1 3 points

Suppose a university decides to test all of its students for the disorder known as Zoom Brain. It is estimated that 10% of students have this disorder. If a student **has** Zoom Brain, the test will have a positive result with probability 0.98. If a student does **not** have Zoom Brain, the test will have a positive result with probability 0.20. If a student has a **positive** result, what is the probability that the student **has** Zoom Brain? Please give your answer to 3 decimal places, and as a number, not a percentage. Write ZB for diseased, ZB^c for not diseased, + for positive test, and − for negative test. By Bayes' rule,

The prob would be higher if ZB were more common or if $P(+|ZB^c)$ were lower.

Type your answer...

$$P(ZB|+) = \frac{P(+|ZB) \times P(ZB)}{P(+|ZB)P(ZB) + P(+|ZB^c)P(ZB^c)} = \frac{.98 \times .10}{.98 \times .10 + .20 \times (1-.10)} = \frac{.098}{.098 + .18} = \frac{.098}{.278} = .353$$

2 3 points

Same situation as the previous question, but now suppose a student has a **negative** result. What is the probability that the student **does not** have Zoom Brain? Please give your answer to 3 decimal places, and as a number, not a percentage.

Type your answer...

$$P(ZB^c|-) = \frac{P(-|ZB^c)P(ZB^c)}{P(-|ZB^c)P(ZB^c) + P(-|ZB)P(ZB)} = \frac{(1-P(+|ZB^c))P(ZB^c)}{(1-P(+|ZB^c))P(ZB^c) + (1-P(+|ZB))P(ZB)} = \frac{.80 \times .90}{.80 \times .90 + .02 \times .10} = \frac{.72}{.72 + .002} = .997$$

3 1 point

Suppose you come across a game at the Hunterdon 4H fair, and you have no idea what the probability of winning is. Your best choice for a prior distribution of winning is

- (●) beta(1,1)
- ( ) beta(7,3)
- ( ) beta(3,7)
- ( ) beta(70, 30)
- ( ) beta(30,70)
- ( ) beta(5,5)
- ( ) beta(50, 50)

beta(1,1) is the uniform distribution, using that for a prior means that a head of time we regard every value of θ, the probability of winning, as equally likely.

4 1 point

Suppose you come across a game at the Hunterdon 4H fair, and you have are very confident that the probability of winning is 0.3. Your best choice for a prior distribution of winning is

- ( ) beta(1,1)
- ( ) beta(7,3)
- ( ) beta(3,7)
- ( ) beta(70,30)
- (●) beta(30,70)
- ( ) beta(5,5)
- ( ) beta(50,50)

The mean of a beta(a, b) distribution is $\frac{a}{a+b}$, so we want $\frac{a}{a+b} = 0.3$. That leaves us with beta (3,7) or beta (30,70). Beta (30,70) has a much sharper and narrower hump, so that corresponds to greater confidence in our prior belief.

5 1 point

Suppose you come across a game at the Hunterdon 4H fair, and you think, but are not confident, that the probability of winning is 0.3. Your best choice for a prior distribution of winning is

- ( ) beta(1,1)
- ( ) beta(7,3)
- (●) beta(3,7)
- ( ) beta(70,30)
- ( ) beta(30,70)
- ( ) beta(5,5)
- ( ) beta(50,50)

Same reasoning as in Q4, but with much less confidence we want a more spread out hump for our prior.

6 1 point

Suppose for the game at the fair you decided on a prior of beta (1, 2), and then won the game 3 times in 3 tries. What would be your posterior distribution for a winning the game? We know that a beta(a, b) prior is updated, after S successes in T tries, to beta(a+S, b+T−S). Here S = 3, T = 3 so the answer is

type your answer...

beta(4,2)

beta(1+3, 2+3−3) = beta(4,2). Be sure to write the name of the distribution, not just the values of a and b.

Notice that the mean shifts from $\frac{1}{3}$, for the prior, to $\frac{4}{6} = \frac{2}{3}$, for the posterior.