Question 4

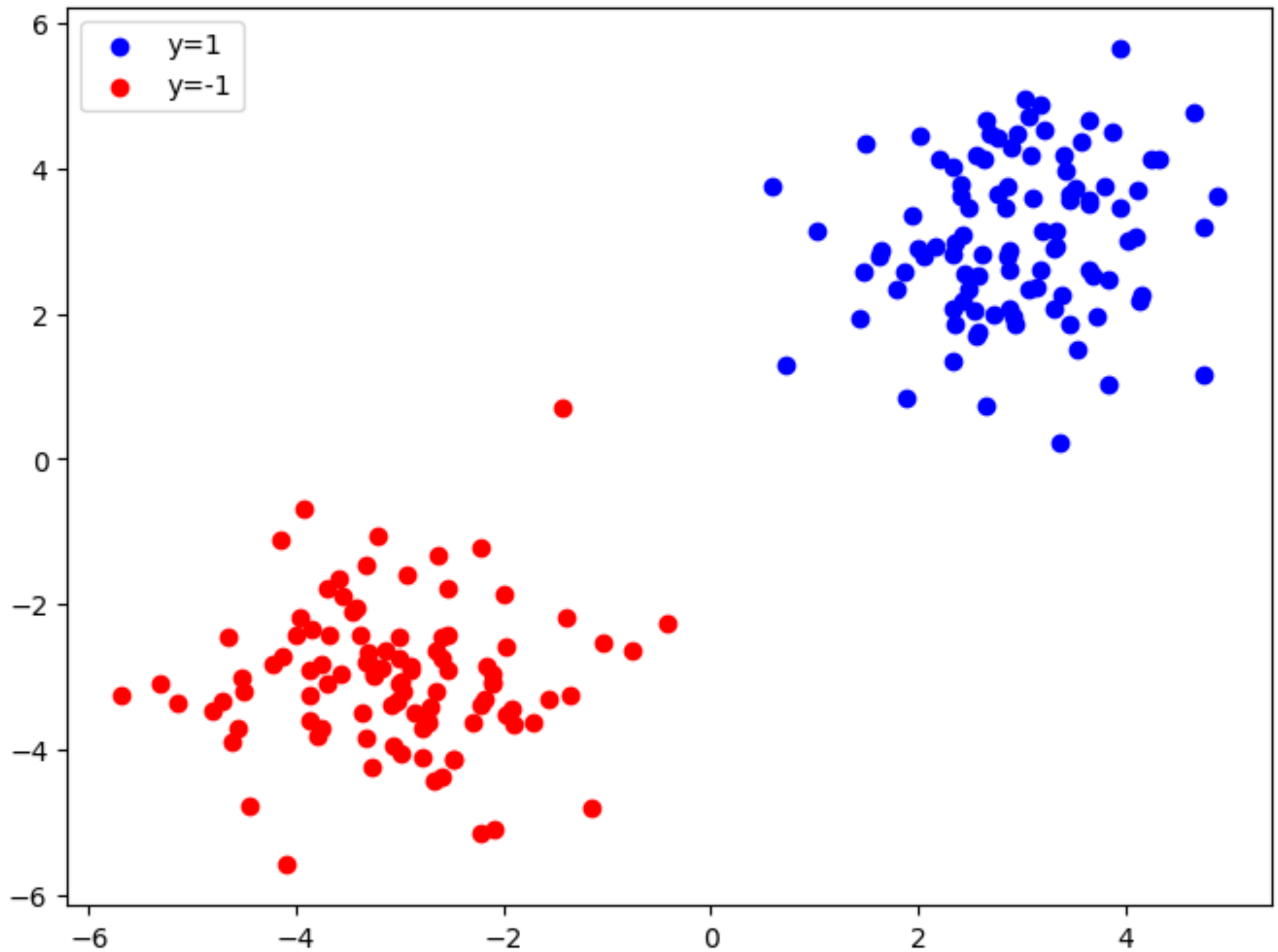
We will implement the SVM algorithm with gradient descent to classify two gaussians in 2D. The dataset is given in `HW7Q4.csv`.

