

Final Practice Problems

CS 323 , SPRING 2019

Problem 1 [10+10 points]

a) Give Newton's method for finding $\sqrt[3]{2}$ by solving $x^3 - 2 = 0$.

b) Find the *first* Newton's iteration to solve for the nonlinear system

$$f_1(x, y) = xy + 1.089 = 0$$

$$f_2(x, y) = x^2 + y^2 - 3.23 = 0$$

with initial value $x_0 = 1, y_0 = 0$.

Problem 2

a) Let

$$A = \begin{bmatrix} 19 & 20 \\ 20 & 21 \end{bmatrix}$$

Find the conditional number of A .

b) We consider the error of the solution x of the linear system $Ax = b$. We showed in class that,

$$\frac{\|x - z\|}{\|x\|} \leq \|A\| \|A^{-1}\| \frac{\|Az - b\|}{\|b\|}.$$

If $b = [1, 1/2]$ and $Az = [1.001, 0.499]$, estimate the relative error $\frac{\|x - z\|}{\|x\|}$.

Problem 3 a) Find the least square fitting by linear polynomial for the given data $(0, 1)$, $(2, 1)$ and $(3, 2)$.

b) Find the interpolating polynomial for the given data $(0, 1)$, $(2, 1)$ and $(3, 2)$ in the Newton's form.

Problem 4

a) Let $f(x)$ be a function defined on the interval $[-1, 1]$ with

$$f(-1) = f(1) = 0, \quad f(-1/2) = f(1/2) = 1, \quad f(0) = 2/3.$$

Find an approximation of $\int_{-1}^1 f(x)dx$ with Simpson's rule.

b) If $|f^{(4)}(x)| < 5$ in $[-1, 1]$, find an error estimate of the approximation in part a).

Problem 5 Use Euler's method to find the an approximation to the solution $y(1/2)$ of the problem

$$y'(x) = -x^2y, \quad y(0) = 1$$

with stepsize $h = 0.5$.

Problem 6 Trapezoid method is a second order implicit method for solving the problem

$$y'(x) = f(x, y), \quad y(0) = y_0.$$

The method reads

$$y_{n+1} = y_n + h[f(x_n, y_n) + f(x_{n+1}, y_{n+1})]/2.$$

Use Trapezoid method to find the approximation to the solution $y(1/2)$ of the problem

$$y'(x) = -x^2y, \quad y(0) = 1$$

with stepsize 0.5.