Bayesian Data Analysis Course Introduction

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Framework of the course

- Introduction
- A very simple model, coin tossing, in depth.
- First, an analytic discussion.
- Next, computational approaches.
- Many other kind of analyses.

Coin tossing

- You've most likely learned only what is called "frequentist" (or traditional) statistics, but frequentist statistics has some shortcomings.
- E.g., suppose you have a coin with probability θ of heads. Suppose you toss it twice and get heads both times. Then by traditional statistics, your estimate of θ would be $\hat{\theta} = 1.0$.
  - Of course your standard error would be large, but do you really think that the probability of heads is 1.0?
  - It feels like you'd be ignoring what you know about coins.

Difficult Frequentist Concepts

- Consider two central concepts from frequentist statistics. Most people can't interpret these correctly.
  - P-values. Can you correctly state what it means if for some statistical test we find $p = 0.02$?
  - Confidence intervals. Can you correctly state what it means that a 95% confidence interval of θ is $(0.4, 0.7)$?

P-value

- A correct interpretation of $p = 0.02$ is that the probability, under the null hypothesis, of seeing a test statistic as large as that observed or even more extreme, is 0.02.
    - However, pretty much everyone, including most scientists, wants it to mean something like “the probability that the null hypothesis is true is 0.02.”

Confidence Intervals

- A correct interpretation of the sentence that a 95% confidence interval of θ is $(0.4, 0.7)$ is that the interval $(0.4, 0.7)$ was determined by a method that will generate an interval that includes the true value of θ 95% of the time.
  - However, pretty much everyone, including most scientists, wants it to mean that the probability that θ lies in $(0.4, 0.7)$ is 95%.
  - In frequentist statistics, θ is fixed, so the probability that θ lies in $(0.4, 0.7)$ is either 0 or 1.

How Bayesian statistics helps

- For both p-values and confidence intervals, people want to give a straightforward probabilistic interpretation.
- We will see that the Bayesian framework will enable us to make statements with straightforward probabilistic interpretations.

Contrast between frequentist and Bayesian statistics

Frequentist	Bayesian
Parameters are fixed	Parameters are random variables
Use probabilities for data	Use probabilities to talk about hypotheses, parameters and data
Conclusions depend on the observed *and* unobserved data	Conclusions depend on the prior probabilities of the parameters and the likelihood of the observed data
Dominant in the 20th Century	Dominant in the 18th and 19th Centuries
Often not computational intensive	Computationally intensive

In the 21st Century, the dominant ethos seems to be “whatever works.”

Bayesian statistics in a nutshell

Express everything in statistics in the language of probability.