Midterm I Practice Problems

CS 323 , Spring 2019

Problem 1 [10+10 points]
a) Give Newton's method for finding $\sqrt[3]{2}$ by solving $x^3 - 2 = 0$.

b) Show that $\sqrt[3]{2}$ is in the interval $[1, 2]$ by mean value theorem. Give a bisection method for finding $\sqrt[3]{2}$.

Problem 2

a) Let

$$A = \begin{bmatrix} 19 & 20 \\ 20 & 21 \end{bmatrix}$$

Find the conditional number of A.

b) We consider the error of the solution x of the linear system $Ax = b$. We showed in class that,

$$\frac{\|x - z\|}{\|x\|} \leq \|A\| \|A^{-1}\| \frac{\|Az - b\|}{\|b\|}.$$

If $b = [1, 1/2]$ and $Az = [1.001, 0.499]$, estimate the relative error $\frac{\|x-z\|}{\|x\|}$.

Problem 3 a) Give a Jacobi method for solving $Ax = b$ where

$$A = \begin{bmatrix} 5 & 2 & 3 \\ 0 & -4 & 2 \\ -1 & 1 & 5 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$$

b) Find the iterate matrix M of Jacobi method in part a) and find the norm $\|M\|$.

c) Does the Jacobi method converge?

Problem 4 a) Give a power method to approximate the eigenvector v of the matrix A associated with the largest eigenvalue where

$$A = \begin{bmatrix} 4 & 2 & -1 \\ 0 & 3 & -2 \\ 0 & 0 & 5 \end{bmatrix} \quad \text{and initial guess} \quad z^0 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

b) Show that the eigenvalues of A in part a) are $\{4, 3, 5\}$. And find the eigenvector v associated with the largest eigenvalue.

c) The error estimate of the power method shows that

$$\|z^m - v\| \approx \|z^0 - v\| \left|\frac{\lambda_2}{\lambda_1}\right|^m \qquad \lambda_i \text{ eigenvalues of } A \text{ and } |\lambda_1| > |\lambda_2| > |\lambda_3| \ldots$$

Estimate the number of iterations of the power method needed such that

$$\|z^m - v\| \leq 10^{-4}.$$