Problem 1 [10+10 points]

a) Give Newton's method for finding $\sqrt[3]{2}$ by solving $x^3 - 2 = 0$.

b) Find the first Newton's iteration to solve for the nonlinear system

$$f_1(x,y) = xy + 1.089 = 0$$

 $f_2(x,y) = x^2 + y^2 - 3.23 = 0$

with initial value $x_0 = 1, y_0 = 0$.

Problem 2

a) Let

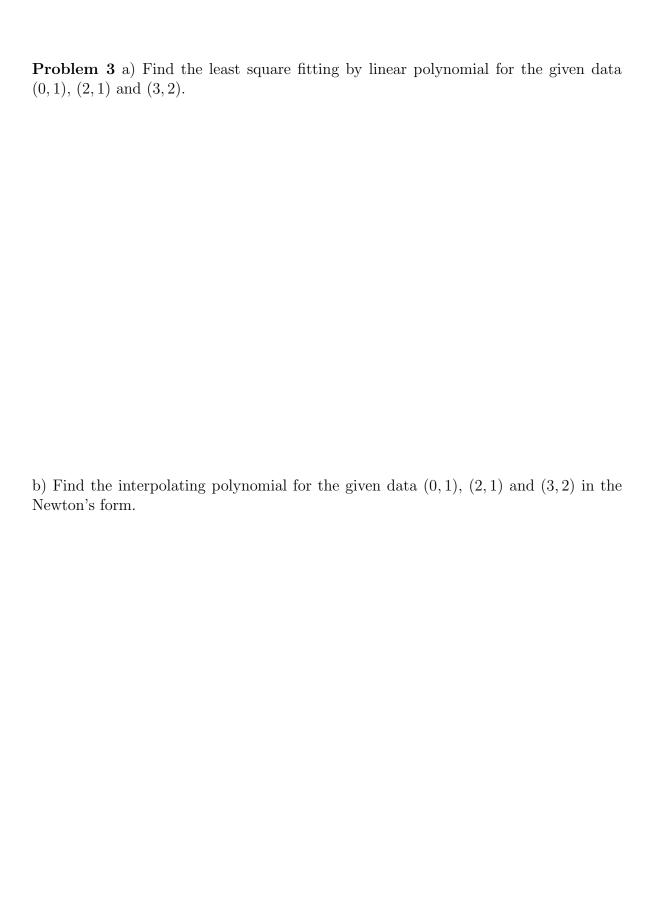
$$A = \begin{bmatrix} 19 & 20 \\ 20 & 21 \end{bmatrix}$$

Find the conditional number of A.

b) We consider the error of the solution x of the linear system Ax = b. We showed in class that,

$$\frac{\|x - z\|}{\|x\|} \le \|A\| \|A^{-1}\| \frac{\|Az - b\|}{\|b\|}.$$

If b = [1, 1/2] and Az = [1.001, 0.499], estimate the relative error $\frac{\|x-z\|}{\|x\|}$.



Problem 4

a) Let f(x) be a function defined on the interval [-1,1] with

$$f(-1) = f(1) = 0$$
, $f(-1/2) = f(1/2) = 1$, $f(0) = 2/3$.

Find an approximation of $\int_{-1}^{1} f(x)dx$ with Simpson's rule.

b) If $|f^{(4)}(x)| < 5$ in [-1, 1], find an error estimate of the approximation in part a).

Problem 5 Use Euler's method to find the an approximation to the solution y(1/2) of the problem

$$y'(x) = -x^2y, \quad y(0) = 1$$

with stepsize h = 0.5.

 $\bf Problem~6$ Trapezoid method is a second order implicit method for soloving the problem

$$y'(x) = f(x, y), \quad y(0) = y_0.$$

The method reads

$$y_{n+1} = y_n + h[f(x_n, y_n) + f(x_{n+1}, y_{n+1})]/2.$$

Use Trapezoid method to find the approximation to the solution y(1/2) of the problem

$$y'(x) = -x^2y, \quad y(0) = 1$$

with stepsize 0.5.