Question 3

Consider the optimization problem:

$$f(x) = \frac{1}{2}\|Ax - b\|^2, \tag{1}$$

where $A \in \mathbb{R}^{20\times 50}$ and $b \in \mathbb{R}^{20}$ are from the dataset `HW7Q3.csv`. The file `HW7Q3.csv` contains the data A and b. The first 50 columns form the matrix A and the last column is the vector b. The vector b is generated by setting $b = Ax^*$ for a vector $x^* \in \mathbb{R}^{20}$ that has 2 nonzeros. Note the linear system $Ax = b$ is underdetermined and has a lot of solutions. Write a projected gradient method for the following optimization problem to find the x^*:

$$\text{minimize } f(x) \quad \text{s.t. } x \text{ has at most 2 nonzeros.}$$

You can experiment with the stepsize to make sure $f(x^{(t)})$ converges to 0. You need to submit the code, the plot of $f(x^{(t)}) - f(x^*) = f(x^{(t)})$, and the indices and values of the nonzero entries of x^* you found.

In [84]:

```python
import numpy as np
import matplotlib.pyplot as plt
```

In [85]:

```python
data = np.loadtxt('HW7Q3.csv', delimiter=',')
A = data[:, :-1]
b = data[:, -1]
```

In [86]:

```python
d = A.shape[1]
x = np.zeros(d)
T = 10
f_values = []
```

In [87]:

```python
def f(x):
    return 1/2 * np.linalg.norm(A @ x - b)**2

def df(x):
    return A.T @ (A @ x - b)
```

In [88]:

```python
s = 2
mu = 5e-2

x = np.zeros(d)
```

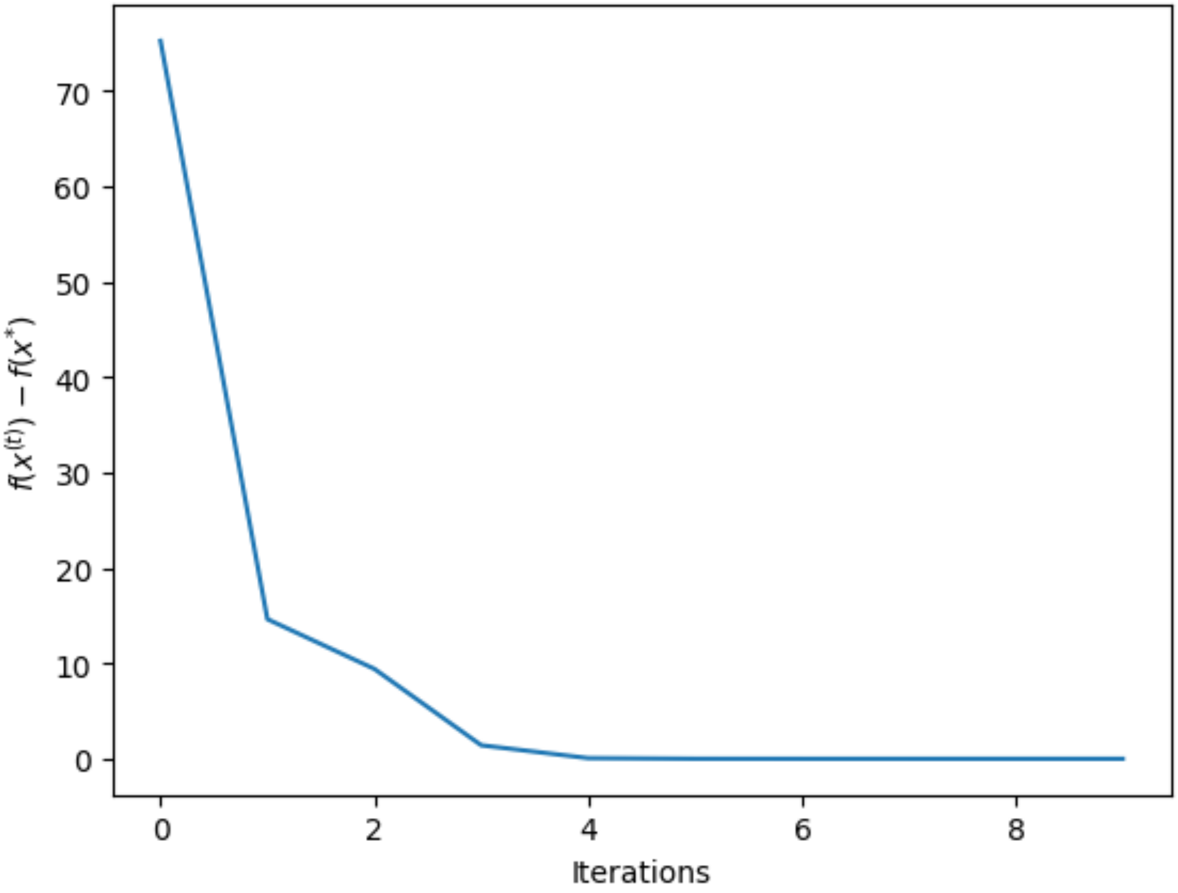
```
for t in range(T):
    f_values.append(f(x))
    y = x - mu * df(x)
    y_plus = np.maximum(y, 0)
    I_s = np.argpartition(y_plus, -s)[-s:]
    x = np.zeros(d)
    x[I_s] = y[I_s]

print(f_values[-1])
```

```
1.4095374442260З6e-07
```

In [90]:

```
plt.plot(f_values)
plt.xlabel('Iterations')
plt.ylabel(r'$f(x^{(t)}) - f(x^{*})$')
plt.show()
```



In [92]:

```
nonzero_ind = np.nonzero(x)[0]
nonzero_val = x[nonzero_ind]
print("Nonzero indices:", nonzero_ind)
print("Nonzero values:", nonzero_val)
```

```
Nonzero indices: [ 0 14]
Nonzero values: [0.99997612 2.99992859]
```