Statement of Purpose

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I am applying to the Ph.D program in Algorithm, Combinatorics, and Optimization (ACO) at Georgia Tech to pursue my interest in combinatorics, especially in extremal graph theory and its applications to adjacent fields like theoretical computer science.

As a mathematics-computer science major at UC San Diego, I dedicated most of my undergraduate studies to mathematics, primarily through honors and graduate-level coursework. While I found beauty across fields in mathematics, I was especially drawn to combinatorics due to its deceiving simplicity and intimate ties to real-world problems. I aspire to become a researcher who not only specializes in combinatorics but also bridges combinatorics with other disciplines. I believe the ACO program at Georgia Tech is the ideal place to achieve this goal.

During my sophomore year, I began my research journey by undertaking independent study under Professor Verstraete, exploring the extensive literature on extremal graph theory. Through the readings, I was exposed to a variety of powerful techniques for tackling extremal graph problems, such as probabilistic methods, stability, and constructions using projective planes, which laid the groundwork for my future research. Currently, I am working on my honors thesis centering on the Double Turán problem, which asks for the maximum possible number of edges across n subgraphs of a complete graph K_n such that no pairwise intersection of these subgraphs contain a specified forbidden structure. After dedicating my summer to studying the triangle-free case, I established a tight upper bound on the number of edges under the stricter condition that each subgraph is induced. Specifically, I showed that the extremal condition is uniquely achieved when all subgraphs are complete bipartite graphs, by recursively expanding the intersection of all subgraphs. This case serves as a stepping stone for the project, and I am currently working on the general triangle-free case.

As I advanced in my studies, I also grew interested in computational complexity theory, which studies combinatorial problems with a computational lens grounded in real-world problems. After a quarter spent working through Sanjeev Arora and Boaz Barak's *Computational Complexity: A Modern Approach* [1], I joined Professor Impagliazzo's research group, studying Multicalibration to address unintended bias in learning models from the perspective of complexity theory. The project opened my eyes to the unexpected connections between theoretical computer science and combinatorics, as it brought the application of combinatorics to a new level by modeling the fairness of algorithms with the random-like structures yielded by Szemeredi's Regularity Lemma. Through the project, I realized the boundless potential of real-world application of combinatorial tools and it adds another layer of meaning to my interest in combinatorics.

My undergraduate experiences opened my appetite for mathematical research, but

it also humbled me with the immense breadth and depth of the discipline. Graduate school is a crucial first step to further proceed in the realm of combinatorics. I plan to continue my research in extremal combinatorics in graduate school, as well as explore adjacent fields such as discrete geometry, random graphs, and topics that I can bridge combinatorics with.

The ACO program at Georgia Tech seems tailor-made for my goals. With its emphasis on combinatorics and interdisciplinary nature with computer science and operation research, I can build solid foundations in various fields while furthering my studies in combinatorics, which would realize my aspiration to become a researcher who connects combinatorics with other disciplines.

The diverse expertise of the faculty members in combinatorics will provide me with comprehensive training in the field. I am particularly interested in collaborating with Professor Tom Bohman on problems related to independent sets in graphs. Through my independent studies, I explored the problem of the lower bound of independent sets in hypergraphs, and I was excited by its extensive applications both within combinatorics and in broader contexts, information theory is one of which. Given Professor Bohman's focus on Shannon capacities of graphs, I believe that working with him on this topic will reveal deeper insights into the connections between combinatorics and information theory. I am also excited by the prospect of collaborating with Professor Boris Bukh. Although my background in discrete geometry is limited, I am intrigued by the easy-to-state yet hard-to-solve nature of problems Professor Bukh worked on, such as his past works on empty axis-paralleled boxes [2] and digital almost nets [3].

Moreover, I want to collaborate with faculty beyond the math department. Although this area diverges from my current research focus, Professor Tuomas Sandholm's projects on real-world applications of game theory and market design are particularly appealing to me. His work on price of fairness in kidney exchange [4], which involves using graph models to determine the cost of fairness in such exchanges, is especially intriguing. It prompts me to wonder whether other combinatorial results could be applied in this domain. I would like to leverage my specialization in combinatorics to contribute to projects such as heart transplantation policy optimization.

I have only begun to scratch the surface of combinatorics, and there are still a lot of topics to explore. I am excited about the prospect of joining Georgia Tech's Ph.D program in ACO to receive comprehensive training in mathematical research, and I am confident in my ability to evolve into a capable researcher and make meaningful contributions.

References

[1] Sanjeev Arora and Boaz Barak. *Computational complexity: a modern approach*. Cambridge University Press, 2009.

[2] Boris Bukh and Ting-Wei Chao. Empty axis-parallel boxes, 2021.

[3] Boris Bukh and Ting-Wei Chao. Digital almost nets, 2022.

[4] John P Dickerson, Ariel D Procaccia, and Tuomas Sandholm. Price of fairness in kidney exchange. In *AAMAS*, pages 1013–1020, 2014.