aresty_data_analysis

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Intertemporal Choice: A Laboratory Investigation of Choice Behavior under Additive and Compound Wealth Growth

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[0]:
```
# Requied libraries/packages
# ==========================
require(MASS)
install.packages("devtools")
library(devtools)
library(ggplot2)
```

[0]:
```
main_data <- read.csv(file="/content/stacked-data-ergodicity.csv",header = TRUE)
summary(main_data)
```

```
   first_name      last_name          login_id             start_time
 Joseph  :  42   Patel   :  63   ac1887       :  21   2/26/2020 12:13:294
 Nicholas:  42   Shah    :  63   acm298       :  21   2/25/2020 10:16:168
 Ryan    :  42   Parekh  :  42   ah1123       :  21   2/26/2020 10:13:147
 Aditya  :  21   Ackerley:  21   aka99        :  21   2/24/2020 10:13:126
 Akil    :  21   Ahmed   :  21   al1104       :  21   2/24/2020 12:10:126
 Alan    :  21   Ahmmed  :  21   aman.s.gupta:  21   2/25/2020 12:13:105
 (Other) :1365   (Other) :1323   (Other)      :1428   (Other)          :588
           end_time       duration        treatment        session
 2/24/2020 10:26:  63   Min.   : 3.00   Min.   :1.000   Min.   :1.000
 2/26/2020 12:27:  63   1st Qu.: 8.00   1st Qu.:1.000   1st Qu.:2.000
 2/24/2020 10:23:  42   Median :12.50   Median :1.000   Median :4.000
 2/24/2020 12:18:  42   Mean   :12.99   Mean   :1.473   Mean   :3.676
 2/25/2020 10:25:  42   3rd Qu.:16.00   3rd Qu.:2.000   3rd Qu.:5.000
 2/25/2020 12:21:  42   Max.   :34.00   Max.   :2.000   Max.   :6.000
 (Other)        :1260
    subject         question
 Min.   : 1.0   Min.   : 1.00
 1st Qu.:19.0   1st Qu.: 6.00
 Median :37.5   Median :11.00
 Mean   :37.5   Mean   :10.98
 3rd Qu.:56.0   3rd Qu.:16.00
```

```
 Max.   :74.0   Max.   :21.00


 A 50% chance of Option A and a 50% chance of Option B.
 Option A: $2.00 every 3 days. A 3 percent rate of interest on accumulated earnings every time
 Option A: $0.29 every 1 days. Accumulations end in 70 days.
 Option A: $0.67 every 1 days. Accumulations end in 30 days.
 Option B: $7.00 every 9 days. Accumulations end in 30 days.
 Option B: $3.00 every 7 days. Accumulations end in 50 days.
 (Other)
     choice            Xa               Xb               H                D
 Min.   :0.000   Min.   :0.290   Min.   :0.430   Min.   :1.000   Min.   :2
 1st Qu.:0.000   1st Qu.:1.330   1st Qu.:2.000   1st Qu.:3.000   1st Qu.:2
 Median :1.000   Median :2.000   Median :3.000   Median :5.000   Median :2
 Mean   :0.546   Mean   :2.248   Mean   :3.349   Mean   :4.946   Mean   :2
 3rd Qu.:1.000   3rd Qu.:2.800   3rd Qu.:4.200   3rd Qu.:7.000   3rd Qu.:2
 Max.   :1.000   Max.   :6.000   Max.   :9.000   Max.   :9.000   Max.   :2

       T             cumXa            cumXb        idealChoice
 Min.   :30.00   Min.   :16.48   Min.   : 9.60   A:709
 1st Qu.:30.00   1st Qu.:19.52   1st Qu.:18.00   B:845
 Median :50.00   Median :20.00   Median :21.00
 Mean   :49.55   Mean   :20.58   Mean   :19.82
 3rd Qu.:70.00   3rd Qu.:21.34   3rd Qu.:21.97
 Max.   :70.00   Max.   :27.82   Max.   :26.27
```

Looking at the above code snippet and its result, we will only focus on our response variable (choice) and all independent variables (Xa, Xb, H, D, T) in this course of action. These results might be useful in future

[0]:
```
# Regression
# ==========
lmod <- lm(choice ~ Xa+Xb+H+D+T , data=main_data)           # linear model␣
 ↪regression
summary(lmod)
```

```
Call:
lm(formula = choice ~ Xa + Xb + H + D + T, data = main_data)

Residuals:
     Min       1Q   Median       3Q      Max
-1.00700 -0.31518  0.04834  0.33846  0.87492

Coefficients: (1 not defined because of singularities)
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.128372   0.075703   1.696   0.0901 .
```

```
Xa          -0.182942   0.116654  -1.568   0.1170
Xb           0.091822   0.073814   1.244   0.2137
H            0.128112   0.012905   9.927   <2e-16 ***
D                  NA         NA      NA       NA
T           -0.002266   0.001415  -1.602   0.1094
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.4186 on 1549 degrees of freedom
Multiple R-squared:  0.2533,Adjusted R-squared:  0.2514
F-statistic: 131.4 on 4 and 1549 DF,  p-value: < 2.2e-16
```

Results above show that, among all of our independent variables, Horizon (H) has the least P-Value, making it the most important variable which affects the response variable the most. We can even ignore the other variables

[0]:
```
anova(lmod)
```

A anova: 5 × 5

	Df <int>	Sum Sq <dbl>	Mean Sq <dbl>	F value <dbl>	Pr(>F) <dbl>
Xa	1	48.816593	48.8165934	278.637384	1.187734e-57
Xb	1	2.291036	2.2910356	13.076868	3.085475e-04
H	1	40.521987	40.5219869	231.293084	8.655774e-49
T	1	0.449608	0.4496080	2.566292	1.093673e-01
Residuals	1549	271.381040	0.1751976	NA	NA

[0]:
```
coef(lmod)
```

(Intercept) 0.128372093392043 **Xa** -0.182941841781605 **Xb** 0.0918220441099204 **H** 0.128112081222266 **D** <NA> **T** -0.00226645355776882

The above result help us come up with the multiple linear regression model

[0]:
```
confint(lmod)               # CI at 95%
confint(lmod, level=.99)    # CI at 99%
```