In [33]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
```

Part A

In `HW7Q4.csv`, the first 100 rows are the data for cluster 1: $(x_i, y_i) \in \mathbb{R}^2 \times \mathbb{R}$, $i = 1, \ldots, 100$, with $y_i = 1$ always. The next 100 rows are the data for cluster 2: $(x_i, y_i) \in \mathbb{R}^2 \times \mathbb{R}$, $i = 101, \ldots, 200$, with $y_i = -1$ always. Create and turn in a scatter plot of the feature vectors, i.e., the x_i's, colored by the label, i.e., y_i's (blue for 1 and red for -1).

In [34]:
```
data = pd.read_csv('HW7Q4.csv', header=None)
data.columns = ['x1', 'x2', 'y']

plt.figure(figsize=(8, 6))
plt.scatter(data[:100]['x1'], data[:100]['x2'], color='blue', label='y=1')
plt.scatter(data[100:]['x1'], data[100:]['x2'], color='red', label='y=-1')
plt.legend()
plt.show()
```



Part B

Create a function for the gradient of the loss

$$L(w) = \frac{1}{2}\|w\|^2 + \sum_{i=1}^{n} \max(0, 1 - y_i\langle x_i, w\rangle)$$

$$\nabla L(w) = w + \sum_{i=1}^{n} -y_i x_i \cdot 1_{1-y_i\langle x_i, w\rangle > 0},$$

where $1_{1-y_i\langle x_i,w\rangle>0} = \begin{cases} 1 & \text{if } 1 - y_i\langle x_i, w\rangle > 0 \\ 0 & \text{else} \end{cases}$. Also, here $n = 200$. To compute the gradient, you'll have to compute an indicator of whether $1 - y_i\langle x_i, w\rangle$ is positive or negative at every point, and sum up the contribution of this term for all points where it's positive.

In [43]:
```
n = len(data)
```

```
def indicator(w, x1, x2, y):
    if 1 - y * (w[0] * x1 + w[1] * x2) > 0:
        return 1
    return 0

def L(w):
    return 1/2 * np.linalg.norm(w)**2 + sum([max(0, 1 - row['y'] * (w[0] * r

