSE$= s^2 = 13.921^2 = 193.8$, the minimum $C_p = 2.5$, and the minimum PRESS$= 10{,}507.4$.

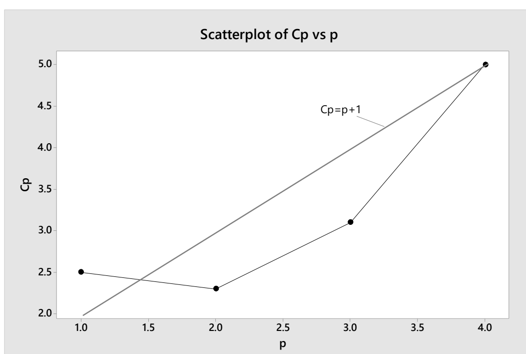
(ii) for the 2 variable model, the maximum $R^2 = 0.247$, minimum MSE$= s^2 = 13.752^2 = 189.1$, the minimum $C_p = 2.3$, and the minimum PRESS$= 10461.0$.

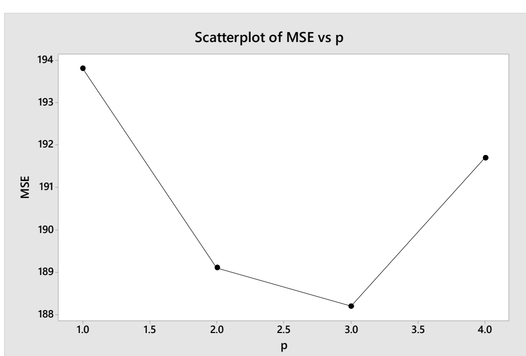
(iii) for the 3 variable model, the maximum $R^2 = 0.266$, minimum MSE$= s^2 = 13.719^2 = 188.2$, the minimum $C_p = 3.1$, and the minimum PRESS$= 10489.3$.

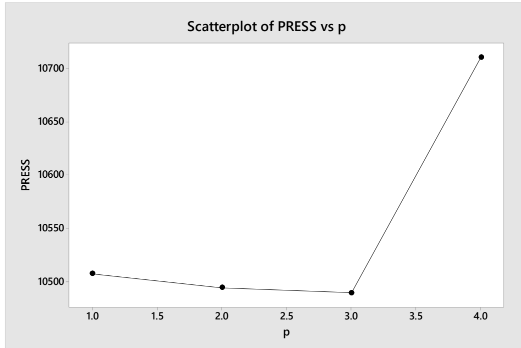
(iv) for the 4 variable model, $R^2 = 0.268$, MSE$= s^2 = 13.846^2 = 191.7$, $C_p = 5.0$, and PRESS$= 10710.4$.

c. Using MINITAB, the plots are:









d. The best variables to use are x_2, x_3, x_4.

6.10 a. Using MINITAB, the Stepwise, Stepwise analysis is:

Regression Analysis: HEATRATE versus SHAFTS, RPM, ... OWER, ENG1, ENG2
Stepwise Selection of Terms

Candidate terms: SHAFTS, RPM, CPRATIO, INLET-TEMP, EXH-TEMP, AIRFLOW, POWER, ENG1, ENG2

	-----Step 1----		-----Step 2----		-----Step 3----		-----Step 4----	
	Coef	P	Coef	P	Coef	P	Coef	P
Constant	9470		16523		14360		13618	
RPM	0.1917	0.000	0.1312	0.000	0.1051	0.000	0.0888	0.000
INLET-TEMP			-5.577	0.000	-9.223	0.000	-9.186	0.000
EXH-TEMP					12.43	0.000	14.36	0.000
AIRFLOW							-0.848	0.057
ENG2								
S		862.007		578.322		464.980		455.114
R-sq		71.23%		87.25%		91.89%		92.35%
R-sq(adj)		70.79%		86.85%		91.50%		91.86%
R-sq(pred)		69.63%		85.89%		90.70%		90.78%
Mallows' Cp		165.09		40.08		5.32		3.65

	-----Step 5----	
	Coef	P
Constant	15004	
RPM	0.0893	0.000
INLET-TEMP	-9.965	0.000
EXH-TEMP	13.26	0.000
AIRFLOW	-0.860	0.050
ENG2	384	0.078
S		447.232
R-sq		92.73%
R-sq(adj)		92.14%
R-sq(pred)		91.02%
Mallows' Cp		2.62

α to enter = 0.15, α to remove = 0.15

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	5	155696225	31139245	155.68	0.000
Error	61	12200983	200016		
Total	66	167897208			

Model Summary

S	R-sq	R-sq(adj)
447.232	92.73%	92.14%

The best predictors are: RPM, INLET-TEMP, and EXH-TEMP.

b. Using MINITAB, the Backward, Stepwise analysis is:

Regression Analysis: HEATRATE versus SHAFTS, RPM, ... OWER, ENG1, ENG2
Backward Elimination of Terms

Candidate terms: SHAFTS, RPM, CPRATIO, INLET-TEMP, EXH-TEMP, AIRFLOW, POWER, ENG1, ENG2

	-----Step 1-----		-----Step 2-----		-----Step 3----		-----Step 4----	
	Coef	P	Coef	P	Coef	P	Coef	P
Constant	15450		15284		15147		15164	
SHAFTS	-130	0.568	-99	0.546	-100	0.536	-69	0.643
RPM	0.0866	0.000	0.0877	0.000	0.0906	0.000	0.0914	0.000
CPRATIO	13.0	0.704	13.3	0.693	16.6	0.603		
INLET-TEMP	-10.47	0.000	-10.48	0.000	-10.46	0.000	-9.781	0.000
EXH-TEMP	13.66	0.000	13.78	0.000	13.84	0.000	12.68	0.000
AIRFLOW	-1.55	0.424	-1.48	0.434	-0.884	0.048	-0.861	0.051
POWER	0.00163	0.727	0.00148	0.746				
ENG1	-63	0.841						
ENG2	298	0.447	360	0.142	391	0.082	374	0.090
S		460.169		456.348		452.877		450.129
R-sq		92.81%		92.81%		92.79%		92.76%
R-sq(adj)		91.68%		91.81%		91.94%		92.04%
R-sq(pred)		89.16%		89.63%		90.05%		90.61%
Mallows' Cp		10.00		8.04		6.14		4.41

	-----Step 5----	
	Coef	P
Constant	15004	
SHAFTS		
RPM	0.0893	0.000
CPRATIO		
INLET-TEMP	-9.965	0.000
EXH-TEMP	13.26	0.000
AIRFLOW	-0.860	0.050
POWER		
ENG1		
ENG2	384	0.078
S		447.232
R-sq		92.73%
R-sq(adj)		92.14%
R-sq(pred)		91.02%

Mallows' Cp 2.62
α to remove = 0.1

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	5	155696225	31139245	155.68	0.000
Error	61	12200983	200016		
Total	66	167897208			

Model Summary

S	R-sq	R-sq(adj)
447.232	92.73%	92.14%

The best predictors are: RPM, INLET-TEMP, and EXH-TEMP.

c. Using best subset selection, the analysis is:

Best Subsets Regression: HEATRATE versus SHAFTS, ... , POWER, ENG1, ENG2
Response is HEATRATE

Vars	R-Sq	R-Sq (adj)	R-Sq (pred)	Mallows Cp	S	SHAFTS	RPM	CPRATIO	INLET-TEMP	EXH-TEMP	AIRFLOW	POWER	ENG 1	ENG 2
1	71.2	70.8	69.6	165.1	862.01		X							
1	64.1	63.5	61.7	221.7	963.13				X					
2	87.3	86.9	85.9	40.1	578.32		X		X					
2	84.8	84.3	80.6	59.7	631.88		X	X						
3	91.9	91.5	90.7	5.3	464.98		X		X	X				
3	90.2	89.7	88.4	19.1	512.27		X	X	X					
4	92.4	91.9	90.8	3.6	455.11		X		X	X	X			
4	92.3	91.8	90.8	4.4	457.95		X		X	X				X
5	92.7	92.1	91.0	2.6	447.23		X		X	X	X			X
5	92.7	92.1	90.9	3.2	449.57		X		X	X		X		X
6	92.8	92.0	90.6	4.4	450.13	X	X		X	X	X			X
6	92.8	92.0	90.8	4.4	450.25		X		X	X	X	X		X
7	92.8	91.9	90.0	6.1	452.88	X	X	X	X	X	X			X
7	92.8	91.9	90.4	6.2	453.08	X	X		X	X	X	X		X
8	92.8	91.8	89.6	8.0	456.35	X	X	X	X	X	X	X		X
8	92.8	91.8	89.6	8.1	456.68	X	X	X	X	X	X		X	X
9	92.8	91.7	89.2	10.0	460.17	X	X	X	X	X	X	X	X	X

The best predictors would be RPM, INLET-TEMP, EXH-TEMP, and AIRFLOW.

d. RPM, INLET-TEMP, and EXH-TEMP are consistently selected as the best predictors. AIRFLOW is marginal, but should be considered.

e. We would use RPM, INLET-TEMP, EXH-TEMP, and possibly AIRFLOW as independent variables for predicting HEATRATE in a multiple regression model, then checking to see if including interaction and higher-order terms would improve the model.

6.11 Using MINITAB, the best subset analysis is:

Best Subsets Regression: MTBE versus WC, AQ, pH, ... Distance, IndPct
Response is MTBE
191 cases used, 32 cases contain missing values

Vars	R-Sq	R-Sq (adj)	R-Sq (pred)	Mallows Cp	S	WC	AQ	pH	Depth	DissOxy	Distance	IndPct
1	20.1	19.7	7.0	0.8	4.9326							X
1	2.4	1.9	0.3	42.4	5.4514				X			
2	21.0	20.2	6.4	0.6	4.9165				X			X
2	20.6	19.8	7.3	1.5	4.9294		X					X
3	21.4	20.1	6.1	1.7	4.9183				X		X	X
3	21.3	20.1	3.9	1.8	4.9194				X	X		X
4	21.7	20.0	3.4	3.1	4.9228				X	X	X	X
4	21.6	19.9	3.8	3.1	4.9237		X		X	X		X
5	21.9	19.8	3.3	4.4	4.9274		X		X	X	X	X
5	21.7	19.6	3.0	4.9	4.9343			X	X	X	X	X
6	22.1	19.5	2.3	6.1	4.9368	X	X		X	X	X	X
6	22.0	19.5	2.9	6.3	4.9387		X	X	X	X	X	X
7	22.1	19.1	1.9	8.0	4.9485	X	X	X	X	X	X	X

Based on the analysis above, the best set of predictor variables would include the percentage of adjacent land allocated to industry and well depth. The model including these 2 variables has the smallest S, and largest adjusted R-Sq.

6.12 No, one would not want to use a stepwise procedure because there are only 2 independent variables. The first step would be to fit a first order model with album publicity and death status as the 2 independent variables. After this model is fit, one would add the interaction term between the 2 independent variables. The next model would include the second-order term for album publicity and the interaction between the second-order term and death status. One would then pick the best model among those fit. If none of the model is a good fit, one would have to look at adding additional independent variables to the model.

Chapter

7

Some Regression Pitfalls

7.1 We only have information about how the independent variables are related to the dependent variable within the range of observed values. We have no idea what the relationship looks like outside the observed range.

7.2 a. Multicollinearity first increases the likelihood of rounding errors in our calculations. Second, the results themselves may not seem to make sense (i.e., a significant F test for model utility but no significant t tests among the individual parameter estimates). A third problem is that a high correlation between variables may affect the sign of the parameter estimates, a positive slope is obtained when a negative relationship was expected.

b. One way to detect multicollinearity is to test for pairwise correlations among the independent variables. Another way to detect multicollinearity is to look for confusing results in the regression analysis. One could also look at the variance inflation factors (VIF). VIF exceeding 10 indicate multicollinearity

c. When multicollinearity is present, one way of dealing with the problem is to remove all but one of the highly correlated variables, possibly by using stepwise regression. When using the model for estimation and prediction only, it is not necessary to drop any independent variables. It is necessary, though, to make sure the values of the x variables fall within the ranges of x used in the experiment. The presence of multicollinearity when making inferences about the β's in the model is a dangerous problem. The solution is to use a designed experiment to break up the patterns of multicollinearity. To reduce rounding errors in polynomial models, it is useful to code the x values so that the correlation between the subsequent powers of x is reduced. Another way of reducing rounding errors due to multicollinearity is to use ridge regression which provides biased estimates of the β's in the model, but these estimates have smaller standard errors than their least squares analogs.

7.3 Another transformation that might work in Example 7.8 would be to use $x = p^2$, $\ln(p)$, or e^{-p}.

7.4 a. This is a designed experiment because the patients were divided into two groups, with one group receiving the opioids and the other group receiving non-opioids.

b. Let $x = \begin{cases} 1 & \text{if opioid group} \\ 0 & \text{if non-opioid group} \end{cases}$. The model would be $E(x) = \beta_0 + \beta_1 x$.

c. $\hat{\beta}_0 = \mu_{\text{non-opioid}} = 3.5$ and $\hat{\beta}_1 = \mu_{\text{opioid}} - \mu_{\text{non-opioid}} = 4.0 - 3.5 = 0.5$.

d. The study was a designed experiment. The results indicated that there was a difference in the mean pain levels between the two groups. Since the sample mean pain level for the opioid group was greater than the sample mean pain level for the non-opioid, it can be inferred that "treatment with opioids was not superior to treatment with non-opioid medication for improving pain."

7.5 a. This is an observational study. None of the subjects were randomly assigned to a group or a particular treatment.

b. No, the researchers' conclusions should not be considered causal in nature because this was not a designed experiment.

7.6 a. Even though $r = 0.983$ is rather large, you must be cautious about deducing a cause-and-effect between the number of females in managerial positions and the number of females with college degrees. The independent variables could not be controlled, but there could be a strong relationship between these two variables.

b. There could be a problem with multicollinearity because several variables are highly correlated with each other.

7.7 a. Based on the correlation matrix, there is no evidence of extreme multicollinearity. None of the pairwise correlations are larger than 0.5 in absolute value. Thus, it does not appear that multicollinearity is a big problem in this exercise.

b. No, there is no evidence of extreme multicollinearity. The global test is significant $(F = 32.47,\ p < 0.001)$ indicating that at least one of the variables contributes to the prediction of urban/rural rating. However, 3 of the 6 independent variables, when tested for significance in predicting urban/rural rating, are significant. Multicollinearity is present if the global test is significant, but none (or very few) of the individual variable tests are significant.

7.8 The "experimental region", which is defined in the text as the range of values of the independent variables in the sample data. Ultimately, we do not want to predict the dependent variable for values of the independent variables outside the experimental region (the extrapolation problem). Since there are two independent variables (depth and ice type), we need to find the "joint" range. Using Minitab, we get the output below:

Descriptive Statistics: depth

Statistics

Variable	icetype	N	Mean	StDev	Minimum	Maximum
depth	First-Year	88	0.13080	0.08026	0.02000	0.36000
	Landfast	196	0.3116	0.1920	0.0000	0.8600
	Multi-Year	220	0.27527	0.12646	0.07000	0.64000

Thus, we should not predict albedo for depth of first-year ice less than 0.02 or greater than 0.36, for depth of landfast ice less than 0.00 or greater than 0.86, and for depth of multi-year ice less than 0.07 or greater than 0.64.

7.9 a. There is no evidence of extreme multicollinearity since aggressive behavior is correlated with all three independent variables and the independent variables are not highly correlated with each other.

b. There could be a problem with multicollinearity for this example. Narcissism is only somewhat correlated with irritability and trait anger, and moderately correlated with aggressive behavior. However, two independent variables are highly correlated with each other – aggressive behavior and irritability $(r = 0.77)$. If these two variables are included in the model, a multicollinearity problem could occur.

7.10 Based on these results, we would say that multicollinearity is a severe problem in this analysis. The model predicts the amount of carbohydrate solubilized during steam processing of peat (y) very well $\left(R^2 = 0.93\right)$. We disagree with the bioengineer's statement. Note that since the given correlations between pairs of independent variables of the model in this problem are large, these indicate that multicollinearity is probably present. If we test each individual independent variable, none would be significant:

$$t_1 = \frac{\hat{\beta}_1}{s_{\hat{\beta}_1}} = \frac{3.2}{2.4} = 1.33 \qquad t_2 = \frac{\hat{\beta}_2}{s_{\hat{\beta}_2}} = \frac{-0.4}{0.6} = -0.67 \qquad t_3 = \frac{\hat{\beta}_3}{s_{\hat{\beta}_3}} = \frac{-1.1}{0.8} = -1.375$$

The rejection region requires $\alpha / 2 = 0.05 / 2 = 0.025$ in each tail of the t distribution with $df = n - (k+1) = 15 - (3+1) = 11$. From Table 2, Appendix D, $t_{0.025} = 2.201$. The rejection region is $t < -2.201$ or $t > 2.201$.

Since none of the observed value of the test statistics fall in the rejection region, no null hypotheses will be rejected. There is insufficient evidence to indicate that any of the independent variables are significant.

The results above, along with the high value of R^2 indicate multicollinearity.

7.11 When fitting the p^{th}-order polynomial regression model, two requirements must be met:

1. The number of levels of x must be greater than or equal to $(p+1)$. For a second-order model $(p = 2)$, we must have at least $p + 1 = 3$ levels of x.
2. The sample size n must be greater than $(p+1)$ in order to allow sufficient degrees of freedom for estimating σ^2. For a second-order model, we must have n greater than $(p+1) = 3$.

Note that for our sample data, requirement #1 is satisfied since $x = 1, 2,$ or 5. However, requirement #2 is not satisfied since $n = 3$. If we attempt to fit this model, we will have $n - (k+ 1) = n - 3 = 0$ df for estimating σ^2 and, thus, will be unable to test model adequacy.

7.12 a. Using MINITAB, the results are:

Regression Analysis: DRESSWT versus LIVEWT

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	6739.6	6739.60	97.75	0.000
Error	7	482.6	68.95		
Total	8	7222.2			

Model Summary

S	R-sq	R-sq(adj)
8.30338	93.32%	92.36%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	5.7	26.3	0.22	0.834
LIVEWT	0.6260	0.0633	9.89	0.000

Regression Equation

DRESSWT = 5.7 + 0.6260 LIVEWT

The fitted model is $\hat{y} = 5.7 + 0.6260x$.

b. Using MINITAB, the results are:

Prediction for DRESSWT

Settings

Variable	Setting
LIVEWT	300

Prediction

Fit	SE Fit	95% CI	95% PI	
193.501	7.69075	(175.316, 211.687)	(166.739, 220.264)	X

X denotes an unusual point relative to predictor levels used to fit the model.

The 95% prediction interval is $(166.739, 220.264)$.

c. No, we would not recommend that the FDA use the interval obtained above to determine whether the dressed weight of 150 pounds is reasonable. The sample data used to find the prediction equation had a range for live weight of 340 to 480. The weight used to predict the dressed weight was a live weight of 300 pounds which is outside the observed range.

7.13 a. Using MINITAB, the results are:

Regression Analysis: y versus x1

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	494.28	494.281	253.37	0.000
Error	23	44.87	1.951		
Total	24	539.15			

Model Summary

S	R-sq	R-sq(adj)
1.39672	91.68%	91.32%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	2.743	0.675	4.06	0.000
x1	0.8010	0.0503	15.92	0.000

Regression Equation

y = 2.743 + 0.8010 x1

The least squares equation is $\hat{y} = 2.743 + 0.8010x_1$.
The test statistic is $t = 15.92$ and the p-value $p = 0.000$. Since the p-value is so small, there is evidence that tar content is a useful predictor of carbon monoxide content.

b. Using MINITAB, the results are:

Regression Analysis: y versus x2

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	462.26	462.256	138.27	0.000
Error	23	76.89	3.343		
Total	24	539.15			

Model Summary

S	R-sq	R-sq(adj)
1.82845	85.74%	85.12%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	1.665	0.994	1.68	0.107
x2	12.40	1.05	11.76	0.000

Regression Equation

y = 1.665 + 12.40 x2

The least squares equation is $\hat{y} = 1.665 + 12.40x_2$.
The test statistic is $t = 11.76$ and the p-value $p = 0.000$. Since the p-value is so small, there is evidence that nicotine content is a useful predictor of carbon monoxide content.

c. Using MINITAB, the results are:

Regression Analysis: y versus x3

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	116.1	116.06	6.31	0.019
Error	23	423.1	18.40		
Total	24	539.2			

Model Summary

S	R-sq	R-sq(adj)
4.28898	21.53%	18.11%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	-11.80	9.72	-1.21	0.237
x3	25.07	9.98	2.51	0.019

Regression Equation

y = -11.80 + 25.07 x3

The least squares equation is $\hat{y} = -11.80 + 25.07x_3$.
The test statistic is $t = 2.51$ and the p-value $p = 0.019$. Since the p-value is so small, there is evidence that weight is a useful predictor of carbon monoxide content.

d. In Example 7.5, the coefficient for tar content is 0.96257, the coefficient for nicotine is -2.63166, and the coefficient for weight is -0.13048. We see the estimated signs for nicotine and weight have changed from the individual t-tests compared to the multiple regression analysis. This indicates a serious multicollinearity problem.

7.14 We would not recommend that the least squares prediction equation be used to predict per capita consumption of motor fuel in 2020 because the year 2020 is outside the observed range of years used to compute the least squares equation. We do not know what the relationship between year and per capita consumption is like outside the observed range.

7.15 a. There may be correlation between the relative error in estimating effort and the company role of estimator which could make the sign of $\hat{\beta}_1$ in the model have an opposite sign than what is expected.

b. If there is no data collected for project leaders with less than 20% accuracy this forces, then there will only be 3 different combinations of company role of estimator-previous accuracy (developer-more than 20% accurate, developer-less than 20% accurate, and project leader-more than 20% accurate). Interaction is defined as: the effect of one variable on the dependent variable is different for different levels of the second variable. If there is only one level accuracy of project leader (more than 20% accuracy), interaction cannot be estimated. Thus, β_3 is not estimable.

7.16 If Nickel is highly correlated with each of the other ten potential independent variables, then multicollinearity could be a problem. You could drop Nickel altogether, or transform (code) this variable to reduce the multicollinearity.

7.17 First, we calculate pairwise correlations among all of the variables. Using MINITAB, the correlations are:

Correlation: SELFESTM, BODYSAT, IMPREAL, GENDER, DESIRE

Correlations

	SELFESTM	BODYSAT	IMPREAL	GENDER
BODYSAT	0.757			
	0.000			
IMPREAL	0.167	0.143		
	0.030	0.062		
GENDER	0.511	0.828	0.065	
	0.000	0.000	0.398	
DESIRE	-0.485	-0.644	0.132	-0.637
	0.000	0.000	0.087	0.000

Cell Contents
Pearson correlation
P-Value

The dependent variable, DESIRE, has a fairly high correlation with all independent variables except IMPREAL with all correlations being negative except the one with IMPREAL. The correlation between SELFEST and BODYSAT is 0.757, which could indicate multicollinearity. Looking at the results of the analysis from Exercise 4.12, we get:

Regression Analysis: DESIRE versus GENDER, SELFESTM, ... T, IMPREAL

Analysis of Variance

Source	DF	Seq SS	Contribution	Adj SS	Adj MS	F-Value	P-Value
Regression	4	827.83	49.76%	827.833	206.958	40.85	0.000
GENDER	1	674.64	40.55%	52.904	52.904	10.44	0.001
SELFESTM	1	57.66	3.47%	8.650	8.650	1.71	0.193
BODYSAT	1	19.62	1.18%	25.569	25.569	5.05	0.026
IMPREAL	1	75.91	4.56%	75.907	75.907	14.98	0.000
Error	165	835.95	50.24%	835.955	5.066		
Total	169	1663.79	100.00%				

Model Summary

S	R-sq	R-sq(adj)	PRESS	R-sq(pred)
2.25087	49.76%	48.54%	885.289	46.79%

Coefficients

Term	Coef	SE Coef	95% CI	T-Value	P-Value	VIF
Constant	14.011	0.775	(12.480, 15.542)	18.07	0.000	
GENDER	-2.186	0.677	(-3.522, -0.851)	-3.23	0.001	3.56
SELFESTM	-0.0479	0.0367	(-0.1204, 0.0245)	-1.31	0.193	2.62
BODYSAT	-0.322	0.143	(-0.606, -0.039)	-2.25	0.026	6.16
IMPREAL	0.493	0.127	(0.242, 0.745)	3.87	0.000	1.03

Regression Equation

DESIRE = 14.011 - 2.186 GENDER - 0.0479 SELFESTM - 0.322 BODYSAT + 0.493 IMPREAL

We see that the overall test for testing for model adequacy indicates that the overall model is adequate for predicting DESIRE $(F = 40.85,\ p = 0.000)$. All signs of the coefficients are what is to be expected. However, the *t*-test for testing if SELFESTM adds to the model is not significant $(p = 0.193)$. Since SELFESTM is highly correlated with BODYSAT, we will rerun the analysis without SELFESTM. The anaylsis is:

Regression Analysis: DESIRE versus GENDER, BODYSAT, IMPREAL

Analysis of Variance

Source	DF	Seq SS	Contribution	Adj SS	Adj MS	F-Value	P-Value
Regression	3	819.18	49.24%	819.18	273.061	53.67	0.000
GENDER	1	674.64	40.55%	44.76	44.763	8.80	0.003
BODYSAT	1	71.52	4.30%	94.36	94.360	18.55	0.000
IMPREAL	1	73.02	4.39%	73.02	73.025	14.35	0.000
Error	166	844.60	50.76%	844.60	5.088		
Total	169	1663.79	100.00%				

Model Summary

S	R-sq	R-sq(adj)	PRESS	R-sq(pred)
2.25566	49.24%	48.32%	883.866	46.88%

Coefficients

Term	Coef	SE Coef	95% CI	T-Value	P-Value	VIF
Constant	13.323	0.570	(12.197, 14.449)	23.35	0.000	
GENDER	-1.911	0.644	(-3.184, -0.639)	-2.97	0.003	3.21
BODYSAT	-0.451	0.105	(-0.658, -0.244)	-4.31	0.000	3.27
IMPREAL	0.483	0.127	(0.231, 0.734)	3.79	0.000	1.03

Regression Equation

DESIRE = 13.323 - 1.911 GENDER - 0.451 BODYSAT + 0.483 IMPREAL

Reviewing the results, we again see that the overall model is adequate for predicting DESIRE $(F = 53.67,\ p = 0.000)$. Signs of the coefficients are to be expected. All remaining terms are significant. If we compare the two models, the estimate of the standard deviation is almost the same when removing SELFESTM $(s_1 = 2.25087,\ s_2 = 2.25566)$. The adjusted R-square is almost the same when removing SELFESTM $\left(R_1^2 = 0.4854,\ R_2^2 = 0.4832\right)$. This indicates that by removing SELFESTM, the model has not changed in its ability to predict DESIRE. Next, we need to see if there are any 2-way interaction terms or second-order terms that contribute to the prediction if DESIRE.

7.18 For the quadratic (second-order) model, $E(y) = \beta_0 + \beta_1 x + \beta_2 x^2$, we have $p = k = 2$ where p is the order of the model and k is the number of β's in the model excluding β_0. To fit the model, we require:

1. At least $p + 1 = 2 + 1 = 3$ different values of x in order to estimate all three β's.
2. Sample size $n > (k + 1) = 3$ in order to have sufficient degrees of freedom for estimating σ^2.

7.19 In order to fit an interaction model, $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2$, we have $p = 2$ and $k = 3$ where p is the order of the model and k is the number of β's in the model excluding β_0. To fit the model, we require:

1. At least $p + 1 = 1 + 1 = 2$ different values of x_1 and at least 2 different values of x_2 in order to estimate all four β's.

2. Sample size $n > (k + 1) = (3 + 1) = 4$ in order to have sufficient degrees of freedom for estimating σ^2.

7.20 For the complete second-order model,

$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 + \beta_4 x_1^2 + \beta_5 x_2^2$$

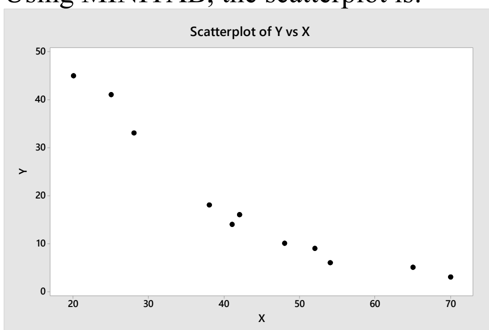
we have $p = 2$ and $k = 5$ where p is the order of the model and k is the number of β's in the model, excluding β_0. To fit the model, we require:

1. At least $p + 1 = 2 + 1 = 3$ different values of x_1 and at least 3 different values of x_2 in order to estimate all five β's.

2. Sample size $n > (k+1) = (5+1) = 6$ in order to have sufficient degrees of freedom for estimating σ^2.

7.21 There are signs of multicollinearity since the VIF values for INLET-TEMP, AIRFLOW, and POWER are greater than 10. In order to modify the model you could drop either POWER or AIRFLOW since their VIF values are very high and redo the analysis. Or you could transform (code) the highly correlated variables to reduce the multicollinearity. Include only one of the three variables.

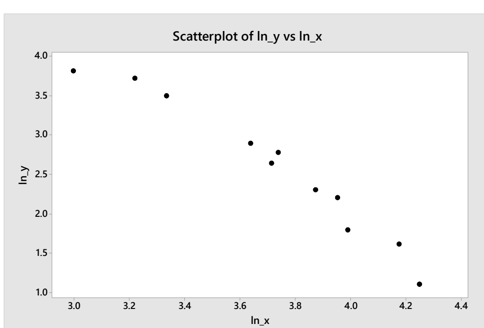
7.22 a. Using MINITAB, the scatterplot is:



There appears to be a curvilinear trend.

b. The ln x and ln y appear in the table. Using MINITAB, the scatterplot of the transformed data is:

x	y	ln x	ln y
54	6	3.989	1.792
42	16	3.738	2.773
28	33	3.332	3.497
38	18	3.638	2.890
25	41	3.219	3.714
70	3	4.248	1.099
48	10	3.871	2.303
41	14	3.714	2.639
20	45	2.996	3.807
52	9	3.951	2.197
65	5	4.174	1.609



The relationship now looks rather linear.

c. Using MINITAB, the results are:

Regression Analysis: ln_y versus ln_x

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	7.3800	7.37999	180.71	0.000
Error	9	0.3675	0.04084		
Total	10	7.7475			

Model Summary

S	R-sq	R-sq(adj)
0.202085	95.26%	94.73%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	10.636	0.603	17.64	0.000
ln_x	-2.170	0.161	-13.44	0.000

Regression Equation

ln_y = 10.636 - 2.170 ln_x

To determine if the model is adequate, we test:

$$H_0: \beta_1 = 0$$
$$H_a: \beta_1 \neq 0$$

The test statistic is $F = 180.71$ and the p-value is $p = 0.000$. Since the p-value is less than $\alpha\left(p = 0.000 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate the model is adequate at $\alpha = 0.05$.

d. When $x = 30$, $\ln x = 3.401$. Then $\widehat{\ln y} = 10.636 - 2.170\left(3.401\right) = 3.256$. Thus, $\hat{y} = e^{3.256} = 25.95$.

7.23 a. Using MINITAB, the correlation between y and x_1 is

Correlation: Y, X1

Correlations

Pearson correlation	0.002
P-value	0.993

The correlation between y and x_1 is $r = 0.002$ and the p-value is $p = 0.993$. Since the p-value is so large, there is no evidence of a linear relationship between y and x_1.

b. Using MINITAB, the correlation between y and x_2 is

Correlation: Y, X2

Correlations

Pearson correlation	0.434
P-value	0.106

The correlation between y and x_2 is $r = 0.434$ and the p-value is $p = 0.106$. Since the p-value is not small, there is no evidence of a linear relationship between y and x_2.

c. Based on the values of the correlation coefficients computed in parts (a) and (b), there is no evidence that the model will be useful for predicting the sale price.

d. Using MINITAB, the results are:

Regression Analysis: Y versus X1, X2

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	2	900.722	450.361	39222.34	0.000
Error	12	0.138	0.011		
Total	14	900.860			

Model Summary

S	R-sq	R-sq(adj)
0.107155	99.98%	99.98%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	-45.154	0.611	-73.85	0.000
X1	3.0970	0.0123	252.31	0.000
X2	1.03186	0.00368	280.08	0.000

Regression Equation

Y = -45.154 + 3.0970 X1 + 1.03186 X2

The least squares line is $\hat{y} = -45.154 + 3.0970x_1 + 1.03186x_2$.

To determine if the model is adequate, we test:

$$H_0: \beta_1 = \beta_2 = 0$$
$$H_a: \text{At least one } \beta_i \neq 0$$

The test statistic is $F = 39{,}222.34$ and the p-value is $p = 0.000$. Since the p-value is so small, H_0 is rejected for any reasonable value of α. There is sufficient evidence to indicate the model is adequate.

$R^2 = 0.9998$. This indicates that 99.98% of the sample variation in the sales process is explained by the model including x_1 and x_2.

e. Using MINITAB, the correlation is:

Correlation: X1, X2

Correlations

Pearson correlation	-0.900
P-value	0.000

Because the correlation between x_1 and x_2 is very close to -1, it implies that x_1 and x_2 are highly correlated.

f. In this case, we would not want to throw out a redundant variable. The models with just one independent variable are not significant. However, the model with both independent variables, even though they are highly correlated, is very significant.

7.24 a. Using MINITAB, the correlations are:

Correlation: MassFlux, HeatFlux, Density
Correlations

	MassFlux	HeatFlux
HeatFlux	-0.000	
	1.000	
Density	-0.180	0.954
	0.476	0.000

Cell Contents
Pearson correlation
P-Value

Yes, there is evidence of multicollinearity because the correlation between density and heat flux is $r = 0.954$. It appears that either density or heat flux should not be included in the model.

b. If the researcher wants to predict bubble diameter when mass flux is 406 $\text{kg/m}^2\text{/s}$ and the heat flux is $1.5\,\mu\text{w/m}^2$, then he should use the model with just mass flux and heat flux.

7.25 Since $df(Error) = 0$, the researcher's sample size is not sufficient to estimate σ^2. Thus, no estimate of the variation in the y values exists and no test of the model adequacy can be made.

Chapter 8

Residual Analysis

8.1 a. Using MINITAB, the results are:

Regression Analysis: Y versus X

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	10.260	10.2601	80.85	0.000
Error	10	1.269	0.1269		
Total	11	11.529			

Model Summary

S	R-sq	R-sq(adj)
0.356244	88.99%	87.89%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	2.588	0.107	24.15	0.000
X	0.5414	0.0602	8.99	0.000

Regression Equation

Y = 2.588 + 0.5414 X

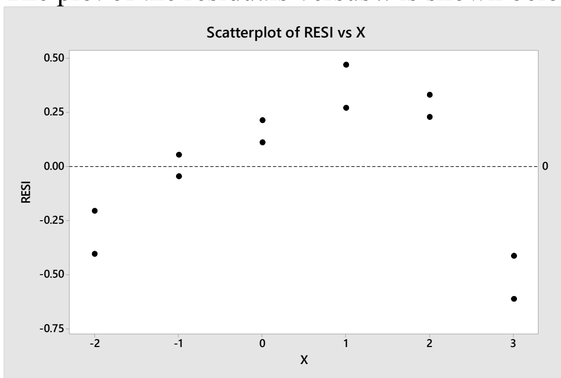
The estimated model is $\hat{y} = 2.588 + 0.5414x$.

b. The predicted values are obtained by substituting the observed values of x into the least squares prediction equation $\hat{y} = 2.588 + 0.5414x$.

The regression residuals are then computed as shown in the table.

x	y	*Predicted value* $\hat{y}$	*Residual* $(y - \hat{y})$
-2	1.1	1.505	-0.405
-2	1.3	1.505	-0.205
-1	2	2.046	-0.046
-1	2.1	2.046	0.054
0	2.7	2.588	0.112
0	2.8	2.588	0.212
1	3.4	3.129	0.271
1	3.6	3.129	0.471
2	4	3.670	0.330
2	3.9	3.670	0.230
3	3.8	4.212	-0.412
3	3.6	4.212	-0.612

c. The plot of the residuals versus x is shown below.



Notice that the plot reveals a quadratic upside down (U-shaped) trend. For small and large values of x, the residuals are negative, while for intermediate values of x, the residuals are positive. This pattern indicates a lack-of-fit in the straight-line model and suggests that a term to introduce curvature in the model is necessary. To verify this, we would fit the quadratic model $E(y) = \beta_0 + \beta_1 x + \beta_2 x^2$ and test for curvature, i.e., test $H_0: \beta_2 = 0$.

8.2 a. Using MINITAB, the results are:

Regression Analysis: Y versus X

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	3303.5	3303.54	191.43	0.000
Error	8	138.1	17.26		
Total	9	3441.6			

Model Summary

S	R-sq	R-sq(adj)
4.15417	95.99%	95.49%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	-3.18	2.75	-1.16	0.281
X	2.491	0.180	13.84	0.000

Regression Equation

Y = -3.18 + 2.491 X

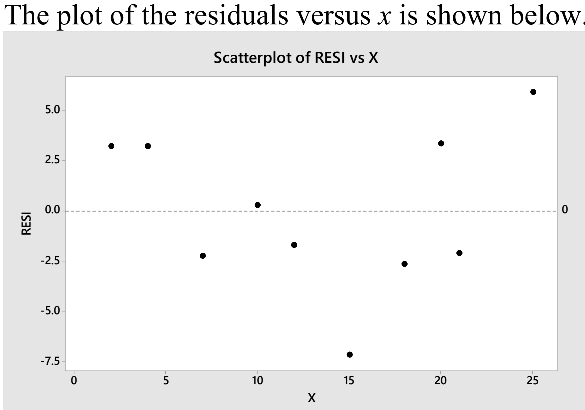
The estimated model is $\hat{y} = -3.18 + 2.491x$.

b. The predicted values are obtained by substituting the observed values of x into the least squares prediction equation $\hat{y} = -3.18 + 2.491x$.

The regression residuals are then computed as shown in the table.

x	y	*Predicted value* $\hat{y}$	*Residual* $(y-\hat{y})$
2	5	1.803	3.197
4	10	6.785	3.215
7	12	14.258	-2.258
10	22	21.731	0.269
12	25	26.713	-1.713
15	27	34.186	-7.186
18	39	41.659	-2.659
20	50	46.640	3.360
21	47	49.131	-2.131
25	65	59.095	5.905

c. The plot of the residuals versus x is shown below.



Notice that the plot reveals a quadratic U-shaped trend. For small and large values of x, the residuals are positive, while for intermediate values of x, the residuals are negative. This pattern indicates a lack-of-fit in the straight-line model and suggests that a term to introduce curvature in the model is necessary. To verify this, we would fit the quadratic model $E(y)=\beta_0+\beta_1 x+\beta_2 x^2$ and test for curvature, i.e., test $H_0:\beta_2=0$.

8.3 a. Using MINITAB, the results are:

Regression Analysis: MILEAGE_Y versus PRESS_X

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	2.403	2.403	0.16	0.699
Error	12	183.434	15.286		
Total	13	185.837			

Model Summary

S	R-sq	R-sq(adj)
3.90976	1.29%	0.00%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	40.4	17.3	2.34	0.038
PRESS_X	-0.207	0.522	-0.40	0.699

Regression Equation

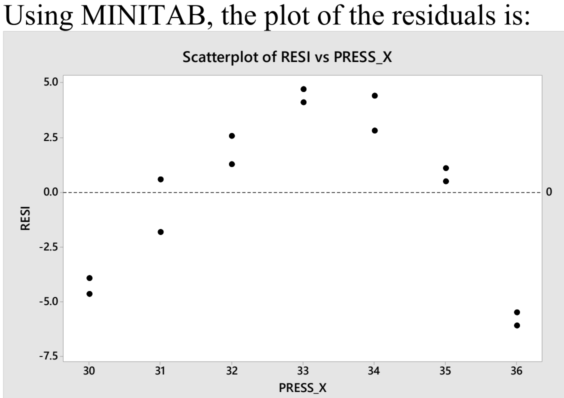
MILEAGE_Y = 40.4 - 0.207 PRESS_X

The least squares prediction equation is $\hat{y} = 40.4 - 0.207x$.

b. The least squares equation above is used to obtain the predicted values and residuals shown below:

x	y	*Predicted value* $\hat{y}$	*Residual* $(y - \hat{y})$
30	29.5	34.136	-4.636
30	30.2	34.136	-3.936
31	32.1	33.929	-1.829
31	34.5	33.929	0.571
32	36.3	33.721	2.579
32	35	33.721	1.279
33	38.2	33.514	4.686
33	37.6	33.514	4.086
34	37.7	33.307	4.393
34	36.1	33.307	2.793
35	33.6	33.100	0.500
35	34.2	33.100	1.100
36	26.8	32.893	-6.093
36	27.4	32.893	-5.493

c. Using MINITAB, the plot of the residuals is:



The residual plot reveals a quadratic upside down (U-shaped) trend. For small and large values of tire pressure, x, the residuals are negative, while for intermediate values of x, the residuals are positive. The quadratic trend suggests the second-order model $E(y) = \beta_0 + \beta_1 x + \beta_2 x^2$, will provide a better fit than the straight line model.

d. Using MINITAB, the results are:

Regression Analysis: MILEAGE_Y versus PRESS_X, X_sq

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	2	172.41	86.204	70.62	0.000
Error	11	13.43	1.221		
Total	13	185.84			

Model Summary

S	R-sq	R-sq(adj)
1.10488	92.77%	91.46%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	-1051.1	92.6	-11.35	0.000
PRESS_X	66.19	5.63	11.76	0.000
X_sq	-1.0060	0.0852	-11.80	0.000

Regression Equation

MILEAGE_Y = -1051.1 + 66.19 PRESS_X - 1.0060 X_sq

The least squares regression equation is $\hat{y} = -1051.1 + 66.19x - 1.0060x^2$.
To determine if the quadratic term has improved model adequacy, test:

$$H_0 : \beta_2 = 0$$
$$H_a : \beta_2 \neq 0$$

The test statistic is $t = -11.80$ and the p-value is $p = 0.000$. Since the p-value is so small, H_0 is rejected for any reasonable value of α. There is sufficient evidence to indicate the quadratic term did improve model adequacy.

8.4 The plots of the residuals against both IC and RTIME appear to be curved. This implies that second-order terms for both IC (IC_sq) and RTIME (RTIME_sq) should be added to the model for a better fit.

8.5 Using MINITAB, the results of fitting the straight-line model are:

Regression Analysis: FAILTIME versus TEMP

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	129663987	129663987	107.32	0.000
Error	20	24163399	1208170		
Total	21	153827386			

Model Summary

S	R-sq	R-sq(adj)
1099.17	84.29%	83.51%

Coefficients

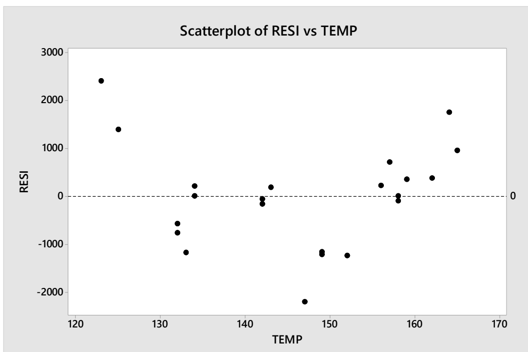
Term	Coef	SE Coef	T-Value	P-Value
Constant	30856	2713	11.37	0.000
TEMP	-191.6	18.5	-10.36	0.000

Regression Equation

FAILTIME = 30856 - 191.6 TEMP

The fitted straight-line model is $\hat{y} = 30{,}856 - 191.6x$.

The plot of the residuals against temperature is:



This plot indicates upward curvature in the relationship between time to failure and solder temperature. In Exercise 4.43c, an upward curvature was detected in the relationship between failure time and solder temperature. These two results agree.

8.6 a. Using MINITAB, the results are:

Regression Analysis: DEMAND versus PRICE

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	1775	1775	0.56	0.471
Error	10	31592	3159		
Total	11	33367			

Model Summary

S	R-sq	R-sq(adj)
56.2066	5.32%	0.00%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	99.8	29.6	3.38	0.007
PRICE	0.0452	0.0603	0.75	0.471

Regression Equation

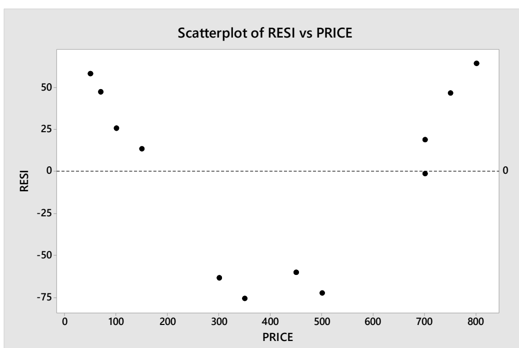
DEMAND = 99.8 + 0.0452 PRICE

The fitted model is $\hat{y} = 99.8 + 0.0452x$.

The residuals are:

x	y	*Predicted value* $\hat{y}$	*Residual* $(y-\hat{y})$
100	130	104.333	25.667
700	150	131.430	18.570
450	60	120.140	-60.140
150	120	106.591	13.409
500	50	122.398	-72.398
800	200	135.947	64.053
70	150	102.978	47.022
50	160	102.075	57.925
300	50	113.366	-63.366
350	40	115.624	-75.624
750	180	133.688	46.312
700	130	131.430	-1.430

b. A plot of the residuals is:



c. There is an U-shape to the plot. This indicates that a second-order term (PRICE-sq) should be added to the model for a better fit.

8.7 a. Using MINITAB, the results are:

Regression Analysis: Man-HRs versus Capacity, Pressure, Boiler, Drum

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	4	230856767	57714192	72.12	0.000
Error	31	24807847	800253		
Total	35	255664615			

Model Summary

S	R-sq	R-sq(adj)
894.569	90.30%	89.04%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	-3784	1205	-3.14	0.004
Capacity	8.749	0.903	9.68	0.000
Pressure	1.927	0.649	2.97	0.006
Boiler	3444	912	3.78	0.001
Drum	2093	306	6.85	0.000

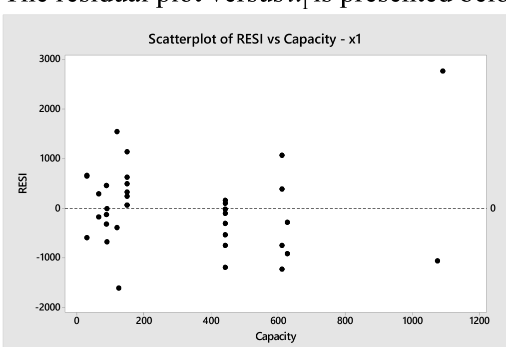
Regression Equation

Man-HRs = -3784 + 8.749 Capacity + 1.927 Pressure + 3444 Boiler + 2093 Drum

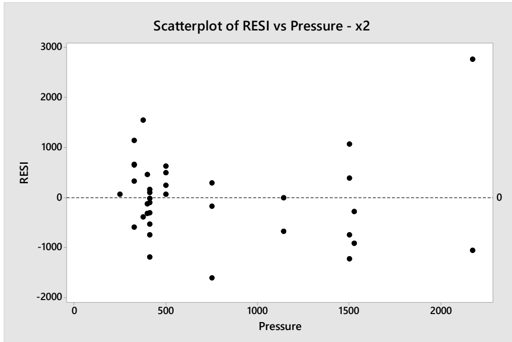
The residuals are:

Residual $(y-\hat{y})$	*Residual* $(y-\hat{y})$	*Residual* $(y-\hat{y})$
-389.45	-751.41	63.61
-177.71	162.59	1135.76
496.24	-15.41	-745.19
-1060.29	-279.46	-9.45
330.40	391.44	656.67
1069.44	238.24	-534.04
453.53	2768.35	-1189.04
-322.47	-1612.67	-103.04
-131.47	1540.91	-303.04
-672.82	290.65	-912.10
-594.69	629.61	-1232.19
97.59	60.25	650.67

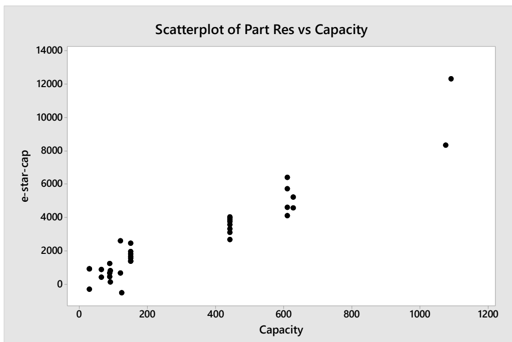
b. The residual plot versus x_1 is presented below; there are no trends:



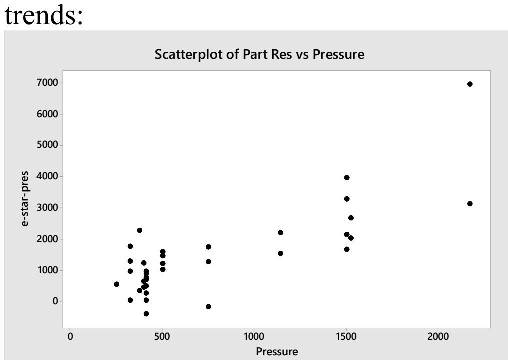
c. The residual plot versus x_2 is presented below; there are no trends:



d. The partial residuals versus x_1 are presented below; there does not appear to be any trends:



e. The partial residuals versus x_2 are presented below; there does not appear to be any trends:



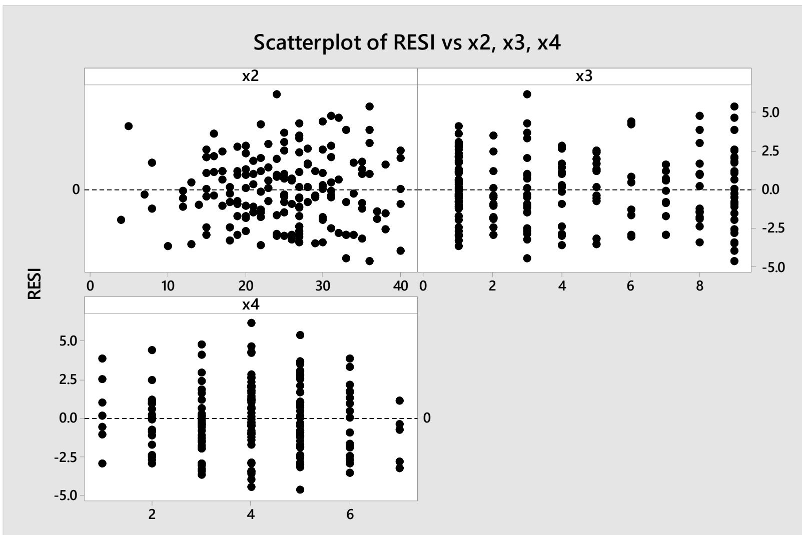
8.8 a. The residuals for fitting the model are:

Residuals $(y-\hat{y})$							
-2.87	-0.43	6.13	0.64	-1.21	-2.95	1.84	3.84
-1.06	1.05	5.34	0.13	-1.72	-0.33	-1.74	-0.92
-2.99	0.79	0.11	-1.04	-1.52	-2.96	0.57	0.65
1.16	3.29	2.02	0.21	-1.15	2.06	-0.83	1.12
1.74	-1.70	1.19	-0.58	1.62	-2.42	-2.46	-1.89
-2.94	-0.45	0.69	0.24	-3.64	-3.46	0.17	-2.84
0.04	4.07	1.04	3.61	1.73	-2.41	1.31	3.46
0.91	-2.72	4.76	-0.27	-1.51	-1.31	-1.22	-1.77
-2.54	-0.76	2.46	-3.51	-0.30	2.65	2.30	-1.11
0.35	1.16	-0.09	1.66	-0.88	3.04	2.53	2.57
-0.39	2.42	-0.53	0.92	-4.66	2.76	3.01	3.84
2.06	-2.57	1.15	-0.54	-1.47	-1.98	-2.95	3.69

-0.96	0.97	-1.87	-0.80	-3.08	4.38	-2.89	-0.94
-3.02	-2.71	-1.79	2.10	1.41	4.19	-1.10	-3.55
1.11	-3.25	-3.98	2.04	1.66	-0.31	-2.43	0.93
-1.44	-2.93	-0.81	2.12	0.09	1.97	2.80	-3.43
-0.51	-1.94	-1.28	4.64	2.79	-0.46	0.47	
0.45	0.99	2.49	-0.75	2.94	-1.96	0.16	
-0.40	1.59	0.99	-0.83	-4.43	-0.15	-2.96	
-0.99	-3.30	4.28	-0.92	-3.19	1.78	0.71	
1.01	0.05	0.56	-1.58	-3.69	0.13	2.54	
-0.61	-1.75	1.63	-0.07	0.04	1.18	1.31	

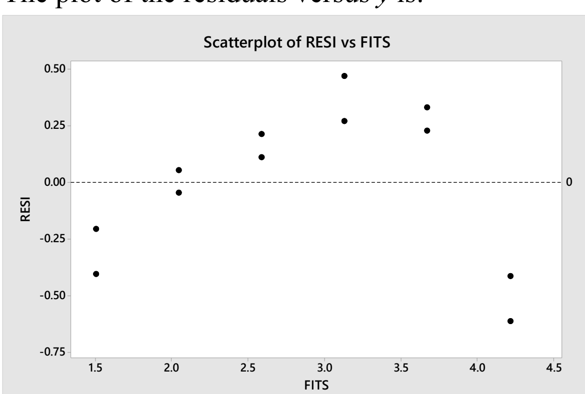
The residuals do sum to 0.

b. Using MINITAB, the residual plots are:



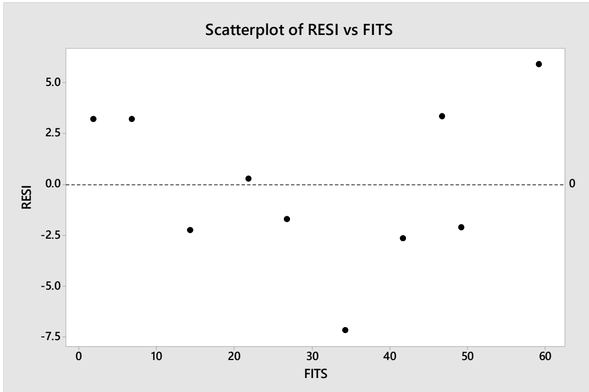
There are no apparent trends to any of the residual plots.

8.9 The plot of the residuals versus $\hat{y}$ is:



The residuals and predicted values are given in the table, Exercise 8.1. The plot reveals the same quadratic trend detected in Exercise 8.1, suggesting that curvature needs to be added to the model. The spread of the residuals, however, does not appear to change as $\hat{y}$ increases. This suggests that the assumption of equal variances is most likely satisfied.

8.10 The plot of the residuals versus $\hat{y}$ is:

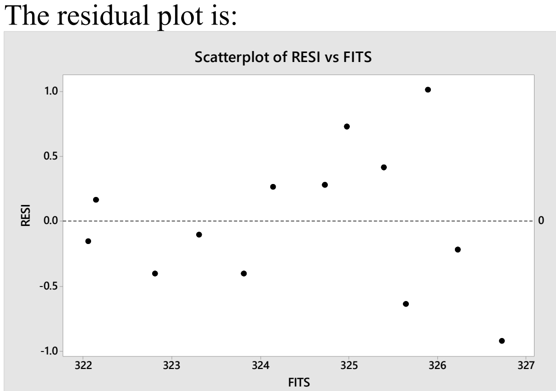


Yes, there appears to be a trend. The assumption of constant variance appears to be satisfied. The pattern suggests adding curvature to the model.

8.11 a. The fitted values and residuals from fitting the first-order model are:

Predicted value $\hat{y}$	*Residual* $(y-\hat{y})$	*Predicted value* $\hat{y}$	*Residual* $(y-\hat{y})$
322.054	-0.154	324.972	0.728
322.137	0.163	325.389	0.411
322.804	-0.404	325.639	-0.639
323.305	-0.105	325.889	1.011
323.805	-0.405	326.223	-0.223
324.138	0.262	326.723	-0.923
324.722	0.278		

The residual plot is:



It appears that as the fitted values increase, the spread of the residual increases. This implies that the assumption of equal variances is not met.

b. Using MINITAB, the results of fitting the first-order model with the heights less than 50 degrees are:

Regression Analysis: Ang1 versus Ht1
Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	7.3990	7.39903	81.26	0.000
Error	5	0.4553	0.09105		
Total	6	7.8543			

Using MINITAB, the results of fitting the first-order model with the heights greater than 50 degrees are:

Regression Analysis: Ang2 versus Ht2
Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	0.08004	0.08004	0.18	0.694
Error	4	1.79330	0.44832		
Total	5	1.87333			

To determine if heteroscedasticity is present, we test:

$$H_0 : \frac{\sigma_2^2}{\sigma_1^2} = 1$$

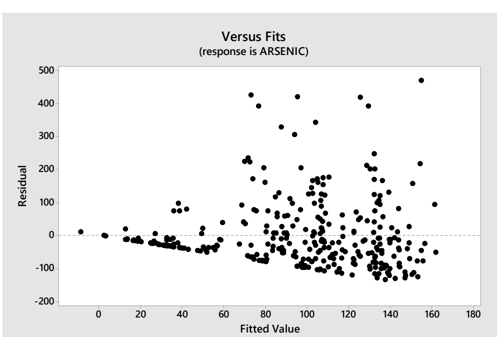
$$H_a : \frac{\sigma_2^2}{\sigma_1^2} \neq 1$$

The test statistic is $F = \frac{MSE_2}{MSE_1} = \frac{0.44832}{0.09105} = 4.924.$

The rejection region requires $\alpha / 2 = 0.05 / 2 = 0.025$ in the upper tail of the F distribution with $\nu_1 = n_2 - 2 = 6 - 2 = 4$ and $\nu_2 = n_1 - 2 = 7 - 2 = 5$. From Table 5, Appendix D, $F_{0.025} = 7.39$. The rejection region is $F > 7.39$.

Since the observed value of the test statistic does not fall in the rejection region $\left(F = 4.924 \not> 7.39\right)$, H_0 is not rejected. There is insufficient evidence to indicate heteroscedasticity is present at $\alpha = 0.05$. This does not support our answer in part a.

8.12 Using MINITAB, the plot of the residuals versus the fits is:



It appears that the spread of the residuals increases as the fitted values increase. This indicates the variances may not be constant.

8.13 Using MINITAB, the results are:

Regression Analysis: HEATRATE versus RPM, CPRATIO, RPM_CPR

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	3	142586570	47528857	118.30	0.000
Error	63	25310639	401756		
Total	66	167897208			

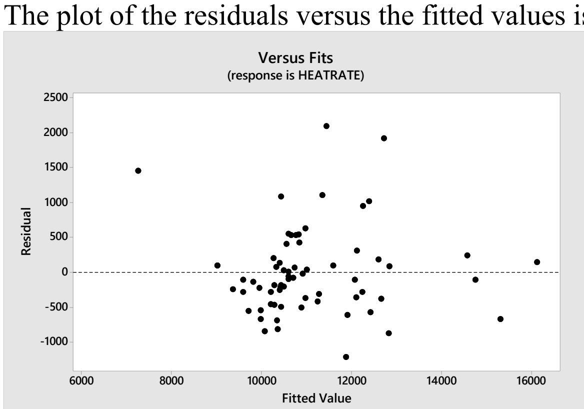
Model Summary

S	R-sq	R-sq(adj)
633.842	84.92%	84.21%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	12065	419	28.83	0.000
RPM	0.1697	0.0347	4.89	0.000
CPRATIO	-146.1	26.7	-5.48	0.000
RPM_CPR	-0.00242	0.00312	-0.78	0.440

The plot of the residuals versus the fitted values is:



There is no indication that the assumption of constant variance is violated.

8.14 a. Using MINITAB, the results are:

Regression Analysis: LEASEFEE versus SIZE, SIZE_SQ

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	2	18096	9047.9	14.35	0.000
Error	17	10719	630.5		
Total	19	28815			

Model Summary

S	R-sq	R-sq(adj)
25.1105	62.80%	58.42%

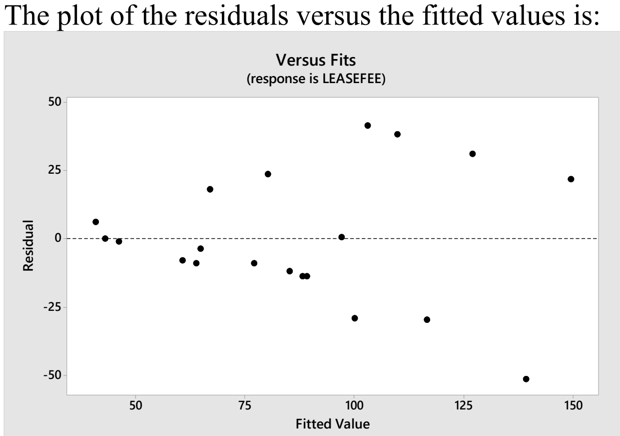
Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	-44.1	88.3	-0.50	0.624
SIZE	11.5	14.2	0.81	0.427
SIZE_SQ	-0.064	0.546	-0.12	0.908

The residuals and fitted values are:

Predicted value $\hat{y}$	Residual $(y-\hat{y})$	Predicted value $\hat{y}$	Residual $(y-\hat{y})$
99.99	-29.29	109.74	38.26
60.75	-8.05	66.92	18.08
139.15	-51.55	149.29	21.91
43.04	0.16	97.04	0.46
80.11	23.69	126.96	31.14
46.20	-1.10	88.12	-13.92
116.49	-29.69	40.94	6.06
85.13	-11.83	63.84	-9.14
102.93	41.37	77.09	-9.09
64.87	-3.57	89.12	-13.92

The plot of the residuals versus the fitted values is:



It appears that the residuals become more spread out as the fitted values increase. This indicates that the assumption of constant variance may be violated. This indicates that the data may be multiplicative in nature.

c. Using MINITAB with SIZE less than or equal to 12, the results are:

Regression Analysis: LEASEFEE1 versus SIZE1, SIZE_SQ1

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	2	2120	1060.1	5.69	0.034
Error	7	1304	186.3		
Total	9	3424			

Using MINITAB with SIZE greater than 12, the results are:

Regression Analysis: LEASEFEE2 versus SIZE2, SIZE_SQ2

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	2	4759	2380	1.82	0.230
Error	7	9135	1305		
Total	9	13894			

To determine if heteroscedasticity is present, we test:

$$H_0: \frac{\sigma_2^2}{\sigma_1^2} = 1$$

$$H_a: \frac{\sigma_2^2}{\sigma_1^2} \neq 1$$

The test statistic is $F = \frac{MSE_2}{MSE_1} = \frac{1305}{186.3} = 7.00.$

The rejection region requires $\alpha / 2 = 0.05/2 = 0.025$ in the upper tail of the F distribution with $\nu_1 = n_2 - 3 = 10 - 3 = 7$ and $\nu_2 = n_1 - 3 = 10 - 3 = 7$. From Table 5, Appendix D, $F_{0.025} = 4.99$. The rejection region is $F > 4.99$.

Since the observed value of the test statistic falls in the rejection region $(F = 7.00 > 4.99)$, H_0 is rejected. There is sufficient evidence to indicate heteroscedasticity is present at $\alpha = 0.05$.

d. The estate should transform the dependent variable and refit the model to correct the constant variance assumption problem.

8.15 a. Yes, the assumption of equal variances appears to be violated.

b. One could use the transformation $y^* = \sqrt{y}$.

8.16 a. To predict the number of breakdowns during the midnight shift when the temperature of the plant is 87° and there is one inexperienced worker, we substitute $x_1 = 0$, $x_2 = 1$, $x_3 = 87$, and $x_4 = 1$ into the prediction equation for the transformed model:

$$\hat{y}^* = 1.3 + 0.008(0) - 0.13(1) + 0.0025(87) + 0.26(1) = 1.6475$$

Therefore, $\hat{y} = (\hat{y}^*)^2 = 1.6475^2 = 2.714.$

b. The transformation $\hat{y} = (\hat{y}^*)^2$ is applied to each endpoint of the prediction interval:

$$(1.965)^2 = 3.861, (2.125)^2 = 4.516$$

The 95% prediction interval for y is $(3.861, 4.516)$.

c. It is not possible to obtain the 95% confidence interval for $E(y)$ from a 95% confidence interval for $E(y^*)$ because the mean of the square root of y is not equal to the square root of y, i.e., $E(\sqrt{y}) \neq \sqrt{E(y)}$. In general, confidence intervals for the mean of a transformed response cannot be transformed back to the original scale.

8.17 a. Using MINITAB, the results are:

Regression Analysis: BUYPROP versus AGE

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	0.915920	0.915920	267.03	0.000
AGE	1	0.915920	0.915920	267.03	0.000
Error	8	0.027440	0.003430		
Lack-of-Fit	3	0.003440	0.001147	0.24	0.866
Pure Error	5	0.024000	0.004800		
Total	9	0.943360			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.0585662	97.09%	96.73%	96.16%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	0.9400	0.0321	29.30	0.000	
AGE	-0.2140	0.0131	-16.34	0.000	1.00

Regression Equation

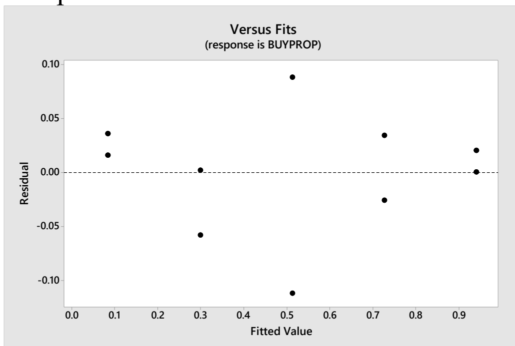
BUYPROP = 0.9400 - 0.2140 AGE

The fitted model is $\hat{y} = 0.9400 - 0.2140x_1$.

b. The residuals and fitted values are:

x	y	*Predicted value* $\hat{y}$	*Residual* $(y-\hat{y})$
0	0.94	0.940	0.000
0	0.96	0.940	0.020
1	0.70	0.726	-0.026
1	0.76	0.726	0.034
2	0.60	0.512	0.088
2	0.40	0.512	-0.112
3	0.24	0.298	-0.058
3	0.30	0.298	0.002
4	0.12	0.084	0.036
4	0.10	0.084	0.016

The plot of the residuals versus the fitted values is:



c. The residual plot reveals a football-shaped pattern (smaller variances for small and large values of $\hat{y}$ and larger variances for intermediate values of $\hat{y}$). This indicates unequal variances.

d. An appropriate variance-stabilizing transformation when the responses are generated by a binomial experiment is $y^* = \sin^{-1}\sqrt{y}$.

e. Using MINITAB, the results of fitting the transformed data are:

Regression Analysis: Arcsin(y) versus AGE

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	1.24646	1.24646	287.88	0.000
Error	8	0.03464	0.00433		
Total	9	1.28110			

Model Summary

S	R-sq	R-sq(adj)
0.0658010	97.30%	96.96%

Coefficients

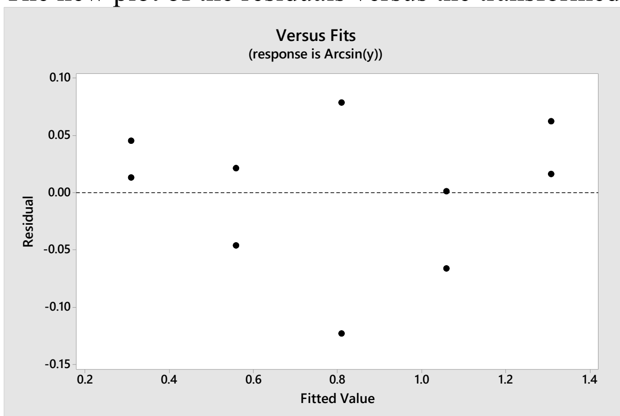
Term	Coef	SE Coef	T-Value	P-Value
Constant	1.3074	0.0360	36.27	0.000
AGE	-0.2496	0.0147	-16.97	0.000

The transformed data, residuals and fitted values are:

x	Arcsin(y)	*Predicted value* $\hat{y}^*$	*Residual* $(y^* - \hat{y}^*)$
0	1.3233	1.3074	0.0160
0	1.3694	1.3074	0.0621
1	0.9912	1.0577	-0.0666
1	1.0588	1.0577	0.0011
2	0.8861	0.8081	0.0780
2	0.6847	0.8081	-0.1233
3	0.5120	0.5584	-0.0464

3	0.5796	0.5584	0.0212
4	0.3537	0.3088	0.0450
4	0.3218	0.3088	0.0130

The new plot of the residuals versus the transformed predicted values is:



The plot still reveals a football-shaped pattern.

8.18 The inferences that are derived from the regression analysis tend to remain valid even when the normality assumptions are violated.

8.19 a. To compute the estimated expected residual for Bangkok under the assumption of normality, with MSE = 0.01, we use:

$$A = \frac{i - 0.375}{n + 0.25} = \frac{126 - 0.375}{126 + 0.25} = 0.9950 \quad \text{where } i = \text{rank of residual and } n = \text{total sample size}$$

$Z(A) = z$-value with A area to the left. For Bangkok, $Z(A) = Z(0.9950) = 2.58$

Thus, for Bangkok, $E(\hat{\varepsilon}_i) = \sqrt{MSE}\left[Z(A)\right] = \sqrt{0.01}(2.58) = 0.258$. The rest of the estimated expected residuals for selected cities are:

City	residual	i	A	Z	E
Bangkok	0.510	126	0.9950	2.58	0.258
Paris	0.228	110	0.8683	1.12	0.112
London	0.033	78	0.6149	0.30	0.030
Warsaw	-0.132	32	0.2505	-0.67	-0.670
Lagos	-0.392	2	0.0129	-2.23	-0.223

b. From the plots in the Exercise, it appears that the assumption of normal errors is satisfied.

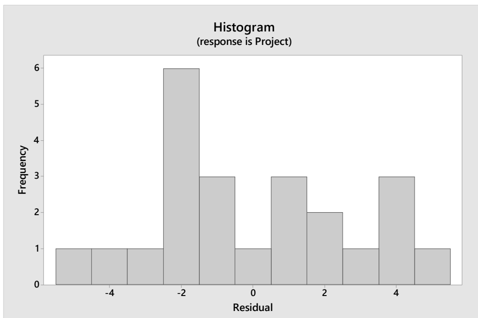
8.20 Using MINITAB, the results are:

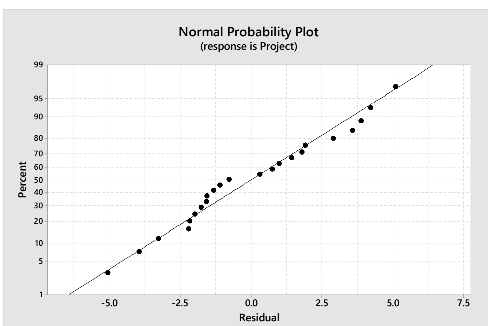
Regression Analysis: Project versus IntraPers, StressMan, Mood

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	3	70.29	23.429	2.66	0.077
Error	19	167.08	8.794		
Total	22	237.37			

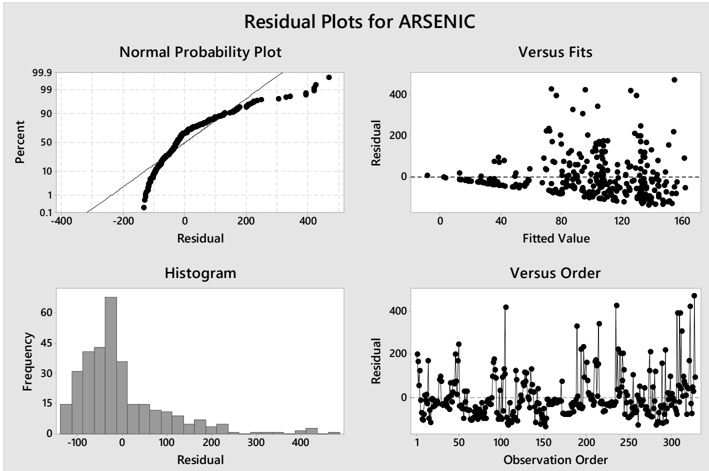
A histogram of the residuals and a normal probability plot are:



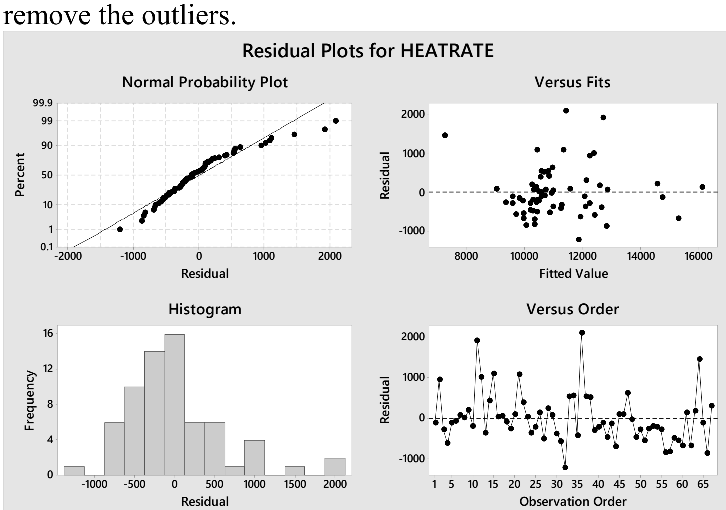


The histogram is fairly mound-shaped and the normal probability plot is very close to a straight line. There is no indication that the assumption of normality is violated.

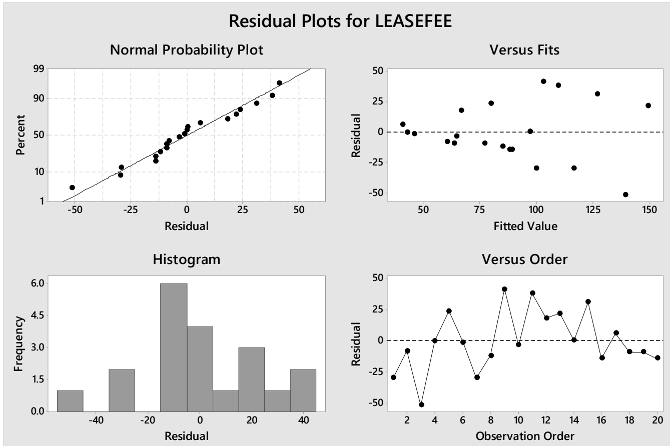
8.21 As seen in the graphs below, the normality assumption does not appear to be satisfied. The normal probability plot in the upper left corner is not a straight line. In addition, the histogram of the residuals in the lower left corner is skewed to the right. Both of these indicate that the data do not meet the assumption of normality. We may have to use a normalizing transformation of the data.



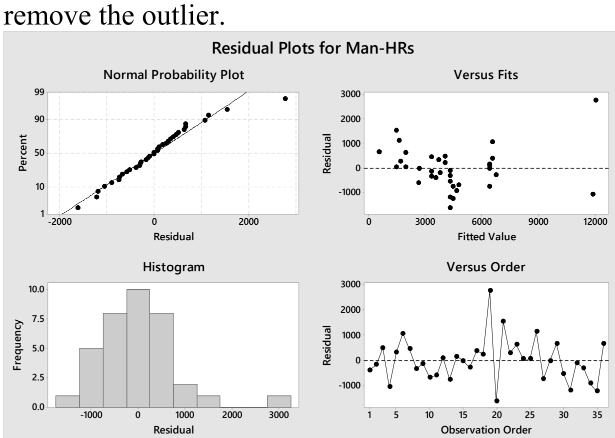
8.22 As seen in the graphs below, the normality assumption does not appear to be satisfied. The normal probability plot in the upper left corner is not a straight line. There appears to be at least one outlier. In addition, the histogram of the residuals in the lower left corner is somewhat skewed to the right. Both of these indicate that the data do not meet the assumption of normality. We may have to use a normalizing transformation of the data or remove the outliers.



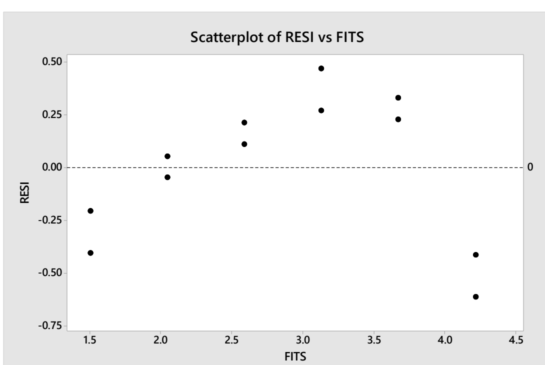
8.23 As seen in the graphs below, the normality assumption appears to be satisfied. The normal probability plot in the upper left corner is close to a straight line. In addition, the histogram of the residuals in the lower left corner is somewhat mound-shaped. Both of these indicate that the data appear to meet the assumption of normality.



8.24 As seen in the graphs below, the normality assumption does not appear to be satisfied. The normal probability plot in the upper left corner is close to a straight line except for one point. There appears to one outlier. In addition, the histogram of the residuals in the lower left corner is somewhat skewed to the right. Both of these indicate that the data may not meet the assumption of normality. We may have to use a normalizing transformation of the data or remove the outlier.



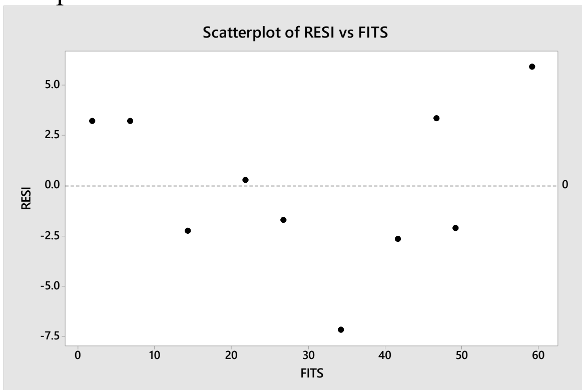
8.25 The plot of the residuals versus the fitted values is:



To look for outliers, we compute: $3s = 3\sqrt{\text{MSE}} = 3\sqrt{0.1267} = 1.07$

Looking down the fourth column (residual) of the table shown in the solution to Exercise 8.1, we see that none of the residuals exceed $3s$, or 1.07, in absolute value. No outliers are present in the data.

8.26 The plot of the residuals versus the fitted values is:



To look for outliers, we compute: $3s = 3\sqrt{\text{MSE}} = 3\sqrt{17.2557} = 12.462$

Looking down the fourth column (residual) of the table shown in the solution to Exercise 8.2, we see that none of the residuals exceed 3*s*, or 12.462, in absolute value. No outliers are present in the data.

8.27 Three methods of identifying influential observations could be:

1. If an observation has a residual larger than 3*s* (in absolute value)
2. The leverage of an observation could be high – greater than $h_i > \frac{2(k+1)}{n}$
3. Cook's distance test: if $D_i > F_{\nu_1 = k+1, [n-(k+1)]}$

The methods will not always detect the same influential observations. For example, leverage statistics will not detect outliers that result from coding errors in the dependent variable.

8.28 An observation is considered to be influential if $h_i > \frac{2(k+1)}{n} = \frac{2(1+1)}{13} = 0.308$. No observations have a leverage point greater than 0.308. Thus, there do not appear to be any influential observations.

8.29 An observation is influential if the Cook's distance falls above the 50th percentile of the *F* distribution with $\nu_1 = k+1 = 4+1 = 5$; $\nu_2 = n-(k+1) = 90-(4+1) = 85$.

$F_{5,85}(0.50) = 0.87725$. Observation #8 has a Cook's Distance of 2.40433 which is greater than 0.87725. Thus, observation #8 is influential. In addition, the zDELRES value for observation #8 is 10.35824. This is extremely large, which again indicates that observation #8 is influential.