A matrix: 6 × 2 of type dbl

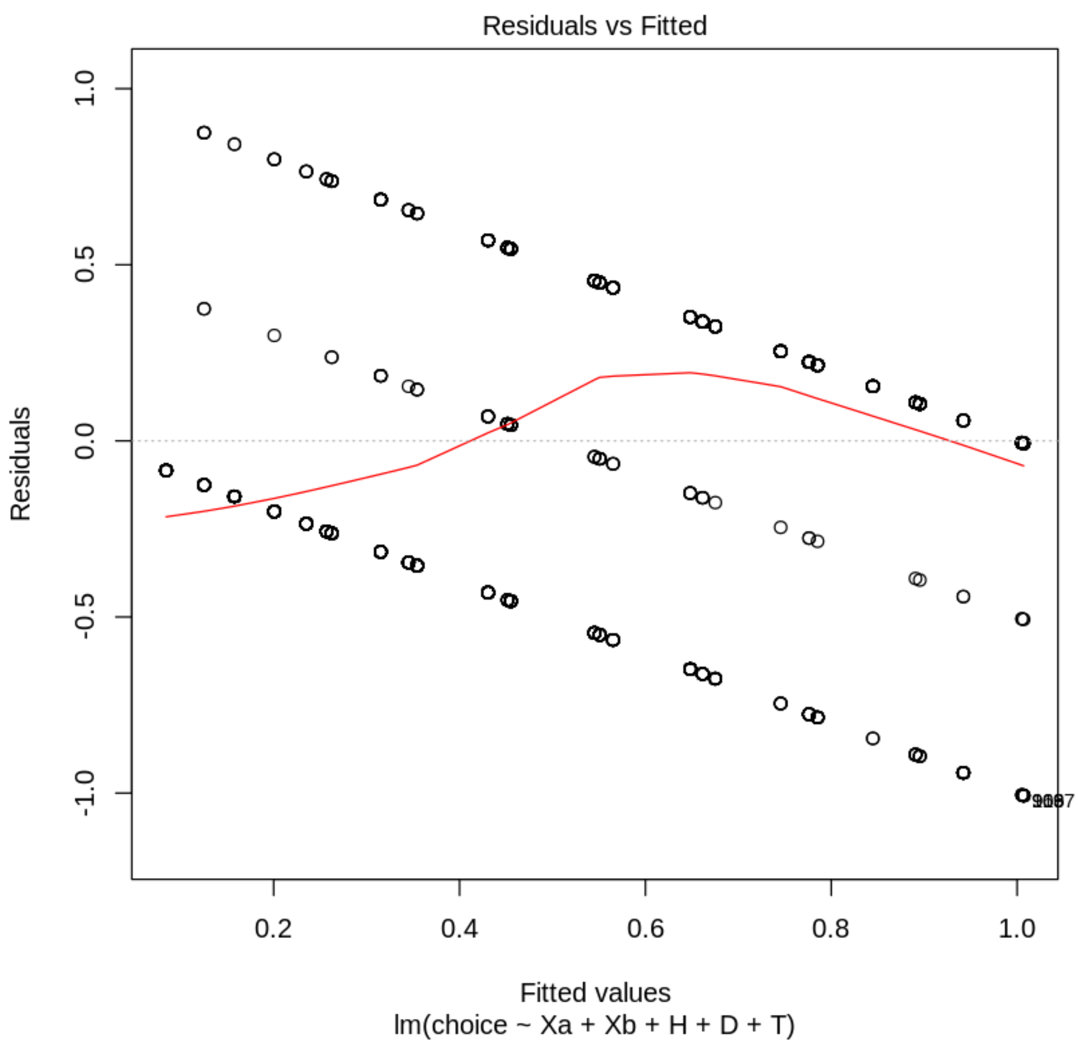
	2.5 %	97.5 %
(Intercept)	-0.020118423	0.2768626099
Xa	-0.411757689	0.0458740057
Xb	-0.052963646	0.2366077338
H	0.102798400	0.1534257627
D	NA	NA
T	-0.005041571	0.0005086641

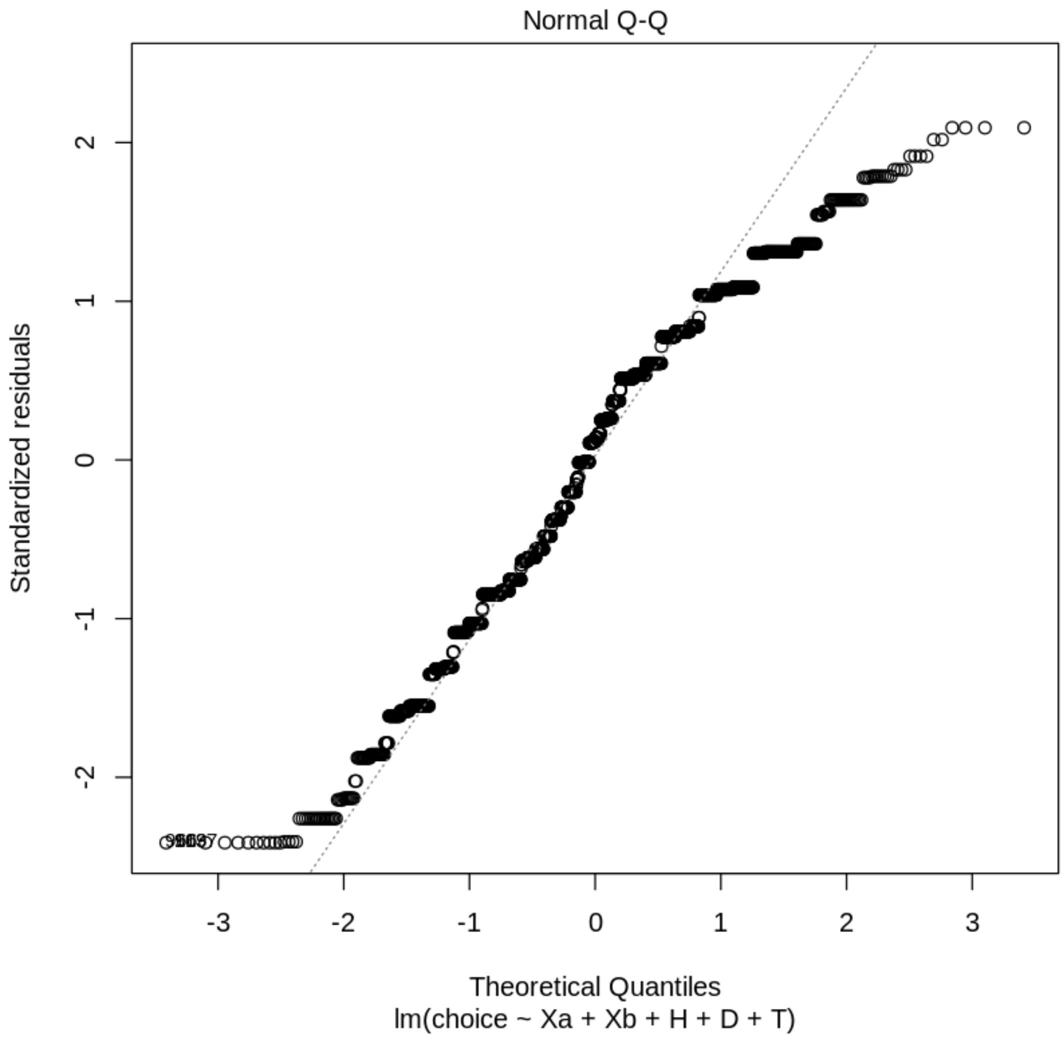
A matrix: 6 × 2 of type dbl

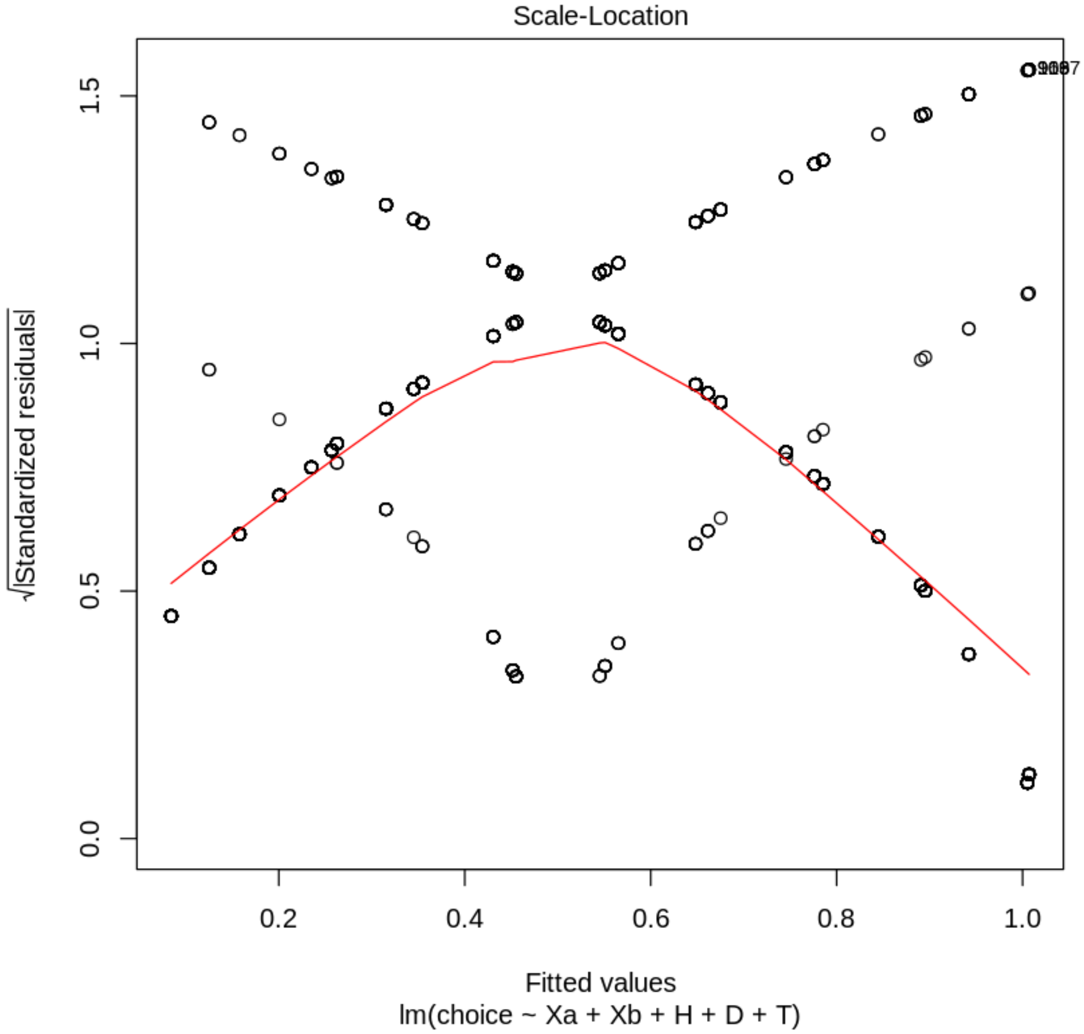
	0.5 %	99.5 %
(Intercept)	-0.066865600	0.323609787
Xa	-0.483792559	0.117908875
Xb	-0.098544485	0.282188573
H	0.094829250	0.161394912
D	NA	NA
T	-0.005915222	0.001382315

