MATH 173A: Homework #5

Due on Mov 19, 2024 at 23:59pm

Professor Cloninger

Ray Tsai

A16848188

Problem 1

Consider the function $f(x_1, x_2) = (2x_1 - 1)^4 + (x_1 + x_2 - 1)^2$. Recall the minimizer is $(\frac{1}{2}, \frac{1}{2})$ for this problem. Starting at $x^{(0)} = (0, 0)$, perform two steps of Newton's method to find $x^{(2)}$. Show your work.

Proof. Note that

$$\nabla f(x_1, x_2) = \begin{bmatrix} 8(2x_1 - 1)^3 + 2(x_1 + x_2 - 1) \\ 2(x_1 + x_2 - 1) \end{bmatrix},$$
$$\nabla^2 f(x_1, x_2) = \begin{bmatrix} 48(2x_1 - 1)^2 + 2 & 2 \\ 2 & 2 \end{bmatrix}.$$

Starting at $x^{(0)} = (0, 0)$, we have

$$[\nabla^2 f(x^{(0)})]^{-1} = \frac{1}{48} \begin{bmatrix} 1 & -1 \\ -1 & 25 \end{bmatrix}, \quad \nabla f(x^{(0)}) = \begin{bmatrix} -10 \\ -2 \end{bmatrix},$$
$$x^{(1)} = x^{(0)} - [\nabla^2 f(x^{(0)})]^{-1} \nabla f(x^{(0)}) = \frac{1}{6} \begin{bmatrix} 1 \\ 5 \end{bmatrix}$$
$$[\nabla^2 f(x^{(1)})]^{-1} = \frac{1}{64} \begin{bmatrix} 3 & -3 \\ -3 & 35 \end{bmatrix}, \quad \nabla f(x^{(1)}) = \frac{1}{48} \begin{bmatrix} -\frac{64}{27} \\ 0 \end{bmatrix},$$
$$x^{(2)} = x^{(1)} - [\nabla^2 f(x^{(1)})]^{-1} \nabla f(x^{(1)}) = \frac{1}{18} \begin{bmatrix} 5 \\ 13 \end{bmatrix}.$$

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