8.30 a. Using MINITAB, the residuals and fitted values are:

AGE	NUMBIDS	PRICE	FITS	RESI
127	13	1235	1396.49037	-161.49037
115	12	1080	1157.65049	-77.65049
127	7	845	880.77246	-35.77246
150	9	1522	1345.71163	176.28837
156	6	1047	1164.29612	-117.29612

182	11	1979	1925.31597	53.68403
156	12	1822	1680.01403	141.98597
132	10	1253	1202.33428	50.66572
137	9	1297	1180.08417	116.91583
113	9	946	874.31039	71.68961
137	15	1713	1695.80208	17.19792
117	11	1024	1097.17866	-73.17866
137	8	1147	1094.13119	52.86881
153	6	1092	1126.07440	-34.07440
117	13	1152	1269.08463	-117.08463
126	10	1336	1125.89084	210.10916
170	14	2131	2030.28804	100.71196
182	8	1550	1667.45702	-117.45702
162	11	1884	1670.50449	213.49551
184	10	2041	1864.84414	176.15586
143	6	845	998.66866	-153.66866
159	9	1483	1460.37680	22.62320
108	14	1055	1240.37244	-185.37244
175	8	1545	1578.27300	-33.27300
108	6	729	552.74857	176.25143
179	9	1792	1715.18828	76.81172
111	15	1175	1364.54715	-189.54715
187	8	1593	1731.15989	-138.15989
111	7	785	676.92327	108.07673
115	7	744	727.88557	16.11443
194	5	1356	1562.48496	-206.48496
168	7	1262	1403.13600	-141.13600

b. Using Excel, the mean of the residuals is 0 and the variance of the residuals is16,668.598. This is close to MSE in the output of 17,818. The difference is because the variance of the residuals is calculated with 31 in the denominator, while MSE is calculated with 29 in the denominator.

c. $2s = 2\left(\sqrt{MSE}\right) = 2\left(\sqrt{16{,}668.598}\right) = 258.2$ None of the observations fall outside 2s from 0.

d. The diagnostics are:

PRICE	FITS	RESI	SRES	TRES	HI	COOK	DFIT
1235	1396.4904	-161.4904	-1.2637	-1.2774	0.0835	0.0485	-0.3855
1080	1157.6505	-77.6505	-0.6071	-0.6004	0.0819	0.0110	-0.1793
845	880.7725	-35.7725	-0.2800	-0.2755	0.0836	0.0024	-0.0832
1522	1345.7116	176.2884	1.3430	1.3627	0.0330	0.0205	0.2519
1047	1164.2961	-117.2961	-0.9168	-0.9142	0.0814	0.0248	-0.2722
1979	1925.3160	53.6840	0.4277	0.4216	0.1158	0.0080	0.1526

1822	1680.0140	141.9860	1.1024	1.1067	0.0691	0.0301	0.3015
1253	1202.3343	50.6657	0.3871	0.3813	0.0385	0.0020	0.0763
1297	1180.0842	116.9158	0.8922	0.8890	0.0363	0.0100	0.1725
946	874.3104	71.6896	0.5609	0.5541	0.0831	0.0095	0.1669
1713	1695.8021	17.1979	0.1399	0.1375	0.1523	0.0012	0.0583
1024	1097.1787	-73.1787	-0.5676	-0.5608	0.0671	0.0077	-0.1504
1147	1094.1312	52.8688	0.4057	0.3998	0.0469	0.0027	0.0887
1092	1126.0744	-34.0744	-0.2663	-0.2620	0.0812	0.0021	-0.0779
1152	1269.0846	-117.0846	-0.9229	-0.9205	0.0968	0.0304	-0.3013
1336	1125.8908	210.1092	1.6121	1.6602	0.0467	0.0424	0.3673
2131	2030.2880	100.7120	0.8285	0.8239	0.1707	0.0471	0.3738
1550	1667.4570	-117.4570	-0.9232	-0.9208	0.0916	0.0287	-0.2924
1884	1670.5045	213.4955	1.6492	1.7023	0.0595	0.0573	0.4281
2041	1864.8441	176.1559	1.3960	1.4203	0.1064	0.0774	0.4901
845	998.6687	-153.6687	-1.2043	-1.2141	0.0863	0.0457	-0.3731
1483	1460.3768	22.6232	0.1730	0.1700	0.0399	0.0004	0.0346
1055	1240.3724	-185.3724	-1.4994	-1.5340	0.1422	0.1242	-0.6245
1545	1578.2730	-33.2730	-0.2588	-0.2546	0.0724	0.0017	-0.0712
729	552.7486	176.2514	1.4551	1.4850	0.1766	0.1514	0.6877
1792	1715.1883	76.8117	0.6005	0.5937	0.0817	0.0107	0.1771
1175	1364.5471	-189.5471	-1.5589	-1.6003	0.1703	0.1663	-0.7250
1593	1731.1599	-138.1599	-1.0959	-1.0999	0.1081	0.0485	-0.3829
785	676.9233	108.0767	0.8685	0.8647	0.1309	0.0379	0.3356
744	727.8856	16.1144	0.1285	0.1263	0.1169	0.0007	0.0459
1356	1562.4850	-206.4850	-1.7078	-1.7695	0.1796	0.2129	-0.8280
1262	1403.1360	-141.1360	-1.0964	-1.1003	0.0699	0.0301	-0.3018

No observation appears to be overly influential in the analysis.

8.31 To detect outliers, we look for observations in the standardized residual stem-and-leaf display that fall beyond 3.00 or −3.00. None of the standardized residuals fall beyond 3.00 or −3.00. No outliers are detected.

8.32 a. Using MINITAB, the results are:

Regression Analysis: HOURS versus WAGES

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	17500	17500	1.02	0.331
Error	13	222828	17141		
Total	14	240328			

Model Summary

S	R-sq	R-sq(adj)
130.922	7.28%	0.15%

Coefficients

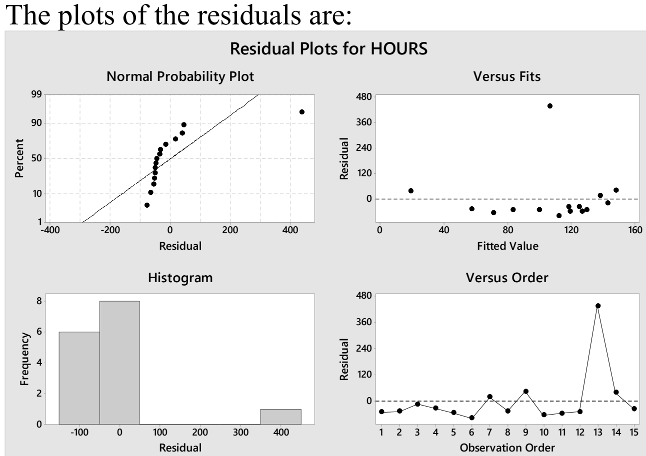
Term	Coef	SE Coef	T-Value	P-Value
Constant	223	120	1.86	0.086
WAGES	-9.60	9.50	-1.01	0.331

Regression Equation

HOURS = 223 - 9.60 WAGES

The least squares prediction equation is $\hat{y} = 223 - 9.60x$.

b. The plots of the residuals are:



From the plots, it is evident that there is one extreme outlier.

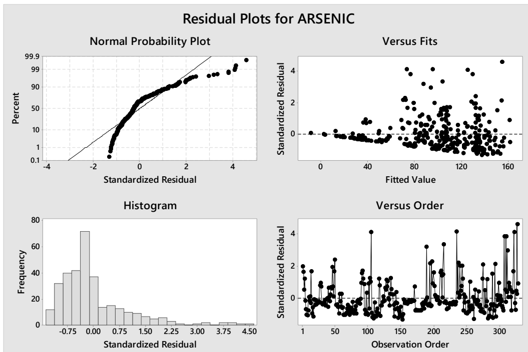
c. Since the observation is really not representative of an employee in the population, it should be deleted from the data.

d. The diagnostics are:

HOURS	FITS	RESI	SRES	TRES	HI	COOK	DFIT
49	99.7433	-50.7433	-0.4017	-0.3884	0.0692	0.0060	-0.1059
36	83.4217	-47.4217	-0.3811	-0.3682	0.0968	0.0078	-0.1206
127	142.9477	-15.9477	-0.1316	-0.1265	0.1430	0.0014	-0.0517
91	124.7059	-33.7059	-0.2693	-0.2594	0.0858	0.0034	-0.0795
72	126.6261	-54.6261	-0.4374	-0.4234	0.0900	0.0095	-0.1332
34	112.2246	-78.2246	-0.6191	-0.6038	0.0686	0.0141	-0.1639
155	138.1472	16.8528	0.1376	0.1323	0.1243	0.0013	0.0498
11	57.4990	-46.4990	-0.3979	-0.3847	0.2033	0.0202	-0.1943
191	147.7482	43.2518	0.3614	0.3490	0.1644	0.0128	0.1548
6	70.9404	-64.9404	-0.5344	-0.5192	0.1385	0.0230	-0.2082
63	118.9453	-55.9453	-0.4445	-0.4303	0.0757	0.0081	-0.1231
79	129.5064	-50.5064	-0.4060	-0.3926	0.0972	0.0089	-0.1288
543	106.4640	436.5360	3.4513	11.4612	0.0667	0.4254	3.0631
57	19.0951	37.9049	0.4104	0.3968	0.5022	0.0849	0.3986
82	117.9852	-35.9852	-0.2857	-0.2753	0.0743	0.0033	-0.0780

The diagnostics highlighted above indicate that employee #13 had much influence on the regression analysis.

8.33 The plots of the standardized residuals is below. There are several standardized residuals greater than 3. The wells associated with these values and are considered outliers are:



WELLID	SRES
3482	3.1908
3887	3.3311
1296	3.8101
2049	3.8163
1861	4.0767
4567	4.1069
4353	4.1274
7893	4.5714

Any value larger than $2\bar{h} = 2\left(\frac{3+1}{328}\right) = 0.0244$ is very influential. A MINTAB spreadsheet of influential residual values and their leverage values are presented below.

WELLID	**HI2**
2219	0.069532
3710	0.056000
7954	0.050824
2220	0.045784
7918	0.039874
4950	0.036918
3510	0.034970
3928	0.034760
3409	0.032901
59	0.026768
3932	0.026737
794	0.026523
4352	0.025846
784	0.024845
2993	0.024772

None of these outliers appear to be influential. Possibly delete these.

8.34 Using MINITAB, the results are

Regression Analysis: HEATRATE versus RPM, CPRATIO, RPM_CPR

Analysis of Variance

Source	DF	Seq SS	Contribution	Adj SS	Adj MS	F-Value	P-Value
Regression	3	142586570	84.92%	142586570	47528857	118.30	0.000
Error	63	25310639	15.08%	25310639	401756		
Total	66	167897208	100.00%				

Model Summary

S	R-sq	R-sq(adj)	PRESS	R-sq(pred)
633.842	84.92%	84.21%	33199917	80.23%

Coefficients

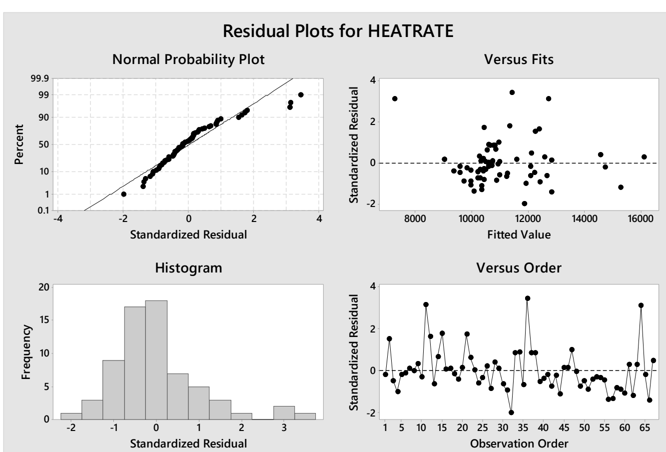
Term	Coef	SE Coef	95% CI	T-Value	P-Value	VIF
Constant	12065	419	(11229, 12902)	28.83	0.000	
RPM	0.1697	0.0347	(0.1004, 0.2390)	4.89	0.000	9.74
CPRATIO	-146.1	26.7	(-199.3, -92.8)	-5.48	0.000	2.10
RPM_CPR	-0.00242	0.00312	(-0.00866, 0.00381)	-0.78	0.440	7.68

Fits and Diagnostics for Unusual Observations

Obs	HEATRATE	Fit	SE Fit	95% CI	Resid	Std Resid	Del Resid	HI	Cook's D
11	14628	12711	165	(12381, 13040)	1917	3.13	3.38	0.067823	0.18
28	14796	14562	278	(14007, 15117)	234	0.41	0.41	0.192206	0.01
36	13523	11428	172	(11085, 11771)	2095	3.43	3.78	0.073224	0.23
61	16243	16105	410	(15286, 16925)	138	0.28	0.28	0.418768	0.01
62	14628	15296	289	(14719, 15873)	-668	-1.18	-1.19	0.207450	0.09
64	8714	7259	427	(6405, 8112)	1455	3.11	3.35	0.454080	2.01

Obs	DFITS		
11	0.91248	R	
28	0.19912		X
36	1.06185	R	
61	0.24002		X
62	-0.60798		X
64	3.05565	R	X

R Large residual
X Unusual X



From the results above, it appears that there are 3 outliers – observations 11, 36, and 64. All have standardized residuals greater than 3. A "high" value of leverage is given by $h_i > \frac{2(k+1)}{n} = \frac{2(3+1)}{67} = \frac{8}{67} = 0.119$. Analysis of the three outliers indicates that observation 64 is also influential. Possibly delete these or see if the model should be revised.

8.35 Residual correlation leads to smaller than usual standard errors of betas, which results in inflated *t*-statistics for testing model parameters.

8.36 a. From Table 7, $d_{\mathrm{L}} = 1.21$ and $d_{\mathrm{U}} = 1.65$.

b. From Table 8, $d_{\mathrm{L}} = 1.25$ and $d_{\mathrm{U}} = 1.34$.

c. From Table 7, $d_{\mathrm{L}} = 1.16$ and $d_{\mathrm{U}} = 1.80$.

8.37 a. To determine if positive first-order autocorrelation exists, we test:

H_0: No residual correlation
H_a: Positive residual correlation

b. The *p*-value is $p < 0.0001$. Since the *p*-value is so small, H_0 is rejected. There is sufficient evidence to indicate positive first-order autocorrelation exists for any reasonable value of α.

8.38 To determine if the model contains positively correlated errors, we test:

H_0: No residual correlation
H_a: Positive residual correlation

The test statistic is $d = 1.77$.

There is no *n* given and $k = 5$. Using Table 7, with $n = 100$, the rejection region is $d < d_{\mathrm{L},0.05} = 1.57$. The nonrejection region is $d > d_{\mathrm{U},0.05} = 1.78$. The inconclusive region is $d_{\mathrm{L},0.05} \le d \le d_{\mathrm{U},0.05} \Rightarrow 1.57 \le d \le 1.78$.

Since $d = 1.77$ falls in the inconclusive region, no decision can be made.

8.39 a. For Bank 1, $R^2 = 0.914$. Thus, 91.4% of the variation in the deposit share values for Bank 1 is explained by the model containing expenditures on promotion-related activities, expenditures on service-related activities, and expenditures on distribution related-activities.

For Bank 2, $R^2 = 0.721$. Thus, 72.1% of the variation in the deposit share values for Bank 2 is explained by the model containing expenditures on promotion-related activities, expenditures on service-related activities, and expenditures on distribution-related activities.

For Bank 3, $R^2 = 0.926$. Thus, 92.6% of the variation in the deposit share values for Bank 3 is explained by the model containing expenditures on promotion-related activities, expenditures on service-related activities, and expenditures on distribution-related activities.

For Bank 4, $R^2 = 0.827$. Thus, 82.7% of the variation in the deposit share values for Bank 4 is explained by the model containing expenditures on promotion-related activities, expenditures on service-related activities, and expenditures on distribution-related activities.

For Bank 5, $R^2 = 0.270$. Thus, 27.0% of the variation in the deposit share values for Bank 5 is explained by the model containing expenditures on promotion-related activities, expenditures on service-related activities, and expenditures on distribution-related activities.

For Bank 6, $R^2 = 0.616$. Thus, 61.6% of the variation in the deposit share values for Bank 6 is explained by the model containing expenditures on promotion-related activities, expenditures on service-related activities, and expenditures on distribution-related activities.

For Bank 7, $R^2 = 0.962$. Thus, 96.2% of the variation in the deposit share values for Bank 7 is explained by the model containing expenditures on promotion-related activities, expenditures on service-related activities, and expenditures on distribution-related activities.

For Bank 8, $R^2 = 0.495$. Thus, 49.5% of the variation in the deposit share values for Bank 8 is explained by the model containing expenditures on promotion-related activities, expenditures on service-related activities, and expenditures on distribution-related activities.

For Bank 9, $R^2 = 0.500$. Thus, 50.0% of the variation in the deposit share values for Bank 9 is explained by the model containing expenditures on promotion-related activities, expenditures on service-related activities, and expenditures on distribution-related activities.

b. To test for the overall adequacy of the models, we test:

$$H_0: \beta_1 = \beta_2 = \beta_3 = 0$$
$$H_a: \text{At least one } \beta_i \neq 0, i = 1, 2, 3$$

For Bank 1, the p-value for the global F test statistic is $p = 0.000$. Since this p-value is so small, we reject H_0. There is sufficient evidence to indicate that the overall model is adequate at $\alpha = 0.05$.

For Bank 2, the p-value for the global F test statistic is $p = 0.004$. Since this p-value is so small, we reject H_0. There is sufficient evidence to indicate that the overall model is adequate at $\alpha = 0.05$.

For Bank 3, the p-value for the global F test statistic is $p = 0.000$. Since this p-value is so small, we reject H_0. There is sufficient evidence to indicate that the overall model is adequate at $\alpha = 0.05$.

For Bank 4, the p-value for the global F test statistic is $p = 0.000$. Since this p-value is so small, we reject H_0. There is sufficient evidence to indicate that the overall model is adequate at $\alpha = 0.05$.

For Bank 5, the p-value for the global F test statistic is $p = 0.155$. Since this p-value is not so small, we do not reject H_0. There is insufficient evidence to indicate that the overall model is adequate at $\alpha = 0.05$.

For Bank 6, the p-value for the global F test statistic is $p = 0.012$. Since this p-value is so small, we reject H_0. There is sufficient evidence to indicate that the overall model is adequate at $\alpha = 0.05$.

For Bank 7, the p-value for the global F test statistic is $p = 0.000$. Since this p-value is so small, we reject H_0. There is sufficient evidence to indicate that the overall model is adequate at $\alpha = 0.05$.

For Bank 8, the p-value for the global F test statistic is $p = 0.014$. Since this p-value is so small, we reject H_0. There is sufficient evidence to indicate that the overall model is adequate at $\alpha = 0.05$.

For Bank 9, the p-value for the global F test statistic is $p = 0.011$. Since this p-value is so small, we reject H_0. There is sufficient evidence to indicate that the overall model is adequate at $\alpha = 0.05$.

Thus, the model is adequate at $\alpha = 0.05$ for all banks except bank 5.

c. To determine if autocorrelation is present, we test:

H_0 : No residual correlation
H_{a} : Residual correlation exists

For $\alpha = 0.10$, the rejection region is $d < d_{\text{L},\alpha/2}$ or $(4-d) < d_{\text{L},\alpha/2}$. From Table 7, Appendix D, for $n = 20$ and $k = 3$, $d_{\text{L},0.05} = 1.00$. The rejection region is $d < 1.00$ or $(4-d) < 1.00$. If $d > d_{\text{U},0.05}$ or $(4-d) > d_{\text{U},0.05}$, then H_0 is not rejected. If $d_{\text{L},0.05} < d < d_{\text{U},0.05}$ or $d_{\text{L},0.05} < (4-d) < d_{\text{U},0.05}$, no conclusion is reached. From Table 7, Appendix D, with $n = 20$ and $k = 3$, $d_{\text{U},0.05} = 1.68$.
For Bank 1, $d = 1.3$ and $4 - d = 4 - 1.3 = 2.7$. Since $1.00 < d = 1.3 < 1.68$, no conclusion can be reached.

For Bank 2, $d = 3.4$ and $4 - d = 4 - 3.4 = 0.6$. Since $4 - d = 0.6 < 1.00, H_0$ is rejected. There is sufficient evidence to indicate autocorrelation is present.

For Bank 3, $d = 2.7$ and $4 - d = 4 - 2.7 = 1.3$. Since $1.00 < 4 - d = 1.3 < 1.68$, no conclusion can be reached.

For Bank 4, $d = 1.9$ and $4 - d = 4 - 1.9 = 2.1$. Since $d = 1.9 > 1.68, H_0$ is not rejected. There is insufficient evidence to indicate autocorrelation is present.

For Bank 5, $d = 0.85$ and $4 - d =$ 4-0.85 $= 3.15$. Since $d = 0.85 < 1.00, H_0$ is rejected. There is sufficient evidence to indicate autocorrelation is present.

For Bank 6, $d = 1.8$ and $4 - d =$ 4-1.8 $= 2.2$. Since $d = 1.8 > 1.68, H_0$ is not rejected. There is insufficient evidence to indicate autocorrelation is present.

For Bank 7, $d = 2.5$ and $4 - d =$ 4-2.5 $= 1.5$. Since $1.00 < 4 - d = 1.5 < 1.68$, no conclusion can be reached.

For Bank 8, $d = 2.3$ and $4 - d = 4 - 2.3 = 1.7$. Since $4 - d = 1.7 > 1.68, H_0$ is not rejected. There is insufficient evidence to indicate autocorrelation is present.

For Bank 9, $d = 1.1$ and $4 - d = 4 - 1.1 = 2.9$. Since $1.00 < d = 1.1 < 1.68$, no conclusion can be reached.

Thus, reject H_0 (two-tailed at $\alpha = 0.10$) for banks 2, 5; fail to reject H_0 for banks 4, 6, 8; test is inconclusive for banks 1, 3, 7, 9.

8.40 a. To determine if the model contributes information for the prediction of y, we test:

$$H_0: \beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = 0$$
$$H_a: \text{At least one } \beta_i \neq 0$$

The test statistic is $F = \left[\frac{R^2}{1-R^2}\right]\left[\frac{n-(k+1)}{k}\right] = \left[\frac{0.856}{1-0.856}\right]\left[\frac{144-(5+1)}{5}\right] = 164.07.$

The rejection region requires $\alpha = 0.05$ in the upper tail of the F distribution with $\nu_1 = k = 5$ and $\nu_2 = n-(k+1) = 144-(5+1) = 138$. Using Excel, $F_{0.05} = 2.28$. The rejection region is $F > 2.28$.

Since the observed value of the test statistic falls in the rejection region $(F = 164.07 > 2.28)$, H_0 is rejected. There is sufficient evidence to indicate the model contributes information for the prediction of y at $\alpha = 0.05$.

b. To determine if the regression errors are positively correlated, we test:

H_0 : No residual correlation
H_a : Positive autocorrelation exists

The test statistic is $d = 1.01$.

For the rejection region, $k = 5$ and $n = 144$. Using Table 7, Appendix D, $d_{L,0.05} \approx 1.57$ and $d_{U,0.05} \approx 1.78$.

Rejection region: $d < 1.57$
Nonrejection region: $d > 1.78$
Inconclusive region: $1.57 \leq (4 - d) \leq 1.78$

Since the observed vale of the test statistic falls in the rejection region $(d = 1.01 < 1.57)$, H_0 is rejected. There is sufficient evidence to indicate that positive autocorrelation exists at $\alpha = 0.05$.

c. Since autocorrelation was found to exist, the conclusion about the model adequacy is suspect. A model that accounts for first-order autocorrelation in the error terms should be considered.

8.41 a. Yes, the residuals tend to have a long periods of positive and negative runs.

b. The Durbin-Watson statistic is $d = 0.105868$. To determine if the residuals are uncorrelated, we test:

H_0 : No residual autocorrelation
H_a : Autocorrelation exists

For $\alpha = 0.10$, the rejection region is $d < d_{L,\alpha/2}$ or $(4 - d) < d_{L,\alpha/2}$. From Table 7, Appendix D, for $n = 38$ and $k = 1$, $d_{L,0.05} = 1.43$. The rejection region is $d < 1.43$ or $(4 - d) < 1.43$. If $d > d_{U,0.05}$ or $(4 - d) > d_{U,0.05}$, then H_0 is not rejected. Any other results indicates that no conclusion is reached. From Table 7, Appendix D, with $n = 38$ and $k = 1$, $d_{U,0.05} = 1.54$.

Since the observed value of the test statistic falls in the rejection region $(d = 0.105868 < 1.43)$, H_0 is rejected. There is sufficient evidence to indicate that autocorrelation exists among the residuals at $\alpha = 0.10$.

c. We must assume that the error terms are normally distributed.

8.42 a. Using MINITAB, the results are:

Regression Analysis: Policies versus t

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	36730	36730.2	64.21	0.000
t	1	36730	36730.2	64.21	0.000
Error	30	17161	572.0		
Total	31	53891			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
23.9174	68.16%	67.09%	64.35%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	410.42	8.66	47.40	0.000	
t	-3.669	0.458	-8.01	0.000	1.00

Regression Equation

Policies = 410.42 - 3.669 t

Fits and Diagnostics for Unusual Observations

Obs	Policies	Fit	Resid	Std Resid	
23	374.00	326.02	47.98	2.05	R

R Large residual

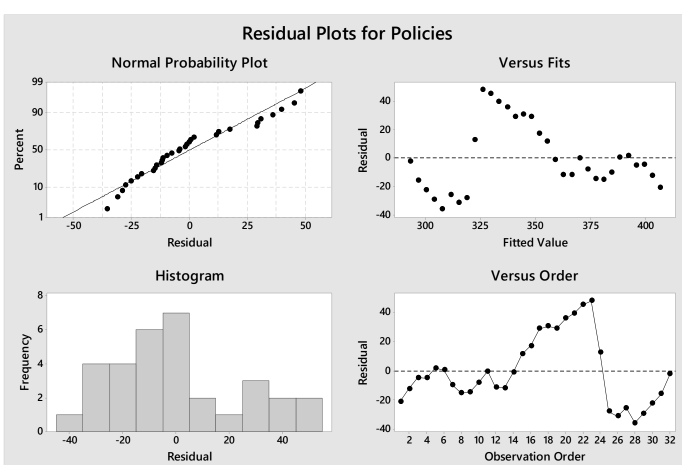
Durbin-Watson Statistic

Durbin-Watson Statistic = 0.258560

The fitted model is $\hat{y} = 410.42 - 3.669t$.

The model is adequate $\left(F = 64.21,\ p = 0.000\right)$. The R^2 value is $R^2 = 0.6816$. This is not a particularly high value and indicates we might want to look for additional variables to predict y.

b. The residual plots are:



Yes. The plot of residuals against time shows significantly positive and negative runs.

c. To determine if positive autocorrelation exists among the residuals, we test:

H_0 : No residual correlation
H_a : Positive autocorrelation exists

The test statistic is $d = 0.258560$.

For the rejection region, $k = 1$ and $n = 32$. Using Table 7, Appendix D, $d_{L,0.05} = 1.37$ and $d_{U,0.05} = 1.50$

Rejection region: $d < 1.37$
Nonrejection region: $d > 1.50$
Inconclusive region: $1.37 \leq (4 - d) \leq 1.50$

Since the observed vale of the test statistic falls in the rejection region $(d = 0.25856 < 1.37)$, H_0 is rejected. There is sufficient evidence to indicate that positive autocorrelation exists at $\alpha = 0.05$.

d. The model is not valid due to the positive autocorrelation in the residuals.

8.43 a. Using MINITAB, the results are:

Regression Analysis: Sales versus Time

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	12880305	12880305	4.44	0.047
Time	1	12880305	12880305	4.44	0.047
Error	22	63843919	2901996		
Total	23	76724223			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
1703.52	16.79%	13.01%	1.14%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	1668	718	2.32	0.030	
Time	105.8	50.2	2.11	0.047	1.00

Regression Equation

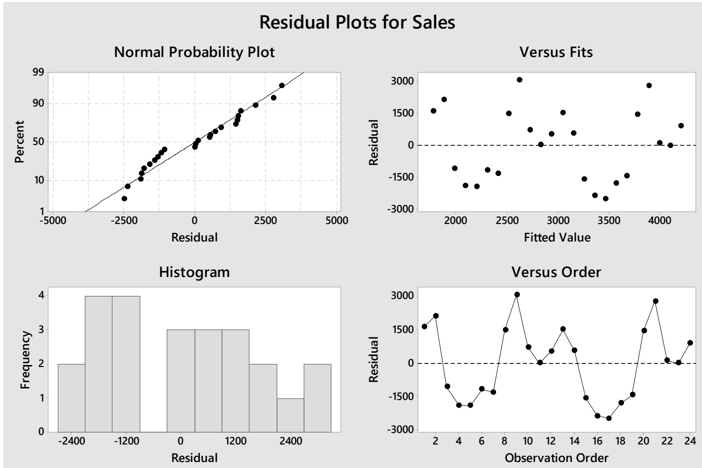
Sales = 1668 + 105.8 Time

Durbin-Watson Statistic

Durbin-Watson Statistic = 0.845030

The model is statistically useful for predicting monthly sales $(F = 4.44,\ p = 0.047)$.

b. The residual plots are:



Yes, there is a tendency for the residuals to group alternately into positive and negative clusters, so there is evidence that autocorrelation is present.

c. To determine if autocorrelation exists among the residuals, we test:

H_0 : No residual correlation
H_a : Autocorrelation exists

The test statistic is $d = 0.845030$.

No α is given so we will use $\alpha = 0.10$. For the rejection region, $k = 1$ and $n = 24$. Using Table 7, Appendix D, $d_{L,0.05} = 1.27$ and $d_{U,0.05} = 1.45$

Rejection region: $d < 1.27$ or $(4 - d) < 1.27$
Nonrejection region: $d > 1.45$ or $(4 - d) > 1.45$
Inconclusive region: Any other region.

Since the observed value of the test statistic falls in the rejection region $(d = 0.845030 < 1.27)$, H_0 is rejected. There is sufficient evidence to indicate that autocorrelation exists at $\alpha = 0.10$.

8.44 a. To determine if the model is useful for predicting future spot exchange rates, we test:

$H_0: \beta_1 = 0$
$H_a: \beta_1 \neq 0$

The test statistic is $t = 41.9$.

The rejection region requires $\alpha / 2 = 0.05 / 2 = 0.025$ in the upper tail of the t distribution with $df = n - 2 = 81 - 2 = 79$. Using Table 2, Appendix D, $t_{0.025} = 1.99$. The rejection region is $t < -1.99$ or $t > 1.99$.

Since the observed value of the test statistic falls in the rejection region $(t = 41.9 > 1.99)$, H_0 is rejected. There is sufficient evidence to indicate that the model is useful for predicting future spot exchange rates at $\alpha = 0.05$.

b. $s = 0.0249$. We expect most of the observations to fall within $2s$ or $2(0.0249) = 0.0498$ units of the least squares line.

$R^2 = 0.957$. This means that 95.7% of the sums of squares of the deviations of the logarithms of the spot exchange rate values about their means is attributable to the linear relationship between logarithms of the spot exchange rates and the logarithms of the forward rate.

c. To determine if autocorrelation exists, we test:

H_0 : No autocorrelation
H_a : Positive autocorrelation exists

The test statistic is $d = 0.962$.

For the rejection region, $k = 1$ and $n = 81$. Using Table 7, Appendix D, $d_{L,0.05} = 1.61$ and $d_{U,0.05} = 1.66$.

Rejection region: $d < 1.61$
Nonrejection region: $d > 1.66$
Inconclusive region: $1.61 \le (4 - d) \le 1.66$

Since the observed vale of the test statistic falls in the rejection region $(d = 0.962 < 1.61)$, H_0 is rejected. There is sufficient evidence to indicate that positive autocorrelation exists at $\alpha = 0.05$.

d. Because autocorrelation was detected, we should not use the least squares model. A time series model that accounts for first-order autocorrelation should be used.

8.45 a. The curvilinear trend implies a misspecified model; a quadratic term is missing.

b. The spread of the residuals increasing as $\hat{y}$ increases implies unequal variances.

c. A residual lying more than 3 standard deviations from the mean implies an outlier is present.

d. The spread of the residuals increasing and then decreasing as $\hat{y}$ increases implies unequal variances.

e. The histogram indicates the errors are not normally distributed.

8.46 a. Using MINITAB, the results are:

Regression Analysis: IMPROVE versus COST, COST-SQ

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	2	1368.78	684.39	33.08	0.000
Error	7	144.82	20.69		
Total	9	1513.60			

Model Summary

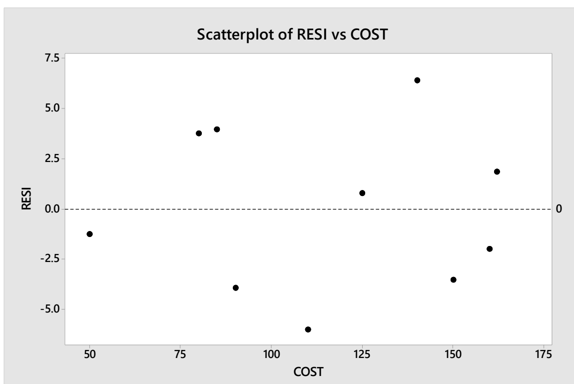
S	R-sq	R-sq(adj)	R-sq(pred)
4.54855	90.43%	87.70%	80.07%

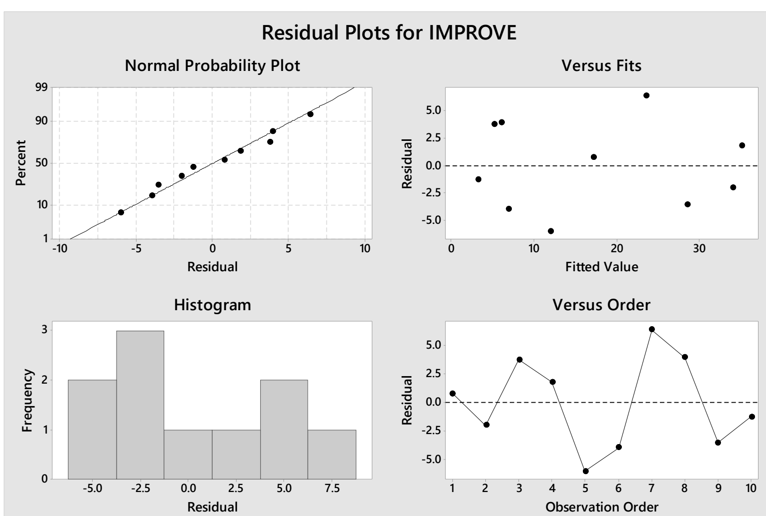
Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	10.7	14.6	0.73	0.488	
COST	-0.282	0.281	-1.00	0.349	50.27
COST-SQ	0.00267	0.00125	2.13	0.071	50.27

Regression Equation

IMPROVE = 10.7 - 0.282 COST + 0.00267 COST-SQ





There are no apparent trends and there is no evidence of outliers.

b. The influence diagnostics are listed in the following table. There

IMPROVE	FITS	RESI	SRES	HI	COOK	DFIT
18	17.2073	0.7927	0.1955	0.2057	0.0033	0.0924
32	34.0037	-2.0037	-0.5423	0.3401	0.0505	-0.3682
9	5.2310	3.7690	0.9294	0.2051	0.0743	0.4669
37	35.1612	1.8388	0.5175	0.3898	0.0570	0.3904
6	12.0128	-6.0128	-1.5141	0.2377	0.2383	-0.9545
3	6.9572	-3.9572	-0.9787	0.2098	0.0848	-0.5025
30	23.6042	6.3958	1.5387	0.1649	0.1558	0.7780
10	6.0273	3.9727	0.9783	0.2030	0.0812	0.4919
25	28.5367	-3.5367	-0.8665	0.1947	0.0605	-0.4174
2	3.2586	-1.2586	-0.7129	0.8494	0.9553	-1.6276

The leverage (H), Cook's D, and DFIT all indicate that observation #10 has a large influence on the analysis.

8.47 Using MINITAB, the results are:

Regression Analysis: ACIDPCT versus OXIDANT

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	2.414	2.4136	17.08	0.001
Error	17	2.402	0.1413		
Total	18	4.816			

Model Summary

S	R-sq	R-sq(adj)
0.375918	50.12%	47.18%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	-0.024	0.246	-0.10	0.924
OXIDANT	0.01958	0.00474	4.13	0.001

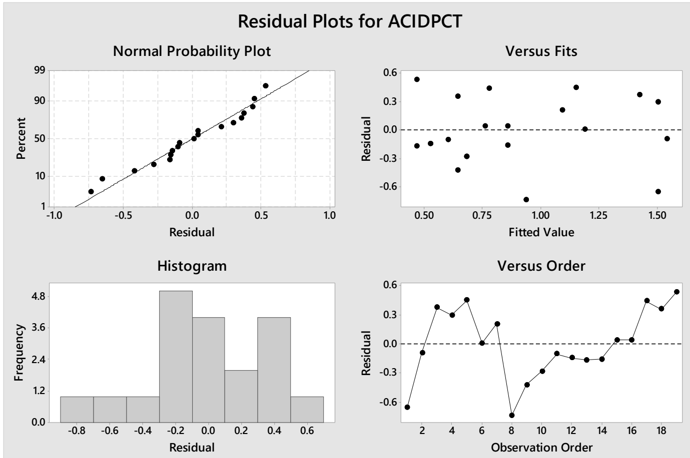
Regression Equation

ACIDPCT = -0.024 + 0.01958 OXIDANT

Fits and Diagnostics for Unusual Observations

Obs	ACIDPCT	Fit	Resid	Std Resid	
8	0.200	0.936	-0.736	-2.01	R

R Large residual



The assumptions are reasonably satisfied since the residual plot on the upper right corner shows scatter of the residuals and the normal probability plot of the residuals appears normal.

8.48 To determine if the residuals are positively correlated, we test:

H_0 : No residual correlation
H_a : Positive residual correlation

The test statistic is $d = 0.776$.

Using Table 7 in Appendix D, with $k = 2$ and $n = 15$, $d_{L,0.05} = 0.95$ and $d_{U,0.05} = 1.54$.
Rejection region: $d < 0.95$
Nonrejection region: $d > 1.54$
Inconclusive region: Any other result

Since the observed value of the test statistic falls in the rejection region $(d = 0.776 < 0.95)$, H_0 is rejected. There is sufficient evidence to indicate the residuals are positively correlated at $\alpha = 0.05$.

8.49 Using MINITAB, the results are:

Regression Analysis: LOS versus FACTORS
Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	126.6	126.584	28.68	0.000
FACTORS	1	126.6	126.584	28.68	0.000
Error	48	211.8	4.413		
Total	49	338.4			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
2.10077	37.40%	36.10%	30.62%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	3.306	0.673	4.91	0.000	
FACTORS	0.01475	0.00276	5.36	0.000	1.00

Regression Equation

LOS = 3.306 + 0.01475 FACTORS

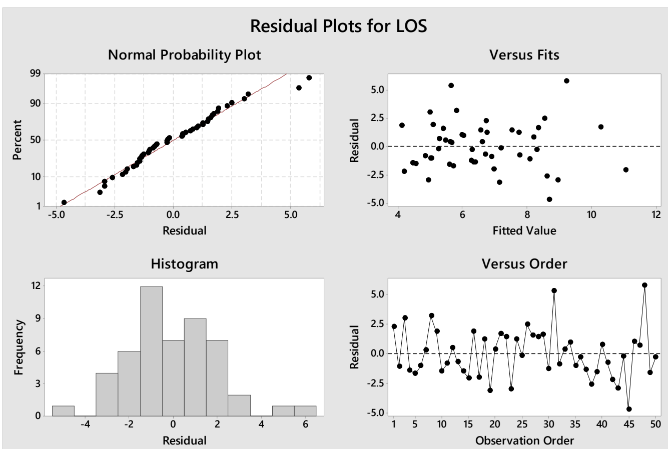
Fits and Diagnostics for Unusual Observations

Obs	LOS	Fit	Resid	Std Resid	
15	9.000	11.052	-2.052	-1.08	X
21	12.000	10.270	1.730	0.88	X
31	11.000	5.637	5.363	2.59	R
45	4.000	8.677	-4.677	-2.29	R
48	15.000	9.223	5.777	2.86	R

R Large residual
X Unusual X

Durbin-Watson Statistic

Durbin-Watson Statistic = 2.12974



From the plot of the residuals versus the fitted values (upper right corner), there is no pattern. This indicates the model is correctly specified and that the assumption of constant variance is valid. The normal probability plot is almost a straight line, indicating that the assumption of normal errors is probably valid. Three observations have standardized residuals greater than 2.28 in absolute value. These are suspect outliers. The graph in the lower right corner does not indicate the data have autocorrelation.

The diagnostics for selected residuals are:

OBS	LOS	FACTORS	FITS	RESI	SRES	TRES	HI	COOK	DFIT
15	9	525	11.0523	-2.0523	-1.0794	-1.0813	0.1809	0.1286	-0.5081
21	12	472	10.2703	1.7297	0.8827	0.8806	0.1299	0.0582	0.3403

Observations have high leverage if $h_i > \frac{2(k+1)}{n} = \frac{2(1+1)}{50} = 0.08$. Two observations (#15 and #21) have high leverage. No other observations have unusual values.

Our general conclusion to the residual analysis is that the model should be used as specified.

8.50 a. Using MINITAB, the results are:

Regression Analysis: ABSRATE versus QTR1DUM, ... 2DUM, QTR3DUM

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	3	0.14754	0.049180	20.14	0.000
Error	16	0.03908	0.002443		
Total	19	0.18662			

Model Summary

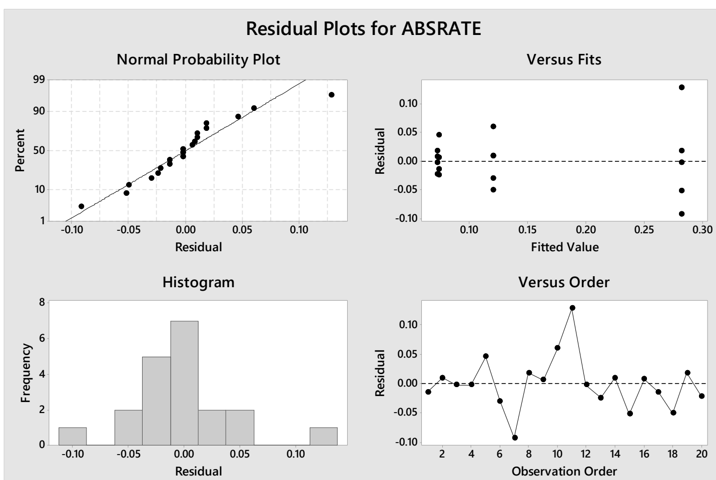
S	R-sq	R-sq(adj)
0.0494217	79.06%	75.13%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	0.0720	0.0221	3.26	0.005
QTR1DUM	0.0020	0.0313	0.06	0.950
QTR2DUM	0.0480	0.0313	1.54	0.144
QTR3DUM	0.2100	0.0313	6.72	0.000

Regression Equation

ABSRATE = 0.0720 + 0.0020 QTR1DUM + 0.0480 QTR2DUM + 0.2100 QTR3DUM



The fitted model is $\hat{y} = 0.0720 + 0.0020x_1 + 0.0480x_2 + 0.2100x_3$.

b. Because the response variable is a rate, we anticipate problems with the equal variance assumption. The plot of the residuals versus the fitted values in the upper right corner of the plots in part **a** indicate that the assumption of equal variances is violated.

c. The recommended model will transform the dependent variable using $y^* = \sin^{-1}\sqrt{y}$. The y^* values will then be used as the dependent variables.

d. Using MINITAB to fit the transformed data, the results are:

Regression Analysis: y* versus QTR1DUM, QTR2DUM, QTR3DUM
Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	3	0.27243	0.090811	22.35	0.000
Error	16	0.06502	0.004064		
Total	19	0.33746			

Model Summary

S	R-sq	R-sq(adj)
0.0637486	80.73%	77.12%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	0.2705	0.0285	9.49	0.000
QTR1DUM	0.0017	0.0403	0.04	0.967
QTR2DUM	0.0792	0.0403	1.96	0.067
QTR3DUM	0.2862	0.0403	7.10	0.000

Regression Equation

y* = 0.2705 + 0.0017 QTR1DUM + 0.0792 QTR2DUM + 0.2862 QTR3DUM

The fitted model is $\hat{y}^* = 0.2705 + 0.0017x_1 + 0.0792x_2 + 0.2862x_3$.

The transformed model has slightly better adequacy $\left(R^2 = 0.807 \text{ over } R^2 = 0.791\right)$.

8.51 a. Using MINITAB, the results are:

Regression Analysis: LIFE_Y versus GEST_X
Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	146.3	146.32	2.96	0.146
Error	5	247.1	49.42		
Total	6	393.4			

Model Summary

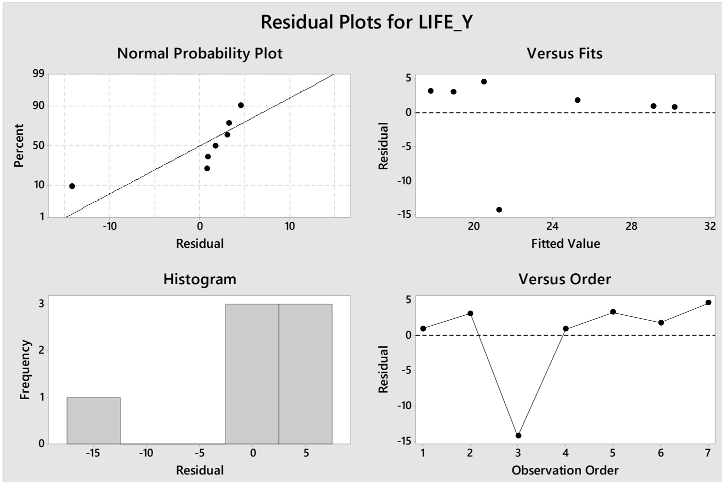
S	R-sq	R-sq(adj)	R-sq(pred)
7.03012	37.19%	24.63%	4.05%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	-3.9	16.0	-0.25	0.816	
GEST_X	0.0820	0.0477	1.72	0.146	1.00

Regression Equation

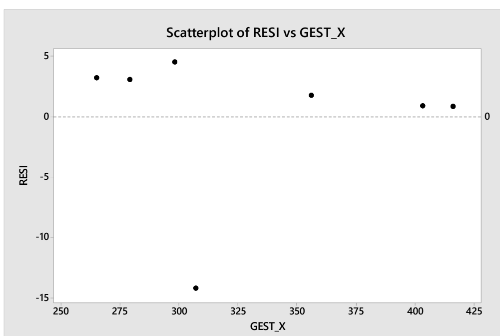
LIFE_Y = -3.9 + 0.0820 GEST_X



The fitted model is $\hat{y} = -3.9 + 0.0820x$.

b. $R^2 = 0.3719$ which indicates that only 37.19% of the sample variance is explained by the model. There is insufficient evidence to indicate the model is adequate for predicting y $(F = 2.96,\ p = 0.146)$. From the plots above, notice that the residuals are not normally distributed nor scattered.

c. The plot of the residuals versus x is:



d. Horse #3 has a residual that lies outside the $0 \pm 2s \Rightarrow 0 \pm 2(7.03) \Rightarrow (-14.06,\ 14.06)$ interval.

e. Using MINITAB, the results are:

Regression Analysis: Y versus X
Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	81.506	81.5057	130.71	0.000
Error	4	2.494	0.6236		
Total	5	84.000			

Model Summary

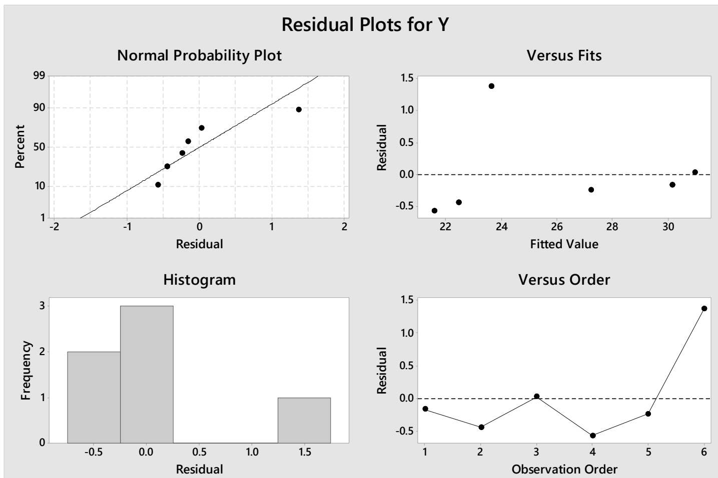
S	R-sq	R-sq(adj)	R-sq(pred)
0.789666	97.03%	96.29%	94.35%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	5.07	1.86	2.73	0.053	
X	0.06227	0.00545	11.43	0.000	1.00

Regression Equation

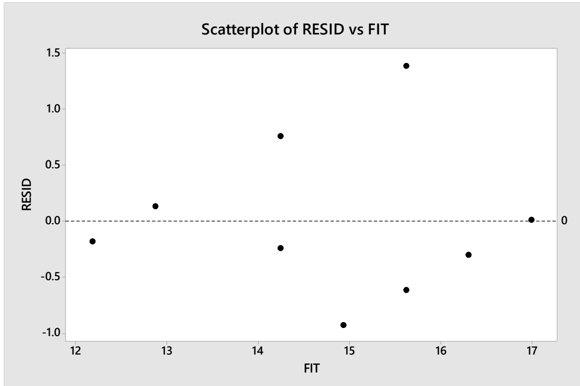
Y = 5.07 + 0.06227 X



For the model with the deleted observation, $R^2 = 0.9703$ which indicates that 97.03% of the sample variance is explained by the model. There is sufficient evidence to indicate the model is adequate for predicting y $(F = 130.71,\ p = 0.000)$. Also notice improvement in the residual plot.

8.52 a. Using the given prediction equation, the residuals were calculated and plotted:

Market Share	Predicted Mkt. Share	Residual
15	14.241	0.759
17	16.989	0.011
17	15.615	1.385
13	12.867	0.133
12	12.180	-0.18
14	14.928	-0.928
16	16.302	-0.302
14	14.241	-0.241
15	15.615	-0.615



b. The assumption of homoscedasticity is probably violated. Viewing the residual plot above confirms this belief.

c. The $y^* = \sin^{-1}\sqrt{y}$. transformation is suggested for correcting the variance assumption problems. Using MINITAB, the results are:

Regression Analysis: Y* versus ADVEXP

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	0.003968	0.003968	36.52	0.001
Error	7	0.000760	0.000109		
Total	8	0.004728			

Model Summary

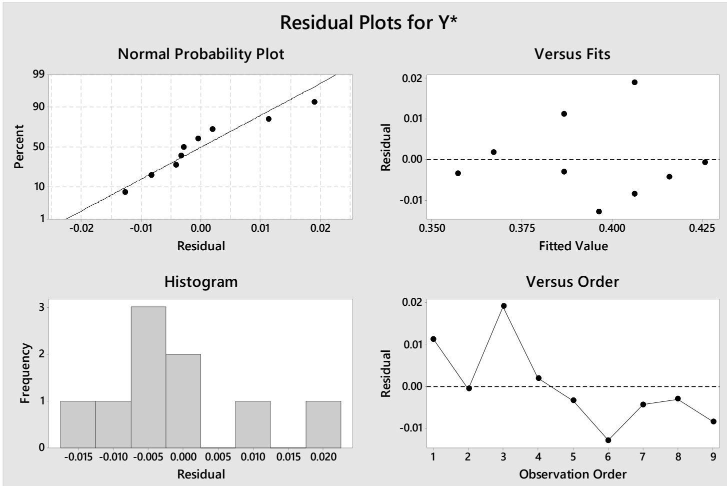
S	R-sq	R-sq(adj)
0.0104231	83.92%	81.62%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	0.1617	0.0386	4.19	0.004
ADVEXP	0.00977	0.00162	6.04	0.001

Regression Equation

Y* = 0.1617 + 0.00977 ADVEXP



The fitted model is $\hat{y}^* = 0.1617 + 0.00977x$.

The plot of the residuals versus the fitted values indicates very little has changed from the transformation. The residual plot looks very similar to the plot in part a.

8.53 Using MINITAB, the results are:

Regression Analysis: RATE versus EST, EST_SQ

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	2	18138955261	9069477631	8.91	0.002
Error	21	21371254395	1017678781		
Total	23	39510209656			

Model Summary

S	R-sq	R-sq(adj)
31901.1	45.91%	40.76%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	-288	8049	-0.04	0.972
EST	1.39	3.65	0.38	0.706
EST_SQ	0.000035	0.000097	0.36	0.722

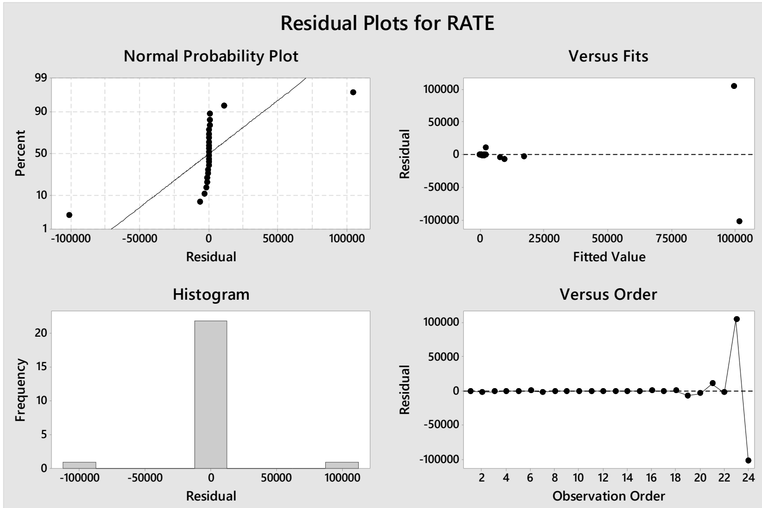
Regression Equation

RATE = -288 + 1.39 EST + 0.000035 EST_SQ

Fits and Diagnostics for Unusual Observations

Obs	RATE	Fit	Resid	Std Resid		
22	14889	17170	-2281	-0.11		X
23	203864	99366	104498	4.57	R	X
24	15	101371	-101356	-4.57	R	X

R Large residual
X Unusual X



There is a possible violation of constant variance as observed in the residuals graphs above. When plotting the residuals versus the fitted values, there should not be any type of pattern. The upper graph to the right definitely displays a pattern in the residuals. Therefore, the constant variance assumption is violated to a degree. Observations 23 and 24 are clearly outliers. Analysis of their leverage and Cook's D diagnostics indicates that they are also influential. You may want to remove these observations from the model and study them separately.

Chapter 9

Special Topics in Regression (Optional)

9.1 a. For this case, $k = 15$ is the knot value (i.e., the value of x at which the slope changes). Since there is no discontinuity in the linear relationship, the appropriate model is:

$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 (x_1 - 15) x_2$$

where $x_1 = x$ and $x_2 = \begin{cases} 1 & \text{if } x > 15 \\ 0 & \text{if not} \end{cases}$

b. For $x \le 15$, $x_2 = 0$, and the model is $E(y) = \beta_0 + \beta_1 x$.

Thus, $\beta_0 = y$-intercept and $\beta_1 =$ slope.

For $x > 15$, $x_2 = 1$, the model is:

$$E(y) = \beta_0 + \beta_1 x + \beta_2 (x - 15) = (\beta_0 - 15\beta_2) + (\beta_1 + \beta_2) x$$

Thus, $(\beta_0 - 15\beta_2) = y$-intercept and $(\beta_1 + \beta_2) =$ slope.

c. If the two slopes are identical, then we must have $\beta_2 = 0$. Therefore, to test for a difference between the two slopes, we test:

$H_0: \beta_2 = 0$ against $H_a: \beta_2 \ne 0$

using a t test.

9.2 a. For this case, $k_1 = 1.45$ and $k_2 = 5.20$ are the knot values. Since there is no discontinuity in the linear relationship, the appropriate model is:

$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 (x_1 - 1.45) x_2 + \beta_3 (x_1 - 5.20) x_3$$

where $x_1 = x$ and $x_2 = \begin{cases} 1 & \text{if } x_1 > 1.45 \\ 0 & \text{if not} \end{cases}$ and $x_3 = \begin{cases} 1 & \text{if } x_1 > 5.20 \\ 0 & \text{if not} \end{cases}$

b. For $x \le 1.45$, $x_2 = 0$, and $x_3 = 0$, the model is $E(y) = \beta_0 + \beta_1 x_1$.

Thus, $\beta_0 = y$-intercept and $\beta_1 =$ slope.

For $1.45 < x \le 5.20$, $x_2 = 1$, and $x_3 = 0$, the model is

$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 (x_1 - 1.45) = (\beta_0 - 1.45\beta_2) + (\beta_1 + \beta_2) x_1$$

Thus, $(\beta_0 - 1.45\beta_2) = y$-intercept and $(\beta_1 + \beta_2) = $ slope.

For $x > 5.20$, $x_2 = 1$, and $x_3 = 1$, the model is

$$\begin{aligned} E(y) &= \beta_0 + \beta_1 x_1 + \beta_2 (x_1 - 1.45) + \beta_3 (x_1 - 5.20) \\ &= (\beta_0 - 1.45\beta_2 - 5.20\beta_3) + (\beta_1 + \beta_2 + \beta_3) x_1 \end{aligned}$$

Thus, $(\beta_0 - 1.45\beta_2 - 5.20\beta_3) = y$-intercept and $(\beta_1 + \beta_2 + \beta_3) = $ slope.

c. If all three of the slopes are identical, the model specifies that $\beta_2 = \beta_3 = 0$ Therefore, to test for a difference in at least two of the three slopes, we test:

$H_0: \beta_2 = \beta_3 = 0$
$H_a:$ At least one $\beta_i \neq 0$
using a partial F test.

9.3 a. For this case, $k = 320$ is the knot value. Since there is discontinuity in the linear relationship, the appropriate linear model is:

$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 (x_1 - 320) x_2 + \beta_3 x_2$$

where $x_1 = x$ and $x_2 = \begin{cases} 1 & \text{if } x > 320 \\ 0 & \text{if not} \end{cases}$

b. For $x \leq 320$, $x_2 = 0$, and the model is $E(y) = \beta_0 + \beta_1 x$

Thus, $\beta_0 = y$-intercept and $\beta_1 = $ slope.

For $x > 320$, $x_2 = 1$, and the model is:

$$E(y) = \beta_0 + \beta_1 x + \beta_2 (x - 320) + \beta_3 = (\beta_0 - 320\beta_2 + \beta_3) + (\beta_1 + \beta_2) x$$

Thus, $(\beta_0 - 320\beta_2 + \beta_3) = y$-intercept and $(\beta_1 + \beta_2) = $ slope.

c. To test for a difference between the two lines, we test:

$H_0: \beta_2 = \beta_3 = 0$
$H_a:$ At least one $\beta_i \neq 0$
using a nested F test.

9.4 a. When age is less than the change point, $x_1 = 0$ and $x_2 = 1$. The equation is $E(\text{BMI}) = \beta_0 + \beta_1 (\text{AGE}) x_2 + \beta_2 (\text{AGE} - \text{CP}) x_1 = \beta_0 + \beta_1 (\text{AGE})$.
The slope of the line is β_1.

b. When age is greater than the change point, $x_1 = 1$ and $x_2 = 0$. The equation is $E(\text{BMI}) = \beta_0 + \beta_1(\text{AGE})x_2 + \beta_2(\text{AGE} - \text{CP})x_1 = \beta_0 + \beta_2(\text{AGE} - \text{CP})$.
The slope of the line is β_2.

9.5 a. For this case, $k_1 = 50$ and $k_2 = 100$ are the knot values. Since there is no discontinuity in the linear relationship, the appropriate model is:

$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2(x_1 - 50)x_2 + \beta_3(x_1 - 100)x_3$$

where $x_1 = x$ and $x_2 = \begin{cases} 1 & \text{if } x_1 > 50 \\ 0 & \text{if not} \end{cases}$ and $x_3 = \begin{cases} 1 & \text{if } x_1 > 100 \\ 0 & \text{if not} \end{cases}$

b. For $x \le 50$, $x_2 = 0$, and $x_3 = 0$, the model is $E(y) = \beta_0 + \beta_1 x_1$.

Thus, $\beta_0 = y$-intercept and $\beta_1 =$ slope.

For $50 < x \le 100$, $x_2 = 1$, and $x_3 = 0$, the model is

$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2(x_1 - 50) = (\beta_0 - 50\beta_2) + (\beta_1 + \beta_2)x_1$$

Thus, $(\beta_0 - 50\beta_2) = y$-intercept and $(\beta_1 + \beta_2) =$ slope.

For $x > 100$, $x_2 = 1$, and $x_3 = 1$, the model is

$$\begin{aligned} E(y) &= \beta_0 + \beta_1 x_1 + \beta_2(x_1 - 50) + \beta_3(x_1 - 100) \\ &= (\beta_0 - 50\beta_2 - 100\beta_3) + (\beta_1 + \beta_2 + \beta_3)x_1 \end{aligned}$$

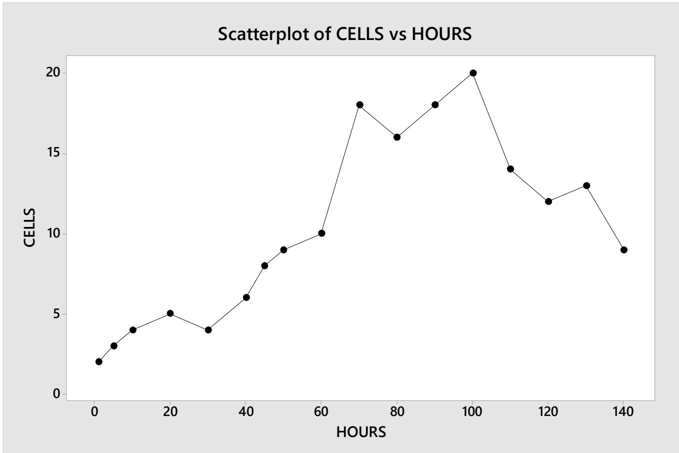
Thus, $(\beta_0 - 50\beta_2 - 100\beta_3) = y$-intercept and $(\beta_1 + \beta_2 + \beta_3) =$ slope.

c. For this case, $k_1 = 50$, $k_2 = 75$, and $k_3 = 100$ are the knot values. Since there is no discontinuity in the linear relationship, the appropriate model is:

$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2(x_1 - 50)x_2 + \beta_3(x_1 - 75)x_3 + \beta_4(x_1 - 100)x_4$$

where $x_1 = x$ and $x_2 = \begin{cases} 1 & \text{if } x_1 > 50 \\ 0 & \text{if not} \end{cases}$, $x_3 = \begin{cases} 1 & \text{if } x_1 > 75 \\ 0 & \text{if not} \end{cases}$, and $x_4 = \begin{cases} 1 & \text{if } x_1 > 100 \\ 0 & \text{if not} \end{cases}$

9.6 a. A plot of the cells against the number of hours is:



There appears to be an increasing trend until hours reach approximately 70, then a decreasing trend after hours exceeds 70.

b. For this case, $k = 70$ is the knot value. Since there is no discontinuity in the linear relationship, the appropriate model is:

$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2^*$$

where $x_1 = x$, $x_2^* = (x_1 - 70)x_2$, and $x_2 = \begin{cases} 1 & \text{if } x \geq 70 \\ 0 & \text{if } x < 70 \end{cases}$

c. Using MINITAB, the results are:

Regression Analysis: CELLS versus X1, X2*

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	2	415.2	207.598	26.48	0.000
Error	14	109.7	7.839		
Total	16	524.9			

Model Summary

S	R-sq	R-sq(adj)
2.79981	79.09%	76.11%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	0.17	1.53	0.11	0.914
X1	0.2221	0.0340	6.53	0.000
X2*	-0.2729	0.0631	-4.33	0.001

Regression Equation

CELLS = 0.17 + 0.2221 X1 - 0.2729 X2*

The least squares line is $\hat{y} = 0.17 + 0.2221x_1 - 0.2729x_2^*$.

d. To determine if the overall model is useful for predicting number of cells, we test:

$$H_0: \beta_1 = \beta_2 = 0$$
$$H_a: \text{ At least one } \beta_i \neq 0$$

The test statistic is $F = 26.48$ and the p-value is $p = 0.000$. Since the p-value is less than $\alpha\left(p = 0.000 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate that the overall model is useful for predicting the number of cells at $\alpha = 0.05$.

e. Less than 70 hours: Slope $= \beta_1 = 0.2221$
More than 70 hours: Slope $= \beta_1 + \beta_2 = 0.2221 - 0.2729 = -0.0508$

f. To determine if the slopes differ, we test:

$$H_0: \beta_2 = 0$$
$$H_a: \beta_2 \neq 0$$

The test statistic is $t = -4.33$ and the p-value is $p = 0.001$. Since the p-value is less than $\alpha\left(p = 0.001 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate that that the slopes differ for less than 70 hours and more than 70 hours at $\alpha = 0.05$.

9.7 a. Knot values are 4 and 7.

b. The hypothesized model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2\left(x_1 - 4\right)x_2 + \beta_3\left(x_1 - 7\right)x_3$.
where $x_1 = x$, $x_2 = \begin{cases} 1 & \text{if } x > 4 \\ 0 & \text{if not} \end{cases}$, and $x_3 = \begin{cases} 1 & \text{if } x > 7 \\ 0 & \text{if not} \end{cases}$

c.

	$x \leq 4$	$4 < x \leq 7$	$x > 7$
Slope	β_1	$\left(\beta_1 + \beta_2\right)$	$\left(\beta_1 + \beta_2 + \beta_3\right)$

d. For every 1-point increase in performance over range $x \leq 4$, satisfaction is estimated to increase 5.05 units. For every 1-point increase in performance over the range $4 < x \leq 7$, satisfaction is estimated to increase 0.59 units. For every 1-point increase in performance over the range $7 < x \leq 10$, satisfaction is estimated to increase 1.45 units.

9.8 a. Yes, since there appears to be an increase in the slope starting at 1.5 cubic meters.

b. Since there is no discontinuity in the linear relationship, the appropriate model is:

$$E\left(y\right) = \beta_0 + \beta_1 x_1 + \beta_2 x_2^*$$

where $x_1 = x$, $x_2^* = \left(x_1 - 1.5\right)x_2$, and $x_2 = \begin{cases} 1 & \text{if } x_1 > 1.5 \\ 0 & \text{if } x_1 \leq 1.5 \end{cases}$

c. Using MINITAB, the results are:

Regression Analysis: BedLoad versus X1, X2*

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	2	0.007991	0.003996	118.39	0.000
Error	73	0.002464	0.000034		
Total	75	0.010455			

Model Summary

S	R-sq	R-sq(adj)
0.0058094	76.44%	75.79%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	-0.00487	0.00223	-2.19	0.032
X1	0.01192	0.00208	5.73	0.000
X2*	0.05812	0.00814	7.14	0.000

Regression Equation

BedLoad = -0.00487 + 0.01192 X1 + 0.05812 X2*

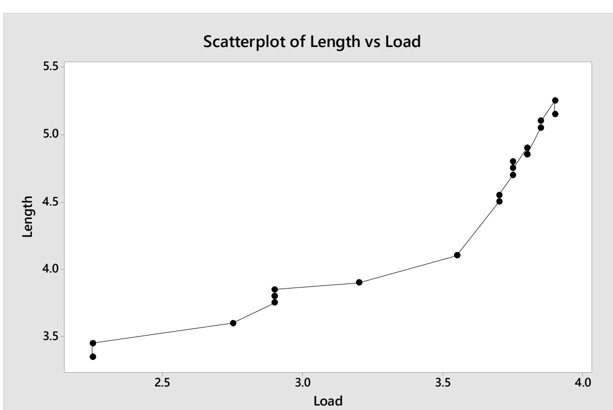
To determine the adequacy of the model, we test:

$$H_0: \beta_1 = \beta_2 = 0$$
$$H_a: \text{ At least one } \beta_i \neq 0$$

The test statistic is $F = 118.39$ and the p-value is $p = 0.000$. Since the p-value is less than $\alpha\left(p = 0.000 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate that that the overall model is useful for predicting the number of cells at $\alpha = 0.05$.

d. For every 1 cubic meter per second increase in discharge rate for flows less than 1.5 cubic meters per second, the bedload transport rate is estimated to increase by 0.01192. For every 1 cubic meter per second increase in discharge rate for flows greater than or equal to 1.5 cubic meters per second, the bedload transport rate is estimated to increase by $0.01192 + 0.05812 = 0.07004$.

9.9 a. The plot of the data is:



Yes, there is a knot at 3.55.

b. Since there is no discontinuity in the linear relationship, the appropriate model is:

$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2^*$$

where $x_1 = \text{load}$, $x_2^* = (x_1 - 3.55)x_2$ and $x_2 = \begin{cases} 1 & \text{if } x > 3.55 \\ 0 & \text{if not} \end{cases}$

c. Using MINITAB, the results are:

Regression Analysis: Length versus X1, X2*

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	2	7.27925	3.63962	1371.01	0.000
Error	17	0.04513	0.00265		
Total	19	7.32437			

Model Summary

S	R-sq	R-sq(adj)
0.0515237	99.38%	99.31%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	2.221	0.115	19.25	0.000
X1	0.5287	0.0393	13.46	0.000
X2*	2.629	0.162	16.25	0.000

Regression Equation

Length = 2.221 + 0.5287 X1 + 2.629 X2*

$$\hat{y} = 2.221 + 0.5287x_1 + 2.629x_2^*.$$

To determine the adequacy of the model, we test:

$$H_0: \beta_1 = \beta_2 = 0$$
$$H_a: \text{ At least one } \beta_i \neq 0$$

The test statistic is $F = 1371.01$ and the p-value is $p = 0.000$. Since the p-value is less than $\alpha\ (p = 0.000 < 0.05)$, H_0 is rejected. There is sufficient evidence to indicate that that the overall model is useful for predicting crack length at $\alpha = 0.05$.

9.10 The fitted regression line is $\hat{y} = 320.63639 + 0.08338x$. For $y_p = 325$,

$$\hat{x} = \frac{y_p - \hat{\beta}_0}{\hat{\beta}_1} = \frac{325 - 320.63639}{0.08338} = 52.334$$

9.11 The fitted regression line is $\hat{y} = -27.237 + 0.558x$. For $y_p = 75$,

$$\hat{x} = \frac{y_p - \hat{\beta}_0}{\hat{\beta}_1} = \frac{75 - (-27.237)}{0.558} = 183.22$$

For $\alpha = 0.05$ with $df = n - 2 = 8 - 2 = 6$, $t_{0.025} = 2.447$. From the printout, $s = 5.269$ and $\hat{\beta}_1 = 0.558$.

$$SS_{xx} = \sum x^2 - n(\bar{x})^2 = 245{,}080 - 8(174.25)^2 = 2175.5$$

The 95% confidence interval is

$$\hat{x} \pm t_{\alpha/2}\left(\frac{s}{\hat{\beta}_1}\right)\sqrt{1 + \frac{1}{n} + \frac{(\hat{x} - \bar{x})^2}{SS_{xx}}} \Rightarrow 183.22 \pm 2.447\left(\frac{5.269}{0.558}\right)\sqrt{1 + \frac{1}{8} + \frac{(183.22 - 174.25)^2}{2175.5}}$$

$$\Rightarrow 183.22 \pm 24.91 \Rightarrow (158.31,\ 208.13)$$

We are 95% confident that the maximal oxygen uptake value will be between 158.31 and 208.13 when the mean heart rate is at 75% of the maximum heart rate.

9.12 We want to predict the pectin amount, x, required to yield a sweetness index of $y_p = 5.8$. The fitted regression line is $\hat{y} = 6.252 - 0.002311x$. The estimate of x is calculated as follows:

$$\hat{x} = \frac{y_p - \hat{\beta}_0}{\hat{\beta}_1} = \frac{5.8 - (6.252)}{-0.00231} = 195.7$$

For $\alpha = 0.05$ with $df = n - 2 = 24 - 2 = 22$, $t_{0.025} = 2.074$. From the printout, $s = 0.214998$ and $\hat{\beta}_1 = 6.252$.

$$SS_{xx} = \sum x^2 - n(\bar{x})^2 = 1{,}641{,}115 - 24\left(\frac{6167}{24}\right)^2 = 56{,}452.96$$

The 95% confidence interval is

$$\hat{x} \pm t_{\alpha/2}\left(\frac{s}{\hat{\beta}_1}\right)\sqrt{1 + \frac{1}{n} + \frac{(\hat{x} - \bar{x})^2}{SS_{xx}}} \Rightarrow 195.7 \pm 2.074\left(\frac{0.215}{-0.00231}\right)\sqrt{1 + \frac{1}{24} + \frac{(195.7 - 256.96)^2}{56{,}452.96}}$$

$$\Rightarrow 195.7 \pm 203.2 \Rightarrow (-7.5,\ 398.9)$$

We are 95% confident that the amount of pectin required to yield a sweetness index of 5.8 will fall between 0 and 398.9 parts per million.

9.13 Using MINITAB, the results are:

Regression Analysis: MASS versus TIME

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	89.79	89.7942	122.19	0.000
Error	21	15.43	0.7349		
Total	22	105.23			

Model Summary

S	R-sq	R-sq(adj)
0.857257	85.33%	84.64%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	5.221	0.296	17.64	0.000
TIME	-0.1140	0.0103	-11.05	0.000

Regression Equation

MASS = 5.221 - 0.1140 TIME

The fitted regression line is $\hat{y} = 5.221 - 0.114x$. We want to predict the time, x, required to yield a spillage mass of $y_p = 5$.

$$\hat{x} = \frac{y_p - \hat{\beta}_0}{\hat{\beta}_1} = \frac{5 - 5.221}{-0.114} = 1.94$$

For $\alpha = 0.05$ with $df = n - 2 = 23 - 2 = 21$, $t_{0.025} = 2.080$. From the printout, $s = 0.857257$ and $\hat{\beta}_1 = -0.114$.

$$SS_{xx} = \sum x^2 - n(\bar{x})^2 = 18{,}936 - 23\left(\frac{526}{23}\right)^2 = 6906.609$$

The 95% confidence interval is

$$\hat{x} \pm t_{\alpha/2}\left(\frac{s}{\hat{\beta}_1}\right)\sqrt{1 + \frac{1}{n} + \frac{(\hat{x} - \bar{x})^2}{SS_{xx}}} \Rightarrow 1.94 \pm 2.08\left(\frac{0.857257}{-0.114}\right)\sqrt{1 + \frac{1}{23} + \frac{(1.94 - 22.87)^2}{6906.609}}$$

$$\Rightarrow 1.94 \pm 16.46 \Rightarrow (-14.52, 18.40)$$

We are 95% confident that the amount of time used to produce 5 pounds of spillage will fall between 0 and 18.40 minutes.

9.14 Using MINITAB, the results are:

Regression Analysis: DAMAGE versus DISTANCE

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	841.77	841.766	156.89	0.000
Error	13	69.75	5.365		
Total	14	911.52			

Model Summary

S	R-sq	R-sq(adj)
2.31635	92.35%	91.76%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	10.28	1.42	7.24	0.000
DISTANCE	4.919	0.393	12.53	0.000

Regression Equation

DAMAGE = 10.28 + 4.919 DISTANCE

The fitted regression line is $\hat{y} = 10.28 + 4.919x$. We want to predict the distance, x, for a residential fire causing damage of $y_p = 18.2$.

$$\hat{x} = \frac{y_p - \hat{\beta}_0}{\hat{\beta}_1} = \frac{18.2 - 10.28}{4.919} = 1.61$$

For $\alpha = 0.10$ with $df = n - 2 = 15 - 2 = 13$, $t_{0.05} = 1.771$. From the printout, $s = 2.31635$ and $\hat{\beta}_1 = 4.919$.

$$SS_{xx} = \sum x^2 - n(\bar{x})^2 = 196.16 - 15\left(\frac{49.2}{15}\right)^2 = 34.784$$

The 90% confidence interval is

$$\hat{x} \pm t_{\alpha/2}\left(\frac{s}{\hat{\beta}_1}\right)\sqrt{1 + \frac{1}{n} + \frac{(\hat{x} - \bar{x})^2}{SS_{xx}}} \Rightarrow 1.61 \pm 1.771\left(\frac{2.31635}{4.919}\right)\sqrt{1 + \frac{1}{15} + \frac{(1.61 - 3.28)^2}{34.784}}$$

$$\Rightarrow 1.61 \pm 0.893 \Rightarrow (0.717, 2.503)$$

We are 90% confident that the distance from a fire where the damage was \$18,200 will fall between 0.717 and 2.503 miles.

9.15 a. Using MINITAB, the results are:

Regression Analysis: DECREASE versus DOSAGE

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	265.15	265.152	107.14	0.000
Error	6	14.85	2.475		
Total	7	280.00			

Model Summary

S	R-sq	R-sq(adj)
1.57313	94.70%	93.81%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	-2.03	1.60	-1.27	0.252
DOSAGE	6.061	0.586	10.35	0.000

Regression Equation

DECREASE = -2.03 + 6.061 DOSAGE

The fitted model is $\hat{y} = -2.03 + 6.061x$.

b. To determine if the model is adequate, we test:

$$H_0: \beta_1 = 0$$
$$H_a: \beta_1 \neq 0$$

The test statistic is $t = 10.35$ and the p-value is $p = 0.000$. Since the p-value is less than $\alpha\left(p = 0.000 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate that that the overall model is useful for predicting decrease in pulse rate at $\alpha = 0.05$.

c. We want to predict dosage, x, for a reduction in pulse rate of $y_p = 10$.

$$\hat{x} = \frac{y_p - \hat{\beta}_0}{\hat{\beta}_1} = \frac{10 - (-2.03)}{6.061} = 1.985$$

For $\alpha = 0.05$ with $df = n - 2 = 8 - 2 = 6$, $t_{0.025} = 2.447$. From the printout, $s = 1.57313$ and $\hat{\beta}_1 = 6.061$.

$$SS_{xx} = \sum x^2 - n\left(\bar{x}\right)^2 = 59.75 - 8\left(\frac{20.5}{8}\right)^2 = 7.21875$$

The 95% confidence interval is

$$\hat{x} \pm t_{\alpha/2}\left(\frac{s}{\hat{\beta}_1}\right)\sqrt{1 + \frac{1}{n} + \frac{(\hat{x} - \bar{x})^2}{SS_{xx}}}$$

$$\Rightarrow 1.985 \pm 2.447\left(\frac{1.57313}{6.061}\right)\sqrt{1 + \frac{1}{8} + \frac{(1.985 - 2.5625)^2}{7.21875}}$$

$$\Rightarrow 1.985 \pm 0.687 \Rightarrow (1.298,\ 2.672)$$

We are 95% confident that to reduce a patient's pulse rate by 10 beats/minute, the dosage will be between 1.298 cc and 2.672 cc.

9.16 a. Use weights $w_i = \frac{1}{x_i^2}$ b. Use weights $w_i = \frac{1}{\sqrt{x_i}}$ c. Use weights $w_i = \frac{1}{x_i}$

d. Use weights $w_i = \frac{1}{1/n_i} = n_i$ e. Use weights $w_i = \frac{1}{1/x_i} = x_i$

9.17 a. Using MINITAB, the results are:

Regression Analysis: DEFECTS versus SPEED

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	940.9	940.90	20.45	0.001
Error	13	598.0	46.00		
Total	14	1538.9			

Model Summary

S	R-sq	R-sq(adj)
6.78252	61.14%	58.15%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	-3.37	6.67	-0.50	0.622
SPEED	0.1940	0.0429	4.52	0.001

Regression Equation

DEFECTS = -3.37 + 0.1940 SPEED

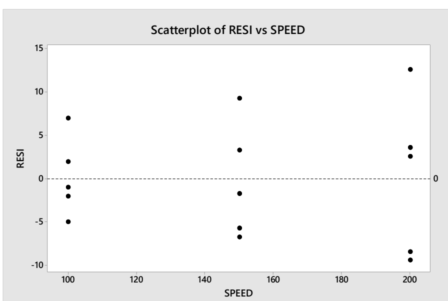
The fitted model is $\hat{y} = -3.37 + 0.194x$.

To determine if the model is useful for predicting the number of defectives, we test:

$$H_0: \beta_1 = 0$$
$$H_a: \beta_1 \neq 0$$

The test statistic is $t = 4.52$ and the p-value is $p = 0.001$. Since the p-value is less than $\alpha\left(p = 0.001 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate that that the model is useful for predicting the number of defectives at $\alpha = 0.05$.

b. The plot of the residuals versus speed is:



The assumption of constant variance appears to be violated. As speed increases, the spread of the residuals increases.

c. Let $\hat{\varepsilon}_i = (y_i - \hat{y}_i)$ represent the residual for the ith-observation. To obtain the variance of the 5 residuals at each level of x, we use the short-cut formula for the sample variance.

$$s^2 = \frac{\sum_{i=1}^{5} \hat{\varepsilon}_i^2 - \frac{\left(\sum_{i=1}^{5} \hat{\varepsilon}_i\right)^2}{5}}{5-1}$$

The results are shown in the following table:

$\bar{x}$	**Variance of Residuals** s^2	$s^2/\bar{x}$	$s^2/\bar{x}^2$	$s^2/\sqrt{\bar{x}}$
100	20.7	0.2070	0.00207	2.070
150	44.3	0.2953	0.00197	3.617
200	84.3	0.4215	0.00211	5.961

Note that when the variance s^2 is divided by $\bar{x}^2$ for each level of x, the result is a constant (approximately 0.002). Therefore, the variance is proportional to $\bar{x}^2$, and the appropriate weights to use in a weighted least squares are:

$$w_i = \frac{1}{\bar{x}_i^2}$$

d. Using MINITAB, the results of the weighted least squares is:

Regression Analysis: DEFECTS versus SPEED

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	0.04224	0.042240	22.30	0.000
Error	13	0.02462	0.001894		
Total	14	0.06686			

Model Summary

S	R-sq	R-sq(adj)
0.0435195	63.18%	60.34%

Coefficients

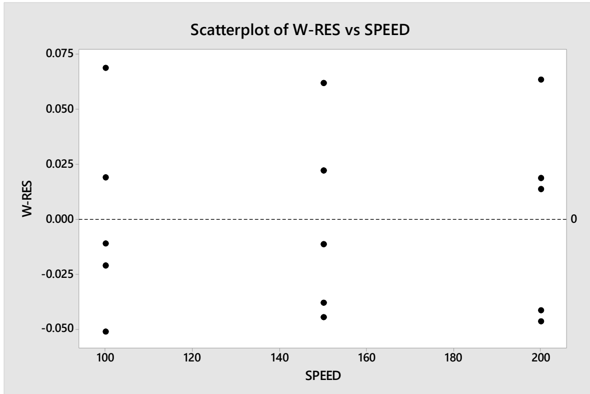
Term	Coef	SE Coef	T-Value	P-Value
Constant	-3.06	5.41	-0.57	0.581
SPEED	0.1919	0.0406	4.72	0.000

Regression Equation

DEFECTS = -3.06 + 0.1919 SPEED

The standard deviation of the unweighted least squares slope is 0.0429 while the standard deviation of the weighted least squares slope is 0.0406, a slight improvement.

e. The weighted residuals are found using $\sqrt{w_i}\left(y_i - \hat{y}_i\right)$, where $\hat{y}_i$ is the predicted value of y_i obtained using the weight w_i in the weighted least squares regression. The plot of the weighted residuals is:



From the residual plot, it is evident that the spread of the residuals for the three values of x is constant. Thus, it appears that weighted least squares has corrected the problem of unequal variances.

