Established ~ 1935 by Carlo Emilio Bonferroni

Bonferroni <u>Joint</u> Confidence Intervals $\underline{(1-\alpha)\times 100\%}$ CIs

Start with Joint CIs on B_1, B_2.

Let A = event that CI on B_1 does not cover B_1. $P(A)=\alpha$

Let B = event that CI on B_2 does not cover B_2. $P(B)=\alpha$

$P(\text{CI on } B_1 \text{ covers } B_1 \text{ and CI on } B_2 \text{ covers } B_2) = P(\bar{A} \cap \bar{B})$.

However, $P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B})$ by De Morgan's Law.

$$P(\overline{A \cup B}) = 1 - P(A \cup B) = 1 - [P(A) + P(B) - P(A \cap B)]$$

$$= 1 - P(A) - P(B) + P(A \cap B) \geq 1 - P(A) - P(B) = 1 - 2\alpha$$

$\therefore$ Compute $(1-\alpha/2)\times 100\%$ CI on B_1 <u>and</u>

$(1-\alpha/2)\times 100\%$ CI on B_2.

The confidence associated with <u>both</u> CIs covering their parameters $\geq 1 - 2(\alpha/2) = \underline{\underline{1-\alpha}}$

Bonferroni Joint CIs are known to be <u>conservative</u>.

This method works for $\underline{\underline{m}}$ joint CIs.

Compute $(1-\alpha/m)\times 100\%$ CIs on each of $\underline{m}$ parameters.

Familywise/Experimentwise confidence $\geq 1 - \frac{\alpha}{m}(m) = \boxed{1-\alpha}$

$\therefore$ for $\underline{m=2}$ Compute
$$\begin{cases} \hat{B}_1 \pm t_{\alpha/4;\, n-p}\, s(\hat{B}_1) \\ \hat{B}_2 \pm t_{\alpha/4;\, n-p}\, s(\hat{B}_2) \end{cases}$$