ECE 271A: Quiz #1

Due on October 16, 2023 at 11:59pm

Professor Vasconcelos

Ray Tsai

A16848188

Part A

Using the training data in `TrainingSamplesDCT_8.mat`, what are reasonable estimates for the prior probabilities?

Solution

We assume that `TrainingSampleDCT_FG` and `TrainingSampleDCT_BG` represents a set of complete images collectively. Then, given that there are 250 rows in `TrainingSampleDCT_FG` and 1053 rows in `TrainingSampleDCT_BG`, we use the ratio of the size of samples to give an estimate for the prior probabilities, namely

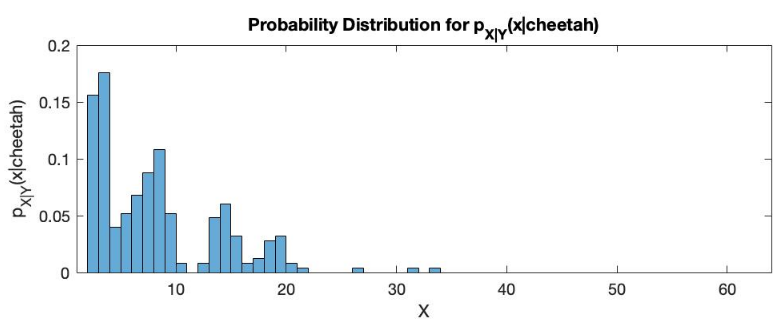
$$\mathbb{P}_Y(cheetah) = \frac{250}{1053 + 250} \approx 19\%,$$
$$\mathbb{P}_Y(grass) = \frac{1053}{1053 + 250} \approx 81\%.$$

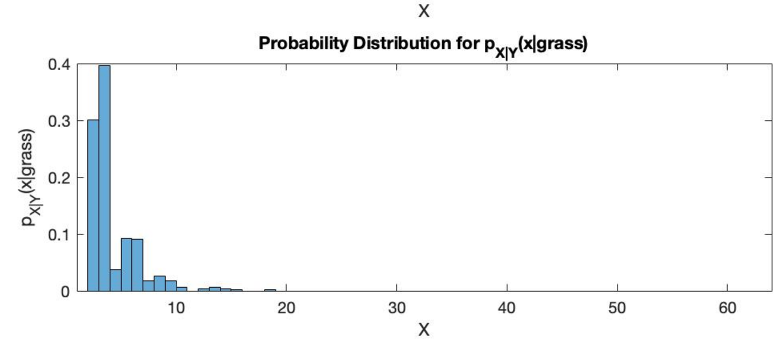
Part B

Using the training data in `TrainingSamplesDCT_8.mat`, compute and plot the index histograms $P_{X|Y}(x|cheetah)$ and $P_{X|Y}(x|grass)$.

Solution

We collect feature X, the position of the coefficient of the second largest magnitude, from each row vector and normalize it to obtained the index histogram for $P_{X|Y}(x|cheetah)$ and $P_{X|Y}(x|grass)$:





Part C

For each block in the image `cheetah.bmp`, compute the feature X (index of the DCT coefficient with 2nd greatest energy). Compute the state variable Y using the minimum probability of error rule based on the probabilities obtained in part a and b. Store the state in an array A. Using the commands `imagesc` and `colormap(gray(255))`, create a picture of that array.

Solution

Let $g(x)$ be our decision function. Assume our loss function $L[g(x), y] = \begin{cases} 1, & g(x) \neq y \\ 0, & g(x) = y \end{cases}$. By the minimum probability of error rule, the optimal decision function is

$$\begin{aligned} g^*(x) &= \arg\max_i \mathbb{P}_{Y|X}(i|x) \\ &= \arg\max_i \mathbb{P}_{X|Y}(x|i)\mathbb{P}_Y(i) \\ &= \begin{cases} cheetah, & \frac{\mathbb{P}_{X|Y}(grass|x)}{\mathbb{P}_{X|Y}(cheetah|x)} < \frac{\mathbb{P}_Y(cheetah)}{\mathbb{P}_Y(grass)} \\ grass, & \text{otherwise} \end{cases} \\ &= \begin{cases} cheetah, & \frac{\mathbb{P}_{X|Y}(grass|x)}{\mathbb{P}_{X|Y}(cheetah|x)} < 0.24 \\ grass, & \text{otherwise,} \end{cases} \end{aligned}$$

for the 0-1 loss function. We then use function $g^*(x)$ to assign a state to each pixel and obtian our pridiction mask array A. Resulting picture of array A is shown at the bottom of the page.