9.18 Using MINITAB, the results are:

Regression Analysis: SALARY versus EXP, EXPSQ

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	2	13723582237	6861791118	103.99	0.000
Error	47	3101279310	65984666		
Total	49	16824861546			

Model Summary

S	R-sq	R-sq(adj)
8123.09	81.57%	80.78%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	20242	4423	4.58	0.000
EXP	522	617	0.85	0.401
EXPSQ	53.0	19.6	2.71	0.009

Regression Equation

SALARY = 20242 + 522 EXP + 53.0 EXPSQ

To determine the proper weights, we use the following:

Group	Range of x	$\bar{x}_j$	s_j^2	$\frac{s_j^2}{\bar{x}_j}$	$\frac{s_j^2}{\bar{x}_j^2}$	$\frac{s_j^2}{\sqrt{\bar{x}_j}}$
1	$1 \le x \le 10$	4.25	15,525,326.75	3,653,018.06	859,533.66	7,530,889.65
2	$11 \le x \le 20$	16.864	57,582,147.82	3,414,574.80	202,481.52	14,022,073.71
3	$21 \le x \le 30$	25.05	93,581,188.29	3,735,775.98	149,132.77	18,697,549.45

The column where the values are similar is the column $\frac{s_j^2}{\bar{x}_j}$. Thus, we would suggest using the weight of $w_j = \frac{1}{\bar{x}_j}$.

9.19 a. Using MINITAB, the results are:

Regression Analysis: ARSENIC versus DEPTH-FT

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	237209	237209	20.75	0.000
Error	325	3715353	11432		
Total	326	3952562			

Model Summary

S	R-sq	R-sq(adj)
106.920	6.00%	5.71%

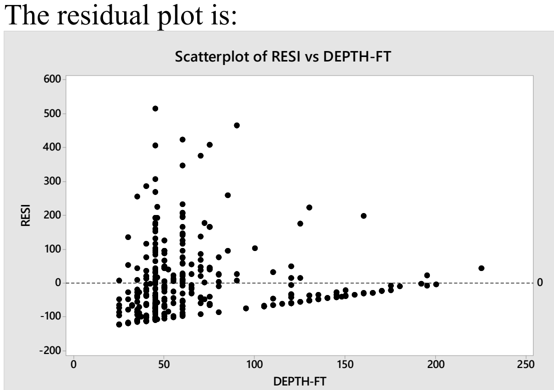
Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	140.6	11.8	11.93	0.000
DEPTH-FT	-0.669	0.147	-4.56	0.000

Regression Equation

ARSENIC = 140.6 - 0.669 DEPTH-FT

The residual plot is:



The assumption of constant variance appears to be violated. As the depth increases, the spread of the variance decreases. However, since there are very few observations for large values of x (depth), it is difficult to determine if there really is a problem with non-constant variance.

b. To determine the proper weights, we use the following:

Grp	Range	$\bar{x}_j$	s_j^2	$s_j^2/\bar{x}_j$	$s_j^2/\bar{x}_j^2$	$s_j^2/\sqrt{\bar{x}_j}$	$s_j^2(\bar{x}_j)$	$s_j^2(\bar{x}_j)^2$	$s_j^2\sqrt{\bar{x}_j}$
1	$25 \le x \le 85$	49.826	12787.730	256.6473	5.1509	1811.6115	637,162.54	31,747,316.03	90,265.51
2	$90 \le x \le 155$	124.362	7193.473	57.8430	0.4651	645.0519	894,595.21	111,253,711.75	80,219.99
3	$160 \le x \le 225$	178.563	3162.062	17.7084	0.0992	236.6329	564,625.62	100,820,961.99	42,253.77

Since the spread of the residuals decreases as the depth increases, the weight will have to be a function of $1/\bar{x}_j$ rather than a function of $\bar{x}_j$. From the table above, it appears that the best weight would be $1/\left(1/\bar{x}_j\right) = \bar{x}_j$.

c. Using $\bar{x}_j$ as the weight, the results are:

Regression Analysis: ARSENIC versus DEPTH-FT

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	31681581	31681581	46.14	0.000
Error	325	223180138	686708		
Total	326	254861719			

Model Summary

S	R-sq	R-sq(adj)
828.679	12.43%	12.16%

Coefficients

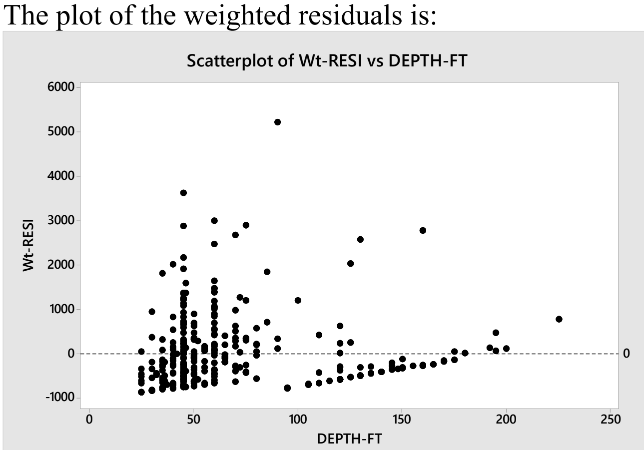
Term	Coef	SE Coef	T-Value	P-Value
Constant	143.6	11.3	12.67	0.000
DEPTH-FT	-0.749	0.110	-6.79	0.000

Regression Equation

ARSENIC = 143.6 - 0.749 DEPTH-FT

Comparing the original model in part **a** to this model, we find that the R^2 has increased from 0.0600 to 0.1243. In addition, the standard error of the slope has decreased from 0.147 to 0.110.

The plot of the weighted residuals is:



Although the weighted least squares regression has a slightly better fit, the problem of non-constant variance does not appear to be corrected.

9.20 a. Using MINITAB, the results are:

Regression Analysis: HEATRATE versus INLET-TEMP

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	107601712	107601712	116.00	0.000
Error	65	60295496	927623		
Total	66	167897208			

Model Summary

S	R-sq	R-sq(adj)
963.132	64.09%	63.54%

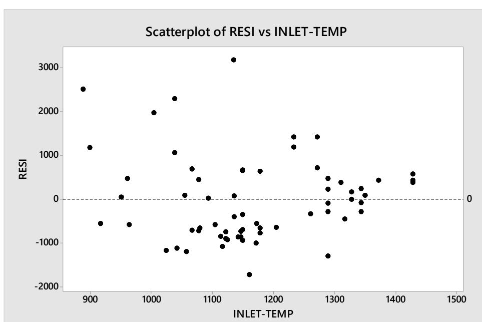
Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	21977	1020	21.55	0.000
INLET-TEMP	-9.291	0.863	-10.77	0.000

Regression Equation

HEATRATE = 21977 - 9.291 INLET-TEMP

The residual plot is:



It appears that as the inlet temperature increases, the spread of the residuals tends to decrease. However, there really does not appear to be much of a problem.

b. To determine the proper weights, we use the following:

Group	Range	$\bar{x}_j$	s_j^2	$s_j^2 / \bar{x}_j$	$s_j^2 / \bar{x}_j^2$	$s_j^2 / \sqrt{\bar{x}_j}$
1	$888 \le x \le 1057$	987.08	1,755,598.27	1778.583	1.8019	55,879.13
2	$1058 \le x \le 1135$	1104.07	1,159,125.55	1049.869	0.9509	34,884.53
3	$1136 \le x \le 1177$	1158.00	492,545.24	425.341	0.3673	14,474.11
4	$1178 \le x \le 1310$	1268.33	714,271.65	563.158	0.4440	20,056.11
5	$1311 \le x \le 1427$	1373.38	96,384.44	70.180	0.0511	2,600.82

No weight looks particularly good since there did not appear to be a real problem with non-constant variance. Thus, we would suggest using the weight of $w_j = \dfrac{1}{\bar{x}_j}$.

c. Using MINITAB, the results of the weighted least squares are:

Regression Analysis: HEATRATE versus INLET-TEMP

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	96622	96622.4	113.60	0.000
Error	65	55285	850.5		
Total	66	151908			

Model Summary

S	R-sq	R-sq(adj)
29.1641	63.61%	63.05%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	22273	1046	21.30	0.000
INLET-TEMP	-9.545	0.896	-10.66	0.000

Regression Equation

HEATRATE = 22273 - 9.545 INLET-TEMP

Comparing the original model in part **a** to this model, we find that the R^2 has decreased from 0.6409 to 0.6361. In addition, the standard error of the slope has increased from 0.863 to 0.896. As noted earlier, there really does not appear to be much of a problem with the assumption of non-constant variance. Using the weighted least squares has not improved the model.

9.21 When the response (i.e., dependent) variable y is recorded as 0 or 1, three major problems arise:

1. The assumption of normal errors is violated.
2. The assumption of equal error variances is violated.
3. Predicted values may take on nonsensical values (i.e., negative values or values greater than 1).

The first two problems are the most serious, since we have little or no faith in any inferences derived from an ordinary least squares regression model when the standard regression assumptions are violated. There is a violation of constant error variance assumption; violation of assumption of normal errors; predicted y is not bounded between 0 and 1.

9.22 **Stage 1:**

Using MINITAB, the results of the first stage are:

Regression Analysis: OWN versus INCOME

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	0.6684	0.6684	3.10	0.095
Error	18	3.8816	0.2156		
Total	19	4.5500			

Model Summary

S	R-sq	R-sq(adj)
0.464373	14.69%	9.95%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	-0.182	0.319	-0.57	0.576
INCOME	0.000012	0.000007	1.76	0.095

Regression Equation

OWN = -0.182 + 0.000012 INCOME

The least squares equation is $\hat{y} = -0.182 + 0.000012x$. The predicted values are:

Obs. i	$\hat{y}_i$	Obs. i	$\hat{y}_i$
1	0.251	11	0.324
2	0.190	12	0.183
3	0.492	13	0.312
4	0.315	14	0.156
5	0.536	15	0.385
6	0.204	16	0.244
7	0.235	17	0.201
8	0.166	18	0.767
9	0.494	19	0.297
10	0.796	20	0.451

Stage 2:

The predicted values above are used to calculate the appropriate weights for weighted least squares regression, where $w_i = \dfrac{1}{\hat{y}_i\left(1-\hat{y}_i\right)}$.

Using MINITAB, the weighted least squares regression results are:

Regression Analysis: OWN versus INCOME

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	3.675	3.675	3.53	0.076
Error	18	18.718	1.040		
Total	19	22.394			

Model Summary

S	R-sq	R-sq(adj)
1.01975	16.41%	11.77%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	-0.205	0.284	-0.72	0.480
INCOME	0.000012	0.000006	1.88	0.076

Regression Equation

OWN = -0.205 + 0.000012 INCOME

The weighted least squares equation is $\hat{y} = -0.205 + 0.000012x$.
To determine if the model is useful for predicting y, we test:

$$H_0: \beta_1 = 0$$
$$H_a: \beta_1 > 0$$

The test statistic is $t = 1.88$ and the p-value is $p = 0.076/2 = 0.038$. Since the p-value is less than $\alpha\left(p = 0.038 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate that the model is useful for predicting y at $\alpha = 0.05$.

9.23 a. $\beta_0 =$ y-intercept since $x_1 = 0$ is not in the observed range of years of higher education
$\beta_1 =$ change in the probability of being hired for each additional year of education, all other variables held constant.
$\beta_2 =$ change in the probability of being hired for each additional year of experience, all other variables held constant.
$\beta_3 =$ difference in the probability of being hired between males and females, all other variables held constant.

b. **Stage 1:**

Using MINITAB, the results are:

Regression Analysis: HIRE versus EDUC, EXP, GENDER
Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	3	3.279	1.0929	9.27	0.000
Error	24	2.828	0.1179		
Total	27	6.107			

Model Summary

S	R-sq	R-sq(adj)
0.343298	53.69%	47.90%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	-0.844	0.281	-3.00	0.006
EDUC	0.1069	0.0423	2.53	0.018
EXP	0.0886	0.0207	4.28	0.000
GENDER	0.485	0.138	3.51	0.002

Regression Equation

HIRE = -0.844 + 0.1069 EDUC + 0.0886 EXP + 0.485 GENDER

The least squares equation is $\hat{y} = -0.844 + 0.1069x_1 + 0.0886x_2 + 0.485x_3$. The predicted values are:

Obs. i	1	2	3	4	5	6	7	8	9	10
$\hat{y}_i$	-0.026	0.068	0.814	0.548	-0.328	0.277	0.245	-0.062	-0.114	0.897
Obs. i	11	12	13	14	15	16	17	18	19	20
$\hat{y}_i$	0.245	0.454	-0.239	0.418	0.511	0.152	0.496	0.371	0.204	0.157
Obs. i	21	22	23	24	25	26	27	28		
$\hat{y}_i$	0.026	0.282	0.939	0.381	0.100	0.371	0.954	0.861		

Stage 2:

The predicted values above are used to calculate the appropriate weights for weighted least squares regression, where $w_i = \dfrac{1}{\hat{y}_i(1-\hat{y}_i)}$. Note: Where the predicted values are less than 0, we will replace the values with 0.01.

Using MINITAB, the weighted least squares regression results are:

Regression Analysis: HIRE versus EDUC, EXP, GENDER

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	3	53.93	17.9752	21.79	0.000
Error	24	19.80	0.8250		
Total	27	73.73			

Model Summary

S	R-sq	R-sq(adj)
0.908294	73.14%	69.79%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	-0.528	0.144	-3.66	0.001
EDUC	0.0750	0.0271	2.77	0.011
EXP	0.0747	0.0139	5.36	0.000
GENDER	0.3912	0.0976	4.01	0.001

Regression Equation

HIRE = -0.528 + 0.0750 EDUC + 0.0747 EXP + 0.3912 GENDER

The weighted least squares equation is $\hat{y} = -0.528 + 0.0750x_1 + 0.0747x_2 + 0.3912x_3$.

To determine if the model is useful for predicting y, we test:

$$H_0: \beta_1 = \beta_2 = \beta_3 = 0$$
$$H_a: \text{At least one } \beta_i \neq 0$$

The test statistic is $F = 21.79$ and the p-value is $p = 0.000$. Since the p-value is less than $\alpha\left(p = 0.000 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate that the model is useful for predicting y at $\alpha = 0.05$.

d. To determine if gender is an important predictor of hiring status, we test:

$H_0: \beta_3 = 0$
$H_a: \beta_3 \neq 0$

The test statistic is $t = 4.01$ and the p-value is $p = 0.001$. Since the p-value is less than $\alpha\left(p = 0.001 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate that gender is an important predictor of hiring status y. Since the estimate of β_3 is positive, it appears that males have a higher probability of being hired than females.

e. Using MINITAB, the 95% confidence interval for $E\left(y\right)$ when $x_1 = 4, x_2 = 3,$ and $x_3 = 0$:

Prediction for HIRE

Settings

Variable	Setting
EDUC	4
EXP	3
GENDER	0

Prediction

Fit	SE Fit	95% CI	95% PI
-0.0037493	0.0450864	(-0.0968032, 0.0893045)	(-0.212196, 0.204697)

Weight = 101.01

The confidence interval is $\left(-0.0968, 0.0893\right)$. That is, we estimate the probability of being hired $E\left(y\right)$, for female applicants $\left(x_3 = 0\right)$ with $x_1 = 4$ years of education and $x_2 = 3$ years of experience to fall between -0.0968 and 0.0893. Note that the interval includes negative numbers, which are nonsensical probabilities.

9.24 a. $\pi = P\left(y = 1\right)$ for each value of x. For this problem π is the probability that the household owns a digital organizer given the households annual income.

b. $\text{odds} = \dfrac{\pi}{1-\pi}$ This represents the odds of that a household owns a digital organizer.

c. To determine if the model is useful in the prediction of PC ownership, we test:

$H_0: \beta_1 = 0$
$H_a: \beta_1 \neq 0$

The test statistic is $\chi^2 = 2.4533$ and the p-value is $p = 0.117$. Since the p-value is not less than $\alpha\left(p = 0.117 \not< 0.10\right)$, H_0 is not rejected. There is sufficient evidence to indicate that the annual income of the head of the household is a useful predictor of PC ownership at $\alpha = 0.10$.

9.25 a. $\pi =$ probability that the rock sample is from the Maryland nappe.

b. The model is $\pi^* = \beta_0 + \beta_1 x_1$ where $\pi^* = \ln\left(\dfrac{\pi}{(1-\pi)}\right)$

c. β_1 is the change in the log-odds of a Maryland nappe for every 1-unit increase in FIA.

d. $\hat{y} = \dfrac{\exp(\beta_0 + 80\beta_1)}{\left[1+\exp(\beta_0 + 80\beta_1)\right]}$

9.26 a. To determine if the overall model is statistically useful for predicting geese flight response, we test:

$$H_0: \beta_1 = \beta_2 = 0$$
$$H_a: \text{At least one } \beta_i \neq 0$$

The test statistic is $\chi^2 = 259.18$ and the p-value is $p = 0.000$. Since the p-value is less than $\alpha\ (p = 0.000 < 0.01)$, H_0 is rejected. There is sufficient evidence to indicate the overall model is statistically useful for predicting geese flight response at $\alpha = 0.01$.

b. To determine if flight response of the geese depends on altitude of the helicopter, we test:

$$H_0: \beta_1 = 0$$
$$H_a: \beta_1 \neq 0$$

The test statistic is $\chi^2 = 9.38$ and the p-value is $p = 0.002$. Since the p-value is less than $\alpha\ (p = 0.002 < 0.01)$, H_0 is rejected. There is sufficient evidence to indicate the flight response of the geese depends on altitude of the helicopter at $\alpha = 0.01$.

c. To determine if flight response of the geese depends on lateral distance of the helicopter from the flock, we test:

$$H_0: \beta_2 = 0$$
$$H_a: \beta_2 \neq 0$$

The test statistic is $\chi^2 = 252.48$ and the p-value is $p = 0.000$. Since the p-value is less than $\alpha\ (p = 0.002 < 0.01)$, H_0 is rejected. There is sufficient evidence to indicate the flight response of the geese depends on lateral distance of the helicopter from the flock at $\alpha = 0.01$.

d. The fitted model is $\hat{\pi}^* = 2.395 + 0.1965x_1 - 0.2388x_2$. When $x_1 = 6$ and $x_2 = 3$,

$$\hat{\pi} = \frac{e^{(2.395+0.1965(6)-0.2388(3))}}{\left[1+e^{(2.395+0.1965(6)-0.2388(3))}\right]} = 0.946$$

9.27 a. Using MINITAB, the results are:

Binary Logistic Regression: HIRE versus EDUC, EXP, GENDER

Response Information

Variable	Value	Count	
HIRE	1	9	(Event)
	0	19	
	Total	28	

Deviance Table

Source	DF	Adj Dev	Adj Mean	Chi-Square	P-Value
Regression	3	20.430	6.8100	20.43	0.000
EDUC	1	6.112	6.1117	6.11	0.013
EXP	1	14.878	14.8775	14.88	0.000
GENDER	1	11.321	11.3209	11.32	0.001
Error	24	14.735	0.6140		
Total	27	35.165			

Model Summary

Deviance R-Sq	Deviance R-Sq(adj)	AIC
58.10%	49.57%	22.73

Coefficients

Term	Coef	SE Coef	VIF
Constant	-14.25	6.08	
EDUC	1.155	0.602	1.83
EXP	0.910	0.429	3.64
GENDER	5.60	2.60	3.55

Odds Ratios for Continuous Predictors

	Odds Ratio	95% CI
EDUC	3.1736	(0.9747, 10.3331)
EXP	2.4839	(1.0708, 5.7618)
GENDER	271.4232	(1.6526, 44579.8931)

To determine if the model is adequate, we test:

$$H_0 : \beta_1 = \beta_2 = \beta_3 = 0$$
$$H_a : \text{At least one } \beta_i \neq 0$$

The test statistic is $\chi^2 = 20.43$ and the p-value is $p = 0.000$. Since the p-value is less than $\alpha\left(p = 0.000 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate the overall model is adequate at $\alpha = 0.05$.

b. To determine if gender is an important predictor of hiring status, we test:

$$H_0 : \beta_3 = 0$$
$$H_a : \beta_3 \neq 0$$

The test statistic is $\chi^2 = 11.32$ and the *p*-value is $p = 0.001$. Since the *p*-value is less than $\alpha\left(p = 0.001 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate gender is an important predictor of hiring status at $\alpha = 0.05$.

c. Using MINITAB, the results are:

Prediction for HIRE

Settings

Variable	Setting
EDUC	4
EXP	0
GENDER	1

Prediction

Fitted Probability	SE Fit	95% CI
0.0175489	0.0318498	(0.0004778, 0.400267)

When $x_1 = 4,\ x_2 = 0,$ and $x_3 = 1,$ the 95% confidence interval for the mean response is $\left(0.0004778,\ 0.400267\right)$.

9.28 From Exercise 6.11, the two variables selected as significant were well depth and percentage of adjacent land allocated to industry. Using MINITAB, the results are:

Binary Logistic Regression: MTBE1 versus Depth, IndPct

Response Information

Variable	Value	Count	
MTBE1	1	63	(Event)
	0	128	
	Total	191	

Deviance Table

Source	DF	Adj Dev	Adj Mean	Chi-Square	P-Value
Regression	2	10.153	5.077	10.15	0.006
Depth	1	5.108	5.108	5.11	0.024
IndPct	1	3.817	3.817	3.82	0.051
Error	188	232.060	1.234		
Total	190	242.214			

Model Summary

Deviance R-Sq	Deviance R-Sq(adj)	AIC
4.19%	3.37%	238.06

Coefficients

Term	Coef	SE Coef	VIF
Constant	-1.447	0.316	
Depth	0.00625	0.00280	1.01
IndPct	0.0538	0.0279	1.01

Odds Ratios for Continuous Predictors

	Odds Ratio	95% CI
Depth	1.0063	(1.0008, 1.0118)
IndPct	1.0552	(0.9991, 1.1145)

To determine if the model is adequate for predicting detectible levels of MTBE, we test:

$H_0: \beta_1 = \beta_2 = 0$
$H_a:$ At least one $\beta_i \neq 0$

The test statistic is $\chi^2 = 10.15$ and the p-value is $p = 0.006$. Since the p-value is less than $\alpha\ (p = 0.006 < 0.05)$, H_0 is rejected. There is sufficient evidence to indicate the overall model is adequate for predicting detectible levels of MTBE at $\alpha = 0.05$.

Depth is a significant predictor of detectible levels of MTBE $\left(\chi^2 = 5.11,\ p = 0.024\right)$ and the percentage of adjacent land allocated to industry is very close to being a significant predictor of detectible levels of MTBE $\left(\chi^2 = 3.82,\ p = 0.051\right)$. Thus, we would recommend using this model.

9.29 a. $\pi =$ probability of landfast ice.

b. The model is $\pi^* = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$, where $\pi^* = \ln\left(\frac{\pi}{(1-\pi)}\right)$.

c. Using MINITAB, the results are:

Binary Logistic Regression: y versus x1, x2, x3

Response Information

Variable	Value	Count	
y	1	196	(Event)
	0	220	
	Total	416	

Deviance Table

Source	DF	Adj Dev	Adj Mean	Chi-Square	P-Value
Regression	3	70.46	23.485	70.46	0.000
x1	1	29.06	29.056	29.06	0.000
x2	1	64.06	64.059	64.06	0.000
x3	1	63.95	63.948	63.95	0.000
Error	412	504.86	1.225		
Total	415	575.31			

Model Summary

Deviance R-Sq	Deviance R-Sq(adj)	AIC
12.25%	11.73%	512.86

Coefficients

Term	Coef	SE Coef	VIF
Constant	0.296	0.434	
x1	4.128	0.809	1.34
x2	47.12	7.48	27.28
x3	-31.14	4.73	26.27

Odds Ratios for Continuous Predictors

	Odds Ratio	95% CI
x1	62.0648	(12.7053, 303.1827)
x2	2.91936E+20	(1.26237E+14, 6.75132E+26)
x3	0.0000	(0.0000, 0.0000)

The fitted prediction equation is $\hat{\pi}^* = 0.296 + 4.128x_1 + 47.12x_2 - 31.14x_3$.

d. To determine if the overall model is adequate, we test:

$$H_0: \beta_1 = \beta_2 = \beta_3 = 0$$
$$H_a: \text{At least one } \beta_i \neq 0$$

The test statistic is $\chi^2 = 70.46$ and the p-value is $p = 0.000$. Since the p-value is less than $\alpha\,(p = 0.000 < 0.05)$, H_0 is rejected. There is sufficient evidence to indicate the overall model is adequate in predicting the probability of landfast ice at $\alpha = 0.05$.

e. The model is $\pi^* = \beta_0 + \beta_1x_1 + \beta_2x_2 + \beta_3x_3 + \beta_4x_1x_2 + \beta_5x_1x_3 + \beta_6x_2x_3$.

f. Using MINITAB, the results of fitting the complete model are:

Binary Logistic Regression: y versus x1, x2, x3, x1x2, x1x3, x2x3

Response Information

Variable	Value	Count	
y	1	196	(Event)
	0	220	
	Total	416	

Deviance Table

Source	DF	Adj Dev	Adj Mean	Chi-Square	P-Value
Regression	6	102.649	17.1082	102.65	0.000
x1	1	0.985	0.9850	0.99	0.321
x2	1	0.733	0.7331	0.73	0.392
x3	1	37.632	37.6318	37.63	0.000
x1x2	1	1.567	1.5666	1.57	0.211
x1x3	1	0.065	0.0646	0.06	0.799
x2x3	1	26.783	26.7829	26.78	0.000
Error	409	472.664	1.1557		
Total	415	575.313			

Model Summary

Deviance R-Sq	Deviance R-Sq(adj)	AIC
17.84%	16.80%	486.66

Coefficients

Term	Coef	SE Coef	VIF
Constant	6.10	1.44	
x1	-3.00	3.01	18.12
x2	10.6	12.5	63.74
x3	-39.69	7.73	55.66
x1x2	50.5	39.6	126.24
x1x3	-6.1	24.1	167.21
x2x3	56.2	12.1	33.77

Odds Ratios for Continuous Predictors

	Odds Ratio	95% CI
x1	0.0500	(0.0001, 18.2084)
x2	38499.6206	(0.0000, 1.71183E+15)
x3	0.0000	(0.0000, 0.0000)
x1x2	8.46393E+21	(0.0000, 3.99376E+55)
x1x3	0.0021	(0.0000, 6.51110E+17)
x2x3	2.66545E+24	(1.45796E+14, 4.87299E+34)

The fitted prediction equation is
$\hat{\pi}^* = 6.10 - 3.00x_1 + 10.6x_2 - 39.69x_3 + 50.5x_1x_2 - 6.1x_1x_3 + 56.2x_2x_3$.

g. To determine if the interaction terms contribute to the model, we test:

$H_0: \beta_4 = \beta_5 = \beta_6 = 0$
$H_a:$ At least one $\beta_i \neq 0$

The test statistic is the difference in the χ^2 values from fitting the complete model and the reduced model. The test statistic is $\chi^2 = \chi^2_{\text{Complete}} - \chi^2_{\text{Reduced}}$ $= 102.65 - 70.46 = 32.19$.

Using $\alpha = 0.01$, the critical value is $\chi^2_{3,0.01} = 11.3449$. The rejection region is $\chi^2 > 11.3449$. Since the observed value of the test statistic falls in the rejection region $\left(\chi^2 = 32.19 > 11.3449\right)$, H_0 is rejected. There is sufficient evidence to indicate at least one of the interaction terms contribute to the model at $\alpha = 0.01$.

9.30 First, we must code the variables. Let $y = \begin{cases} 1 \text{ if sweet} \\ 0 \text{ if sour} \end{cases}$ and $x = \begin{cases} 1 \text{ if angular} \\ 0 \text{ if rounded} \end{cases}$

The model is $\pi^* = \beta_0 + \beta_1 x$, where $\pi^* = \ln\left(\frac{\pi}{(1-\pi)}\right)$.

Using MINITAB, the results of the logistic regression are:

Binary Logistic Regression: Y versus X

Response Information

Variable	Value	Count	
Y	1	42	(Event)
	0	38	
	Total	80	

Deviance Table

Source	DF	Adj Dev	Adj Mean	Chi-Square	P-Value
Regression	1	43.46	43.4637	43.46	0.000
Error	78	67.24	0.8620		
Total	79	110.70			

Model Summary

Deviance R-Sq	Deviance R-Sq(adj)	AIC
39.26%	38.36%	71.24

Coefficients

Term	Coef	SE Coef	VIF
Constant	-1.551	0.416	
X	3.497	0.634	1.00

Odds Ratios for Continuous Predictors

	Odds Ratio	95% CI
X	33.0000	(9.5280, 114.2941)

The fitted prediction equation is $\hat{\pi}^* = -1.551 + 3.497x$.
To determine if the model is adequate for predicting sweet-tasting food, we test:

$$H_0: \beta_1 = 0$$
$$H_a: \beta_1 \neq 0$$

The test statistic is $\chi^2 = 43.46$ and the p-value is $p = 0.000$. Since the p-value is less than $\alpha\ (p = 0.000 < 0.05)$, H_0 is rejected. There is sufficient evidence to indicate the overall model is adequate in predicting sweet-tasting food at $\alpha = 0.05$.

9.31 a. Using MINITAB, the descriptive statistics are:

Descriptive Statistics: LOS

Statistics

Variable	N	Mean	Variance
LOS	50	6.540	6.907

The mean (6.540) is very similar to the variance (6.907). Thus, Poisson regression is justified.

b. The lengths of stay are whole numbers or count data. The value of length of stay cannot be negative, unlike normal data with a mean and variance about the same.

c. The Poisson regression model would be $E(y) = e^{\beta_0} \cdot e^{\beta_1 x}$ or $\ln\{E(y)\} = \beta_0 + \beta_1 x$.

d. Using MINITAB, the results of fitting a Poisson model are:

Poisson Regression Analysis: LOS versus FACTORS
Deviance Table

Source	DF	Adj Dev	Adj Mean	Chi-Square	P-Value
Regression	1	18.39	18.3883	18.39	0.000
Error	48	32.64	0.6800		
Total	49	51.03			

Model Summary

Deviance R-Sq	Deviance R-Sq(adj)	AIC
36.03%	34.07%	219.77

Coefficients

Term	Coef	SE Coef	VIF
Constant	1.392	0.130	
FACTORS	0.002094	0.000478	1.00

Goodness-of-Fit Tests

Test	DF	Estimate	Mean	Chi-Square	P-Value
Deviance	48	32.64133	0.68003	32.64	0.956
Pearson	48	32.69350	0.68111	32.69	0.955

e. To determine if the model is adequate, we test:

$H_0: \beta_1 = 0$
$H_a: \beta_1 \neq 0$

The test statistic is $\chi^2 = 18.39$ and the p-value is $p = 0.000$. Since the p-value is less than $\alpha\ (p = 0.000 < 0.05)$, H_0 is rejected. There is sufficient evidence to indicate the overall model is adequate in predicting length of stay at $\alpha = 0.05$.

f. $\beta_1 = 0.002094$. For each additional factor, the ln of the length of stay is estimated to increase by 0.002094. A more practical interpretation is found by computing $e^{0.002094} = 1.002$. The length of stay is estimated to increase by $100(1.002 - 1)\% = 0.2\%$ for each additional factor.

9.32 There is sufficient evidence to indicate the model is adequate for predicting the number of flycatchers killed $(\chi^2 = 12.30,\ p = 0.000)$. To determine if the model was correctly specified, we look at the "goodness-of-fit" test based on $n-(k+1) = 14-(1+1) = 12$ degrees of freedom. If the model is correctly specified, then χ^2 / df will be close to 1. For this example, $\chi^2 / df = 10.18111/12 = 0.848$. Since this is close to 1, it indicates the model is correctly specified. In addition, the p-value for the "goodness-of-fit" test is $p = 0.600$. Since the p-value is larger than $\alpha = 0.05$, this again indicates that the model is correctly specified.

$\beta_1 = 0.0684$. For each additional percent of nest box occupancy, the ln of the number of flycatchers killed is estimated to increase by 0.0684. A more practical interpretation is found by computing $e^{0.0684} = 1.071$. The length of stay is estimated to increase by $100(1.071-1)\% = 7.1\%$ for each additional percent of nest box occupancy.

9.33 a. The Poisson regression model would be $E(y) = e^{\beta_0} \cdot e^{\beta_1 x_1} \cdot e^{\beta_2 x_2} \cdot e^{\beta_3 x_3} \cdot e^{\beta_4 x_4}$ or $\ln\{E(y)\} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4$.

b. To determine if the model was correctly specified, we look at the "goodness-of-fit" test based on $n-(k+1) = 14-(1+1) = 12$ degrees of freedom. If the model is correctly specified, then χ^2 / df will be close to 1. For this example, $\chi^2 / df = 664/126 = 5.270$. Since this is not close to 1, it indicates the model is not correctly specified. In addition, the *p*-value for the "goodness-of-fit" test is $p = 0.001$. Since the *p*-value is not larger than $\alpha = 0.05$, this again indicates that the model is not correctly specified.

c. $\beta_1 = 0.368$. The difference in the ln of the number of flycatchers killed between the Papilionidae species and the Hesperidae species is estimated to be 0.368. A more practical interpretation is found by computing $e^{0.368} = 1.445$. The number of flycatchers killed is estimated to be $100(1.445-1)\% = 44.5\%$ greater for the Papilionidae species than the Hesperidae species.

9.34 $\beta_1 = 0.1148$. For each 1- unit increase in the composite score, the ln of the number of suicides increases by 0.1148. A more practical interpretation is found by computing $e^{0.1148} = 1.122$. The number of suicides increases by $100(1.122-1)\% = 12.2\%$ for each additional unit increase in the composite score.

9.35 a. The Poisson regression model would be $E(y) = e^{\beta_0} \cdot e^{\beta_1 x_1} \cdot e^{\beta_2 x_2}$ or $\ln\{E(y)\} = \beta_0 + \beta_1 x_1 + \beta_2 x_2$, where $x_1 =$ level of sensation seeking and

$$x_2 = \begin{cases} 1 & \text{if male} \\ 0 & \text{if female} \end{cases}$$

b. $\beta_1 = 0.2315$. For each 1- point increase in the sensation seeking level, the ln of the number of alcoholic drinks consumed is estimated to increase by 0.2315, holding gender constant. A more practical interpretation is found by computing $e^{0.2315} = 1.260$. The number of alcoholic drinks consumed is estimated to increase by $100(1.260-1)\% = 26.0\%$ for each additional point increase in the sensation seeking level, holding gender constant.

c. There is sufficient evidence to indicate the model is adequate for predicting the number of alcoholic drinks consumed $\left(\chi^2 = 227.3,\ p < 0.0001\right)$.

d. To determine if the interaction terms contribute to the model, we test:

$H_0: \beta_3 = 0$
$H_a: \beta_3 \neq 0$

The test statistic is the difference in the χ^2 values from fitting the complete model and the reduced model. The test statistic is $\chi^2 = \chi^2_{\text{Complete}} - \chi^2_{\text{Reduced}}$.

e. The interaction model would be $\ln\{E(y)\} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2$.
The male model is $\ln\{E(y)\} = \beta_0 + \beta_1 x_1 + \beta_2 + \beta_3 x_1 = (\beta_0 + \beta_2) + (\beta_1 + \beta_3) x_1$.
The female model is $\ln\{E(y)\} = \beta_0 + \beta_1 x_1$.

For Males: The percentage change in the number of drinks consumed for a 1-point increase in sensation seeking level is $\left(e^{\beta_1 + \beta_3} - 1\right)100\%$.

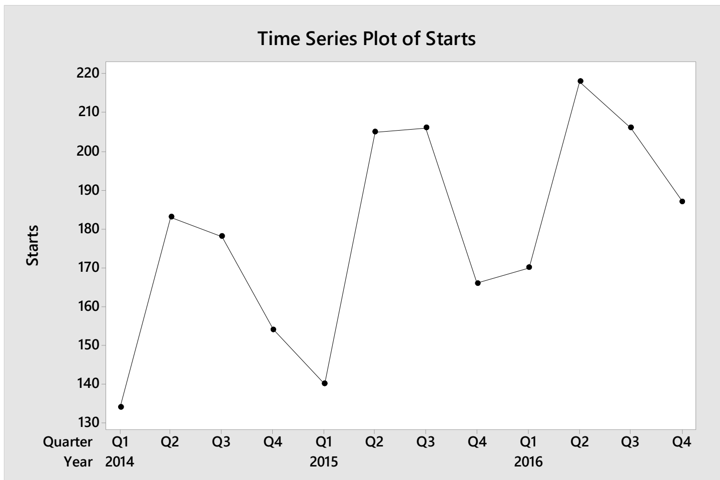
For Females: The percentage change in the number of drinks consumed for a 1-point increase in sensation seeking level is $\left(e^{\beta_1} - 1\right)100\%$.

Chapter

10

Time Series Modeling and Forecasting

10.1 a. The yearly time series plot is shown below. Yes, there appears to be a long-term trend, with housing starts increasing over time. Yes, there also appears to be seasonal variation with housing starts peaking in the second quarter of each year.



b. To compute a moving average using four points, it is important to note that midpoint of the sum is between the second and third time point. The convention of "dropping it down one line", thereby using the third time point in the series as the starting point, is used.

The first moving average in this series which can be calculated in this manner will be from the third quarter in 2014. The general formula for calculating the sum of the four values is:

$$L_t = y_{t-2} + y_{t-1} + y_t + y_{t+1}.$$

To calculate the moving average we divide the resulting sum by 4. For example, for the third quarter in 2014, the total is calculated using:

$$L_{2014,3} = y_{2014,1} + y_{2014,2} + y_{2014,3} + y_{2014,4} = 134 + 183 + 178 + 154 = 649$$

The moving average is $M_{2014,3} = \frac{L_{2014,3}}{4} = \frac{649}{4} = 162.25$

The moving average in the fourth quarter of 2014 is calculated in a similar manner, i.e.:

$$L_{2014,4} = y_{2014,2} + y_{2014,3} + y_{2014,4} + y_{2015,1} = 183 + 178 + 154 + 140 = 655$$

The moving average is $M_{2014,4} = \frac{L_{2014,4}}{4} = \frac{655}{4} = 163.75$

The remaining moving averages may be calculated in a similar manner and are shown in the table below:

Year	Quarter	Housing Starts, y_t	L_t	M_t
2014	1	134		
2014	2	183		
2014	3	178	649	162.25
2014	4	154	655	163.75
2015	1	140	677	169.25
2015	2	205	705	176.25
2015	3	206	717	179.25
2015	4	166	747	186.75
2016	1	170	760	190.00
2016	2	218	760	190.00
2016	3	206	781	195.25
2016	4	187		

c. Yes, the moving average when superimposed on the time series plot, illustrates the increasing trend. The seasonal effect indicated in part (a) has been smoothed out when using the 4-point moving average.

d. When calculating the seasonal index, we begin by finding the ratio of y_t divided by the corresponding moving average, M_t. The seasonal index for a given quarter is the average of these ratios multiplied by 100. For quarter 1, the ratios are shown below:

Year	Quarter	y_t	M_t	y_t/M_t
2015	1	140	169.25	0.827
2016	1	170	190.00	0.895

The seasonal index for the first quarter is therefore: $\left(\frac{0.827+0.895}{2}\right)100 = 86.1$

e. For quarter 2, the ratios are shown in the table below:

Year	Quarter	y_t	M_t	y_t/M_t
2015	2	205	176.25	1.163
2016	2	218	190.00	1.147

The seasonal index for the second quarter is therefore: $\left(\frac{1.163+1.147}{2}\right)100 = 115.5$

f. To forecast the number of housing starts in the first quarter of 2017, we first extend the moving average to find $M_{2017,1}$ which is approximately 205. To adjust the forecast for seasonal variation, multiply the future moving average value, $M_{2017,1} = 205$ by the seasonal index for quarter 1, then divide by 100:

$$F_{2017,1} = \frac{M_{2017,1}(\text{Seasonal index for quarter 1})}{100} = \frac{205(86.1)}{100} = 176.5$$

Similarly, for quarter 2 we extend the moving average to find $M_{2017,2}$ which is estimated to be 210. To adjust the forecast for seasonal variation, $M_{2017,2} = 210$ by the seasonal index for quarter 2, then divide by 100:

$$F_{2017,2} = \frac{M_{2017,2}(\text{Seasonal index for quarter 2})}{100} = \frac{210(115.5)}{100} = 242.6$$

10.2 a. To calculate the exponentially smoothed time series, note that the first value in the series is merely the first value of the time series. The remaining values in the smoothed series are calculated using the formula:

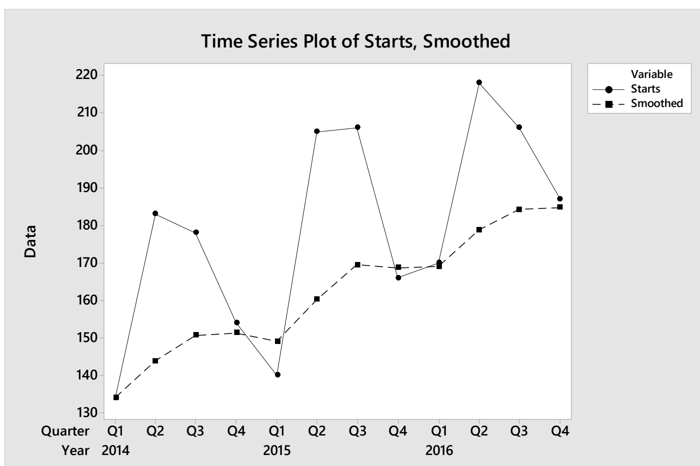
$E_t = wy_t + (1-w)E_{t-1}$, where w is the smoothing constant, and in this case we use $w = 0.2$.

For 2014,1: $E_{2014,1} = y_{2014,1} = 134$
For 2014,2: $E_{2014,2} = w(y_{2014,2}) + (1-w)E_{2014,1} = 0.2(183) + (1-0.2)134 = 143.80$
For 2014,3: $E_{2014,3} = w(y_{2014,3}) + (1-w)E_{2014,2} = 0.2(178) + (1-0.2)143.80 = 150.64$

The rest of the values of the exponentially smoothed series are found in a similar manner and are shown in the table below:

Year	Quarter	Housing Starts	Exponentially Smoothed Value, E_t
2014	1	134	134.00
2014	2	183	143.80
2014	3	178	150.64
2014	4	154	151.31
2015	1	140	149.05
2015	2	205	160.24
2015	3	206	169.39
2015	4	166	168.71
2016	1	170	168.97
2016	2	218	178.78
2016	3	206	184.22
2016	4	187	184.78

The results from MINITAB are shown below. Note that in the "Options" setting the initial smoothed value was calculated from the first "K=1" values, i.e. only the first data point (134). A graph of the smoothed values is shown below



b. As exponential smoothing forecasts are obtained by using the most recent exponentially smoothed value, it follows that $F_{2017,1} = F_{2017,2} = E_{2016,4} = 184.78$.

c. As the data clearly indicate a seasonal trend, we include $P = 4$ time periods in the cycle (i.e. quarterly data).

There are 12 values in the time series, so $F_{2017,1}$ corresponds with the thirteenth time point, and $F_{2017,2}$ corresponds to the fourteenth time point.

To compute the Holt-Winters forecasting model with both trend and seasonal components with $w = 0.2$, $v = 0.5$, and $u = 0.7$, we use:

$E_t = y_t$ when $t = 2$
$E_t = wy_t + (1-w)(E_{t-1} + T_{t-1})$ when $t = 3,4,5,6$ (i.e. up to the value of $P+2=6$)
$E_t = w(y_t / S_{t-P}) + (1-w)(E_{t-1} + T_{t-1})$ when $t > 6$

$T_t = y_2 - y_1$ when $t = 2$
$T_t = v(E_t - E_{t-1}) + (1-v)T_{t-1}$ when $t > 2$

$S_t = y_t / E_t$ when $t = 3,4,5,6$ (i.e. up to the value of $P+2=6$)
$S_t = u(y_t / E_t) + (1-u)S_{t-P}$ when $t > 6$

Therefore, for $t = 2$:
$E_2 = y_2 = 183$
$T_2 = y_2 - y_1 = 183 - 134 = 49$
$S_2 = y_2 / E_2 = 183/183 = 1$

For $t = 3$:
$E_3 = 0.2y_3 + (1-0.2)(E_2 + T_2) = 0.2(178) + 0.8(183+49) = 221.2$
$T_3 = 0.5(E_3 - E_2) + (1-0.5)T_2 = 0.5(221.2 - 183) + 0.5(49) = 43.6$
$S_3 = y_3 / E_3 = 178/221.2 = 0.8047$

The remaining values are shown in the table below:

Year	Quarter	Housing Starts	E_t	T_t	S_t
2014	1	134			
2014	2	183	183.00	49.0000	1.000
2014	3	178	221.20	43.6000	0.805
2014	4	154	242.64	32.5200	0.635
2015	1	140	248.13	19.0040	0.564
2015	2	205	254.71	12.7908	0.805
2015	3	206	265.20	11.6407	0.785
2015	4	166	273.78	10.1117	0.615
2016	1	170	287.37	11.8525	0.583
2016	2	218	293.55	9.0158	0.761
2016	3	206	294.53	4.9958	0.725
2016	4	187	300.45	5.4582	0.620

To forecast the first quarter of 2017 (i.e. y_{13}) we use:

$$F_{13} = F_{2017,1} = (E_{12} + T_{12})S_{13-4} = (300.45 + 5.4582)(0.583) = 178.34$$

To forecast the second quarter of 2017 (i.e. y_{14}) we use:

$$F_{14} = F_{2017,2} = (E_{12} + 2T_{12})S_{14-4} = (300.45 + 2(5.4582))(0.761) = 236.95$$

10.3 a. The forecasts using the three approaches are summarized below:

	Actual Number of Housing Starts (in thousands)	Moving Average Forecast (in thousands)	Exponential Smoothing Forecast (in thousands)	Holt-Winters Forecast (in thousands)
2017 Q1	181	176.5	184.78	178.34
2017 Q2	238	242.6	184.78	236.95

For the Moving Average, we find that the MAD is:

$$\text{MAD} = \left(|181 - 176.5| + |238 - 242.6|\right)/2 = 4.55$$

For the Exponential Smoothing, we find that the MAD is:

$$\text{MAD} = \left(|181 - 184.78| + |238 - 184.78|\right)/2 = 28.5$$

For the Holt-Winters approach, compute the MAD as:

$$\text{MAD} = \left(|181 - 178.34| + |238 - 236.95|\right)/2 = 1.855$$

b. For the moving average approach we find that the RMSE is:

$$\text{RMSE} = \sqrt{\frac{(181-176.5)^2 + (238-242.6)^2}{2}} = 4.55$$

For the exponential smoothing approach we find that the RMSE is:

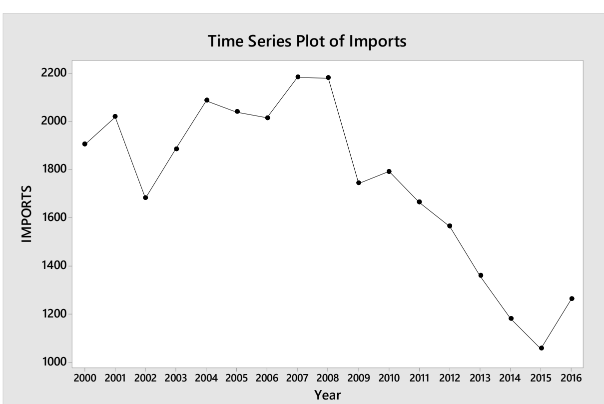
$$\text{RMSE} = \sqrt{\frac{(181-184.78)^2 + (238-184.78)^2}{2}} = 37.73$$

For the Holt-Winters approach we find that the RMSE is:

$$\text{RMSE} = \sqrt{\frac{(181-178.34)^2 + (238-236.95)^2}{2}} = 2.02$$

c. As the MAD and RMSE values are lowest for the Holt-Winters approach, it is the most accurate.

10.4 a. Using MINITAB, the graph is:



Yes, there appears to be a downward trend over time.

b. To compute a moving average using three points, the midpoint of the sum is the second time point. The first moving average in this series which can be calculated in this manner will be from 2001.

The general formula for calculating the sum of the three values is:
$L_t = y_{t-1} + y_t + y_{t+1}$.

To calculate the moving average we divide the resulting sum by 3. For example, for the 2001 moving average point we first compute the total as:

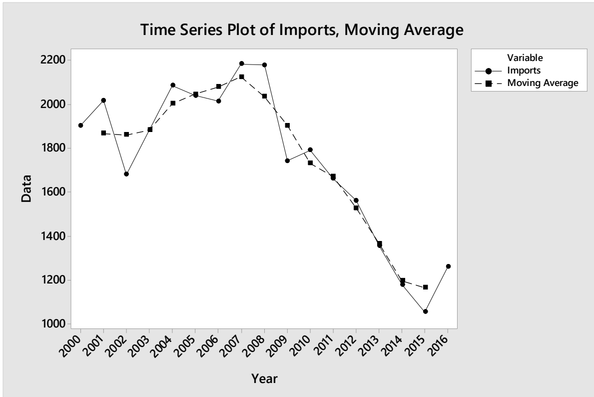
$$L_{2001} = y_{2000} + y_{2001} + y_{2002} = 1904 + 2018 + 1681 = 5603$$

The moving average is $M_{2001} = \dfrac{L_{2001}}{3} = \dfrac{5603}{3} = 1867.67$

The remaining moving averages are calculated in a similar manner and are shown in the table below:

Year	Imports	3-point Moving Total, L_t	3-point Moving Average, M_t
2000	1904		
2001	2018	5603	1867.67
2002	1681	5583	1861.00
2003	1884	5651	1883.67
2004	2086	6009	2003.00
2005	2039	6139	2046.33
2006	2014	6236	2078.67
2007	2183	6376	2125.33
2008	2179	6105	2035.00
2009	1743	5713	1904.33
2010	1791	5197	1732.33
2011	1663	5017	1672.33
2012	1563	4584	1528.00
2013	1358	4102	1367.33
2014	1181	3595	1198.33
2015	1056	3498	1166.00
2016	1261		

A time series plot is shown for the Imports and Moving Average results is shown below:



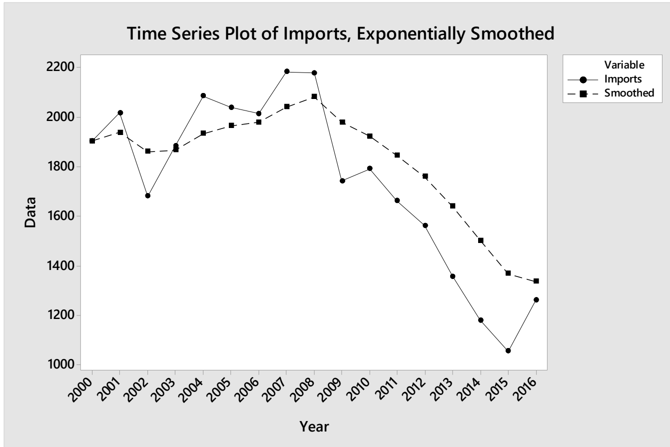
c. To calculate the exponentially smoothed time series, note that the first value in the series is merely the first value of the time series. The remaining values in the smoothed series are calculated using the formula:

$E_t = wy_t + (1-w)E_{t-1}$, where w is the smoothing constant, and in this case we use $w = 0.3$.

For 2000: $E_{2000} = y_{2000} = 1904$
For 2001: $E_{2001} = wy_{2001} + (1-w)E_{2000} = 0.3(2018) + (1-0.3)1904 = 1938.2$

The rest of the values of the exponentially smoothed series are found in a similar manner and are shown in the table below:

Year	Imports	E_t
2000	1904	1904
2001	2018	1938.20
2002	1681	1861.04
2003	1884	1867.93
2004	2086	1933.35
2005	2039	1965.04
2006	2014	1979.73
2007	2183	2040.71
2008	2179	2082.20
2009	1743	1980.44
2010	1791	1923.61
2011	1663	1845.43
2012	1563	1760.70
2013	1358	1639.89
2014	1181	1502.22
2015	1056	1368.36
2016	1261	1336.15



d. Using the moving average method, extending the moving average graph to 2017 suggests a forecast around 1100.

e. As exponential smoothing forecasts are obtained by using the most recent exponentially smoothed value, it follows that $F_{2017} = E_{2016} = 1336.15$.

f. Using the Holt-Winters Method, note that we need to be concerned with the exponential component and the time trend component only (no seasonal component in this case).

$E_t = y_t$ and $T_t = y_t - y_{t-1}$ when $t = 2$
$E_2 = y_2 = 2018$ and $T_2 = y_t - y_{t-1} = 2018 - 1904 = 114$

When $t > 2$ we use:
$E_t = w(y_t) + (1-w)(E_{t-1} + T_{t-1})$ and $T_t = v(E_t - E_{t-1}) + (1-v)T_{t-1}$
$E_3 = (0.3)(1681) + (1-0.3)(2018+114) = 1996.7$ and
$T_3 = (0.8)(1996.7 - 2018) + (1-0.8)114 = 5.76$

In this manner, the remaining exponentially smoothed values and trend values are calculated and shown in the table below:

Year	Imports	E_t	T_t
2000	1904		
2001	2018	2018.00	114.00
2002	1681	1996.70	5.76
2003	1884	1966.92	-22.67
2004	2086	1986.78	11.35
2005	2039	2010.39	21.16
2006	2014	2026.28	16.95
2007	2183	2085.16	50.49
2008	2179	2148.66	60.90
2009	1743	2069.59	-51.08
2010	1791	1950.26	-105.68

2011	1663	1790.10	-149.26
2012	1563	1617.49	-167.94
2013	1358	1422.09	-189.91
2014	1181	1216.82	-202.19
2015	1056	1027.04	-192.26
2016	1261	962.64	-89.97

To forecast 2017 (i.e.y_{19}), we use:
$F_{18} = (E_{17} + T_{17}) = 962.64 - 89.97 = 872.67$

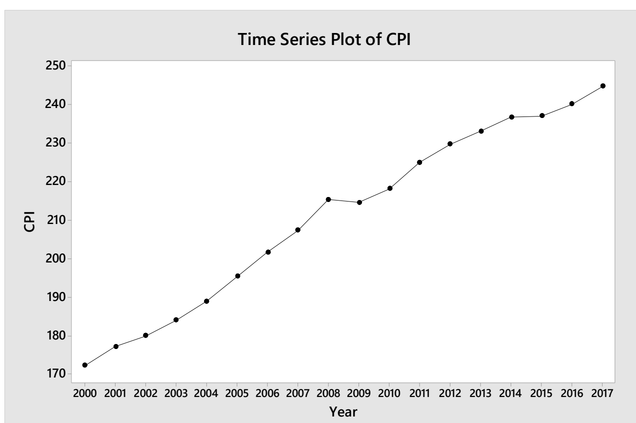
g. Summarizing the results in the tables below, we see that the exponentially smoothed forecast provides the best estimate as it has the smallest error:

Year	Imports	Moving Average Forecast	Error
2017	1227	1100	127

Year	Imports	Exponential Smoothed Forecast	Error
2017	1227	1336.15	-109.15

Year	Imports	Holt-Winters Forecast	Error
2017	1227	872.67	354.33

10.5 a. Yes, there is a long-term increasing trend as shown in the time series plot below.



b. To compute a moving average using five points, the midpoint of the sum is the third time point. The first moving average in this series which can be calculated in this manner will be from 2002.

The general formula for calculating the sum of the five values is:
$L_t = y_{t-2} + y_{t-1} + y_t + y_{t+1} + y_{t+2}$

To calculate the moving average we divide the resulting sum by 5. For example, for the 2002 moving average point

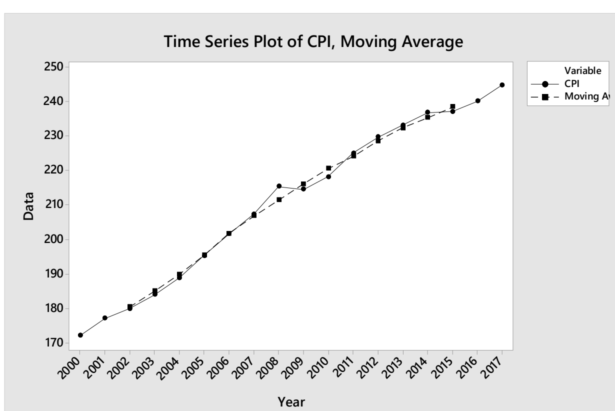
$$L_{2002} = y_{2000} + y_{2001} + y_{2002} + y_{2003} + y_{2004} = 172.2 + 177.1 + 179.9 + 184.0 + 188.9 = 902.1$$

The moving average is $M_{2002} = \frac{L_{2002}}{2} = \frac{902.1}{5} = 180.42$

The remaining moving averages may be calculated in a similar manner, and are shown in the table below, and the graph below:

Year	CPI	L_t	M_t
2000	172.2		
2001	177.1		
2002	179.9	902.1	180.42
2003	184	925.2	185.04
2004	188.9	949.7	189.94
2005	195.3	977.1	195.42
2006	201.6	1008.4	201.68
2007	207.3	1034	206.80
2008	215.3	1056.8	211.36
2009	214.5	1080.1	216.02
2010	218.1	1102.4	220.48
2011	224.9	1120.1	224.02
2012	229.6	1142.3	228.46
2013	233	1161.2	232.24
2014	236.7	1176.3	235.26
2015	237	1191.4	238.28
2016	240		
2017	244.7		

The time series plot for the CPI and the moving average results is shown below:



Extending the moving average line appears to be approximately 15 units higher than the final smoothed value of 238.28, hence the forecast for 2020 is approximately 253.28.

c. To calculate the exponentially smoothed time series, note that the first value in the series is merely the first value of the time series. The remaining values in the smoothed series are calculated using the formula:

$E_t = wy_t + (1-w)E_{t-1}$, where w is the smoothing constant, and in this case we use $w = 0.4$.

For 2000: $E_{2000} = y_{2000} = 172.2$
For 2001: $E_{2001} = wy_{2001} + (1-w)E_{2000} = 0.4(177.1) + 0.6(172.2) = 174.16$

The rest of the values of the exponentially smoothed series are found in a similar manner and are shown in the table below:

Year	CPI	E_t
2000	172.2	172.20
2001	177.1	174.16
2002	179.9	176.46
2003	184	179.47
2004	188.9	183.24
2005	195.3	188.07
2006	201.6	193.48
2007	207.3	199.01
2008	215.3	205.52
2009	214.5	209.11
2010	218.1	212.71
2011	224.9	217.59
2012	229.6	222.39
2013	233	226.63
2014	236.7	230.66
2015	237	233.20
2016	240	235.92
2017	244.7	239.43

As exponential smoothing forecasts are obtained by using the most recent exponentially smoothed value, it follows that $F_{2020} = E_{2017} = 239.43$.

d. Using the Holt-Winters Method, note that we need to be concerned with the exponential component, and the time trend components only (no seasonal component in this case).

We know that $E_t = y_t$ and $T_t = y_t - y_{t-1}$ when $t = 2$, so
$E_2 = y_2 = 177.1$ and $T_2 = y_2 - y_1 = 177.1 - 172.2 = 4.9$
Now, when $t > 2$ we use:

$E_t = wy_t + (1-w)(E_{t-1} + T_{t-1})$ and $T_t = v(E_t - E_{t-1}) + (1-v)T_{t-1}$
Therefore, $E_3 = 0.4(179.9) + (1-0.4)(177.1 + 4.9) = 181.16$ and
$T_3 = 0.5(181.16 - 177.1) + (1-0.5)4.9 = 4.48$

The remaining exponentially smoothed values and trend values are calculated in a similar manner and are shown in the table below:

t	Year	CPI	E_t	T_t
1	2000	172.2		
2	2001	177.1	177.1	4.90
3	2002	179.9	181.16	4.48
4	2003	184	184.98	4.15
5	2004	188.9	189.04	4.10
6	2005	195.3	194.01	4.54
7	2006	201.6	199.77	5.15
8	2007	207.3	205.87	5.62
9	2008	215.3	213.02	6.39
10	2009	214.5	217.44	5.41
11	2010	218.1	220.95	4.46
12	2011	224.9	225.20	4.36
13	2012	229.6	229.57	4.36
14	2013	233	233.56	4.18
15	2014	236.7	237.32	3.97
16	2015	237	239.58	3.11
17	2016	240	241.61	2.57
18	2017	244.7	244.39	2.68

As the forecast for 2020 is time point 21 (i.e. 3 time periods after the 2017 value) the forecast at time $t = 21$ is given by:

$$F_{2020} = E_{2017} + 3(T_{2017}) = 244.39 + 3(2.68) = 252.43$$

10.6 a. To compute a moving average using four points, it is important to note that the midpoint of the sum is between the second and third time point. The convention of "dropping it down one line" is used, thereby using the third time point in the series as the starting point. The first moving average in this series which can be calculated in this manner will be from the third quarter in 2008. The general formula for calculating the sum of the four values is:

$$L_t = y_{t-2} + y_{t-1} + y_t + y_{t+1}$$

To calculate the moving average we divide the resulting sum by 4. For example, for the third quarter in 2008,

$$L_{2008,3} = 1322.70 + 1280.00 + 1166.36 + 903.25 = 4672.32$$

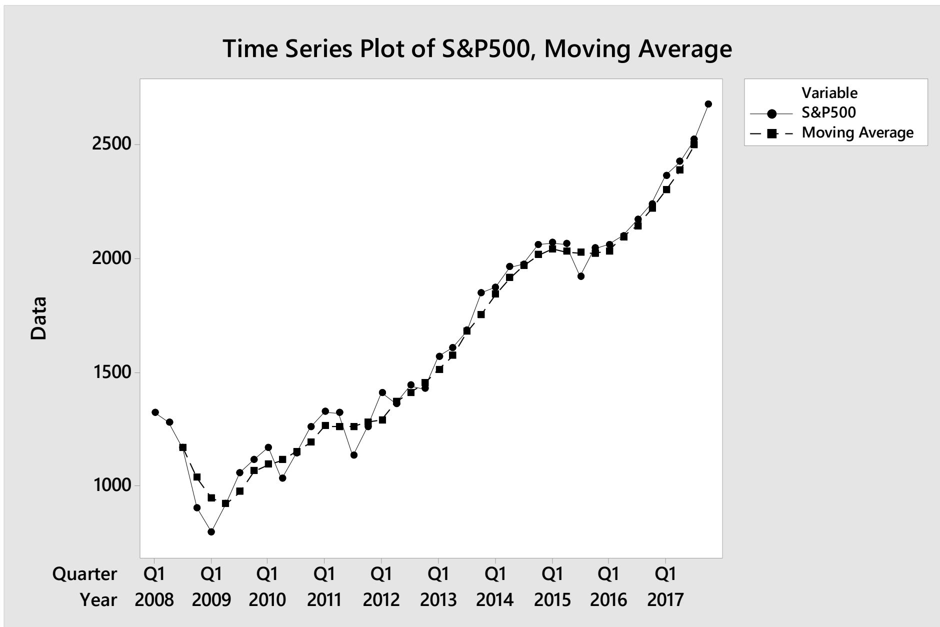
The moving average is $M_{2008,3} = L_{2008,3}/4 = 4672.32/4 = 1168.08$.

The remaining moving averages are calculated in a similar manner and are shown in the table below:

t	Year	Quarter	S&P500	L_t	M_t
1	2008	1	1322.70		
2	2008	2	1280.00		
3	2008	3	1166.36	4672.32	1168.08
4	2008	4	903.25	4147.48	1036.87
5	2009	1	797.87	3786.80	946.70
6	2009	2	919.32	3677.52	919.38
7	2009	3	1057.08	3889.37	972.34
8	2009	4	1115.10	4260.93	1065.23
9	2010	1	1169.43	4372.32	1093.08
10	2010	2	1030.71	4456.44	1114.11
11	2010	3	1141.20	4598.98	1149.74
12	2010	4	1257.64	4755.37	1188.84
13	2011	1	1325.83	5045.30	1261.33
14	2011	2	1320.64	5035.52	1258.88
15	2011	3	1131.42	5035.49	1258.87
16	2011	4	1257.60	5118.13	1279.53
17	2012	1	1408.47	5159.65	1289.91
18	2012	2	1362.16	5468.90	1367.23
19	2012	3	1440.67	5637.48	1409.37
20	2012	4	1426.19	5798.20	1449.55
21	2013	1	1569.19	6042.32	1510.58
22	2013	2	1606.28	6283.20	1570.80
23	2013	3	1681.55	6705.37	1676.34
24	2013	4	1848.36	7008.52	1752.13
25	2014	1	1872.34	7362.47	1840.62
26	2014	2	1960.23	7653.21	1913.30
27	2014	3	1972.29	7863.75	1965.94
28	2014	4	2058.90	8059.31	2014.83
29	2015	1	2067.89	8162.19	2040.55
30	2015	2	2063.11	8109.93	2027.48
31	2015	3	1920.03	8094.97	2023.74
32	2015	4	2043.94	8086.82	2021.70
33	2016	1	2059.74	8122.56	2030.64
34	2016	2	2098.86	8370.81	2092.70
35	2016	3	2168.27	8565.70	2141.42
36	2016	4	2238.83	8868.68	2217.17
37	2017	1	2362.72	9193.23	2298.31

38	2017	2	2423.41	9544.32	2386.08
39	2017	3	2519.36	9979.10	2494.77
40	2017	4	2673.61		

b. The effects of the recession can be seen in the time series plot, with the decline in 2009 by a bull market the rest of the time in the S&P 500. It is difficult to distinguish any seasonal effects as distinct from the overall trends marking the bull and bear market periods.



c. A subjective assessment of the forecast for the 1st quarter of 2018 would need to rely on the continued upward trend exhibited at the end of the time period. An estimate of 2660 would be reasonable.

d. To calculate the exponentially smoothed time series, note that the first value in the series is merely the first value of the time series. The remaining values in the smoothed series are calculated using the formula:

$E_t = wy_t + (1-w)E_{t-1}$, where w is the smoothing constant, and in this case we use $w = 0.3$.

For 2008,1: $E_{2008,1} = y_{2008,1} = 1322.70$

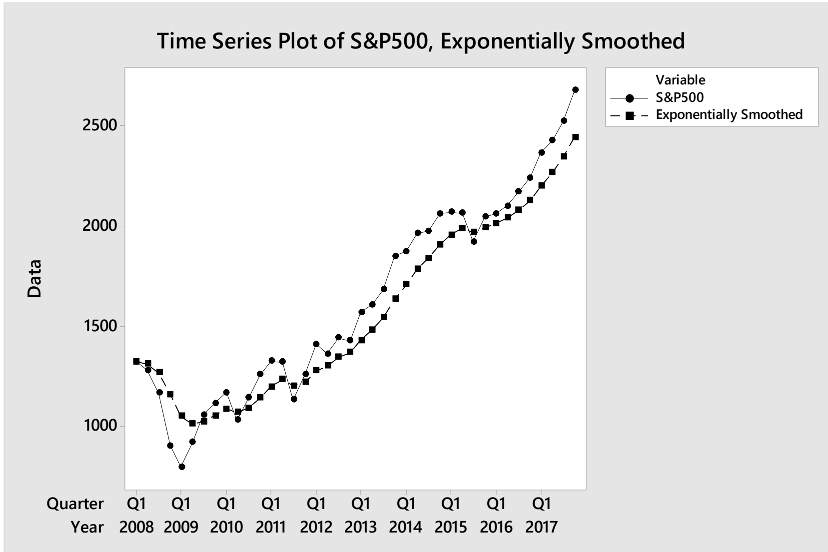
For 2008,2: $E_{2008,2} = wy_{2008,2} + (1-w)E_{2008,1} = 0.3(1280.00) + 0.7(1322.70) = 1309.89$

For 2008,3: $E_{2008,3} = wy_{2008,3} + (1-w)E_{2008,2} = 0.3(1166.36) + 0.7(1309.89) = 1266.83$

The rest of the values of the exponentially smoothed series are found in a similar manner and are shown in the table below:

t	Year	Quarter	S&P500	E_t
1	2008	1	1322.70	1322.70
2	2008	2	1280.00	1309.89
3	2008	3	1166.36	1266.83
4	2008	4	903.25	1157.76
5	2009	1	797.87	1049.79
6	2009	2	919.32	1010.65
7	2009	3	1057.08	1024.58
8	2009	4	1115.10	1051.73
9	2010	1	1169.43	1087.04
10	2010	2	1030.71	1070.14
11	2010	3	1141.20	1091.46
12	2010	4	1257.64	1141.31
13	2011	1	1325.83	1196.67
14	2011	2	1320.64	1233.86
15	2011	3	1131.42	1203.13
16	2011	4	1257.60	1219.47
17	2012	1	1408.47	1276.17
18	2012	2	1362.16	1301.97
19	2012	3	1440.67	1343.58
20	2012	4	1426.19	1368.36
21	2013	1	1569.19	1428.61
22	2013	2	1606.28	1481.91
23	2013	3	1681.55	1541.80
24	2013	4	1848.36	1633.77
25	2014	1	1872.34	1705.34
26	2014	2	1960.23	1781.81
27	2014	3	1972.29	1838.95
28	2014	4	2058.90	1904.94
29	2015	1	2067.89	1953.82
30	2015	2	2063.11	1986.61
31	2015	3	1920.03	1966.63
32	2015	4	2043.94	1989.83
33	2016	1	2059.74	2010.80
34	2016	2	2098.86	2037.22
35	2016	3	2168.27	2076.53
36	2016	4	2238.83	2125.22
37	2017	1	2362.72	2196.47
38	2017	2	2423.41	2264.55
39	2017	3	2519.36	2340.99
40	2017	4	2673.61	2440.78

A plot of the smoothed series and the S & P 500 values is:



e. As exponential smoothing forecasts are obtained by using the most recent exponentially smoothed value, it follows that $F_{2018,1} = E_{2017,4} = 2440.78$.

f. There are 40 values in the time series, so $F_{2018,1}$ corresponds with the forty-first time point, and $P = 4$ (i.e. we are using quarterly data). To assess which formula is to be used for the exponentially smoothed component, the trend component, and the seasonal component note that we use:

$E_t = y_t$ when $t = 2$
$E_t = wy_t + (1-w)(E_{t-1} + T_{t-1})$ when $t = 3, 4, ..., 6$ (i.e. up to the value of $P + 2 = 6$)
$E_t = w(y_t/S_{t-P}) + (1-w)(E_{t-1} + T_{t-1})$ when $t > 6$

$T_t = y_2 - y_1$ when $t = 2$
$T_t = v(E_t - E_{t-1}) + (1-v)T_{t-1}$ when $t > 2$

$S_t = y_t/E_t$ when $t = 2, 3, ..., 6$ (i.e. up to the value of $P + 2 = 6$)
$S_t = u(y_t/E_t) + (1-u)S_{t-P}$ when $t > 6$

Therefore, for $t = 2$:
$E_2 = y_2 = 1280.00$
$T_2 = 1280.00 - 1322.70 = -42.70$
$S_2 = y_2/E_2 = 1280.00/1280.00 = 1$

For $t = 3$:
$E_3 = 0.3y_3 + (1-0.3)(E_2 + T_2) = 0.3(1166.36) + 0.7(1280.00 + (-42.70)) = 1216.02$
$T_3 = 0.8(E_3 - E_2) + (1-0.8)T_2 = 0.8(1216.02 - 1280.00) + 0.2(-42.70) = -59.73$
$S_3 = y_3/E_3 = 1166.36/1216.02 = 0.959$

The remaining values are shown in the table below:

t	Year	Quarter	S&P500	E_t	T_t	S_t
1	2008	1	1322.70			
2	2008	2	1280.00	1280.00	-42.70	1.000
3	2008	3	1166.36	1216.02	-59.73	0.959
4	2008	4	903.25	1080.38	-120.46	0.836
5	2009	1	797.87	911.31	-159.35	0.876
6	2009	2	919.32	802.16	-119.18	1.146
7	2009	3	1057.08	808.71	-18.60	1.133
8	2009	4	1115.10	953.21	111.88	1.003
9	2010	1	1169.43	1146.27	176.83	0.948
10	2010	2	1030.71	1195.98	75.13	1.004
11	2010	3	1141.20	1191.91	11.77	1.045
12	2010	4	1257.64	1218.76	23.83	1.017
13	2011	1	1325.83	1289.44	61.31	0.988
14	2011	2	1320.64	1340.17	52.84	0.995
15	2011	3	1131.42	1299.83	-21.71	0.958
16	2011	4	1257.60	1265.51	-31.80	1.006
17	2012	1	1408.47	1291.25	14.24	1.039
18	2012	2	1362.16	1324.68	29.59	1.011
19	2012	3	1440.67	1399.19	65.53	0.994
20	2012	4	1426.19	1450.79	54.38	0.994
21	2013	1	1569.19	1506.52	55.47	1.041
22	2013	2	1606.28	1569.80	61.72	1.017
23	2013	3	1681.55	1649.70	76.26	1.007
24	2013	4	1848.36	1765.85	108.17	1.021
25	2014	1	1872.34	1851.65	90.28	1.026
26	2014	2	1960.23	1937.38	86.64	1.015
27	2014	3	1972.29	2004.66	71.15	0.995
28	2014	4	2058.90	2058.32	57.16	1.010
29	2015	1	2067.89	2085.58	33.24	1.009
30	2015	2	2063.11	2093.21	12.75	1.000
31	2015	3	1920.03	2052.97	-29.64	0.965
32	2015	4	2043.94	2023.20	-29.75	1.010
33	2016	1	2059.74	2008.02	-18.09	1.017
34	2016	2	2098.86	2022.55	8.00	1.019
35	2016	3	2168.27	2095.31	59.81	1.000
36	2016	4	2238.83	2173.36	74.41	1.020
37	2017	1	2362.72	2270.26	92.40	1.029
38	2017	2	2423.41	2367.39	96.18	1.021
39	2017	3	2519.36	2480.29	109.56	1.008
40	2017	4	2673.61	2599.08	116.94	1.024

The forecast for quarter 1 of 2018, where $t = 41$ $(n = 40)$ is given by:

$$F_t = (E_n + T_n)S_{n+1-P} = (E_{40} + T_{40})S_{40+1-4} = (E_{40} + T_{40})S_{37}$$
$$= (2599.08 + 116.94)1.029 = 2794.78$$

10.7 a. To compute a moving average using three points, the midpoint of the sum is the second time point. The first moving average in this series which can be calculated in this manner will be from 2001.

The general formula for calculating the sum of the three values is:
$L_t = y_{t-1} + y_t + y_{t+1}$

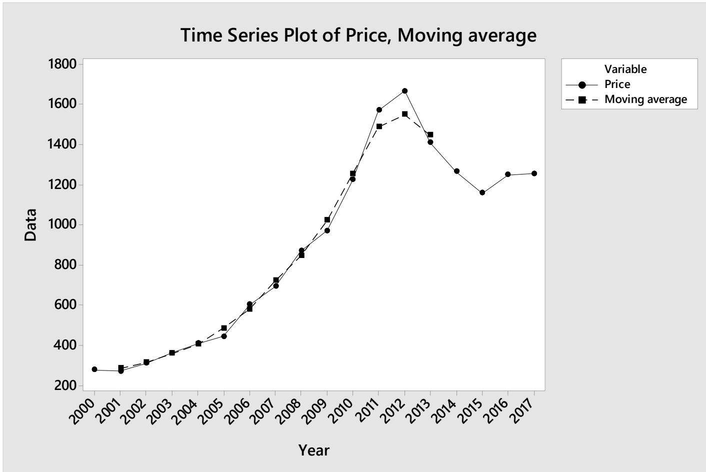
To calculate the moving average we divide the resulting sum by 3. For example, for the 2001 moving average point

$$L_{2001} = y_{2000} + y_{2001} + y_{2002} = 279 + 271 + 310 = 860$$

The moving average, $M_{2001} = L_{2001}/3 = 860/3 = 286.67$. The remaining moving averages may be calculated in a similar manner, and are shown in the table below:

Year	Price	L_t	M_t
2000	279		
2001	271	860	286.67
2002	310	944	314.67
2003	363	1083	361.00
2004	410	1218	406.00
2005	445	1458	486.00
2006	603	1743	581.00
2007	695	2170	723.33
2008	872	2539	846.33
2009	972	3069	1023.00
2010	1225	3769	1256.33
2011	1572	4466	1488.67
2012	1669	4652	1550.67
2013	1411	4346	1448.67
2014	1266		
2015	1160		
2016	1251		
2017	1257		

The plot of the price and moving average series is:



There appears to be a long term upward trend to the data.

b. To forecast the value in 2014, stretch the moving average line from 2013. This value is roughly 1311. Subsequent decreases of 100 units seem reasonable when projecting from 2014, hence the forecast for 2015 is $1311-100=1211$, for 2016 is $1211-100=1111$, and for 2017 is $1111-100=1011$.

c. To calculate the exponentially smoothed time series, note that the first value in the series is merely the first value of the time series. The remaining values in the smoothed series are calculated using the formula:

$E_t = wy_t + (1-w)E_{t-1}$, where w is the smoothing constant, and in this case we use $w = 0.8$.

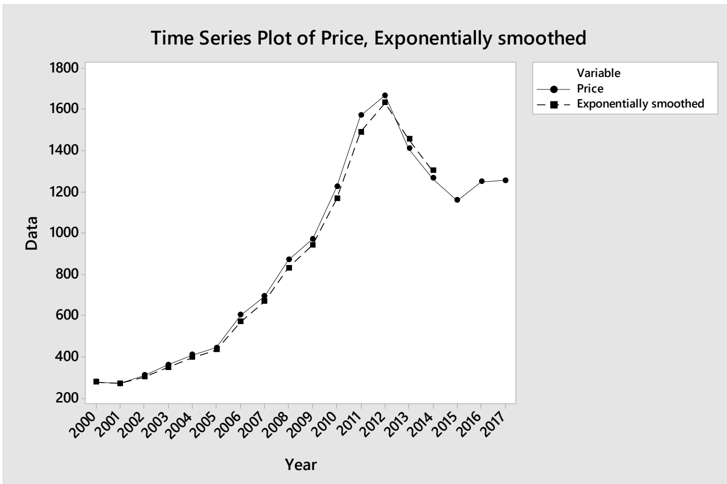
For 2000: $E_{2000} = y_{2000} = 279$
For 2001: $E_{2001} = 0.8\left(y_{2001}\right) + (1-0.8)E_{2000} = 0.8(271) + 0.2(279) = 272.6$

The rest of the values of the exponentially smoothed series are found in a similar manner and are shown in the table below:

Year	Price	E_t
2000	279	279.00
2001	271	272.60
2002	310	302.52
2003	363	350.90
2004	410	398.18
2005	445	435.64
2006	603	569.53
2007	695	669.91
2008	872	831.58
2009	972	943.92
2010	1225	1168.78

2011	1572	1491.36
2012	1669	1633.47
2013	1411	1455.49
2014	1266	1303.90
2015	1160	
2016	1251	
2017	1257	

A time series plot of the gold prices and smoothed values is shown below:



d. Note that the forecast is obtained from the most recent exponentially smoothed value, hence:

$$F_{2015} = F_{2016} = F_{2017} = E_{2014} = 1303.90$$

e. Using the Holt-Winters Method, note that we need to be concerned with the exponential component and the time trend components only (no seasonal component in this case).

