Example 1.3 Illustration of a Randomization Test

Consider an experiment in which two treatments were randomly assigned to seven experimental units. Four experimental units received treatment A and three experimental units received treatment B with the following set of responses:

Unit:	1	2	3	4	5	6	7
Treatment:	B	B	A	A	B	A	A
Response:	14	16	19	17	15	13	17

If there is no difference between the effects of treatments A and B then they are merely labels on the experimental units and do not affect the results. Under the null hypothesis the response for unit 4 would be 17 regardless of the treatment applied.

The A and B labels (four A's and three B's) may be allocated to the seven experimental units in $7!/4!3! = 35$ possible arrangements.

These are the 35 experiments possible if the treatments are randomly allocated to the units.

All 35 of the possible arrangements are shown in Table 1.2 along with the difference between the group means $(\bar{y}_A - \bar{y}_B)$ based on the labels assigned to the units in each arrangement. The 35 differences, $(\bar{y}_A - \bar{y}_B)$, are 35 possible differences that occur under the null hypothesis. They make up the randomization distribution under the null hypothesis.

Consider an alternative hypothesis H_a: $\mu_A - \mu_B \neq 0$ to the null hypothesis H_0: $\mu_A - \mu_B = 0$. The arrangement of the actual experiment is arrangement 30 with a mean difference, $(\bar{y}_A - \bar{y}_B) = 1.50$. An absolute difference of 1.50 or larger occurs with 13 arrangements. Under the null hypothesis an absolute mean difference of 1.50 or larger occurs with a frequency of $\frac{13}{35}$, yielding a significance level of 0.37. On the basis of the observed results of the experiment, arrangement 30, there is no reason to reject the null hypothesis.

Under the assumption of a true null hypothesis, H_0: $\mu_A - \mu_B = 0$, the randomization test enabled an evaluation of the test statistic from the actual experiment, $(\bar{y}_A - \bar{y}_B) = 1.50$, against the values for $(\bar{y}_A - \bar{y}_B)$ from all other members of the population of 35 possible experiments.

Table 1.2 Thirty-five possible arrangements of four A's and three B's to seven experimental units with the mean difference $\bar{y}_A - \bar{y}_B$.

Arrangement	*Unit*:	1	2	3	4	5	6	7	
	Response:	14	16	19	17	15	13	17	$\bar{y}_A - \bar{y}_B$
1		A	A	B	B	A	A	B	−3.17
2		A	B	B	A	A	A	B	−2.58
3		A	B	B	B	A	A	A	−2.58
4		A	A	B	A	B	A	B	−2.00
5		A	A	B	B	B	A	A	−2.00
6		A	B	A	B	A	A	B	−1.42
7		A	B	B	A	B	A	A	−1.42
8		B	A	B	A	A	A	B	−1.42
9		B	A	B	B	A	A	A	−1.42
10		A	A	A	B	B	A	B	−0.83
11		A	A	B	A	A	B	B	−0.83
12		A	A	B	B	A	B	A	−0.83
13		B	B	B	A	A	A	A	−0.83
14		A	B	A	A	B	A	B	−0.25
15		A	B	A	B	B	A	A	−0.25
16		A	B	B	A	A	B	A	−0.25
17		B	A	A	B	A	A	B	−0.25
18		B	A	B	A	B	A	A	−0.25
19		A	A	A	B	A	B	B	0.33
20		A	A	B	A	B	B	A	0.33
21		B	B	A	A	A	A	B	0.33
22		B	B	A	B	A	A	A	0.33
23		A	B	A	A	A	B	B	0.92
24		A	B	A	B	A	B	A	0.92
25		B	A	A	A	B	A	B	0.92
26		B	A	A	B	B	A	A	0.92
27		B	A	B	A	A	B	A	0.92
28		A	A	A	A	B	B	B	1.50
29		A	A	A	B	B	B	A	1.50
30		B	B	A	A	B	A	A	1.50
31		A	B	A	A	B	B	A	2.08
32		B	A	A	A	A	B	B	2.08
33		B	A	A	B	A	B	A	2.08
34		B	B	A	A	A	B	A	2.67
35		B	A	A	A	B	B	A	3.25