

Math 109 HW 1

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1.

Proposition 1. $x = -1, y = -2$ is a counter example to the statement “For all real numbers x, y with $x \geq y$, we have $x^2 \leq y^2$.”

Proof. $x > y$, but $x^2 = 1 > y^2 = 4$, which contradicts the given statement. \square

2. (a) $\neg(P \vee (\neg Q)) \equiv (\neg P) \wedge Q$

(b) $\neg(P \rightarrow (\neg Q)) \equiv \neg((\neg P) \vee (\neg Q)) \equiv P \wedge Q$

(c) $\neg(\forall x (\exists y (S(x, y) \text{ is true.}))) \equiv \exists x (\forall y (S(x, y) \text{ is false.}))$

(d) $\neg(\exists x (\forall y (S(x, y) \text{ is true.}))) \equiv \forall x (\exists y (S(x, y) \text{ is false.}))$

3. The negation of “for every real number x , there exists a real number y such that $x + y = 0$ ” is “there exists a real number x , such that $x + y \neq 0$ for every real number y .”

4. The negation of “if z is a real number such that there is a real number x with $xz = 0$, then $z = 0$ ” is “ z is a real number such that there is a real number x with $xz = 0$ and $z \neq 0$.”

5.

Proposition 2. “if a person gets at least an 98% in the class, then they get an A+”

(a) i. converse: “If a person gets an A+, then they get at least an 98% in the class.”

ii. contrapositive: “If a person does not get an A+, then they get lesser than 98% in the class.”

iii. negation: “A person gets at least an 98% in the class and they does not get an A+”.

(b) If a person gets an 97% in the class and gets an A+, then this scenario makes (iii) and (ii) true but (i) false.

If a person does not gets an A+ and gets an 98% in the class, then this scenario makes (iii) true but (ii) false.

Therefore, the three statements written above are different.

- (c) Below is a truth table of the original statement and statement (ii).

gets at least an 98%	gets an A+	original	ii.
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

Therefore, according to the truth table, the original statement is equivalent to statement (ii).

6. (a) There is not enough information to tell because $P \rightarrow Q$ does not imply whether P is true or false.
- (b) There is not enough information to tell because $P \rightarrow Q$ does not imply whether Q is true or false.
- (c) There is not enough information to tell because $P \rightarrow Q$ does not imply $P \wedge Q$.
- (d) This statement is false because $P \wedge (\neg Q) \equiv \neg((\neg P) \vee Q) \equiv \neg(P \rightarrow Q)$.
- (e) There is not enough information to tell because $P \rightarrow Q$ does not imply $Q \rightarrow P$.
- (f) This statement is true because a statement is equivalent to it's contrapositive.
- (g) This statement is false because a statement is opposite to it's negation.