## The Likelihood Ratio Test Procedure

Used in logistic regression to test hypotheses concerning the regression coefficients.

Let  $L_I$  be the maximum value of the likelihood of the data without the additional assumption. In other words,  $L_I$  is the likelihood of the data with all the parameters unrestricted and maximum likelihood estimates substituted for these parameters.

Let  $L_0$  be the maximum value of the likelihood when the parameters are restricted (and reduced in number) based on the assumption. Assume k parameters were lost (i.e.,  $L_0$  has k less parameters than  $L_I$ ).

Form the ratio  $\frac{\lambda}{l} = L_0/L_1$ . This ratio is always between 0 and 1 and the less likely the assumption is, the smaller  $\frac{\lambda}{l}$  will be. This can be quantified at a given confidence level as follows:

- 1. Calculate  $\chi^2 = -2 \ln \lambda$ . The smaller  $\lambda$  is, the larger  $\chi^2$  will be.
- 2. We can tell when  $\chi^2$  is significantly large by comparing it to the upper  $100 \times (1^{-\alpha})$  percentile point of a Chi Square distribution with k degrees of freedom.  $\chi^2$  has an approximate Chi-Square distribution with k degrees of freedom and the approximation is usually good, even for small sample sizes.
- 3. The likelihood ratio test computes  $\frac{\chi^2}{\chi^2}$  and rejects the assumption if  $\frac{\chi^2}{\chi^2}$  is larger than a Chi-Square percentile with k degrees of freedom, where the percentile corresponds to the confidence level chosen by the analyst.

**Note**: While Likelihood Ratio test procedures are very useful and widely applicable, the computations are difficult to perform by hand, especially for censored data, and appropriate software is necessary.