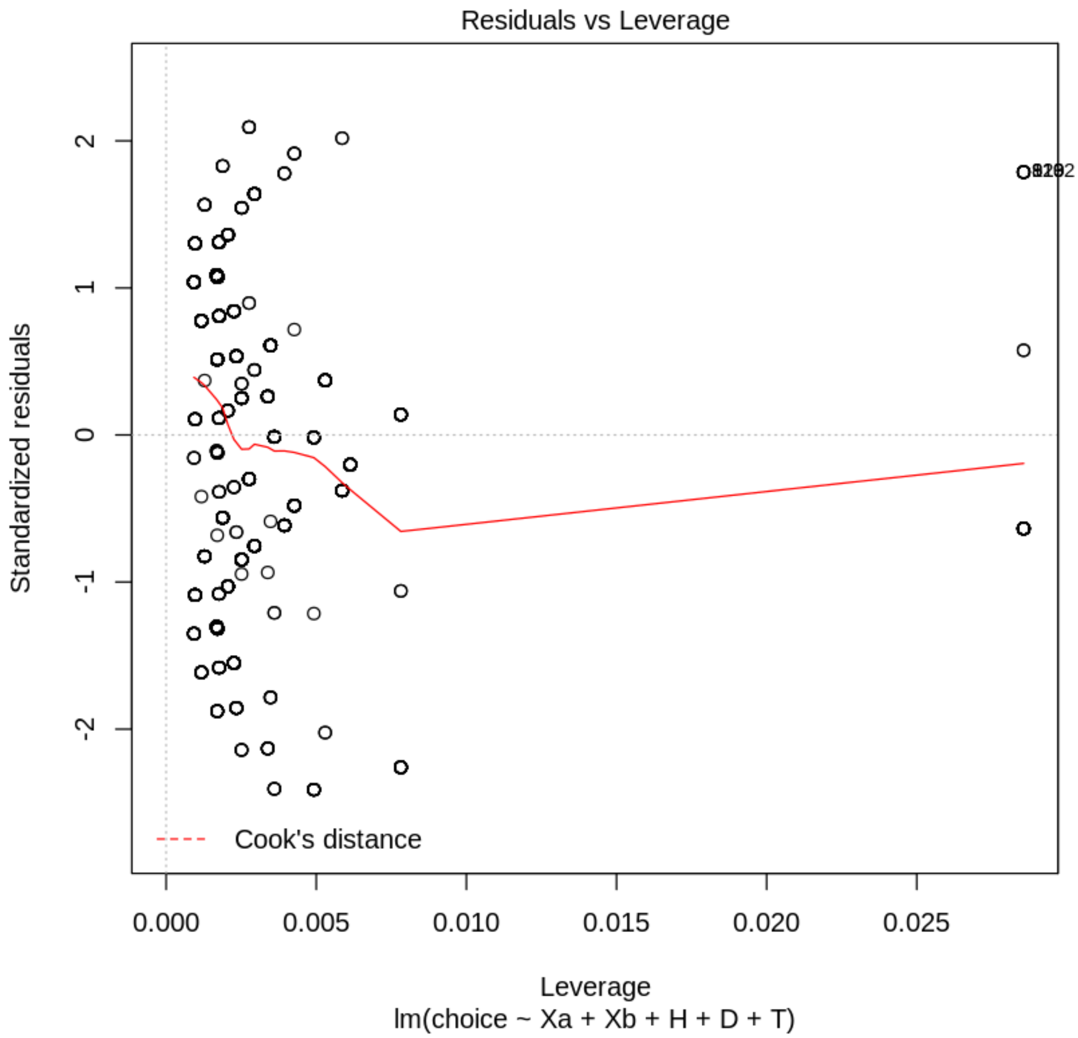
We have computed the 95% and 99% confidence interval for our model. One can confirm the results by comparing the P-Values. This result can be very useful for the hypothesis testing, as it will help to determine whether to ignore the null hypothesis or not

[0]:
```
plot(lmod)
```




Normal Q-Q
Standardized residuals
2
1
0
-1
-2
-3
-2
-1
0
1
2
3
Theoretical Quantiles
lm(choice ~ Xa + Xb + H + D + T)


Scale-Location
√|Standardized residuals|
0.0
0.5
1.0
1.5
0.2
0.4
0.6
0.8
1.0
Fitted values
lm(choice ~ Xa + Xb + H + D + T)



These are the graphs generated by the regression analysis. Each graph helps us understand different things about residuals and distribution of the data. Here is what each graph represents:

1. ***Residual(errors) vs Fitted:*** linearity of the modal and shows if residuals have non-linear patterns
2. ***Normal Q-Q plot (Normal Quantile-Quantile plot):*** shows if residuals are normally distributed
3. ***Scale-Location:*** shows if residuals are spread equally along the ranges of predictors
4. ***Residuals vs Leverage:*** helps us to find influential cases (robust regression can also help)

[0]:
```
lmod_g <- glm(choice ~ Xa+Xb+H+D+T , data=main_data)        # generalized
 ↪linear model
summary(lmod_g)
```

```
Call:
glm(formula = choice ~ Xa + Xb + H + D + T, data = main_data)

Deviance Residuals:
     Min        1Q    Median        3Q       Max
-1.00700  -0.31518   0.04834   0.33846   0.87492

Coefficients: (1 not defined because of singularities)
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.128372   0.075703   1.696   0.0901 .
Xa          -0.182942   0.116654  -1.568   0.1170
Xb           0.091822   0.073814   1.244   0.2137
H            0.128112   0.012905   9.927   <2e-16 ***
D                  NA         NA      NA       NA
T           -0.002266   0.001415  -1.602   0.1094
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for gaussian family taken to be 0.1751976)

    Null deviance: 363.46  on 1553  degrees of freedom
Residual deviance: 271.38  on 1549  degrees of freedom
AIC: 1710.2

Number of Fisher Scoring iterations: 2
```

Here is a generalized linear modal. The modal lmod was a simple linear modal, where this modal helps us understand the exponentiality of the distribution

[0]:
```
anova(lmod_g)
```

A anova: 6 × 4

	Df <int>	Deviance <dbl>	Resid. Df <int>	Resid. Dev <dbl>
NULL	NA	NA	1553	363.4603
Xa	1	48.816593	1552	314.6437
Xb	1	2.291036	1551	312.3526
H	1	40.521987	1550	271.8306
D	0	0.000000	1550	271.8306
T	1	0.449608	1549	271.3810

[0]:
```
confint(lmod_g)              # CI at 95%
confint(lmod_g, level=.99)   # CI at 99%
```

Waiting for profiling to be done…

A matrix: 6 × 2 of type dbl

	2.5 %	97.5 %
(Intercept)	-0.020002396	0.2767465833
Xa	-0.411578898	0.0456952149
Xb	-0.052850514	0.2364946020
H	0.102818179	0.1534059832
D	NA	NA
T	-0.005039403	0.0005064957

```
Waiting for profiling to be done...
```

