CS 323

Homework # 5: due May 2

Problem 1 Use the Euler's method with step size $h = 1/2$ to compute approximations to $y(1/2)$ and $y(1)$, where $y(x)$ is the solution of the Initial Value Problem

$$y' = 1 - 8xy, \quad y(0) = 0.$$

b) Repeat part a) using a Taylor series methods of order two.
c) Repeat part a) using Heun's method

Solution a)

$$\begin{aligned} y_1 &= y_0 + h * f(x_0, y_0) = 0 + 0.5 * 1 = 0.5 \\ y_2 &= y_1 + h * f(x_1, y_1) = 0.5 + 0.5 * (-1) = 0 \end{aligned}$$

b)

$$\begin{aligned} y_1 &= y_0 + h * f(x_0, y_0) + \frac{h^2}{2}(f_x + f_y f)(x_0, y_0) = 0 + 0.5 * 1 + 0.5^3 * 0 = 0.5 \\ y_2 &= y_1 + h * f(x_1, y_1) + \frac{h^2}{2}(f_x + f_y f)(x_1, y_1) = 0.5 + 0.5 * (-1) + 0.5^3 * 0 = 0 \end{aligned}$$

c)

$$\begin{aligned} v_1 &= f(x_0, y_0) = 1, \quad v_2 = f(x_1, y_0 + h * v_1) = -1 \\ y_1 &= y_0 + h * (v_1 + v_2)/2 = 0 + 0.5 * 0 = 0 \\ v_1 &= f(x_1, y_1) = 1, \quad v_2 = f(x_2, y_1 + h * v_1) = -3 \\ y_2 &= y_1 + h * (v_1 + v_2)/2 = -0.5 \end{aligned}$$

Problem 2 In this problem, we use Euler's method to approximate the solution $y(x)$ of the Initial Value problem

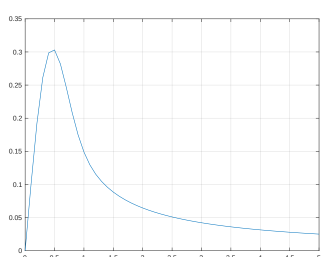
$$y' = 1 - 8xy, \quad y(0) = 0.$$

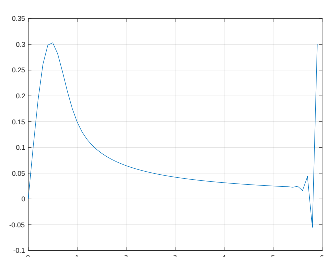
a) Find an approximate solution on the interval $[0, 5]$ with stepsize $h = 0.1$ and plot the solution by using the following command

```
clear
x0 = 0;
y0 = 0;
xfinal = 5;
h = 0.1;
n = (xfinal - x0)/h;
n = round(n);
f = @(x,y) 1 - 8*x*y;
x = linspace(x0,xfinal,n+1);
y = zeros(1,n+1);
y(1) = y0;
for i = 1:n
    y(i+1) = y(i) + h*f(x(i), y(i));
end
plot(x,y)
[x(end),y(end)]
grid on                 % Use this statement to see the values more clearly
```

b) Find approximation solution on the interval $[0, 5.9]$ and plot its solution replace `xfinal`$= 5$ by `xfinal`$= 5.9$. We know that the exact solution is a smooth function. Is the numerical solution is an accurate approximation to the exact solution $y(5.9)$?

Solution





On the interval $[0, 5.9]$, the numerical solution oscillates and does not give an accurate approximation of the exact solution.