When $t = 2$, $E_t = y_t$ and $T_t = y_t - y_{t-1}$
Thus, $E_2 = y_2 = 271$ and $T_2 = 271 - 279 = -8$

When $t > 2$, we use:

$$E_t = wy_t + (1-w)(E_{t-1} + T_{t-1}) \text{ and } T_t = v(E_t - E_{t-1}) + (1-v)(T_{t-1})$$

Therefore, $E_3 = 0.8(310) + 0.2(271 + (-8)) = 300.6$
$T_3 = 0.4(300.6 - 271) + 0.6(-8) = 7.04$

In this manner the remaining exponentially smoothed values, and trend values may be calculated as shown in the table below:

Year	Price	E_t	T_t
2000	279		
2001	271	271.00	-8.00
2002	310	300.60	7.04
2003	363	351.93	24.76
2004	410	403.34	35.42
2005	445	443.75	37.42
2006	603	578.63	76.40
2007	695	687.01	89.19
2008	872	852.84	119.85
2009	972	972.14	119.63
2010	1225	1198.35	162.26
2011	1572	1529.72	229.91
2012	1669	1687.13	200.90
2013	1411	1506.41	48.25
2014	1266	1323.73	-44.12
2015	1160		
2016	1251		
2017	1257		

To forecast the price of gold from 2014 as a baseline (i.e. from $n = 15$), note that:

$$F_{2015} = F_{16} = E_{15} + T_{15} = 1323.73 - 44.12 = 1279.61$$
$$F_{2016} = F_{17} = E_{15} + 2T_{15} = 1323.73 + 2(-44.12) = 1235.49$$
$$F_{2017} = F_{18} = E_{15} + 3T_{15} = 1323.73 + 3(-44.12) = 1191.37$$

f. Summarizing the results in the tables below we see that the Holt-Winters forecast provides the best estimate:

Year	Gold Price	Moving Average Forecast	Error
2015	1160	1211	-51
2016	1251	1111	140
2017	1257	1011	246

Year	Gold Price	Exponential Smoothing Forecast	Error
2015	1160	1303.90	-143.9
2016	1251	1303.90	-52.9
2017	1257	1303.90	-46.9

Year	Gold Price	Holt-Winters Forecast	Error
2015	1160	1279.61	-119.61
2016	1251	1235.49	15.51
2017	1257	1191.37	65.63

As well as assessing the errors, the RMSE and MAD values also indicate the superiority of the Holt-Winters approach over the two others, to provide the most accurate forecasts. The results are summarized below. Note that the RMSE and MAD values are both lowest for the Holt-Winters approach.

Forecasting Approach	MAD	RMSE
Moving average	145.67	166.05
Exponential smoothing	81.23	92.57
Holt-Winters	66.92	79.28

10.8 a. Using MINITAB, the results are:

Regression Analysis: RATE versus t

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	44.802	44.8016	205.18	0.000
Error	21	4.585	0.2184		
Total	22	49.387			

Model Summary

S	R-sq	R-sq(adj)
0.467281	90.72%	90.27%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	8.101	0.189	42.93	0.000
t	-0.2104	0.0147	-14.32	0.000

Regression Equation

RATE = 8.101 - 0.2104 t

The fitted regression line is $\hat{y} = 8.101 - 0.2104t$.

b. Using MINITAB, the results are:

Prediction for RATE

Regression Equation

RATE = 8.101 - 0.2104 t

Settings

Variable	Setting
t	23

Prediction

Fit	SE Fit	95% CI	95% PI
3.26123	0.201403	(2.84238, 3.68007)	(2.20304, 4.31941)

The forecasted average mortgage interest rate for 2018 is $\hat{y} = 8.101 - 0.2104(23) = 3.26$. From the printout above, the 95% prediction interval is $(2.203, 4.319)$.

10.9 a. Using MINITAB, the results are:

Regression Analysis: PRICE versus t

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	2.500	2.500	0.84	0.374
Error	16	47.774	2.986		
Total	17	50.274			

Model Summary

S	R-sq	R-sq(adj)
1.72797	4.97%	0.00%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	10.332	0.782	13.22	0.000
t	0.0718	0.0785	0.92	0.374

Regression Equation

PRICE = 10.332 + 0.0718 t

The regression equation $\hat{y} = 10.332 + 0.0718t$.

b. The MINITAB output above indicates that the model is not significant for predicting price based on year $(F = 0.84,\ p = 0.374)$. In addition, the R^2 value is $R^2 = 0.0497$. This indicates that only 4.97% of the variability in gas prices is explained by the linear relationship between gas price and time.

c. Using MINITAB, the results are:

Prediction for PRICE

Regression Equation

PRICE = 10.332 + 0.0718 t

Settings

Variable	Setting
t	18

Prediction

Fit	SE Fit	95% CI	95% PI
11.6252	0.849748	(9.82384, 13.4266)	(7.54314, 15.7073)

Settings

Variable	Setting
t	19

Prediction

Fit	SE Fit	95% CI	95% PI
11.6971	0.919417	(9.74799, 13.6461)	(7.54768, 15.8464)

For 2018 the 95% prediction interval is $(7.543, 15.707)$.

For 2019 the 95% prediction interval is $(7.548, 15.846)$.

This indicates that the natural gas prices did not fluctuate much in one year.

d. We have no information as to what the relationship looks like outside the observed range. Using a regression model assumes that the relationship stays the same outside the observed range. In addition, extrapolation gives no account for cyclical trends. Using some kind of moving average, exponential smoothing, or the Holt-Winters model would be better.

10.10 a. Using MINITAB, the results are:

Regression Analysis: Policies versus t

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	27438	27438.1	42.38	0.000
Error	25	16186	647.4		
Total	26	43624			

Model Summary

S	R-sq	R-sq(adj)
25.4448	62.90%	61.41%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	395.39	9.53	41.50	0.000
t	-4.093	0.629	-6.51	0.000

Regression Equation

Policies = 395.39 - 4.093 t

The fitted model is $\hat{y} = 395.39 - 4.093t$.

b. For 2017, the forecast is $\hat{y} = 395.39 - 4.093(27) = 284.88$.

For 2018, the forecast is $\hat{y} = 395.39 - 4.093(28) = 280.79$.

c. Using MINITAB, the results are:

Prediction for Policies

Regression Equation

Policies = 395.39 - 4.093 t

Settings

Variable	Setting
t	27

Prediction

Fit	SE Fit	95% CI	95% PI
284.886	10.0723	(264.142, 305.630)	(228.525, 341.247)

Settings

Variable	Setting
t	28

Prediction

Fit	SE Fit	95% CI	95% PI
280.793	10.6261	(258.908, 302.678)	(224.003, 337.584)

For 2017, the 95% prediction interval is $(228.53, 341.25)$.
For 2018, the 95% prediction interval is $(224.00, 337.58)$.

10.11 a. The regression model is $E(Y_i) = \beta_0 + \beta_1 t + \beta_2 Q_1 + \beta_3 Q_2 + \beta_4 Q_3$.

b. Using MINITAB, the results are:

Regression Analysis: Index versus t, Q1, Q2, Q3
Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	4	318560	79640.1	117.82	0.000
Error	15	10140	676.0		
Total	19	328700			

Model Summary

S	R-sq	R-sq(adj)
25.9994	96.92%	96.09%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	119.9	16.9	7.07	0.000
t	16.51	1.03	16.07	0.000
Q1	262.3	16.7	15.68	0.000
Q2	222.8	16.6	13.45	0.000
Q3	105.5	16.5	6.40	0.000

Regression Equation
Index = 119.9 + 16.51 t + 262.3 Q1 + 222.8 Q2 + 105.5 Q3

The fitted regression line is $\hat{y} = 119.9 + 16.51t + 262.3Q_1 + 222.8Q_2 + 105.5Q_3$.

Notice that the model is useful in predicting the index value based on the p-values for each variable.

c. The assumptions of independent errors are in doubt.

d. Using MINITAB, the results are:

Prediction for Index
Regression Equation
Index = 119.9 + 16.51 t + 262.3 Q1 + 222.8 Q2 + 105.5 Q3

Settings

Variable	Setting
t	21
Q1	1
Q2	0
Q3	0

Prediction

Fit	SE Fit	95% CI	95% PI
728.95	16.9495	(692.823, 765.077)	(662.798, 795.102)

Settings

Variable	Setting
t	22
Q1	0
Q2	1
Q3	0

Prediction

Fit	SE Fit	95% CI	95% PI
705.95	16.9495	(669.823, 742.077)	(639.798, 772.102)

Settings

Variable	Setting
t	23
Q1	0
Q2	0
Q3	1

Prediction

Fit	SE Fit	95% CI	95% PI
605.15	16.9495	(569.023, 641.277)	(538.998, 671.302)

Settings

Variable	Setting
t	24
Q1	0
Q2	0
Q3	0

Prediction

Fit	SE Fit	95% CI	95% PI
516.15	16.9495	(480.023, 552.277)	(449.998, 582.302)

The 95% prediction intervals for 2019 are $(662.80, 795.10)$, $(639.80, 772.10)$, $(539.00, 671.30)$, and $(450.00, 582.30)$ for each of the four quarters.

10.12 a. A possible model would be $E(Y_i) = \beta_0 + \beta_1 t + \beta_2 Q_1 + \beta_3 Q_2 + \beta_4 Q_3$

where $Q_1 = \begin{cases} 1 & \text{if Quarter 1} \\ 0 & \text{if not} \end{cases}$ $Q_2 = \begin{cases} 1 & \text{if Quarter 2} \\ 0 & \text{if not} \end{cases}$ $Q_3 = \begin{cases} 1 & \text{if Quarter 3} \\ 0 & \text{if not} \end{cases}$

and $t = 1$ corresponds to year 2008, quarter 1.

b. Using MINITAB, the results are:

Regression Analysis: S&P500 versus T, Q1, Q2, Q3

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	4	8804463	2201116	82.13	0.000
T	1	8759292	8759292	326.83	0.000
Q1	1	6236	6236	0.23	0.633
Q2	1	156	156	0.01	0.940
Q3	1	2372	2372	0.09	0.768
Error	35	938041	26801		
Total	39	9742503			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
163.711	90.37%	89.27%	86.89%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	786.3	71.7	10.97	0.000	
T	40.73	2.25	18.08	0.000	1.01
Q1	35.5	73.5	0.48	0.633	1.51
Q2	5.6	73.4	0.08	0.940	1.51
Q3	-21.8	73.2	-0.30	0.768	1.50

Regression Equation

S&P500 = 786.3 + 40.73 T + 35.5 Q1 + 5.6 Q2 - 21.8 Q3

The fitted model is $\hat{y} = 786.3 + 40.73t + 35.5Q_1 + 5.6Q_2 - 21.8Q_3$.

c. Using MINITAB, the results are:

Prediction for S&P500

Regression Equation

S&P500 = 786.3 + 40.73 T + 35.5 Q1 + 5.6 Q2 - 21.8 Q3

Settings

Variable	Setting
T	41
Q1	1
Q2	0
Q3	0

Prediction

Fit	SE Fit	95% CI	95% PI
2491.68	71.6721	(2346.18, 2637.18)	(2128.88, 2854.49)

Settings

Variable	Setting
T	42
Q1	0
Q2	1
Q3	0

Prediction

Fit	SE Fit	95% CI	95% PI
2502.54	71.6721	(2357.04, 2648.04)	(2139.73, 2865.34)

Settings

Variable	Setting
T	43
Q1	0
Q2	0
Q3	1

Prediction

Fit	SE Fit	95% CI	95% PI
2515.89	71.6721	(2370.39, 2661.39)	(2153.08, 2878.69)

Settings

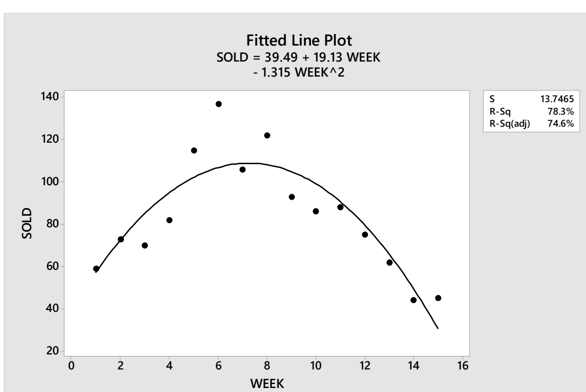
Variable	Setting
T	44
Q1	0
Q2	0
Q3	0

Prediction

Fit	SE Fit	95% CI	95% PI
2578.41	71.6721	(2432.91, 2723.91)	(2215.60, 2941.21)

The 95% prediction intervals for 2018 are $(2128.88,\ 2854.49),(2139.73,\ 2865.34),(2153.08,\ 2878.69),$ and $(2215.60,\ 2941.21)$ for each of the four quarters.

10.13 a. Using MINITAB, the plot is:



There is evidence that there is a quadratic trend.

b. Using MINITAB, the results are:

Regression Analysis: SOLD versus t, t-sq
Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	2	8163	4081.4	21.60	0.000
Error	12	2268	189.0		
Total	14	10430			

Model Summary

S	R-sq	R-sq(adj)
13.7465	78.26%	74.64%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	39.5	12.2	3.22	0.007
T	19.13	3.52	5.43	0.000
t-sq	-1.315	0.214	-6.15	0.000

Regression Equation

SOLD = 39.5 + 19.13 t - 1.315 t-sq

The least squares prediction equation is $\hat{y}_t = 39.5 + 19.13t - 1.315t^2$.

c. The least squares prediction equation $\hat{y}_t$, plotted in the graph above, appears to provide an adequate fit to the quadratic secular trend. The coefficient of determination is $R^2 = 0.7826$ and the test statistic for checking model adequacy is $F = 21.60$ $(p-\text{value} = 0.000)$.

d. Using MINITAB, the results are:

Prediction for SOLD
Regression Equation

SOLD = 39.5 + 19.13 t - 1.315 t-sq

Settings

Variable	Setting
t	16
t-sq	256

Prediction

Fit	SE Fit	95% CI	95% PI
8.85934	12.2445	(-17.8191, 35.5377)	(-31.2505, 48.9692)

A 95% prediction interval for home sales in week 16 is $(-31.25, 48.97)$. Note the negative lower bound and the large width of the interval. Although the quadratic model is statistically adequate (see part (c)), the wide 95% prediction interval reveals that the model may not be very practical for forecasting.

10.14 a. The model would be $E(y_t) = \beta_0 + \beta_1 x_{1t} + \beta_2 x_{2t} + \beta_3 x_{3t} + \beta_4 x_{4t} + \beta_5 x_{5t}$.

b. $R^2 = 0.91$. This indicates that 91% of the variation in the percentage of the two-party vote won by the incumbent party's candidate is explained by the model.

c. $$F = \frac{R^2/k}{\left(1-R^2\right)/\left[n-(k+1)\right]} = \frac{0.91/5}{(1-0.91)/\left[24-(5+1)\right]} = 36.4.$$

To determine if the model is adequate, we test:

$H_0: \beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = 0$
$H_a:$ At least one $\beta_i \neq 0$

The test statistic is $F = 36.4$. The rejection region requires $\alpha = 0.05$ in the upper tail of the F distribution with $\nu_1 = k = 5$ and $\nu_2 = n-(k+1) = 24-(5+1) = 18$. From Table 4, Appendix D, $F = 2.77$. The rejection region is $F > 2.77$.

Since the observed value of the test statistic falls in the rejection region $(F = 36.4 > 2.77)$, H_0 is rejected. There is sufficient evidence to indicate that the model is adequate at $\alpha = 0.05$.

d. There is sufficient evidence to indicate that there is a linear relationship between the percentage of the two-party vote won by the incumbent party's candidate and the fiscal policy of the incumbent party at $\alpha = 0.05$, adjusting for the other variables in the model.

e. There is sufficient evidence to indicate that there is a linear relationship between the percentage of the two-party vote won by the incumbent party's candidate and the duration of the incumbent party at $\alpha = 0.05$, adjusting for the other variables in the model.

f. There is sufficient evidence to indicate that there is a linear relationship between the percentage of the two-party vote won by the incumbent party's candidate and the party of the incumbent party at $\alpha = 0.05$, adjusting for the other variables in the model.

g. There is sufficient evidence to indicate that there is a linear relationship between the percentage of the two-party vote won by the incumbent party's candidate and the GDP trend in election year at $\alpha = 0.05$, adjusting for the other variables in the model.

h. $\beta_5 = 0.66$. For each additional unit increase in the growth rate of the GDP in the first 3 quarters of the election year, the percentage of the two-party vote won by the incumbent party's candidate is estimated to increase by 0.66, holding all other variables constant.

i. $s = 2.36$. We would expect most of the observations to fall within $2s = 2(2.36) = 4.72$ units of their predicted values.

j. Yes. The model is adequate, the value of R^2 is quite large, and the independent variables are significant.

10.15 a. The model would be $E(y_t) = \beta_0 + \beta_1 x_{1,t}$.

b. The model would be $E(y_t) = \beta_0 + \beta_1 x_{1,t} + \beta_2 x_{2,t} + \beta_3 x_{3,t} + \beta_4 x_{4,t}$.

c. The model would be
$E(y_t) = \beta_0 + \beta_1 x_{1,t} + \beta_2 x_{2,t} + \beta_3 x_{3,t} + \beta_4 x_{4,t} + \beta_5 Q_1 + \beta_6 Q_2 + \beta_7 Q_3$,

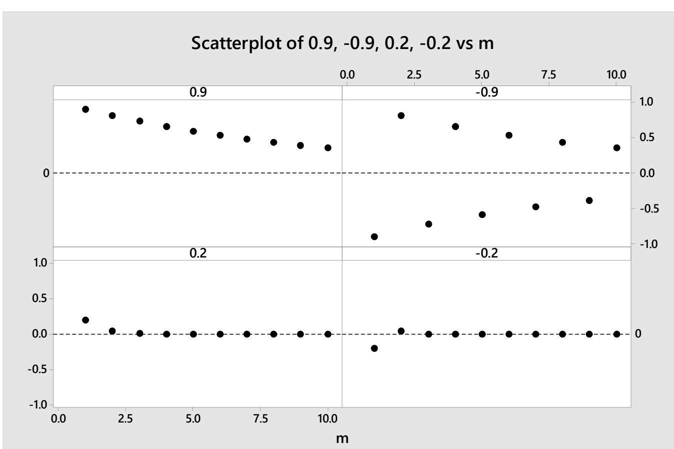
where $Q_1 = \begin{cases} 1 & \text{if Quarter 1} \\ 0 & \text{if not} \end{cases}$ $\quad Q_2 = \begin{cases} 1 & \text{if Quarter 2} \\ 0 & \text{if not} \end{cases}$ $\quad Q_3 = \begin{cases} 1 & \text{if Quarter 3} \\ 0 & \text{if not} \end{cases}$

10.16 The autocorrelations can be calculated using Autocorrelation$(R_t, R_{t+m}) = \varphi^m$.

a. Using the calculations above, the first ten autocorrelations for the four models are shown below:

Autocorrelation	$\varphi = 0.9$	$\varphi = -0.9$	$\varphi = 0.2$	$\varphi = -0.2$
R_t, R_{t+1}	0.9	-0.9	0.2	-0.2
R_t, R_{t+2}	0.81	0.81	0.04	0.04
R_t, R_{t+3}	0.729	-0.729	0.008	-0.008
R_t, R_{t+4}	0.6561	0.6561	0.0016	0.0016
R_t, R_{t+5}	0.5905	-0.5905	0.00032	-0.00032
R_t, R_{t+6}	0.5314	0.5314	0.000064	0.000064
R_t, R_{t+7}	0.4783	-0.4783	0.0000128	-0.0000128
R_t, R_{t+8}	0.4305	0.4305	0.0000026	0.0000026
R_t, R_{t+9}	0.3874	-0.3874	0.0000005	-0.0000005
R_t, R_{t+10}	0.3487	0.3487	0.0000001	0.0000001

b. The plots are:



c. For larger values of φ, the rate at which the correlation diminishes is small, when compared to smaller values of φ. This implies the residuals will tend to be more correlated.

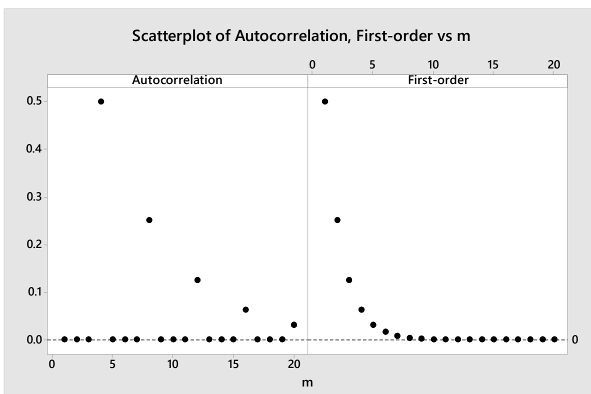
10.17 a. The nonzero autocorrelations are those for values of $m = 4, 8, 12, 16,$ and 20:

$$\text{Autocorrelation }(R_t, R_{t+4}) = \varphi^{4/4} = \varphi^1 = 0.5$$
$$\text{Autocorrelation }(R_t, R_{t+8}) = \varphi^{8/4} = \varphi^2 = 0.25$$
$$\text{Autocorrelation }(R_t, R_{t+12}) = \varphi^{12/4} = \varphi^3 = 0.125$$
$$\text{Autocorrelation }(R_t, R_{t+16}) = \varphi^{16/4} = \varphi^4 = 0.0625$$
$$\text{Autocorrelation }(R_t, R_{t+20}) = \varphi^{20/4} = \varphi^5 = 0.03125$$

Thus, the first twenty autocorrelations for this model are:

0, 0, 0, 0.5, 0, 0, 0, 0.25, 0, 0, 0, 0.125, 0, 0, 0, 0.0625, 0, 0, 0, 0.03125

b. The plots are:



10.18 Since the autocorrelation $(R_t, R_{t+1}) = -0.6$, this implies that $\varphi = -0.6$. The model is $E(y) = \beta_0 + \beta_1 t + R_t$ where $R_t = -0.6R_{t-1} + \varepsilon_t$.

10.19 The general form of the pth-order autoregressive model is:

$$R_t = \varphi_1 R_{t-1} + \varphi_2 R_{t-2} + \cdots + \varphi_p R_{t-p} + \varepsilon_t$$

Thus, for $p = 4$, we have: $R_t = \varphi_1 R_{t-1} + \varphi_2 R_{t-2} + \varphi_3 R_{t-3} + \varphi_4 R_{t-4} + \varepsilon_t$

10.20 a. The first order model is $E(y_t) = \beta_0 + \beta_1 t$.

b. The model including a quadratic trend is $E(y_t) = \beta_0 + \beta_1 t + \beta_2 t^2$.

c. The autoregressive model for R_t is $R_t = \varphi R_{t-1} + \varepsilon_t$.

10.21 a. The first-order model is $E(y_t) = \beta_0 + \beta_1 x_{1t} + \beta_2 x_{2t} + \beta_3 x_{3t} + \beta_4 t$.

b. The interaction model allows for interaction between each independent variable and time is $E(y_t) = \beta_0 + \beta_1 x_{1t} + \beta_2 x_{2t} + \beta_3 x_{3t} + \beta_4 t + \beta_5 x_{1t} t + \beta_6 x_{2t} t + \beta_7 x_{3t} t$.

c. We postulate a first-order autoregressive model for R_t as $R_t = \varphi R_{t-1} + \varepsilon_t$,

where $|\varphi| < 1$ and ε_t is a white noise process. This model is appropriate because we expect that residuals at consecutive time points will be positively correlated (i.e., they will tend to have the same sign), and the correlation will diminish as the time distance between two points increases.

10.22 a. The proposed model would be $E(y_t) = \beta_0 + \beta_1 t + \beta_2 x_1 + \beta_3 x_2 + \beta_4 x_3 + R_t$

where $x_1 = \begin{cases} 1 & \text{if fall} \\ 0 & \text{if not} \end{cases}$ $\quad x_2 = \begin{cases} 1 & \text{if winter} \\ 0 & \text{if not} \end{cases}$ $\quad x_3 = \begin{cases} 1 & \text{if spring} \\ 0 & \text{if not} \end{cases}$

b. The proposed model would be $R_t = \varphi R_{t-1} + \varepsilon_t$.

c. The full model would be $y_t = \beta_0 + \beta_1 t + \beta_2 x_1 + \beta_3 x_2 + \beta_4 x_3 + \varphi R_{t-1} + \varepsilon_t$.

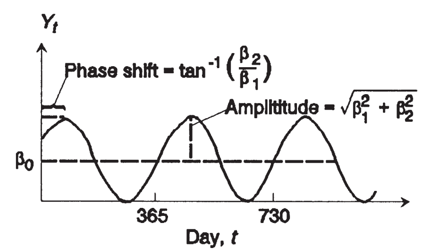
d. The full model with interaction terms would be
$y_t = \beta_0 + \beta_1 t + \beta_2 x_1 + \beta_3 x_2 + \beta_4 x_3 + \beta_5 x_1 y + \beta_6 x_2 t + \beta_7 x_3 t + \varphi R_{t-1} + \varepsilon_t$.

10.23 a. The model for daily price with mean $E(y_t)$ may be written as:

$$E(y_t) = \beta_0 + \beta_1\left(\cos\frac{2\pi t}{365}\right) + \beta_2\left(\sin\frac{2\pi t}{365}\right)$$

This cyclic model, with a period of 365 days, allows for seasonality, with the expected peaks and valleys remaining the same from year to year.

b. The amplitude (magnitude of the seasonal effect) and phase shift (which determines the location in time of the peaks and valleys) are functions of β_1 and β_2. The value of β_0 is the overall mean of the time series.



c. The interaction model is:

$$E(y_t) = \beta_0 + \beta_1\left(\cos\frac{2\pi t}{365}\right) + \beta_2\left(\sin\frac{2\pi t}{365}\right) + \beta_3 t + \beta_4 t\left(\cos\frac{2\pi t}{365}\right) + \beta_5 t\left(\sin\frac{2\pi t}{365}\right)$$

The presence of interaction indicates seasonal effects increase or decrease with time (not the same from year to year).

d. We expect that residuals on consecutive days would be positively correlated. Thus, it is unreasonable to assume that the random error component, R_t, is white noise. Instead, we propose a first-order autoregressive model for R_t :

$$R_t = \varphi R_{t\text{-}1} + \varepsilon_t, \qquad \text{where } |\varphi| < 1 \text{ and } \varepsilon_t \text{ is a white noise process.}$$

10.24 a. The model including a curvilinear relationship would be $E(y_t) = \beta_0 + \beta_1 x_t + \beta_2 x_t^2$.

b. The first-order autoregressive model for R_t is $R_t = \varphi R_{t-1} + \varepsilon_t$.

c. The full model would be $y_t = \beta_0 + \beta_1 x_t + \beta_2 x_t^2 + \varphi R_{t-1} + \varepsilon_t$.

10.25 a. The time series model that includes a straight-line long-term trend and autocorrelated residuals would be $y_t = \beta_0 + \beta_1 t + \varphi R_{t-1} + \varepsilon_t$.

b. Using the SAS printout, the least squares prediction equation is $\hat{y}_t = 14767 + 88.1634t + 0.6468\hat{R}_{t-1}$.

c. $\hat{\beta}_0 = 14767$: This is an estimate of where the regression line will cross the y_t axis

$\hat{\beta}_1 = 88.1634$: The estimated increase in GDP for each additional increase of one time period is 88.1634.

$\hat{\varphi} = 0.6468$: Since $\hat{\varphi}$ is positive, it implies that the time series residuals are positively autocorrelated.

d. $R^2 = 0.9948$ This means that 99.48 % of the sum of the squares of the deviations of the GDP values about their means is attributable to the time series model.

$s = 54.17$ We would expect most of the observed GDP values to fall within $2s = 2(54.17) = 108.34$ units of their predicted values.

10.26 a. The model would be $y_t = \beta_0 + \beta_1 t + \varphi R_{t-1} + \varepsilon_t$.

b. Using SAS, the results are:

```
                          The AUTOREG Procedure

                     Dependent Variable    RATE


                     Ordinary Least Squares Estimates

SSE                 4.58538172    DFE                     21
MSE                    0.21835    Root MSE           0.46728
SBC                 34.4518812    AIC             32.1808928
Regress R-Square        0.9072    Total R-Square      0.9072
Durbin-Watson           1.0603
```

Variable	DF	Estimate	Standard Error	t Value	Approx Pr > |t|
Intercept	1	8.3109	0.2014	41.27	<.0001
T	1	-0.2104	0.0147	-14.32	<.0001

```
                    Estimates of Autocorrelations

Lag    Covariance     Correlation    -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1

  0        0.1994        1.000000    |                    |********************|
  1        0.0872        0.437370    |                    |*********           |
```

Preliminary MSE 0.1612

Estimates of Autoregressive Parameters

Lag	Coefficient	Standard Error	t Value
1	-0.437370	0.201085	-2.18

Yule-Walker Estimates

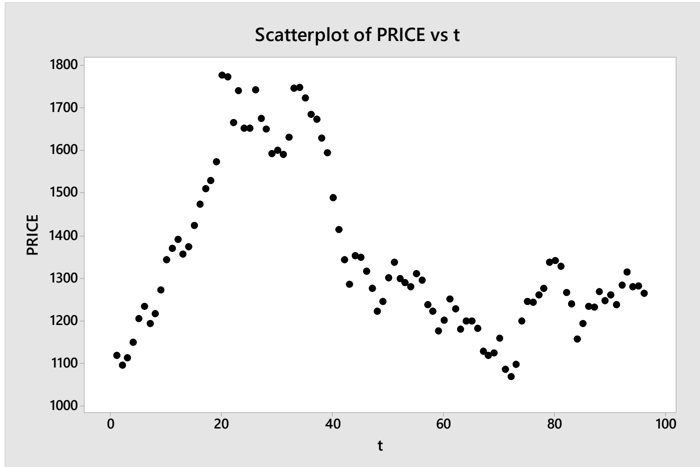
SSE	3.63827923	DFE	20
MSE	0.18191	Root MSE	0.42651
SBC	32.4783553	AIC	29.0718727
Regress R-Square	0.8179	Total R-Square	0.9263
Durbin-Watson	1.5436		

Variable	DF	Estimate	Standard Error	t Value	Approx Pr > |t|
Intercept	1	8.2552	0.3011	27.42	<.0001
T	1	-0.2048	0.0216	-9.48	<.0001

As indicated in the SAS output above, the estimate of β_0 is 8.2552, the estimate of β_1 is -0.2048, and the estimate of φ is 0.4374 (note: the estimate of φ from SAS is defined so that φ takes the opposite sign from the model specified above).

To assess whether the residuals are autocorrelated, an approximate *t*-test of the null hypothesis $H_0 : \varphi = 0$ yields $t = 2.18$. With 20 degrees of freedom for error, the *p*-value is approximately 0.0414. Thus, the value is significant at the 5% level. The first order autoregressive model does describe the residual correlation well.

10.27 a. Using MINITAB, the plot of the data is:



Yes, as indicated in the scatterplot there appears to be a quadratic trend.

b. A time series model to incorporate a long-term quadratic trend and autocorrelated residuals takes the form $y_t = \beta_0 + \beta_1 t + \beta_2 t^2 + \varphi R_{t-1} + \varepsilon_t$.

c. Using SAS, the results are:

```
                         The AUTOREG Procedure

                       Dependent Variable    PRICE

                    Ordinary Least Squares Estimates

      SSE                  2652179.62    DFE                      93
      MSE                       28518    Root MSE          168.87291
      SBC                  1267.87748    AIC              1260.18444
      Regress R-Square         0.2535    Total R-Square       0.2535
      Durbin-Watson            0.0877

                                        Standard                 Approx
    Variable        DF      Estimate       Error    t Value    Pr > |t|

    Intercept        1          1373     52.8028      26.00      <.0001
    T                1        4.4607      2.5127       1.78      0.0791
    TSQ              1       -0.0763      0.0251      -3.04      0.0031


                     Estimates of Autocorrelations

Lag    Covariance     Correlation    -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1

  0       27626.9        1.000000    |                    |********************|
  1       25921.3        0.938265    |                    |*******************  |

                      Preliminary MSE      3305.8

                 Estimates of Autoregressive Parameters

                                          Standard
                 Lag      Coefficient        Error     t Value

                   1        -0.938265     0.036064      -26.02
```

Yule-Walker Estimates

SSE	229532.569	DFE	92
MSE	2495	Root MSE	49.94917
SBC	1039.6441	AIC	1029.38671
Regress R-Square	0.0269	Total R-Square	0.9354
Durbin-Watson	1.4982		

Variable	DF	Estimate	Standard Error	t Value	Approx Pr > |t|
Intercept	1	1213	142.7479	8.50	<.0001
T	1	9.1386	6.3942	1.43	0.1563
TSQ	1	-0.0989	0.0627	-1.58	0.1180

(i) From the SAS output, we obtain the following estimated values (note: the estimate of φ from SAS is defined so that φ takes the opposite sign from the model specified in part b.

$$\hat{\beta}_0 = 1213$$
$$\hat{\beta}_1 = 9.1386$$
$$\hat{\beta}_2 = -0.0989$$
$$\hat{\varphi} = 0.9383$$

Hence the equation takes the form:
$$\hat{y} = 1213 + 9.1386t - 0.0989t^2 + 0.9383\hat{R}_{t-1}$$

(ii) Note from the SAS output that the value of the Regression R^2 is 0.2535, and the Total R^2 is 0.9354.

(iii) To test the whether the t^2 coefficient is significantly different from zero, we find from the SAS output that $t = -1.58$, and the associated p-value is $p = 0.1180$. Since 0.1180 is not less than a significance level of 0.05 we cannot reject the null hypothesis and conclude there is insufficient evidence of a quadratic long term trend.

10.28 a. For $t = 31$:

$$\hat{R}_{30} = y_{30} - \hat{y}_{30} = 82 - (10 + 2.5t) = 82 - (10 + 2.5(30)) = -3$$

$$\hat{y}_{31} = 10 + 2.5(31) + 0.64\hat{R}_{30} = 87.5 + 0.64(-3) = 85.58$$

For $t = 32$:

$$\hat{R}_{31} = 0.64\hat{R}_{30} = 0.64(-3) = -1.92$$

$$\hat{y}_{32} = 10 + 2.5(32) + 0.64\hat{R}_{31} = 90 + 0.64(-1.92) = 88.77$$

For $t = 33$:

$$\hat{R}_{32} = 0.64\hat{R}_{31} = 0.64(-1.92) = -1.2288$$

$$\hat{y}_{33} = 10 + 2.5(33) + 0.64\hat{R}_{32} = 92.5 + 0.64(-1.2288) = 91.71$$

b. The form of the prediction interval is $\hat{y}_{n+m} \pm 2\sqrt{MSE\left(1+\hat{\varphi}^2+\hat{\varphi}^4+\cdots++\hat{\varphi}^{2(m-1)}\right)}$ where m is the number of steps ahead.

For $t = 31,\ m = 1$.

The approximate 95% prediction interval is:
$$\hat{y}_{31} \pm 2\sqrt{MSE(1)} \Rightarrow 85.58 \pm 2\sqrt{4.3} \Rightarrow 85.58 \pm 4.15 \Rightarrow (81.43,\ 89.73)$$

For $t = 32,\ m = 2$.

The approximate 95% prediction interval is:
$$\hat{y}_{32} \pm 2\sqrt{MSE\left(1+\hat{\varphi}^2\right)} \Rightarrow 88.77 \pm 2\sqrt{4.3\left(1+0.64^2\right)} \Rightarrow 88.77 \pm 4.92$$
$$\Rightarrow (83.85,\ 93.69)$$

For $t = 33,\ m = 3$.

The approximate 95% prediction interval is:
$$\hat{y}_{33} \pm 2\sqrt{MSE\left(1+\hat{\varphi}^2+\hat{\varphi}^4\right)} \Rightarrow 91.71 \pm 2\sqrt{4.3\left(1+0.64^2+0.64^4\right)} \Rightarrow 91.71 \pm 5.21$$
$$\Rightarrow (86.50,\ 96.92)$$

10.29 a. We begin the forecasting procedure by calculating the estimated residual for the last observation in the series, i.e., $\hat{R}_n$, where

$$\hat{R}_n = y_n - \hat{y}_n = y_n - (\hat{\beta}_0 + \hat{\beta}_1 n + \hat{\beta}_2 n^2)$$

Substituting $n = 48,\ \hat{\beta}_0 = 220,\ \hat{\beta}_1 = 17,\ \hat{\beta}_2 = -0.3,$ and $y_{48} = 350$, we have:

$$\hat{R}_{48} = y_{48} - [\hat{\beta}_0 + \hat{\beta}_1(48) + \hat{\beta}_2(48)^2] = 350 - [220 + 17(48) - .3(48)^2]$$
$$= 350 - 334.8 = 5.2$$

To obtain the forecasts, we use the formulas:

$$\hat{R}_t = 0.81\hat{R}_{t-1}$$
$$\hat{y}_t = 220 + 17t - 0.3t^2 + \hat{R}_t$$

$t = 49$: $\hat{R}_{49} = 0.81\hat{R}_{48} = 0.81(5.2) = 4.212$

$\hat{y}_{49} = 220 + 17(49) - 0.3(49)^2 + 4.212 = 336.91$

$t = 50$: $\hat{R}_{50} = 0.81\hat{R}_{49} = 0.81(4.212) = 3.412$

$\hat{y}_{50} = 220 + 17(50) - 0.3(50)^2 + 3.412 = 323.41$

$t = 51$: $\hat{R}_{51} = .81\hat{R}_{48} = 0.81(3.412) = 2.763$

$\hat{y}_{51} = 220 + 17(51) - 0.3(51)^2 + 2.763 = 309.46$

b. An approximate 95% prediction interval for the forecast m time periods into the future is given by the formula:

$$\hat{y}_{n+m} \pm 2\sqrt{\text{MSE}\left(1 + \hat{\varphi}^2 + \hat{\varphi}^4 + \cdots + \hat{\varphi}^{2(m-1)}\right)}$$

Since $n = 48$, we want to obtain prediction intervals for the forecasts $m = 1, m = 2,$ and $m = 3$ time periods (quarters) into the future.

Approximate 95% Prediction Intervals

$t = 49$: $\hat{y}_{49} \pm 2\sqrt{\text{MSE}}$

$(m = 1) \Rightarrow 336.91 \pm 2\sqrt{10.5} \Rightarrow 336.91 \pm 6.48 \Rightarrow (330.43,\ 343.39)$

$t = 50$: $\hat{y}_{50} \pm 2\sqrt{\text{MSE}\left(1 + \hat{\varphi}^2\right)}$

$(m = 2) \Rightarrow 323.41 \pm 2\sqrt{10.5\left(1 + 0.81^2\right)} \Rightarrow 323.41 \pm 8.34 \Rightarrow (315.07,\ 331.75)$

$t = 51$: $\hat{y}_{51} \pm 2\sqrt{\text{MSE}\left(1 + \hat{\varphi}^2 + \hat{\varphi}^4\right)}$

$(m = 3) \Rightarrow 309.46 \pm 2\sqrt{10.5\left(1 + 0.81^2 + 0.81^4\right)} \Rightarrow 309.46 \pm 9.36 \Rightarrow (300.10,\ 318.82)$

10.30 An approximate 95% prediction interval for the forecast m time periods into the future is given by the formula:

$$\hat{y}_{n+m} \pm 2\sqrt{\text{MSE}\left(1 + \hat{\varphi}^2 + \hat{\varphi}^4 + \cdots + \hat{\varphi}^{2(m-1)}\right)}$$

For $t = 29$:

$$\hat{R}_{28} = y_{28} - \hat{y}_{28} = 17286.50 - (14767 + 88.1634t)$$
$$= 17286.50 - (14767 + 88.1634(28)) = 50.9248$$

$$\hat{y}_{29} = 14767 + 88.1634(29) + 0.6468\hat{R}_{28}$$
$$= 14767 + 88.1634(29) + 0.6468(50.9248) = 17356.68$$

The approximate 95% prediction interval is

$$(m=1) \Rightarrow \hat{y}_{29} \pm 2\sqrt{\text{MSE}} \Rightarrow 17356.68 \pm 2\sqrt{2934}$$
$$\Rightarrow 17356.68 \pm 108.33 \Rightarrow (17248.35,\ 17465.01)$$

For $t = 30$:

$$\hat{R}_{29} = 0.6468\hat{R}_{28} = 0.6468(50.9248) = 32.9382$$

$$\hat{y}_{30} = 14767 + 88.1634(30) + 0.6468\hat{R}_{29}$$
$$= 14767 + 88.1634(30) + 0.6468(32.9382) = 17433.21$$

The approximate 95% prediction interval is

$$(m=2) \Rightarrow \hat{y}_{30} \pm 2\sqrt{\text{MSE}\left(1+\hat{\varphi}^2\right)} \Rightarrow 17433.21 \pm 2\sqrt{2934\left(1+0.6468^2\right)}$$
$$\Rightarrow 17433.21 \pm 129.02 \Rightarrow (17304.19,\ 17562.23)$$

For $t = 31$:

$$\hat{R}_{30} = 0.6468\hat{R}_{29} = 0.6468(32.9382) = 21.3044$$

$$\hat{y}_{31} = 14767 + 88.1634(31) + 0.6468\hat{R}_{30}$$
$$= 14767 + 88.1634(31) + 0.6468(21.3044) = 17513.85$$

The approximate 95% prediction interval is

$$(m=3) \Rightarrow \hat{y}_{31} \pm 2\sqrt{\text{MSE}\left(1+\hat{\varphi}^2+\hat{\varphi}^4\right)}$$
$$\Rightarrow 17513.85 \pm 2\sqrt{2934\left(1+0.6468^2+0.6468^4\right)}$$
$$\Rightarrow 17513.85 \pm 136.75 \Rightarrow (17377.10,\ 17650.60)$$

For $t = 32$:

$$\hat{R}_{31} = 0.6468\hat{R}_{30} = 0.6468(21.3044) = 13.7797$$

$$\hat{y}_{32} = 14767 + 88.1634(32) + 0.6468\hat{R}_{31}$$
$$= 14767 + 88.1634(32) + 0.6468(13.7797) = 17597.14$$

The approximate 95% prediction interval is

$$(m=4)\Rightarrow \hat{y}_{32}\pm 2\sqrt{\text{MSE}\left(1+\hat{\varphi}^2+\hat{\varphi}^4+\hat{\varphi}^6\right)}$$
$$\Rightarrow 17597.14\pm 2\sqrt{2934\left(1+0.6468^2+0.6468^4+0.6468^6\right)}$$
$$\Rightarrow 17597.14\pm 139.85\Rightarrow (17457.29,\ 17736.99)$$

10.31 To forecast the average mortgage interest rate for 2018, i.e. a value one step ahead from 2017, takes the form:

$$F_{n+1}=\hat{\beta}_0+\hat{\beta}_1 t+\hat{\varphi}\hat{R}_n$$

$$\hat{R}_n=y_n-\hat{y}_n\Rightarrow \hat{R}_{23}=y_{23}-\hat{y}_{23}=3.99-\left(8.2552-0.2048(23)\right)=0.4452$$

$$F_{24}=8.2552-0.2048(24)+0.4374(0.4452)=3.53$$

The approximate 95% confidence interval is

$$\hat{y}_{24}\pm 2\sqrt{MSE}\Rightarrow 3.53\pm 2\sqrt{0.18191}\Rightarrow 3.53\pm 0.853\Rightarrow (2.677,\ 4.383)$$

10.32 Note that the time series model from 10.27 takes the form:
$\hat{y}_t=1213+9.1386t-0.0989t^2+0.9383\hat{R}_{t-1}$

To forecast gold prices in January and February of 2018 (i.e. the 97th and 98th values in the time series) we first require an estimate of the last residual R_{96}

$$\hat{R}_{96}=y_{96}-\left[\hat{\beta}_0+\hat{\beta}_1(96)+\hat{\beta}_2(96)^2\right]=1264.5-\left[1213+9.1386(96)-0.0989(96)^2\right]=85.6568$$

Therefore the one-step ahead forecast (i.e. the gold price estimate for Jan 2018) is given by

$$F_{97}=\hat{\beta}_0+\hat{\beta}_1 t+\hat{\beta}_2 t^2+\hat{\varphi}\hat{R}_{96}=1213+9.1386(97)-0.0989(97)^2+0.9383(85.6568)=1249.27$$

The two-step ahead forecast (i.e. the gold price estimate for February 20189) is given by

$$F_{98}=\hat{\beta}_0+\hat{\beta}_1 t+\hat{\beta}_2 t^2+\hat{\varphi}^2\hat{R}_{96}=1213+9.1386(98)-0.0989(98)^2+0.9383^2(85.6568)=1234.16$$

The approximate 95% prediction interval for gold prices in January 2018 is:

$$F_{97}\pm 2\sqrt{MSE}\Rightarrow 1249.27\pm 2\sqrt{2495}\Rightarrow 1249.27\pm 99.90\Rightarrow (1149.37,\ 1349.17)$$

The approximate 95% prediction interval for gold prices in February 2018 is:

$$F_{98}\pm 2\sqrt{MSE\left(1+\hat{\varphi}^2\right)}\Rightarrow 1234.16\pm 2\sqrt{2495\left(1+0.9383^2\right)}\Rightarrow 1234.16\pm 136.99\Rightarrow (1097.17,\ 1371.15)$$

10.33 a. The hypothesized model would be $E(y_t)=\beta_0+\beta_1 t+\beta_2 t^2$.

b. We would not expect the random error term to be uncorrelated. Economic conditions would surely determine the price of gold at any point in time. Since these economic conditions would most likely be correlated from one year to the next, it seems likely that the random error component in the model would also be correlated.

c. The hypothesized model would be $R_t = \varphi R_{t-1} + \varepsilon_t$.

d. The combined model would be $y_t = \beta_0 + \beta_1 t + \beta_2 t^2 + \varphi R_{t-1} + \varepsilon_t$

10.34 a. To compute a moving average using three points, the midpoint of the sum is the second time point. The first moving average in this series which can be calculated in this manner will be from 1996.

The general formula for calculating the sum of the three values is:
$L_t = y_{t-1} + y_t + y_{t+1}$.

To calculate the moving average we divide the resulting sum by 3. For example, for the 1996 moving average point we first compute the total as:

$$L_{1996} = y_{1995} + y_{1996} + y_{1997} = 7.93 + 7.81 + 7.60 = 23.34$$

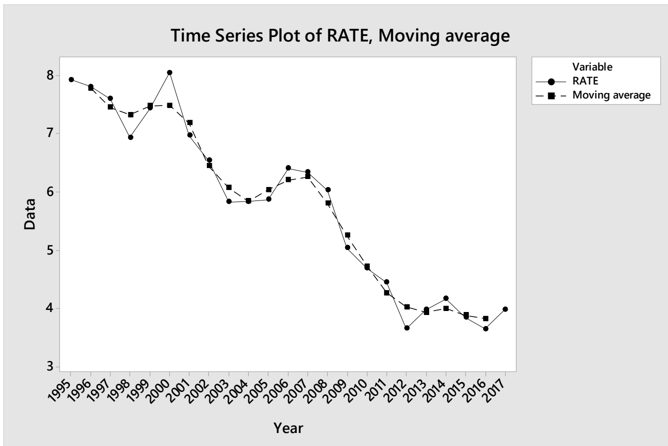
The moving average is $M_{1996} = \frac{L_{1996}}{3} = \frac{23.34}{3} = 7.78$

The remaining moving averages are calculated in a similar manner and are shown in the table below:

YEAR	RATE	L_t	M_t
1995	7.93		
1996	7.81	23.34	7.780
1997	7.60	22.35	7.450
1998	6.94	21.98	7.327
1999	7.44	22.43	7.477
2000	8.05	22.46	7.487
2001	6.97	21.56	7.187
2002	6.54	19.34	6.447
2003	5.83	18.21	6.070
2004	5.84	17.54	5.847
2005	5.87	18.12	6.040
2006	6.41	18.62	6.207
2007	6.34	18.78	6.260
2008	6.03	17.41	5.803
2009	5.04	15.76	5.253
2010	4.69	14.18	4.727
2011	4.45	12.80	4.267
2012	3.66	12.09	4.030
2013	3.98	11.81	3.937

2014	4.17	12.00	4.000
2015	3.85	11.67	3.890
2016	3.65	11.49	3.830
2017	3.99		

A plot of the time series and the moving averages is:



It appears that a reasonable estimate of the interest rate for 2018 would be about 3.6%. This is a bit higher than the estimated value of 3.26 from Exercise 10.8b.

b. To calculate the exponentially smoothed time series, note that the first value in the series is merely the first value of the time series. The remaining values in the smoothed series are calculated using the formula:

$E_t = wy_t + (1-w)E_{t-1}$, where w is the smoothing constant, and in this case we use $w = 0.2$.

For 1995: $E_{1995} = y_{1995} = 7.93$

For 1996: $E_{1996} = wy_{1996} + (1-w)E_{1995} = 0.2(7.81) + (1-0.2)7.93 = 7.906$

The rest of the values of the exponentially smoothed series are found in a similar manner and are shown in the table below:

YEAR	RATE	E_t
1995	7.93	7.930
1996	7.81	7.906
1997	7.60	7.845
1998	6.94	7.664
1999	7.44	7.619
2000	8.05	7.705
2001	6.97	7.558
2002	6.54	7.355
2003	5.83	7.050

2004	5.84	6.808
2005	5.87	6.620
2006	6.41	6.578
2007	6.34	6.531
2008	6.03	6.430
2009	5.04	6.152
2010	4.69	5.860
2011	4.45	5.578
2012	3.66	5.194
2013	3.98	4.951
2014	4.17	4.795
2015	3.85	4.606
2016	3.65	4.415
2017	3.99	4.330

As exponential smoothing forecasts are obtained by using the most recent exponentially smoothed value, it follows that $F_{2018} = E_{2017} = 4.330$. This value is markedly higher than the estimate of 3.26 from Exercise 10.8b.

c. Using the Holt-Winters Method, note that we need to be concerned with the exponential component and the time trend components only (no seasonal component in this case).

When $t = 2$, $E_t = y_t$ and $T_t = y_t - y_{t-1}$
Thus, $E_2 = y_2 = 7.81$ and $T_2 = 7.81 - 7.93 = -0.12$

When $t > 2$, we use:

$$E_t = wy_t + (1-w)(E_{t-1} + T_{t-1}) \text{ and } T_t = v(E_t - E_{t-1}) + (1-v)(T_{t-1})$$

Therefore, $E_3 = 0.2(7.6) + 0.8(7.81 + (-0.12)) = 7.672$
$T_3 = 0.5(7.672 - 7.81) + 0.5(-0.12) = -0.129$

In this manner the remaining exponentially smoothed values, and trend values may be calculated as shown in the table below:

YEAR	RATE	E_t	T_t
1995	7.93		
1996	7.81	7.810	-0.120
1997	7.60	7.672	-0.129
1998	6.94	7.422	-0.189
1999	7.44	7.274	-0.169
2000	8.05	7.295	-0.074
2001	6.97	7.170	-0.099
2002	6.54	6.965	-0.152

2003	5.83	6.616	-0.251
2004	5.84	6.260	-0.303
2005	5.87	5.940	-0.312
2006	6.41	5.784	-0.234
2007	6.34	5.709	-0.155
2008	6.03	5.649	-0.107
2009	5.04	5.442	-0.157
2010	4.69	5.165	-0.217
2011	4.45	4.849	-0.267
2012	3.66	4.398	-0.359
2013	3.98	4.027	-0.365
2014	4.17	3.764	-0.314
2015	3.85	3.530	-0.274
2016	3.65	3.335	-0.235
2017	3.99	3.278	-0.146

To forecast the price of gold from 2017 as a baseline (i.e. from $n = 23$), note that:

$$F_{2018} = F_{24} = E_{23} + T_{23} = 3.278 + \left(-0.146\right) = 3.132$$

This value is markedly lower than the estimate of 3.26 from Exercise 10.8b.

d. A straight-line, first-order autoregressive model is of the form $y_t = \beta_0 + \beta_1 t + \beta_2 t^2 + \varphi R_{t-1} + \varepsilon_t$. Using SAS, the results are:

```
                         The AUTOREG Procedure

                       Dependent Variable    RATE

                   Ordinary Least Squares Estimates

       SSE                   4.58538172    DFE                      21
       MSE                      0.21835    Root MSE            0.46728
       SBC                   34.4518812    AIC              32.1808928
       Regress R-Square          0.9072    Total R-Square      0.9072
       Durbin-Watson             1.0603


                                      Standard                 Approx
    Variable        DF     Estimate      Error    t Value    Pr > |t|

    Intercept        1       8.3109     0.2014      41.27      <.0001
    T                1      -0.2104     0.0147     -14.32      <.0001


                      Estimates of Autocorrelations

 Lag    Covariance     Correlation    -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1

   0        0.1994        1.000000    |                    |********************|
   1        0.0872        0.437370    |                    |*********           |

                       Preliminary MSE      0.1612


                  Estimates of Autoregressive Parameters
```

Lag	Coefficient	Standard Error	t Value
1	-0.437370	0.201085	-2.18

Yule-Walker Estimates

SSE	3.63827923	DFE	20
MSE	0.18191	Root MSE	0.42651
SBC	32.4783553	AIC	29.0718727
Regress R-Square	0.8179	Total R-Square	0.9263
Durbin-Watson	1.5436		

Variable	DF	Estimate	Standard Error	t Value	Approx Pr > |t|
Intercept	1	8.2552	0.3011	27.42	<.0001
T	1	-0.2048	0.0216	-9.48	<.0001

To obtain a forecast for the 2018 interest rate, we note that the three-step ahead forecast (i.e. the 246th time point, which would be for the year 2018) is given by

$$F_{23+1} = \hat{\beta}_0 + \hat{\beta}_1 t + \hat{\varphi}\hat{R}_{23}$$

Note: $\hat{R}_n = y_n - \hat{y}_n$

Therefore, $\hat{R}_{23} = y_{23} - \hat{y}_{23} = 3.99 - \left(8.2552 - 0.2048(23)\right) = 0.4452$

It follows that $F_{24} = \hat{\beta}_0 + \hat{\beta}_1 t + \hat{\varphi}\hat{R}_{23} = 8.2552 - 0.2048(26) + 0.43737(0.4452) = 3.125$

To compute the approximate 95% confidence interval, the one-step ahead forecast takes the form:

$$\hat{y}_{n+3} \pm 2\sqrt{MSE(1)} \Rightarrow 2.968 \pm 2\sqrt{0.18191(1)} \Rightarrow 2.968 \pm 0.853 \Rightarrow (2.115,\ 3.821)$$

10.35 a. The model would be $E(y) = \beta_0 + \beta_1 t + \beta_2 M_1 + \beta_3 M_2 + \cdots + \beta_{12} M_{11}$

where $M_1 = \begin{cases} 1 & \text{if January} \\ 0 & \text{if not} \end{cases}$ $M_2 = \begin{cases} 1 & \text{if February} \\ 0 & \text{if not} \end{cases}$ … $M_{11} = \begin{cases} 1 & \text{if November} \\ 0 & \text{if not} \end{cases}$

b. Using MINITAB, the results are:

Regression Analysis: PHX_ROOMS versus MONTH_T, M1, ... , M10, M11

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	12	4171.50	347.625	38.58	0.000
Error	11	99.12	9.011		
Total	23	4270.62			

Model Summary

S	R-sq	R-sq(adj)
3.00189	97.68%	95.15%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	43.13	2.81	15.36	0.000
MONTH_T	0.354	0.102	3.47	0.005
M1	23.90	3.21	7.46	0.000
M2	42.04	3.17	13.26	0.000
M3	38.69	3.14	12.32	0.000
M4	25.33	3.11	8.14	0.000
M5	19.48	3.09	6.31	0.000
M6	9.12	3.06	2.98	0.013
M7	-0.23	3.05	-0.08	0.941
M8	-1.58	3.03	-0.52	0.612
M9	11.06	3.02	3.67	0.004
M10	22.21	3.01	7.38	0.000
M11	17.85	3.00	5.94	0.000

Regression Equation

PHX_ROOMS = 43.13 + 0.354 MONTH_T + 23.90 M1 + 42.04 M2 + 38.69 M3 + 25.33 M4 + 19.48 M5 + 9.12 M6 - 0.23 M7 - 1.58 M8 + 11.06 M9 + 22.21 M10 + 17.85 M11

The fitted model is

$$\hat{y} = 43.13 + 0.354t + 23.90M_1 + 42.04M_2 + 38.69M_3 + 25.33M_4 + 19.48M_5 + 9.12M_6 - 0.23M_7 - 1.58M_8 + 11.06M_9 + 22.21M_{10} + 17.85M_{11}$$

c. We must fit the reduced model:

Regression Analysis: PHX_ROOMS versus MONTH_T
Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	96.14	96.14	0.51	0.484
Error	22	4174.49	189.75		
Total	23	4270.63			

The test statistic is $F = \dfrac{(SSE_R - SSE_C)/(k-g)}{SSE_C/[n-(k+1)]} = \dfrac{(4174.49 - 99.12)/(12-1)}{99.12/[24-(12+1)]} = 41.11.$

The rejection region requires $\alpha = 0.05$ in the upper tail of the F distribution with $\nu_1 = k - g = 12 - 1 = 11$ and $\nu_2 = n - (k+1) = 24 - (12+1) = 11$. From Table 4, Appendix D, $F_{0.05} \approx 2.82$. The rejection region is $F > 2.82$.

Since the observed value of the test statistic falls in the rejection region $(F = 41.11 > 2.82)$, H_0 is rejected. There is sufficient evidence to indicate the monthly variables are useful predictors of occupancy rates.

d. Using MNIITAB, the results are:

Prediction for PHX_ROOMS
Settings

Variable	Setting
MONTH_T	25
M1	1
M2	0
M3	0
M4	0
M5	0
M6	0
M7	0
M8	0
M9	0
M10	0
M11	0

Prediction

Fit	SE Fit	95% CI	95% PI
75.875	2.80801	(69.6946, 82.0554)	(66.8278, 84.9222)

The 95% prediction interval is $(66.828, 84.922)$.

e. The model would be $E(y) = \beta_0 + \beta_1 t + \beta_2 M_1 + \beta_3 M_2 + \cdots + \beta_{12} M_{11}$

where $M_1 = \begin{cases} 1 & \text{if January} \\ 0 & \text{if not} \end{cases}$ $M_2 = \begin{cases} 1 & \text{if February} \\ 0 & \text{if not} \end{cases}$ $\ldots$ $M_{11} = \begin{cases} 1 & \text{if November} \\ 0 & \text{if not} \end{cases}$

Using MINITAB, the results are:

Regression Analysis: ATL_ROOMS versus MONTH_T, M1, ... 9, M10, M11
Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	12	976.83	81.403	9.03	0.000
Error	11	99.12	9.011		
Total	23	1075.96			

Model Summary

S	R-sq	R-sq(adj)
3.00189	90.79%	80.74%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	42.12	2.81	15.00	0.000
MONTH_T	0.271	0.102	2.65	0.023
M1	17.48	3.21	5.45	0.000
M2	21.71	3.17	6.85	0.000
M3	25.94	3.14	8.26	0.000
M4	23.67	3.11	7.61	0.000
M5	20.40	3.09	6.61	0.000
M6	19.63	3.06	6.41	0.000
M7	21.85	3.05	7.18	0.000

M8	21.58	3.03	7.12	0.000
M9	17.31	3.02	5.74	0.000
M10	26.04	3.01	8.66	0.000
M11	15.77	3.00	5.25	0.000

Regression Equation

ATL_ROOMS = 42.12 + 0.271 MONTH_T + 17.48 M1 + 21.71 M2 + 25.94 M3 + 23.67 M4 + 20.40 M5 + 19.63 M6 + 21.85 M7 + 21.58 M8 + 17.31 M9 + 26.04 M10 + 15.77 M11

The fitted model is

$$\hat{y} = 42.12 + 0.271t + 17.48M_1 + 21.71M_2 + 25.94M_3 + 23.67M_4 + 20.40M_5 + 19.63M_6 + 21.85M_7 + 21.58M_8 + 17.31M_9 + 26.04M_{10} + 15.77M_{11}$$

We must fit the reduced model:

Regression Analysis: ATL_ROOMS versus MONTH_T

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	1.43	1.426	0.03	0.866
Error	22	1074.53	48.842		
Total	23	1075.96			

The test statistic is $F = \dfrac{(SSE_R - SSE_C)/(k-g)}{SSE_C/[n-(k+1)]} = \dfrac{(1074.53 - 99.12)/(12-1)}{99.12/[24-(12+1)]} = 9.85.$

The rejection region requires $\alpha = 0.05$ in the upper tail of the F distribution with $\nu_1 = k - g = 12 - 1 = 11$ and $\nu_2 = n - (k+1) = 24 - (12+1) = 11$. From Table 4, Appendix D, $F_{0.05} \approx 2.82$. The rejection region is $F > 2.82$.

Since the observed value of the test statistic falls in the rejection region $(F = 9.85 > 2.82)$, H_0 is rejected. There is sufficient evidence to indicate the monthly variables are useful predictors of occupancy rates.

Using MNIITAB, the results are:

Prediction for ATL_ROOMS

Settings

Variable	Setting
MONTH_T	25
M1	1
M2	0
M3	0
M4	0
M5	0
M6	0
M7	0
M8	0
M9	0
M10	0
M11	0

Prediction

Fit	SE Fit	95% CI	95% PI
66.375	2.80801	(60.1946, 72.5554)	(57.3278, 75.4222)

The 95% prediction interval is $(57.328, 75.422)$.

f. The new model is $y_t = \beta_0 + \beta_1 t + \beta_2 M_1 + \beta_3 M_2 + \cdots + \beta_{12} M_{11} + \varphi R_{t-1} + \varepsilon_t$.

g. For Phoenix, using SAS, the results are:

```
                         The AUTOREG Procedure

                   Dependent Variable    PHX_ROOMS


                   Ordinary Least Squares Estimates

       SSE                      99.125    DFE                     11
       MSE                     9.01136    Root MSE           3.00189
       SBC                  143.463618    AIC             128.148918
       Regress R-Square         0.9768    Total R-Square      0.9768
       Durbin-Watson            1.7963


                                          Standard                Approx
    Variable        DF      Estimate         Error    t Value    Pr > |t|

    Intercept        1       43.1250        2.8080      15.36      <.0001
    MONTH_T          1        0.3542        0.1021       3.47      0.0053
    M1               1       23.8958        3.2052       7.46      <.0001
    M2               1       42.0417        3.1709      13.26      <.0001
    M3               1       38.6875        3.1395      12.32      <.0001
    M4               1       25.3333        3.1111       8.14      <.0001
    M5               1       19.4792        3.0858       6.31      <.0001
    M6               1        9.1250        3.0638       2.98      0.0126
    M7               1       -0.2292        3.0450      -0.08      0.9414
    M8               1       -1.5833        3.0296      -0.52      0.6116
    M9               1       11.0625        3.0175       3.67      0.0037
    M10              1       22.2083        3.0088       7.38      <.0001
    M11              1       17.8542        3.0036       5.94      <.0001


                     Estimates of Autocorrelations

 Lag    Covariance     Correlation    -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1

   0        4.1302        1.000000    |                    |********************|
   1        0.4095        0.099149    |                    |**                  |


                     Preliminary MSE      4.0896


                Estimates of Autoregressive Parameters

                                          Standard
                  Lag     Coefficient        Error    t Value

                    1       -0.099149     0.314670      -0.32


                        Yule-Walker Estimates

       SSE                  98.1427958    DFE                     10
       MSE                     9.81428    Root MSE           3.13278
       SBC                  146.412555    AIC             129.919801
```

Regress R-Square	0.9744	Total R-Square	0.9770
Durbin-Watson	1.9274		

Variable	DF	Estimate	Standard Error	t Value	Approx Pr > |t|
Intercept	1	43.1183	3.0330	14.22	<.0001
MONTH_T	1	0.3536	0.1166	3.03	0.0126
M1	1	23.8767	3.2953	7.25	<.0001
M2	1	42.0501	3.3345	12.61	<.0001
M3	1	38.6992	3.3046	11.71	<.0001
M4	1	25.3459	3.2715	7.75	<.0001
M5	1	19.4923	3.2416	6.01	0.0001
M6	1	9.1388	3.2155	2.84	0.0175
M7	1	-0.2148	3.1935	-0.07	0.9477
M8	1	-1.5684	3.1754	-0.49	0.6320
M9	1	11.0780	3.1604	3.51	0.0057
M10	1	22.2243	3.1371	7.08	<.0001
M11	1	17.8691	2.9877	5.98	0.0001

For Atlanta, using SAS, the results are:

The AUTOREG Procedure

Dependent Variable   ATL_ROOMS

Ordinary Least Squares Estimates

SSE	99.125	DFE	11
MSE	9.01136	Root MSE	3.00189
SBC	143.463618	AIC	128.148918
Regress R-Square	0.9079	Total R-Square	0.9079
Durbin-Watson	2.2705		

Variable	DF	Estimate	Standard Error	t Value	Approx Pr > |t|
Intercept	1	42.1250	2.8080	15.00	<.0001
MONTH_T	1	0.2708	0.1021	2.65	0.0225
M1	1	17.4792	3.2052	5.45	0.0002
M2	1	21.7083	3.1709	6.85	<.0001
M3	1	25.9375	3.1395	8.26	<.0001
M4	1	23.6667	3.1111	7.61	<.0001
M5	1	20.3958	3.0858	6.61	<.0001
M6	1	19.6250	3.0638	6.41	<.0001
M7	1	21.8542	3.0450	7.18	<.0001
M8	1	21.5833	3.0296	7.12	<.0001
M9	1	17.3125	3.0175	5.74	0.0001
M10	1	26.0417	3.0088	8.66	<.0001
M11	1	15.7708	3.0036	5.25	0.0003

Estimates of Autocorrelations

```
Lag    Covariance     Correlation    -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0        4.1302        1.000000    |                    |********************|
  1       -0.6296       -0.152427    |                 ***|                    |
```

Preliminary MSE   4.0342

Estimates of Autoregressive Parameters

Lag	Coefficient	Standard Error	t Value
1	0.152427	0.312533	0.49

Yule-Walker Estimates

SSE	96.6905473	DFE	10
MSE	9.66905	Root MSE	3.10951
SBC	146.068395	AIC	129.575641
Regress R-Square	0.9150	Total R-Square	0.9101
Durbin-Watson	2.0317		

Variable	DF	Estimate	Standard Error	t Value	Approx Pr > |t|
Intercept	1	42.1651	2.8057	15.03	<.0001
MONTH_T	1	0.2728	0.0932	2.93	0.0151
M1	1	17.5558	3.4381	5.11	0.0005
M2	1	21.6325	3.2691	6.62	<.0001
M3	1	25.8826	3.2652	7.93	<.0001
M4	1	23.6063	3.2379	7.29	<.0001
M5	1	20.3340	3.2171	6.32	<.0001
M6	1	19.5611	3.1983	6.12	0.0001
M7	1	21.7883	3.1823	6.85	<.0001
M8	1	21.5156	3.1678	6.79	<.0001
M9	1	17.2424	3.1634	5.45	0.0003
M10	1	25.9717	3.1132	8.34	<.0001
M11	1	15.6855	3.3747	4.65	0.0009

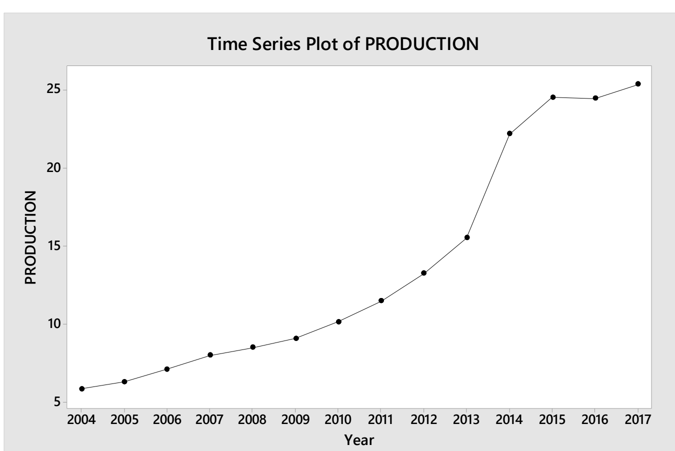
h. We would probably not recommend using the autoregressive model for Phoenix because $R^2 = 0.9768$ for the linear regression model and $R^2 = 0.9770$ for the autoregressive model. There is little improvement.
We would probably not recommend using the autoregressive model for Atlanta because $R^2 = 0.9079$ for the linear regression model and $R^2 = 0.9101$ for the autoregressive model. There is little improvement.

10.36 a. The model including a curvilinear relationship would be $E(y_t) = \beta_0 + \beta_1 x_t + \beta_2 x_t^2$.

b. The first-order autoregressive model for R_t is $R_t = \varphi R_{t-1} + \varepsilon_t$.

c. The full model would be $y_t = \beta_0 + \beta_1 x_t + \beta_2 x_t^2 + R_t$ or
$y_t = \beta_0 + \beta_1 x_t + \beta_2 x_t^2 + \varphi R_{t-1} + \varepsilon_t$.

10.37 a. Using MINITAB, the time series plot is:



There appears to be a positive linear trend to the data.

b. The model would be $E(y_t) = \beta_0 + \beta_1 x_t$.

c. Using MINITAB, the results are:

Regression Analysis: PRODUCTION versus t
Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	632.22	632.222	107.37	0.000
Error	12	70.66	5.888		
Total	13	702.88			

Model Summary

S	R-sq	R-sq(adj)
2.42655	89.95%	89.11%

Coefficients

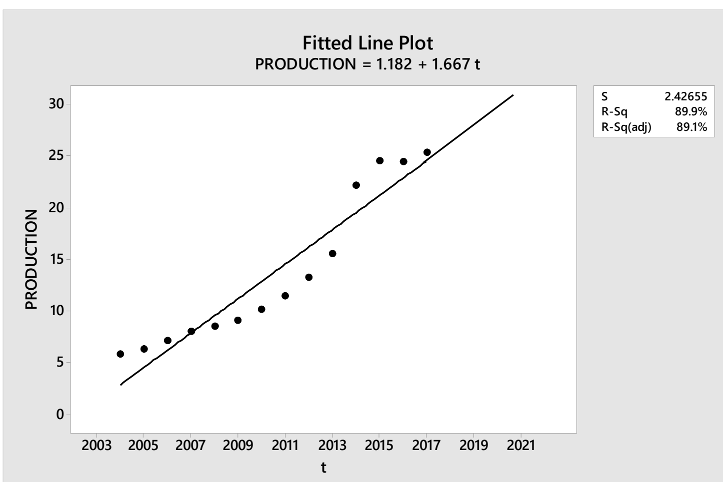
Term	Coef	SE Coef	T-Value	P-Value
Constant	1.18	1.37	0.86	0.405
t	1.667	0.161	10.36	0.000

Regression Equation

PRODUCTION = 1.18 + 1.667 t

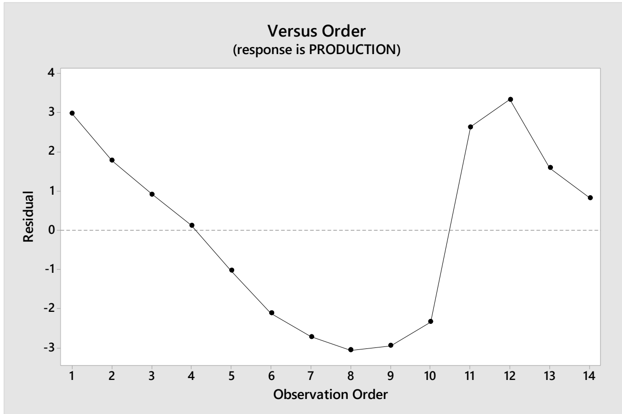
The fitted model is $\hat{y}_t = 1.18 + 1.667t$.

d. Using MINITAB, the plot is:



$\hat{y}_{17} = 1.18 + 1.667(17) = 29.519$. Forecasting 3 years out is questionable.

e. The residual plot is:



Since the residual tend to follow a line, there is evidence that there is residual autocorrelation.

f. Using MINITAB, the Durbin-Watson statistic is:

Durbin-Watson Statistic

Durbin-Watson Statistic = 0.496837

The rejection region for a two-tailed test with $k = 1$ and $\alpha = 0.02$ is $d < d_{L,0.01} \approx 0.78$ or $(4 - d) < d_{L,0.01} \approx 0.78$. Since the observed value of the test statistic falls in the rejection region $(d = 0.4969 < 0.78)$, H_0 is rejected. There is sufficient evidence to indicate residual autocorrelation exists at $\alpha = 0.05$.

g. The autoregressive model would be $y_t = \beta_0 + \beta_1 t + \varphi R_{t-1} + \varepsilon_t$.

Using SAS, the results are:

```
                        The AUTOREG Procedure

                  Dependent Variable    PRODUCTION


                  Ordinary Least Squares Estimates

     SSE                  70.6578901    DFE                     12
     MSE                     5.88816    Root MSE           2.42655
     SBC                  67.6714879    AIC             66.3933733
     Regress R-Square         0.8995    Total R-Square      0.8995
     Durbin-Watson            0.4968

                                          Standard              Approx
  Variable         DF      Estimate         Error    t Value    Pr > |t|

  Intercept         1        1.1815        1.3698       0.86      0.4053
  T                 1        1.6670        0.1609      10.36      <.0001


                      Estimates of Autocorrelations

Lag    Covariance     Correlation    -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1

  0        5.0470        1.000000    |                    |********************|
  1        3.4512        0.683806    |                    |**************      |
```

```
                    Preliminary MSE      2.6871

               Estimates of Autoregressive Parameters

                                         Standard
            Lag     Coefficient             Error    t Value

              1       -0.683806          0.220002      -3.11

                         Yule-Walker Estimates

   SSE                  32.3649457    DFE                      11
   MSE                     2.94227    Root MSE            1.71530
   SBC                  60.0100531    AIC              58.0928811
   Regress R-Square         0.7932    Total R-Square       0.9540
   Durbin-Watson            1.1793

                                         Standard                 Approx
Variable         DF      Estimate           Error    t Value    Pr > |t|
Intercept         1        2.3068          2.2186       1.04      0.3208
T                 1        1.5770          0.2428       6.49      <.0001
```

The fitted model is $\hat{y}_t = 2.3068 + 1.577t + 0.6838\hat{R}_{t-1}$. To assess whether the residuals are autocorrelated, an approximate *t*-test of the null hypothesis $H_0 : \varphi = 0$ yields $t = 3.11$. With 11 degrees of freedom for error, the *p*-value is approximately 0.0099. Thus, the value is significant at the 5% level. There is evidence that residual autocorrelation exists.

h. To obtain a forecast for the 2020 interest rate, we note that the three-step ahead forecast (i.e. the 17th time point, which would be for the year 2020) is given by

$$F_{14+3} = \hat{\beta}_0 + \hat{\beta}_1 t + \hat{\varphi}^3 \hat{R}_{14}$$

Note: $\hat{R}_n = y_n - \hat{y}_n$

Therefore, $\hat{R}_{14} = y_{14} - \hat{y}_{14} = 25.35 - \left(2.3068 + 1.577(14)\right) = 0.9652$

It follows that $F_{17} = \hat{\beta}_0 + \hat{\beta}_1 t + \hat{\varphi}^3 \hat{R}_{14} = 2.3068 + 1.577(17) + 0.6838^3\left(0.9652\right) = 29.424$

To compute the approximate 95% confidence interval, the three-step ahead forecast takes the form:

$$\hat{y}_{n+3} \pm 2\sqrt{MSE\left(1 + \hat{\varphi}^2 + \hat{\varphi}^4\right)} \Rightarrow 29.424 \pm 2\sqrt{2.94227\left(1 + 0.6838^2 + 0.6838^4\right)}$$
$$\Rightarrow 29.424 \pm 4.455 \Rightarrow \left(24.969,\ 33.879\right)$$

This forecast is preferred because it takes the autocorrelation into account.

10.38 a. To test for positive autocorrelation, we test:

H_0 : No autocorrelation
H_a : Positive autocorrelation

The test statistic is d = value from printout for a one-tailed alternative. The rejection region is $d < d_{L,\alpha}$. From Table 8, Appendix D, with $\alpha = 0.05$, $n = 40$, and $k = 2$, $d_{L,0.05} = 1.39$. The rejection region is $d < 1.39$.

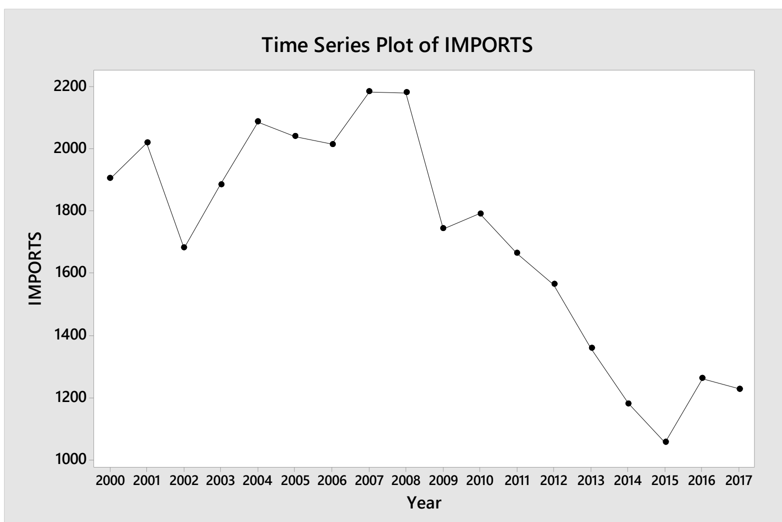
b. Since the test statistic falls in the rejection region $(d = 1.14 < 1.39)$, H_0 is rejected. There is sufficient evidence to indicate the regression errors are positively correlated at $\alpha = 0.05$.

10.39 a. The model would be $E(y_t) = \beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 x$, where $x = \begin{cases} 1 & \text{if January - April} \\ 0 & \text{if not} \end{cases}$

b. We would add the interaction terms to the model:

$$E(y_t) = \beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 x + \beta_1 tx + \beta_2 t^2 x$$

10.40 a. Using MINITAB, the time series plot is:



b. A straight-line autoregressive model takes the form $y_t = \beta_0 + \beta_1 t + \varphi R_{t-1} + \varepsilon_t$.

c. The SAS printout for the model appears below.

```
                         The AUTOREG Procedure

                       Dependent Variable    IMPORTS


                      Ordinary Least Squares Estimates

SSE                  853210.202    DFE                      16
MSE                       53326    Root MSE          230.92345
SBC                  250.657541    AIC              248.876798
Regress R-Square         0.6203    Total R-Square       0.6203
Durbin-Watson            0.6648
```

Variable	DF	Estimate	Standard Error	t Value	Approx Pr > |t|
Intercept	1	2222	113.5594	19.57	<.0001
T	1	-53.6357	10.4911	-5.11	0.0001

Estimates of Autocorrelations

Lag	Covariance	Correlation	-1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
0	47400.6	1.000000	| |******************** |
1	29672.9	0.626002	| |************* |

Preliminary MSE 28825.3

Estimates of Autoregressive Parameters

Lag	Coefficient	Standard Error	t Value
1	-0.626002	0.201349	-3.11

Yule-Walker Estimates

SSE	486083.25	DFE	15
MSE	32406	Root MSE	180.01542
SBC	243.918026	AIC	241.24691
Regress R-Square	0.3488	Total R-Square	0.7837
Durbin-Watson	1.8717		

Variable	DF	Estimate	Standard Error	t Value	Approx Pr > |t|
Intercept	1	2147	192.1166	11.18	<.0001
T	1	-48.1630	16.9915	-2.83	0.0126

The fitted autoregressive model is $\hat{y}_t = 2147 - 48.163t + 0.626\hat{R}_{t-1}$.

To assess whether the residuals are autocorrelated, an approximate *t*-test of the null hypothesis $H_0: \varphi = 0$ yields a *t*-statistic of $t = 3.11$. With 15 degrees of freedom for the error, the *p*-value is $2(0.0036) = 0.0072$. Since the *p*-value is less than $\alpha\left(p = 0.0072 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate the autoregressive model describes the residual correlation well.

d. The fitted autoregressive model is $\hat{y}_t = 2147 - 48.163t + 0.626\hat{R}_{t-1}$.

e. To forecast the amount of oil in 2019, i.e. two steps ahead of the most recent value (2017), we use:

$$F_{18+2} = \hat{\beta}_0 + \hat{\beta}_1 t + \hat{\varphi}^2 \hat{R}_{18}$$

Note: $\hat{R}_{18} = y_t - \hat{y}_t = 1227 - \left(2147 - 48.163(18)\right) = -53.066$

Thus, $F_{20} = 2147 - 48.163(20) + 0.626^2(-53.066) = 1162.945$

To compute the approximate 95% forecasting interval, we use:

$$\hat{y}_{n+2} \pm 2\sqrt{MSE\left(1+\hat{\varphi}^2\right)} \Rightarrow 1162.945 \pm 2\sqrt{32406\left(1+0.626^2\right)}$$
$$\Rightarrow 1162.945 \pm 424.759 \Rightarrow (938.186, 1587.704)$$

10.41 a. Let μ_{PRE} = mean difference between the April rates of return of the two stocks on the exchange with the largest and smallest returns in a year in the pre-tax period and let μ_{POST} = mean difference between the April rates of return of the two stocks on the exchange with the largest and smallest returns in a year in the post-tax period.