Part D

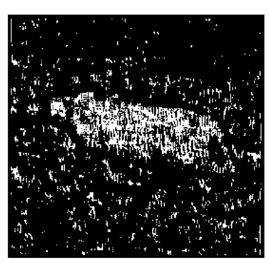
The array A contains a mask that indicates which blocks contain grass and which contain the cheetah. Compare it with the ground truth provided in image `cheetah_mask.bmp` and compute the probability of error of your algorithm.

Solution By comparing our prediction with the truth, we calculate the error rate with the following formula

$$\text{Error rate} = \frac{\text{number of pixels of different values}}{\text{number of pixels}}.$$

Results shown in the following figure.

Truth



Prediction

Error Rate: 17.22%

MATLAB Code

```
load('../dataset/TrainingSamplesDCT_8.mat');
zigzag = load('../dataset/Zig-Zag Pattern.txt');
cheetah = imread('../dataset/cheetah.bmp');
cheetah_mask = imread('../dataset/cheetah_mask.bmp');
target = im2double(cheetah);
mask = im2double(cheetah_mask);

training_BG = TrainsampleDCT_BG;
training_FG = TrainsampleDCT_FG;
zigzag = zigzag + 1;

[row_BG, col_BG] = size(training_BG);
[row_FG, col_FG] = size(training_FG);
[row_TG, col_TG] = size(target);

padded_target = zeros(row_TG + 7, col_TG + 7);
padded_target(5:4 + row_TG, 5:4 + col_TG) = target;

% pick cheetah if (p(x | grass) / p(x | cheetah)) < threshold
prior_BG = row_BG / (row_BG + row_FG);
prior_FG = row_FG / (row_BG + row_FG);
threshold = prior_FG / prior_BG;

feature_BG = zeros(64, 1);
feature_FG = zeros(64, 1);

for r = 1:1:row_BG
    maxVal = max(training_BG(r));
    secVal = 0;
    secPos = 0;
    for c = 1:1:col_BG
        if (training_BG(r, c) < maxVal && training_BG(r, c) > secVal)
            secVal = training_BG(r, c);
            secPos = c;
        end
    end
    feature_BG(secPos) = feature_BG(secPos) + 1;
end

for r = 1:1:row_FG
    maxVal = max(training_FG(r));
    secVal = 0;
    secPos = 0;
    for c = 1:1:col_FG
        if (training_FG(r, c) < maxVal && training_FG(r, c) > secVal)
            secVal = training_FG(r, c);
            secPos = c;
```

```
        end
    end
    feature_FG(secPos) = feature_FG(secPos) + 1;
end

cprob_BG = feature_BG / sum(feature_BG);
cprob_FG = feature_FG / sum(feature_FG);
A = zeros(row_TG, col_TG);

for r = 1:row_TG
    for c = 1:col_TG
        block = padded_target(r:r + 7, c:c + 7);
        dctBlock = abs(dct2(block));
        maxVal = max(max(dctBlock));
        secVal = 0;
        x = 0;
        for i = 1:8
            for j = 1:8
                if dctBlock(i, j) < maxVal && dctBlock(i, j) > secVal
                    secVal = dctBlock(i, j);
                    x = zigzag(i, j);
                end
            end
        end
        A(r, c) = int8(cprob_BG(x)/cprob_FG(x) <= threshold);
    end
end

subplot(1, 2, 1);
imagesc(mask);
axis off
colormap(gray(255));
axis equal tight;

subplot(1, 2, 2);
imagesc(A);
axis off
colormap(gray(255));
axis equal tight;

error = 0;
for r = 1:row_TG
    for c = 1:col_TG
        if (A(r, c) ~= mask(r, c))
            error = error + 1;
        end
    end
end
error_rate = error / (row_TG * col_TG);
```