A matrix: 6 × 2 of type dbl

	0.5 %	99.5 %
(Intercept)	-0.066625040	0.32336923
Xa	-0.483421869	0.11753818
Xb	-0.098309926	0.28195401
H	0.094870259	0.16135390
D	NA	NA
T	-0.005910727	0.00137782

In the above snippets, we looked at generalized linear modal and its anova along with the confidence intervals

[0]:
```
lmod_gnb <- glm.nb(choice ~ Xa+Xb+H+D+T , data=main_data)   # negative binomial␣
↪model
```

[0]:
```
summary(lmod_gnb)
```

```
Call:
glm.nb(formula = choice ~ Xa + Xb + H + D + T, data = main_data,
    init.theta = 20592.51732, link = log)

Deviance Residuals:
     Min        1Q    Median        3Q       Max
-1.52843  -0.82247   0.05077   0.46607   1.20075

Coefficients: (1 not defined because of singularities)
             Estimate Std. Error z value Pr(>|z|)
(Intercept) -1.450862   0.281308  -5.158  2.5e-07 ***
Xa          -0.514161   0.526948  -0.976    0.329
Xb           0.285997   0.339990   0.841    0.400
H            0.240630   0.041925   5.740  9.5e-09 ***
D                  NA         NA      NA       NA
T           -0.005057   0.005276  -0.958    0.338
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for Negative Binomial(20592.52) family taken to be 1)

    Null deviance: 1026.87  on 1553  degrees of freedom
```

```
Residual deviance:  855.72  on 1549  degrees of freedom
AIC: 2543.7

Number of Fisher Scoring iterations: 1


              Theta:  20593
          Std. Err.:  68329
Warning while fitting theta: iteration limit reached

 2 x log-likelihood:  -2531.744
```

[0]:
```
anova(lmod_gnb)
```

```
Warning message in anova.negbin(lmod_gnb):
"tests made without re-estimating 'theta'"
```

A anova: 6 × 5

	Df	Deviance	Resid. Df	Resid. Dev	Pr(>Chi)
	<int>	<dbl>	<int>	<dbl>	<dbl>
NULL	NA	NA	1553	1026.8667	NA
Xa	1	82.2917733	1552	944.5749	1.174098e-19
Xb	1	4.8276907	1551	939.7472	2.800612e-02
H	1	83.1065249	1550	856.6407	7.774863e-20
D	0	0.0000000	1550	856.6407	NA
T	1	0.9220767	1549	855.7186	3.369303e-01

[0]:
```
confint(lmod_gnb)              # CI at 95%
confint(lmod_gnb, level=.99)   # CI at 99%
```

```
Waiting for profiling to be done…
```

A matrix: 6 × 2 of type dbl

	2.5 %	97.5 %
(Intercept)	-2.0067405	-0.903729261
Xa	-1.6512186	0.437843122
Xb	-0.3220629	1.026101982
H	0.1589407	0.323315416
D	NA	NA
T	-0.0154445	0.005246005

```
Waiting for profiling to be done…
```

A matrix: 6 × 2 of type dbl

	0.5 %	99.5 %
(Intercept)	-2.18335716	-0.733467723
Xa	-2.05922968	0.708823322
Xb	-0.49258754	1.294168864
H	0.13346256	0.349516468
D	NA	NA
T	-0.01872919	0.008469432

In the above snippets, we also looked at negative binomial distribution model and its anova along with the confidence intervals

Now we are considering different types of regression modals

[0]:
```
install.packages("olsrr")
```

[0]:
```
# Different types of regression
# ==============================
require(olsrr)
main_data <- read.csv(file="/stacked-data-ergodicity.csv",header = TRUE)
lmod <- lm(choice ~ Xa+Xb+H+D+T , data=main_data)           # linear model␣
 ↪regression
k <- ols_step_all_possible(lmod)     # all possible regressions
```

Here's the ouput generated by R for k in the above snippet:

```
   Index N   Predictors    R-Square Adj. R-Square Mallow's Cp
3      1 1            H 0.251406437  0.2509240962    3.012463
2      2 1           Xb 0.137924879  0.1373694180  238.438312
1      3 1           Xa 0.134310675  0.1337528854  245.936244
5      4 1            T 0.000129974 -0.0005142721  524.303987
4      5 1            D 0.000000000  0.0000000000  522.573628
7      6 2         Xa H 0.251644988  0.2506799914    4.517572
10     7 2         Xb H 0.251573823  0.2506087346    4.665209
14     8 2          H T 0.251406636  0.2504413317    5.012051
13     9 2          H D 0.251406437  0.2509240962    3.012463
9     10 2         Xa T 0.205578190  0.2045537901  100.086537
12    11 2         Xb T 0.205295974  0.2042712108  100.672014
6     12 2        Xa Xb 0.140614075  0.1395059054  234.859375
11    13 2         Xb D 0.137924879  0.1373694180  238.438312
8     14 2         Xa D 0.134310675  0.1337528854  245.936244
15    15 2          D T 0.000129974 -0.0005142721  524.303987
20    16 3       Xa H T 0.252594638  0.2511480467    4.547454
23    17 3       Xb H T 0.252155056  0.2507076139    5.459399
16    18 3      Xa Xb H 0.252103531  0.2506559893    5.566292
19    19 3       Xa H D 0.251644988  0.2506799914    4.517572
22    20 3       Xb H D 0.251573823  0.2506087346    4.665209
25    21 3        H D T 0.251406636  0.2504413317    5.012051
18    22 3      Xa Xb T 0.205838121  0.2043010331  101.547291
21    23 3       Xa D T 0.205578190  0.2045537901  100.086537
```

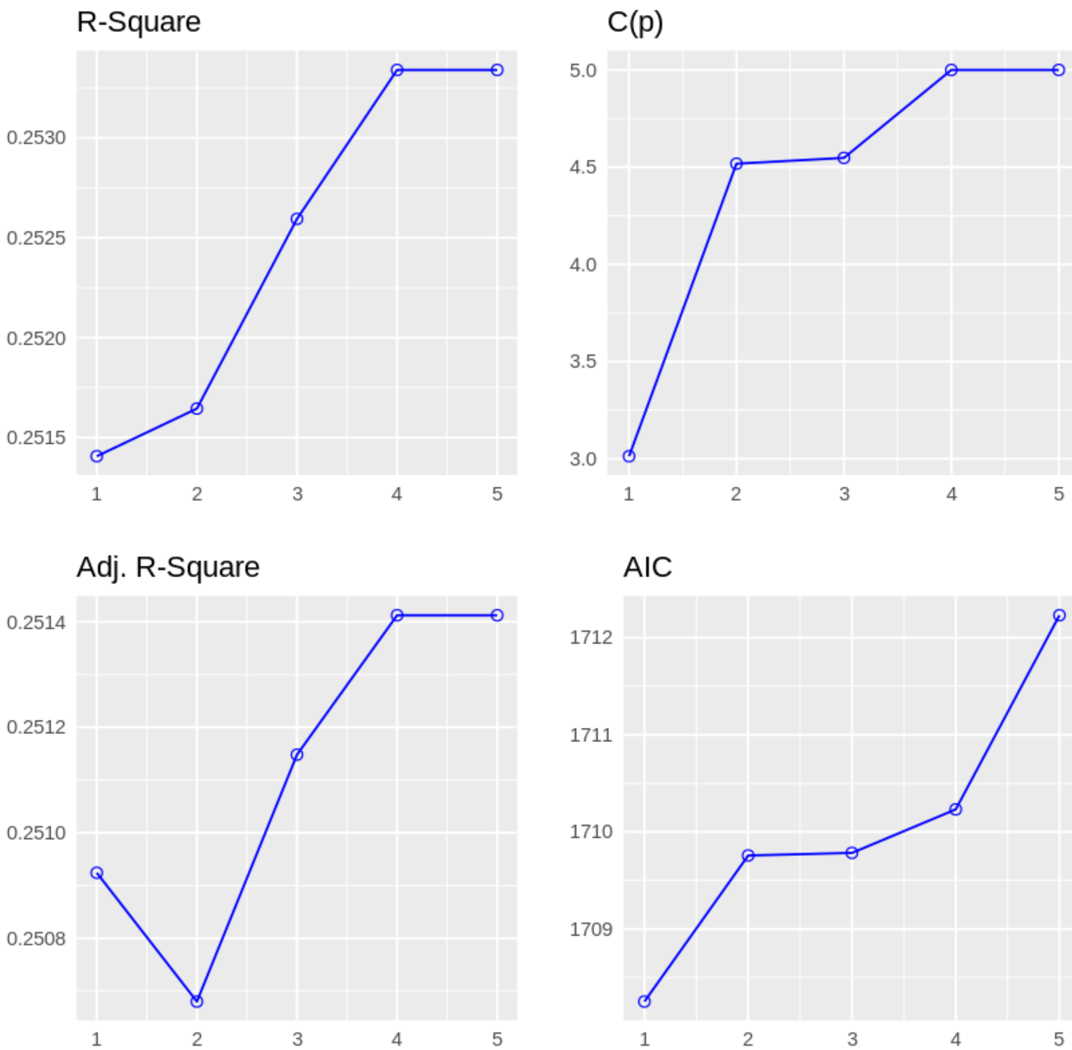
```
24     24 3       Xb D T 0.205295974  0.2042712108  100.672014
17     25 3      Xa Xb D 0.140614075  0.1395059054  234.859375
27     26 4    Xa Xb H T 0.253340552  0.2514124450    5.000000
29     27 4     Xa H D T 0.252594638  0.2511480467    4.547454
30     28 4     Xb H D T 0.252155056  0.2507076139    5.459399
26     29 4    Xa Xb H D 0.252103531  0.2506559893    5.566292
28     30 4    Xa Xb D T 0.205838121  0.2043010331  101.547291
31     31 5 Xa Xb H D T 0.253340552  0.2514124450    5.000000
```