def dL(w):
    return w + sum([-row['y'] * np.array([row['x1'], row['x2']]) * indicator
```

Part C

Setting the step size $\mu = 10^{-4}$ and starting at $w^{(0)} = (-1, 1)$, run 1000 iterations of gradient descent. You will create two plots.

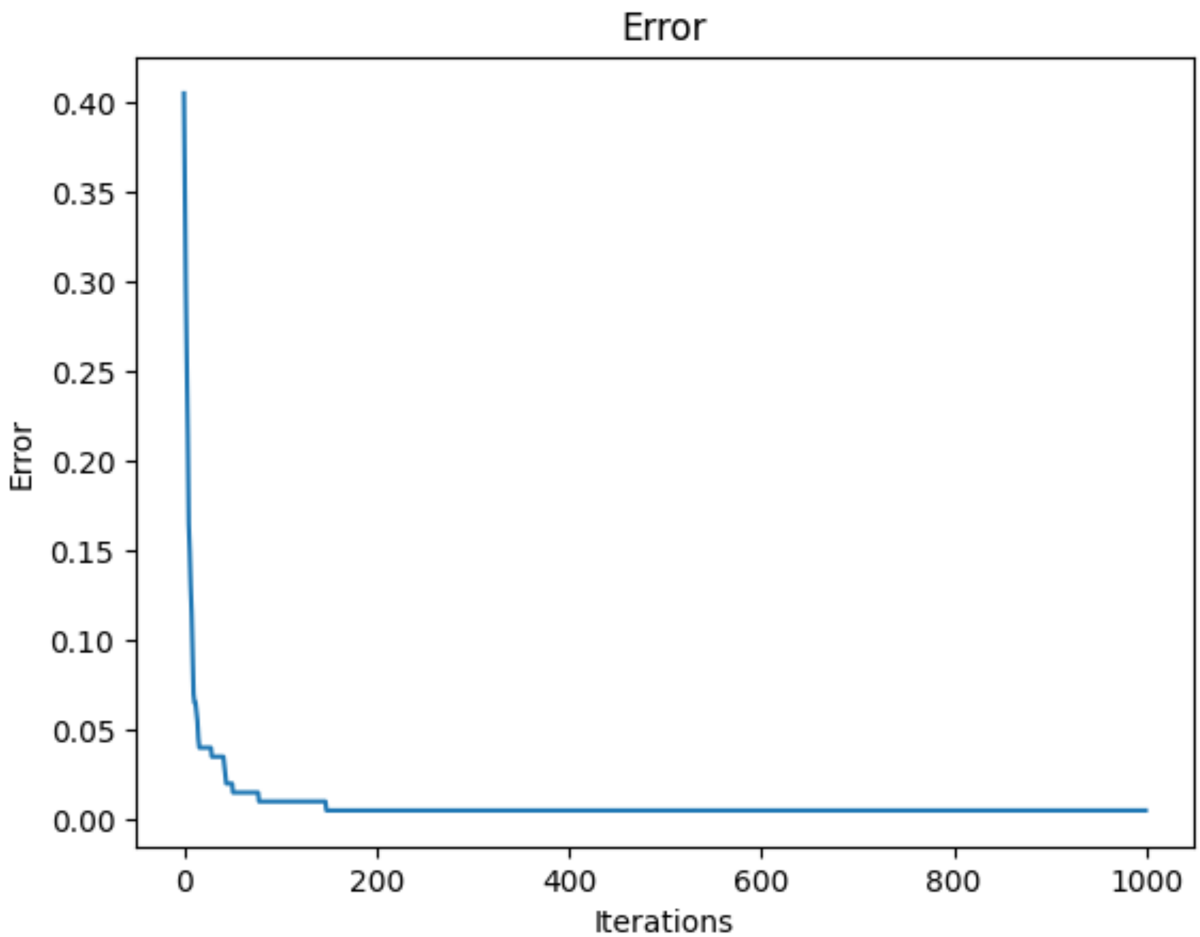
i. Plot the classification error (averaged over all the