Then, $\beta_1 = \mu_{\text{POST}} - \mu_{\text{PRE}}$

b. $\beta_0 = \mu_{\text{PRE}}$

c. During the pre-tax period, $D_t = 0$. Thus, $\hat{y}_t = -0.55 + 3.08(0) = -0.55$

d. During the post-tax period, $D_t = 1$. Thus, $\hat{y}_t = -0.55 + 3.08(1) = 2.53$

10.42 a. To determine if the quarterly number of pension plan qualifications increase as a decreasing rate over time, we test:

$$H_0: \beta_2 = 0$$
$$H_a: \beta_2 < 0$$

The test statistic is $t = -1.39$.

The rejection region requires $\alpha = 0.05$ in the lower tail of the *t*-distribution with $df = n - 2 = 107 - 2 = 105$. From Table 2, Appendix D, $t_{0.05} = 1.645$. The rejection region is $t < -1.645$.

Since the observed value of the test statistic does not fall in the rejection region $(t = -1.39 \not< -1.645)$, H_0 is not rejected. There is insufficient evidence to indicate that quarterly number of pension plan qualifications increases at a decreasing rate over time at $\alpha = 0.05$.

b. Substituting $t = 108$ into the prediction equation, we obtain:

$$\hat{y}_{108} = 6.19 + 0.039(108) - 0.00024(108)^2 = 7.603$$

Since y_t is the natural log of the number of pension plan qualifications, we must take the antilog of $\hat{y}_{108}$ to obtain the forecast: Forecast $= e^{\hat{y}_{108}} = e^{7.603} = 2003.48$

c. To determine if the quarterly number of profit-sharing plan qualifications increase as a decreasing rate over time, we test:

$$H_0: \beta_2 = 0$$
$$H_a: \beta_2 < 0$$

The test statistic is $t = -1.61$.

The rejection region requires $\alpha = 0.05$ in the lower tail of the t-distribution with $df = n - 2 = 107 - 2 = 105$. From Table 2, Appendix D, $t_{0.05} = 1.645$. The rejection region is $t < -1.645$.

Since the observed value of the test statistic does not fall in the rejection region $\left(t = -1.61 \not< -1.645\right)$, H_0 is not rejected. There is insufficient evidence to indicate that quarterly number of profit-sharing plan qualifications increases at a decreasing rate over time at $\alpha = 0.05$.

d. Substituting $t = 108$ into the prediction equation, we obtain:

$$\hat{y}_{108} = 6.22 + 0.035(108) - 0.00021(108)^2 = 7.551$$

The forecast is the antilog of $\hat{y}_{108}$: Forecast $= e^{\hat{y}_{108}} = e^{7.551} = 1901.81$

10.43 a. $\hat{\beta}_0 = 3.54$: This is the estimate of where the regression line will cross the y_t axis.

$\hat{\beta}_1 = 0.039$: The estimated increase in the mean logarithm of the number of pension plan terminations for each additional time period is 0.039.

$\hat{\varphi} = 0.40$: Since $\hat{\varphi}$ is positive, it implies that the time series residuals are positively correlated.

b. $\hat{\beta}_0 = 3.45$: This is the estimate of where the regression line will cross the y_t axis.

$\hat{\beta}_1 = 0.038$: The estimated increase in the mean logarithm of the number of profit-sharing plan terminations for each additional time period is 0.038.

$\hat{\varphi} = 0.22$: Since $\hat{\varphi}$ is positive, it implies that the time series residuals are positively correlated.

c. $\hat{R}_{107} = y_{107} - \hat{y}_{107} = 7.5 - \left(3.54 + 0.039(107)\right) = -0.213$

Substituting $t = 108$ into the prediction equation, we obtain:

$$\hat{y}_{108} = 3.54 + 0.039(108) + 0.40(-0.213) = 7.667$$

The forecast is the antilog of $\hat{y}_{108}$: Forecast $= e^{\hat{y}_{108}} = e^{7.667} = 2136.66$

d. To compute the approximate 95% forecasting interval, we use:

$$\hat{y}_{n+1} \pm 2\sqrt{MSE} \Rightarrow 7.667 \pm 2\sqrt{0.0440} \Rightarrow 7.667 \pm 0.4195 \Rightarrow (7.247,\ 8.087)$$

The limits are the antilogs of the endpoints: $\left(e^{7.247},\ e^{8.087}\right) \Rightarrow (1403.89,\ 3251.92)$

e. $\hat{R}_{107} = y_{107} - \hat{y}_{107} = 7.6 - \left(3.45 + 0.038(107)\right) = 0.084$

Substituting $t = 108$ into the prediction equation, we obtain:

$$\hat{y}_{108} = 3.45 + 0.038(108) + 0.22(0.084) = 7.572$$

The forecast is the antilog of $\hat{y}_{108}$: Forecast $= e^{\hat{y}_{108}} = e^{7.572} = 1943.02$

To compute the approximate 95% forecasting interval, we use:

$$\hat{y}_{n+1} \pm 2\sqrt{MSE} \Rightarrow 7.572 \pm 2\sqrt{0.0402} \Rightarrow 7.572 \pm 0.4010 \Rightarrow (7.171,\ 7.973)$$

The limits are the antilogs of the endpoints: $\left(e^{7.171},\ e^{7.973}\right) \Rightarrow (1301.15,\ 2901.55)$

Chapter **11**

Principles of Experimental Design

11.1 a. The two factors that affect the quantity of information in an experiment are noise (variability) and volume (n).

b. Block designs remove noise from an extraneous source of variation.

11.2 a. The experimental units are accounting alumni of a large southwestern university.

b. The response variable is the accountant's income rating scores.

c. The factors are gender and the Mach rating scores of alumni.

d. High, moderate, and low are the Mach rating score levels while the gender levels are male or female.

e. The treatments are (Female, High), (Female, Moderate), (Female, Low), (Male, High), (Male, Moderate), (Male, Low).

11.3 a. The experimental units are the cockatiels.

b. Yes, this experiment is a completely randomized design. The birds were randomly divided into 3 groups and each group received a different treatment.

c. The factor is the liquid group.

d. There are three levels of the group variable – Group 1 received purified water, Group 2 received purified water and liquid sucrose, and Group 3 received purified water and liquid sodium chloride.

e. There are 3 treatments in the study. Because there is only one factor, the treatments are the same as the factor levels.

f. The response variable is the total water consumption.

g. The regression model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2$,

where $x_1 = \begin{cases} 1 \text{ if group 1} \\ 0 \text{ if not} \end{cases}$ $\quad x_2 = \begin{cases} 1 \text{ if group 2} \\ 0 \text{ if not} \end{cases}$

11.4 a. The competence level is the response variable.

b. The factor is the time of study (one week before the training, 2 days after the training, or 2 months after the training).

c. There are three treatments in the study, because there is only one factor (time of study) and three levels (one week before, 2 days after, 2 months after). When there is only one factor, the treatments are the same as the factor levels.

d. The design of the experiment is a randomized block design.

e. The blocks in the experiment are the 222 employees.

11.5 a. The model for the first observation for Appraiser $B\left(x_1 = 0, x_2 = 1, x_3 = 0\right)$ is:

$$y_{B1} = \beta_0 + \beta_2 + \beta_4 + \varepsilon_{B1}$$

The models for the rest of the observations for Appraiser B are:

$$\begin{aligned}
y_{B2} &= \beta_0 + \beta_2 + \beta_5 + \varepsilon_{B2} \\
y_{B3} &= \beta_0 + \beta_2 + \beta_6 + \varepsilon_{B3} \\
y_{B4} &= \beta_0 + \beta_2 + \beta_7 + \varepsilon_{B4} \\
y_{B5} &= \beta_0 + \beta_2 + \beta_8 + \varepsilon_{B5} \\
y_{B6} &= \beta_0 + \beta_2 + \beta_9 + \varepsilon_{B6} \\
y_{B7} &= \beta_0 + \beta_2 + \beta_{10} + \varepsilon_{B7} \\
y_{B8} &= \beta_0 + \beta_2 + \beta_{11} + \varepsilon_{B8} \\
y_{B9} &= \beta_0 + \beta_2 + \beta_{12} + \varepsilon_{B9} \\
y_{B10} &= \beta_0 + \beta_2 + \varepsilon_{B10}
\end{aligned}$$

The average of the 10 observations for Appraiser B is:

$$\bar{y}_B = \beta_0 + \beta_2 + \frac{\beta_4 + \beta_5 + \cdots + \beta_{12}}{10} + \bar{\varepsilon}_B$$

b. The models for the observations for Appraiser $D\left(x_1 = 0, x_2 = 0, x_3 = 0\right)$ are:

$$\begin{aligned}
y_{D1} &= \beta_0 + \beta_4 + \varepsilon_{D1} \\
y_{D2} &= \beta_0 + \beta_5 + \varepsilon_{D2} \\
y_{D3} &= \beta_0 + \beta_6 + \varepsilon_{D3} \\
y_{D4} &= \beta_0 + \beta_7 + \varepsilon_{D4} \\
y_{D5} &= \beta_0 + \beta_8 + \varepsilon_{D5} \\
y_{D6} &= \beta_0 + \beta_9 + \varepsilon_{D6} \\
y_{D7} &= \beta_0 + \beta_{10} + \varepsilon_{D7} \\
y_{D8} &= \beta_0 + \beta_{11} + \varepsilon_{D8} \\
y_{D9} &= \beta_0 + \beta_{12} + \varepsilon_{D9} \\
y_{D10} &= \beta_0 + \varepsilon_{D10}
\end{aligned}$$

The average of the 10 observations for Appraiser D is:

$$\bar{y}_D = \beta_0 + \frac{\beta_4 + \beta_5 + \cdots + \beta_{12}}{10} + \bar{\varepsilon}_D$$

c. $$\left(\bar{y}_B - \bar{y}_D\right) = \left(\beta_0 + \beta_2 + \frac{\beta_4 + \cdots + \beta_{12}}{10} + \bar{\varepsilon}_B\right) - \left(\beta_0 + \frac{\beta_4 + \cdots + \beta_{12}}{10} + \bar{\varepsilon}_D\right)$$
$$= \beta_2 + \left(\bar{\varepsilon}_B - \bar{\varepsilon}_D\right)$$

11.6 a. The treatments in this study are the 5 food/beverage items.

b. A diagram of the design is:

Taste Testers

1	2	3	…	200
BC	CC	PP	…	GF
CC	OJ	BC	…	PP
GF	PP	GF	…	BC
OJ	GF	CC	…	OJ
PP	BC	OJ	…	CC

c. The linear model is
$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_6 + \cdots + \beta_{203} x_{203}$$

where $x_1 = \begin{cases} 1 \text{ if BC} \\ 0 \text{ if not} \end{cases}$ $x_2 = \begin{cases} 1 \text{ if CC} \\ 0 \text{ if not} \end{cases}$ $x_3 = \begin{cases} 1 \text{ if GF} \\ 0 \text{ if not} \end{cases}$ $x_4 = \begin{cases} 1 \text{ if OJ} \\ 0 \text{ if not} \end{cases}$

$x_5 = \begin{cases} 1 \text{ if tester 1} \\ 0 \text{ if not} \end{cases}$ $x_6 = \begin{cases} 1 \text{ if tester 6} \\ 0 \text{ if not} \end{cases}$ … $x_{203} = \begin{cases} 1 \text{ if tester 199} \\ 0 \text{ if not} \end{cases}$

11.7 The factorial design allows for the interaction of two variables to be investigated.

11.8 a. The factors are yeast extract (baker's yeast or brewer's yeast), temperature (45, 48, 51, or 54 degrees Celsius).

b. Autolysis yield is the response variable.

c. There are eight treatments: each of the yeast extract-temperature combinations. (baker's yeast, 45°), (baker's yeast, 48°), (baker's yeast, 51°), (baker's yeast, 54°), (brewer's yeast, 45°), (brewer's yeast, 48°), (brewer's yeast, 51°), (brewer's yeast, 54°).

d. A randomized block design is employed.

11.9 a. The experimental units for this study are the students.

b. This experiment is a factorial designed experiment. The students were first divided into three groups based on their class standing. Then, within each group, the students were randomly assigned to one of two types of preparation for the final.

c. There are two factors in this experiment. One factor is class standing and the other factor is type of preparation.

d. Class standing has three levels – low, medium, and high. Type of preparation has two levels – review session and practice exam.

e. There are 6 treatments in this experiment – (Low, Review), (Medium, Review), (High, Review), (Low, Practice Exam), (Medium, Practice Exam), and (High, Practice Exam).

f. The response variable is the final exam score.

11.10 a. The tablet dissolution time is the response variable.

b. The three factors are binding agent with two levels (khaya gum, PVP), binding concentration with two levels (0.5%, 4.0%), and relative density with two levels (low, high).

c. There are eight treatments possible in the study.

(khaya gum, 0.5% binding concentration, low relative density)
(PVP, 0.5% binding concentration, low relative density)
(khaya gum, 4.0% binding concentration, low relative density)
(PVP, 4.0% binding concentration, low relative density)
(khaya gum, 0.5% binding concentration, high relative density)
(PVP, 0.5% binding concentration, high relative density)
(khaya gum, 4.0% binding concentration, high relative density)
(PVP, 4.0% binding concentration, high relative density)

11.11 a. This is a 2×2 factorial design.

b. The response variable is final sales price.

c. There are 4 treatments in this experiment: (high adaptation, high knowledge), (high adaptation, low knowledge), (low adaptation, high knowledge), (low adaptation, low knowledge).

d. This design will allow for the test for interaction. Since there are observations at each level combination of the 2 factors, there are enough observations to test for interaction.

e. The linear model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2$

where $x_1 = \begin{cases} 1 & \text{if adaptation is high} \\ 0 & \text{if not} \end{cases}$ $\quad x_2 = \begin{cases} 1 & \text{if knowledge is high} \\ 0 & \text{if not} \end{cases}$

11.12 a. This is not a complete factorial experiment since the treatments do not include all $3 \times 3 = 9$ factor-level combinations.

b. Interaction between two factors implies the effect of one factor on the dependent variable depends on the level of the second factor. There are 3 levels of factor A at B_1. However, there is only 1 level of A at B_2 and only 1 level of A at B_3. Thus, we cannot measure the effect of A at levels B_2 and B_3. Therefore, we cannot determine if the effect of A differs at different levels of B.

11.13 a. The complete factorial model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_1 x_2 + \beta_5 x_1 x_3$

where $x_1 = \begin{cases} 1 & \text{if level 1} \\ 0 & \text{if level 2} \end{cases}$ Factor #1

$$\left.\begin{aligned} x_2 &= \begin{cases} 1 & \text{if level1} \\ 0 & \text{if not} \end{cases} \\ x_3 &= \begin{cases} 1 & \text{if level 2} \\ 0 & \text{if not} \end{cases} \end{aligned}\right\} \text{Factor \#2}$$

b. The complete factorial model is

$$\begin{aligned} E(y) = {} & \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_1 x_2 + \beta_7 x_1 x_3 + \beta_8 x_1 x_4 \\ & + \beta_9 x_1 x_5 + \beta_{10} x_2 x_4 + \beta_{11} x_2 x_5 + \beta_{12} x_3 x_4 + \beta_{13} x_3 x_5 + \beta_{14} x_1 x_2 x_4 \\ & + \beta_{15} x_1 x_2 x_5 + \beta_{16} x_1 x_3 x_4 + \beta_{17} x_1 x_3 x_5, \end{aligned}$$

where $x_1 =$ quantitative factor A; $x_2, x_3 =$ dummy variables for qualitative factor B; $x_4, x_5 =$ are dummy variables for qualitative factor C.

11.14 The factorial design allows for the interaction of variables to be investigated.

11.15 If a randomized block design is used to investigate the effect of two qualitative factors, there will be only one observation per treatment. There will not be enough degrees of freedom (or observations) to test for interaction between the two factors.

11.16 Replication allows for an estimate of the error term in a factorial design.

11.17 To solve for the number of replicates, r, we want to solve the equation:

$$t_{\alpha/2} \frac{s_{\hat{\beta}_3}}{\sqrt{r}} = B$$

We know the estimate of $s_{\hat{\beta}_3}$ is 3, $B = 2$, and $\alpha = 0.10$, $\alpha / 2 = 0.05$.

To determine the value of $t_{0.05}$, we need to know the degrees of freedom. We know the degrees of freedom for t will be $df(\text{Error}) = n - 4 = 4r - 4 = 4(r-1)$. At minimum, we require 2 replicates, so we will start with $r = 2$. Thus, with $df = 4$, $t_{0.05} = 2.132$. Substituting into the formula, we get:

$$2.132\left(\frac{3}{\sqrt{r}}\right)=2 \Rightarrow 2.132\left(\frac{3}{2}\right)=\sqrt{r} \Rightarrow r=\frac{2.132^2(3)^2}{2^2}= 10.23$$

Since this number is quite a bit larger than the 2 replicates that we used to get the original t value, we will redo the problem using the $t_{0.05}$ with $df=4(r-1)=4(10-1)=36$. This value is $t_{0.05} \approx 1.69$.

$$1.690\left(\frac{3}{\sqrt{r}}\right)=2 \Rightarrow 1.690\left(\frac{3}{2}\right)=\sqrt{r} \Rightarrow r=\frac{1.690^2(3)^2}{2^2}= 6.43 \approx 7$$

Using 7 as the number of replicates, $df=4(r-1)=4(7-1)=24$ and $t_{0.05}=1.711$.

$$1.711\left(\frac{3}{\sqrt{r}}\right)=2 \Rightarrow 1.711\left(\frac{3}{2}\right)=\sqrt{r} \Rightarrow r=\frac{1.711^2(3)^2}{2^2}=6.59 \approx 7$$

Thus, we should use 7 replicates.

11.18 Since we wish to estimate the difference between 2 treatment means to within 10 units, the formula for the half-width is $t_{\alpha/2}s\sqrt{2/b}$.

Setting this equal to our desired half-width yields:

$$t_{\alpha/2}s\sqrt{2/b}=10 \Rightarrow t_{\alpha/2}^2 s^2(2/b)=10^2 \Rightarrow 2t_{\alpha/2}^2 s^2=10^2 b \Rightarrow b=\frac{2t_{\alpha/2}^2 s^2}{10^2}$$

Since the degrees of freedom for $t_{\alpha/2}$ is $df=(a-1)(b-1)=(2-1)(b-1)=(b-1)$, we will use a conservative value of $df=10$ to find $t_{\alpha/2}$. For $\alpha=0.05$, $t_{\alpha/2}=t_{0.025}$. From Table 2, Appendix D, $t_{0.025}=2.228$. The specified number of blocks necessary is:

$$b=\frac{2t_{\alpha/2}^2 s^2}{10^2}=\frac{2(2.228)^2(15)^2}{10^2}=22.34 \approx 23$$

For $b=24$, $df=23$, $t_{0.025}=2.069$.

$$b=\frac{2t_{\alpha/2}^2 s^2}{10^2}=\frac{2(2.069)^2(15)^2}{10^2}=19.26 \approx 20$$

For $b=20$, $df=19$, $t_{0.025}=2.093$.

$$b=\frac{2t_{\alpha/2}^2 s^2}{10^2}=\frac{2(2.093)^2(15)^2}{10^2}=19.71 \approx 20$$

Thus, we should use 20 blocks.

11.19 Determine the number of observations per treatment (n) and the standard deviation of the estimate.

11.20 The steps in the design of an experiment that affect the volume of the signal are:

1) Selecting the factors
2) Choosing the treatments
3) Determining the sample size

11.21 The variation produced by extraneous variables can be reduced in step 4 by the process known as blocking. We observe all the treatments within relatively homogeneous blocks of experimental material. By assigning the treatments within blocks, we may decrease the noise (variability) in an experiment.

11.22 In both designs, there is one factor of primary interest to study. In the randomized block design, a second factor is suspected of contributing information to the response variable. The block design is advantageous when such a contribution really does exist.

11.23 Factor 1 contains 2 levels $\Rightarrow A_1$ and A_2.
Factor 2 contains 4 levels $\Rightarrow B_1, B_2, B_3$ and B_4.

There are $2 \times 4 = 8$ possible factor-level combinations. They are:

$$A_1B_1,\ A_1B_2,\ A_1B_3,\ A_1B_4,\ A_2B_1,\ A_2B_2,\ A_2B_3,\ A_2B_4$$

11.24 The complete model is

$$\begin{aligned}E(y) = {} & \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_1 x_2 + \beta_7 x_1 x_3 + \beta_8 x_1 x_4 \\ & + \beta_9 x_1 x_5 + \beta_{10} x_2 x_3 + \beta_{11} x_2 x_4 + \beta_{12} x_2 x_5 + \beta_{13} x_1 x_2 x_3 + \beta_{14} x_1 x_2 x_4 + \beta_{15} x_1 x_2 x_5\end{aligned}$$

where $x_1 = \left\{\begin{matrix} 1 & \text{if level 1} \\ 0 & \text{if level 2} \end{matrix}\right\}$ Factor 1 $\quad x_2 = \left\{\begin{matrix} 1 & \text{if level 1} \\ 0 & \text{if level 2} \end{matrix}\right\}$ Factor 2

$$\left.\begin{matrix} x_3 = \begin{cases} 1 & \text{if level 1} \\ 0 & \text{if not} \end{cases} \\ x_4 = \begin{cases} 2 & \text{if level 2} \\ 0 & \text{if not} \end{cases} \\ x_5 = \begin{cases} 3 & \text{if level 3} \\ 0 & \text{if not} \end{cases} \end{matrix}\right\} \text{Factor 3}$$

Since there are $2 \times 2 \times 4 = 16$ treatments, the number of parameters (i.e. $\beta_0, \beta_1, \ldots, \beta_{15}$) required in the estimation of the model will also be 16. Therefore, 16 degrees of freedom will be needed in the estimation of the betas. Due to the fact that the experiment only has one replicate, $n = 16$ will be the sample size. The number of degrees of freedom left over will be $n -$ (number of model parameters that are estimated) $= 16 - 16 = 0$.

11.25 The model with all main effects but no interactions is:

$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5$$

where $x_1 = \begin{cases} 1 & \text{if level 1} \\ 0 & \text{if level 2} \end{cases}\Bigg\}$ Factor 1 $\quad x_2 = \begin{cases} 1 & \text{if level 1} \\ 0 & \text{if level 2} \end{cases}\Bigg\}$ Factor 2

$$\left.\begin{aligned} x_3 &= \begin{cases} 1 & \text{if level 1} \\ 0 & \text{if not} \end{cases} \\ x_4 &= \begin{cases} 2 & \text{if level 2} \\ 0 & \text{if not} \end{cases} \\ x_5 &= \begin{cases} 3 & \text{if level 3} \\ 0 & \text{if not} \end{cases} \end{aligned}\right\} \text{Factor 3}$$

The number of degrees of freedom for estimating σ^2 is $n-(k+1) = 16-(5+1) = 10$.

11.26 a. There are two treatments in this experiment: the two auditing methods.

b. For each beer brand, recent samples should be collected. Each of the two auditing methods should be used for each of the samples.

c. The complete model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \cdots + \beta_{10} x_{10}$

where $x_1 = \begin{cases} 1 & \text{if method 1} \\ 0 & \text{if method 2} \end{cases} \quad x_2 = \begin{cases} 1 & \text{if brand 1} \\ 0 & \text{if not} \end{cases} \quad x_3 = \begin{cases} 1 & \text{if brand 2} \\ 0 & \text{if not} \end{cases}$

$$x_4 = \begin{cases} 1 & \text{if brand 3} \\ 0 & \text{if not} \end{cases} \quad \ldots \quad x_{10} = \begin{cases} 1 & \text{if brand 9} \\ 0 & \text{if not} \end{cases}$$

11.27 a. The three factors in the study are training method, practice session, and task consistency.

b. Both training method and practice session are qualitative variables, while task consistency is quantitative.

c. Let the two training methods be represented by:

C: continuously consistent and A: adjusted consistent

Let the six practice sessions be represented by:

1, 2, 3, 4, 5, 6

The treatments are the $2 \times 6 \times 4 = 48$ combinations of these three factors:

C, 1, 100%	C, 4, 100%	A, 1, 100%	A, 4, 100%
C, 1, 67%	C, 4, 67%	A, 1, 67%	A, 4, 67%

C, 1, 50%	C, 4, 50%	A, 1, 50%	A, 4, 50%
C, 1, 33%	C, 4, 33%	A, 1, 33%	A, 4, 33%
C, 2, 100%	C, 5, 100%	A, 2, 100%	A, 5, 100%
C, 2, 67%	C, 5, 67%	A, 2, 67%	A, 5, 67%
C, 2, 50%	C, 5, 50%	A, 2, 50%	A, 5, 50%
C, 2, 33%	C, 5, 33%	A, 2, 33%	A, 5, 33%
C, 3, 100%	C, 6, 100%	A, 3, 100%	A, 6, 100%
C, 3, 67%	C, 6, 67%	A, 3, 67%	A, 6, 67%
C, 3, 50%	C, 6, 50%	A, 3, 50%	A, 6, 50%
C, 3, 33%	C, 6, 33%	A, 3, 33%	A, 6, 33%

11.28 a. The three treatments of the experiment are the three computer runs (STAAD-III run 1, STAAD-III run 2, and DRIFT).

b. Using each building level as a block and the three computer runs as treatments, the experiment would consist of running each treatment on each block. The diagram below would contain the drift ratios for each of the $21 \times 3 = 63$ computer runs:

Blocks	**Treatments**		
Building Levels	**STAAD-III Run 1**	**STAAD-III Run 2**	**DRIFT**
1			
2			
3			
⋮	⋮	⋮	⋮
21			

c. The model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \cdots + \beta_{22} x_{22}$

where $x_1 = \begin{cases} 1 & \text{STAAD-III Run 1} \\ 0 & \text{if not} \end{cases}$ $x_2 = \begin{cases} 1 & \text{if STAAD-III Run 2} \\ 0 & \text{if not} \end{cases}$ $x_3 = \begin{cases} 1 & \text{if floor 1} \\ 0 & \text{if not} \end{cases}$

$x_4 = \begin{cases} 1 & \text{if floor 2} \\ 0 & \text{if not} \end{cases}$ . . . $x_{22} = \begin{cases} 1 & \text{if floor 20} \\ 0 & \text{if not} \end{cases}$

The Analysis of Variance for Designed Experiments

Chapter 12

12.1 Recall in Exercise 11.3 we first number the cockatiels from 1 to 15. Then using Table 7, Appendix C, we will select 2-digit numbers until we have 5 different 2-digit numbers between 1 and 15. Those cockatiels assigned the first 5 numbers will be assigned to group 1, those assigned to the next 5 numbers will be assigned to group 2, and those remaining will be assigned to group 3.

Using Excel, generate numbers between 1 and 15. You must generate more than 15 numbers because of duplicates. An example of numbers generated is:

8, 2, 15, 4, 7, 6, 8, 12, 3, 13, 8, 11, 4, 7, 10, 4

The first 5 unique numbers will be assigned to group 1: 8, 2, 15, 4, 7

The next 5 unique numbers will be assigned to group 2: 6, 12, 3, 13, 11

The remaining numbers will be assigned to group 3: 1, 5, 9, 10, 14

12.2 a. $df(\text{Total}) = df(\text{Treatment}) + df(\text{Error})$
$\Rightarrow df(\text{Error}) = df(\text{Total}) - df(\text{Treatment}) = 34 - 4 = 30$

$$SSE = SS(Total) - SST = 62.4 - 24.7 = 37.7$$

$$MST = \frac{SST}{p-1} = \frac{24.7}{4} = 6.175 \qquad MSE = \frac{SSE}{n-p} = \frac{37.7}{30} = 1.257$$

$$F = \frac{MST}{MSE} = \frac{6.175}{1.257} = 4.91$$

The completed table is

SOURCE	*df*	SS	MS	*F*
Treatments	4	24.7	6.175	4.91
Error	30	37.7	1.257	
Total	34	62.4		

b. Since there are $p-1=4$ df for treatments, there are $p=5$ treatments involved in the experiment.

c. To determine if there is a difference among the treatment means, we test:

$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$
H_a: At least two treatment means differ

The test statistic is $F = \frac{MST}{MSE} = 4.91.$

The rejection region requires $\alpha = 0.10$ in the upper tail of the F distribution with $\nu_1 = p-1 = 4$ and $\nu_2 = n-p = 30$. From Table 3, Appendix D, $F_{0.10} = 2.14$. The rejection region is $F > 2.14$.

Since the observed value of the test statistic falls in the rejection region $(F = 4.91 > 2.14)$, H_0 is rejected. There is sufficient evidence to indicate at least two treatment means differ at $\alpha = 0.10$.

12.3 a. The linear model is: $E(y) = \beta_0 + \beta_1 x$ where $x = \begin{cases} 1 & \text{if Treatment 1} \\ 0 & \text{if Treatment 2} \end{cases}$

b. Some preliminary calculations are:

$$n = 13 \qquad \sum x = 7 \qquad \sum y = 128 \qquad \sum xy = 64$$
$$\sum x^2 = 7 \qquad \sum y^2 = 1294$$

$$\text{SS}_{xx} = 7 - \frac{7^2}{13} = 3.230769231 \qquad \text{SS}_{xy} = 64 - \frac{7(128)}{13} = -4.92307692$$

$$\text{SS}_{yy} = 1294 - \frac{128^2}{13} = 33.692308$$

$$\hat{\beta}_1 = \frac{\text{SS}_{xy}}{\text{SS}_{xx}} = \frac{-4.92307692}{3.230769231} = -1.523809523 \approx -1.524$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} = \frac{128}{13} - (-1.523809523)\left(\frac{7}{13}\right) = 10.666666667 \approx 10.667$$

The fitted model is $\hat{y} = 10.667 - 1.524x$.

$$\text{SSE} = \text{SS}yy - \hat{\beta}_1 \text{SS}_{xy} = 33.692308 - (-1.523809523)(-4.92307692) = 26.19047651$$

$$s^2 = \frac{\text{SSE}}{n-2} = \frac{26.19047651}{13-2} = 2.38095241 \quad s = \sqrt{2.38095241} = 1.543$$

To determine if there is a difference between the treatment means, we test:

$$H_0: \beta_1 = 0$$
$$H_a: \beta_1 \neq 0$$

The test statistic is $t = \frac{\hat{\beta}_1}{s_{\hat{\beta}_1}} = \frac{-1.524}{\frac{1.543}{\sqrt{3.23077}}} = -1.775.$

The rejection region requires $\alpha / 2 = 0.05/2 = 0.025$ in each tail of the t distribution with $df = n - 2 = 13 - 2 = 11$. From Table 2, Appendix D, $t_{0.025} = 2.201$. The rejection region is $t < -2.201$ or $t > 2.201$.

Since the observed value of the test statistic does not fall in the rejection region $\left(t = -1.775 \nless -2.201\right)$, H_0 is not rejected. There is insufficient evidence to indicate a difference in the treatment means at $\alpha = 0.05$.

Note: We could also test the hypothesis using the F test. To calculate the F statistic, we must first calculate r^2.

$$r^2 = 1 - \frac{\text{SSE}}{\text{SS}_{yy}} = 1 - \frac{26.1905}{33.6923} = 0.2227$$

Test statistic: $$F = \frac{r^2 / p}{\left(1 - r^2\right) / \left[n - (p+1)\right]} = \frac{0.2227 / 1}{(1 - 0.2227) / \left[13 - (1+1)\right]} = 3.15$$

Rejection region: For $\alpha = 0.05$, $v_1 = p = 1$, $v_2 = n - (p+1) = 11$
$F_{0.05} = 4.84$. Reject H_0 if $F > 4.84$.

The conclusion is the same as using the t test.

12.4 a. From the text: $$\text{MST} = \frac{n_1 \left(\bar{x}_1 - \bar{x}\right)^2 + n_2 \left(\bar{x}_2 - \bar{x}\right)^2}{2 - 1}$$

Some preliminary calculations are:

$$\bar{x}_1 = \frac{\sum x_1}{n_1} = \frac{10 + 7 + \cdots + 9}{7} = \frac{64}{7} = 9.1429$$

$$\bar{x}_2 = \frac{\sum x_2}{n_2} = \frac{12 + 8 + \cdots + 1}{6} = \frac{64}{6} = 10.6667$$

$$\bar{x} = \frac{\sum x}{n} = \frac{10 + 7 + \cdots + 9 + 12 + 8 + \cdots + 11}{13} = \frac{128}{13} = 9.8462$$

$$MST = \frac{7(9.1429 - 9.8462)^2 + 6(10.6667 - 9.8462)^2}{1} = 7.502$$

The mean square for treatments measures the variability among the two population means.

b. From the text:

$$MSE = s^2 = \frac{\sum_{i=1}^{n_1}(x_{i1}-\bar{x}_1)^2 + \sum_{i=1}^{n_2}(x_{i2}-\bar{x}_2)^2}{n_1+n_2-2}$$

$$= \frac{\left[(10-9.1429)^2+\cdots+(9-9.1429)^2\right]+\left[(12-9.1429)^2+\cdots+(11-9.1429)^2\right]}{7+6-2}$$

$$= \frac{10.8571+15.3333}{11} = 2.381$$

The mean square for error measures the variability within samples.

c. The degrees of freedom associated with MST is the numerator df, $\nu_1 = p-1 = 2-1 = 1$.

d. The degrees of freedom associated with MSE is the denominator df, $\nu_1 = n-p = 13-2 = 11$.

e. The test statistic is $F = \frac{MST}{MSE} = \frac{7.502}{2.381} = 3.15$ which has 1 and 11 df.

f. The ANOVA table is merely a structured presentation of the results of the calculations for an analysis of variance. It lists, in tabular form, the sources of variability, degrees of freedom, sums of squares for each source, mean squares for those sources in the F test and the F statistic. The table is:

SOURCE	*df*	SS	MS	*F*
Treatments	1	7.502	7.502	3.15
Error	11	26.190	2.381	
Total	12	33.692		

g. The rejection region requires $\alpha = 0.05$ in the upper tail of the F distribution with $\nu_1 = p-1 = 2-1 = 1$ and $\nu_1 = n-p = 13-2 = 11$. From Table 4, Appendix D, $F_{0.05} = 4.84$. The rejection region is $F > 4.84$.

h. Since the observed value of the test statistic does not fall in the rejection region $(F = 3.15 \not> 4.84)$, H_0 is not rejected. There is insufficient evidence to indicate there is a difference in the treatment means at $\alpha = 0.05$.

12.5 a. Some preliminary calculations:

$$\bar{y}_1 = \frac{64}{7} = 9.143 \qquad \bar{y}_2 = \frac{64}{6} = 10.667$$

$$s_1^2 = \frac{\sum y_1^2 - \frac{\left(\sum y_1\right)^2}{n_1}}{n_1-1} = \frac{596 - \frac{64^2}{7}}{7-1} = 1.8095$$

$$s_2^2 = \frac{\sum y_2^2 - \frac{\left(\sum y_2\right)^2}{n_2}}{n_2 - 1} = \frac{698 - \frac{64^2}{6}}{6-1} = 3.0667$$

$$s_{\text{p}}^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2} = \frac{(7-1)1.8095 + (6-1)3.0667}{7+6-2}$$

$$H_0: \mu_1 = \mu_2$$
$$H_{\text{a}}: \mu_1 \neq \mu_2$$

The test statistic is $t = \dfrac{\overline{y}_1 - \overline{y}_2}{\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \dfrac{9.143 - 10.667}{\sqrt{2.381\left(\frac{1}{7} + \frac{1}{6}\right)}} = -1.775$

The rejection region requires $\alpha / 2 = 0.05/2 = 0.025$ is each tail of the t distribution with $df = n_1 + n_2 - 2 = 7 + 6 - 2 = 11$. From Table 2, Appendix D, $t_{0.025} = 2.201$. The rejection region is $t < -2.201$ or $t > 2.201$.

Since the observed value of the test statistic does not fall in the rejection region $\left(t = -1.775 \not< -2.201\right)$, H_0 is not rejected. There is insufficient evidence to indicate a difference in the treatment means at $\alpha = 0.05$.

b. $t^2 = (-1.775)^2 = 3.15 = F$

c. The analysis of variance F test for comparing two population means is always a two-tailed test with the alternative hypothesis $H_{\text{a}}: \mu_1 \neq \mu_2$.

12.6 a. The group of 252 students were randomly assigned to one of three groups. The groups also contained the same sample size.

b. The dependent variable is the amount that a student is willing to pay (WTP) for insuring a valuable painting. The treatment for each group is the kind of information provided concerning the probabilities theft and fire.

c. To determine if there are differences in the mean WTP values for the three groups, we test:

$$H_0: \mu_1 = \mu_2 = \mu_3$$
H_a: At least two treatment means differ

d. Answers will differ. First, assign a number from 1 to 252 to each of the subjects. Then, generate random numbers between 1 and 252 (one must generate many more than 252 numbers to account for duplicates). The subjects with numbers corresponding to the first 84 unique numbers will be assigned to group 1, the subjects with the next 84 unique numbers will be assigned to group 2, and the rest of the subjects will be assigned to group 3. A set of generated numbers is:

49	60	11	172	153	186	92	45	215	168	190	43	210	83
40	118	31	234	25	142	78	82	65	19	37	235	133	119
169	171	76	249	186	67	213	50	210	60	38	89	160	191
244	203	250	93	76	6	10	104	83	65	23	144	71	36
207	156	156	132	186	149	38	9	186	206	115	174	88	21
47	63	106	37	213	30	182	84	135	65	138	137	248	227
10	122	124	128	42	115	244	21	142	99	155	209	92	214
61	84	168	50	227	178	239	133	2	164	189	41	56	41
7	19	181	223	168	183	53	219	16	38	210	26	246	112
230	223	169	150	6	44	109	223	242	48	116	211	68	95
213	26	231	33	46	82	106	160	240	162	237	75	3	138
107	69	114	50	153	42	22	84	130	179	29	45	159	244
113	122	57	38	170	12	198	28	243	195	75	29	251	110
250	121	225	223	28	161	89	21	146	31	112	89	217	196
217	219	234	106	162	222	151	32	174	176	180	128	25	37
184	224	234	127	170	162	108	46	155	56	163	171	61	31
174	155	38	136	242	186	228	64	197	208	208	199	93	31
108	210	236	88	158	22	128	68	220	48	41	63	227	226
157	47	236	232	205	210	34	181	130	45	160	53	181	36
166	239	34											

The first 84 unique numbers (going across) are:

6	9	10	11	19	21	23	25	30	31	36	37	38	40	42
43	45	47	49	50	60	61	63	65	67	71	76	78	82	83
84	88	89	92	93	99	104	106	115	118	119	120	122	124	128
132	133	135	137	138	142	144	149	153	155	156	160	168	169	171
172	174	178	186	190	191	201	203	206	207	208	209	210	213	214
215	227	234	235	239	244	248	249	250						

The subjects with these numbers will be assigned to group 1.

The next 84 unique numbers are:

2	3	7	12	16	22	26	28	29	32	33	34	35	41	44
46	48	53	56	57	64	68	69	75	90	95	101	107	108	109
110	112	113	114	116	121	127	130	136	146	150	151	157	158	159
161	162	163	164	166	170	176	179	180	181	183	184	189	195	196
197	198	199	205	211	217	219	220	222	223	224	225	226	228	230
231	232	236	237	240	242	243	246	251						

The subjects with these numbers will be assigned to group 2.

The subjects with the remaining numbers will be assigned to group 3:

1	4	5	8	13	14	15	17	18	20	24	27	39	51	52
54	55	58	59	62	66	70	72	73	74	77	79	80	81	85
86	87	91	94	96	97	98	100	102	103	105	111	117	123	125
126	129	131	134	139	140	141	143	145	147	148	152	154	165	167
173	175	177	182	185	187	188	192	193	194	200	202	204	212	216
218	221	229	233	238	241	245	247	252						

12.7 a. There are four ANOVAs. The treatments for each ANOVA are the 5 self-assessed twitter skill level. For each ANOVA, the Hypotheses tested are:

$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$
H_a: At least two treatment means differ

The response variables for the four ANOVAs are:

Males, Twitter skill level: Response variable is # tweets
Females, Twitter skill level: Response variable is # tweets
Males, Twitter skill level: Continue using tweet level
Females, Twitter skill level: Continue using tweet level

b. **Males, Twitter skill level**: **Response variable is # tweets**
The test statistic is $F = 1.17$ and the p-value is $p = 0.331$. Since the p-value is not less than $\alpha\left(p = 0.331 \not< 0.05\right)$, H_0 is not rejected. There is insufficient evidence to indicate a difference among the mean number of tweets for the 5 twitter levels for the males at $\alpha = 0.05$.

Females, Twitter skill level: **Response variable is # tweets**
The test statistic is $F = 0.56$ and the p-value is $p = 0.731$. Since the p-value is not less than $\alpha\left(p = 0.731 \not< 0.05\right)$, H_0 is not rejected. There is insufficient evidence to indicate a difference among the mean number of tweets for the 5 twitter levels for the females at $\alpha = 0.05$.

Males, Twitter skill level: **Response variable is Continue using tweet level**
The test statistic is $F = 2.21$ and the p-value is $p = 0.062$. Since the p-value is not less than $\alpha\left(p = 0.062 \not< 0.05\right)$, H_0 is not rejected. There is insufficient evidence to indicate a difference among the mean levels of continue to use twitter for the 5 twitter levels for the males at $\alpha = 0.05$.

Females, Twitter skill level: **Response variable is Continue using tweet level**
The test statistic is $F = 3.34$ and the p-value is $p = 0.006$. Since the p-value is less than $\alpha\left(p = 0.0062 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate a difference among the mean levels of continue to use twitter for the 5 twitter levels for the females at $\alpha = 0.05$.

12.8 a. The experimental units are the 324 adult TV viewers.

b. The dependent variable is the brand name recalled in commercial messages with scores ranging from no brands recalled to all nine brands recalled.

c. The factor is the type of TV program that was watched. The treatment is a particular type of TV program.

d. The means presented are only sample means. You also need to take into consideration the variation within each group as well as the variation between groups.

e. The test statistic $F = 20.45$ and the p–value is $p = 0.000$.

f. Since the p–value is less than $\alpha\left(p = 0.000 < 0.01\right)$, H_0 is rejected. There is sufficient evidence to conclude that the treatment means differ at $\alpha = 0.01$.

12.9 a. The type of experiment is a completely randomized design. The students were randomly assigned to one of three conditions.

b. The experimental units are the college students. The dependent variable is the tanning attitude index. The three treatments are the three tanning conditions – model with a tan, model without a tan, and no model.

c. The hypotheses are

$H_0: \mu_1 = \mu_2 = \mu_3$
H_a: At least two treatment means differ

d. The means presented are the sample means. We must take into consideration the variation within the treatments as well as the variation between the groups.

e. The test statistic is $F = 3.60$ and the p–value is $p = 0.03$. Since the p–value is less than $\alpha\left(p = 0.03 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to conclude that the treatment means differ at $\alpha = 0.05$.

f. We must assume that all the treatment probability distributions are normal and that the population variances of all the treatments are equal.

12.10 a. The hypotheses are:

$H_0: \mu_1 = \mu_2 = \mu_3 = \cdots = \mu_7$
H_a: At least two treatment means differ

b. The test statistic is $F = 1.60$ and the p–value is $p = 0.174$. Since the p–value is not less than $\alpha\left(p = 0.174 \not< 0.10\right)$, H_0 is not rejected. There is insufficient evidence to conclude that the mean LUST values differ among the seven states at $\alpha = 0.10$.

12.11 To determine if the mean recall percentage differs for student-drivers in the four groups, we test:

$H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4$
H_a : At least two treatment means differ

The test statistic is $F = 5.388$ and the p–value is $p = 0.004$. Since the p–value is less than $\alpha\left(p = 0.004 < 0.01\right)$, H_0 is rejected. There is sufficient evidence to conclude that the mean recall percentage differs for student-drives in the four groups at $\alpha = 0.01$.

12.12 a. This is a completely randomized design.

b. Using MINITAB, the results are:

One-way ANOVA: SCORE versus GROUP
Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
GROUP	3	0.9506	0.31688	10.29	0.000
Error	40	1.2317	0.03079		
Total	43	2.1824			

To determine if there are differences among the mean task scores for the four groups, we test:

$H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4$
H_a : At least two treatment means differ

The test statistic is $F = 10.29$ and the p–value is $p = 0.000$. Since the p–value is less than $\alpha\left(p = 0.000 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to conclude that the mean recall percentage differs for students in the four groups at $\alpha = 0.05$.

c. We must assume that the probability distributions for each of the treatments are normal and that the population variances are equal for all the treatments.

12.13 Using MINITAB, the results are:

One-way ANOVA: ETHANOL versus TEMP
Analysis of Variance

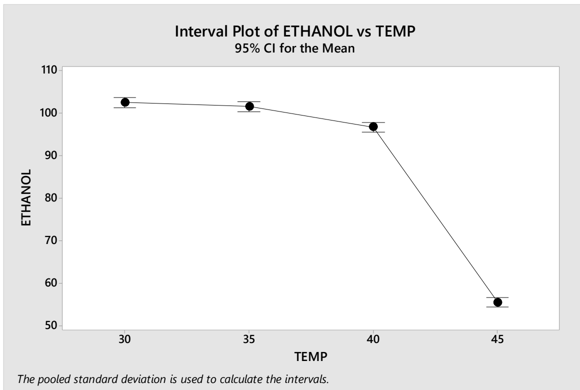
Source	DF	Adj SS	Adj MS	F-Value	P-Value
TEMP	3	4589.47	1529.82	2026.26	0.000
Error	8	6.04	0.76		
Total	11	4595.51			

To determine if high temperatures inhibit mean concentration of ethanol, we test:

$H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4$
H_a : At least two treatment means differ

The test statistic is $F = 2026.26$ and the p–value is $p = 0.000$. Since the p–value is less than $\alpha\left(p = 0.000 < 0.10\right)$, H_0 is rejected. There is sufficient evidence to conclude that the mean concentration of ethanol in the four temperature groups differ at $\alpha = 0.10$.

The interval plot of the means is:



As the temperature increases, the mean concentration of ethanol decreases. There is evidence that high temperatures inhibit mean concentration of ethanol.

12.14 a. This is a completely randomized design was used. The treatments are the types of medicine given (DM, Honey, and none).

b. The ANOVA table from MINITAB is:

One-way ANOVA: TotalScore versus Treatment
Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Treatment	2	318.5	159.255	17.51	0.000
Error	102	927.7	9.095		
Total	104	1246.2			

To determine if the mean improvement scores among the three treatment groups, we test:

$$H_0: \mu_1 = \mu_2 = \mu_3$$
$$H_a: \text{At least two treatment means differ}$$

The test statistic is $F = 17.51$ and the p–value is $p = 0.000$. Since the p–value is so small, H_0 is rejected. There is sufficient evidence to conclude that the mean improvement scores among the three treatment groups differ for any reasonable value of α.

12.15 a. In order to complete the ANOVA table we need to start with computing the grand total for the anxiety level for all groups: $\sum\sum x = 273.0 + 97.5 + 35.0 = 405.5$.

Next we compute $CM = \frac{\left(\sum\sum x\right)^2}{n} = \frac{(405.5)^2}{76} = 2163.6.$

b. $SST = \frac{T_1^2}{n_1} + \frac{T_2^2}{n_2} + \frac{T_3^2}{n_3} - CM = \frac{(273)^2}{26} + \frac{(97.5)^2}{25} + \frac{(35)^2}{25} - 2163.6 = 1132.15$

c. $$SSE = (n_1 - 1)s_1^2 + (n_2 - 1)s_2^2 + (n_3 - 1)s_3^2$$
$$= (26-1)7.6^2 + (25-1)7.5^2 + (25-1)7.5^2 = 4144$$

d. $SS(\text{Total}) = SSE + SST = 4144 + 1132.15 = 5276.15$

e. Some preliminary calculations:

$$MST = \frac{SST}{p-1} = \frac{1132.15}{2} = 566.075 \quad MSE = \frac{SSE}{n-p} = \frac{4144}{73} = 56.77$$

$$F = \frac{MST}{MSE} = \frac{566.075}{56.77} = 9.97$$

The ANOVA table is:

SOURCE	*df*	SS	MS	*F*
Treatments	2	1132.15	566.075	9.97
Error	73	4144.00	56.77	
Total	75	5276.15		

f. To determine if differences in the mean drops in anxiety levels exist among patients in the three groups, we test:

$H_0: \mu_1 = \mu_2 = \mu_3$
H_a: At least two treatment means differ

The test statistic is $F = 9.97$.

The rejection region requires $\alpha = 0.05$ in the upper tail of the F distribution with $\nu_1 = p - 1 = 2$ and $\nu_2 = n - p = 73$. From Table 4, Appendix D, $F_{0.05} \approx 3.13$. The rejection region is $F > 3.13$.

Since the observed value of the test statistic falls in the rejection region $(F = 9.97 > 3.13)$, H_0 is rejected. There is sufficient evidence to indicate there are differences in the mean drops in anxiety levels exist among patients in the three groups at $\alpha = 0.05$.

g. The model would be $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2$

where $x_1 = \begin{cases} 1 & \text{if Group T} \\ 0 & \text{if not} \end{cases}$ $\quad x_2 = \begin{cases} 1 & \text{if Group V} \\ 0 & \text{if not} \end{cases}$

h. $\hat{\beta}_0 = \text{mean for Group C} = 1.4$

$\hat{\beta}_1 = \text{difference in means between Group T and Group C} = 10.5 - 1.4 = 9.1$

$\hat{\beta}_2 = \text{difference in means between Group V and Group C} = 3.9 - 1.4 = 2.5$

12.16 To determine if there is a difference among the mean AI/Be ratios for the five boreholes an ANOVA was computed. The MINITAB results are given below:

One-way ANOVA: RATIO versus BOREHOLE
Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
BOREHOLE	4	5.836	1.4589	7.25	0.001
Error	21	4.225	0.2012		
Total	25	10.061			

The test statistic is $F = 7.25$ and the p–value is $p = 0.001$. Since the p–value is less than $\alpha\left(p = 0.001 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate that differences exist among the mean AI/Be ratios for the five boreholes at $\alpha = 0.05$.

12.17 a. The hypotheses are:

$H_0: \mu_1 = \mu_2 = \mu_3$

H_a: At least two treatment means differ

b. Using MINITAB, the ANOVA table is:

One-way ANOVA: IMPROVE versus ASSIST
Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
ASSIST	2	6.643	3.322	0.45	0.637
Error	72	527.357	7.324		
Total	74	534.000			

c. The test statistic is $F = 0.45$ and the p–value is $p = 0.637$. Since the p–value is not small, H_0 is not rejected. There is insufficient evidence to indicate that differences exist among the mean knowledge gains of the three treatment groups for any reasonable value of α.

12.18 a. $MST = \dfrac{SST}{n-1} = \dfrac{27.1}{3} = 9.033 \qquad MSB = \dfrac{SSB}{b-1} \Rightarrow SSB = MSB\left(b-1\right) = 14.90\left(5\right) = 74.50$

$$n = bp = \left(5+1\right)\left(3+1\right) = 24 \Rightarrow df\left(Error\right) = n - b - p + 1 = 24 - \left(5+1\right) - \left(3+1\right) + 1 = 15$$

$$df\left(Total\right) = n - 1 = 24 - 1 = 23$$

$SS(Total) = SST + SSB + SSE = 27.1 + 74.5 + 33.4 = 135.0$

$MSE = \dfrac{SSE}{n-b-p+1} = \dfrac{33.427.1}{15} = 2.227 \quad F(Treatments) = \dfrac{MST}{MSE} = \dfrac{9.033}{2.227} = 4.06$

$F(Blocks) = \dfrac{MSB}{MSE} = \dfrac{14.90}{2.227} = 6.69$

The completed ANOVA table is:

SOURCE	*df*	SS	MS	*F*
Treatments	3	27.0	9.033	4.06
Blocks	5	74.5	14.90	6.69
Error	15	33.4	2.2227	
Total	23	135.0		

b. To determine if a difference exists among the treatment means, we test:

$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$
H_a : At least two treatment means differ

The test statistic is $F = \dfrac{MST}{MSE} = 4.06$.

The rejection region requires $\alpha = 0.01$ in the upper tail of the F distribution with $\nu_1 = p - 1 = 3$ and $\nu_2 = n - p - b + 1 = 24 - 4 - 6 + 1 = 15$. From Table 6, Appendix D, $F_{0.01} = 5.42$. The rejection region is $F > 5.42$.

Since the observed value of the test statistic does not fall in the rejection region $(F = 4.06 \not> 5.42)$, H_0 is not rejected. There is insufficient evidence to indicate there are differences among the treatment means at $\alpha = 0.01$.

c. To see if blocking was useful, we test:

H_0 : There is no difference amnog the block means
H_a : At least two block means differ

The test statistic is $F = \dfrac{MSB}{MSE} = 6.69$.

The rejection region requires $\alpha = 0.01$ in the upper tail of the F distribution with $\nu_1 = b - 1 = 5$ and $\nu_2 = n - p - b + 1 = 24 - 4 - 6 + 1 = 15$. From Table 4, Appendix D, $F_{0.01} = 4.56$. The rejection region is $F > 4.56$.

Since the observed value of the test statistic falls in the rejection region $(F = 6.69 > 4.56)$, H_0 is rejected. There is sufficient evidence to indicate that blocking was useful. There is sufficient evidence that there were differences among the block means at $\alpha = 0.01$.

d. The form of the confidence interval for $(\mu_A - \mu_B)$ is:

$$(\bar{T}_A - \bar{T}_B) \pm t_{\alpha/2} s \sqrt{\frac{2}{b}} \quad \text{where } s = \sqrt{MSE}$$

From part a, $MSE = 2.227 \Rightarrow s = \sqrt{2.227} = 1.492$. For confidence coefficient 0.95, $\alpha = 0.05$ and $\alpha/2 = 0.05/2 = 0.025$. From Table 2, Appendix D, $t_{0.025} = 2.131$ with $df = n - p - b + 1 = 15$.

The 95% confidence interval is

$$(9.7 - 12.1) \pm 2.131(1.492)\sqrt{\frac{2}{6}} \Rightarrow -2.4 \pm 1.836 \Rightarrow (-4.236, -0.564)$$

e. The form of the confidence interval for $(\mu_B - \mu_D)$ is:

$$(\bar{T}_B - \bar{T}_D) \pm t_{\alpha/2} s \sqrt{\frac{2}{b}}$$

The 95% confidence interval is

$$(12.1 - 9.3) \pm 2.131(1.492)\sqrt{\frac{2}{6}} \Rightarrow 2.8 \pm 1.836 \Rightarrow (0.964, 4.636)$$

12.19 a. The same subjects were used in parts A and B of this study (the same 28 people in Group 2 of study 1). Therefore, a randomized block design was used.

b. WTP is the response variable, the treatment variables are Groups A and B, the blocks are subjects.

c. For comparing treatment means, the hypotheses are:

$$H_0: \mu_A = \mu_B$$
$$H_a: \mu_A \neq \mu_B$$

12.20 a. The competence level of each trainee was measured at three different times in the study. Thus, the trainees are the blocks and the time periods are the treatments.

b. Since both p-values are provided, we can test for differences among the treatment means and test for usefulness of blocking.

c. The hypotheses for comparing the treatment means are:

$H_0: \mu_1 = \mu_2 = \mu_3$
H_a: At least two treatment means differ

The hypotheses for comparing the block means are

H_0: There is no difference amnog the block means
H_a: At least two block means differ

d. The p-value for testing for differences in treatment means is $p = 0.001$. Since the p-value is so small, H_0 is rejected. There is sufficient evidence to indicate that the means for the treatment groups differ.

The p-value for testing for differences in block means is $p = 0.001$. Since the p-value is so small, H_0 is rejected. There is sufficient evidence to indicate that blocking was useful.

12.21 To determine if the presence of plants has an effect on stress levels, we test:

$H_0: \mu_1 = \mu_2 = \mu_3$
H_a: At least two treatment means differ

The test statistic is $F = 0.019$ and the p-value is $p = 0.981$. Since the p-value is not less than $\alpha \left(p = 0.981 \not< 0.05\right)$, H_0 is not rejected. There is insufficient evidence to indicate that the presence of plants has an effect on stress levels at $\alpha = 0.05$.

To determine if blocking was effective, we test:

H_0: There is no difference amnog the block means
H_a: At least two block means differ

The test statistic is $F = 0.635$ and the p-value is $p = 0.754$. Since the p-value is not less than $\alpha \left(p = 0.754 \not< 0.05\right)$, H_0 is not rejected. There is insufficient evidence to indicate that blocking was effective at $\alpha = 0.05$.

12.22 a. To determine if the mean Wong scores differ among the nine research methodology dimensions, we test:

$H_0: \mu_1 = \mu_2 = \mu_3 \cdots = \mu_9$
H_a: At least two treatment means differ

b. All of the nine dimensions were measured on each of the 13 papers. Thus, the observations are not independent of each other, which is an assumption in the completely randomized design. The data were collected as a randomized block design and thus, must be analyzed as a randomized block design.

c. The test statistic for testing if there are differences in the treatment means is $F = 5.40$ and the p-value is $p < 0.0001$. Since the p-value is so small, H_0 is rejected.

There is sufficient evidence to indicate that there are differences among the 9 treatment means for any reasonable value of α.

The test statistic for testing is blocking was useful is $F = 1.46$ and the p-value is $p = 0.1520$. Since the p-value is not small, H_0 is not rejected. There is insufficient evidence to indicate that blocking was useful.

12.23 a. The dependent variable is solar energy. The treatments are the 4 solar panel conditions. The blocks are the 12 months.

b. To determine if the mean solar energy values generated by the 4 panel configurations differ, we test:

$$H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4$$
$$H_a : \text{At least two treatment means differ}$$

c. The test statistic is $F = 115.54$ and the p-value is $p = 0.000$.

d. Since the p-value is so small, H_0 is rejected. There is sufficient evidence to indicate that the mean solar energy values generated by the 4 panel configurations differ at any reasonable value of α.

12.24 This is a randomized complete block design with the candidates as the treatments and the subjects as the blocks. Using MINITAB, the results are:

Analysis of Variance for SuitScore

Source	DF	SS	MS	F	P
Applicant	2	110.6	55.30	5.04	0.018
Subject	9	5486.3	609.59	55.59	0.000
Error	18	197.4	10.97		
Total	29	5794.3			

To determine if there are differences in the mean ratings for the 3 candidates, we test:

$$H_0 : \mu_1 = \mu_2 = \mu_3$$
$$H_a : \text{At least two treatment means differ}$$

The test statistic is $F = 5.04$ and the p-value is $p = 0.018$. Since the p-value is less than $\alpha \left(p = 0.018 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate that there are differences in the mean ratings for the 3 candidates at $\alpha = 0.05$.

To determine which candidate received the highest mean rating, we will form confidence intervals for differences in pairs of treatment means. The means for the 3 candidates are:

Means

Applicant	N	SuitScore
H	10	54.6
L	10	49.9
N	10	52.4

The form of the 95% confidence interval for comparing high and neutral candidates $(\mu_H - \mu_N)$ is:

$$\left(\bar{T}_H - \bar{T}_N\right) \pm t_{\alpha/2} s \sqrt{\frac{2}{b}} \quad \text{where } s = \sqrt{MSE}$$

For confidence coefficient 0.95, $\alpha = 0.05$ and $\alpha / 2 = 0.05 / 2 = 0.025$. From Table 2, Appendix D, $t_{0.025} = 2.101$ with $df = n - p - b + 1 = 18$.

The 95% confidence interval is

$$(54.6 - 52.4) \pm 2.101\sqrt{10.97}\sqrt{\frac{2}{10}} \Rightarrow 2.2 \pm 3.11 \Rightarrow (-0.91,\ 5.31)$$

Since the interval contains 0, there is no significant difference in the mean ratings for the high and neutral candidates.

The 95% confidence interval for comparing high and low candidates $(\mu_H - \mu_L)$ is

$$(54.6 - 49.9) \pm 2.101\sqrt{10.97}\sqrt{\frac{2}{10}} \Rightarrow 4.7 \pm 3.11 \Rightarrow (1.59,\ 7.81)$$

Since the interval does not contain 0, there is a significant difference in the mean ratings for the high and low candidates.

The 95% confidence interval for comparing neutral and low candidates $(\mu_N - \mu_L)$ is

$$(52.4 - 49.9) \pm 2.101\sqrt{10.97}\sqrt{\frac{2}{10}} \Rightarrow 2.5 \pm 3.11 \Rightarrow (-0.61,\ 5.61)$$

Since the interval contains 0, there is no significant difference in the mean ratings for the neutral and low candidates.

The candidate receiving the highest mean rating is either the high or neutral candidate. There was no significant difference in the mean ratings for these two candidates.

12.25 a. The complete model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \ldots + \beta_{10} x_{10}$, where x_1, x_2 and x_3 are dummy variables for interventions (treatments), and $x_4, x_5, \ldots x_{10}$ are dummy variables for boxers (blocks).

b. The reduced model is $E(y) = \beta_0 + \beta_4 x_4 + \beta_5 x_5 + \ldots + \beta_{10} x_{10}$, where $x_4, x_5, \ldots x_{10}$ are dummy variables for boxers (blocks).

c. The reduced model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$, where x_1, x_2 and x_3 are dummy variables for interventions (treatments).

12.26 a. The complete ANOVA table is:

SOURCE	*df*	SS	MS	*F*
Treatments	3	15754	5251	4.16
Blocks	7	117044	16721	13.24
Error	21	26525	1263	
Total	31	159323		

b. To determine if there are differences in the punching power means of the four interventions, we test:

$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$
H_a: At least two treatment means differ

The test statistic is $F = 4.16$.

The rejection region requires $\alpha = 0.05$ in the upper tail of the F distribution with $\nu_1 = p - 1 = 3$ and $\nu_2 = n - p - b + 1 = 32 - 4 - 8 + 1 = 21$. From Table 4, Appendix D, $F_{0.05} = 3.07$. The rejection region is $F > 3.07$.

Since the observed value of the test statistic falls in the rejection region $(F = 4.16 > 3.07)$, H_0 is rejected. There is sufficient evidence to indicate there are differences in the punching power means of the four interventions at $\alpha = 0.05$.

c. To determine if there are differences in the punching power means of the boxers, we test:

$H_0: \mu_1 = \mu_2 = \cdots = \mu_8$
H_a: At least two block means differ

The test statistic is $F = 13.24$.

The rejection region requires $\alpha = 0.05$ in the upper tail of the F distribution with $\nu_1 = b - 1 = 7$ and $\nu_2 = n - p - b + 1 = 32 - 4 - 8 + 1 = 21$. From Table 4, Appendix D, $F_{0.05} = 2.49$. The rejection region is $F > 2.49$.

Since the observed value of the test statistic falls in the rejection region $(F = 13.24 > 2.49)$, H_0 is rejected. There is sufficient evidence to indicate there are differences in the mean punching power among the eight boxers at $\alpha = 0.05$.

12.27 a. The treatments are the 5 food/beverage items. The blocks are the subjects. The dependent variable is the 9-point taste rating. This is a randomized complete block design because each subject rated each of the food/beverage items.

b. To determine if there are differences in the mean 9-point taste ratings among the 5 items, we test:

$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$
H_a: At least two treatment means differ

The test statistic is $F = 434.21$ and the p-value is $p < 0.0001$. Since the p-value is so small, H_0 is rejected. There is sufficient evidence to indicate that there are differences in the mean 9-point taste ratings among the 5 items at any reasonable value of α.

To determine if blocking was useful, we test:

$H_0 : \mu_1 = \mu_2 = \cdots = \mu_{200}$
H_a : At least two block means differ

The test statistic is $F = 1.13$ and the p-value is $p = 0.1369$. Since the p-value is not small, H_0 is not rejected. There is insufficient evidence to indicate that there are differences in the mean 9-point taste ratings among the 200 subjects at any reasonable value of α. Thus, blocking was not useful.

c. Using MINITAB, the results for the gLMS rating scale are:

Analysis of Variance for gLMS

Source	DF	SS	MS	F	P
TRT	4	579034	144759	165.51	0.000
Taster_1	199	165962	834	0.95	0.655
Error	796	696205	875		
Total	999	1441201			

To determine if there are differences in the mean gLMS taste ratings among the 5 items, we test:

$H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$
H_a : At least two treatment means differ

The test statistic is $F = 165.51$ and the p-value is $p = 0.000$. Since the p-value is so small, H_0 is rejected. There is sufficient evidence to indicate that there are differences in the mean gLMS taste ratings among the 5 items at any reasonable value of α.

To determine if blocking was useful, we test:

$H_0 : \mu_1 = \mu_2 = \cdots = \mu_{200}$
H_a : At least two block means differ

The test statistic is $F = 0.95$ and the p-value is $p = 0.655$. Since the p-value is not small, H_0 is not rejected. There is insufficient evidence to indicate that there are differences in the mean gLMS taste ratings among the 200 subjects at any reasonable value of α. Thus, blocking was not useful.

12.28 a. $df(A) = a - 1 = 3 - 1 = 2 \quad df(Error) = ab(r-1) = 3(2)(4-1) = 18$
$df(Total) = abr - 1 = 3(2)(4) - 1 = 23$

$$MS(AB) = \frac{SS(AB)}{(a-1)(b-1)} \Rightarrow SS(AB) = MS(AB)[(a-1)(b-1)] = 2.5(3-1)(2-1) = 5$$

$$MSE = \frac{SSE}{ab(r-1)} \Rightarrow SSE = MSE[ab(r-1)] = 2.0(3)(2)(4-1) = 36$$

$$SS(B) = SS(Total) - SS(A) - SS(AB) - SSE = 700 - 100 - 5 - 36 = 559$$

$$MS(A) = \frac{SS(A)}{a-1} = \frac{100}{2} = 50 \qquad MS(B) = \frac{SS(B)}{b-1} = \frac{559}{1} = 559$$

$$F(A) = \frac{MS(A)}{MSE} = \frac{50}{2} = 25 \qquad F(B) = \frac{MS(B)}{MSE} = \frac{559}{2} = 279.5$$

$$F(AB) = \frac{MS(AB)}{MSE} = \frac{2.5}{2} = 1.25$$

The complete ANOVA table is

SOURCE	*df*	SS	MS	*F*
A	2	100	50	25
B	1	559	559	279.5
AB	2	5	2.5	1.25
Error	18	36	2	
Total	23	700		

b. To determine if interaction exists between factor A and factor B, we test:

H_0: No interaction between factors A and B
H_a: Factors A and B interact

The test statistic is $F = \frac{MS(AB)}{MSE} = 1.25$.

The rejection region requires $\alpha = 0.05$ in the upper tail of the F distribution with $\nu_1 = (a-1)(b-1) = 2$ and $\nu_2 = ab(r-1) = 18$. From Table 4, Appendix D, $F_{0.05} = 3.55$. The rejection region is $F > 3.55$.

Since the observed value of the test statistic does not fall in the rejection region $(F = 1.25 \not> 3.55)$, H_0 is not rejected. There is insufficient evidence to indicate interaction exists between factor A and factor B at $\alpha = 0.05$.

c. To test for the main effect of factor A, we test:

H_0: There are no differences among the main effect means of factor A
H_a: At least two main effect A means differ

The test statistic is $F = \dfrac{MS(A)}{MSE} = 25.$

The rejection region requires $\alpha = 0.05$ in the upper tail of the F distribution with $\nu_1 = a - 1 = 2$ and $\nu_2 = ab(r-1) = 18$. From Table 4, Appendix D, $F_{0.05} = 3.55$. The rejection region is $F > 3.55$.

Since the observed value of the test statistic falls in the rejection region $(F = 25 > 3.55)$, H_0 is rejected. There is sufficient evidence to indicate a difference exists among the main effect means of factor A at $\alpha = 0.05$.

d. To test for the main effect of factor B, we test:

H_0: There are no differences among the main effect means of factor B
H_a: At least two main effect B means differ

The test statistic is $F = \dfrac{MS(B)}{MSE} = 279.5.$

The rejection region requires $\alpha = 0.05$ in the upper tail of the F distribution with $\nu_1 = b - 1 = 1$ and $\nu_2 = ab(r-1) = 18 + 1 = 21$. From Table 4, Appendix D, $F_{0.05} = 4.41$. The rejection region is $F > 4.41$.

Since the observed value of the test statistic falls in the rejection region $(F = 279.5 > 4.41)$, H_0 is rejected. There is sufficient evidence to indicate a difference exists among the main effect means of factor B at $\alpha = 0.05$.

12.29 a. This is a complete 2×2 factorial design.

b. There are a total of $2 \times 2 = 4$ treatments. The 4 treatments are

(High adaptation, High knowledge) (High adaptation, Low knowledge)
(Low adaptation, High knowledge) (Low adaptation, Low knowledge)

c. If factor interaction is detected, then the effect of one factor on the dependent variable depends on the level of the second factor.

d. We can conclude that there is a difference in the mean final sales price between the high knowledge and low knowledge buying processes.

12.30 a. There are a total of $2 \times 2 = 4$ treatments. The 4 treatments are

(male, STEM) (female, STEM) (male, non-STEM) (female, non-STEM)

b. If discipline and gender interact, then the effect of discipline on satisfaction depends on the level of gender.

c. Yes. The STEM women have higher satisfaction than STEM men. However, non-STEM women have lower satisfaction than non-STEM men.

d. The ANOVA table would be:

SOURCE	*df*	SS	MS	*F*
Discipline	1			
Gender	1			
DxG	1			
Error	211			
Total	214			

e. The test statistic is $F = 4.10$ and the p-value is $p = 0.04$. Since the p-value is less than $\alpha\left(p = 0.04 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate that gender and discipline interact at $\alpha = 0.05$.

12.31 a. $df(\text{AGE}) = (a-1) = (3-1) = 2$ $\quad df(\text{BOOK}) = (b-1) = (3-1) = 2$
$df(\text{AGE}\times\text{BOOK}) = (a-1)(b-1) = (3-1)(3-1) = 4$
$df(\text{ERROR}) = ab(r-1) = 3(3)(12-1) = 99$

b. There are $3\times 3 = 9$ treatments. They are:

(18, Photos) (18, Drawings) (18, Control)
(24, Photos) (24, Drawings) (24, Control)
(30, Photos) (30, Drawings) (30, Control)

c. To determine if Age and Book interact, we test:

H_0: No interaction between Age and Book
H_a: Age and Book interact

The test statistic is $F = 2.99$ and the p-value is $p < 0.05$. Since the p-value is small, H_0 is rejected. There is sufficient evidence to indicate that Age and Book interact at $\alpha = 0.05$.

d. No, the tests for the main effects are not needed if the interaction is significant.

12.32 a. The VR display device is one source of variation with $df = (a-1) = (3-1) = 2$. Another source of variation is the auxiliary lateral image with $df = (b-1) = (2-1) = 1$. Interaction between the two factors is also a source of variation with $df = (a-1)(b-1) = (3-1)(2-1) = 2$. Unexplained error is the last source of variation with $df = ab(r-1) = 3(2)(2-1) = 6$.

b. There are a total of $3 \times 2 = 6$ treatments.

c. The first test to be performed is for the interaction of the two factors. The p-value is $p = 0.411$. Since the p-value is not less than $\alpha\left(p = 0.411 < 0.05\right)$, H_0 is not rejected. There is insufficient evidence to indicate display and auxiliary lateral image interact at $\alpha = 0.05$. Since the interaction is not significant, the tests for main effects are performed.

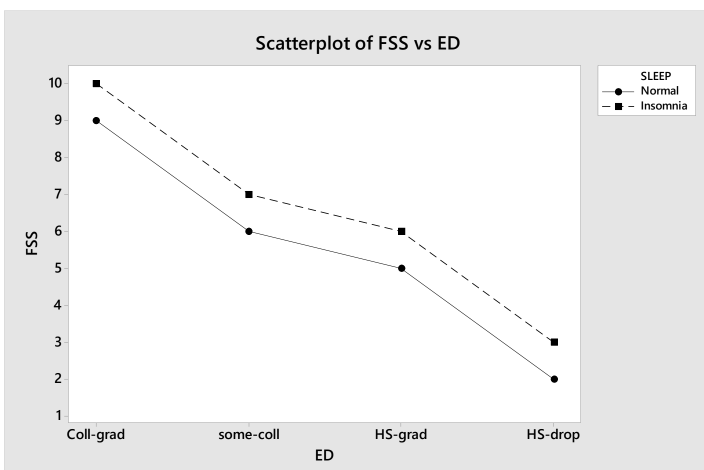
The p-value for the main effect of display is $p = 0.045$. Since the p-value is less than $\alpha\left(p = 0.045 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate a difference among the main effect means of display at $\alpha = 0.05$.

The p-value for the main effect of auxiliary lateral image is $p = 0.003$. Since the p-value is less than $\alpha\left(p = 0.003 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate a difference among the main effect means of auxiliary lateral image at $\alpha = 0.05$.

12.33 a. There are a total of $2 \times 4 = 8$ treatments. The 8 treatments are:

(normal sleeper, college graduate)	(normal sleeper, some college)
(normal sleeper, high school graduate)	(normal sleeper, high school dropout)
(chronic insomnia, college graduate)	(chronic insomnia, some college)
(chronic insomnia, high school graduate)	(chronic insomnia, high school dropout)

b. The difference between mean FSS values for normal sleepers and insomniacs is independent of education level. A possible graph is:



c. This implies that the population mean FFS scores for insomnia adults was statistically greater than that for normal adults: $\mu_{\text{Insomnia}} > \mu_{\text{Normal}}$

d. This implies that the there is a difference in the mean FSS scores among the 4 levels of education.

12.34 a. A 2×2 factorial design was used for this study. The two factors are color and type of questions, each at 2 levels. There are $2 \times 2 = 4$ treatments. The 4 treatments are:

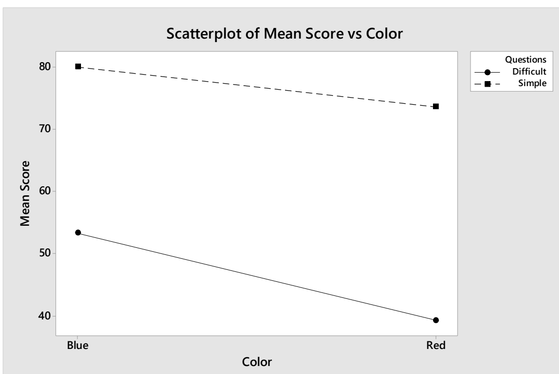
(blue, difficult) (blue, simple) (red, difficult) (red, simple)

b. The complete model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2$

where $x_1 = \begin{cases} 1 & \text{if blue} \\ 0 & \text{if red} \end{cases}$ $x_2 = \begin{cases} 1 & \text{if different} \\ 0 & \text{if simple} \end{cases}$

c. Since the p-value is small $p < 0.03$, H_0 is rejected. There is sufficient evidence that Color and Question interact. This implies that the effect of Question on the mean exam score depends on the color.

d. Using MINITAB, the graph is:



Looking at the graph, the lines are not parallel. The difference between the Blue and Red "simple" questions is not as great as the difference between the Blue and Red "difficult" questions. The effect of color on the exam scores depends on the level of difficulty.

12.35 a. The linear model would be

$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_3 x_1 x_3 + \beta_4 x_1 x_4 + \beta_3 x_2 x_3 + \beta_4 x_2 x_4$$

where $x_1 = \begin{cases} 1 & \text{if once} \\ 0 & \text{if not} \end{cases}$ $x_2 = \begin{cases} 1 & \text{if twice} \\ 0 & \text{if not} \end{cases}$ $x_3 = \begin{cases} 1 & \text{if 5} \\ 0 & \text{if not} \end{cases}$ $x_4 = \begin{cases} 1 & \text{if 10} \\ 0 & \text{if not} \end{cases}$

b. Using MINITAB, the results are:

Regression Analysis: VegHT versus x1, x2, x3, x4, x1x3, x1x4, x2x3, x2x4

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	8	556.11	69.513	31.36	0.000
Error	27	59.84	2.216		
Total	35	615.95			

Model Summary

S	R-sq	R-sq(adj)
1.48875	90.28%	87.41%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	26.025	0.744	34.96	0.000
x1	-9.97	1.05	-9.48	0.000
x2	0.13	1.05	0.12	0.906
x3	-8.03	1.05	-7.62	0.000
x4	-2.72	1.05	-2.59	0.015
x1x3	9.05	1.49	6.08	0.000
x1x4	2.55	1.49	1.71	0.098
x2x3	4.22	1.49	2.84	0.009
x2x4	-0.18	1.49	-0.12	0.907

Regression Equation

VegHT = 26.025 - 9.97 x1 + 0.13 x2 - 8.03 x3 - 2.72 x4 + 9.05 x1x3 + 2.55 x1x4 + 4.22 x2x3 - 0.18 x2x4

To determine if at least one of the interaction terms are significant, we must first run a model without the interaction terms. Using MINITAB, the results are:

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	4	465.9	116.470	24.06	0.000
Error	31	150.1	4.841		
Total	35	615.9			

To determine if at least one of the interaction terms are significant, we test:

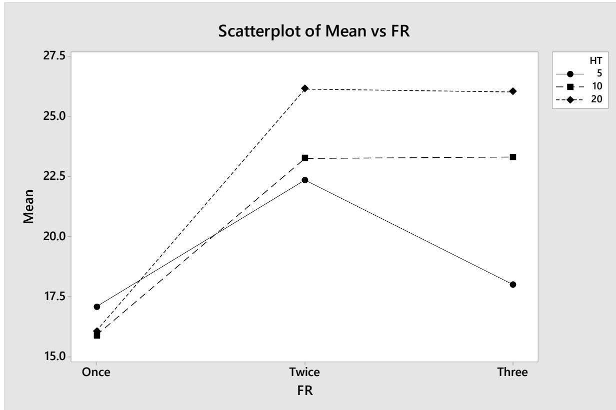
$$H_0: \beta_5 = \beta_6 = \beta_7 = \beta_8$$
$$H_a: \text{At least 1 } \beta_i \neq 0$$

The test statistic is $F = \dfrac{(SSE_R - SSE_C)/(k-g)}{MSE_C} = \dfrac{(150.1-59.84)/(8-4)}{2.216} = 10.18.$

The rejection region requires $\alpha = 0.05$ in the upper tail of the F distribution with $\nu_1 = k - g = 8 - 4 = 4$ and $\nu_2 = 36 - (8+1) = 27$. From Table 4, Appendix D, $F_{0.05} = 2.73$. The rejection region is $F > 2.73$.

Since the observed value of the test statistic falls in the rejection region $(F = 10.18 > 2.73)$, H_0 is rejected. There is sufficient evidence to indicate that at least one of the interaction terms is significant at $\alpha = 0.05$. The interaction terms x_1x_3 and x_2x_3 both have very small p-values, indicating that they are significant. Since the interaction terms are significant, there is no need to test for main effects.

Thus, the effect of mowing frequency on the end of year height depends on the level of mowing height. A plot of the means is:



This graph displays the interaction between Frequency and Height, as the lines are not parallel.

c. Let x_1 = mowing height and x_2 = mowing frequency. If we just use quantitative terms for mowing height and mowing frequency, the complete model would be $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 + \beta_4 x_1^2 + \beta_5 x_2^2 + \beta_6 x_1^2 x_2 + \beta_7 x_1 x_2^2 + \beta_8 x_1^2 x_2^2$.

d. Using MINITAB, the results are:

Regression Analysis: VegHT versus MowHT, MowFreq, ... , HTSQ_FRSQ

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	8	556.11	69.513	31.36	0.000
Error	27	59.84	2.216		
Total	35	615.95			

Model Summary

S	R-sq	R-sq(adj)
1.48875	90.28%	87.41%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	2.0	10.9	0.19	0.854
MowHT	0.14	2.10	0.07	0.946
MowFreq	24.3	12.3	1.97	0.059
HT_FR	-1.25	2.39	-0.52	0.606
HT-SQ	-0.0230	0.0809	-0.28	0.778
FR_SQ	-7.21	3.05	-2.36	0.026
HTSQ_FR	0.0652	0.0919	0.71	0.484
HT_FRSQ	0.605	0.591	1.02	0.315
HTSQ_FRSQ	-0.0250	0.0227	-1.10	0.281

Regression Equation

VegHT = 2.0 + 0.14 MowHT + 24.3 MowFreq - 1.25 HT_FR - 0.0230 HT-SQ - 7.21 FR_SQ + 0.0652 HTSQ_FR + 0.605 HT_FRSQ - 0.0250 HTSQ_FRSQ

The fitted equation is

$$\hat{y} = 2.0 + 0.14x_1 + 24.3x_2 - 1.25x_1x_2 - 0.023x_1^2 - 7.12x_2^2 + 0.0652x_1^2x_2$$
$$+ 0.605x_1x_2^2 - 0.025x_1^2x_2^2$$

e. To determine if the third and fourth order terms can be dropped from the model, we must fit a reduced model. Using MINITAB, the results are:

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	5	537.38	107.476	41.04	0.000
Error	30	78.57	2.619		
Total	35	615.95			

To determine if the third and fourth order terms can be dropped from the model, we test:

$H_0: \beta_6 = \beta_7 = \beta_8$
$H_a:$ At least 1 $\beta_i \neq 0$

The test statistic is $F = \dfrac{(SSE_R - SSE_C)/(k-g)}{MSE_C} = \dfrac{(78.57 - 59.84)/(8-5)}{2.216} = 2.82.$

The rejection region requires $\alpha = 0.05$ in the upper tail of the F distribution with $\nu_1 = k - g = 8 - 5 = 3$ and $\nu_2 = 36 - (8+1) = 27$. From Table 4, Appendix D, $F_{0.05} = 2.96$. The rejection region is $F > 2.96$.