[24]:
```
k <- ols_step_best_subset(lmod, print_plot = TRUE)    # best subset␣
↪regregression
plot(k)
```

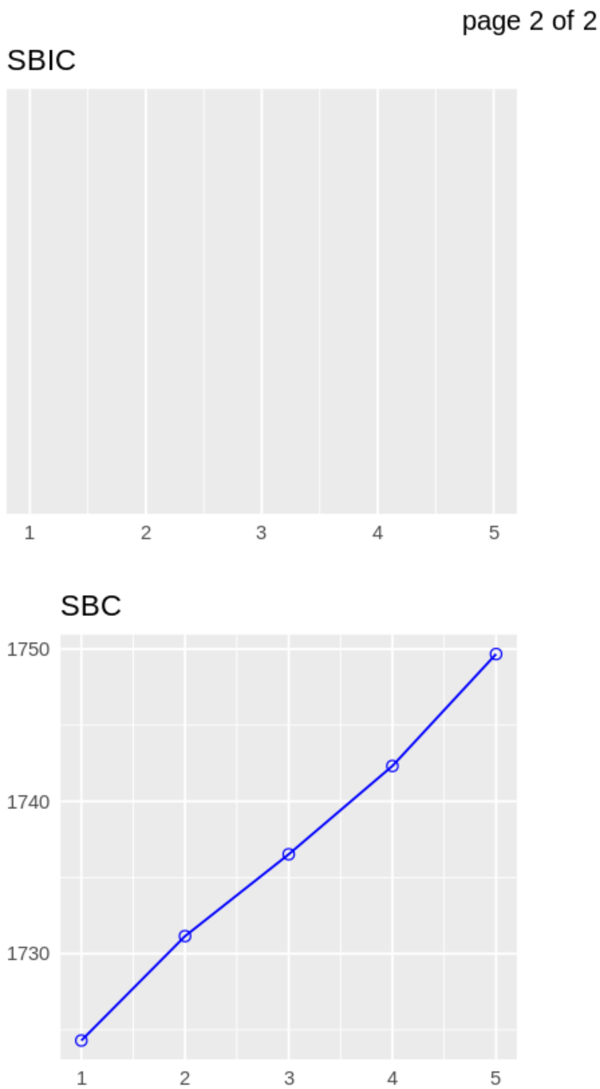
```
[[1]]
NULL

[[2]]
NULL
```

page 1 of 2



R-Square
0.2530
0.2525
0.2520
0.2515
1
2
3
4
5
C(p)
5.0
4.5
4.0
3.5
3.0
1
2
3
4
5
Adj. R-Square
0.2514
0.2512
0.2510
0.2508
1
2
3
4
5
AIC
1712
1711
1710
1709
1
2
3
4
5



Here is the output generated by R for k in the above snippet:

```
Best Subsets Regression
--------------------------
Model Index    Predictors
--------------------------
     1         H
     2         Xa H
     3         Xa H T
     4         Xa Xb H T
     5         Xa Xb H D T
--------------------------

                           Subsets Regression Summary
```

```
--------------------------------------------------------------------------------
                          Adj.        Pred
Model     R-Square    R-Square    R-Square     C(p)        AIC        SBIC
--------------------------------------------------------------------------------
  1        0.2514      0.2509      0.2496    3.0125    1708.2523       NA
  2        0.2516      0.2507      0.2487    4.5176    1709.7570       NA
  3        0.2526      0.2511      0.2488    4.5475    1709.7837       NA
  4        0.2533      0.2514      0.2486    5.0000    1710.2321       NA
  5        0.2533      0.2514      0.2486    5.0000    1712.2321       NA
--------------------------------------------------------------------------------

Model         SBC          MSEP        FPE        HSP       APC
--------------------------------------------------------------------------------
  1       1724.2980    272.4346     0.1755     1e-04    0.7505
  2       1731.1513    272.5235     0.1757     1e-04    0.7513
  3       1736.5267    272.3534     0.1757     1e-04    0.7513
  4       1742.3236    272.2574     0.1758     1e-04    0.7515
  5       1749.6722    272.2574     0.1758     1e-04    0.7515


 AIC: Akaike Information Criteria
 SBIC: Sawa's Bayesian Information Criteria
 SBC: Schwarz Bayesian Criteria
 MSEP: Estimated error of prediction, assuming multivariate normality
 FPE: Final Prediction Error
 HSP: Hocking's Sp
 APC: Amemiya Prediction Criteria
```

[26]:

```r
# forward stepwise regression
# ============================
k <- ols_step_forward_p(lmod,details=TRUE)
k
```

```
Forward Selection Method
---------------------------

Candidate Terms:

1. Xa
2. Xb
3. H
4. D
5. T

We are selecting variables based on p value…


Note: model has aliased coefficients
```

```
      sums of squares computed by model comparison


Forward Selection: Step 1

- H

                        Model Summary
--------------------------------------------------------------
R                       0.501       RMSE                0.419
R-Squared               0.251       Coef. Var          76.684
Adj. R-Squared          0.251       MSE                 0.175
Pred R-Squared          0.250       MAE                 0.358
--------------------------------------------------------------
 RMSE: Root Mean Square Error
 MSE: Mean Square Error
 MAE: Mean Absolute Error

                                ANOVA
----------------------------------------------------------------------
               Sum of
              Squares          DF    Mean Square       F         Sig.
----------------------------------------------------------------------
Regression     91.376           1         91.376    521.221    0.0000
Residual      272.084        1552          0.175
Total         363.460        1553
----------------------------------------------------------------------

                                 Parameter Estimates
--------------------------------------------------------------------------------
------
      model     Beta    Std. Error    Std. Beta      t        Sig      lower
upper
--------------------------------------------------------------------------------
------
(Intercept)    0.009         0.026                  0.368    0.713    -0.041
0.060
          H    0.108         0.005        0.501    22.830    0.000     0.099
0.118
--------------------------------------------------------------------------------
------


Note: model has aliased coefficients
      sums of squares computed by model comparison
```

```
No more variables to be added.

Variables Entered:

+ H


Final Model Output
------------------

                        Model Summary
--------------------------------------------------------------
R                       0.501       RMSE                0.419
R-Squared               0.251       Coef. Var          76.684
Adj. R-Squared          0.251       MSE                 0.175
Pred R-Squared          0.250       MAE                 0.358
--------------------------------------------------------------
 RMSE: Root Mean Square Error
 MSE: Mean Square Error
 MAE: Mean Absolute Error

                                ANOVA
----------------------------------------------------------------------
              Sum of
              Squares         DF    Mean Square       F          Sig.
----------------------------------------------------------------------
Regression     91.376          1         91.376    521.221    0.0000
Residual      272.084       1552          0.175
Total         363.460       1553
----------------------------------------------------------------------

                                 Parameter Estimates
--------------------------------------------------------------------------------
------
      model     Beta    Std. Error    Std. Beta      t        Sig      lower
upper
--------------------------------------------------------------------------------
------
(Intercept)    0.009         0.026                 0.368    0.713    -0.041
0.060
          H    0.108         0.005        0.501    22.830    0.000     0.099
0.118
--------------------------------------------------------------------------------
------

                            Selection Summary
-------------------------------------------------------------------------
```

```
        Variable                        Adj.
Step    Entered        R-Square     R-Square     C(p)         AIC          RMSE
------------------------------------------------------------------------
   1    H                0.2514       0.2509    3.0125    1708.2523    0.4187
------------------------------------------------------------------------
```