points) as a function of the iterations. The classification of x_i is determined by $\text{sign}(\langle x_i, w \rangle)$.

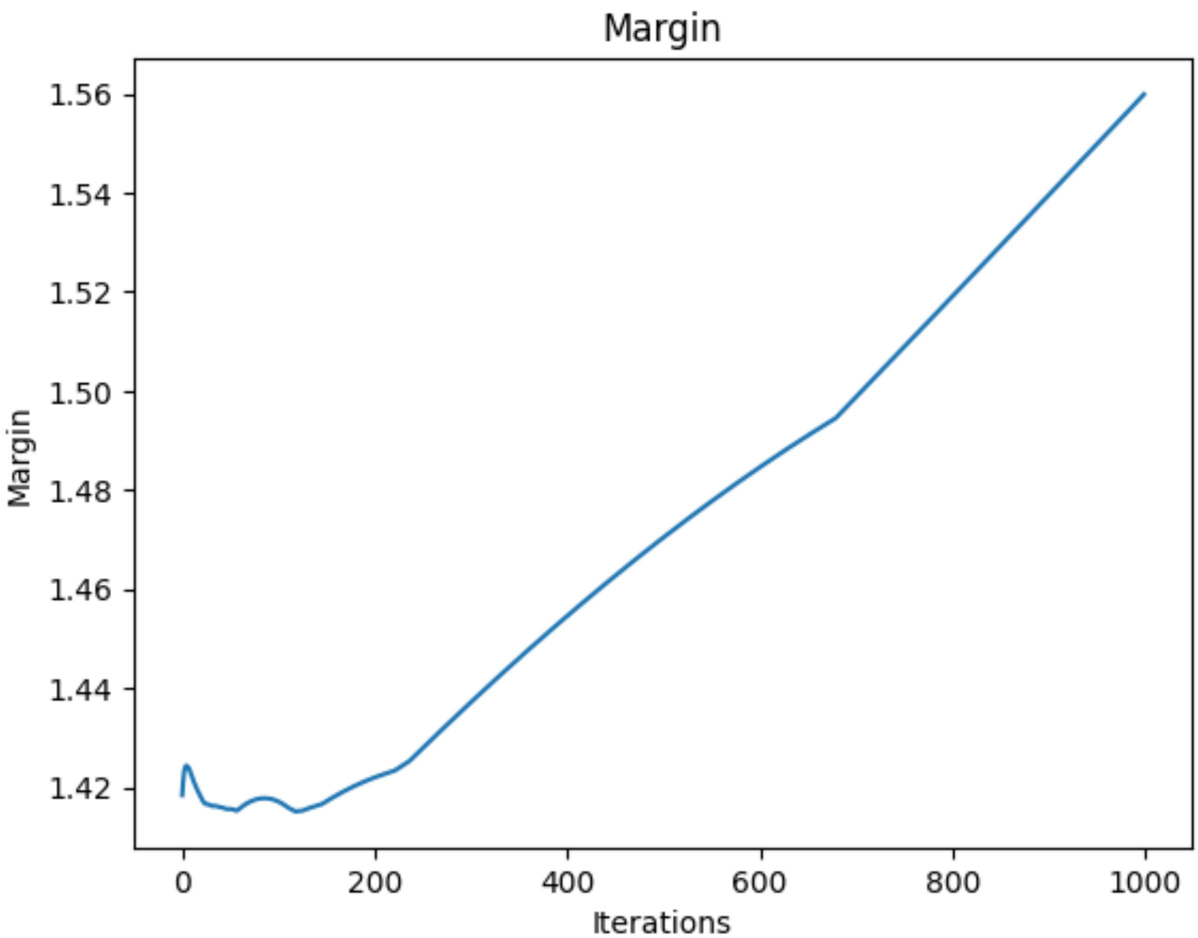
ii. Plot the margin $\frac{2}{\|w\|}$ as a function of the iterations. This shows how much of a gap you have between the classes you've learned.

In [ ]:
```
T = 1000
mu = 1e-4