Since the observed value of the test statistic does not fall in the rejection region $(F = 2.82 \not> 2.96)$, H_0 is not rejected. There is insufficient evidence to indicate that any of the third or fourth order terms are necessary at $\alpha = 0.05$. Thus, the third and fourth order terms can be dropped from the model.

12.36 a. There are two factors – housing systems and egg weight. There are a total of $4 \times 2 = 8$ treatments. The 8 treatments are:

(cage, medium)	(barn, medium)	(free range, medium)	(organic, medium)
(cage, large)	(barn, large)	(free range, large)	(organic, large)

b. Using MINITAB, the results are:

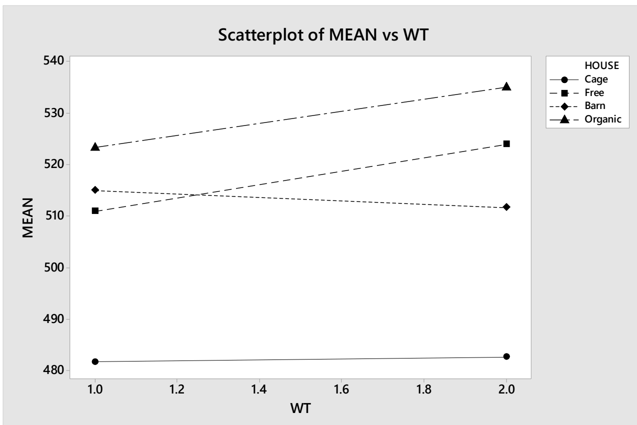
Analysis of Variance for OVERRUN

Source	DF	SS	MS	F	P
HOUSING	3	7249.5	2416.49	19.88	0.000
WTCLASS	1	187.0	187.04	1.54	0.233
HOUSING*WTCLASS	3	288.8	96.26	0.79	0.516
Error	16	1944.7	121.54		
Total	23	9670.0			

c. To determine if housing system and egg weight interact, we test:

H_0 : No interaction between Housing system and Egg weight
H_a : Housing system and Egg weight interact

The test statistic is $F = 0.79$ and the p-value is $p = 0.516$. Since the p-value is not less than $\alpha\left(p = 0.516 \nless 0.05\right)$, H_0 is not rejected. There is insufficient evidence to indicate that housing system and egg weight interact at $\alpha = 0.05$. This implies that the effect of housing system on the mean whipping capacity of the eggs does not depend on the weight of the eggs. A plot of the means is:

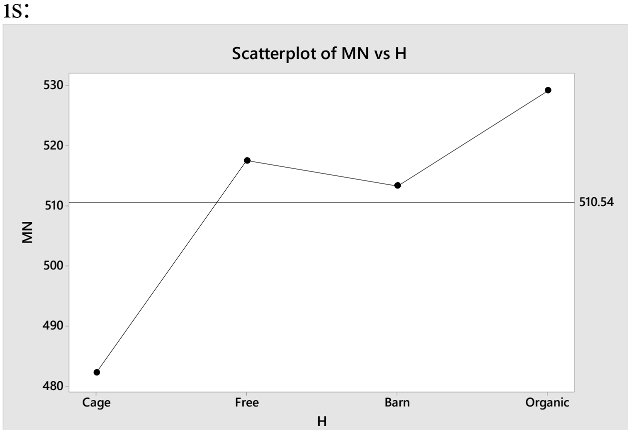


Even though the lines are not exactly parallel, they are not significantly different from parallel.

d. To determine if there are differences in mean whipping capacity among the four housing systems, we test:

H_0 : No difference in Housing system means
H_a : At least two Housing system means differ

The test statistic is $F = 19.88$ and the p-value is $p = 0.000$. Since the p-value is less than $\alpha\left(p = 0.000 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate that there is a difference among the housing system means at $\alpha = 0.05$. This means that the mean whipping capacity of eggs is affected by the housing system. A plot of the means is:



It appears that the mean whipping capacity for cage eggs is lower than the means of the other 3 housing systems.

e. To determine if there are differences in mean whipping capacity among the two weights, we test:

H_0: No difference in weight means
H_a: The two weight means differ

The test statistic is $F = 1.54$ and the *p*-value is $p = 0.233$. Since the *p*-value is not less than $\alpha\left(p = 0.233 \nless 0.05\right)$, H_0 is not rejected. There is insufficient evidence to indicate that there is a difference between the two weight means at $\alpha = 0.05$. This means that the mean whipping capacity of eggs is not affected by the egg weight.

12.37 a. There are a total of $2 \times 2 = 4$ treatments. The 4 treatments are:

(extra-large, cut paper) (extra-large, no cut paper)
(half-sheet, cut paper) (half-sheet, no cut paper)

b. To determine if paper size and paper distortion interact, we test:

H_0: No interaction between paper size and paper distortion
H_a: Paper size and paper distortion interact

The test statistic is $F = 7.52$ and the *p*-value is $p < 0.01$. Since the *p*-value is less than $\alpha\left(p < 0.01 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate that paper size and paper distortion interact at $\alpha = 0.05$. This means that the effect of paper size on the mean extent to which the paper was like garbage depends on the level of paper distortion.

c. No, the researchers should not conduct the main effect tests for paper size and paper distortion. The effect of the interaction may cover up the main effects.

12.38 There are a total of $2 \times 2 = 4$ treatments. The 4 treatments are:

(violent, volunteer) (violent, psychology class)
(non-violent, volunteer) (non-violent, psychology class)

Using MINITAB, the results are:

Analysis of Variance for SCORE

Source	DF	SS	MS	F	P
SONG	1	5.8907	5.8907	26.11	0.000
POOL	1	0.1307	0.1307	0.58	0.450
SONG*POOL	1	0.3527	0.3527	1.56	0.216
Error	56	12.6320	0.2256		
Total	59	19.0060			

To determine if song and pool interact, we test:

H_0 : No interaction between song and pool
H_a : Song and pool interact

The test statistic is $F = 1.56$ and the p-value is $p = 0.216$. Since the p-value is not less than $\alpha\left(p = 0.216 \not< 0.05\right)$, H_0 is not rejected. There is insufficient evidence to indicate that song and pool interact at $\alpha = 0.05$. Since the interaction is not significant, we can test for the main effects.

To determine if there is a difference in the mean aggressive cognition scores between the two levels of song, we test:

$H_0 : \mu_1 = \mu_2$
$H_a : \mu_1 \neq \mu_2$

The test statistic is $F = 26.11$ and the p-value is $p = 0.000$. Since the p-value is less than $\alpha\left(p = 0.000 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate that there is a difference in the mean aggressive cognition scores between the two levels of song at $\alpha = 0.05$. The mean for violent songs $\left(\mu_1 = 3.5433\right)$ is significantly greater than the mean for non-violent songs $\left(\mu_2 = 2.9167\right)$.

To determine if there is a difference in the mean aggressive cognition scores between the two pools of subjects, we test:

$H_0 : \mu_1 = \mu_2$
$H_a : \mu_1 \neq \mu_2$

The test statistic is $F = 0.58$ and the p-value is $p = 0.450$. Since the p-value is not less than $\alpha\left(p = 0.450 \not< 0.05\right)$, H_0 is not rejected. There is insufficient evidence to indicate that there is a difference in the mean aggressive cognition scores between the two pool levels at $\alpha = 0.05$. The mean for volunteers $\left(\mu_1 = 3.1833\right)$ is not significantly different from the mean for the psychology class $\left(\mu_2 = 3.277\right)$.

12.39 Using MINITAB, the ANOVA results are:

Analysis of Variance for RATING

Source	DF	SS	MS	F	P
PREP	1	54.73	54.735	14.40	0.000
STANDING	2	16.50	8.250	2.17	0.118
PREP*STANDING	2	13.47	6.735	1.77	0.174
Error	126	478.95	3.801		
Total	131	563.66			

First, we test for the presence of interaction between preparation and standing. To determine if preparation and class standing interact to affect rating, we test:

H_0 : Preparation and standing do not interact
H_a : Preparation and standing do interact

The test statistic is $F = 1.77$ and the p-value is $p = 0.174$. Since the p-value is not small, H_0 is not rejected. There is insufficient evidence to indicate preparation and standing interact to affect rating.

Since there was no evidence of interaction between preparation and standing, we test for the main effects. To determine if there are differences in the mean ratings between the two types of preparation, we test:

H_0 : Mean ratings for the two levels of preparation are the same
H_a : Mean ratings for the two levels of preparation are not the same

The test statistic is $F = 14.40$ and the p-value is $p = 0.000$. Since the p-value is very small, H_0 is rejected. There is sufficient evidence to indicate the mean ratings are different for those taking the practice exam and those taking the review session.

To determine if there are differences in the mean ratings among the three class standings, we test:

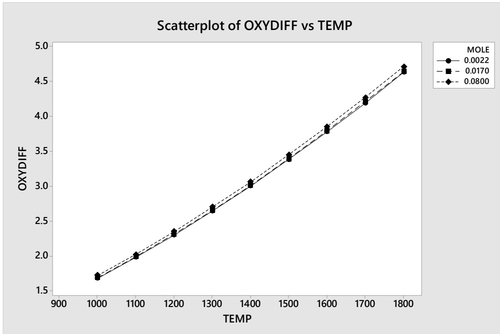
H_0 : Mean ratings for the three levels of class standing are the same
H_a :Mean ratings for the three levels of class standing are not the same

The test statistic is $F = 2.17$ and the p-value is $p = 0.118$. Since the p-value is not small, H_0 is not rejected. There is insufficient evidence to indicate the mean ratings are different among the three levels of class standing.

Since there was a difference between the two levels of preparation, we need to find which level leads to a higher rating. Since there are only two levels, we simply need to find the means for both levels. The mean rating for the practice exam is 5.3030 and the mean rating for the review session is 4.0152. Thus, those taking the practice exam rated the preparation higher than those who attended the review session.

12.40 a. To use the traditional analysis of variance, we need repetitions for each factor-level combinations. In this problem, we have only one observation per factor-level combination.

b. A plot of the data is shown below:



It appears that a first order model would be appropriate for this data.

c. The interaction model would be $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2$ where x_1 = temperature and x_2 = mole fraction.

d. When no interaction is present, the relationship between the dependent variable (rate of combustion) and one independent variable (say temperature) is independent of the second independent variable (mole fraction) or the effect of temperature on rate of combustion does not depend on the value of mole fraction.

e. To determine if interaction exists between mole fraction and temperature, we test:

$$H_0: \beta_3 = 0$$
$$H_a: \beta_3 \neq 0$$

The test statistic is $t = 0.55$ and the p-value is $p = 0.591$. Since the p-value is not less than $\alpha\left(p = 0.591 \nless 0.05\right)$,, H_0 is not rejected. There is insufficient evidence to indicate interaction exists between mole fraction and temperature at $\alpha = 0.05$.

f. The least squares prediction equation is $\hat{y} = -2.0956 + 0.003684x_1 - 0.24x_2 + 0.00073x_1x_2$.

g. When $x_1 = 1300$ and $x_2 = 0.017$,
$\hat{y} = -2.0956 + 0.003684(1300) - 0.24(0.017) + 0.00073(1300)(0.017) = 2.706$.

h. The 95% confidence interval is $(2.677, 2.735)$. We are 95% confident that the mean diffusivity when the temperature is 1,300°K and the mole fraction of water is 0.017 will fall between 2.677 and 2.735.

12.41 a. The complete 2×2 factorial model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2$

where $x_1 = \begin{cases} 1 & \text{if low load} \\ 0 & \text{if high load} \end{cases}$ $\quad x_2 = \begin{cases} 1 & \text{if ambiguous} \\ 0 & \text{if common} \end{cases}$

b. $\hat{\beta}_0 = \bar{y}_{\text{High/Common}} = 6.3$

$$\hat{\beta}_1 = \bar{y}_{\text{Low/Common}} - \bar{y}_{\text{High/Common}} = 7.8 - 6.3 = 1.5$$
$$\hat{\beta}_2 = \bar{y}_{\text{Ambig/High}} - \bar{y}_{\text{Common/High}} = 6.1 - 6.3 = -0.2$$
$$\hat{\beta}_3 = \left(\bar{y}_{\text{Low/Ambig}} - \bar{y}_{\text{High/Ambig}}\right) - \left(\bar{y}_{\text{Low/Common}} - \bar{y}_{\text{High/Common}}\right)$$
$$= (18 - 6.1) - (7.8 - 6.3) = 11.9 - 1.5 = 10.4$$

c. The totals of the four categories are:

$$T_{11} = n_{11}\bar{y}_{11} = 25(18) = 450 \qquad T_{12} = n_{12}\bar{y}_{12} = 25(7.8) = 195$$
$$T_{21} = n_{21}\bar{y}_{21} = 25(6.1) = 152.5 \quad T_{22} = n_{22}\bar{y}_{22} = 25(6.3) = 157.5$$

The sum of all the observations is
$T_{11} + T_{12} + T_{21} + T_{22} = 450 + 195 + 152.5 + 157.5 = 955$

$$CM = \frac{955^2}{100} = 9120.25$$

d. $A_1 = T_{11} + T_{12} = 450 + 195 = 645 \quad A_2 = T_{21} + T_{22} = 152.5 + 157.5 = 310$

$$SS(A) = SS(\text{Load}) = \frac{\sum A_i^2}{br} - CM = \frac{645^2 + 310^2}{2(25)} - 9120.25 = 1122.25$$

$$B_1 = T_{11} + T_{21} = 450 + 152.5 = 602.5 \quad B_2 = T_{12} + T_{22} = 195 + 157.5 = 352.5$$

$$SS(B) = SS(\text{Name}) = \frac{\sum B_i^2}{ar} - CM = \frac{602.5^2 + 352.5^2}{2(25)} - 9120.25 = 625$$

$$SS(AB) = SS(\text{Load} \times \text{Name}) = \frac{\sum\sum AB_{ij}^2}{r} - SS(A) - SS(B) - CM$$
$$= \frac{450^2}{25} + \frac{195^2}{25} + \frac{152.5^2}{25} + \frac{157.5^2}{25} - 1122.25 - 625 - 9120.25 = 676$$

e.
$$s_{11}^2 = 15^2 = 225 \Rightarrow \sum\left(y_{11k} - \bar{y}_{11}\right)^2 = (r-1)s_{11}^2 = (25-1)225 = 5400$$
$$s_{12}^2 = 9.5^2 = 90.25 \Rightarrow \sum\left(y_{12k} - \bar{y}_{12}\right)^2 = (r-1)s_{12}^2 = (25-1)90.25 = 2166$$
$$s_{21}^2 = 9.5^2 = 90.25 \Rightarrow \sum\left(y_{21k} - \bar{y}_{21}\right)^2 = (r-1)s_{21}^2 = (25-1)90.25 = 2166$$
$$s_{22}^2 = 10^2 = 100 \Rightarrow \sum\left(y_{22k} - \bar{y}_{22}\right)^2 = (r-1)s_{22}^2 = (25-1)100 = 2400$$

f. $SSE = \sum\sum\sum\left(y_{ijk} - \bar{y}_{ij}\right)^2 = 5400 + 2166 + 2166 + 2400 = 12132$

g. $SS(Total) = SS(A) + SS(B) + SS(AB) + SSE$
$$= 1122.25 + 625 + 676 + 12132 = 14555.25$$

h. The ANOVA table will be:

SOURCE	*df*	SS	MS	*F*
Load	1	1122.25	1122.25	8.88
Name	1	625.00	625.00	4.95
Load x Name	1	676.00	676.00	5.35
Error	96	12132.00	126.375	
Total	99	14555.25		

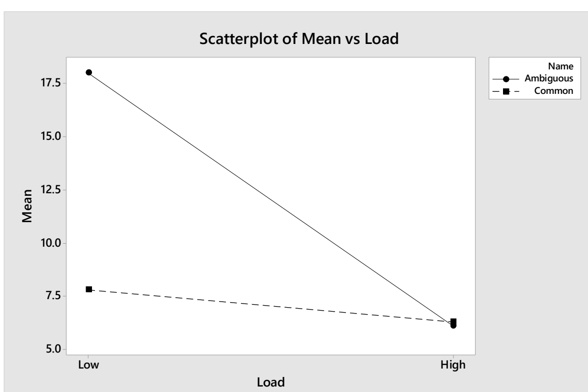
To determine if interaction between load and name exists, we test:

H_0 : Load and name do not interact
H_a : Load and name do interact

The test statistic is $F = 5.35$.

The rejection region requires $\alpha = 0.05$ in the upper tail of the F distribution with $\nu_1 = (a-1)(b-1) = 1$ and $\nu_2 = ab(r-1) = 96$. From Table 4, Appendix D, $F_{0.05} = 3.94$. The rejection region is $F > 3.94$.

Since the observed value of the test statistic falls in the rejection region $(F = 5.35 > 3.94)$, H_0 is rejected. There is sufficient evidence to indicate interaction between load and name exists at $\alpha = 0.05$. Since the interaction is present, we do not perform the main effect tests. A graph of the treatment means is:



It is obvious that there is no difference in the mean number of jelly beans taken by consumers between flavor names when the cognitive load is high. There is a significant difference in the mean number of jelly beans taken by consumers between flavor names when the cognitive load is low. Those consumers who had the ambiguous name took a higher mean number of jelly beans than consumers who had the common name.

i. We must assume that the population distribution of the observations for any factor level combination is approximately normal and that the variance is constant for all factor level combinations.

12.42 a. The ANOVA table is:

SOURCE	*df*
A	2
B	3
C	1
AB	6
AC	2
BC	3
ABC	6
Error	120
Total	143

b. To determine if any of the interactions are significant, we test:

H_0: No interactions exist
H_a: At least one interaction exists

The test statistic is $F = \dfrac{MS(\text{Interactions})}{MSE} = \dfrac{0.73}{0.14} = 5.21$.

The rejection region requires $\alpha = 0.05$ in the upper tail of the F distribution with $\nu_1 = 17$ and $\nu_2 = 120$. From Table 4, Appendix D, $F_{0.05} = 1.75$. The rejection region is $F > 1.75$.

Since the observed value of the test statistic falls in the rejection region $(F = 5.21 > 1.75)$, H_0 is rejected. There is sufficient evidence to indicate at least one interaction term is significant at $\alpha = 0.05$.

c. To determine if any of the AB interaction exists, we test:

H_0: No AB interaction exists
H_a: The AB interaction exists

The test statistic is $F = \dfrac{MS(\text{AB})}{MSE} = \dfrac{0.39}{0.14} = 2.79$.

The rejection region requires $\alpha = 0.05$ in the upper tail of the F distribution with $\nu_1 = 6$ and $\nu_2 = 120$. From Table 4, Appendix D, $F_{0.05} = 2.17$. The rejection region is $F > 2.17$.

Since the observed value of the test statistic falls in the rejection region $(F = 2.79 > 2.17)$, H_0 is rejected. There is sufficient evidence to indicate the AB interaction exists at $\alpha = 0.05$. This means that the effect of paper stock on the whiteness of fine bond paper depends on the bleach.

d. $SS(\text{Interactions}) = df(\text{Interactions}) \times MS(\text{Interactions}) = 17(0.73) = 12.41$

$SSE = df(\text{Error}) \times MSE = 120(0.14) = 16.8$

$$\begin{aligned} SS(Total) &= SS(A) + SS(B) + SS(C) + SS(\text{Interactions}) + SSE \\ &= 2.35 + 2.71 + 0.72 + 12.41 + 16.8 = 34.99 \end{aligned}$$

$$R^2 = \frac{SS(Total) - SSE}{SS(Total)} = \frac{34.99 - 16.8}{34.99} = 0.52$$

There is a 52% reduction in the sample variation of the whiteness measures about their mean when we use the factors *A*, *B*, and *C* and their interactions in a $3 \times 4 \times 2$ factorial design.

12.43 a. The dependent variable is the tablet dissolution time.

b. The three factors are (1) the nature of the binding agent (khaya gum, PVP), (2) the concentration of the binding agent (0.5%, 4.0%), and (3) the relative density (low, high).

c. This is a $2 \times 2 \times 2$ factorial design experiment.

d. There are $2 \times 2 \times 2 = 8$ possible treatments.

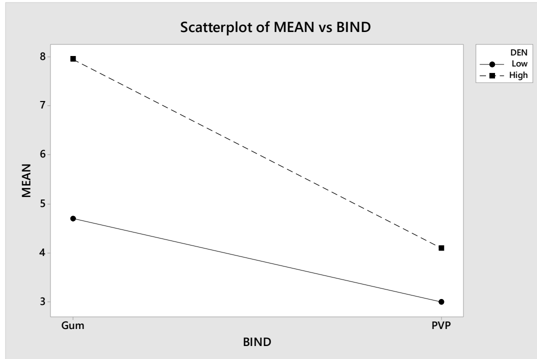
(khaya gum, 0.5% binding concentration, low relative density)
(khaya gum, 4.0% binding concentration, low relative density)
(khaya gum, 0.5% binding concentration, high relative density)
(khaya gum, 4.0% binding concentration, high relative density)
(PVP, 0.5% binding concentration, low relative density)
(PVP, 0.5% binding concentration, high relative density)
(PVP, 4.0% binding concentration, low relative density)
(PVP, 4.0% binding concentration, high relative density)

e. The model is

$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_1 x_2 + \beta_5 x_1 x_3 + \beta_6 x_2 x_3 + \beta_7 x_1 x_2 x_3$$

where $x_1 = \begin{cases} 1 & \text{if Khaya gum} \\ 0 & \text{if PVP} \end{cases}$ $x_2 = \begin{cases} 1 & \text{if } 0.5\% \\ 0 & \text{if } 4\% \end{cases}$ $x_3 = \begin{cases} 1 & \text{if low density} \\ 0 & \text{if high density} \end{cases}$

f. The results do suggest that there is an interaction between binding agent and relative density since the difference in the means between the high and low density for the gum binding agent $(7.95 - 4.70 = 3.25)$ is much greater than the difference in the means between the high and low density for the PVP binding agent $(4.10 - 3.00 = 1.10)$.



12.44 a. The complete model is

$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_1 x_2 + \beta_6 x_1 x_3 + \beta_7 x_1 x_4 + \beta_8 x_2 x_3$$
$$+ \beta_9 x_2 x_4 + \beta_{10} x_1 x_2 x_3 + \beta_{10} x_1 x_2 x_4$$

where $x_1 = \begin{cases} 1 \text{ if CMC} \\ 0 \text{ if FTF} \end{cases}$ $x_2 = \begin{cases} 1 \text{ if HE} \\ 0 \text{ if LE} \end{cases}$ $x_3 = \begin{cases} 1 \text{ if MM} \\ 0 \text{ if not} \end{cases}$ $x_4 = \begin{cases} 1 \text{ if FF} \\ 0 \text{ if not} \end{cases}$

b. The partial ANOVA table are:

SOURCE	*df*
Group	1
Task	1
Gender Pair	2
Group x Task	1
Group x Gender Pair	2
Task x Gender Pair	2
Group x Task x Gender Pair	2
Error	12
Total	23

c. The difference in means between the levels of any one factor do not appear to differ based on the combinations of the other factors.

d. The differences in mean relational intimacy scores between CMC and face-to-face groups varied with the type of task.

e. The gender pair of the participants did not affect the mean relational intimacy score.

12.45 a. The complete model is

$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_1 x_2 + \beta_6 x_1 x_3 + \beta_7 x_1 x_4 + \beta_8 x_2 x_3$$
$$+ \beta_9 x_2 x_4 + \beta_{10} x_3 x_4 + \beta_{11} x_1 x_2 x_3 + \beta_{12} x_1 x_2 x_4 + \beta_{13} x_1 x_3 x_4 + \beta_{14} x_2 x_3 x_4 + \beta_{15} x_1 x_2 x_3 x_4$$

where $x_1 = \{1$ if foaming agents SABO, 0 if not$\}$
$x_2 = \{1$ if low level of Agent-to-mineral, 0 if not$\}$
$x_3 = \{1$ if low level of collector-to-mineral, 0 if not$\}$
$x_4 = \{1$ if low level of liquid-to-solid, 0 if not$\}$

b. The complete model in part (a) cannot be fit since there will be no degrees of freedom for error if there are no replications; $df\,(\text{Error}) = 0$.

c. $$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_1 x_2 + \beta_6 x_1 x_3 + \beta_7 x_1 x_4 + \beta_8 x_2 x_3 + \beta_9 x_2 x_4 + \beta_{10} x_3 x_4$$

d. Using MINITAB, the results are:

Regression Analysis: PCTCOPPER versus x1, x2, x3, x4, ... x3, x2x4, x34x

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	10	4.11851	0.411851	17.71	0.003
x1	1	0.03802	0.038025	1.64	0.257
x2	1	0.09302	0.093025	4.00	0.102
x3	1	0.11056	0.110556	4.75	0.081
x4	1	0.09000	0.090000	3.87	0.106
x1x2	1	0.03331	0.033306	1.43	0.285
x1x3	1	0.18276	0.182756	7.86	0.038
x1x4	1	0.07981	0.079806	3.43	0.123
x2x3	1	0.00141	0.001406	0.06	0.816
x2x4	1	0.01891	0.018906	0.81	0.409
x3x4	1	0.00181	0.001806	0.08	0.792
Error	5	0.11628	0.023256		
Total	15	4.23479			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.1525	97.25%	91.76%	71.88%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	7.828	0.126	61.91	0.000	
x1	-0.195	0.153	-1.28	0.257	4.00
x2	-0.305	0.153	-2.00	0.102	4.00
x3	-0.333	0.153	-2.18	0.081	4.00
x4	-0.300	0.153	-1.97	0.106	4.00
x1x2	-0.183	0.153	-1.20	0.285	3.00
x1x3	-0.427	0.153	-2.80	0.038	3.00
x1x4	-0.282	0.153	-1.85	0.123	3.00
x2x3	-0.038	0.153	-0.25	0.816	3.00
x2x4	0.137	0.153	0.90	0.409	3.00
x3x4	0.043	0.153	0.28	0.792	3.00

Regression Equation

PCTCOPPER = 7.828 - 0.195 x1 - 0.305 x2 - 0.333 x3 - 0.300 x4 - 0.183 x1x2 - 0.427 x1x3 - 0.282 x1x4 - 0.038 x2x3 + 0.137 x2x4 + 0.043 x3x4

The fitted regression equation is

$$\hat{y} = 7.828 - 0.195x_1 - 0.305x_2 - 0.333x_3 - 0.300x_4 - 0.183x_1x_2 - 0.427x_1x_3 - 0.282x_1x_4 - 0.038x_2x_3 + 0.137x_2x_4 + 0.043x_3x_4$$

e. First, we must fit the reduced model $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4$.

Regression Analysis: PCTCOPPER versus x1, x2, x3, x4

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	4	3.8005	0.95013	24.07	0.000
x1	1	1.6448	1.64481	41.66	0.000
x2	1	0.4796	0.47956	12.15	0.005
x3	1	1.1827	1.18266	29.96	0.000
x4	1	0.4935	0.49351	12.50	0.005
Error	11	0.4343	0.03948		
Total	15	4.2348			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.198693	89.75%	86.02%	78.30%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	8.016	0.111	72.17	0.000	
x1	-0.6412	0.0993	-6.45	0.000	1.00
x2	-0.3462	0.0993	-3.49	0.005	1.00
x3	-0.5438	0.0993	-5.47	0.000	1.00
x4	-0.3513	0.0993	-3.54	0.005	1.00

Regression Equation

PCTCOPPER = 8.016 - 0.6412 x1 - 0.3462 x2 - 0.5438 x3 - 0.3513 x4

To determine if any of the interaction terms are significant, we test:

$H_0: \beta_5 = \beta_6 = \beta_7 = \beta_8 = \beta_9 = \beta_{10} = 0$
$H_a:$ At least one $\beta_i \neq 0$

The test statistic is $F = \dfrac{(SSE_R - SSE_C)/6}{MSE_C} = \dfrac{(0.4343 - 0.11628)/6}{0.023256} = 2.279$

The rejection region requires $\alpha = 0.05$ in the upper tail of the F distribution with $\nu_1 = 6$ and $\nu_2 = 5$ degrees of freedom. From Table 4, Appendix D, $F_{0.05} = 4.95$. The rejection region is $F > 4.95$.

Since the observed value of the test statistic does not fall in the rejection region $(F = 2.279 \not> 4.95)$, H_0 is not rejected. There is insufficient evidence to indicate that any of the interaction terms are significant at $\alpha = 0.05$.

f. Since the interaction terms are not significant, the main effect tests should be run. Using the reduced model, all of the main effects are significant. The p-values associated with all of the main effects are all less than or equal to 0.005.

12.46 a. First, we must fit the reduced model. Using MINITAB, the results are:

Regression Analysis: YIELD versus TIME, ALLOY, MATERIAL
Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	4	1579.4	394.85	20.81	0.000
Error	19	360.5	18.97		
Total	23	1939.9			

Model Summary

S	R-sq	R-sq(adj)
4.35585	81.42%	77.50%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	34.54	1.99	17.37	0.000
TIME				
25	2.21	2.18	1.02	0.322
50	-2.00	2.18	-0.92	0.370
ALLOY				
INCONEL	9.59	1.78	5.39	0.000
MATERIAL				
ROLLED	12.63	1.78	7.10	0.000

Regression Equation

YIELD = 34.54 + 0.0 TIME_0 + 2.21 TIME_25 - 2.00 TIME_50 + 0.0 ALLOY_INCOLOY
+ 9.59 ALLOY_INCONEL + 0.0 MATERIAL_DRAWN
+ 12.63 MATERIAL_ROLLED

To determine if any of the interactions are significant, we test:

$H_0: \beta_5 = \beta_6 = \beta_7 = \beta_8 = \beta_9 = \beta_{11} = 0$
$H_a:$ At least one $\beta_i \neq 0$

The test statistic is $F = \dfrac{(SSE_R - SSE_C)/7}{MSE_C} = \dfrac{(360.5 - 8.145)/7}{0.67875} = 74.16$

The rejection region requires $\alpha = 0.05$ in the upper tail of the F distribution with $\nu_1 = 7$ and $\nu_2 = 12$ degrees of freedom. From Table 4, Appendix D, $F_{0.05}$ =2.91. The rejection region is $F > 2.91$.

Since the observed value of the test statistic falls in the rejection region $(F = 74.16 > 2.91)$, H_0 is rejected. There is sufficient evidence to indicate that at least one of the interaction terms is significant at $\alpha = 0.05$.

b. From the printout, Alloy x Material has an F value of 500.56 with a p-value of < 0.0001. Also, Alloy x Time has an F value of 5.88 with a p-value of 0.0166. Both of these interactions would be significant at $\alpha = 0.05$.

12.47 a. For cold drawn incoloy alloy, $x_2 = 0$ and $x_3 = 0$.

$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_1^2$$

b. For cold drawn inconel alloy, $x_2 = 1$ and $x_3 = 0$.

$$E(y) = (\beta_0 + \beta_3) + (\beta_1 + \beta_6)x_1 + (\beta_2 + \beta_9)x_1^2$$

c. For cold rolled inconel alloy, $x_2 = 1$ and $x_3 = 1$.

$$\begin{aligned}E(y) &= \beta_0 + \beta_1 x_1 + \beta_2 x_1^2 + \beta_3 + \beta_4 + \beta_5 + \beta_6 x_1 + \beta_7 x_1 + \beta_8 x_1 + \beta_9 x_1^2 \\ &\quad + \beta_{10} x_1^2 + \beta_{11} x_1^2 \\ &= (\beta_0 + \beta_3 + \beta_4 + \beta_5) + (\beta_1 + \beta_6 + \beta_7 + \beta_8)x_1 \\ &\quad + (\beta_2 + \beta_9 + \beta_{10} + \beta_{11})x_1^2\end{aligned}$$

d. Using MINITAB, the results are:

Regression Analysis: YIELD versus x1, x1_sq, x2, x3, x1x2, ... , x1_sqx2x3

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	11	1931.73	175.612	258.73	0.000
Error	12	8.14	0.679		
Total	23	1939.88			

Model Summary

S	R-sq	R-sq(adj)
0.823863	99.58%	99.20%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	31.150	0.583	53.47	0.000
x1	0.1530	0.0594	2.58	0.024
x1_sq	-0.00396	0.00114	-3.47	0.005
x2	17.050	0.824	20.70	0.000
x3	19.100	0.824	23.18	0.000
x1x2	0.1510	0.0840	1.80	0.097
x1x3	0.0170	0.0840	0.20	0.843
x2x3	-14.30	1.17	-12.27	0.000
x1x2x3	-0.080	0.119	-0.67	0.514
x1_sqx2	-0.00356	0.00161	-2.21	0.048
x1_sqx3	0.00060	0.00161	0.37	0.717
x1_sqx2x3	0.00120	0.00228	0.53	0.609

Regression Equation

YIELD = 31.150 + 0.1530 x1 - 0.00396 x1_sq + 17.050 x2 + 19.100 x3 + 0.1510 x1x2
+ 0.0170 x1x3 - 14.30 x2x3 - 0.080 x1x2x3 - 0.00356 x1_sqx2
+ 0.00060 x1_sqx3
+ 0.00120 x1_sqx2x3

The prediction equation is:

$$\begin{aligned}\hat{y} &= 31.15 + 0.153x_1 - 0.00396x_1^2 + 17.05x_2 + 19.1x_3 - 14.3x_2x_3 + 0.151x_1x_2 \\ &\quad + 0.017x_1x_3 - 0.08x_1x_2x_3 - 0.00356x_1^2x_2 + 0.0006x_1^2x_3 + 0.0012x_1^2x_2x_3\end{aligned}$$

e. For inconel alloy and cold rolled, $x_2 = 1$ and $x_3 = 1$.

$$\begin{aligned}\hat{y} &= 31.15 + 0.153x_1 - 0.00396x_1^2 + 17.05(1) + 19.1(1) - 14.3(1)(1) + 0.151x_1(1) \\ &\quad + 0.017x_1(1) - 0.08x_1(1)(1) - 0.00356x_1^2(1) + 0.0006x_1^2(1) + 0.0012x_1^2(1)(1) \\ &= 53 + 0.241x_1 - 0.00572x_1^2\end{aligned}$$

For incoloy alloy and cold rolled, $x_2 = 0$ and $x_3 = 1$.

$$\begin{aligned}\hat{y} &= 31.15 + 0.153x_1 - 0.00396x_1^2 + 17.05(0) + 19.1(1) - 14.3(0)(1) + 0.151x_1(0) \\ &\quad + 0.017x_1(1) - 0.08x_1(0)(1) - 0.00356x_1^2(0) + 0.0006x_1^2(1) + 0.0012x_1^2(0)(1) \\ &= 50.25 + 0.17x_1 - 0.00336x_1^2\end{aligned}$$

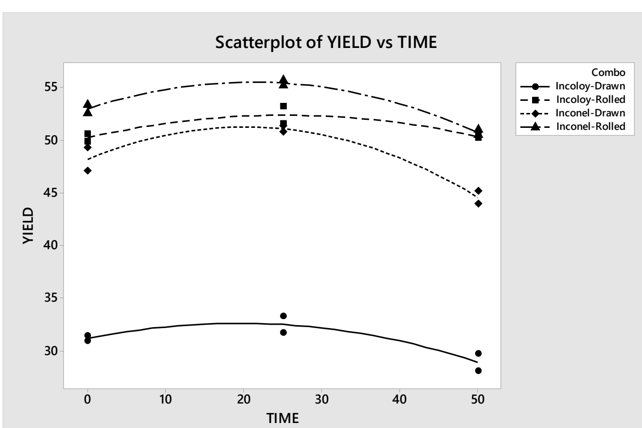
For inconel alloy and cold drawn, $x_2 = 1$ and $x_3 = 0$.

$$\begin{aligned}\hat{y} &= 31.15 + 0.153x_1 - 0.00396x_1^2 + 17.05(1) + 19.1(0) - 14.3(1)(0) + 0.151x_1(1) \\ &\quad + 0.017x_1(0) - 0.08x_1(1)(0) - 0.00356x_1^2(1) + 0.0006x_1^2(0) + 0.0012x_1^2(1)(0) \\ &= 48.2 + 0.304x_1 - 0.00752x_1^2\end{aligned}$$

For incoloy alloy and cold drawn, $x_2 = 0$ and $x_3 = 0$.

$$\begin{aligned}\hat{y} &= 31.15 + 0.153x_1 - 0.00396x_1^2 + 17.05(0) + 19.1(0) - 14.3(0)(0) + 0.151x_1(0) \\ &\quad + 0.017x_1(0) - 0.08x_1(0)(0) - 0.00356x_1^2(0) + 0.0006x_1^2(0) + 0.0012x_1^2(0)(0) \\ &= 31.15 + 0.153x_1 - 0.00396x_1^2\end{aligned}$$

f. The plot is:



12.48 Using MINITAB, the results are:

Regression Analysis: YIELD versus x1, x1_sq
Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	2	71.04	35.52	0.40	0.676
Error	21	1868.84	88.99		
Total	23	1939.88			

Model Summary

S	R-sq	R-sq(adj)
9.43357	3.66%	0.00%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	45.65	3.34	13.69	0.000
x1	0.217	0.340	0.64	0.530
x1_sq	-0.00514	0.00654	-0.79	0.440

Regression Equation

YIELD = 45.65 + 0.217 x1 - 0.00514 x1_sq

To determine if differences exist among the second-order models relating $E(y)$ to x_1 for the four categories of alloy type and material condition, we test:

$$H_0: \beta_3 = \beta_4 = \cdots = \beta_{11} = 0$$
$$H_a: \text{ At least one } \beta_i \neq 0$$

The test statistic is $F = \dfrac{(SSE_R - SSE_C)/9}{MSE_C} = \dfrac{(1868.84 - 8.14)/9}{0.679} = 304.48$

The rejection region requires $\alpha = 0.05$ in the upper tail of the F distribution with $\nu_1 = 9$ and $\nu_2 = 12$ degrees of freedom. From Table 4, Appendix D, $F_{0.05}$ =2.80. The rejection region is $F > 2.80$.

Since the observed value of the test statistic falls in the rejection region $(F = 304.48 > 2.80)$, H_0 is rejected. There is sufficient evidence to indicate differences exist among the second-order models relating $E(y)$ to x_1 for the four categories of alloy type and material condition at $\alpha = 0.05$.

12.49 The first test is to determine if the three-way interaction is significant. To determine if the three-way interaction term is significant, we test:

H_0 : The three-way interaction does not exist
H_a : The three-way interaction does exist

The test statistic is $F = 7.83$ and the p-value is $p = 0.013$. Since the p-value is very small, H_0 is rejected. There is sufficient evidence to indicate the three-way interaction is significant. Thus, no additional tests need to be performed. Because the three-way interaction is significant, the factors do not impact load-to-tensile-strength ratio independently. The effect of one factor on the load-to-tensile-strength ratio depends on the combination of the other two factors.

12.50 a. For a complete $3\times3\times3\times3$ factorial experiment, there would be $3\times3\times3\times3=81$ different treatments. The treatments follow, with the order of factors (IC, CC, TEMP, TIME):

(1, 5, 35, 6) (1, 5, 35, 8) (1, 5, 35, 10) (1, 5, 50, 6) (1, 5, 50, 8) (1, 5, 50, 10)
(1, 5, 65, 6) (1, 5, 65, 8) (1, 5, 65, 10) (1, 10, 35, 6) (1, 10, 35, 8) (1, 10, 35, 10)
(1, 10, 50, 6) (1, 10, 50, 8) (1, 10, 50, 10) (1, 10, 65, 6) (1, 10, 65, 8) (1, 10, 65, 10)
(1, 15, 35, 6) (1, 15, 35, 8) (1, 15, 35, 10) (1, 15, 50, 6) (1, 15, 50, 8) (1, 15, 50, 10)
(1, 15, 65, 6) (1, 15, 65, 8) (1, 15, 65, 10)
(2, 5, 35, 6) (2, 5, 35, 8) (2, 5, 35, 10) (2, 5, 50, 6) (2, 5, 50, 8) (2, 5, 50, 10)
(2, 5, 65, 6) (2, 5, 65, 8) (2, 5, 65, 10) (2, 10, 35, 6) (2, 10, 35, 8) (2, 10, 35, 10)
(2, 10, 50, 6) (2, 10, 50, 8) (2, 10, 50, 10) (2, 10, 65, 6) (2, 10, 65, 8) (2, 10, 65, 10)
(2, 15, 35, 6) (2, 15, 35, 8) (2, 15, 35, 10) (2, 15, 50, 6) (2, 15, 50, 8) (2, 15, 50, 10)
(2, 15, 65, 6) (2, 15, 65, 8) (2, 15, 65, 10)
(3, 5, 35, 6) (3, 5, 35, 8) (3, 5, 35, 10) (3, 5, 50, 6) (3, 5, 50, 8) (3, 5, 50, 10)
(3, 5, 65, 6) (3, 5, 65, 8) (3, 5, 65, 10) (3, 10, 35, 6) (3, 10, 35, 8) (3, 10, 35, 10)
(3, 10, 50, 6) (3, 10, 50, 8) (3, 10, 50, 10) (3, 10, 65, 6) (3, 10, 65, 8) (3, 10, 65, 10)
(3, 15, 35, 6) (3, 15, 35, 8) (3, 15, 35, 10) (3, 15, 50, 6) (3, 15, 50, 8) (3, 15, 50, 10)
(3, 15, 65, 6) (3, 15, 65, 8) (3, 15, 65, 10)

b. First, we define the dummy variables:

$$x_1=\begin{cases}1 & \text{if IC is 1}\\ 0 & \text{if not}\end{cases}\quad x_2=\begin{cases}1 & \text{if IC is 2}\\ 0 & \text{if not}\end{cases}\quad x_3=\begin{cases}1 & \text{if CC is 5}\\ 0 & \text{if not}\end{cases}\quad x_4=\begin{cases}1 & \text{if CC is 10}\\ 0 & \text{if not}\end{cases}$$

$$x_5=\begin{cases}1 & \text{if TEMP is 35}\\ 0 & \text{if not}\end{cases}\quad x_6=\begin{cases}1 & \text{if TEMP is 50}\\ 0 & \text{if not}\end{cases}\quad x_7=\begin{cases}1 & \text{if TIME is 6}\\ 0 & \text{if not}\end{cases}$$

$$x_8=\begin{cases}1 & \text{if TIME is 8}\\ 0 & \text{if not}\end{cases}$$

The complete model is

$$
\begin{aligned}
E(y) = {} & \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_6 + \beta_7 x_7 + \beta_8 x_8 \\
& + \beta_9 x_1 x_3 + \beta_{10} x_1 x_4 + \beta_{11} x_1 x_5 + \beta_{12} x_1 x_6 + \beta_{13} x_1 x_7 + \beta_{14} x_1 x_8 \\
& + \beta_{15} x_2 x_3 + \beta_{16} x_2 x_4 + \beta_{17} x_2 x_5 + \beta_{18} x_2 x_6 + \beta_{19} x_2 x_7 + \beta_{20} x_2 x_8 \\
& + \beta_{21} x_3 x_5 + \beta_{22} x_3 x_6 + \beta_{23} x_3 x_7 + \beta_{24} x_3 x_8 + \beta_{25} x_4 x_5 + \beta_{26} x_4 x_6 + \beta_{27} x_4 x_7 \\
& + \beta_{28} x_4 x_8 + \beta_{29} x_5 x_7 + \beta_{30} x_5 x_8 + \beta_{31} x_6 x_7 + \beta_{32} x_6 x_8 + \beta_{33} x_1 x_3 x_5 + \beta_{34} x_1 x_3 x_6 \\
& + \beta_{35} x_1 x_3 x_7 + \beta_{36} x_1 x_3 x_8 + \beta_{37} x_1 x_4 x_5 + \beta_{38} x_1 x_4 x_6 + \beta_{39} x_1 x_4 x_7 + \beta_{40} x_1 x_4 x_8 \\
& + \beta_{41} x_1 x_5 x_7 + \beta_{42} x_1 x_5 x_8 + \beta_{43} x_1 x_6 x_7 + \beta_{44} x_1 x_6 x_8 + \beta_{45} x_2 x_5 x_7 + \beta_{46} x_2 x_5 x_8 \\
& + \beta_{47} x_2 x_6 x_7 + \beta_{48} x_2 x_6 x_8 + \beta_{49} x_2 x_3 x_5 + \beta_{50} x_2 x_3 x_6 + \beta_{51} x_2 x_3 x_7 + \beta_{52} x_2 x_3 x_8 \\
& + \beta_{53} x_2 x_4 x_5 + \beta_{54} x_2 x_4 x_6 + \beta_{55} x_2 x_4 x_7 + \beta_{56} x_2 x_4 x_8 + \beta_{57} x_3 x_5 x_7 + \beta_{58} x_3 x_5 x_8 \\
& + \beta_{59} x_3 x_6 x_7 + \beta_{60} x_3 x_6 x_8 + \beta_{61} x_4 x_5 x_7 + \beta_{62} x_4 x_5 x_8 + \beta_{63} x_4 x_6 x_7 + \beta_{64} x_4 x_6 x_8 \\
& + \beta_{65} x_1 x_3 x_5 x_7 + \beta_{66} x_1 x_3 x_5 x_8 + \beta_{67} x_1 x_3 x_6 x_7 + \beta_{68} x_1 x_3 x_6 x_8 \\
& + \beta_{69} x_1 x_4 x_5 x_7 + \beta_{70} x_1 x_4 x_5 x_8 + \beta_{71} x_1 x_4 x_6 x_7 + \beta_{72} x_1 x_4 x_6 x_8 \\
& + \beta_{73} x_2 x_3 x_5 x_7 + \beta_{74} x_2 x_3 x_5 x_8 + \beta_{75} x_2 x_3 x_6 x_7 + \beta_{76} x_2 x_3 x_6 x_8 \\
& + \beta_{77} x_2 x_4 x_5 x_7 + \beta_{78} x_2 x_4 x_5 x_8 + \beta_{79} x_2 x_4 x_6 x_7 + \beta_{80} x_2 x_4 x_6 x_8
\end{aligned}
$$

c. The partial ANOVA table is:

SOURCE	*df*
A	2
B	2
C	2
D	2
AB	4
AC	4
AD	4
BC	4
BD	4
CD	4
ABC	8
ABD	8
ACD	8
BCD	8
ABCD	16
Error	81
Total	161

d. No, it would not be possible to check for all factor interactions. There would not be enough degrees of freedom to complete all the tests.

e. We must assume that the factors are continuous. Using MINITAB, the main effects results are:

Regression Analysis: Efficiency versus IC, CC, Temp, Time

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	4	2208.2	552.1	4.38	0.091
Error	4	503.6	125.9		
Total	8	2711.8			

Model Summary

S	R-sq	R-sq(adj)
11.2206	81.43%	62.86%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	97.3	27.4	3.55	0.024
IC	-5.72	4.58	-1.25	0.280
CC	-3.357	0.916	-3.66	0.022
Temp	0.433	0.305	1.42	0.229
Time	-1.68	2.29	-0.74	0.503

Regression Equation

Efficiency = 97.3 - 5.72 IC - 3.357 CC + 0.433 Temp - 1.68 Time

From the results above, the only factor that appears to be significant is Cardanol Concentration or CC. The test statistic is $F = -3.66$ with a p-value of $p = 0.022$. Cardanol Concentration is the only factor that has a p-value less than $\alpha = 0.05$.

12.51 Based on the summary of results, there is no significant difference in the mean attitude toward tanning between the tanned model condition and the no model condition. However, both these means are significantly higher than the no tan model condition mean.

12.52 Based on the summary of results, there is no difference in the mean competency level between two days after and two months after. However, both these means are significantly higher than before training mean.

12.53 a. The treatments in this experiment are the 7 different kinds of cheese.

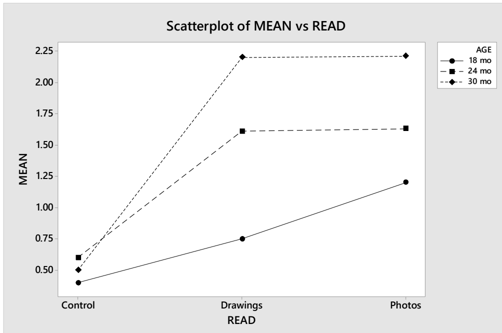
b. The dependent variable is the color change index.

c. The p-value was $p < 0.05$. Since the p-value is less than $\alpha = 0.05$, H_0 is rejected. There is sufficient evidence to indicate a difference in the mean color change index among the 7 cheeses.

d. There will be a combination of 7 things taken 2 at a time or $\binom{7}{2} = 21$ different pairwise comparisons.

e. The mean color change index for mozzarella cheese is significantly higher than the means of all the other cheeses. The mean color change index for provolone cheese is significantly higher than the means of all the other cheeses except mozzarella. There is no difference in the mean color change index between cheddar cheese and gruyére cheese, but both means are significantly greater than the means for edam cheese, emmental cheese, and Colby cheese. There is no significant difference in the mean color change index for edam cheese and emmental cheese, but both means are significantly greater than the mean for Colby cheese.

12.54 A graph of the means is:



For AGE = 18 months:
The mean reenactment score for photos is significantly higher than the mean reenactment score for control. No other significant differences exist.

For AGE = 24 months:
There is no significant difference in the mean reenactment score between photos and drawings. However, both of these means are significantly greater than the mean reenactment score for control.

For AGE = 30 months:
There is no significant difference in the mean reenactment score between photos and drawings. However, both of these means are significantly greater than the mean reenactment score for control.

12.55 The test statistic is $F = 35.5$ and the p-value is $p < 0.001$. Since the p-value is so small, H_0 is rejected. There is sufficient evidence to indicate there are differences in mean NPPS scores among the 5 journals. From the display, there is no significant difference in the mean NPPS scores among Lancet, JAMA and NEJM. However, these three means are all significantly greater than the means for BMJ and AIM. There is no significant difference in the mean NPPS scores between BMJ and AIM.

12.56 The mean FSS value for high school dropouts is significantly higher than the mean FSS values for the other three education levels. There are no significant differences in the mean FSS values among college graduates, some college, and high school graduates.

12.57 No, Tukey's multiple comparison method should not be applied. If no differences were found, there is no need to find where the differences exist.

12.58 a. The 95% confidence interval for $\left(\mu_{\text{CAGE}} - \mu_{\text{BARN}}\right)$ is $(-49.38, -12.96)$. We are 95% confident that the mean whipping capacity for cage eggs is between 12.96% and 49.38% less than the mean whipping capacity for barn eggs.

b. The 95% confidence interval for $(\mu_{\text{CAGE}} - \mu_{\text{FREE}})$ is $(-53.54, -17.12)$. We are 95% confident that the mean whipping capacity for cage eggs is between 17.12% and 53.54% less than the mean whipping capacity for free eggs.

c. The 95% confidence interval for $(\mu_{\text{CAGE}} - \mu_{\text{ORGANIC}})$ is $(-65.21, -28.79)$. We are 95% confident that the mean whipping capacity for cage eggs is between 28.79% and 65.21% less than the mean whipping capacity for organic eggs.

d. The 95% confidence interval for $(\mu_{\text{BARN}} - \mu_{\text{FREE}})$ is $(-22.38, 14.04)$. Since 0 is in the interval, we are 95% confident that there is no difference in the mean whipping capacity between barn and free eggs.

e. The 95% confidence interval for $(\mu_{\text{BARN}} - \mu_{\text{ORGANIC}})$ is $(-34.04, 2.38)$. Since 0 is in the interval, we are 95% confident that there is no difference in the mean whipping capacity between barn and organic eggs.

f. The 95% confidence interval for $(\mu_{\text{FREE}} - \mu_{\text{ORGANIC}})$ is $(-29.88, 6.54)$. Since 0 is in the interval, we are 95% confident that there is no difference in the mean whipping capacity between free and organic eggs.

g. Based on the confidence intervals, the rankings are:

Rank	1	2	3	4
	Cage	<u>Barn</u>	<u>Free</u>	<u>Organic</u>

We are 95% confident that the mean whipping capacity for cage eggs is significantly lower than the mean whipping capacity for the 3 other housing systems.

12.59 a. The pairs of means that are significantly different are:

WHAT-B and any of WHO-A, WHO-B, and HOW-A;

WHO-C or HOW-C and either of WHO-B or WHO-A.

$\mu_{\text{What-B}} > (\mu_{\text{How-A}}, \mu_{\text{Who-B}}, \mu_{\text{Who-A}})$; $(\mu_{\text{Who-C}}, \mu_{\text{How-C}}) > (\mu_{\text{Who-B}}, \mu_{\text{Who-A}})$

These seven pairs are significantly different since they do not have the same letter next to the mean.

b. The experiment error rate is $\alpha = 0.05$. This means that the probability of declaring at least 2 means different when they are not different is 0.05.

12.60 Using MINITAB, the results of Tukey's multiple comparison procedure are:

Tukey Pairwise Comparisons
Grouping Information Using the Tukey Method and 95% Confidence

GROUP	N	Mean	Grouping	
AR	11	0.4400	A	
P	11	0.4000	A	
AC	11	0.2655	A	
A	11	0.0636		B

Means that do not share a letter are significantly different.

From this analysis, the mean word completion task score for the alcohol only group is significantly less than the means for the other three groups. There is no significant difference in the mean completion task scores among the groups alcohol plus caffeine, placebo, and alcohol plus reward.

12.61 a. We would expect the Bonferroni multiple comparison procedure to yield fewer significant differences in means. If the sample sizes are the same, then the critical value for the Tukey's multiple comparison procedure will be less than that for the Bonferroni method.

b. We would expect the Scheffe's multiple comparison procedure to yield fewer significant differences in means. If the sample sizes are the same, then the critical value for the Tukey's multiple comparison procedure will be less than that for the Scheffe method.

12.62 The mean response for cut, half-sized paper is significantly higher than the means for the other 3 combinations. There are no significant differences in the mean responses among the other 3 treatment combinations.

12.63 a. To determine if there are differences in the mean dental fear scores among the three groups, we test:

$H_0: \mu_1 = \mu_2 = \mu_3$
$H_a:$ At least two treatment means differ

The test statistic is $F = 4.43$ and the p-value is $p < 0.05$. Since the p-value is less than 0.05, we reject H_0. There is sufficient evidence to indicate there are differences in the mean dental fear scores among the three groups at $\alpha = 0.05$.

b. First, we arrange the means in order from the largest to the smallest. Then we draw a line between the means that are not significantly different. The results are:

Control	Slide	Questionnaire
41.8	43.1	53.8

Thus, $\mu_Q > (\mu_S, \mu_C)$.

12.64 Using MINITAB, the results of Tukey's multiple comparison procedure are:

Tukey Pairwise Comparisons

Grouping Information Using the Tukey Method and 95% Confidence

Treatment	N	Mean	Grouping		
H	35	10.714	A		
DM	33	8.333		B	
C	37	6.514			C

Means that do not share a letter are significantly different.

From the above output, there is evidence that all the population means are significantly different. The mean response for the honey group is greater than the means for the other two groups. The mean response for the DM group is significantly greater than the control group. Tukey's multiple comparison procedure will find more differences than either Scheffe's procedure or Bonferonni's procedure. Although the sample sizes are not the same, they are very similar. Thus, we used Tukey's multiple comparison procedure.

12.65 a. The p-value is $p = 0.001$. Since the p-value is so small, there is evidence to indicate there is a difference in the mean number of alternatives among the 3 emotional states.

b. In order to use Tukey's multiple comparisons procedure the sample sizes should be equal and in this case this does not apply. The design is not balanced.

c. An experiment-wise error rate of 0.05 represents the probability of declaring at least one pair of means different when they are not.

d. The mean response for Guilt is significantly larger than the means for the other 2 emotional states; $\mu_{\text{Guilt}} > \left(\mu_{\text{Neutral}}, \mu_{\text{Angry}}\right)$.

12.66 a. There are a combination of four things taken two at a time or $\binom{4}{2} = 6$ pairwise comparisons possible.

b. An experiment-wise error rate of 0.05 represents the probability of declaring at least one pair of means different when they are not.

c. Each pairwise comparison is made at the $\alpha / 6 = 0.05 / 6 = 0.0083$ level.

d. The mean soluble magnesium level for Sourdough is significantly higher than the means for the rest of the treatments. There is no difference in the mean soluble magnesium levels between yeast and control. These two means are significantly smaller than the means of the other two treatments.

12.67 Using MINITAB, the results of Tukey's multiple comparison procedure are:

Tukey Pairwise Comparisons
Grouping Information Using the Tukey Method and 90% Confidence

BOREHOLE	N	Mean	Grouping		
UMRB-2	6	4.017	A		
UMRB-3	7	3.761	A		
UMRB-1	7	3.497	A	B	
SD	3	2.780		B	C
SWRA	3	2.643			C

Means that do not share a letter are significantly different.

Even though the sample sizes are not exactly the same, the modified Tukey's procedure is used. It tends to be more conservative than the actual Tukey's comparison procedure.

There is no difference in the mean A1/Be ratios between treatments UMRB-2 and UMRB-3. These two means are all significantly greater than the mean A1/Be ratios for treatments SD and SWRA. The mean A1/Be ratio for UMRB-1 is greater than the mean A1/Be ratio for treatment C.

12.68 a. $$MS(P) = \frac{SS(P)}{df} \Rightarrow SS(P) = (df)MS(P) = 1(1.55) = 1.55$$

$$MS(D) = \frac{SS(D)}{df} \Rightarrow SS(D) = (df)MS(D) = 1(22.26) = 22.26$$

$$MS(PD) = \frac{SS(PD)}{df} \Rightarrow SS(PD) = (df)MS(PD) = 1(0.61) = 0.61$$

$$MSE = \frac{SSE}{df} \Rightarrow SSE = (df)MSE = 80(1.43) = 114.4$$

$$SS(Total) = SS(P) + SS(D) + SS(PD) + SSE = 1.55 + 22.26 + 0.61 + 114.4 = 138.82$$

$$F_P = \frac{MS(P)}{MSE} = \frac{1.55}{1.43} = 1.08 \qquad F_D = \frac{MS(D)}{MSE} = \frac{22.26}{1.43} = 15.57$$

$$F_{PD} = \frac{MS(PD)}{MSE} = \frac{0.61}{1.43} = 0.43$$

The ANOVA table is

SOURCE	*df*	SS	MS	*F*
P	1	1.55	1.55	1.08
D	1	22.26	22.26	15.57
PD	1	0.61	0.61	0.43
Error	80	114.40	1.43	
Total	83	138.82		

b. We first test to see if interaction between P and D is significant:

H_0: No PD interaction exists
H_a: The PD interaction exists

The test statistic is $F = \frac{MS(PD)}{MSE} = \frac{0.61}{1.43} = 0.43$.

The rejection region requires $\alpha = 0.05$ in the upper tail of the F distribution with $\nu_1 = 1$ and $\nu_2 = 80$. From Table 4, Appendix D, $F_{0.05} \approx 3.97$. The rejection region is $F > 3.97$.

Since the observed value of the test statistic does not fall in the rejection region $(F = 0.43 \ngtr 3.97)$, H_0 is not rejected. There is insufficient evidence to indicate that prior information and degree of choice interact to affect satisfaction ratings at $\alpha = 0.05$.

Since the interaction is not significant, we run the tests on the main effects.

To determine if there is a difference in the mean satisfaction ratings between the two levels of prior information, we test:

$H_0: \mu_{P_1} = \mu_{P_2}$
$H_a: \mu_{P_1} \neq \mu_{P_2}$

The test statistic is $F = \frac{MS(P)}{MSE} = \frac{1.55}{1.43} = 1.08$.

The rejection region requires $\alpha = 0.05$ in the upper tail of the F distribution with $\nu_1 = 1$ and $\nu_2 = 80$. From Table 4, Appendix D, $F_{0.05} \approx 3.97$. The rejection region is $F > 3.97$.

Since the observed value of the test statistic does not fall in the rejection region $(F = 1.08 \ngtr 3.97)$, H_0 is not rejected. There is insufficient evidence to indicate that there is a difference in the mean satisfaction ratings between the two levels of prior information at $\alpha = 0.05$.

To determine if there is a difference in the mean satisfaction ratings between the two levels of degree of choice, we test:

$H_0: \mu_{D_1} = \mu_{D_2}$
$H_a: \mu_{D_1} \neq \mu_{D_2}$

The test statistic is $F = \dfrac{MS(D)}{MSE} = \dfrac{22.26}{1.43} = 15.57.$

The rejection region requires $\alpha = 0.05$ in the upper tail of the F distribution with $\nu_1 = 1$ and $\nu_2 = 80$. From Table 4, Appendix D, $F_{0.05} \approx 3.97$. The rejection region is $F > 3.97$.

Since the observed value of the test statistic falls in the rejection region $(F = 15.57 > 3.97)$, H_0 is rejected. There is sufficient evidence to indicate that there is a difference in the mean satisfaction ratings between the two levels of degree of choice at $\alpha = 0.05$.

c. We have $p = 4$ information – degree of choice combinations. Also, $\nu = df(error) = 80$, $n_t = 21$, and $s = \sqrt{MSE} = \sqrt{1.43} = 1.196$.

For $\alpha = 0.05$, $q_{0.05}(4,80) \approx 3.72$ and $\omega = q_{0.05}(4,80)\dfrac{s}{\sqrt{n_t}} = 3.72\dfrac{(1.196)}{\sqrt{21}} = 0.971$

The four sample means ranked in order are:

(Low/Trad)	(Low/Real)	(High/Trad)	(High/Real)
4.82	5.33	6.06	6.20

The pairs of means that are significantly different are: (High/Real, Low/Trad) and (High/Trad, Low/Trad).

d. We have $p = 4$ information – degree of choice combinations. Also, $\nu = df(error) = 80$ and $n_t = 21$.

For $\alpha = 0.05$, $F_{0.05}(3,80) \approx 2.73$

$$S_{ij} = \sqrt{(p-1)F_{0.05}(3,80)MSE\left(\frac{1}{n_i} + \frac{1}{n_j}\right)} = \sqrt{3(2.73)1.43\left(\frac{1}{21} + \frac{1}{21}\right)} = 1.06$$

The four sample means ranked in order are:

(Low/Trad)	(Low/Real)	(High/Trad)	(High/Real)
4.82	5.33	6.06	6.20

The pairs of means that are significantly different are: (High/Real, Low/Trad) and (High/Trad, Low/Trad).

e. For $p = 4$ means, the number of pairwise comparisons to be made is

$$g = \frac{p(p-1)}{2} = \frac{4(3)}{2} = 6.$$ We need to find $t_{\alpha/[2g]} = t_{0.05/[2(6)]} = t_{0.0042} = 2.703$ for the t distribution based on $\nu = df(error) = 80$.

$$B_{ij} \approx t_{0.0042} s \sqrt{\frac{1}{n_i} + \frac{1}{n_j}} = 2.703(1.196)\sqrt{\frac{1}{21} + \frac{1}{21}} = 0.998$$

The four sample means ranked in order are:

(Low/Trad)	(Low/Real)	(High/Trad)	(High/Real)
4.82	5.33	6.06	6.20

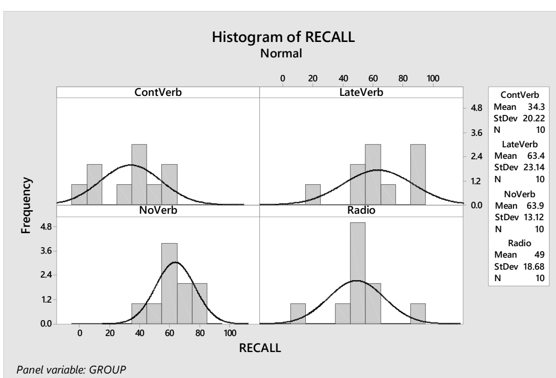
The pairs of means that are significantly different are: (High/Real, Low/Trad) and (High/Trad, Low/Trad).

f. These results from parts c, d, and e are all the same.

12.69 The assumptions for the completely randomized design are:

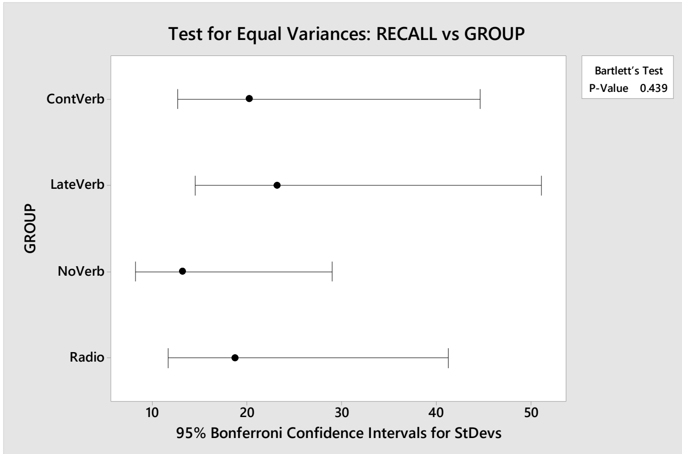
1. All four probability distributions of the recall percentages are normally distributed.
2. The population variances of the recall percentages are equal.

First, we'll check the assumption of normality. For this design, we have enough observations in each of the four treatments to check for normality in each. The histograms for each group are displayed below:



The graphs do not look particularly normal. However, the ANOVA procedure is pretty robust with respect to normality. That is, the analyses perform fairly well even when the normality assumption fails.

We next perform Bartlett's test for the equality of variances. The MINITAB printout gives this test as one of the tests conducted in the analysis.



To determine if the variances differ, we test:

$H_0: \sigma_1^2 = \sigma_2^2 = \sigma_3^2 = \sigma_4^2$
H_a: At least two variances differ

The p-value is $p = 0.439$. Since the p-value is not small, H_0 is not rejected. There is insufficient evidence to indicate the variances are different at $\alpha = 0.05$.

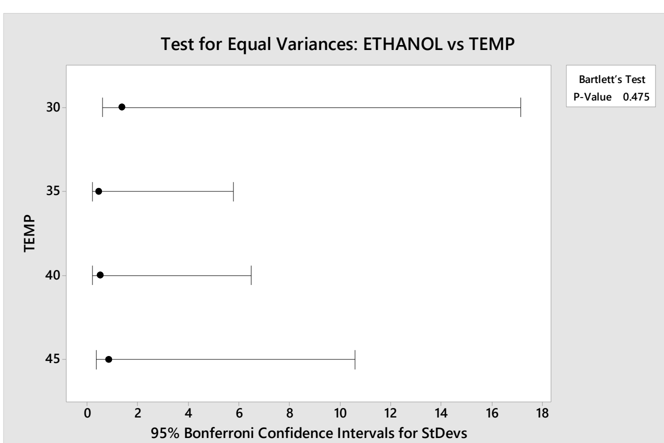
The data are not real different from normal and there is no evidence to indicate the variances differ. Thus, we should use the ANOVA procedures to analyze the data.

12.70 The assumptions for the completely randomized design are:

1. All four probability distributions of the ethanol concentrations are normally distributed.
2. The population variances of the ethanol concentrations are equal.

First, we'll check the assumption of normality. For this design, we only have 3 observations per treatment. There are not enough data points to check for normality.

We next perform Bartlett's test for the equality of variances. The MINITAB printout gives this test as one of the tests conducted in the analysis.



To determine if the variances differ, we test:

$H_0: \sigma_1^2 = \sigma_2^2 = \sigma_3^2 = \sigma_4^2$
H_a: At least two variances differ

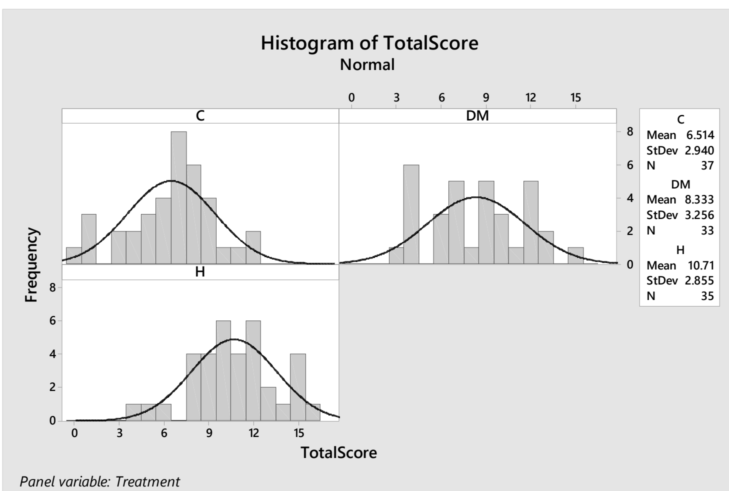
The p-value is $p = 0.475$. Since the p-value is not small, H_0 is not rejected. There is insufficient evidence to indicate the variances are different at $\alpha = 0.05$.

We cannot test for normality, but there is no evidence to indicate the variances differ. Thus, we should use the ANOVA procedures to analyze the data.

12.71 The assumptions for the completely randomized design are:

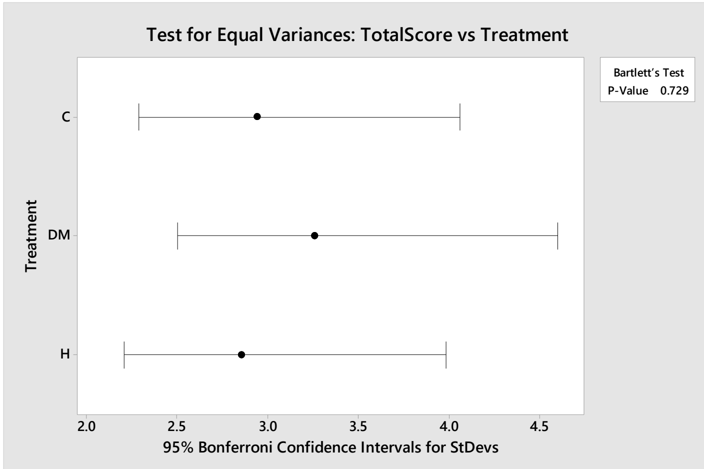
1. All three probability distributions of parents' ratings on their children's cough ratings are normally distributed.
2. The population variances of the parents' ratings on their children's cough ratings are equal.

First, we'll check the assumption of normality. For this design, we have enough observations in each of the three treatments to check for normality in each. The histograms for each group are displayed below:



The graphs look fairly normal. However, the ANOVA procedure is pretty robust with respect to normality. That is, the analyses perform fairly well even when the normality assumption fails.

We next perform Bartlett's test for the equality of variances. The MINITAB printout gives this test as one of the tests conducted in the analysis.



To determine if the variances differ, we test:

$H_0: \sigma_1^2 = \sigma_2^2 = \sigma_3^2$
H_a: At least two variances differ

The p-value is $p = 0.729$. Since the p-value is not small, H_0 is not rejected. There is insufficient evidence to indicate the variances are different at $\alpha = 0.05$.

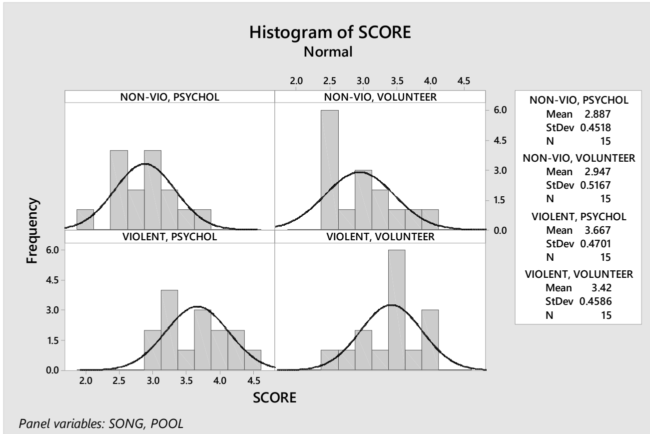
The data are fairly normal and there is no evidence to indicate the variances differ. Thus, we should use the ANOVA procedures to analyze the data.

12.72 The assumptions for the factorial design are:

1. The four probability distributions of aggressive cognition score corresponding to the four violent/non-violent songs and the pool of student combinations are normally distributed.

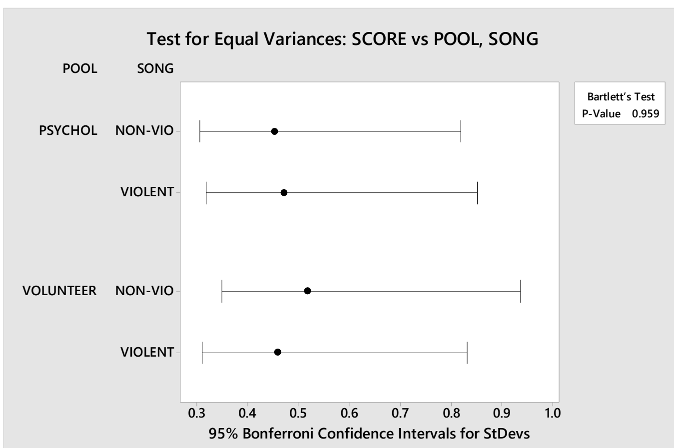
2. The population variances of the aggressive cognition score distributions are equal for the four violent/non-violent songs and the pool of student combinations.

First, we'll check the assumption of normality. For this design, we have enough observations in each of the four treatment combinations to check for normality in each. The histograms for each group are displayed below:



The graphs indicate a distribution that is not too normal. Remember that the ANOVA procedures are fairly robust with respect to normality. That is, the analysis is valid even if the data are not exactly normally distributed.

We next perform Bartlett's test for the equality of variances. The MINITAB printout gives this test as one of the tests conducted in the analysis.



To determine if the variances differ, we test:

$$H_0: \sigma_1^2 = \sigma_2^2 = \sigma_3^2 = \sigma_4^2$$
$$H_a: \text{ At least two variances differ}$$

The p-value is $p = 0.959$. Since the p-value is not small, H_0 is not rejected. There is insufficient evidence to indicate the variances are different at $\alpha = 0.05$.

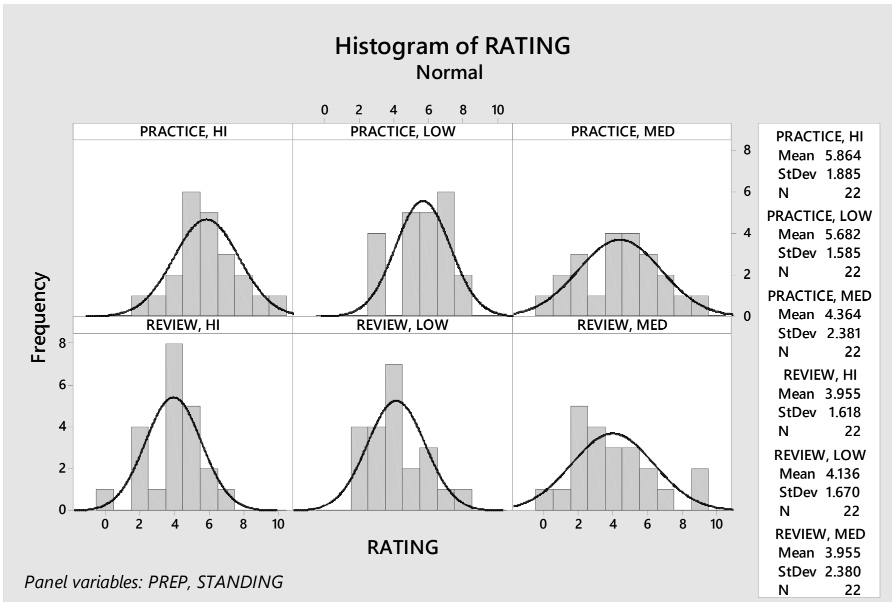
The data are fairly normal and there is no evidence to indicate the variances differ. Thus, we should use the ANOVA procedures to analyze the data.

12.73 The assumptions for the factorial design are:

1. The six probability distributions of exam preparation scores corresponding to the six class standing and type of preparation combinations are normally distributed.

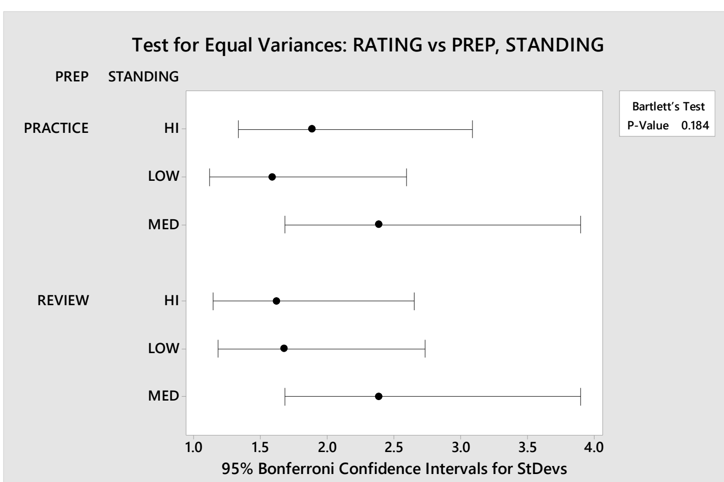
2. The population variances of the exam preparation scores distributions are equal for the six class standing and type of preparation combinations.

First, we'll check the assumption of normality. For this design, we have enough observations in each of the six treatment combinations to check for normality in each. The histograms for each group are displayed below:



The graphs indicate distributions that are somewhat normal. Remember that the ANOVA procedures are fairly robust with respect to normality. That is, the analysis is valid even if the data are not exactly normally distributed.

We next perform Bartlett's test for the equality of variances. The MINITAB printout gives this test as one of the tests conducted in the analysis.



To determine if the variances differ, we test:

$H_0: \sigma_1^2 = \sigma_2^2 = \sigma_3^2 = \sigma_4^2 = \sigma_5^2 = \sigma_6^2$
H_a: At least two variances differ

The p-value is $p = 0.184$. Since the p-value is not small, H_0 is not rejected. There is insufficient evidence to indicate the variances are different at $\alpha = 0.05$.

The data are fairly normal and there is no evidence to indicate the variances differ. Thus, we should use the ANOVA procedures to analyze the data.

12.74 a. The experimental design used is a completely randomized design.

b. There are 4 treatments in this experiment. The four treatments are the 4 "colonies" to which the robots were assigned: 3, 6, 9, or 12 robots per colony.

c. To determine if the mean energy expended (per robot) of the four different colony sizes differed, we test:

$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$
H_a: At least two treatment means differ

d. The test statistic is $F = 7.70$ and the p-value is $p < 0.001$. Since the p-value is less than $\alpha\ (p < 0.001 < 0.05)$, H_0 is rejected. There is sufficient evidence to indicate that mean energy expended (per robot) of the four different colony sizes differed at $\alpha = 0.05$.

e. There are $c = \frac{p(p-1)}{2} = \frac{4(4-1)}{2} = 6$ pairwise comparisons. The mean energy expended for group size 12 is significantly less than the mean energy expended for any of the other size colonies. No other significant differences exist. $\mu_{12} < (\mu_3, \mu_6, \mu_9)$

12.75 a. To determine if the average heights of male Australian school children differ among the three age groups, we test:

$H_0: \mu_1 = \mu_2 = \mu_3$
H_a: At least two treatment means differ

b. The model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2$.

where $x_1 = \begin{cases} 1 & \text{if youngest third} \\ 0 & \text{if not} \end{cases}$ $\quad x_2 = \begin{cases} 1 & \text{if middle third} \\ 0 & \text{if not} \end{cases}$

c. The test statistic is $F = 4.57$ and the p-value is $p = 0.01$. Since the p-value is less than $\alpha\ (p = 0.01 < 0.05)$, H_0 is rejected. There is sufficient evidence to indicate the average heights of male Australian school children differ among the three age groups at $\alpha = 0.05$.

d. To determine if the average heights of female Australian school children differ among the three age groups, we test:

$H_0: \mu_1 = \mu_2 = \mu_3$
H_a: At least two treatment means differ

The test statistic is $F = 0.85$ and the p-value is $p = 0.43$. Since the p-value is not less than $\alpha\left(p = 0.43 \nless 0.05\right)$, H_0 is not rejected. There is insufficient evidence to indicate the average heights of female Australian school children differ among the three age groups at $\alpha = 0.05$.

e. For the boys, differences were found in the mean standardized heights among the three tertiles. For the girls, no differences were found in the mean standardized heights among the three tertiles.

f. There is a no significant difference between the standardized height means for the middle and youngest boys. However, the standardized height mean for the oldest boys is significantly lower than the means for the two other groups.

g. No significant differences were found among the means of the three groups of girls. Therefore, we do not need to find where the differences are.