[27]:
```
plot(k)
```

```
geom_path: Each group consists of only one observation. Do you need to adjust
the group aesthetic?

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the group aesthetic?

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the group aesthetic?
```
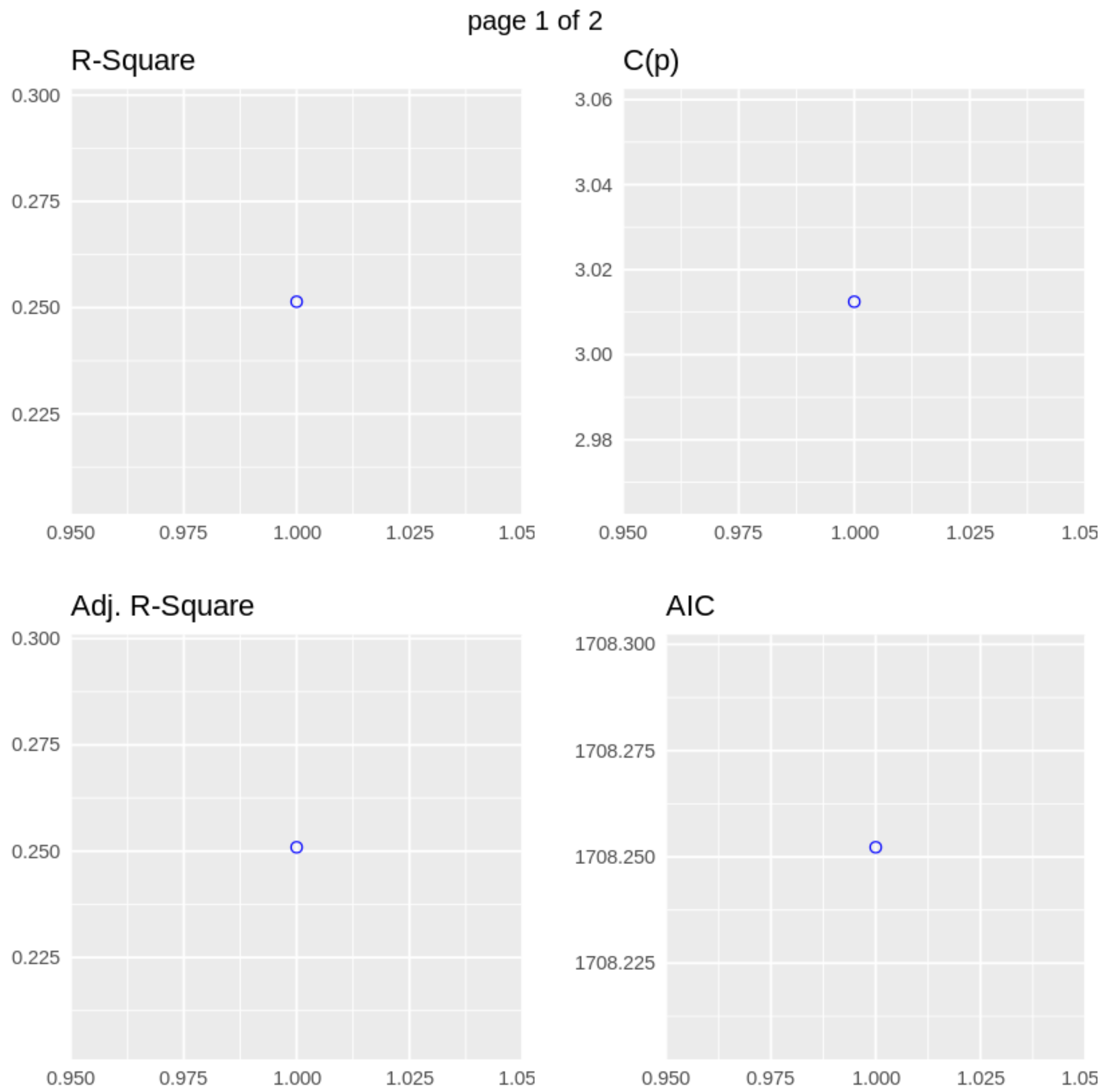
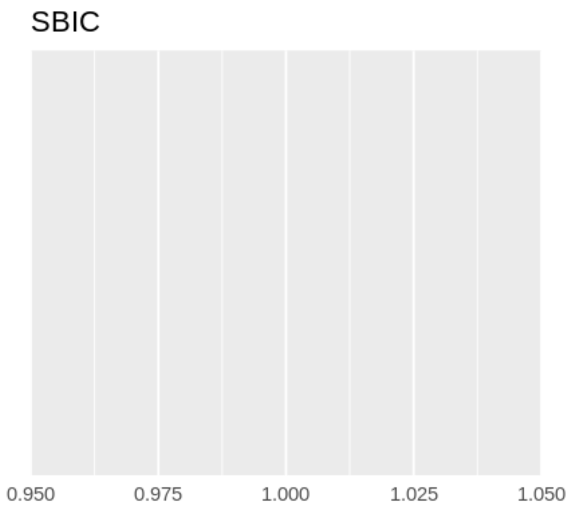
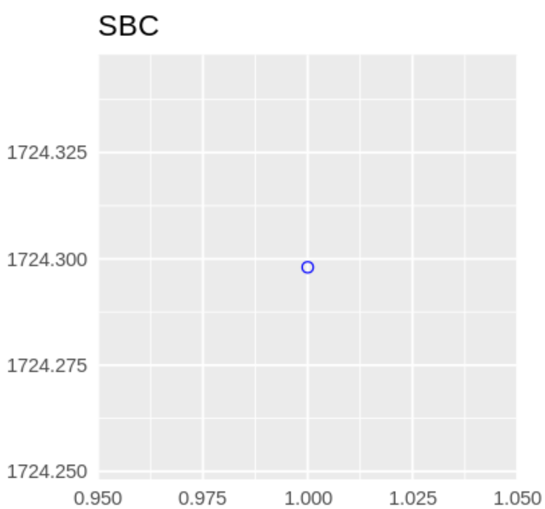

```
[[1]]
NULL

[[2]]
NULL
```


page 1 of 2
R-Square
0.300
0.275
0.250
0.225
0.950
0.975
1.000
1.025
1.05
C(p)
3.06
3.04
3.02
3.00
2.98
0.950
0.975
1.000
1.025
1.05
Adj. R-Square
0.300
0.275
0.250
0.225
0.950
0.975
1.000
1.025
1.05
AIC
1708.300
1708.275
1708.250
1708.225
0.950
0.975
1.000
1.025
1.05

page 2 of 2





Explanation: *Build regression model from a set of candidate predictor variables by entering predictors based on p values, in a stepwise manner until there is no variable left to enter any more. The model should include all the candidate predictor variables. If details is set to TRUE, each step is displayed.*