def error(w):
    sum = 0
    for _, row in data.iterrows():
        if np.sign(w[0] * row['x1'] + w[1] * row['x2']) != row['y']:
            sum += 1
    return sum / n

w = np.array([-1, 1])
error_val = []
margin_val = []

for t in range(T):
    w = w - mu * dL(w)
    error_val.append(error(w))
    margin_val.append(2 / np.linalg.norm(w))
```

In [ ]:
```
plt.plot(error_val)
plt.xlabel('Iterations')
plt.ylabel('Error')
plt.title('Error')
```

```
plt.show()

plt.plot(margin_val)
plt.xlabel('Iterations')
plt.ylabel('Margin')
plt.title('Margin')

plt.show()
```

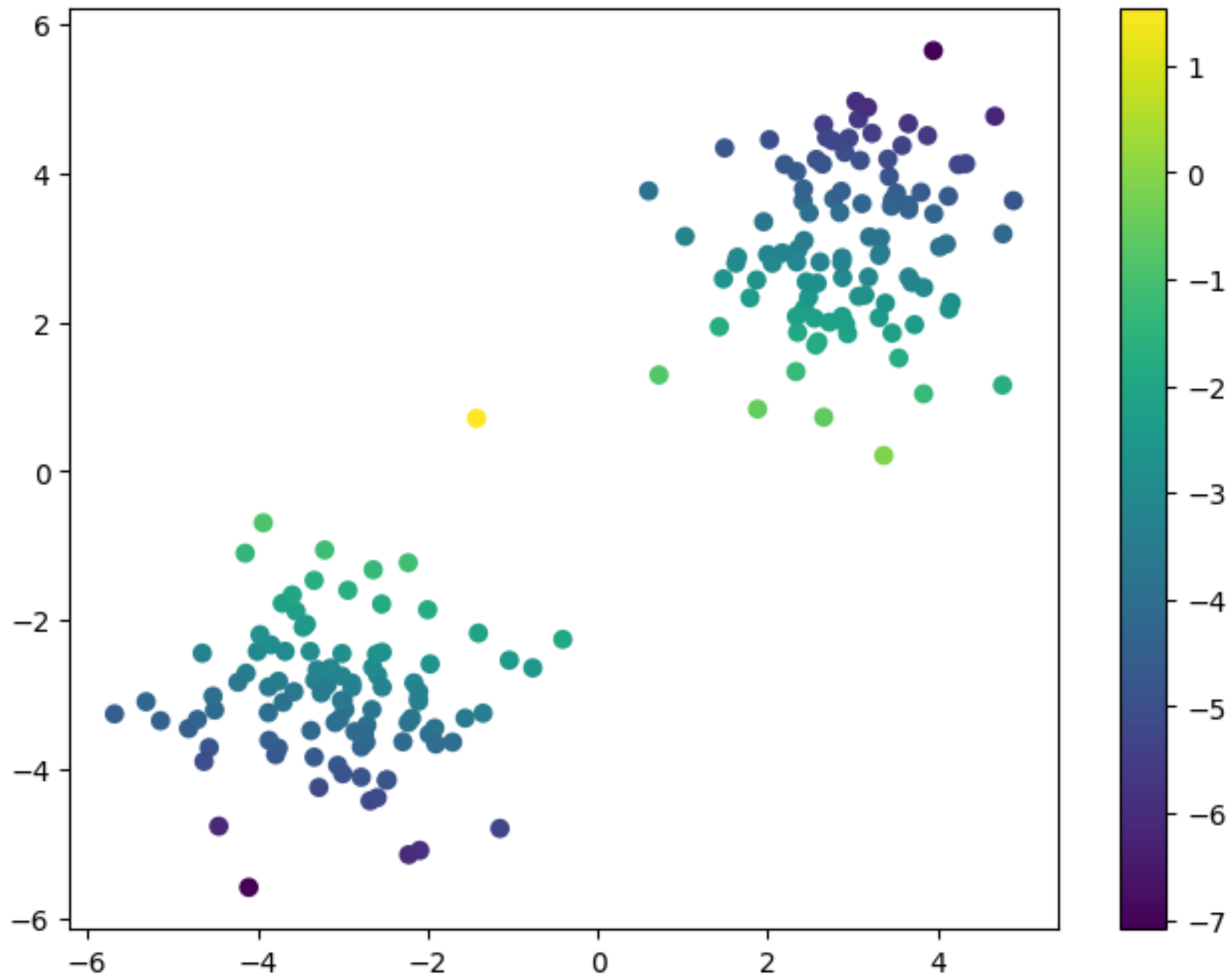




Part D

Create another scatter plot of your data, but this time color the points by the function $f(x_i) = 1 - y_i \cdot \langle x_i, w \rangle$. The numbers closest to 0 (positive numbers or largest negative numbers) will show you which points were "most important" in determining the classification.

In [54]:
```
f_values = [1 - row['y'] * (w[0] * row['x1'] + w[1] * row['x2']) for _, row

plt.figure(figsize=(8, 6))
plt.scatter(data['x1'], data['x2'], c=f_values, label='f(x_i)')
plt.colorbar()
plt.show()
```


6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
1
0
−1
−2
−3
−4
−5
−6
−7