AP Statistics Tutorial

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Negative Binomial Distribution

In this lesson, we cover the negative binomial distribution and the geometric distribution. As we will see, the geometric distribution is a special case of the negative binomial distribution.

Negative Binomial Experiment

A **negative binomial experiment** is a statistical experiment that has the following properties:

- The experiment consists of x repeated trials.
- Each trial can result in just two possible outcomes. We call one of these outcomes a success and the other, a failure.
- The probability of success, denoted by P, is the same on every trial.
- The trials are independent; that is, the outcome on one trial does not affect the outcome on other trials.
- The experiment continues until r successes are observed, where r is specified in advance.

Consider the following statistical experiment. You flip a coin repeatedly and count the number of times the coin lands on heads. You continue flipping the coin until it has landed 5 times on heads. This is a negative binomial experiment because:

- The experiment consists of repeated trials. We flip a coin repeatedly until it has landed 5 times on heads.
- Each trial can result in just two possible outcomes - heads or tails.
- The probability of success is constant - 0.5 on every trial.
- The trials are independent; that is, getting heads on one trial does not affect whether we get heads on other trials.
- The experiment continues until a fixed number of successes have occurred; in this case, 5 heads.

Notation

The following notation is helpful, when we talk about negative binomial probability.

- x: The number of trials required to produce r successes in a negative binomial experiment.
- r: The number of successes in the negative binomial experiment.
- P: The probability of success on an individual trial.
- Q: The probability of failure on an individual trial. (This is equal to 1 - P.)
- $b^*(x; r, P)$: Negative binomial probability - the probability that an x-trial negative binomial experiment results in the rth success on the xth trial, when the probability of success on an individual trial is P.
- ${}_nC_r$: The number of combinations of n things, taken r at a time.

Negative Binomial Distribution

A **negative binomial random variable** is the number *X* of repeated trials to produce *r* successes in a negative binomial experiment. The probability distribution of a negative binomial random variable is called a **negative binomial distribution**. The negative binomial distribution is also known as the **Pascal distribution**.

Suppose we flip a coin repeatedly and count the number of heads (successes). If we continue flipping the coin until it has landed 2 times on heads, we are conducting a negative binomial experiment. The negative binomial random variable is the number of coin flips required to achieve 2 heads. In this example, the number of coin flips is a random variable that can take on any integer value between 2 and plus infinity. The negative binomial probability distribution for this example is presented below.

Number of coin flips	Probability
2	0.25
3	0.25
4	0.1875
5	0.125
6	0.078125
7 or more	0.109375

Negative Binomial Probability

The **negative binomial probability** refers to the probability that a negative binomial experiment results in $r - 1$ successes after trial $x - 1$ and r successes after trial x. For example, in the above table, we see that the negative binomial probability of getting the second head on the sixth flip of the coin is 0.078125.

Given x, r, and P, we can compute the negative binomial probability based on the following formula:

Negative Binomial Formula. Suppose a negative binomial experiment consists of x trials and results in r successes. If the probability of success on an individual trial is P, then the negative binomial probability is:

$$b^*(x; r, P) = {}_{x-1}C_{r-1} * P^r * (1 - P)^{x - r}$$

The Mean of the Negative Binomial Distribution

If we define the mean of the negative binomial distribution as the average number of trials required to produce r successes, then the mean is equal to:

$$\mu = r / P$$

where μ is the mean number of trials, r is the number of successes, and P is the probability of a success on any given trial.

Alternative Views of the Negative Binomial Distribution

As if statistics weren't challenging enough, the above definition is not the only definition for the negative binomial distribution. Two common alternative definitions are:

- The negative binomial random variable is R, the number of successes before the binomial experiment results in k failures. The mean of R is:

$$\mu_R = kP/Q$$

- The negative binomial random variable is K, the number of failures before the binomial experiment results in r successes. The mean of K is:

$$\mu_K = rQ/P$$

The moral: If someone talks about a negative binomial distribution, find out how they are defining the negative binomial random variable.

On this web site, when we refer to the negative binomial distribution, we are talking about the definition presented earlier. That is, we are defining the negative binomial random variable as *X*, the total number of trials required for the binomial experiment to produce *r* successes.

Geometric Distribution

The **geometric distribution** is a special case of the negative binomial distribution. It deals with the number of trials required for a single success. Thus, the geometric distribution is negative binomial distribution where the number of successes (*r*) is equal to 1.

An example of a geometric distribution would be tossing a coin until it lands on heads. We might ask: What is the probability that the first head occurs on the third flip? That probability is referred to as a **geometric probability** and is denoted by g(*x*; *P*). The formula for geometric probability is given below.

Geometric Probability Formula. Suppose a negative binomial experiment consists of *x* trials and results in one success. If the probability of success on an individual trial is *P*, then the geometric probability is:

$$g(x; P) = P * Q^{x - 1}$$

Sample Problems

The problems below show how to apply your new-found knowledge of the negative binomial distribution (see Example 1) and the geometric distribution (see Example 2).

Negative Binomial Calculator

As you may have noticed, the negative binomial formula requires some potentially time-consuming computations. The Negative Binomial Calculator can do this work for you - quickly, easily, and error-free. Use the Negative Binomial Calculator to compute negative binomial probabilities and geometric probabilities. The calculator is free. It can be found under the Stat Tables tab, which appears in the header of every Stat Trek web page.

Negative Binomial Calculator

Example 1

Bob is a high school basketball player. He is a 70% free throw shooter. That means his probability of making a free throw is 0.70. During the season, what is the probability that Bob makes his third free throw on his fifth shot?

Solution: This is an example of a negative binomial experiment. The probability of success (*P*) is 0.70, the number of trials (*x*) is 5, and the number of successes (*r*) is 3.

To solve this problem, we enter these values into the negative binomial formula.

$$b^*(x; r, P) = {}_{x-1}C_{r-1} * P^r * Q^{x - r}$$

$$b^*(5; 3, 0.7) = {}_4C_2 * 0.7^3 * 0.3^2$$

$$b^*(5; 3, 0.7) = 6 * 0.343 * 0.09 = 0.18522$$

Thus, the probability that Bob will make his third successful free throw on his fifth shot is 0.18522.

Example 2

Let's reconsider the above problem from Example 1. This time, we'll ask a slightly different question: What is the probability that Bob makes his first free throw on his fifth shot?

Solution: This is an example of a geometric distribution, which is a special case of a negative binomial distribution. Therefore, this problem can be solved using the negative binomial formula or the geometric formula. We demonstrate each approach below, beginning with the negative binomial formula.

The probability of success (*P*) is 0.70, the number of trials (*x*) is 5, and the number of successes (*r*) is 1. We enter these values into the negative binomial formula.

$$b^*(x; r, P) = {}_{x-1}C_{r-1} * P^r * Q^{x - r}$$

$$b^*(5; 1, 0.7) = {}_4C_0 * 0.7^1 * 0.3^4$$

$$b^*(5; 3, 0.7) = 0.00567$$

Now, we demonstate a solution based on the geometric formula.

$$g(x; P) = P * Q^{x - 1}$$

$$g(5; 0.7) = 0.7 * 0.3^4 = 0.00567$$

Notice that each approach yields the same answer.