Metropolis Algorithm Exercise

Situation: Our *target distribution* is, rather strangely, proportional to

θ	unstandardized posterior
100	0
101	4
102	2
103	3
104	0

The posterior of all other values of θ is 0.

Goal: Simulate this posterior using the Metropolis algorithm (clearly we could just standardize the posterior instead, but the idea is to get a sense of how the algorithm works).

Tools: A coin and the attached table of random numbers from the Uniform$(0, 1)$ distribution.

Directions: Record the following steps as you do them in the table on the next page.

1. Divide your age by 3, take the remainder, and add 101 (so the result is 101, 102, or 103). That is your starting value for θ. This process corresponds to the *starting distribution*.

2. For each row,

   (a) Toss the coin. If you observe heads, set θ^* to $\theta + 1$. If you observe tails, set θ^* to $\theta - 1$. This process corresponds to the *proposal distribution*.

   (b) Calculate the ratio of the posteriors at the proposed point compared to the current point.
   $$r = \frac{P(\theta^*|data)}{P(\theta^{t-1}|data)}.$$

   (c) Look at the next number in the random number table and call it U.

   (d) Set $\theta = \theta^*$ if $U \leq r$ and fill in the appropriate circle to denote that. If $U > r$, don't fill in a circle at all.

3. The filled in circles under 101, 102, and 103 represent our sample from the underlying distribution.

θ^*	r	U	Accept or reject?	θ 101	102	103
not applicable	not applicable	not applicable	not applicable	○	○	○
______	______	______	______	○	○	○
______	______	______	______	○	○	○
______	______	______	______	○	○	○
______	______	______	______	○	○	○
______	______	______	______	○	○	○
______	______	______	______	○	○	○
______	______	______	______	○	○	○
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______	______	______	______	○	○	○
______	______	______	______	○	○	○
______	______	______	______	○	○	○
______	______	______	______	○	○	○
______	______	______	______	○	○	○
______	______	______	______	○	○	○

Table of random digits.

```
0.543 0.520 0.260 0.758
0.388 0.827 0.146 0.329
0.927 0.506 0.106 0.060
0.694 0.188 0.031 0.322
0.558 0.463 0.592 0.239
```