12.76 a. If the samples represent random and independent selections from the populations associated with the four hair colors, then a completely randomized design has been used.

b. Using MINITAB, the results are:

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
COLOR	3	1361	453.58	6.79	0.004
Error	15	1002	66.79		
Total	18	2363			

To determine if there are differences in the mean pain thresholds among the people with the four types of hair color, we test:

$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$
H_a: At least two treatment means differ

The test statistic is $F = 6.79$ and the p-value is $p = 0.004$. Since the p-value is less than $\alpha\left(p = 0.004 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate there are differences in the mean pain thresholds among the people with the four types of hair color at $\alpha = 0.05$.

c. The observed significance level is $p = 0.004$. Since the p-value is less than $\alpha\left(p = 0.004 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate there are differences in the mean pain thresholds among the people with the four types of hair color at $\alpha = 0.05$.

d. The assumptions necessary to assure the validity of the test are as follows:

1. The probability distributions of the pain threshold scores is normal for each hair color.
2. The variances of the probability distributions of the pain threshold scores for each hair color are equal.
3. The samples are selected randomly and independently.

12.77 Using MINITAB, the results are:

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
GROUP	2	6110	3054.9	7.69	0.001
Error	136	54046	397.4		
Total	138	60156			

To determine if the mean percentages of names recalled differ for the three name retrieval methods, we test:

$H_0: \mu_1 = \mu_2 = \mu_3$
H_a: At least two treatment means differ

The test statistic is $F = 7.69$ and the p-value is $p = 0.001$. Since the p-value is less than $\alpha\left(p = 0.001 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate the mean percentages of names recalled differ for the three name retrieval methods at $\alpha = 0.05$.

12.78 a. Using MINITAB, the results are:

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
SOLVENT	2	3.305	1.65270	24.51	0.000
Error	29	1.955	0.06743		
Total	31	5.261			

b. To determine if differences exist among the mean sorption rates of the three organic solvent types, we test:

$H_0: \mu_1 = \mu_2 = \mu_3$
H_a: At least two treatment means differ

The test statistic is $F = 24.51$ and the p-value is $p = 0.000$. Since the p-value is less than $\alpha\left(p = 0.000 < 0.10\right)$, H_0 is rejected. There is sufficient evidence to indicate differences exist among the mean sorption rates of the three organic solvent types at $\alpha = 0.10$.

c. For $p = 3$ means, the number of pairwise comparisons to be made is
$g = \frac{p(p-1)}{2} = \frac{3(2)}{2} = 3$. We need to find $t_{\alpha/[2g]} = t_{0.10/[2(3)]} = t_{0.0167} = 2.233$ for the t distribution based on $\nu = df(error) = 29$.

$$B_{12} \approx t_{0.0167} s \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = 2.233\sqrt{0.06743}\sqrt{\frac{1}{8} + \frac{1}{9}} = 0.282$$

$$B_{13} \approx t_{0.0167} s \sqrt{\frac{1}{n_1} + \frac{1}{n_3}} = 2.233\sqrt{0.06743}\sqrt{\frac{1}{8} + \frac{1}{15}} = 0.254$$

$$B_{23} \approx t_{0.0167} s \sqrt{\frac{1}{n_2} + \frac{1}{n_3}} = 2.233\sqrt{0.06743}\sqrt{\frac{1}{9} + \frac{1}{15}} = 0.244$$

The three sample means ranked in order are:

Esters	Aromatics	Chloroalkanes
0.330	0.942	1.006

First, we compare the means of Chloroalkanes and Esters. The critical value is 0.244. The difference between the means of these two groups is greater than 0.244. Therefore, there is a significant difference between the two means.

Next, we compare the means of Chloroalkanes and Aromatics. The critical value is 0.282. The difference between the means of these two groups is less than 0.282. Therefore, there is not a significant difference between the two means.

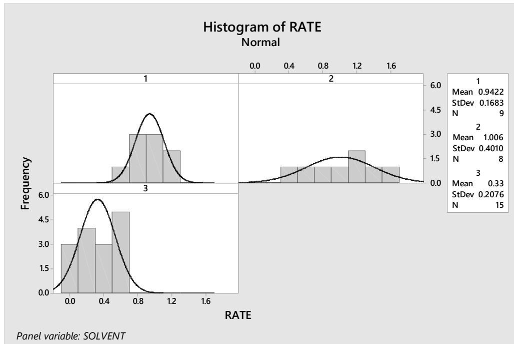
Finally, we compare the means of Aromatics and Esters. The critical value is 0.254. The difference between the means of these two groups is greater than 0.254. Therefore, there is a significant difference between the two means.

The pairs of means that are significantly different are: (Chloroalkanes, Esters) and (Aromatics, Esters).

d. The assumptions for the completely randomized design are:

1. All three probability distributions of sorption rates corresponding to the three solvents are normally distributed.
2. The population variances of the sorption rates of the three solvents are equal.

First, we'll check the assumption of normality. For this design, we have few observations in each of the three treatments (solvents). The histograms of the three solvents are:



The graphs indicate distributions that are not too normal. Remember that the ANOVA procedures are fairly robust with respect to normality. That is, the analysis is valid even if the data are not exactly normally distributed.

We next perform Bartlett's test for the equality of variances:

First, we must compute the sample variances for each of the three solvents:

Aromatics: $s_1^2 = 0.0283$

Chloroalkanes: $s_2^2 = 0.1608$

Esters: $s_3^2 = 0.0431$

$$\bar{s}^2 = \frac{\sum(n_i - 1)s_i^2}{\sum(n_i - 1)} = \frac{(9-1)(0.0283)+(8-1)(0.1608)+(15-1)(0.0431)}{(9-1)+(8-1)+(15-1)}$$

$$= \frac{1.9554}{29} = 0.0674$$

The test is as follows:

$H_0: \sigma_1^2 = \sigma_2^2 = \sigma_3^2$

H_a: At least two variances differ

The test statistic is $B = \dfrac{\left[\sum (n_i - 1)\ln \bar{s}^2\right] - \sum (n_i - 1)\ln s_i^2}{1 + \dfrac{1}{3(p-1)}\left\{\sum \dfrac{1}{(n_i - 1)} - \dfrac{1}{\sum (n_i - 1)}\right\}}$

$$B = \frac{29\ln(0.0674) - \left[(9-1)\ln(0.0283) + (8-1)\ln(0.1608) + (15-1)\ln(0.0431)\right]}{1 + \dfrac{1}{3(3-1)}\left\{\dfrac{1}{(9-1)} + \dfrac{1}{(8-1)} + \dfrac{1}{(15-1)} - \dfrac{1}{29}\right\}} = 6.77$$

The rejection region requires $\alpha = 0.05$ in the upper tail of the χ^2 distribution with $df = p - 1 = 2$. From Table 9, Appendix D, $\chi^2_{0.05} = 5.99147$. The rejection region is $\chi^2 > 5.99147$.

Since the observed value of the test statistic falls in the rejection region $\left(\chi^2 = 6.77 > 5.99147\right)$, H_0 is rejected. There is sufficient evidence to indicate at least two of the variances are different at $\alpha = 0.05$. Thus, the analysis may not be valid since the assumption of equal variances is not met.

12.79 Using MINITAB, the results of the analyses are:

Analysis of Variance for AbsRate

Source	DF	SS	MS	F	P
Day	4	94.20	23.55	2.00	0.118
Week	8	575.23	71.90	6.10	0.000
Error	32	376.94	11.78		
Total	44	1046.37			

To determine if the mean absentee rate differs by the day of the week, we test:

$$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$$
$$H_a: \text{At least two means differ}$$

The test statistic is $F = 2.00$ and the p-value is $p = 0.118$. Since the p-value is not small, H_0 is not rejected. There is insufficient evidence to indicate the mean absentee rate differs by the day of the week. Since there are no differences among the days of the week, no multiple comparison procedure is necessary.

To determine if blocking was effective, we test:

$$H_0: \mu_1 = \mu_2 = \cdots = \mu_9$$
$$H_a: \text{At least two means differ}$$

The test statistic is $F = 6.10$ and the p-value is $p = 0.000$. Since the p-value is so small, H_0 is rejected. There is sufficient evidence to indicate blocking was effective.

12.80 a. This is a randomized block design. The blocks are the 12 plots of land. The treatments are the three methods used on the shrubs: fire, clipping, and control. The response variable is the mean number of flowers produced. The experimental units are the 36 shrubs.

b. The layout of the design is:

		Treatments		
	Plot 1	Fire	Clipping	Control
Blocks	Plot 2	Fire	Clipping	Control
	:			
	:			
	Plot 12	Fire	Clipping	Control

c. The model for this design is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \cdots + \beta_{13} x_{13}$,

$$\text{where } x_1 = \begin{cases} 1 & \text{if fire} \\ 0 & \text{if not} \end{cases} \quad x_2 = \begin{cases} 1 & \text{if clipping} \\ 0 & \text{if not} \end{cases} \quad x_3 = \begin{cases} 1 & \text{if block 1} \\ 0 & \text{if not} \end{cases} \quad x_4 = \begin{cases} 1 & \text{if block 2} \\ 0 & \text{if not} \end{cases}$$

$$\ldots\ x_{13} = \begin{cases} 1 & \text{if block 11} \\ 0 & \text{if not} \end{cases}$$

d. To determine if there are differences in the mean number of flowers per plant among the three treatments, we test:

$H_0: \mu_1 = \mu_2 = \mu_3$
H_a: At least two treatment means are different

The test statistic is $F = 5.42$ and the p-value is $p = 0.009$. Since the p-value is so small, H_0 is rejected. There is sufficient evidence to indicate that there are differences in the mean number of flowers per plant among the three treatments.

e. The mean number of flowers produced by plants with the burning treatment is significantly greater than the mean number of flowers produced by plants with the control treatment. No other significant differences exist.

12.81 Using MINITAB, the results are:

Analysis of Variance for CORRATE

Source	DF	SS	MS	F	P
SYSTEM	3	9.5833	3.1944	34.12	0.000
EXP-TIME	2	63.1050	31.5525	337.06	0.000
Error	6	0.5617	0.0936		
Total	11	73.2500			

To determine if there are differences in the mean corrosion rates among the four epoxy treatments, we test:

$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$
H_a: At least two treatment means are different

The test statistic is $F = 34.12$ and the p-value is $p = 0.000$. Since the p-value is so small, H_0 is rejected. There is sufficient evidence to indicate that there are differences in the mean corrosion rates among the four epoxy treatments.

12.82 a. The two factors are wash-up events (three different events) and algae type or strata (coarse-branching, medium-branching, fine-branching, and hydroid algae).

b. There are a total of $3 \times 4 = 12$ treatments. The treatments are the 12 different event-strata combinations.

c. There are two samples from each 12 event/strata combination.

d. The total sample size is 24.

e. The mussel density is the response variable.

f. The first test to be run is the test to determine if the interaction between event and strata is significant. To determine if strata and event interact to affect mussel density, we test:

H_0 : Event and strata do not interact
H_a : Event and strata do interact

The test statistic is $F = 1.91$ and the p-value is $p > 0.05$. Since the p-value is not less than $\alpha\left(p > 0.05 \not< 0.05\right)$, , H_0 is not rejected. There is insufficient evidence to indicate strata and event interact to affect mussel density at $\alpha = 0.05$.

g. To determine if the mean mussel density differs among the three events, we test:

$H_0 : \mu_1 = \mu_2 = \mu_3$
H_a : At least two event means are different

The test statistic is $F = 0.35$ and the p-value is $p > 0.05$. Since the p-value is not less than $\alpha\left(p > 0.05 \not< 0.05\right)$, H_0 is not rejected. There is insufficient evidence to indicate the mean mussel density differs among the three events at $\alpha = 0.05$.

To determine if the mean mussel density differs among the four strata, we test:

$H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4$
H_a : At least two event means are different

The test statistic is $F = 217.33$ and the p-value is $p < 0.05$. Since the p-value is less than $\alpha\left(p < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate the mean mussel density differs among the four strata at $\alpha = 0.05$.

h. The mean mussel density for the hydroid algae is significantly greater than the mean mussel density of the other 3 algae types. The mean mussel density for the fine-branching algae is significantly greater than the mean mussel density of the coarse-branching and the medium-branching algae. There is no significant difference in the mean mussel density between the coarse-branching and the medium branching algae.

12.83 a. This is a 6×6 factorial design.

b. There are 2 factors – level of coagulant and pH level. The factor "level of coagulation" has 6 levels – 5, 10, 20, 50, 100, and 200 milligrams per liter. The factor "pH level" has 6 levels – 4, 5, 6, 7, 8, and 9. There are $6 \times 6 = 36$ treatments for this study. Each treatment is a combination of one level of coagulation and one level of pH. An example of a treatment is coagulation level 5 and pH level 4. The treatments are:

(5, 4)	(5, 5)	(5,6)	(5,7)	(5,8)	(5,9)
(10, 4)	(10, 5)	(10,6)	(10,7)	(10,8)	(10,9)
(20, 4)	(20, 5)	(20,6)	(20,7)	(20,8)	(20,9)

(50, 4) (50, 5) (50,6) (50,7) (50,8) (50,9)
(100, 4) (100, 5) (100,6) (100,7) (100,8) (100,9)
(200, 4) (200, 5) (200,6) (200,7) (200,8) (200,9)

12.84 a. The data analysis method seems reasonable. There are probably large differences among the infants, which we can control for using a randomized complete block design.

b. To determine if the mean listening times differ for the two types of sentences, we test:

$$H_0 : \mu_C = \mu_I$$
$$H_a : \mu_C \neq \mu_I$$

The test statistic is $F = 25.7$ and the p-value is $p < 0.001$. Since the p-value is so small, H_0 is rejected. There is sufficient evidence to indicate the mean listening times differ for the two types of sentences for $\alpha < 0.001$.

c. The mean listening time of consistent and inconsistent sentences are paired for each infant. We know that $t = \sqrt{F}$. For this example, $\sqrt{F} = \sqrt{25.7} = 5.07 = t$.

d. $$SSE = (n_1 - 1)s_1^2 + (n_2 - 1)s_2^2 = (16-1)2.6^2 + (16-1)2.16^2 = 171.384$$

$$SST = n_1(\bar{x}_1 - \bar{x})^2 + n_2(\bar{x}_2 - \bar{x})^2 = 16(6.3 - 7.65)^2 + 16(9.0 - 7.65)^2 = 58.32$$

$$MSE = \frac{SSE}{n-p} = \frac{171.384}{32-2} = 5.7128 \qquad MST = \frac{SST}{p-1} = \frac{58.32}{2-1} = 58.32$$

$$F = \frac{MST}{MSE} = \frac{58.32}{5.7128} = 10.209$$

This test statistic provides weaker evidence of a difference between treatment means than the test in part b because it was not able to remove that source of variation due to differences among infants.

e. We do not need to control for experimentwise error because there are only two treatments.

12.85 a.

SOURCE	*df*	*F*	*p*-VALUE
Aggressiveness (A)	1	16.43	< 0.001
Alcohol Condition (C)	2	6.00	< 0.01
A x C	2		
Error	129		
Total	134		

b. To determine if the mean severity of conflict differs for the different levels of aggressiveness, we test:

$$H_0: \mu_{\text{aggressive}} = \mu_{\text{nonaggressive}}$$
$$H_a: \mu_{\text{aggressive}} \neq \mu_{\text{nonaggressive}}$$

The test statistic is $F = 16.43$ and the p-value is $p < 0.001$. Since the p-value is so small, H_0 is rejected. There is sufficient evidence to conclude that the mean severity of conflict differs for the two levels of aggressiveness for $\alpha < 0.001$.

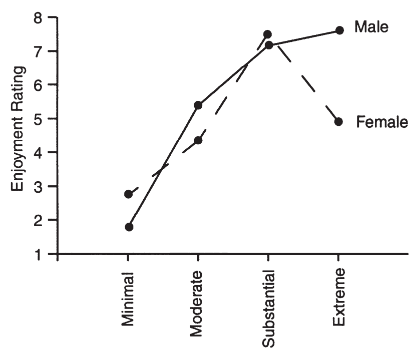
c. To determine if the mean severity of conflict differs among the three alcoholic conditions, we test:

$$H_0: \mu_1 = \mu_2 = \mu_3$$
$$H_a: \text{At least two treatment means differ}$$

The test statistic is $F = 6.00$ and the p-value is $p < 0.01$. Since the p-value is so small, H_0 is rejected. There is sufficient evidence to conclude that the mean severity of conflict differs among the three alcoholic conditions for $\alpha < 0.01$.

d. The main effect tests assume no interaction.

12.86 a.



The pattern in the graph suggests interaction between suspense and gender. The difference between the mean ratings of the two genders varies depending on the suspense category. Notice that the lines cross twice.

b. To determine if interaction exists between suspense and gender, we test:

$$H_0: \text{Gender and Suspense do not interact}$$
$$H_a: \text{Gender and Suspense interact}$$

The test statistic is $F = 4.42$ and the p-value is $p = 0.007$. Since the p-value is so small, H_0 is rejected. There is sufficient evidence to conclude that interaction exists between suspense and gender for $\alpha < 0.007$.

c. No. Since interaction between gender and suspense exists, it means that the difference between the mean enjoyment levels for males and females differs depending on the suspense level of the game.

12.87 a. There are two factors. The factor luckiness has three levels: lucky, unlucky, and uncertain. The factor competition condition has two levels: competitive and noncompetitive.

b. There are three tests:

To determine if interaction between Luckiness and Competition exists, we test:

H_0 : Luckiness and Competition do not interact
H_a : Luckiness and Competition interact

The test statistic is $F = 0.72$ and the p-value is $p = 0.72$. Since the p-value is not small, H_0 is not rejected. There is insufficient evidence to indicate that luckiness and condition interact to affect the mean percentage of coin-flips correctly guessed for any reasonable level of α.

To determine if the mean percentage guessed correctly differs among the three levels of luckiness, we test:

$H_0 : \mu_L = \mu_{UL} = \mu_{UC}$
H_a : At least two treatment means differ

The test statistic is $F = 1.39$ and the p-value is $p = 0.26$. Since the p-value is not small, H_0 is not rejected. There is insufficient evidence to indicate the mean percentage guessed correctly differs among the three levels of luckiness for any reasonable level of α.

To determine if differences exist in the mean percentages guessed correctly between the two competition levels, we test:

$H_0 : \mu_C = \mu_N$
$H_a : \mu_C \neq \mu_N$

The test statistic is $F = 2.84$ and the p-value is $p = 0.10$. Since the p-value is not small, H_0 is not rejected. There is insufficient evidence to indicate the mean percentage guessed correctly differs between the two levels of competition for $\alpha < 0.10$.

12.88 a. Using MINITAB, the results are:

Analysis of Variance for WEIGHT

Source	DF	SS	MS	F	P
RATSIZE	1	8.06789	8.06789	141.18	0.000
DIET	1	0.01243	0.01243	0.22	0.645
RATSIZE*DIET	1	0.03643	0.03643	0.64	0.432
Error	24	1.37151	0.05715		
Total	27	9.48827			

b. To determine if interaction between diet and size exists, we test:

H_0 : Diet and Size do not interact
H_a : Diet and Size interact

The test statistic is $F = 0.64$ and the p-value is $p = 0.432$. Since the p-value is not less than α $(p = 0.432 \nless 0.01)$, H_0 is not rejected. There is insufficient evidence to indicate that diet and size interact to affect the mean weight of kidneys at $\alpha = 0.01$.

To determine if the mean weight differs for the two diets, we test:

$H_0 : \mu_R = \mu_S$
$H_a : \mu_R \neq \mu_S$

The test statistic is $F = 0.22$ and the p-value is $p = 0.645$. Since the p-value is not less than α $(p = 0.645 \nless 0.01)$, H_0 is not rejected. There is insufficient evidence to indicate the mean weight differs for the two diets at $\alpha = 0.01$.

To determine if the mean weight differs for the two sizes, we test:

$H_0 : \mu_L = \mu_O$
$H_a : \mu_L \neq \mu_O$

The test statistic is $F = 141.18$ and the p-value is $p = 0.000$. Since the p-value is less than α $(p = 0.000 \nless 0.01)$, H_0 is rejected. There is sufficient evidence to indicate the mean weight differs for the two sizes at $\alpha = 0.01$.

12.89 Using MINITAB, the results are:

Analysis of Variance for FEEDBACK

Source	DF	SS	MS	F	P
SUBJECT	1	8381	8381.0	25.50	0.000
CONFED	1	8151	8151.0	24.80	0.000
SUBJECT*CONFED	1	20839	20839.2	63.40	0.000
Error	36	11834	328.7		
Total	39	49205			

To determine if interaction between subject and confederate exists, we test:

H_0 : Subject and Confederate do not interact
H_a : Subject and Confederate interact

The test statistic is $F = 63.40$ and the p-value is $p = 0.000$. Since the p-value is so small, H_0 is rejected. There is sufficient evidence to indicate that subject and confederate interact to impact the mean MUM effect for any reasonable value of α.

Since interaction is significant, no tests for main effects are necessary. Now, compare the four treatment means using Tukey's Multiple comparison procedure. The means of the four treatments are:

Means

SUBJECT*CONFED	N	FEEDBACK
NotVis Failure	10	72.2
NotVis Success	10	89.3
Visible Failure	10	146.8
Visible Success	10	72.6

The critical value is $\omega = q_{\alpha}(p,\nu)\frac{s}{\sqrt{n_t}}$, where $p=4$, $s=\sqrt{MSE}=\sqrt{328.7}=18.13$, $\nu = df\ error = 36$, $n_t = 10$, and $q_{\alpha}(p,\nu) =$ critical value from Studentized range distribution. From Table 11, Appendix D, $q_{0.05}(4,36)=3.82$. Thus,

$$\omega = q_{\alpha}(p,\nu)\frac{s}{\sqrt{n_t}} = 3.82\frac{18.13}{\sqrt{10}} = 21.90.$$

The ordered means are:

NotVis/Fail	Vis/Succ	NotVis/Succ	Vis/Fail
72.2	72.6	89.3	146.8

Subtracting 21.90 from the mean of Vis/Fail, we get $146.8 - 21.90 = 124.9$. All the other means are less than 124.9. Thus, the mean MUM effect of Vis/Fail is significantly greater than the mean MUM effect for all other treatments. None of the means for the other three treatments are more than 21.90 units apart. Thus, there is no difference in the mean MUM effect among the treatments NotVis/Fail. Vis/Succ, and NotVis/Succ.

12.90 a. The model is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_1 x_2 + \beta_7 x_1 x_3$,

where $x_1 = \begin{cases} 1 & \text{if pH} = 4.5 \\ 0 & \text{if not} \end{cases}$ $\quad x_2 = \begin{cases} 1 & \text{if Soil depth} = 15-30 \\ 0 & \text{if not} \end{cases}$

$x_3 = \begin{cases} 1 & \text{if Soil depth} = 30-46 \\ 0 & \text{if not} \end{cases}$ $\quad x_4 = \begin{cases} 1 & \text{if Date} = \text{June 16} \\ 0 & \text{if not} \end{cases}$

$x_5 = \begin{cases} 1 & \text{if Date} = \text{June 30} \\ 0 & \text{if not} \end{cases}$

b. Using MINITAB, the results are:

Analysis of Variance for SOILPH

Source	DF	SS	MS	F	P
DEPTH	2	0.067144	0.033572	3.37	0.076
ACIDPH	1	0.030422	0.030422	3.06	0.111
DEPTH*ACIDPH	2	0.007811	0.003906	0.39	0.685
DATE	2	0.381478	0.190739	19.17	0.000
Error	10	0.099522	0.009952		
Total	17	0.586378			

c. To determine if pH level of acid rain and soil depth interact, we test:

H_0 : pH level and depth do not interact
H_a : pH level and depth interact

The test statistic is $F = 0.39$ and the p-value is $p = 0.685$. Since the p-value is not less than $\alpha\left(p = 0.685 \not< 0.05\right)$, H_0 is not rejected. There is insufficient evidence to indicate that pH level of acid rain and soil depth interact at $\alpha = 0.05$.

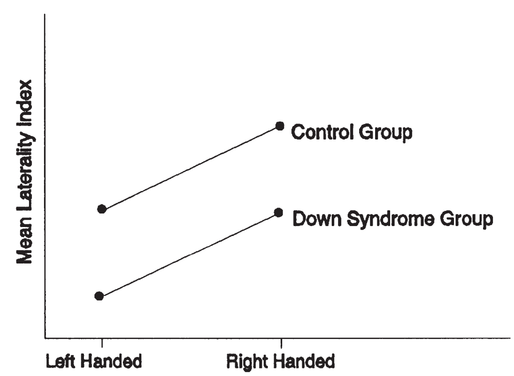
d. To determine whether blocking over time was effective, we test:

H_0 : There are no differences among the block means
H_a : At least two block means differ

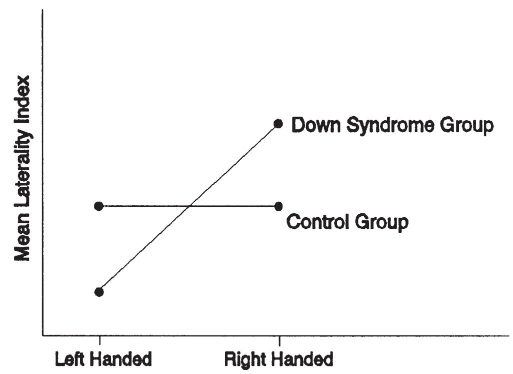
The test statistic is $F = 19.17$ and the p-value is $p = 0.000$. Since the p-value is less than $\alpha\left(p = 0.000 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate blocking over time was effective at $\alpha = 0.05$.

12.91 a. There are $2 \times 2 = 4$ treatments in this experiment: (Down syndrome, left handed), (Down syndrome, right handed), (Control, left handed), and (Control, right handed).

b. A graph that would indicate no interaction could look like the following:



c. A graph that would indicate interaction could look like the following:



d. Since the p-value is so small $(p < 0.05)$, H_0 is rejected. There is sufficient evidence to indicate that group and handedness interact at $\alpha = 0.05$. This means that the effect of handedness on the laterality index depends on which group the person comes from.

e. The mean laterality index for the Down syndrome, left handed group is significantly less than the mean laterality index of the three other groups: $\mu_{\text{DL}} < (\mu_{\text{DR}}, \mu_{\text{CL}}, \mu_{\text{CR}})$.

f. The experimentwise error rate was $\alpha = 0.05$. This is the probability of saying that at least one pair of means is different when it is not.

12.92 a. The response variable is the job satisfaction of a driver.

b. The two factors are career stage and time spent on the road.

c. The treatments are the six combinations of the factor levels:

(early, short) (early, long) (mid-career, short) (mid-career, long) (late, short) (late, long)

d. The first step in the analysis is to determine if career stage and time spent on the road interact, we test:

H_0 : Career stage and time do not interact
H_a : Career stage and time interact

The test statistic is $F = 1.59$ and the p-value is $p > 0.05$. Since the p-value is not less than $\alpha\ (p > 0.05 \not< 0.05)$, H_0 is not rejected. There is insufficient evidence to indicate that career stage and time spent on the road interact at $\alpha = 0.05$. Since the interaction is not significant, the tests for the main effects are appropriate.

To determine if the mean job satisfaction of the two levels of time spent on the road differ, we test:

$H_0 : \mu_{\text{S}} = \mu_{\text{L}}$
$H_a : \mu_{\text{S}} \neq \mu_{\text{L}}$

The test statistic is $F = 0.19$ and the p-value is $p > 0.05$. Since the p-value is not less than $\alpha\left(p > 0.05 \nless 0.05\right)$, H_0 is not rejected. There is insufficient evidence to indicate that the mean job satisfaction of the two levels of time spent on the road differ at $\alpha = 0.05$.

To determine if the mean job satisfaction of the three levels of career stage differ, we test:

$H_0: \mu_{\text{E}} = \mu_{\text{M}} = \mu_{\text{L}}$
H_a: At least two treatment means differ

The test statistic is $F = 26.67$ and the p-value is $p \le 0.001$. Since the p-value is less than $\alpha\left(p \le 0.001 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate that the mean job satisfaction of the three levels of career stage differ at $\alpha = 0.05$.

e. No, the impact of road time on job satisfaction is not different depending on the career stage of the driver because the interaction was not significant.

f. Yes, there is evidence to indicate that career stage affects the job satisfaction of truck drivers because the main effect test for career stage is significant.

g. To compare the 3 career stages, there are $c = \dfrac{3(3-1)}{2} = 3$ pairwise comparisons. The adjusted α level to use is $\alpha / c = 0.09 / 3 = 0.03$.

h. The means organized in order are:

Late	Middle	Early
3.36	3.38	3.47

Any pair of means that differ by more than 0.06 are significantly different. Thus, the mean job satisfaction for Early is significantly higher than the mean job satisfaction for Middle and Late. There are no other significant differences.

12.93 a. To determine if there are differences in the mean performance levels among the five firm types, we test:

$H_0: \mu_{\text{R}} = \mu_{\text{D}} = \mu_{\text{P}} = \mu_{\text{A}} = \mu_{\text{B}}$
H_a: At least two of the five means differ

Conclusion: Reject H_0 at $\alpha = 0.05$. There is sufficient evidence to indicate at least two of the mean performance levels differ among the 5 groups.

b. $\mu_{\text{R}} < \left(\mu_{\text{P}}, \mu_{\text{D}}, \mu_{\text{A}}, \mu_{\text{B}}\right)$, $\mu_{\text{P}} < \left(\mu_{\text{D}}, \mu_{\text{A}}, \mu_{\text{B}}\right)$ and $\mu_{\text{D}} < \mu_{\text{B}}$

12.94 Using MINITAB, the results are:

Analysis of Variance for WATER

Source	DF	SS	MS	F	P
EXPOSURE	2	8200.4	4100.19	856.52	0.000
TEMP	2	4376.7	2188.36	457.14	0.000
EXPOSURE*TEMP	4	103.3	25.82	5.39	0.003
Error	27	129.3	4.79		
Total	35	12809.6			

To determine if exposure and temperature interact to affect the percentage of water removed, we test:

H_0 : Exposure and temperature do not interact
H_a : Exposure and temperature interact

The test statistic is $F = 5.39$ and the p-value is $p = 0.003$. Since the p-value is so small, H_0 is rejected. There is sufficient evidence to indicate that exposure and temperature interact to affect the percentage of water removed for any level of $\alpha > 0.003$.

Since the interaction is significant, no main effect tests should be done. There are two ways to proceed from this point. One way is to look at all $3 \times 3 = 9$ treatments together to determine which treatments significantly differ. The other is to hold one factor fixed at a specific level and compare all levels of the second factor to determine any significant differences. Both choices utilize a multiple comparison procedure and are left to the reader to compute.

12.95 a. There are two factors in this problem with two levels each:

Factor A - Confirmation of accounts receivable, with levels completed and not completed.
Factor B - Verification of sales transactions, with levels completed and not completed.

There are 4 treatments in this problem in which the treatments correspond to all possible combinations of the two factors:

Let AC = confirmation of accounts receivable completed
AN = confirmation of accounts receivable not completed
VC = verification of sales transactions completed
VN = verification of sales transactions not completed

The four treatments are then (AC, VC), (AC, VN), (AN, VC), and (AN,VN)

b. If the two factors interact, then the effect of accounts receivable on the mean misstatement risk depends on the verification level.

c. Yes. If interaction does not exist, the two lines drawn would be parallel. In this case, the two lines are not parallel. The difference between the mean misstatement risks for completed and not completed confirmation of accounts receivable for completed verification of sales transaction is much smaller than the difference between the mean misstatement risks for completed and not completed confirmation of accounts receivable for not completed verification of sales transaction.

12.96 a. Using MINITAB, the results are:

Analysis of Variance for ITEMS

Source	DF	SS	MS	F	P
TEMP	3	38.830	12.9433	38.35	0.000
RATE	3	95.620	31.8733	94.44	0.000
TEMP*RATE	9	15.490	1.7211	5.10	0.002
Error	16	5.400	0.3375		
Total	31	155.340			

b. Both factors, rate and temperature, are quantitative. Consequently, the complete factorial model for the 4×4 factorial experiment is:

$$\begin{aligned} E(y) = {} & \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1^2 + \beta_4 x_2^2 + \beta_5 x_1 x_2 + \beta_6 x_1 x_2^2 + \beta_7 x_1^3 \\ & + \beta_8 x_2^3 + \beta_9 x_1^2 x_2 + \beta_{10} x_1 x_2^3 + \beta_{11} x_1^2 x_2^2 + \beta_{12} x_1^3 x_2 + \beta_{13} x_1^2 x_2^3 \\ & + \beta_{14} x_1^3 x_2^2 + \beta_{15} x_1^3 x_2^3 \end{aligned}$$

where $x_1 =$ rate of incoming components and $x_2 =$ room temperature. Note that the model has $4 \times 4 = 16$ parameters.

c. To determine if there are differences among the 16 treatments, we must first compute SST by combining $SS(Temp)$, $SS(Rate)$, $SS(Temp \times Rate)$.

$$SST = SS(Temp) + SS(Rate) + SS(Temp \times Rate) = 38.83 + 95.62 + 15.49 = 149.94$$

$$MST = \frac{SST}{(3+3+9)} = \frac{149.94}{15} = 9.996 \quad F = \frac{MST}{MSE} = \frac{9.996}{0.3375} = 29.62$$

To determine if there are differences in the mean parts per minute among the 16 treatments, we test:

$H_0: \mu_1 = \mu_2 = \mu_3 = \cdots = \mu_{16}$
H_a: At least two treatment means differ

or

$H_0: \beta_1 = \beta_2 = \beta_3 = \cdots = \beta_{15} = 0$
H_a: At least one $\beta_i \neq 0$

The test statistic is $F = 29.62$.

The rejection region requires $\alpha = 0.05$ in the upper tail of the F distribution with $\nu_1 = p - 1 = 16 - 1 = 15$ and $\nu_2 = n - p = 32 - 16 = 16$. From Table 4, Appendix D, $F_{0.05} = 2.35$. The rejection region is $F > 2.35$.

Since the observed value of the test statistic falls in the rejection region $(F = 29.62 > 2.35)$, H_0 is rejected. There is sufficient evidence to indicate that there are differences in the mean parts per minute among the 16 treatments at $\alpha = 0.05$.

d. To determine if arrival rate and room temperature interact to affect worker productivity, we test:

H_0 : Rate and temperature do not interact
H_a : Rate and temperature interact

The test statistic is $F = 5.10$ and the p-value is $p = 0.002$. Since the p-value is less than $\alpha\left(p = 0.002 < 0.05\right)$, H_0 is rejected. There is sufficient evidence to indicate that arrival rate and room temperature interact to affect worker productivity at $\alpha = 0.05$.

e. $R^2 = 1 - \dfrac{SSE}{SS(Total)} = 1 - \dfrac{5.40}{155.34} = 0.9652$

The linear model explains approximately 96.52% of the sample variation of the y-values about their mean.

f. A regression analysis would produce an equation which could be used to predict worker productivity for specific values of arrival rate and room temperature.

12.97 a. The complete second-order model is:

$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 + \beta_4 x_1^2 + \beta_5 x_2^2$$

b. Using MINITAB, the results are:

Regression Analysis: ITEMS versus x1, x2, x1_sq, x2_sq, x1x2

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	5	130.81	26.1614	27.73	0.000
Error	26	24.53	0.9436		
Total	31	155.34			

Model Summary

S	R-sq	R-sq(adj)
0.971383	84.21%	81.17%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	29.856	0.349	85.61	0.000
x1	0.5600	0.0768	7.29	0.000
x2	-0.1625	0.0768	-2.12	0.044
x1_sq	-0.2750	0.0429	-6.41	0.000
x2_sq	-0.2313	0.0429	-5.39	0.000
x1x2	-0.1135	0.0343	-3.30	0.003

Regression Equation

ITEMS = 29.856 + 0.5600 x1 - 0.1625 x2 - 0.2750 x1_sq - 0.2313 x2_sq - 0.1135 x1x2

The prediction equation is:

$$\hat{y} = 29.86 + 0.56x_1 - 0.1625x_2 - 0.1135x_1x_2 - 0.275x_1^2 - 0.2313x_2^2$$

c. The SSE's are different because the models are different. In Exercise 12.96, a model of higher than second order was fit.

d. $R^2 = 1 - \frac{SSE}{SS(Total)} = 1 - \frac{24.53}{155.34} = 0.842$. This implies that 84.2% of the sample variation in the productivity scores is explained by the complete second order model containing arrival rate and temperature.

e. To determine if the complete factorial model provides more information for predicting y than the second-order model, we test:

$$H_0: \beta_6 = \beta_7 = \ldots = \beta_{15} = 0$$
$$H_a: \text{At least one } \beta_i \neq 0$$

The test statistic is $F = \frac{(SSE_R - SSE_C)/(k-g)}{SSE_C/[n-(k+1)]} = \frac{(24.53-5.4)/(15-5)}{5.4/[32-(15+1)]} = 5.67$.

The rejection region requires $\alpha = 0.05$ in the upper tail of the F distribution with $\nu_1 = k - g = 15 - 5 = 10$ and $\nu_2 = n - p = 32 - 16 = 16$. From Table 4, Appendix D, $F_{0.05} = 2.49$. The rejection region is $F > 2.49$.

Since the observed value of the test statistic falls in the rejection region $(F = 5.67 > 2.49)$, H_0 is rejected. There is sufficient evidence to indicate the complete factorial model provides more information for predicting y than the second-order model at $\alpha = 0.05$.

12.98 a. The complete model would be

$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 + \beta_4 x_3 + \beta_5 x_4 + \cdots + \beta_{12} x_{11},$$

where $x_1 =$ arrival rate, $x_2 =$ room temperature, and $x_3 = \begin{cases} 1 & \text{if worker 1} \\ 0 & \text{if not} \end{cases}$

$x_4 = \begin{cases} 1 & \text{if worker 2} \\ 0 & \text{if not} \end{cases}$ . . . $x_{11} = \begin{cases} 1 & \text{if worker 9} \\ 0 & \text{if not} \end{cases}$

b. Using MINITAB, the results are:

Analysis of Variance for TIME

Source	DF	SS	MS	F	P
TEMP	1	1.8490	1.84900	46.18	0.000
RATE	1	0.7290	0.72900	18.21	0.000
TEMP*RATE	1	2.4010	2.40100	59.97	0.000
WORKER	9	5.1950	0.57722	14.42	0.000
Error	27	1.0810	0.04004		
Total	39	11.2550			

To compare the 4 treatment means, we must compute *SST*:

$$SST = SS(Temp) + SS(Rate) + SS(Temp \times Rate) = 1.849 + 0.729 + 2.401 = 4.979$$

$$MST = \frac{SST}{(1+1+1)} = \frac{4.979}{3} = 1.6597 \quad F = \frac{MST}{MSE} = \frac{1.6597}{0.04004} = 41.45$$

To determine if there are differences in the mean parts per minute among the 4 treatments, we test:

$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$
H_a: At least two treatment means differ

or

$H_0: \beta_1 = \beta_2 = \beta_3 = 0$
H_a: At least one $\beta_i \neq 0$

The test statistic is $F = 41.45$.

The rejection region requires $\alpha = 0.05$ in the upper tail of the F distribution with $\nu_1 = p - 1 = 4 - 1 = 3$ and $\nu_2 = n - p - b + 1 = 40 - 4 - 10 + 1 = 27$. From Table 4, Appendix D, $F_{0.05} = 2.96$. The rejection region is $F > 2.96$.

Since the observed value of the test statistic falls in the rejection region $(F = 41.45 > 2.96)$, H_0 is rejected. There is sufficient evidence to indicate that there are differences in the mean parts per minute among the 4 treatments at $\alpha = 0.05$.

c. To determine if arrival rate and temperature interact to affect assembly time, we test:

H_0: Rate and temperature do not interact
H_a: Rate and temperature interact

The test statistic is $F = 59.97$ and the p-value is $p = 0.000$. Since the p-value is less than $\alpha\ (p = 0.000 < 0.05)$, H_0 is rejected. There is sufficient evidence to indicate that arrival rate and room temperature interact to affect assembly time at $\alpha = 0.05$.

d. Using MINITAB to fit the regression model, the results are:

Regression Analysis: TIME versus x1, x2, x1x2, x3, x4, x5, ... , x9, x10, x11
Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	12	10.174	0.84783	21.18	0.000
Error	27	1.081	0.04004		
Total	39	11.255			

Model Summary

S	R-sq	R-sq(adj)
0.200093	90.40%	86.13%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	9.76	1.51	6.47	0.000
x1	-15.24	1.90	-8.01	0.000
x2	-0.1040	0.0200	-5.20	0.000
x1x2	0.1960	0.0253	7.74	0.000
x3	-0.150	0.141	-1.06	0.298
x4	-0.275	0.141	-1.94	0.062
x5	0.400	0.141	2.83	0.009
x6	0.150	0.141	1.06	0.298
x7	0.475	0.141	3.36	0.002
x8	0.525	0.141	3.71	0.001
x9	0.250	0.141	1.77	0.089
x10	0.775	0.141	5.48	0.000
x11	-0.400	0.141	-2.83	0.009

Regression Equation

TIME = 9.76 - 15.24 x1 - 0.1040 x2 + 0.1960 x1x2 - 0.150 x3 - 0.275 x4 + 0.400 x5 + 0.150 x6
+ 0.475 x7 + 0.525 x8 + 0.250 x9 + 0.775 x10 - 0.400 x11

The fitted regression line is

$$\hat{y} = 9.76 - 15.24x_1 - 0.104x_2 + 0.196x_1x_2 - 0.15x_3 - 0.275x_4 + 0.4x_5 + 0.15x_6 + 0.475x_7 + 0.525x_8 + 0.25x_9 + 0.775x_{10} - 0.4x_{11}$$

When $x_2 = 70$, the equation becomes

$$\begin{aligned}\hat{y} &= 9.76 - 15.24x_1 - 0.104(70) + 0.196(70)x_1 - 0.15x_3 - 0.275x_4 + 0.4x_5 \\ &\quad + 0.15x_6 + 0.475x_7 + 0.525x_8 + 0.25x_9 + 0.775x_{10} - 0.4x_{11} \\ &= 2.48 - 1.52x_1 - 0.15x_3 - 0.275x_4 + 0.4x_5 \\ &\quad + 0.15x_6 + 0.475x_7 + 0.525x_8 + 0.25x_9 + 0.775x_{10} - 0.4x_{11}\end{aligned}$$

Thus, for each unit change in arrival rate, x_1, the mean assembly time decreases by an estimated 1.52 units. If x_1 increases by only 0.5 units (0.5 to 1.0), the mean assembly time decreases by an estimated $1.52 / 2 = 0.76$ units. The estimated loss then is 0.76 units.

12.99 Using MINITAB, the results are:

Analysis of Variance for FAILURES

Source	DF	SS	MS	F	P
BURNIN	8	27.974	3.4968	54.63	0.000
INSLEVEL	2	43.084	21.5421	336.54	0.000
BURNIN*INSLEVEL	16	97.554	6.0971	95.25	0.000
Error	54	3.457	0.0640		
Total	80	172.069			

To determine if interaction exists between inspection levels and burn-in hours, we test:

H_0 : Inspection levels and burn-in hours do not interact
H_a : Inspection levels and burn-in hours interact

The test statistic is $F = 95.25$ and the p-value is $p = 0.000$. Since the p-value is so small, H_0 is rejected. There is sufficient evidence to indicate that inspection levels and burn-in hours interact to affect early part failure.

Since interaction exists, no main effect tests are run. The next step is to find which treatment combination gives the optimal detection of early part failure. This could be done by comparing all 27 treatments, or by comparing the 3 inspection levels at each level of burn-in hours.

12.100 a. The complete model is

$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_1 x_2 + \beta_6 x_1 x_3 + \beta_7 x_1 x_4 + \beta_8 x_2 x_3 + \beta_9 x_2 x_4 + \beta_{10} x_3 x_4 + \beta_{11} x_1 x_2 x_3 + \beta_{12} x_1 x_2 x_4 + \beta_{13} x_1 x_3 x_4 + \beta_{14} x_2 x_3 x_4 + \beta_{15} x_1 x_2 x_3 x_4$$

b. With $n = 32$ and 16 parameters, there will be $32 - 16 = 16$ degrees of freedom available to estimate σ^2.

c. Using MINITAB, the results are:

Regression Analysis: LIGHT versus x1, x2, x3, x4, x1x2, ... 3x4, x1x2x3x4

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	15	745.47	49.698	40.78	0.000
Error	16	19.50	1.219		
Total	31	764.97			

Model Summary

S	R-sq	R-sq(adj)
1.10397	97.45%	95.06%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	10.031	0.195	51.40	0.000
x1	4.719	0.195	24.18	0.000
x2	-0.344	0.195	-1.76	0.097
x3	0.219	0.195	1.12	0.279
x4	-0.156	0.195	-0.80	0.435
x1x2	0.094	0.195	0.48	0.637
x1x3	0.156	0.195	0.80	0.435
x1x4	0.281	0.195	1.44	0.169
x2x3	-0.156	0.195	-0.80	0.435
x2x4	-0.031	0.195	-0.16	0.875
x3x4	0.781	0.195	4.00	0.001
x1x2x3	-0.219	0.195	-1.12	0.279

x1x2x4	-0.094	0.195	-0.48	0.637
x1x3x4	-0.031	0.195	-0.16	0.875
x2x3x4	0.156	0.195	0.80	0.435
x1x2x3x4	0.094	0.195	0.48	0.637

Regression Equation

LIGHT = 10.031 + 4.719 x1 - 0.344 x2 + 0.219 x3 - 0.156 x4 + 0.094 x1x2 + 0.156 x1x3
+ 0.281 x1x4 - 0.156 x2x3 - 0.031 x2x4 + 0.781 x3x4 - 0.219 x1x2x3 -
0.094 x1x2x4
- 0.031 x1x3x4 + 0.156 x2x3x4 + 0.094 x1x2x3x4

To determine if any of the factors contribute information for the prediction of y, we test:

$$H_0: \beta_1 = \beta_2 = \cdots = \beta_{15} = 0$$
$$H_a: \text{At least one } \beta_i \neq 0$$

The test statistic is $F = 40.78$ and the p-value is $p = 0.000$. Since the p-value is less than $\alpha \left(p = 0.000 < 0.05 \right)$, H_0 is rejected. There is sufficient evidence to indicate that at least one factor contributes to the prediction of y at $\alpha = 0.05$.

d. Examination of the individual t statistics shows that Foil $\left(t = 24.18,\ p = 0.000 \right)$ and Shift$\times$Operator interaction $\left(t = 4.00,\ p = 0.001 \right)$ appear to affect the amount of light, y, in the flashbulbs

Case Study

1

Legal Advertising – Does It Pay?

1. When calculating the correlation between the number of new personal injury cases and the number of new worker's compensation cases, we can see that there is a very small correlation of 0.086. This does not indicate a significant advantage to partner A $(p-\text{value}=0.560)$. When the correlation is computed for each individual partner versus the advertising expenditures, there is a significant correlation for partner A $(p-\text{value}=0.000)$ but not for partner B $(p-\text{value}=0.725)$.

   Rationale: The larger p-value means that the correlation is statistically indistinguishable from zero.

Correlation: NewPI, NewWC
Correlations

Pearson correlation	0.086
P-value	0.560

Correlation: AdvExp6, NewPI
Correlations

Pearson correlation	0.539
P-value	0.000

Correlation: AdvExp6, NewWC
Correlations

Pearson correlation	0.056
P-value	0.725

2. The standard deviations for the two partners are presented below (highlighted in the regression analyses). Note that partner A has nearly the same standard deviation (9.67521) as partner B (9.62296). If partner A were to present only standard deviations, without the corresponding p-values, it might suggest that the advertising produced similar effects in WC and PI cases. However, the prediction equation for partner A is significantly useful for predicting PI cases:

$$H_0 : \beta_1 = 0$$
$$H_a : \beta_1 \neq 0; \quad p\text{-value} = 0.000 < \alpha = 0.05.$$

Regression Analysis: NewPI versus AdvExp6
Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	1530	1529.52	16.34	0.000
Error	40	3744	93.61		
Total	41	5274			

Model Summary

S	R-sq	R-sq(adj)
9.67521	29.00%	27.23%

However, partner B's model is not significant in predicting new WC cases: $p\text{-value} = 0.725 \nless \alpha = 0.05$.

Regression Analysis: NewWC versus AdvExp6

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	11.58	11.58	0.13	0.725
Error	40	3704.06	92.60		
Total	41	3715.64			

Model Summary

S	R-sq	R-sq(adj)
9.62296	0.31%	0.00%

3. The standard deviations for the number of new personal injury cases and the standard deviation for the number of new worker's compensation cases are presented below:

Descriptive Statistics: NewPI, NewWC

Statistics

Variable	N	Mean	SE Mean	StDev
NewPI	48	19.15	1.61	11.12
NewWC	48	25.29	1.33	9.20

Even though partner B has a smaller standard deviation when only looking at the number of cases individually, this does not take into account the variability of the different complexity of each law situation (personal injury versus worker's compensation). Partner A would not benefit from reporting this data without looking at the complete analysis presented above since his/her standard deviation is larger. In fact, the standard deviation for partner A is reduced in the linear regression model, while the standard deviation for partner B is nearly the same with or without the linear regression model.

Rationale: This suggests that the variability of the predicted number of new WC cases is not affected by a model of linear regression on advertising revenues, which further supports partner B's argument.

Case Study
2

Modeling the Sale Prices of Residential Properties in Four Neighborhoods

1. The data that was used in the testing represent the population of home sales from January 2017 to January 2018. Changes in variables in the future (neighborhoods, economic conditions, etc.) could cause Model 3 to be unsuccessful if applied to future data.

2. The MINITAB analysis on the full model is presented below without data-splitting:

Regression Analysis: SALES versus LAND, IMP, ... , IMP_DAV, IMP_HUN

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	11	138325114	12575010	1304.43	0.000
LAND	1	12485	12485	1.30	0.256
IMP	1	157327	157327	16.32	0.000
NBHD_CHEV	1	3088	3088	0.32	0.572
NBHD_DAV	1	135541	135541	14.06	0.000
NBHD_HUN	1	694	694	0.07	0.789
LAN_CHEV	1	3503	3503	0.36	0.547
LAN_DAV	1	4653	4653	0.48	0.488
LAN_HUN	1	2048	2048	0.21	0.645
IMP_CHEV	1	111	111	0.01	0.914
IMP_DAV	1	12530	12530	1.30	0.255
IMP_HUN	1	13665	13665	1.42	0.234
Error	448	4318837	9640		
Total	459	142643951			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
98.1848	96.97%	96.90%	96.06%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	52.0	33.3	1.56	0.119	
LAND	1.074	0.944	1.14	0.256	1662.21
IMP	0.874	0.216	4.04	0.000	138.93
NBHD_CHEV	22.5	39.7	0.57	0.572	11.95
NBHD_DAV	-142.1	37.9	-3.75	0.000	11.15
NBHD_HUN	10.7	39.8	0.27	0.789	14.12
LAN_CHEV	0.601	0.997	0.60	0.547	104.18
LAN_DAV	0.657	0.945	0.69	0.488	1926.65
LAN_HUN	-0.49	1.06	-0.46	0.645	61.05
IMP_CHEV	-0.026	0.240	-0.11	0.914	55.94
IMP_DAV	0.249	0.219	1.14	0.255	153.10
IMP_HUN	0.343	0.288	1.19	0.234	38.39

A MINITAB analysis using a data-splitting (cross-validation) technique is presented below:

PLS Regression: SALES versus LAND, IMP, NBHD_CHEV, ... V, IMP_HUN

Method

Cross-validation	Leave-one-out
Components to evaluate	Set
Number of components evaluated	10
Number of components selected	5

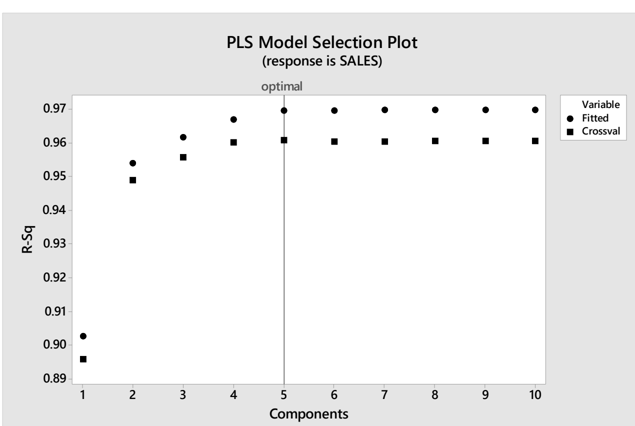
Analysis of Variance for SALES

Source	DF	SS	MS	F	P
Regression	5	138286780	27657356	2881.79	0.000
Residual Error	454	4357171	9597		
Total	459	142643951			

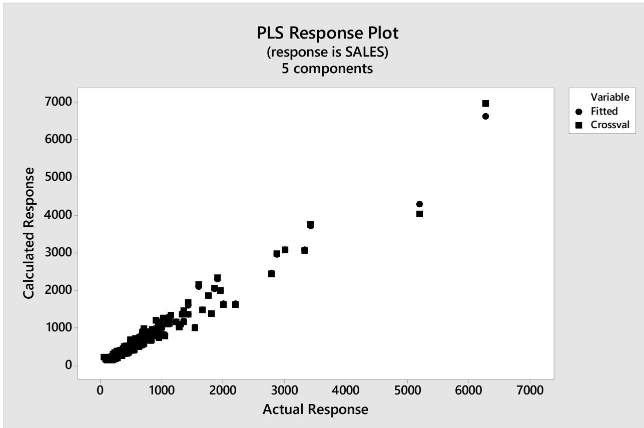
Model Selection and Validation for SALES

Components	X Variance	Error	R-Sq	PRESS	R-Sq (pred)
1	0.370079	13906074	0.902512	14855407	0.895857
2	0.580067	6558542	0.954022	7283059	0.948942
3	0.761535	5478646	0.961592	6307724	0.955780
4	0.932499	4726635	0.966864	5699576	0.960043
5	0.957388	4357171	0.969454	5605032	0.960706
6		4339185	0.969580	5659421	0.960325
7		4326828	0.969667	5644446	0.960430
8		4326171	0.969672	5623187	0.960579
9		4322847	0.969695	5623916	0.960574
10		4318905	0.969722	5622934	0.960581

The first line of the above output shows the number of components in the optimal model, which is defined as the model with the highest predicted R^2_{pred}. MINITAB selected the five-component model as the optimal model, with a predicted $R^2_{\text{pred}} = 0.9607$. Because you fit the same number of components as there are predictors, you can compare the goodness-of-fit and goodness-of-prediction statistics for the PLS models and the least squares solution. The X-variance indicates the amount of variance in the predictors that is explained by the model. Notice that the five component model explains 95.7% of the variance in the predictors. The model selection plot is a graphical display of the Model Selection and Validation table. The vertical line indicates that the optimal model has five components.



Because the points are in a linear pattern, from the bottom left-hand corner to the top right-hand corner, the response plot indicates that the model fits the data adequately. Although there are differences between the fitted and cross-validated fitted responses, none are severe enough to indicate an extreme leverage point.



3. The descriptive statistics for all the neighborhoods are presented below. Notice that DAVISISLE has the largest sales, appraised land value and appraised improvement value.

Descriptive Statistics: SALES, x1, x2

Statistics

Variable	NBHD	N	Mean	StDev	Minimum	Median	Maximum
SALES	ARBORGRN	70	357910	85856	135000	361750	515000
	AVILA	22	973436	318515	615000	946250	1900000
	CHEVAL	91	497477	240926	202000	430000	1425000
	CWOODVILL	103	372263	101702	47800	361500	750000
	DAVISISLE	94	1074009	959320	325000	761000	6275000
	HERITAGE	139	276007	58907	158000	280000	470000
	HUNGREEN	114	345736	140790	175000	330000	1050000
	HYDEPARK	38	871105	311114	319000	835300	1537500
	NORTHDALE	76	276776	37802	155000	284500	357500
	TAMPALMS	154	428696	276391	190000	368500	3100000
	TOWN&CTY	161	185048	49893	54200	180900	420000
	YBORCITY	202	108433	53650	18500	94350	264000
x1-LAND	ARBORGRN	70	72294	22544	37895	70194	162162
	AVILA	22	271919	66530	159782	268043	453600
	CHEVAL	91	99642	56363	37646	80651	291519
	CWOODVILL	103	68292	22450	28875	64167	161523
	DAVISISLE	94	417505	288520	203136	351194	2335608
	HERITAGE	139	51431	20015	6250	50387	114569
	HUNGREEN	114	65612	36936	30087	57427	290716
	HYDEPARK	38	311737	145965	110124	300000	844906
	NORTHDALE	76	50390	11705	32616	47040	91919
	TAMPALMS	154	83330	49498	28671	70843	455872
	TOWN&CTY	161	34632	8870	26010	32181	78951
	YBORCITY	202	14601	6999	4232	12870	58721

x2-IMP	ARBORGRN	70	209293	44813	131492	209647	291754
	AVILA	22	501509	216283	268597	452230	970359
	CHEVAL	91	301837	173169	140847	248780	1235275
	CWOODVILL	103	224425	72544	113448	219777	476562
	DAVISISLE	94	392810	455359	24795	184318	2381464
	HERITAGE	139	158197	40431	80312	155907	248991
	HUNGREEN	114	200977	93798	97689	180009	671956
	HYDEPARK	38	335394	160291	106405	318931	631635
	NORTHDALE	76	167513	38692	107516	161023	285252
	TAMPALMS	154	260913	138229	105573	226316	1092222
	TOWN&CTY	161	109629	38698	48813	104736	353603
	YBORCITY	202	55631	29461	0	49002	155486

Several models were fit to the data:

Model 1 – First-order model with sale price as a function of x_1 = appraised land value and x_2 = appraised improvement value.

Regression Analysis: SALES versus x1, x2

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	2	1.91482E+14	9.57411E+13	12833.43	0.000
Error	1261	9.40743E+12	7460292114		
Total	1263	2.00890E+14			

Model Summary

S	R-sq	R-sq(adj)
86373.0	95.32%	95.31%

Model 2 – First-order model with dummy variables $(x_3 - x_{13})$ for neighborhoods added Model 1.

Regression Analysis: SALES versus x1, x2, x3, x4, x5, x6, ... , x11, x12, x13

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	13	1.91694E+14	1.47457E+13	2004.39	0.000
Error	1250	9.19586E+12	7356685329		
Total	1263	2.00890E+14			

Model Summary

S	R-sq	R-sq(adj)
85771.1	95.42%	95.37%

Model 3 – Model 2 with interactions of appraised land by neighborhoods added.

Regression Analysis: SALES versus x1, x2, x3, x4, x5, x6, ... , x1x12, x1x13

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	24	1.92545E+14	8.02272E+12	1191.24	0.000
Error	1239	8.34440E+12	6734787533		
Total	1263	2.00890E+14			

Model Summary

S	R-sq	R-sq(adj)
82065.8	95.85%	95.77%

Model 4 – Model 2 with interactions of appraised improvement by neighborhoods added.

Regression Analysis: SALES versus x1, x2, x3, x4, x5, x6, ... x2x12, x2x13
Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	24	1.92239E+14	8.00995E+12	1147.21	0.000
Error	1239	8.65083E+12	6982105030		
Total	1263	2.00890E+14			

Model Summary

S	R-sq	R-sq(adj)
83559.0	95.69%	95.61%

Model 5 – Model 2 with interactions of appraised land by neighborhoods and appraised improvement by neighborhoods added.

Regression Analysis: SALES versus x1, x2, x3, x4, x5, x6, ... x2x12, x2x13
Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	35	1.92752E+14	5.50721E+12	831.08	0.000
Error	1228	8.13740E+12	6626544394		
Total	1263	2.00890E+14			

Model Summary

S	R-sq	R-sq(adj)
81403.6	95.95%	95.83%

If we start with Model 5 and compare it to model 2, we can test to see if any of the interaction terms are significant. The test statistic is

$$F = \frac{\left(SSE_R - SSE_C\right)/\left(k-g\right)}{MSE_C} = \frac{\left(9.19586x10^{12} - 8.13740x10^{12}\right)/\left(35-13\right)}{6626544394} = 7.260$$

The p-value associate with $F = 7.260$ is $p = 0.000$. Thus, we can conclude that at least one of the interaction terms is significant.

Next, we test to see if at least one of the appraised land by neighborhood interactions is significant. To do this, we will compare Model 5 to Model 4. The test statistic is

$$F = \frac{\left(SSE_R - SSE_C\right)/\left(k-g\right)}{MSE_C} = \frac{\left(8.65083x10^{12} - 8.13740x10^{12}\right)/\left(35-24\right)}{6626544394} = 7.044$$

The p-value associate with $F = 7.044$ is $p = 0.000$. Thus, we can conclude that at least one of the interaction terms between appraised land and neighborhoods is significant.

Next, we test to see if at least one of the appraised improvements by neighborhood interactions is significant. To do this, we will compare Model 5 to Model 3. The test statistic is

$$F=\frac{\left(SSE_R-SSE_C\right)/\left(k-g\right)}{MSE_C}=\frac{\left(8.3444x10^{12}-8.13740x10^{12}\right)/\left(35-24\right)}{6626544394}=2.840$$

The p-value associate with $F=2.840$ is $p=0.0092$. Thus, we can conclude that at least one of the interaction terms between appraised improvements and neighborhoods is significant.

Thus, the best model would be Model 5. A look at the summary statistics for the 5 models is:

	s	R^2	$R^2{}_{adj}$
Model 1	86,373.0	0.9532	0.9531
Model 2	85,771.1	0.9542	0.9537
Model 3	82,065.8	0.9585	0.9577
Model 4	83,559.0	0.9569	0.9561
Model 5	81,403.6	0.9595	0.9583

One can see that Model 5 has the smallest standard deviation and the largest adjusted R^2 values. The R^2 value of 0.9583 is very impressive. This indicates that 95.83% of the sample variation in the sales values is explained by the model. Although this is statistically significantly better than the adjusted R^2 value of Model 1, $\left(R^2_{adj}=0.9531\right)$, it is not practically different. Again, although this model looks very impressive, it will not be of much value in a predictive sense because of the large standard deviation. Our interpretation is that approximately 95% of the predicted sale price will fall within $\left(2s\right)=2\left(\$81,403.6\right)=\$162,807.2$ of their actual values. This relatively large standard deviation may lead to large errors of prediction for some residential properties if this model is used in practice.

Again, as with the data in the Case Study, additional information to describe the properties such as location, square footage, number of bedrooms, number of bathrooms, condition of the house, etc. and those that describe the market such as interest rates, availability of money, etc. would probably lead to a more accurate predictor of sales.

Note: Model 5 has 35 predictors. If one looks at the individual predictors, many have none significant p-values. However, the dummy variables x_3-x_{13} represent one variable – neighborhoods. Thus, all dummy variables are either in or out of the model. To reduce the number of dummy variables, one could combine similar neighborhoods.

Case Study 3

Deregulation of the Intrastate Trucking Industry

1. Deregulated for $x_3 = 1$:

$$\hat{y} = 12.192 - 0.598x_1 - 0.00598x_2 - 0.01078x_1x_2 + 0.086x_1^2 + 0.00014x_2^2 + 0.677x_4 - 0.275x_1x_4 - 0.026x_2x_4 + 0.013x_1x_2x_4 - 0.782 + 0.0399x_1 - 0.021x_2 - 0.0033x_1x_2$$

$$\Rightarrow 11.41 - 0.5581x_1 - 0.02698x_2 - 0.01408x_1x_2 + 0.086x_1^2 + 0.00014x_2^2 + 0.677x_4 - 0.275x_1x_4 - 0.026x_2x_4 + 0.013x_1x_2x_4$$

Regulated for $x_3 = 0$:

$$\hat{y} = 12.192 - 0.598x_1 - 0.00598x_2 - 0.01078x_1x_2 + 0.086x_1^2 + 0.00014x_2^2 + 0.677x_4 - 0.275x_1x_4 - 0.026x_2x_4 + 0.013x_1x_2x_4$$

$$\hat{y}_{\text{regulated}} - \hat{y}_{\text{deregulated}} = 0.782 - 0.0399x_1 + 0.021x_2 + 0.0033x_1x_2$$

2. Deregulated for $x_3 = 1$, and for $x_4 = 1$, $x_2 = 10$,

$$\hat{y} = 12.192 - 0.598x_1 - 0.00598(10) - 0.01078x_1(10) + 0.086x_1^2 + 0.00014(10)^2 + 0.677(1) - 0.275x_1(1) - 0.026(10)(1) + 0.013x_1(10)(1) - 0.782 + 0.0399x_1 - 0.021(10) - 0.0033x_1(10)$$

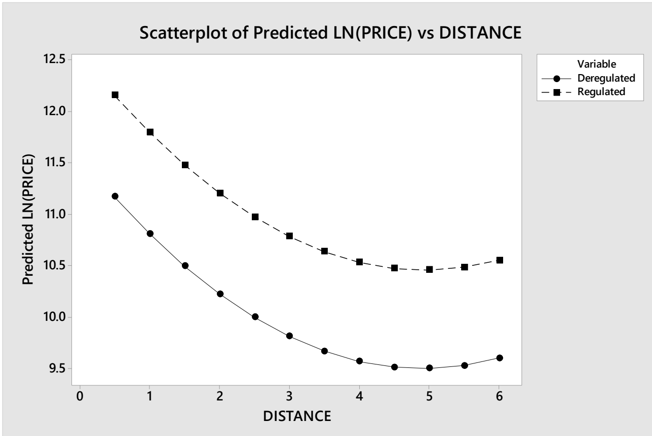
$$\Rightarrow 11.5712 - 0.8439x_1 + 0.086x_1^2$$

Regulated for $x_3 = 0$, and for $x_4 = 1$, $x_2 = 10$,

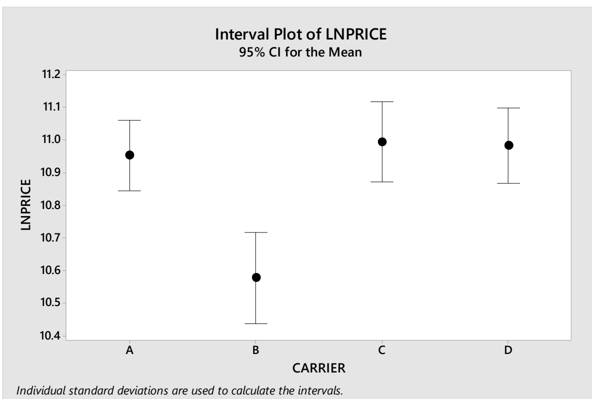
$$\hat{y} = 12.192 - 0.598x_1 - 0.00598(10) - 0.01078x_1(10) + 0.086x_1^2 + 0.00014(10)^2 + 0.677(1) - 0.275x_1(1) - 0.026(10)(1) + 0.013x_1(10)(1)$$

$$\Rightarrow 12.5632 - 0.8508x_1 + 0.086x_1^2$$

$$\hat{y}_{\text{regulated}} - \hat{y}_{\text{deregulated}} = 0.992 - .0069x_1$$



3. First, we will look at an interval plot of LNPRICE with the four carriers. It appears that Carrier B is significantly different from the other Carriers.



To enter the Carrier variable into the model, we define the dummy variables:

$$x_5 = \begin{cases} 1 & \text{if Carrier A} \\ 0 & \text{if not} \end{cases} \qquad x_6 = \begin{cases} 1 & \text{if Carrier B} \\ 0 & \text{if not} \end{cases} \qquad x_7 = \begin{cases} 1 & \text{if Carrier C} \\ 0 & \text{if not} \end{cases}$$

After several Stepwise regressions, a model was developed:

Regression Analysis: LNPRICE versus x1, x2, x3, x4, x5, x6, ... x3x6, x3x7

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	15	177.53	11.8351	129.87	0.000
Error	432	39.37	0.0911		
Total	447	216.90			

Model Summary

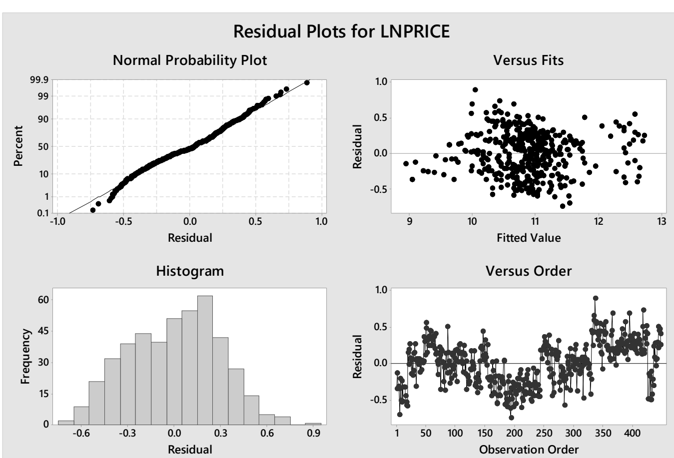
S	R-sq	R-sq(adj)
0.301880	81.85%	81.22%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	12.532	0.110	113.49	0.000
x1	-0.7210	0.0458	-15.76	0.000
x2	-0.02431	0.00358	-6.78	0.000
x3	-0.0045	0.0746	-0.06	0.952
x4	0.3099	0.0979	3.17	0.002
x1_sq	0.08512	0.00622	13.69	0.000
x1x4	-0.1150	0.0335	-3.43	0.001
x5	-0.0383	0.0698	-0.55	0.583
x6	0.0994	0.0695	1.43	0.153
x7	-0.0372	0.0692	-0.54	0.591
x3x5	-0.1172	0.0912	-1.29	0.199
x2x6	-0.01728	0.00465	-3.71	0.000
x3x6	-0.9471	0.0913	-10.37	0.000
x2x7	-0.01233	0.00552	-2.23	0.026
x2x5	-0.00280	0.00462	-0.61	0.544
x3x7	0.065	0.105	0.62	0.538

Thus, all main effects are included – x_1 = distance, x_2 = weight, x_3 = dummy variable for deregulation, x_4 = dummy variable for city origin, and x_5, x_6, x_7 = dummy variables for carrier. In addition, the squared term for distance, the interaction term between distance and city origin, the interaction terms between deregulation and carrier, and the interaction terms between weight and carrier have been included. Notice that some of the terms in the model appear to not be significant. First, if a term is included in a significant interaction, the main effect term must be included – thus, the main effect for deregulation (x_3) must be included. Similarly, x_5, x_6, x_7 are included. In addition, if any of the dummy variables for carrier are include, all must be included. Thus, the terms for the interaction of weight and carrier (x_2x_5, x_2x_6, x_2x_7) must all be included because x_2x_6 is significant. The terms for the interaction of deregulation and carrier (x_3x_5, x_3x_6, x_3x_7) must all be included because x_3x_6 is significant.

Plots of the residuals are:



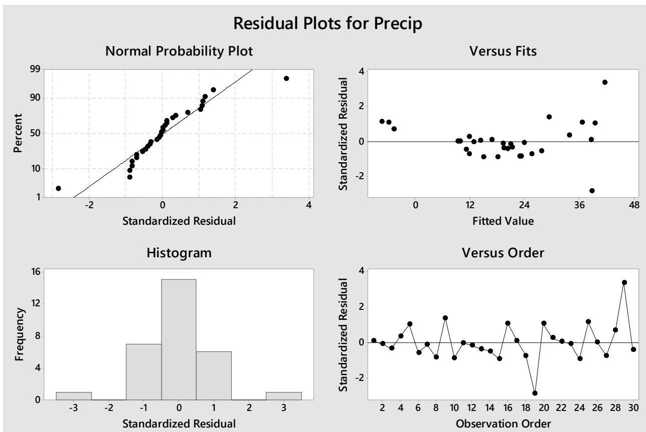
There is no real evidence that the assumptions are not met.

Case Study

4

An Analysis of Rain Levels in California

1. Using MINITAB, the plots of the standardized residuals are:



From the normal probability plot, there appear to be 2 outliers – in the lower left corner and the upper right corner. These two points correspond to observations 19 and 29. In the plot of the standardized residuals versus the fitted values, again there are two observations around 3 standardized deviations from the mean, again corresponding to observations 19 and 29.

Using MINITAB, the statistics for analyzing the residuals are:

Obs	Precip	Predicted	Residual	StdResid	TRES	HI	COOK	DFIT
1	39.57	38.4797	1.0903	0.1078	0.1057	0.1694	0.0006	0.0477
2	23.27	23.9136	-0.6436	-0.0617	-0.0605	0.1169	0.0001	-0.0220
3	18.20	21.2729	-3.0729	-0.3123	-0.3068	0.2138	0.0066	-0.1600
4	37.48	33.7750	3.7050	0.3555	0.3495	0.1182	0.0042	0.1279
5	49.26	39.4605	9.7995	1.0356	1.0371	0.2730	0.1007	0.6356
6	21.82	27.5918	-5.7718	-0.5423	-0.5348	0.0803	0.0064	-0.1580
7	18.07	19.1828	-1.1128	-0.1041	-0.1021	0.0725	0.0002	-0.0285
8	14.17	23.1015	-8.9315	-0.8287	-0.8236	0.0569	0.0104	-0.2023
9	42.63	29.2534	13.3766	1.3957	1.4230	0.2542	0.1660	0.8308
10	13.85	22.8856	-9.0356	-0.8423	-0.8374	0.0656	0.0124	-0.2219
11	9.44	9.3654	0.0746	0.0070	0.0069	0.0745	0.0000	0.0019
12	19.33	20.9364	-1.6064	-0.1509	-0.1481	0.0804	0.0005	-0.0438
13	15.67	19.4445	-3.7745	-0.3501	-0.3441	0.0561	0.0018	-0.0839
14	6.00	11.0967	-5.0967	-0.4715	-0.4643	0.0512	0.0030	-0.1079
15	5.73	14.8859	-9.1559	-0.8910	-0.8874	0.1427	0.0330	-0.3621
16	47.82	36.6194	11.2006	1.0990	1.1036	0.1566	0.0561	0.4755
17	17.95	16.7077	1.2423	0.1181	0.1158	0.1009	0.0004	0.0388

18	18.20	25.4191	-7.2191	-0.7138	-0.7069	0.1696	0.0260	-0.3195
19	10.03	38.7521	-28.7221	-2.8498	-3.3698	0.1752	0.4314	-1.5534
20	4.63	-5.9539	10.5839	1.0809	1.0845	0.2215	0.0831	0.5786
21	14.74	11.8145	2.9255	0.2741	0.2692	0.0753	0.0015	0.0768
22	15.02	14.3149	0.7051	0.0668	0.0655	0.0948	0.0001	0.0212
23	12.36	12.7793	-0.4193	-0.0399	-0.0392	0.1049	0.0000	-0.0134
24	8.26	18.0332	-9.7732	-0.9063	-0.9030	0.0558	0.0121	-0.2195
25	4.05	-7.4478	11.4978	1.1575	1.1655	0.1989	0.0832	0.5808
26	9.94	9.8563	0.0837	0.0082	0.0080	0.1484	0.0000	0.0033
27	4.25	11.7920	-7.5420	-0.7117	-0.7048	0.0883	0.0123	-0.2194
28	1.66	-4.8355	6.4955	0.6884	0.6813	0.2772	0.0454	0.4219
29	74.87	41.5529	33.3171	3.3839	4.4356	0.2129	0.7744	2.3070
30	15.95	20.1703	-4.2203	-0.3995	-0.3929	0.0938	0.0041	-0.1264

The statistics corresponding to observations 19 and 29 are highlighted. These values indicate that these observations are the most influential observations. We recommend leaving them in the model until higher order terms are included in the model.

2. Using MINITAB, the results of fitting Model 1 plus the interaction terms are:

Regression Analysis: Precip versus Altitude, Latitude, ... 1x2, x1x3, x2x3
Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	6	5879	979.81	10.57	0.000
Error	23	2133	92.73		
Total	29	8012			

Model Summary

S	R-sq	R-sq(adj)
9.62961	73.38%	66.43%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	-182.9	37.1	-4.93	0.000
Altitude	-0.0136	0.0192	-0.71	0.486
Latitude	5.67	1.00	5.65	0.000
Distance	1.498	0.532	2.82	0.010
x1x2	0.000463	0.000537	0.86	0.398
x1x3	0.000015	0.000026	0.58	0.569
x2x3	-0.0453	0.0146	-3.09	0.005

To determine if the interaction terms improve the fit of the model, we test:

$$H_0: \beta_4 = \beta_5 = \beta_6 = 0$$
$$H_a: \text{At least one } \beta_i \neq 0$$

The test statistic is $F = \dfrac{\left(SSE_R - SSE_C\right)/\left(k-g\right)}{MSE_C} = \dfrac{\left(3202-2133\right)/\left(6-3\right)}{92.73} = 3.84.$

The p-value associated with this F with $\nu_1 = 3$ and $\nu_2 = 23$ is $p = 0.0230.$ Since the p-value is so small, there is evidence to indicate that at least one of the interactions is useful in predicting the average annual precipitation.

An Investigation of Factors Affecting the Sale Price of Condominium Units Sold at Public Auction

Case Study **5**

1. The results of fitting Model 1 are:

Regression Analysis: PRICE100 versus FLOOR, DIST, VIEW, ... , FURNISH

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	5	118091	23618.2	39.69	0.000
Error	203	120802	595.1		
Total	208	238893			

Model Summary

S	R-sq	R-sq(adj)
24.3943	49.43%	48.19%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	183.57	5.22	35.16	0.000
FLOOR	-3.808	0.748	-5.09	0.000
DIST	1.741	0.375	4.64	0.000
VIEW	40.33	3.46	11.67	0.000
END	4.28	3.60	1.19	0.236
FURNISH	-32.72	9.58	-3.41	0.001

The results of fitting Model 2 are:

Regression Analysis: PRICE100 versus FLOOR, DIST, FL_DI, ... , FURNISH

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	8	138730	17341.2	34.63	0.000
Error	200	100163	500.8		
Total	208	238893			

Model Summary

S	R-sq	R-sq(adj)
22.3789	58.07%	56.39%

CS5-2 An Investigation of Factors Affecting the Sale Piece of Condominium Units Sold at Public Auction

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	209.9	10.1	20.73	0.000
FLOOR	-10.96	3.51	-3.12	0.002
DIST	-5.66	1.79	-3.17	0.002
FL_DI	-0.264	0.153	-1.73	0.085
FL_SQ	1.026	0.348	2.95	0.004
DI_SQ	0.561	0.103	5.44	0.000
VIEW	39.24	3.19	12.30	0.000
END	6.92	3.41	2.03	0.044
FURNISH	-22.03	9.04	-2.44	0.016

The results of fitting Model 3 are:

Regression Analysis: PRICE100 versus FLOOR, DIST, FL_DI, ... , DI_SQ_V
Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	13	150066	11543.5	25.34	0.000
Error	195	88827	455.5		
Total	208	238893			

Model Summary

S	R-sq	R-sq(adj)
21.3430	62.82%	60.34%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	207.2	14.4	14.41	0.000
FLOOR	-11.59	4.90	-2.37	0.019
DIST	-5.52	2.67	-2.07	0.040
FL_DI	-0.058	0.221	-0.26	0.794
FL_SQ	1.235	0.475	2.60	0.010
DI_SQ	0.464	0.155	2.99	0.003
VIEW	45.9	19.3	2.38	0.018
END	6.92	3.33	2.08	0.039
FURNISH	-20.16	8.79	-2.29	0.023
FL_V	0.45	6.54	0.07	0.945
DI_V	-0.12	3.53	-0.03	0.973
FL_DI_V	-0.362	0.298	-1.21	0.226
FL_SQ_V	-0.368	0.647	-0.57	0.570
DI_SQ_V	0.169	0.205	0.82	0.411

2. We define a new dummy variable for sales method: $x_6 = \begin{cases} 1 & \text{if sold at auction} \\ 0 & \text{if sold at fixed price} \end{cases}$

First, fit a Model 3 with the dummy variable for sales method added. This model indicated that adding sales method to Model 3 made a significant improvement.

Regression Analysis: PRICE100 versus FLOOR, DIST, FL_DI, ... _V, SOLD1

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	14	167924	11994.6	32.79	0.000
Error	194	70968	365.8		
Total	208	238893			

Model Summary

S	R-sq	R-sq(adj)
19.1263	70.29%	68.15%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	188.7	13.2	14.35	0.000
FLOOR	-4.61	4.50	-1.03	0.306
DIST	-2.35	2.43	-0.97	0.335
FL_DI	-0.335	0.202	-1.66	0.099
FL_SQ	1.019	0.427	2.39	0.018
DI_SQ	0.291	0.141	2.06	0.041
VIEW	74.3	17.7	4.19	0.000
END	7.96	2.99	2.66	0.008
FURNISH	-16.28	7.90	-2.06	0.041
FL_V	-4.77	5.90	-0.81	0.420
DI_V	-1.63	3.17	-0.51	0.608
FL_DI_V	0.075	0.274	0.27	0.785
FL_SQ_V	-0.267	0.580	-0.46	0.646
DI_SQ_V	0.133	0.184	0.73	0.469
SOLD1	-25.59	3.66	-6.99	0.000

Notice that there are several terms in the model which are not significant, namely the interaction terms with view. Since some of these terms are correlated with each other, we will delete the third-order terms containing view.

