Quiz 3

This quiz is open everything, including notes, textbook, and whatever you might find on the internet. Feel free to use a calculator, whether a physical one or an app on your computer or phone. However, no interaction with other people is allowed. You must complete the quiz within the 24 hour window starting at noon ET on Sept 21. Once you start, you have 60 minutes for the quiz; however, the Canvas timer is set to 65 minutes to provide a cushion in case of technical glitches, and for that matter I don't expect that you will need anywhere close to 60 minutes.

Info for questions 1, 2, and 3.

Suppose that a parameter θ can take only the values shown in the table below. Based on this table, please answer questions 1, 2, and 3.

θ	Prior for θ	$P(data \mid \theta)$
0	1/8	1/2
2	1/2	3/4
4	3/8	1/2

Give your answers to three digits, and be sure to give it as a number, not a percentage.

Unscaled posterior

$\frac{1}{8} \times \frac{1}{2} = \frac{1}{16}$

$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8} = \frac{6}{16}$

$\frac{3}{8} \times \frac{1}{2} = \frac{3}{16}$

$\text{Total} = \frac{1}{16} + \frac{6}{16} + \frac{3}{16} = \frac{10}{16}$

Actual posterior

$\frac{1}{16} / \frac{10}{16} = \frac{1}{10}$

$\frac{6}{16} / \frac{10}{16} = \frac{6}{10}$

$\frac{3}{16} / \frac{10}{16} = \frac{3}{10}$

Note that this column adds to 1.0

1 1 point

What is the posterior probability that $\theta = 0$?

Type your answer... $\frac{1}{10} = .100$

2 1 point

What is the posterior probability that $\theta = 2$?

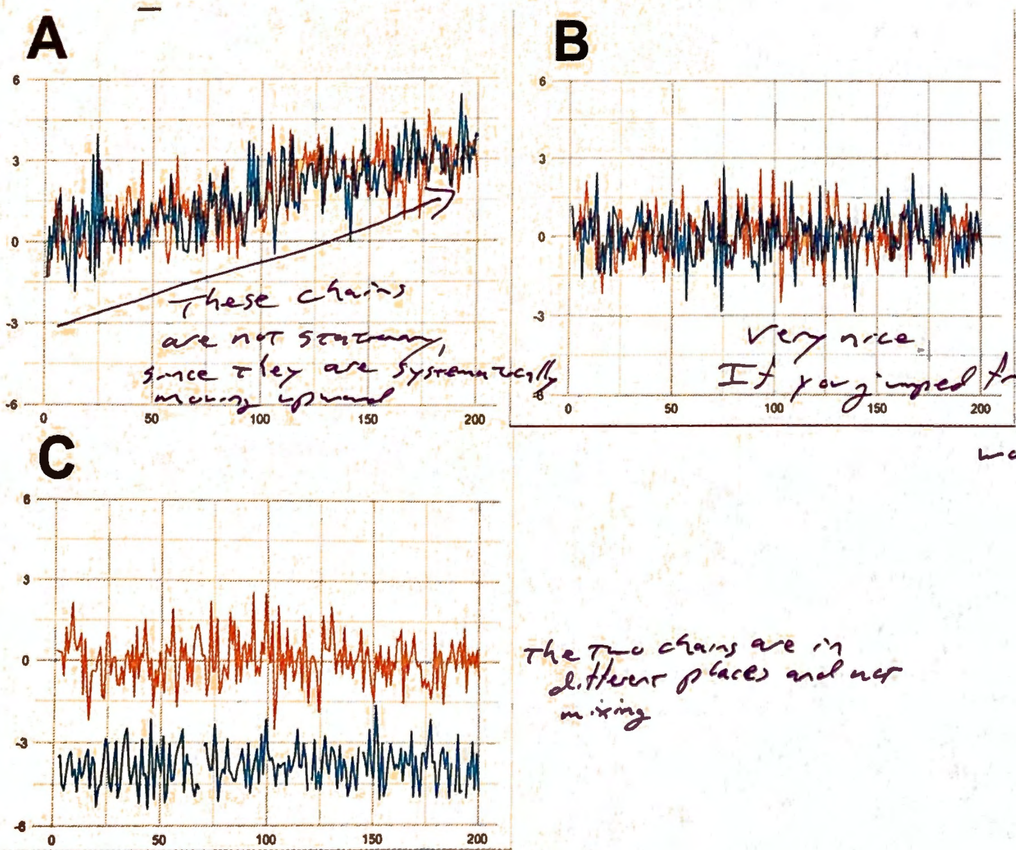
Type your answer... $\frac{6}{10} = .600$

3 1 point

What is the posterior probability that $\theta = 4$?

Type your answer... $\frac{3}{10} = .300$

Consider the following trace plots of MCMC chains, from 3 different analyses. Each panel shows 2 chains (shown in different colors). Please indicate which panels indicate that the algorithm is working well (i.e., that the chains show good behavior).



These chains are not stationary, since they are systematically moving upward

Very nice. If you jumped from one chain to the other you would hardly notice

The two chains are in different places and not mixing

- ○ A, B, and C
- ○ A only
- ● B only
- ○ C only
- ○ A and B
- ○ A and C
- ○ B and C

Info for questions 5, 6, 7, 8

Please consider a posterior distribution that has, for a subset of parameter values, the following values proportional to the posterior probability density:

θ	unstandardized posterior
12	4
13	1
14	2
15	8

For questions 5, 6, and 7, please write the probability that you would jump from theta to theta* when running the Metropolis algorithm. Please give your answer to 3 decimal places.

The probability of jumping from θ to θ^* is

$$r = \min\left(1, \frac{prob(\theta^*)}{prob(\theta)}\right)$$

1 point

theta = 12, theta* = 13 $\quad r = \min(1, \frac{1}{4})$

Type your answer... .250

6 1 point

theta = 12, theta* = 15 $\quad r = \min(1, \frac{8}{4})$

Type your answer... 1.000

7 1 point

theta = 14, theta* = 13 $\quad r = \min(1, \frac{1}{2})$

Type your answer... .500

8 2 points

Suppose you ran the Metropolis algorithm for many iterations, and ended up with 1000 samples with $\theta = 13$. About how many samples with $\theta = 14$ would expect to see. Please give a single number, not a range.

Type your answer...

The Metropolis algorithm will add θs to the sample in roughly ~~the~~ proportion to the distribution. Since it's a proportion, it doesn't matter whether we are talking about the actual posterior or an unscaled version of it. Since $\theta = 14$ has twice the probability of $\theta = 13$, if there are 1,000 samples with $\theta = 13$, there will be about twice that, 2,000, for $\theta = 14$.