[28]:
```
# backward stepwise regression
# ============================
k <- ols_step_backward_p(lmod,details=TRUE)
k
```

```
Backward Elimination Method
---------------------------


Candidate Terms:
```

```
1 . Xa
2 . Xb
3 . H
4 . D
5 . T

We are eliminating variables based on p value…

Note: model has aliased coefficients
      sums of squares computed by model comparison


No more variables satisfy the condition of p value = 0.3


Variables Removed:



Final Model Output
------------------

                        Model Summary
--------------------------------------------------------------
R                       0.503       RMSE                0.419
R-Squared               0.253       Coef. Var          76.659
Adj. R-Squared          0.251       MSE                 0.175
Pred R-Squared          0.249       MAE                 0.357
--------------------------------------------------------------
 RMSE: Root Mean Square Error
 MSE: Mean Square Error
 MAE: Mean Absolute Error

                                ANOVA
----------------------------------------------------------------------
              Sum of
             Squares          DF    Mean Square       F          Sig.
----------------------------------------------------------------------
Regression    92.079           4         23.020    131.393    0.0000
Residual     271.381        1549          0.175
Total        363.460        1553
----------------------------------------------------------------------

                                  Parameter Estimates
--------------------------------------------------------------------------------
-------
```

```
      model       Beta     Std. Error     Std. Beta      t         Sig       lower
upper
--------------------------------------------------------------------------------
-------
(Intercept)     0.128         0.076                    1.696     0.090    -0.020
0.277
         Xa    -0.183         0.117        -0.499     -1.568     0.117    -0.412
0.046
         Xb     0.092         0.074         0.378      1.244     0.214    -0.053
0.237
          H     0.128         0.013         0.592      9.927     0.000     0.103
0.153
          D        NA         0.001         0.000     -1.602     0.109        NA
NA
          T    -0.002            NA        -6.278         NA        NA    -0.005
0.001
--------------------------------------------------------------------------------
-------

[1] "No variables have been removed from the model."
```

Explanation: *Build regression model from a set of candidate predictor variables by removing predictors based on p values, in a stepwise manner until there is no variable left to remove any more. The model should include all the candidate predictor variables. If details is set to TRUE, each step is displayed*

[30]:
```
# sequential (both directions) stepwise regression
# ===============================================
k <- ols_step_both_p(lmod,details=TRUE)
k
```

```
Stepwise Selection Method
---------------------------

Candidate Terms:

1. Xa
2. Xb
3. H
4. D
5. T

We are selecting variables based on p value…


Note: model has aliased coefficients
      sums of squares computed by model comparison
```

Stepwise Selection: Step 1

- H added

Model Summary

R	0.501	RMSE	0.419
R-Squared	0.251	Coef. Var	76.684
Adj. R-Squared	0.251	MSE	0.175
Pred R-Squared	0.250	MAE	0.358

RMSE: Root Mean Square Error
MSE: Mean Square Error
MAE: Mean Absolute Error

ANOVA

	Sum of Squares	DF	Mean Square	F	Sig.
Regression	91.376	1	91.376	521.221	0.0000
Residual	272.084	1552	0.175		
Total	363.460	1553			

Parameter Estimates

model	Beta	Std. Error	Std. Beta	t	Sig	lower	upper
(Intercept)	0.009	0.026		0.368	0.713	-0.041	0.060
H	0.108	0.005	0.501	22.830	0.000	0.099	0.118

Note: model has aliased coefficients
sums of squares computed by model comparison

No more variables to be added/removed.

Final Model Output

Model Summary

R	0.501	RMSE	0.419
R-Squared	0.251	Coef. Var	76.684
Adj. R-Squared	0.251	MSE	0.175
Pred R-Squared	0.250	MAE	0.358

RMSE: Root Mean Square Error
MSE: Mean Square Error
MAE: Mean Absolute Error

ANOVA

	Sum of Squares	DF	Mean Square	F	Sig.
Regression	91.376	1	91.376	521.221	0.0000
Residual	272.084	1552	0.175		
Total	363.460	1553			

Parameter Estimates

model	Beta	Std. Error	Std. Beta	t	Sig	lower	upper
(Intercept)	0.009	0.026		0.368	0.713	-0.041	0.060
H	0.108	0.005	0.501	22.830	0.000	0.099	0.118

Stepwise Selection Summary

Step	Variable	Added/ Removed	R-Square	Adj. R-Square	C(p)	AIC	RMSE
1	H	addition	0.251	0.251	3.0120	1708.2523	0.4187

[31]:
```
plot(k)
```

```
geom_path: Each group consists of only one observation. Do you need to adjust
the group aesthetic?

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the group aesthetic?

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the group aesthetic?

geom_path: Each group consists of only one observation. Do you need to adjust
the group aesthetic?

geom_path: Each group consists of only one observation. Do you need to adjust
the group aesthetic?
```
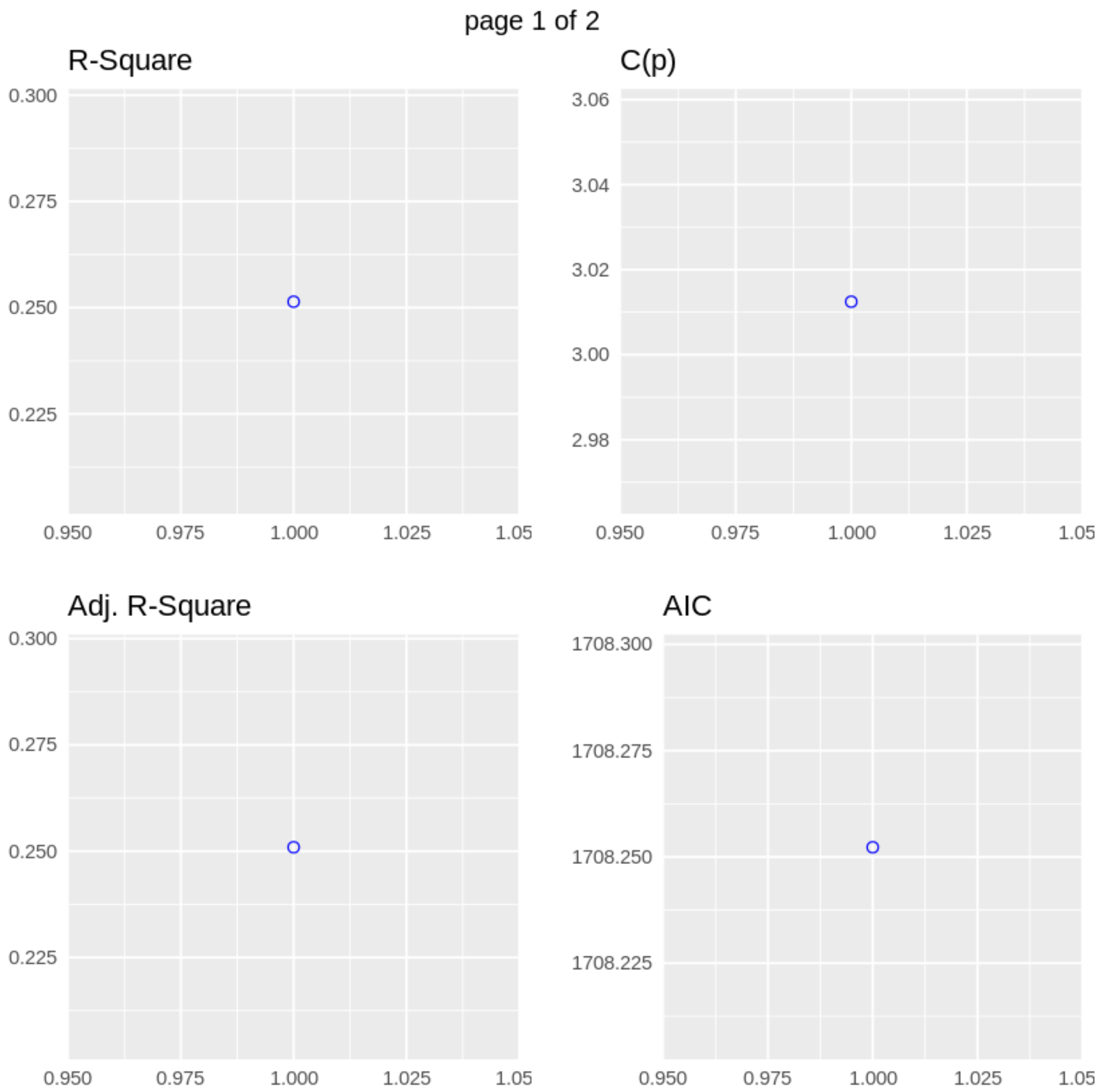
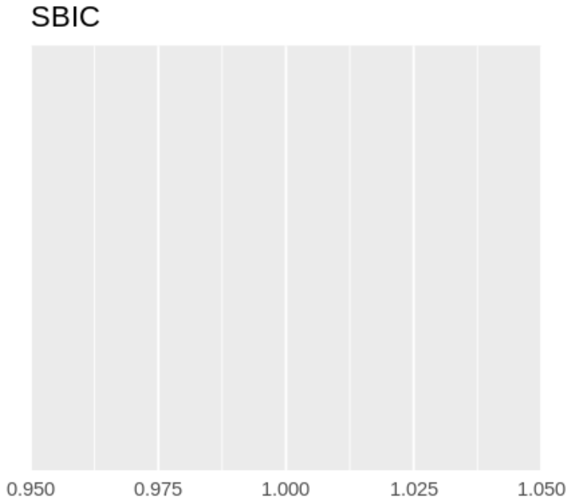
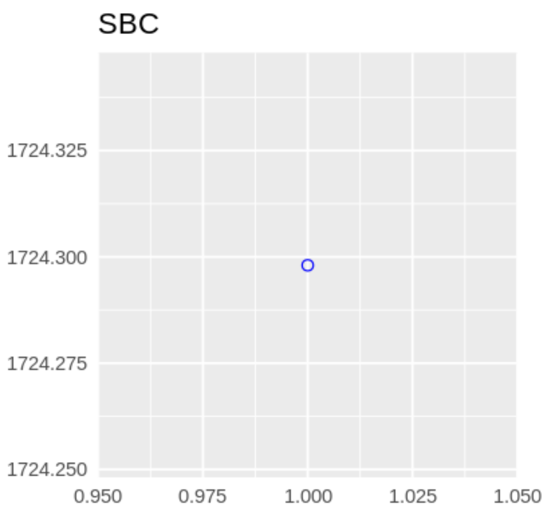

```
[[1]]
NULL

[[2]]
NULL
```