Regression Analysis: PRICE100 versus FLOOR, DIST, FL_DI, ... FL_V, DI_V

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	11	167621	15238.3	42.12	0.000
Error	197	71272	361.8		
Total	208	238893			

Model Summary

S	R-sq	R-sq(adj)
19.0207	70.17%	68.50%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	190.80	9.26	20.60	0.000
FLOOR	-3.74	3.10	-1.21	0.229
DIST	-3.70	1.55	-2.39	0.018
FL_DI	-0.286	0.130	-2.19	0.030
FL_SQ	0.885	0.296	2.99	0.003
DI_SQ	0.3692	0.0923	4.00	0.000
VIEW	71.19	7.89	9.03	0.000
END	7.72	2.92	2.64	0.009
FURNISH	-17.42	7.72	-2.26	0.025
SOLD1	-25.51	3.54	-7.20	0.000
FL_V	-6.63	1.17	-5.65	0.000
DI_V	0.710	0.594	1.20	0.233

Notice by removing the third-order terms, the estimate of *s* has decreased and the adjusted R-squared value has increased.

Next, the term DI_V is removed and interaction terms involving method of sales are added.

Regression Analysis: PRICE100 versus FLOOR, DIST, FL_DI, ... D, SO_FUR

The following terms cannot be estimated and were removed: SO_FUR

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	14	179952	12853.7	42.31	0.000
Error	194	58941	303.8		
Total	208	238893			

Model Summary

S	R-sq	R-sq(adj)
17.4304	75.33%	73.55%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	191.53	8.56	22.38	0.000
FLOOR	-6.38	2.93	-2.18	0.031
DIST	-2.64	1.50	-1.75	0.081
FL_DI	0.073	0.163	0.45	0.653
FL_SQ	0.629	0.304	2.07	0.040
DI_SQ	0.2753	0.0883	3.12	0.002
VIEW	79.99	6.21	12.87	0.000
END	0.32	4.36	0.07	0.942
FURNISH	-14.89	7.18	-2.07	0.039
SOLD1	-14.5	11.2	-1.30	0.197
FL_V	-4.59	1.50	-3.06	0.003
FL_SO	2.69	1.89	1.42	0.157
DI_SO	-2.362	0.729	-3.24	0.001
SO_V	-22.12	6.84	-3.24	0.001
SO_END	12.96	5.66	2.29	0.023

This model has the highest adjusted R-squared value and the smallest value of s among all the models fit. Other models may fit equally well. Note that the interaction term between method of sales and whether the unit was furnished (SO_FUR) could not be estimated and was removed from the analysis. This is because all furnished condos were sold at auction. Thus, the interaction term between method of sales and whether the unit was furnished is perfectly correlated with the furnished variable.

Modeling Daily Peak Electricity Demands

Case Study **6**

1. $$y_t = \beta_0 + \beta_1(x_{1t}-59)x_{2t} + \beta_2(x_{1t}-78)x_{3t} + \beta_3 x_{4t} + \beta_4 x_{5t} + \beta_5(x_{1t}-59)x_{2t}x_{4t}$$
$$+ \beta_6(x_{1t}-59)x_{2t}x_{5t} + \beta_7(x_{1t}-78)x_{3t}x_{4t} + \beta_8(x_{1t}-78)x_{3t}x_{5t}$$

For weekdays: $(x_{4t}=0,\ x_{5t}=0)$

$$\begin{aligned} y_t &= \beta_0 + \beta_1(x_{1t}-59)x_{2t} + \beta_2(x_{1t}-78)x_{3t} + \beta_3(0) + \beta_4(0) + \beta_5(x_{1t}-59)x_{2t}(0) \\ &\quad + \beta_6(x_{1t}-59)x_{2t}(0) + \beta_7(x_{1t}-78)x_{3t}(0) + \beta_8(x_{1t}-78)x_{3t}(0) \\ &= \beta_0 + \beta_1(x_{1t}-59)x_{2t} + \beta_2(x_{1t}-78)x_{3t} \end{aligned}$$

Similarly, for Saturdays: $(x_{4t}=1,\ x_{5t}=0)$

$$\begin{aligned} y_t &= \beta_0 + \beta_1(x_{1t}-59)x_{2t} + \beta_2(x_{1t}-78)x_{3t} + \beta_3(1) + \beta_4(0) + \beta_5(x_{1t}-59)x_{2t}(1) \\ &\quad + \beta_6(x_{1t}-59)x_{2t}(0) + \beta_7(x_{1t}-78)x_{3t}(1) + \beta_8(x_{1t}-78)x_{3t}(0) \\ &= (\beta_0+\beta_3) + (\beta_1+\beta_5)(x_{1t}-59)x_{2t} + (\beta_2+\beta_7)(x_{1t}-78)x_{3t} \end{aligned}$$

Similarly, for Sundays or Holidays: $(x_{4t}=0,\ x_{5t}=1)$

$$\begin{aligned} y_t &= \beta_0 + \beta_1(x_{1t}-59)x_{2t} + \beta_2(x_{1t}-78)x_{3t} + \beta_3(0) + \beta_4(1) + \beta_5(x_{1t}-59)x_{2t}(0) \\ &\quad + \beta_6(x_{1t}-59)x_{2t}(1) + \beta_7(x_{1t}-78)x_{3t}(0) + \beta_8(x_{1t}-78)x_{3t}(1) \\ &= (\beta_0+\beta_4) + (\beta_1+\beta_6)(x_{1t}-59)x_{2t} + (\beta_2+\beta_8)(x_{1t}-78)x_{3t} \end{aligned}$$

To determine if the interaction is significant, we test:

$H_0: \beta_5 = \beta_6 = \beta_7 = \beta_8 = 0$
$H_a:$ At least one $\beta_i \neq 0$

using a partial F test.

2. To forecast the $t=125$ peak demand, we will need an estimate for $\hat{R}_{124}$. Since none exists, we assume that $\hat{R}_{124} = -82.4$.

For $t=125$, $x_{1t}=45$, $x_{2t}=1$, $x_{3t}=0$, $x_{4t}=1$, and $x_{5t}=0$:

$$
\begin{aligned}
\hat{y}_{125} &= 2{,}812.967 - 65.337(45-59)(1) + 83.455(45-78)(0) - 130.828(1) \\
&\quad - 275.551(0) + 0.6475(-82.4) \\
&= 3{,}543.503
\end{aligned}
$$

The approximate 95% prediction interval for the peak demand expected is:

$$
\begin{aligned}
\hat{y}_{125} \pm 1.96\sqrt{MSE} &\Rightarrow 3{,}543.503 \pm 1.96\sqrt{27{,}687.4} \\
&\Rightarrow 3{,}543.503 \pm 326.135 \Rightarrow (3{,}217.368,\ 3{,}869.638)
\end{aligned}
$$

Voice Versus Face Recognition – Does One Follow the Other?

Case Study 7

1. Using MINITAB, the results of fitting the reduced model are:

Regression Analysis: SPEED versus x1, x2

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	2	1292458	646229	29.74	0.000
Error	37	804105	21733		
Total	39	2096563			

Model Summary

S	R-sq	R-sq(adj)
147.420	61.65%	59.57%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	1193.0	40.4	29.55	0.000
x1	-359.4	46.6	-7.71	0.000
x2	-8.8	46.6	-0.19	0.851

Using MINITAB, the results of fitting the complete model are:

Regression Analysis: SPEED versus x1, x2, X1X2

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	3	1443502	481167	26.52	0.000
Error	36	653061	18141		
Total	39	2096563			

Model Summary

S	R-sq	R-sq(adj)
134.687	68.85%	66.26%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	1131.5	42.6	26.57	0.000
x1	-236.5	60.2	-3.93	0.000
x2	114.1	60.2	1.89	0.066
x1x2	-245.8	85.2	-2.89	0.007

To determine if the interaction terms is significant, the test statistic is

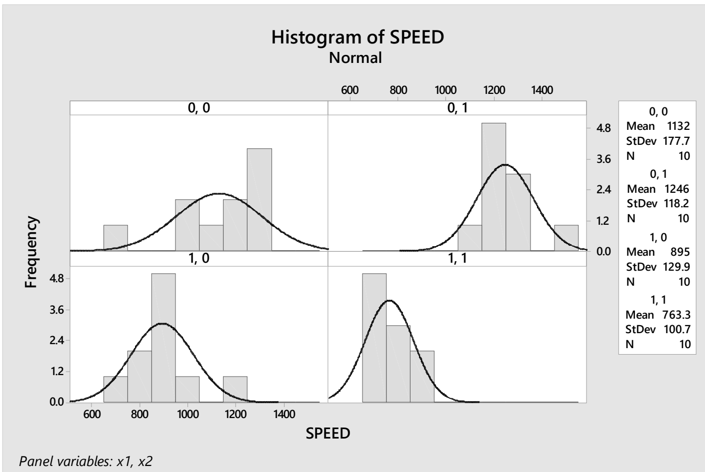
$$F = \frac{\left(SSE_R - SSE_C\right)/\left(k-g\right)}{MSE_c} = \frac{\left(804{,}105 - 653{,}061\right)/\left(3-2\right)}{18{,}141} = 8.33$$

This is the same value as found in CS7.3.

2. The assumptions for the factorial design are:

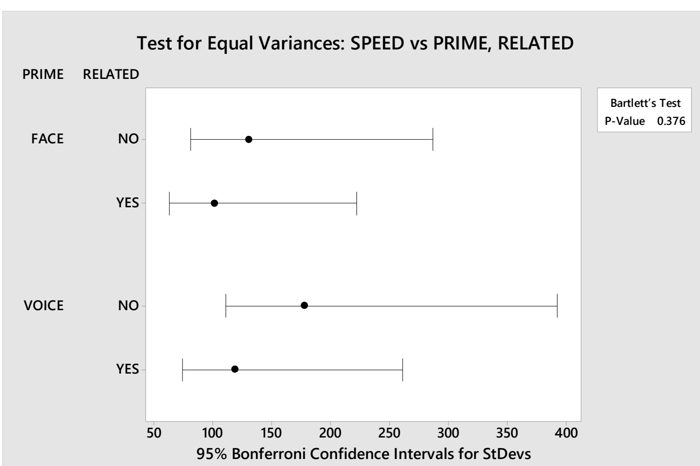
   1. The four probability distributions of response speeds corresponding to the four modality and relational pair combinations are normally distributed.
   2. The population variances of the response speed distributions are equal for the four modality and relational pair combinations.

First, we'll check the assumption of normality. For this design, we have enough observations in each of the four treatment combinations to check for normality in each. The histograms for each group are displayed below:



The graphs indicate distributions that are somewhat normal. Remember that the ANOVA procedures are fairly robust with respect to normality. That is, the analysis is valid even if the data are not exactly normally distributed.

We next perform Bartlett's test for the equality of variances. The MINITAB printout gives this test as one of the tests conducted in the analysis.



To determine if the variances differ, we test:

$$H_0 : \sigma_1^2 = \sigma_2^2 = \sigma_3^2 = \sigma_4^2$$
$$H_a : \text{ At least two variances differ}$$

The p-value is $p = 0.376$. Since the p-value is not small, H_0 is not rejected. There is insufficient evidence to indicate the variances are different at $\alpha = 0.05$.

The data are fairly normal and there is no evidence to indicate the variances differ. Thus, we should use the ANOVA procedures to analyze the data.

3. The design can be described two different ways. It could be considered a paired difference design or it could be described as a randomized complete block design with the subjects as the blocks and the type of pair as the treatments. Analyzing the data using either method will yield the same results. The F from the randomized block design for treatment will be equal to the square of the t from the paired difference analysis. We will use the randomized block design to analyze the data. Using MINITAB, the results for the accuracy rate are:

ANOVA: ACCURACY versus PAIR, SUB

Factor Information

Factor	Type	Levels	Values
PAIR	Fixed	2	RELATED, UNRELATED
SUB	Fixed	20	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20

Analysis of Variance for ACCURACY

Source	DF	SS	MS	F	P
PAIR	1	4.225	4.225	0.89	0.358
SUB	19	417.275	21.962	4.62	0.001
Error	19	90.275	4.751		
Total	39	511.775			

Model Summary

S	R-sq	R-sq(adj)
2.17975	82.36%	63.79%

Means

PAIR	N	ACCURACY
RELATED	20	90.50
UNRELATED	20	89.85

To determine if there is a difference in the mean accuracy rate between related pairs and unrelated pairs, we test:

$$H_0 : \mu_R = \mu_U$$
$$H_a : \mu_R \neq \mu_U$$

The test statistic is $F = 0.89$ and the p-value is $p = 0.358$. Since the p-value is not small, H_0 is not rejected. There is insufficient evidence to indicate a difference in the mean accuracy rate between related pairs and unrelated pairs at any reasonable value of α. The mean accuracy rate for related pairs is 90.50, while the mean accuracy rate for unrelated pairs is 89.85.

Using MINITAB, the results for the speed are:

ANOVA: SPEED versus PAIR, SUB

Factor Information

Factor	Type	Levels	Values
PAIR	Fixed	2	RELATED, UNRELATED
SUB	Fixed	20	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20

Analysis of Variance for SPEED

Source	DF	SS	MS	F	P
PAIR	1	201498	201498	20.29	0.000
SUB	19	174322	9175	0.92	0.567
Error	19	188650	9929		
Total	39	564471			

Model Summary

S	R-sq	R-sq(adj)
99.6442	66.58%	31.40%

Means

PAIR	N	SPEED
RELATED	20	1081.05
UNRELATED	20	1223.00

To determine if there is a difference in the mean response speed between related pairs and unrelated pairs, we test:

$$H_0 : \mu_R = \mu_U$$
$$H_a : \mu_R \neq \mu_U$$

The test statistic is $F = 20.29$ and the p-value is $p = 0.000$. Since the p-value is so small, H_0 is rejected. There is sufficient evidence to indicate a difference in the mean response speed between related pairs and unrelated pairs at any reasonable value of α. The mean response speed for related pairs is 1081.05 while the mean response speed for unrelated pairs is 1223.00.

Appendix

B

The Mechanics of Multiple Regression Analysis

B.1 a. Since **A** is a 2×2 matrix and **B** is a 2×2 matrix, the product **AB** will be a 2×2 matrix, which we will call **S**:

$$\mathbf{AB}=\mathbf{S}$$

$$\begin{bmatrix} 3 & 0 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix}=\begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix}$$

To determine the elements of **S** , the following calculations are required:

$$s_{11}=(3)(2)+(0)(0)=6$$
$$s_{12}=(3)(1)+(0)(-1)=3$$
$$s_{21}=(-1)(2)+(4)(0)=-2$$
$$s_{22}=(-1)(1)+(4)(-1)=-5$$

Hence, $\mathbf{AB}=\begin{bmatrix} 6 & 3 \\ -2 & -5 \end{bmatrix}$

b. $$\mathbf{AC}=\begin{bmatrix} 3 & 0 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 1 & 0 & 3 \\ -2 & 1 & 2 \end{bmatrix}=\begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \end{bmatrix}$$

where
$$s_{11}=(3)(1)+(0)(-2)=3$$
$$s_{12}=(3)(0)+(0)(1)=0$$
$$s_{13}=(3)(3)+(0)(2)=9$$
$$s_{21}=(-1)(1)+(4)(-2)=-9$$
$$s_{22}=(-1)(0)+(4)(1)=4$$
$$s_{23}=(-1)(3)+(4)(2)=5$$

Hence, $\mathbf{AC}=\begin{bmatrix} 3 & 0 & 9 \\ -9 & 4 & 5 \end{bmatrix}$

c. $$\mathbf{BA}=\begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 3 & 0 \\ -1 & 4 \end{bmatrix}=\begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix}$$

$$s_{11}=(2)(3)+(1)(-1)=5$$
$$s_{12}=(2)(0)+(1)(4)=4$$

$$s_{21}=(0)(3)+(-1)(-1)=1$$
$$s_{22}=(0)(0)+(-1)(4)=-4$$

Hence, $\mathbf{BA}=\begin{bmatrix}5 & 4\\ 1 & -4\end{bmatrix}$

B.2 a. $\mathbf{AC}=\begin{bmatrix}3 & 1 & 3\\ 2 & 0 & 4\\ -4 & 1 & 2\end{bmatrix}\begin{bmatrix}3\\0\\2\end{bmatrix}=\begin{bmatrix}3(3)+1(0)+3(2)\\ 2(3)+0(0)+4(2)\\ -4(3)+1(0)+2(2)\end{bmatrix}=\begin{bmatrix}15\\14\\-8\end{bmatrix}$

b. $\mathbf{BC}=\begin{bmatrix}1 & 0 & 2\end{bmatrix}\begin{bmatrix}3\\0\\2\end{bmatrix}=\begin{bmatrix}1(3)+0(0)+2(2)\end{bmatrix}=\begin{bmatrix}7\end{bmatrix}$

c. Since $\mathbf{A}$ is a 3×3 matrix and $\mathbf{B}$ is a 1×3, it is not possible to find $\mathbf{AB}$.

B.3 a. If matrix $\mathbf{A}$ has dimensions $r\times d$ and matrix $\mathbf{B}$ has dimensions $d\times c$, then the product matrix $\mathbf{AB}$ has dimensions $r\times c$. In this exercise, $r=3, d=2,$ and $c=4$:

$$\begin{array}{ccc}\mathbf{A} & \qquad & \mathbf{B}\\ 3\times2 & \qquad & 2\times4\end{array}$$

Thus, $\mathbf{AB}$ is a 3×4 matrix.

b. In order to multiply two matrices, the two inner dimension numbers must be equal:

$$\begin{array}{ccc}\mathbf{B} & \qquad & \mathbf{A}\\ 2\times4 & \qquad & 3\times2\end{array}$$

In this exercise, the number of columns of $\mathbf{B}$ (4) does not equal the number of rows of $\mathbf{A}$ (3); thus, the product $\mathbf{BA}$ does not exist.

B.4 a. $\mathbf{BC}$ is a 1×1 matrix $(1\times3)(3\times1)\Rightarrow(1\times1)$

b. $\mathbf{CB}$ is a 3×3 matrix $(3\times1)(1\times3)\Rightarrow(3\times3)$

c. $$\mathbf{CB} = \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 3(1) & 3(0) & 3(2) \\ 0(1) & 0(0) & 0(2) \\ 2(1) & 2(0) & 2(2) \end{bmatrix} = \begin{bmatrix} 3 & 0 & 6 \\ 0 & 0 & 0 \\ 2 & 0 & 4 \end{bmatrix}$$

B.5 a. $$\mathbf{AB} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -3 & 0 \\ 4 & -1 \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \\ s_{31} & s_{32} \end{bmatrix}$$

where
$$\begin{aligned} s_{11} &= (1)(2) + (0)(-3) + (0)(4) = 2 \\ s_{12} &= (1)(3) + (0)(0) + (0)(-1) = 3 \\ s_{21} &= (0)(2) + (3)(-3) + (0)(4) = -9 \\ s_{22} &= (0)(3) + (3)(0) + (0)(-1) = 0 \\ s_{31} &= (0)(2) + (0)(-3) + (2)(4) = 8 \\ s_{32} &= (0)(3) + (0)(0) + (2)(-1) = -2 \end{aligned}$$

Hence, $\mathbf{AB} = \begin{bmatrix} 2 & 3 \\ -9 & 0 \\ 8 & -2 \end{bmatrix}$

b. $$\mathbf{CA} = \begin{bmatrix} 3 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} & s_{13} \end{bmatrix}$$

where
$$\begin{aligned} s_{11} &= (3)(1) + (0)(0) + (2)(0) = 3 \\ s_{12} &= (3)(0) + (0)(3) + (2)(0) = 0 \\ s_{13} &= (3)(0) + (0)(0) + (2)(2) = 4 \end{aligned}$$

Thus, $\mathbf{CA} = \begin{bmatrix} 3 & 0 & 4 \end{bmatrix}$

c. $$\mathbf{CB} = \begin{bmatrix} 3 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -3 & 0 \\ 4 & -1 \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} \end{bmatrix}$$

where
$$\begin{aligned} s_{11} &= (3)(2) + (0)(-3) + (2)(4) = 14 \\ s_{12} &= (3)(3) + (0)(0) + (2)(-1) = 7 \end{aligned}$$

Thus, $\mathbf{CB} = \begin{bmatrix} 14 & 7 \end{bmatrix}$

B.6 a. $$\mathbf{AB}=\begin{bmatrix}3 & 0 & -1 & 2\end{bmatrix}\begin{bmatrix}2\\-1\\0\\3\end{bmatrix}=\begin{bmatrix}3(2)+0(-1)+(-1)(0)+2(3)\end{bmatrix}=\begin{bmatrix}12\end{bmatrix}$$

b. $$\mathbf{BA}=\begin{bmatrix}2\\-1\\0\\3\end{bmatrix}\begin{bmatrix}3 & 0 & -1 & 2\end{bmatrix}=\begin{bmatrix}2(3) & 2(0) & 2(-1) & 2(2)\\-1(3) & -1(0) & -1(-1) & -1(2)\\0(3) & 0(0) & 0(-1) & 0(2)\\3(3) & 3(0) & 3(-1) & 3(2)\end{bmatrix}=\begin{bmatrix}6 & 0 & -2 & 4\\-3 & 0 & 2 & -2\\0 & 0 & 0 & 0\\9 & 0 & -3 & 6\end{bmatrix}$$

B.7 a. To obtain the product $\mathbf{IA}$, the number of rows of $\mathbf{A}$ (2) must equal the number of columns of $\mathbf{I}$:

$$\begin{array}{ccc}\mathbf{I} & \qquad & \mathbf{A}\\ 2\times 2 & & 2\times 3\end{array}$$

An identity matrix is always square, so $\mathbf{I}$ will have 2 rows and 2 columns:

$$\mathbf{I}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

b. $$\mathbf{IA}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}3 & 0 & 2\\-1 & 1 & 4\end{bmatrix}=\begin{bmatrix}3 & 0 & 2\\-1 & 1 & 4\end{bmatrix}=\mathbf{A}$$

c. To find the product $\mathbf{AI}$, the number of rows and columns of $\mathbf{I}$ must equal the number of columns of $\mathbf{A}$ (3):

$$\begin{array}{ccc}\mathbf{A} & \qquad & \mathbf{I}\\ 2\times 3 & & 3\times 3\end{array}$$

Hence, $\mathbf{I}$ is the 3×3 identity matrix.

$$\mathbf{I}=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

d. $$\mathbf{AI}=\begin{bmatrix}3 & 0 & 2\\-1 & 1 & 4\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}=\begin{bmatrix}3 & 0 & 2\\-1 & 1 & 4\end{bmatrix}=\mathbf{A}$$

B.8 a. $$\mathbf{AB} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{BA} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since $\mathbf{AB} = \mathbf{I} = \mathbf{BA} \Rightarrow \mathbf{A} = \mathbf{B}^{-1}$ and $\mathbf{B} = \mathbf{A}^{-1}$

B.9 It is necessary to show that $\mathbf{AA}^{-1} = \mathbf{I}$ and $\mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$:

$$\mathbf{AA}^{-1} = \begin{bmatrix} 12 & 0 & 0 & 8 \\ 0 & 12 & 0 & 0 \\ 0 & 0 & 8 & 0 \\ 8 & 0 & 0 & 8 \end{bmatrix} \begin{bmatrix} 1/4 & 0 & 0 & -1/4 \\ 0 & 1/12 & 0 & 0 \\ 0 & 0 & 1/8 & 0 \\ -1/4 & 0 & 0 & 3/8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \mathbf{I}$$

where $\mathbf{I}$ is the 4×4 identity matrix. Similarly, it is easy to verify that $\mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$.

B.10 From Theorem B.1, the inverse of a diagonal matrix.

$$\mathbf{D} = \begin{bmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{33} \end{bmatrix} \text{ is } \mathbf{D}^{-1} = \begin{bmatrix} \frac{1}{d_{11}} & 0 & 0 \\ 0 & \frac{1}{d_{22}} & 0 \\ 0 & 0 & \frac{1}{d_{33}} \end{bmatrix}$$

$$\Rightarrow \mathbf{A}^{-1} = \begin{bmatrix} \frac{1}{a_{11}} & 0 & 0 \\ 0 & \frac{1}{a_{22}} & 0 \\ 0 & 0 & \frac{1}{a_{33}} \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{5} & 0 \\ 0 & 0 & \frac{1}{7} \end{bmatrix}$$

B.11 It is necessary to show that $\mathbf{DD}^{-1} = \mathbf{I}$ and $\mathbf{D}^{-1}\mathbf{D} = \mathbf{I}$:

$$\mathbf{DD}^{-1} = \begin{bmatrix} d_{11} & 0 & 0 & \dots & 0 \\ 0 & d_{22} & 0 & \dots & 0 \\ 0 & 0 & d_{33} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & d_{nn} \end{bmatrix} \begin{bmatrix} 1/d_{11} & 0 & 0 & \dots & 0 \\ 0 & 1/d_{22} & 0 & \dots & 0 \\ 0 & 0 & 1/d_{33} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1/d_{nn} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix} = \mathbf{I}$$

where $\mathbf{I}$ is the $n \times n$ identity matrix. Upon multiplication, we find that $\mathbf{DD}^{-1} = \mathbf{I}$ when all $d_{ii} \neq 0$. Similarly, it is easy to verify that $\mathbf{D}^{-1}\mathbf{D} = \mathbf{I}$ when all $d_{ii} \neq 0$.

B.12 To verify Theorem B.2, we will show that $\mathbf{AA}^{-1} = \mathbf{I}$

$$\mathbf{AA}^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} \frac{d}{ad-bc} & \frac{-b}{ad-bc} \\ \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{bmatrix} = \begin{bmatrix} \frac{ad-bc}{ad-bc} & \frac{ab-ab}{ad-bc} \\ \frac{cd-cd}{ad-bc} & \frac{-bc+ad}{ad-bc} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

B.13 The inverse of a 2×2 matrix

$$\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{ is } \mathbf{A}^{-1} = \begin{bmatrix} \frac{d}{ad-bc} & \frac{-b}{ad-bc} \\ \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{bmatrix}$$

Thus, for $\mathbf{A} = \begin{bmatrix} 2 & -1 \\ 2 & 3 \end{bmatrix}$

$$\mathbf{A}^{-1} = \begin{bmatrix} \frac{3}{2(3)-(-1)(2)} & \frac{-(-1)}{2(3)-(-1)(2)} \\ \frac{-2}{2(3)-(-1)(2)} & \frac{2}{2(3)-(-1)(2)} \end{bmatrix} = \begin{bmatrix} 3/8 & 1/8 \\ -1/4 & 1/4 \end{bmatrix}$$

B.14 a. $\mathbf{A} = \begin{bmatrix} 3 & 1 \\ 1 & -1 \end{bmatrix}$ $\quad \mathbf{V} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ $\quad \mathbf{G} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$

b. $$\mathbf{AA}^{-1} = \begin{bmatrix} 3 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} \frac{-1}{-4} & \frac{-1}{-4} \\ \frac{-1}{-4} & \frac{3}{-4} \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{-3}{4} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\mathbf{A}^{-1}\mathbf{A} = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{-3}{4} \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Since $\mathbf{AA}^{-1} = \mathbf{I} = \mathbf{A}^{-1}\mathbf{A} \Rightarrow \ \mathbf{A}^{-1}$ is the inverse of $\mathbf{A}$.

c. $\mathbf{V} = \mathbf{A}^{-1}G = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{-3}{4} \end{bmatrix} \begin{bmatrix} 5 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

B.15 We will first rewrite the system of equations in standard form:

$$\begin{aligned} 10v_1 + 0v_2 + 20v_3 &= 60 \\ 0v_1 + 20v_2 + 0v_3 &= 60 \\ 20v_1 + 0v_2 + 68v_3 &= 176 \end{aligned}$$

a. $\mathbf{A}$ is the matrix of coefficients of the V's:

$$\mathbf{A} = \begin{bmatrix} 10 & 0 & 20 \\ 0 & 20 & 0 \\ 20 & 0 & 68 \end{bmatrix} \quad \mathbf{V} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \quad \mathbf{G} = \begin{bmatrix} 60 \\ 60 \\ 176 \end{bmatrix}$$

b. $\mathbf{AA}^{-1} = \begin{bmatrix} 10 & 0 & 20 \\ 0 & 20 & 0 \\ 20 & 0 & 68 \end{bmatrix} \begin{bmatrix} 17/70 & 0 & -1/14 \\ 0 & 1/20 & 0 \\ -1/14 & 0 & 1/28 \end{bmatrix}$

$$= \begin{bmatrix} 17/7 + 0 - 10/7 & 0+0+0 & -5/7 + 0 + 5/7 \\ 0+0+0 & 0+1+0 & 0+0+0 \\ 34/7 + 0 - 34/7 & 0+0+0 & -10/7 + 0 + 17/7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

c. $\mathbf{V} = \mathbf{A}^{-1}\mathbf{G} = \begin{bmatrix} 17/70 & 0 & -1/14 \\ 0 & 1/20 & 0 \\ -1/14 & 0 & 1/28 \end{bmatrix} \begin{bmatrix} 60 \\ 60 \\ 176 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix}$

The solution may be verified by substitution of $v_1 = 2, v_2 = 3,$ and $v_3 = 2$ into the original system of equations:

$$\begin{aligned} &10(2) + 20(2) - 60 = 0 \\ &20(3) - 60 = 0 \\ &20(2) + 68(2) - 176 = 0 \end{aligned}$$

B.16 a. **Y** is a 5×1 column matrix and **X** is a 5×2 data matrix:

$$\mathbf{Y}=\begin{bmatrix}4\\3\\3\\1\\-1\end{bmatrix} \qquad \mathbf{X}=\begin{bmatrix}1&-2\\1&-1\\1&0\\1&1\\1&2\end{bmatrix}$$

b. $$\mathbf{X'X}=\begin{bmatrix}1&1&1&1&1\\-2&-1&0&1&2\end{bmatrix}\begin{bmatrix}1&-2\\1&-1\\1&0\\1&1\\1&2\end{bmatrix}=\begin{bmatrix}5&0\\0&10\end{bmatrix}$$

$$\mathbf{X'Y}=\begin{bmatrix}1&1&1&1&1\\-2&-1&0&1&2\end{bmatrix}\begin{bmatrix}4\\3\\3\\1\\-1\end{bmatrix}=\begin{bmatrix}10\\-12\end{bmatrix}$$

c. $$\left(\mathbf{X'X}\right)^{-1}=\begin{bmatrix}\frac{1}{5}&0\\0&\frac{1}{10}\end{bmatrix} \quad \text{(from Theorem B.1)}$$

$$\hat{\boldsymbol{\beta}}=\left(\mathbf{X'X}\right)^{-1}\mathbf{X'Y}=\begin{bmatrix}\frac{1}{5}&0\\0&\frac{1}{10}\end{bmatrix}\begin{bmatrix}10\\-12\end{bmatrix}=\begin{bmatrix}2\\-1.2\end{bmatrix}$$

d. $\hat{y}=\hat{\beta}_0+\hat{\beta}_1x=2-1.2x$

B.17 a. **Y** is a 6×1 column matrix and **X** is a 6×2 data matrix:

$$\mathbf{Y}=\begin{bmatrix}1\\2\\2\\3\\5\\5\end{bmatrix} \qquad \mathbf{X}=\begin{bmatrix}1&1\\1&2\\1&3\\1&4\\1&5\\1&6\end{bmatrix}$$

b. $$\mathbf{X'X} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \\ 1 & 5 \\ 1 & 6 \end{bmatrix} = \begin{bmatrix} 6 & 21 \\ 21 & 91 \end{bmatrix}$$

$$\mathbf{X'Y} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 2 \\ 3 \\ 5 \\ 5 \end{bmatrix} = \begin{bmatrix} 18 \\ 78 \end{bmatrix}$$

c. $$\left(\mathbf{XX}\right)^{-1}\left(\mathbf{X'X}\right) = \begin{bmatrix} 13/15 & -7/35 \\ -7/35 & 2/35 \end{bmatrix} \begin{bmatrix} 6 & 21 \\ 21 & 91 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \mathbf{I};$$

similarly, $\left(\mathbf{X'X}\right)\left(\mathbf{X'X}\right)^{-1} = \mathbf{I}$

d. To obtain the least squares estimates, we compute $\hat{\boldsymbol{\beta}} = \left(\mathbf{X'X}\right)^{-1}\mathbf{X'Y}$:

$$\hat{\boldsymbol{\beta}} = \begin{bmatrix} 13/15 & -7/35 \\ -7/35 & 2/35 \end{bmatrix} \begin{bmatrix} 18 \\ 78 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.8571 \end{bmatrix}$$

Thus, $\hat{\beta}_0 = 0$ and $\hat{\beta}_1 = 0.8571$.

e. The prediction equation is:

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x = 0 + 0.8571x = 0.8571x$$

The identical solution was obtained in Exercise 3.6.

B.18 a. $\mathbf{Y}$ is a 10×1 column matrix and $\mathbf{X}$ is a 10×3 data matrix:

b. $$\mathbf{X'X} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -2 & -2 & -1 & -1 & 0 & 0 & 1 & 1 & 2 & 2 \\ 4 & 4 & 1 & 1 & 0 & 0 & 1 & 1 & 4 & 4 \end{bmatrix} \begin{bmatrix} 1 & -2 & 4 \\ 1 & -2 & 4 \\ 1 & -1 & 1 \\ 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 10 & 0 & 20 \\ 0 & 20 & 0 \\ 20 & 0 & 68 \end{bmatrix}$$

$$(\mathbf{X'X})(\mathbf{X'X})^{-1} = \begin{bmatrix} 10 & 0 & 20 \\ 0 & 20 & 0 \\ 20 & 0 & 68 \end{bmatrix} \begin{bmatrix} \frac{17}{70} & 0 & \frac{-1}{14} \\ 0 & \frac{1}{20} & 0 \\ \frac{-1}{14} & 0 & \frac{1}{28} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

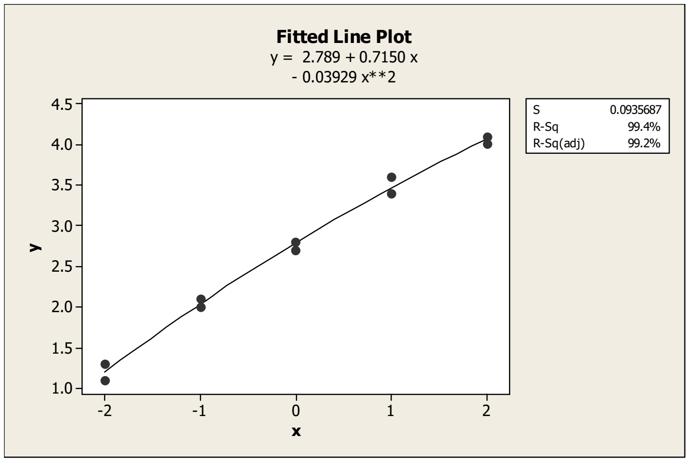
c. A symmetric matrix, $\mathbf{A}$, is one in which $\mathbf{A} = \mathbf{A}'$.

d. $$\mathbf{X'Y} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -2 & -2 & -1 & -1 & 0 & 0 & 1 & 1 & 2 & 2 \\ 4 & 4 & 1 & 1 & 0 & 0 & 1 & 1 & 4 & 4 \end{bmatrix} \begin{bmatrix} 1.1 \\ 1.3 \\ 2.0 \\ 2.1 \\ 2.7 \\ 2.8 \\ 3.4 \\ 3.6 \\ 4.1 \\ 4.0 \end{bmatrix} = \begin{bmatrix} 27.1 \\ 14.3 \\ 53.1 \end{bmatrix}$$

$$\hat{\boldsymbol{\beta}} = (\mathbf{X'X})^{-1}\mathbf{X'Y} = \begin{bmatrix} \frac{17}{70} & 0 & \frac{-1}{14} \\ 0 & \frac{1}{20} & 0 \\ \frac{-1}{14} & 0 & \frac{1}{28} \end{bmatrix} \begin{bmatrix} 27.1 \\ 14.3 \\ 53.1 \end{bmatrix} = \begin{bmatrix} 2.789 \\ 0.715 \\ -0.039 \end{bmatrix}$$

$$\hat{y} = 2.789 + 0.715x - 0.039x^2$$

e.



B.19 From Exercise B.16,

$$\mathbf{X}=\begin{bmatrix}1 & -2\\1 & -1\\1 & 0\\1 & 1\\1 & 2\end{bmatrix}\qquad \mathbf{Y}=\begin{bmatrix}4\\3\\3\\1\\-1\end{bmatrix}\qquad \mathbf{X'X}=\begin{bmatrix}5 & 0\\0 & 10\end{bmatrix}$$

$$\left(\mathbf{X'X}\right)^{-1}=\begin{bmatrix}1/5 & 0\\0 & 1/10\end{bmatrix}\qquad \mathbf{X'Y}=\begin{bmatrix}10\\-12\end{bmatrix}\qquad \hat{\boldsymbol{\beta}}=\begin{bmatrix}2\\-1.2\end{bmatrix}$$

We want to test:

$$H_0:\beta_1=0$$
$$H_a:\beta_1\neq 0$$

Test statistic: $t=\dfrac{\hat{\beta}_1}{s\sqrt{c_{11}}}$

where $s=\sqrt{\dfrac{SSE}{n-(k+1)}}$, $SSE=\mathbf{Y'Y}-\hat{\boldsymbol{\beta}}'\mathbf{X'Y}$, and c_{11} is obtained from the $\left(\mathbf{X'X}\right)^{-1}$ matrix.

We now compute $\mathbf{Y'Y}=\begin{bmatrix}4 & 3 & 3 & 1 & -1\end{bmatrix}\begin{bmatrix}4\\3\\3\\1\\-1\end{bmatrix}=36$;

then, $\boldsymbol{\beta}'\mathbf{X'Y}=\begin{bmatrix}2 & -1.2\end{bmatrix}\begin{bmatrix}10\\-12\end{bmatrix}=34.4$

Hence, $SSE=\mathbf{Y'Y}-\hat{\boldsymbol{\beta}}'\mathbf{X'Y}=36-34.4=1.6$

and $s = \sqrt{\dfrac{SSE}{n-(k+1)}} = \sqrt{\dfrac{1.6}{5-(1+1)}} = \sqrt{0.5333} = 0.7303$

If $\mathbf{X'X}^{-1} = \begin{bmatrix} 1/5 & 0 \\ 0 & 1/10 \end{bmatrix}$, then $c_{11} = 1/10$.

The test statistic is computed as follows: $t = \dfrac{-1.2}{0.7303\sqrt{1/10}} = -5.196$

Rejection region: $\alpha = 0.05,\ df = 3,\ t_{0.025} = 3.182;$
Reject H_0 if $t < -3.182$ or $t > 3.182$

Conclusion: Reject H_0 at $\alpha = 0.05$. There is sufficient evidence that x contributes information for the prediction of y.

B.20 From Exercise B.19, $s = 0.7303$ and $c_{11} = \dfrac{1}{10}$.

The form of the interval is $\hat{\beta}_i \pm t_{\alpha/2} s\sqrt{c_{ii}}$

For confidence coefficient 0.90, $\alpha = 0.10$ and $\alpha/2 = 0.10/2 = 0.05$. From Table 2, Appendix D, with $df = n-(k+1) = 5-2 = 3$, $t_{0.05} = 2.353$. The 90% confidence interval is

$$\hat{\beta}_1 \pm t_{\alpha/2} s\sqrt{c_{11}} \Rightarrow -1.2 \pm 2.353(0.7303)\sqrt{\frac{1}{10}} \Rightarrow -1.2 \pm 0.543 \Rightarrow (-1.743,\ -0.657)$$

B.21 From Exercise B.18,

$$\mathbf{X} = \begin{bmatrix} 1 & -2 & 4 \\ 1 & -2 & 4 \\ 1 & -1 & 1 \\ 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 2 & 4 \end{bmatrix} \qquad \mathbf{Y} = \begin{bmatrix} 1.1 \\ 1.3 \\ 2.0 \\ 2.1 \\ 2.7 \\ 2.8 \\ 3.4 \\ 3.6 \\ 4.1 \\ 4.0 \end{bmatrix} \qquad \mathbf{X'X} = \begin{bmatrix} 10 & 0 & 20 \\ 0 & 20 & 0 \\ 20 & 0 & 68 \end{bmatrix}$$

$$\mathbf{X'X}^{-1} = \begin{bmatrix} 17/70 & 0 & -1/14 \\ 0 & 1/20 & 0 \\ -1/14 & 0 & 1/28 \end{bmatrix} \qquad \mathbf{X'Y} = \begin{bmatrix} 27.1 \\ 14.3 \\ 53.1 \end{bmatrix} \qquad \hat{\boldsymbol{\beta}} = \begin{bmatrix} 2.7885714 \\ 0.715 \\ -0.03928571 \end{bmatrix}$$

$H_0: \beta_2 = 0$
$H_a: \beta_2 \neq 0$

Test statistic: $t = \dfrac{\hat{\beta}_2}{s\sqrt{c_{22}}}$

where $s = \sqrt{\dfrac{SSE}{n-(k+1)}}$, $SSE = \mathbf{Y'Y} - \hat{\boldsymbol{\beta}}'\mathbf{X'Y}$, and c_{22} is obtained from the $(\mathbf{X'X})^{-1}$ matrix.

$$\mathbf{Y'Y} = \begin{bmatrix}1.1 & 1.3 & 2.0 & 2.1 & 2.7 & 2.8 & 3.4 & 3.6 & 4.1 & 4.0\end{bmatrix}\begin{bmatrix}1.1\\1.3\\2.0\\2.1\\2.7\\2.8\\3.4\\3.6\\4.1\\4.0\end{bmatrix} = 83.77$$

$$\hat{\boldsymbol{\beta}}'\mathbf{X'Y} = \begin{bmatrix}2.7885714 & 0.715 & -0.03928571\end{bmatrix}\begin{bmatrix}27.1\\14.3\\53.1\end{bmatrix} = 83.708714$$

Hence, $SSE = \mathbf{Y'Y} - \boldsymbol{\beta}'\mathbf{X'Y} = 83.77 - 83.708714 = 0.061286$

and $s = \sqrt{\dfrac{SSE}{n-(k+1)}} = \sqrt{\dfrac{0.061286}{10-(2+1)}} = \sqrt{0.0087551} = 0.093569$

The test statistic is computed as follows: $t = \dfrac{-0.0392857}{0.093569\sqrt{1/28}} = -2.22$

Rejection region: $\alpha = 0.10$, $df = 7$, $t_{0.05} = 1.895$;
Reject H_0 if $t < -1.895$ or $t > 1.895$

Conclusion: Reject H_0 at $\alpha = 0.10$. There is sufficient evidence to indicate curvature in the model for $E(y)$.

B.22 The 90% confidence interval is given by $\hat{y} \pm t_{0.05} s\sqrt{a'(\mathbf{X'X})^{-1} a}$

where $\hat{y} = 2 - 1.2(1) = 0.8,\ t_{0.05} = 2.353\ (3df),\ s = 0.7307$ (From Exercise B.19), $a = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$,

and $\mathbf{X'X} = \begin{bmatrix} 5 & 0 \\ 0 & 10 \end{bmatrix}$ (From Exercise B.16).

Now, $a'(\mathbf{X'X})^{-1} a = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{5} & 0 \\ 0 & \frac{1}{10} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 0.3$

The 90% confidence interval is

$$\hat{y} \pm t_{0.05} s \sqrt{a(\mathbf{X'X})^{-1} a} \Rightarrow 0.8 \pm 2.353(0.7307)\sqrt{0.3} \Rightarrow 0.8 \pm 0.942 \Rightarrow (-0.142, 1.742)$$

For all values of $x = 1$, we are 90% confident that the mean value of y will fall between -0.142 and 1.742.

B.23 The 90% prediction interval is given by $\hat{y} \pm t_{0.05} s \sqrt{1 + a'(\mathbf{X'X})^{-1} a}$

where $\hat{y} = 2 - 1.2(1) = 0.8,\ t_{0.05} = 2.353\ (3df),\ s = 0.7307$ (From Exercise B.19), $a = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$,

and $\mathbf{X'X} = \begin{bmatrix} 5 & 0 \\ 0 & 10 \end{bmatrix}$ (From Exercise B.16).

Now, $a'(\mathbf{X'X})^{-1} a = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 1/5 & 0 \\ 0 & 1/10 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 0.3$

and substitution yields the desired interval:

$$0.8 \pm 2.353(0.7303)\sqrt{1 + 0.3} \Rightarrow 0.8 \pm 1.959 \Rightarrow (-1.159,\ 2.759)$$

For a future trial of the experiment with $x = 1$, we predict that the y-value will lie between –
-1.159 and 2.759 with 90% confidence.

B.24 The 90% confidence interval is given by $\hat{y} \pm t_{0.05} s \sqrt{a'(\mathbf{X'X})^{-1} a}$

where $\hat{y} = 0.8571(2) = 1.7142,\ t_{0.05} = 2.132\ (4df),\ a = \begin{bmatrix} 1 \\ 2 \end{bmatrix},\ \mathbf{X'X} = \begin{bmatrix} 6 & 21 \\ 21 & 91 \end{bmatrix}$,

$\hat{\boldsymbol{\beta}} = \begin{bmatrix} 0 \\ 0.8571 \end{bmatrix},\ \mathbf{X'Y} = \begin{bmatrix} 18 \\ 78 \end{bmatrix}$. (From Exercise B.17).

Also, $\mathbf{Y'Y} = [1\ 2\ 2\ 3\ 5\ 5]\begin{bmatrix}1\\2\\2\\3\\5\\5\end{bmatrix} = 68$

Thus, $SSE = \mathbf{Y'Y} - \hat{\boldsymbol{\beta}}'\mathbf{X'Y} = 68 - [0\ \ 0.8571]\begin{bmatrix}18\\78\end{bmatrix} = 68 - 66.8538 = 1.1462$

and $s = \sqrt{\dfrac{SSE}{n-(k+1)}} = \sqrt{\dfrac{1.1462}{6-(1+1)}} = 0.535304$

Now, $a'(\mathbf{X'X})^{-1}a = [1\ \ 2]\begin{bmatrix}\frac{13}{15} & \frac{-7}{35}\\ \frac{-7}{35} & \frac{2}{35}\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix} = 0.295238$

The 90% confidence interval is

$$\hat{y} \pm t_{0.05}s\sqrt{a'(\mathbf{X'X})^{-1}a} \Rightarrow 1.7142 \pm 2.132(0.535304)\sqrt{0.295238}$$
$$\Rightarrow 1.7142 \pm 0.6201 \Rightarrow (1.0941,\ 2.3343)$$

For all values of $x = 2$, we are 90% confident that the mean value of y will fall between 1.0941 and 2.3343.

B.25 The 90% prediction interval is given by $\hat{y} \pm t_{0.05}s\sqrt{1 + a'(\mathbf{X'X})^{-1}a}$

where $\hat{y} = 0.8571(2) = 1.7142$, $t_{0.05} = 2.132(4\ df)$,

$$s = \sqrt{\frac{SSE}{n-(k+1)}} = \sqrt{\frac{(\mathbf{Y'Y} - \hat{\boldsymbol{\beta}}'\mathbf{X'Y})}{n-(k+1)}} = \sqrt{\frac{68 - 66.8538}{4}} = \sqrt{\frac{1.1462}{4}} = 0.535304$$

$a = \begin{bmatrix}1\\2\end{bmatrix}$ and $\mathbf{X'X}$ is obtained from Exercise B.17.

Now, $a'(\mathbf{X'X})^{-1}a = [1\ \ 2]\begin{bmatrix}13/15 & -7/35\\ -7/35 & 2/35\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix} = 0.295238$

and substitution yields the desired interval:

$$1.7142 \pm 2.132(0.535304)\sqrt{1 + 0.295238} \Rightarrow 1.7142 \pm 1.29886 \Rightarrow (0.41534,\ 3.01306)$$

For a future trial of the experiment with $x = 2,$ we predict that the y-value will lie between 0.42 and 3.01, with 90% confidence.

B.26 The 90% confidence interval is given by $\hat{y} \pm t_{0.05} s \sqrt{a'(\mathbf{X'X})^{-1} a}$

where $\hat{y} = 2.789 + 0.715(1) - 0.039(1)^2 = 3.465,\ t_{0.05} = 1.895\ (7\,df),\ s = 0.093569$ (From Exercise B.21), $a = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, and $(\mathbf{X'X}) = \begin{bmatrix} 10 & 0 & 20 \\ 0 & 20 & 0 \\ 20 & 0 & 68 \end{bmatrix}$ (From Exercise B.18).

$$\text{Now, } a'(\mathbf{X'X})^{-1} a = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{17}{70} & 0 & \frac{-1}{14} \\ 0 & \frac{1}{20} & 0 \\ \frac{-1}{14} & 0 & \frac{1}{28} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 0.1857143$$

The 90% confidence interval is

$$\hat{y} \pm t_{0.05} s \sqrt{a'(\mathbf{X'X})^{-1} a} \Rightarrow 3.465 \pm 1.895(0.093569)\sqrt{0.1857143}$$
$$\Rightarrow 3.465 \pm 0.0764 \Rightarrow (3.3886,\ 3.5414)$$

For all values of $x = 1,$ we are 90% confident that the mean value of y will fall between 3.3886 and 3.5414.

B.27 The 90% prediction interval is given by $\hat{y} \pm t_{0.05} s \sqrt{1 + a'(\mathbf{X'X})^{-1} a}$

where $\hat{y} = 2.789 + 0.715(1) - 0.039(1)^2 = 3.465,\ t_{0.05} = 1.895(7\ df),$
$s = 0.093569$ (from Exercises B.18 and B.21)

$$\text{and } a'(\mathbf{X'X})^{-1} a = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 17/70 & 0 & -1/14 \\ 0 & 1/20 & 0 \\ -1/14 & 0 & 1/28 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 0.1857143$$

Thus, the required interval is:

$$3.465 \pm 1.895(0.093569)\sqrt{1 + 0.857143} \Rightarrow 3.465 \pm 0.1931 \Rightarrow (3.2719,\ 3.6581)$$

For a future trial of the experiment with $x = 1,$ we predict that the y-value will lie between 3.2719 and 3.6581, with 90% confidence.

B.28 a. $s^2 = \dfrac{SSE}{n - (k+1)} = \dfrac{26.62}{25 - (2+1)} = 1.21$

b. The 95% confidence interval is given by $\hat{y} \pm t_{0.025} s\sqrt{a'(\mathbf{X'X})^{-1} a}$

where $\hat{y} = 2.08 + 8.42(1.10) - 1.65(1.10)^2 = 9.3455$, $t_{0.025} = 2.074\ (22\,df)$, $s = 1.1$,

and $a = \begin{bmatrix} 1 \\ 1.1 \\ 1.21 \end{bmatrix}$.

$$\text{Now, } a'(\mathbf{X'X})^{-1} a = \begin{bmatrix} 1 & 1.1 & 1.21 \end{bmatrix} \begin{bmatrix} 0.020 & -0.010 & 0.015 \\ -0.010 & 0.040 & -0.006 \\ 0.015 & -0.006 & 0.028 \end{bmatrix} \begin{bmatrix} 1 \\ 1.1 \\ 1.21 \end{bmatrix} = 0.1077228$$

The 90% confidence interval is

$$\hat{y} \pm t_{0.025} s\sqrt{a'(\mathbf{X'X})^{-1} a} \Rightarrow 9.3455 \pm 2.074(1.1)\sqrt{0.1077228}$$
$$\Rightarrow 9.3455 \pm 0.7488 \Rightarrow (8.5967, 10.0943)$$

For all values of $x = \$1.10$, we are 95% confident that the mean value of y will fall between 8.5967 and 10.0943.

c. The 95% prediction interval is

$$\hat{y} \pm t_{0.025} s\sqrt{1 + a'(\mathbf{X'X})^{-1} a} \Rightarrow 9.3455 \pm 2.074(1.1)\sqrt{1 + 0.1077228}$$
$$\Rightarrow 9.3455 \pm 2.4011 \Rightarrow (6.9444, 11.7466)$$

For future value rate of $x = \$1.10$, we are 95% confident that the production of that worker will fall between 6.9444 and 11.7466.

d. $R^2 = 1 - \dfrac{SSE}{SS_{yy}} = 1 - \dfrac{26.62}{784.11} = 0.966$

96.6% of the sum of squares of the deviations of the sample production values about their mean can be explained using the regression model.

B.29 a. $\mathbf{Y} = \begin{bmatrix} 1.1 \\ 1.9 \\ 3.0 \\ 3.8 \\ 5.1 \\ 6.0 \end{bmatrix}$, $\mathbf{X} = \begin{bmatrix} 1 & -5 \\ 1 & -3 \\ 1 & -1 \\ 1 & 1 \\ 1 & 3 \\ 1 & 5 \end{bmatrix}$

b. $$\mathbf{X'X} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ -5 & -3 & -1 & 1 & 3 & 5 \end{bmatrix} \begin{bmatrix} 1 & -5 \\ 1 & -3 \\ 1 & -1 \\ 1 & 1 \\ 1 & 3 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 70 \end{bmatrix}$$

$$\mathbf{X'Y} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ -5 & -3 & -1 & 1 & 3 & 5 \end{bmatrix} \begin{bmatrix} 1.1 \\ 1.9 \\ 3.0 \\ 3.8 \\ 5.1 \\ 6.0 \end{bmatrix} = \begin{bmatrix} 20.9 \\ 34.9 \end{bmatrix}$$

c. To obtain the least squares estimates, we first need to obtain $(\mathbf{X'X})^{-1}$.

$$(\mathbf{X'X})^{-1} = \begin{bmatrix} 6 & 0 \\ 0 & 70 \end{bmatrix}^{-1} = \begin{bmatrix} 1/6 & 0 \\ 0 & 1/70 \end{bmatrix}, \text{ by Theorem B.1.}$$

Then, $\hat{\boldsymbol{\beta}} = (\mathbf{X'X})^{-1}\mathbf{X'Y} = \begin{bmatrix} 1/6 & 0 \\ 0 & 1/70 \end{bmatrix} \begin{bmatrix} 20.9 \\ 34.9 \end{bmatrix} = \begin{bmatrix} 3.4833333 \\ 0.4985714 \end{bmatrix}$

Thus, $\hat{\beta}_0 = 3.48333$ and $\hat{\beta}_1 = 0.49857$.

d. The prediction equation is $\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x = 3.4833 + 0.4986x$.

e. $SSE = \mathbf{Y'Y} - \hat{\boldsymbol{\beta}}'(\mathbf{X'Y})$, where

$$\mathbf{Y'Y} = \begin{bmatrix} 1.1 & 1.9 & 3.0 & 3.8 & 5.1 & 6.0 \end{bmatrix} \begin{bmatrix} 1.1 \\ 1.9 \\ 3.0 \\ 3.8 \\ 5.1 \\ 6.0 \end{bmatrix} = 90.27$$

and $\hat{\boldsymbol{\beta}}'(\mathbf{X'Y}) = \begin{bmatrix} 3.4833333 & 0.4985714 \end{bmatrix} \begin{bmatrix} 20.9 \\ 34.9 \end{bmatrix} = 90.201808$

Thus, $SSE = 90.27 - 90.201808 = 0.0681922$, and

$$s^2 = SSE / [n - (k+1)] = 0.0681922 / [6 - (1 + 1)] = 0.0681922 / 4 = 0.0170481$$

f. $H_0: \beta_1 = 0$
$H_a: \beta_1 \neq 0$

Test statistic: $t = \dfrac{\hat{\beta}_1}{s\sqrt{c_{11}}} = \dfrac{0.4986}{\sqrt{0.0170481}\sqrt{1/70}} = 31.95$

[Note: The value of c_{11} is obtained from the $(\mathbf{X'X})^{-1}$ matrix.]

Rejection region: $\alpha = 0.05,\ df = 4,\ t_{0.025} = 2.776$
Reject H_0 if $t < -2.776$ or $t > 2.776$

Conclusion: Reject H_0 at $\alpha = 0.05$. There is sufficient evidence to conclude that the model is useful for predicting y.

g. $r^2 = 1 - \left(SSE / SS_{yy}\right)$, where $SSE = 0.0681922$ and

$$SS_{yy} = \sum y_i^2 - \frac{\left(\sum y_i\right)^2}{n} = 90.27 - \frac{(20.9)^2}{6} = 17.4683333$$

Thus, $r^2 = 1 - (0.0681922 / 17.4683333) = 0.99609.$

Over 99% of the variability of the y-values about their sample mean is accounted for by the least squares model.

h. The 90% confidence interval for $E(y)$ when $x = 0.5$ is given by $\hat{y} \pm t_{0.05} s\sqrt{a'(\mathbf{X'X})^{-1}a}$

where $\hat{y} = 3.4833 + 0.4986(0.5) = 3.7326,\ t_{0.05} = 2.132(4\ df),$
$s = \sqrt{0.0170481} = 0.130568$ [from part (e)]

and $a'(\mathbf{X'X})^{-1}a = \begin{bmatrix} 1 & 0.5 \end{bmatrix} \begin{bmatrix} 1/6 & 0 \\ 0 & 1/70 \end{bmatrix} \begin{bmatrix} 1 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 1/6 & 0.0071429 \end{bmatrix} \begin{bmatrix} 1 \\ 0.5 \end{bmatrix} = 0.1702381$

Thus, the desired confidence interval is:

$$3.7326 \pm 2.132(0.130568)\sqrt{0.1702381} \Rightarrow 3.7326 \pm 0.11486 \Rightarrow (3.6177,\ 3.8475)$$

The mean value of y when $x = 0.5$ is estimated to fall between 3.6177 and 3.8475 with 90% confidence.

B.30 a. $\mathbf{Y} = \begin{bmatrix} 5.2 \\ 5.0 \\ 0.3 \\ -0.1 \\ -1.2 \\ -1.1 \\ 2.2 \\ 2.0 \\ 6.2 \\ 6.1 \end{bmatrix} \qquad \mathbf{X} = \begin{bmatrix} 1 & -2 & 2 \\ 1 & -2 & 2 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \\ 1 & 0 & -2 \\ 1 & 0 & -2 \\ 1 & 1 & -1 \\ 1 & 1 & -1 \\ 1 & 2 & 2 \\ 1 & 2 & 2 \end{bmatrix}$

b. $\mathbf{X'X} = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 28 \end{bmatrix} \Rightarrow (\mathbf{X'X})^{-1} = \begin{bmatrix} \frac{1}{10} & 0 & 0 \\ 0 & \frac{1}{20} & 0 \\ 0 & 0 & \frac{1}{28} \end{bmatrix} \qquad \mathbf{X'Y} = \begin{bmatrix} 24.6 \\ 8.2 \\ 45.2 \end{bmatrix}$

$$\hat{\boldsymbol{\beta}} = (\mathbf{X'X})^{-1}\mathbf{X'Y} = \begin{bmatrix} 2.46 \\ 0.41 \\ 1.614 \end{bmatrix}$$

Thus, $\hat{y} = 2.46 + 0.41x_1 + 1.614x_2$.

c. $\mathbf{Y'Y} = 139.28 \qquad \hat{\boldsymbol{\beta}}'\mathbf{X'Y} = 136.84371 \qquad SSE = \mathbf{Y'Y} - \hat{\boldsymbol{\beta}}'\mathbf{X'Y} = 2.4362857$

$$s^2 = \frac{SSE}{n-(k+1)} = \frac{2.4362857}{10-3} = 0.3480408$$

d. To determine if the model is useful for predicting y, we test:

$H_0: \beta_1 = \beta_2 = 0$
$H_a:$ At least one $\beta_i \neq 0$

$$SS_{yy} = \sum y_i^2 - \frac{\left(\sum y_i\right)^2}{n} = 139.28 - \frac{(24.6)^2}{10} = 78.764$$

Test statistic:

$$F = \frac{(SSyy - SSE)/k}{SSE/\left[n-(k+1)\right]} = \frac{(78.764 - 2.4362857)/2}{2.4362857/7} = \frac{38.1639}{0.3480408} = 109.654$$

Rejection region: $\alpha = 0.05,\ F_{0.05} = 4.74\left(df = 2,7\right)$
Reject H_0 if $F > 4.74$.

Conclusion: Reject H_0 at $\alpha = 0.05$. There is sufficient evidence to indicate that the model is a useful predictor of y.

e. $R^2 = 1 - \dfrac{SSE}{SS_{yy}} = 1 - \dfrac{2.4362857}{78.764} = 0.969$

96.9% of the variation in the strength values about their mean can be explained using the regression model.

f. $H_0: \beta_1 = 0$
$H_a: \beta_1 \neq 0$

Test statistic: $t = \dfrac{\hat{\beta}_1}{s\sqrt{c_{11}}} = \dfrac{0.41}{\sqrt{0.3480408}\sqrt{1/20}} = 3.11$

[Note: The value of c_{11} is obtained from the $\left(\mathbf{X'X}\right)^{-1}$ matrix.]

Rejection region: $\alpha = 0.05,\ df = 7,\ t_{0.025} = 2.365$
Reject H_0 if $t < -2.365$ or $t > 2.365$

Conclusion: Reject H_0 at $\alpha = 0.05$. There is sufficient evidence to indicate that pressure at extrusion is a useful predictor of strength.

g. The 90% confidence interval for is given by $\hat{y} \pm t_{0.05} s\sqrt{a'\left(\mathbf{X'X}\right)^{-1} a}$

where $\hat{y} = 2.46 + 0.41(-2) + 1.614(2) = 4.868,\ t_{0.05} = 1.895(7\ df),$

$s = \sqrt{0.3480408} = 0.5899$ [from part (c)], and $a = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$

Now, $$a'\left(\mathbf{X'X}\right)^{-1} a = \begin{bmatrix} 1 & -2 & 2 \end{bmatrix} \begin{bmatrix} \frac{1}{10} & 0 & 0 \\ 0 & \frac{1}{20} & 0 \\ 0 & 0 & \frac{1}{28} \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix} = 0.4428571$$

Thus, the desired confidence interval is:
$$4.868 \pm 1.895(0.5899)\sqrt{0.4428571} \Rightarrow 4.868 \pm 0.7439 \Rightarrow (4.1241,\ 5.6119)$$

h. The 90% confidence interval for is given by

$$\hat{y} \pm t_{0.05} s\sqrt{1+a'(\mathbf{X'X})^{-1}a} \Rightarrow 5.107 \pm 1.895(0.5899)\sqrt{1.4428571}$$
$$\Rightarrow 5.107 \pm 1.3429 \Rightarrow (3.7641,\ 6.4499)$$

B.31 a. $\mathbf{Y} = \begin{bmatrix} 1.1 \\ .5 \\ 1.8 \\ 2.0 \\ 2.0 \\ 2.9 \\ 3.8 \\ 3.4 \\ 4.1 \\ 5.0 \\ 5.0 \\ 5.8 \end{bmatrix} \qquad \mathbf{X} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 2 \\ 1 & 2 \\ 1 & 3 \\ 1 & 3 \\ 1 & 4 \\ 1 & 4 \\ 1 & 5 \\ 1 & 5 \\ 1 & 6 \\ 1 & 6 \end{bmatrix}$

b. $\mathbf{X'X} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 2 & 3 & 3 & 4 & 4 & 5 & 5 & 6 & 6 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 2 \\ 1 & 2 \\ 1 & 3 \\ 1 & 3 \\ 1 & 4 \\ 1 & 4 \\ 1 & 5 \\ 1 & 5 \\ 1 & 6 \\ 1 & 6 \end{bmatrix} = \begin{bmatrix} 12 & 42 \\ 42 & 182 \end{bmatrix}$

$$\mathbf{X'Y} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 2 & 3 & 3 & 4 & 4 & 5 & 5 & 6 & 6 \end{bmatrix} \begin{bmatrix} 1.1 \\ .5 \\ 1.8 \\ 2.0 \\ 2.0 \\ 2.9 \\ 3.8 \\ 3.4 \\ 4.1 \\ 5.0 \\ 5.0 \\ 5.8 \end{bmatrix} = \begin{bmatrix} 37.4 \\ 163 \end{bmatrix}$$

c. The elements of the $\mathbf{X'X}$ matrix for two replications are equal to the elements of the $\mathbf{X'X}$ matrix for a single replication, multiplied by a factor of 2:

$$\mathbf{X'X} = \begin{bmatrix} 6 & 21 \\ 21 & 91 \end{bmatrix} \text{ for single replication;}$$

$$\mathbf{X'X} = \begin{bmatrix} 12 & 42 \\ 42 & 182 \end{bmatrix} \text{ for two replications.}$$

d. $$(\mathbf{X'X})^{-1} = \begin{bmatrix} 13/15 & -7/35 \\ -7/35 & 2/35 \end{bmatrix} \text{ for one replication;}$$

$$(\mathbf{X'X})^{-1} = \begin{bmatrix} 13/30 & -7/70 \\ -7/70 & 2/70 \end{bmatrix} \text{ for two replications.}$$

Verification: Show that

$$(\mathbf{X'X})^{-1}(\mathbf{X'X}) = \mathbf{I} \text{ and } (\mathbf{X'X})(\mathbf{X'X})^{-1} = \mathbf{I}$$

for two replications:

$$(\mathbf{X'X})^{-1}(\mathbf{X'X}) = \begin{bmatrix} 13/30 & -7/70 \\ -7/70 & 2/70 \end{bmatrix} \begin{bmatrix} 12 & 42 \\ 42 & 182 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \mathbf{I}$$

Similarly, $(\mathbf{X'X})(\mathbf{X'X})^{-1} = \begin{bmatrix} 12 & 42 \\ 42 & 182 \end{bmatrix} \begin{bmatrix} 13/30 & -7/70 \\ -7/70 & 2/70 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \mathbf{I}$

e. We first obtain the least squares estimates:

$$\hat{\boldsymbol{\beta}} = (\mathbf{X'X})^{-1}\mathbf{X'Y} = \begin{bmatrix} 13/30 & -7/70 \\ -7/70 & 2/70 \end{bmatrix} \begin{bmatrix} 37.4 \\ 163 \end{bmatrix} = \begin{bmatrix} -0.0933333 \\ 0.9171429 \end{bmatrix}$$

Thus, the prediction equation is $\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x = -0.09333 + 0.91714x$.

f. $SSE = \mathbf{Y'Y} - \hat{\boldsymbol{\beta}}'\mathbf{X'Y} = 147.56 - 146.00363 = 1.55637;$

$s^2 = SSE / [n - (k+1)] = 1.55637 / (12 - 2) = 1.55637 / 10 = 0.155637$

g. $H_0: \beta_1 = 0$
$H_a: \beta_1 \neq 0$

Test statistic: $t = \dfrac{\hat{\beta}_1}{s\sqrt{c_{11}}}$

where $s = \sqrt{0.115637} = 0.340054$ from part (f) and $c_{11} = 2/70$ from the $(\mathbf{X'X})^{-1}$ matrix in part (d).

Thus, $t = \dfrac{0.9171429}{\sqrt{0.155637}\sqrt{2/70}} = 13.75$

Rejection region: $\alpha = .05$, $df = 10$, $t_{0.025} = 2.228$
Reject H_0 if $t < -2.228$ or $t > 2.228$

Conclusion: Reject H_0 at $\alpha = 0.05$. The data provide sufficient information to indicate that x contributes information for the prediction of y.

h. $r^2 = 1 - \left(SSE / SS_{yy}\right)$, where

$SSE = 1.55637$ [from part (f)]

and

$$SS_{yy} = \sum y_i^2 - \frac{\left(\sum y_i\right)^2}{n} = 147.56 - \frac{(37.4)^2}{12} = 30.996667$$

Thus, $r^2 = 1 - (1.55637 / 30.996667) = 0.9498$.

There is an approximate 95% reduction in the error of prediction obtained by using the least squares equation $\hat{y} = -0.09333 + 0.91714x$, instead of $\bar{y}$, to predict y.

B.32 a. The 90% confidence interval for is given by $\hat{y} \pm t_{0.05} s\sqrt{a'(\mathbf{X'X})^{-1} a}$

where $\hat{y} = -0.09333 + 0.91714(4.5) = 4.0338$, $t_{0.05} = 1.812(10\ df)$,

$s = 0.3401$ [Exercise B.31], and $a = \begin{bmatrix} 1 \\ 4.5 \end{bmatrix}$

Now, $a'(\mathbf{X}'\mathbf{X})^{-1}a = \begin{bmatrix}1 & 4.5\end{bmatrix}\begin{bmatrix}\frac{13}{30} & \frac{-7}{70} \\ \frac{-7}{70} & \frac{2}{70}\end{bmatrix}\begin{bmatrix}1 \\ 4.5\end{bmatrix} = 0.1119048$

The 90% confidence interval is:

$$4.0338 \pm 1.812(0.3401)\sqrt{0.1119048} \Rightarrow 4.0338 \pm 0.2062 \Rightarrow (3.8276,\ 4.2400)$$

For all values in which $x = 4.5,$ we are 90% confident the mean value of y will fall in the interval 3.8276 and 4.2300.

b. The 90% prediction interval for is given by $\hat{y} \pm t_{0.05}s\sqrt{1 + a'(\mathbf{X}'\mathbf{X})^{-1}a}$

Using the same values from part (a), the interval is

$$4.0338 \pm 1.812(0.3401)\sqrt{1 + 0.1119048} \Rightarrow 4.0338 \pm 0.6498 \Rightarrow (3.3840,\ 4.6836)$$

For a future value in which $x = 4.5,$ we are 90% confident that the value of y will fall in the interval 3.3840 and 4.6836.

B.33 a. The **Y** matrix would be $n \times 1 = 18 \times 1.$

b. $$\mathbf{X} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 2 \\ 1 & 2 \\ 1 & 2 \\ 1 & 3 \\ 1 & 3 \\ 1 & 3 \\ 1 & 4 \\ 1 & 4 \\ 1 & 4 \\ 1 & 5 \\ 1 & 5 \\ 1 & 5 \\ 1 & 6 \\ 1 & 6 \\ 1 & 6 \end{bmatrix}$$

For each additional replication, we add one additional row for each value of x.

c. The elements of the $\mathbf{X'X}$ matrix for two replications are equal to the elements of the $\mathbf{X'X}$ matrix for a single replication multiplied by 2. The elements of the $\mathbf{X'X}$ matrix for three replications should be the elements of the $\mathbf{X'X}$ for a single replication multiplied by 3. In general, for *k* replications, the $\mathbf{X'X}$ matrix is *k* times the $\mathbf{X'X}$ matrix for a single replications.

$$\mathbf{X'X} \text{ (for 3 replications)} = 3\begin{bmatrix} 6 & 21 \\ 21 & 91 \end{bmatrix} = \begin{bmatrix} 18 & 63 \\ 63 & 273 \end{bmatrix}$$

The $(\mathbf{X'X})^{-1}$ matrix for *k* replications can be obtained by multiplying the $(\mathbf{X'X})^{-1}$ matrix for a single replication by $\frac{1}{k}$.

$$(\mathbf{X'X})^{-1} \text{ (for 3 replications)} = 1/3\begin{bmatrix} 13/15 & -7/35 \\ -7/35 & 2/35 \end{bmatrix} = \begin{bmatrix} 13/45 & -7/105 \\ -7/105 & 2/105 \end{bmatrix}$$

e. $$a'(\mathbf{X'X})^{-1}a \text{ (for 3 replications)} = \begin{bmatrix} 1 & 4.5 \end{bmatrix}\begin{bmatrix} 13/45 & -7/105 \\ -7/105 & 2/105 \end{bmatrix}\begin{bmatrix} 1 \\ 4.5 \end{bmatrix} = 0.0746$$

$$a'(\mathbf{X'X})^{-1}a \text{ (for 1 replication)} = \begin{bmatrix} 1 & 4.5 \end{bmatrix}\begin{bmatrix} 13/15 & -7/35 \\ -7/35 & 2/35 \end{bmatrix}\begin{bmatrix} 1 \\ 4.5 \end{bmatrix} = 0.2238$$

Notice that

$$a'(\mathbf{X'X})^{-1}a \text{ (for 3 replications)} = 1/3\left[a'(\mathbf{X'X})^{-1}a\right] \text{ (for 1 replication)}$$

In general,

$$a'(\mathbf{X'X})^{-1}a \text{ (for } k \text{ replications)} = 1/k\left[a'(\mathbf{X'X})^{-1}a\right] \text{ (for 1 replication)}$$

f. The form of the confidence interval is $\hat{y} \pm t_{\alpha/2} s\sqrt{a'(\mathbf{X'X})^{-1}a}$

If we assume *s* stays the same, $\alpha = 0.10$, and 1 replicate has 6 observations, then the following is true:

$$\text{For 2 replicates} \Rightarrow \hat{y} \pm t_{0.05,10} s\sqrt{\frac{1}{2}a'(\mathbf{X'X})^{-1}a} \quad (\text{for 1 replicate})$$
$$\Rightarrow \hat{y} \pm 1.812 s\sqrt{\frac{1}{2}}\sqrt{a'(\mathbf{X'X})^{-1}a} \quad (\text{for 1 replicate})$$

$$\text{For 3 replicates} \Rightarrow \hat{y} \pm t_{0.05,16} s\sqrt{\frac{1}{3}a'(\mathbf{X'X})^{-1}a} \quad (\text{for 1 replicate})$$
$$\Rightarrow \hat{y} \pm 1.746 s\sqrt{\frac{1}{3}}\sqrt{a'(\mathbf{X'X})^{-1}a} \quad (\text{for 1 replicate})$$

The width of the interval for 2 replicates is:

$$2(1.812)s\sqrt{\frac{1}{2}}\sqrt{a'(\mathbf{X'X})^{-1}a} = 2.56255s\sqrt{a'(\mathbf{X'X})^{-1}a}$$

The width of the interval for 3 replicates is:

$$2(1.746)s\sqrt{\frac{1}{3}}\sqrt{a'(\mathbf{X'X})^{-1}a} = 2.01611s\sqrt{a'(\mathbf{X'X})^{-1}a}$$

Thus, the width of the interval for 3 replicates is only

$$\frac{2.01611\,s\sqrt{a'(\mathbf{X'X})^{-1}a}}{2.56255\,s\sqrt{a'(\mathbf{X'X})^{-1}a}} = 0.787$$

as wide as that for 2 replicates. The width has been reduced by ≈ 0.213.

Appendix

C

A Procedure for Inverting a Matrix

C.1 a. $\mathbf{A}=\begin{bmatrix}3 & 2\\ 4 & 5\end{bmatrix}$ $\mathbf{I}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

Operation 1 Multiply row 1 by 1/3:

$$\rightarrow\begin{bmatrix}1 & 2/3\\ 4 & 5\end{bmatrix}\begin{bmatrix}1/3 & 0\\ 0 & 1\end{bmatrix}$$

Operation 2 Multiply row 1 by 4 and subtract from row 2:

$$\rightarrow\begin{bmatrix}1 & 2/3\\ 0 & 7/3\end{bmatrix}\begin{bmatrix}1/3 & 0\\ -4/3 & 1\end{bmatrix}$$

Operation 3 Multiply row 2 by 3/7:

$$\rightarrow\begin{bmatrix}1 & 2/3\\ 0 & 1\end{bmatrix}\begin{bmatrix}1/3 & 0\\ -4/7 & 3/7\end{bmatrix}$$

Operation 4 Multiply row 2 by 2/3 and subtract from row 1:

$$\rightarrow\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}5/7 & -2/7\\ -4/7 & 3/7\end{bmatrix}$$

Thus, $\mathbf{A}^{-1}=\begin{bmatrix}5/7 & -2/7\\ -4/7 & 3/7\end{bmatrix}$

Notice: $\mathbf{A}^{-1}\mathbf{A}=\begin{bmatrix}5/7 & -2/7\\ -4/7 & 3/7\end{bmatrix}\begin{bmatrix}3 & 2\\ 4 & 5\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

b. $\mathbf{A}=\begin{bmatrix}3 & 0 & -2\\ 1 & 4 & 2\\ 5 & 1 & 1\end{bmatrix}$ $\mathbf{I}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$

Operation 1 Multiply row 3 by 3 and subtract from it row 1 multiplied by 5.

$$\begin{bmatrix}3 & 0 & -2\\ 1 & 4 & 2\\ 0 & 3 & 13\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ -5 & 0 & 3\end{bmatrix}$$

Operation 2 Multiply row 2 by 3 and subtract row 1 from it.

$$\begin{bmatrix} 3 & 0 & -2 \\ 0 & 12 & 8 \\ 0 & 3 & 13 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -1 & 3 & 0 \\ -5 & 0 & 3 \end{bmatrix}$$

Operation 3 Multiply row 3 by 4 and subtract row 2 from it.

$$\begin{bmatrix} 3 & 0 & -2 \\ 0 & 12 & 8 \\ 0 & 0 & 44 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -1 & 3 & 0 \\ -19 & -3 & 12 \end{bmatrix}$$

Operation 4 Multiply row 2 by 11 and subtract from it row 3 multiplied by 2.

$$\begin{bmatrix} 3 & 0 & -2 \\ 0 & 132 & 0 \\ 0 & 0 & 44 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 27 & 39 & -24 \\ -19 & -3 & 12 \end{bmatrix}$$

Operation 5 Multiply row 1 by 22 and add row 3 to it.

$$\begin{bmatrix} 66 & 0 & 0 \\ 0 & 132 & 0 \\ 0 & 0 & 44 \end{bmatrix} \begin{bmatrix} 3 & -3 & 12 \\ 27 & 39 & -24 \\ -19 & -3 & 12 \end{bmatrix}$$

Operation 6 Divide row 1 by 66.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 132 & 0 \\ 0 & 0 & 44 \end{bmatrix} \begin{bmatrix} \frac{1}{22} & -\frac{1}{22} & \frac{2}{11} \\ 27 & 39 & -24 \\ -19 & -3 & 12 \end{bmatrix}$$

Operation 7 Divide row 2 by 132.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 44 \end{bmatrix} \begin{bmatrix} \frac{1}{22} & -\frac{1}{22} & \frac{2}{11} \\ \frac{9}{44} & \frac{13}{44} & -\frac{2}{11} \\ -19 & -3 & 12 \end{bmatrix}$$

Operation 8 Divide row 3 by 44.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{22} & -\frac{1}{22} & \frac{2}{11} \\ \frac{9}{44} & \frac{13}{44} & -\frac{2}{11} \\ -\frac{19}{44} & -\frac{3}{44} & \frac{3}{11} \end{bmatrix}$$

Thus, $\mathbf{A}^{-1} = \begin{bmatrix} \frac{1}{22} & -\frac{1}{22} & \frac{2}{11} \\ \frac{9}{44} & \frac{13}{44} & -\frac{2}{11} \\ -\frac{19}{44} & -\frac{3}{44} & \frac{3}{11} \end{bmatrix}$

c. $\mathbf{A} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 1 & 1 & 3 \end{bmatrix} \qquad \mathbf{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Operation 1 Subtract row 1 from row 3:

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

Operation 2 Multiply row 2 by 1/2:

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1/2 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

Operation 3 Subtract row 2 from row 3:

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1/2 \\ 0 & 0 & 3/2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ -1 & -1/2 & 1 \end{bmatrix}$$

Operation 4 Multiply row 3 by 2/3:

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1/2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ -2/3 & -1/3 & 2/3 \end{bmatrix}$$

Operation 5 Operate on row 2 by subtracting 1/2 of row 3:

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 1/3 & 2/3 & -1/3 \\ -2/3 & -1/3 & 2/3 \end{bmatrix}$$

Operation 6 Operate on row 1 by subtracting row 3:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 5/3 & 1/3 & -2/3 \\ 1/3 & 2/3 & -1/3 \\ -2/3 & -1/3 & 2/3 \end{bmatrix}$$

Thus, $\mathbf{A}^{-1} = \begin{bmatrix} 5/3 & 1/3 & -2/3 \\ 1/3 & 2/3 & -1/3 \\ -2/3 & -1/3 & 2/3 \end{bmatrix}$

Notice: $\mathbf{A}^{-1}\mathbf{A} = \begin{bmatrix} 5/3 & 1/3 & -2/3 \\ 1/3 & 2/3 & -1/3 \\ -2/3 & -1/3 & 2/3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 1 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

d. $\mathbf{A} = \begin{bmatrix} 4 & 0 & 10 \\ 0 & 10 & 0 \\ 10 & 0 & 5 \end{bmatrix} \qquad \mathbf{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Operation 1 Multiply row 1 by 10/4 and then subtract it from row 3.

$$\begin{bmatrix} 4 & 0 & 10 \\ 0 & 10 & 0 \\ 0 & 0 & -20 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -10/4 & 0 & 1 \end{bmatrix}$$

Operation 2 Divide row 3 by 2 and then add it to row 1.

$$\begin{bmatrix} 4 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & -20 \end{bmatrix} \begin{bmatrix} -1/4 & 0 & 1/2 \\ 0 & 1 & 0 \\ -10/4 & 0 & 1 \end{bmatrix}$$

Operation 3 Divide row 1 by 4, row 2 by 10, and row 3 by -20.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1/16 & 0 & 1/8 \\ 0 & 1/10 & 0 \\ 1/8 & 0 & -1/20 \end{bmatrix}$$

Thus, $\mathbf{A}^{-1} = \begin{bmatrix} -1/16 & 0 & 1/8 \\ 0 & 1/10 & 0 \\ 1/8 & 0 & -1/20 \end{bmatrix}$