page 1 of 2
R-Square
0.300
0.275
0.250
0.225
0.950 0.975 1.000 1.025 1.05
C(p)
3.06
3.04
3.02
3.00
2.98
0.950 0.975 1.000 1.025 1.05
Adj. R-Square
0.300
0.275
0.250
0.225
0.950 0.975 1.000 1.025 1.05
AIC
1708.300
1708.275
1708.250
1708.225
0.950 0.975 1.000 1.025 1.05

page 2 of 2





Explanation: *Build regression model from a set of candidate predictor variables by entering and removing predictors based on p values, in a stepwise manner until there is no variable left to enter or remove any more. The model should include all the candidate predictor variables. If details is set to TRUE, each step is displayed.*

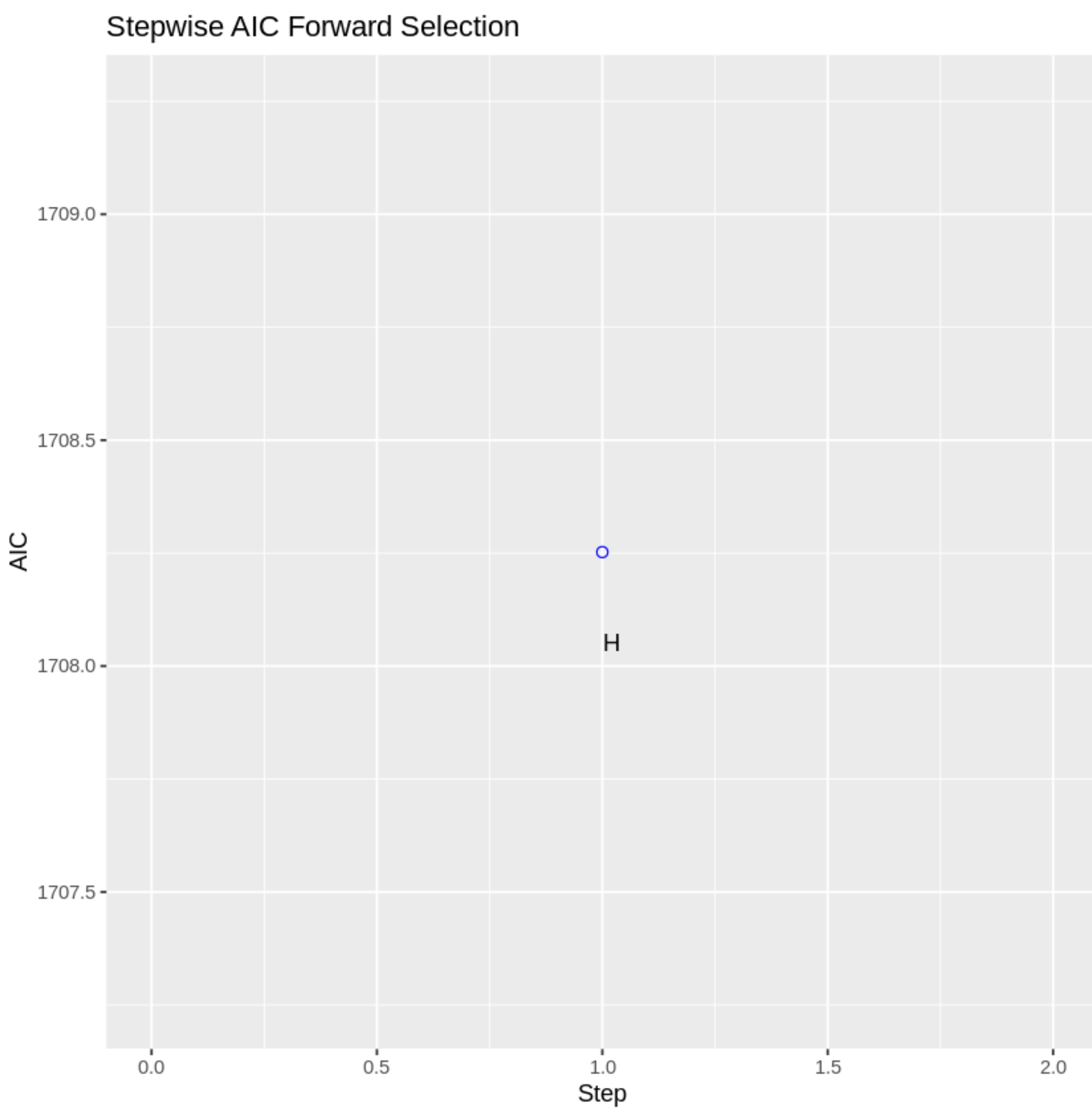
[32]:
```
# Stepwise models based on Akaike Information Criteria criterion
# ==============================================================
k <- ols_step_forward_aic(lmod)       # forward selection
k
```

```
                        Selection Summary
------------------------------------------------------------------
Variable        AIC        Sum Sq       RSS        R-Sq      Adj. R-Sq
------------------------------------------------------------------
```

```
H             1708.252     91.376     272.084     0.25141      0.25092
------------------------------------------------------------------
```

[33]:
```
plot(k)
```

```
geom_path: Each group consists of only one observation. Do you need to adjust
the group aesthetic?
```



[35]:
```
k <- ols_step_both_aic(lmod)          # both sides
k
```

```
                               Stepwise Summary
-----------------------------------------------------------------------------
Variable      Method         AIC          RSS        Sum Sq      R-Sq      Adj. R-Sq
-----------------------------------------------------------------------------
H            addition     1708.252     272.084     91.376     0.25141      0.25092
-----------------------------------------------------------------------------
```

[37]:
```
plot(k)
```

```
geom_path: Each group consists of only one observation. Do you need to adjust
the group aesthetic?
```