Statement of Purpose

Ray Tsai

I am applying to the Master's program in Computer Science at Stanford University, specializing in theoretical computer science to translate my mathematical foundation, particularly in combinatorics, to abilities that can address real-world problems. My primary interests lie in approximation algorithms and cryptography, where the interplay between theory and application is most apparent, and where I can contribute to impactful advancements in areas like resource optimization and secure data sharing.

As a mathematics-computer science major at UC San Diego, I placed most of my attention to mathematics in my undergraduate studies, primarily through honors and graduate-level coursework. While I appreciate the beauty of pure mathematics, I wasn't satisfied pursuing abstract mathematics for its own sake. I instead looked for a field in mathematics with a closer connection to the real world and turned to graph theory, the study of relationships that can model a wide range of practical problems effectively.

During my sophomore year, I began my research journey by directly reading under Professor Jacques Verstraete, exploring the vast literature on extremal graph theory. Through the readings, I was exposed to a variety of powerful techniques for tackling extremal graph problems, such as probabilistic methods, stability, and finite geometric constructions, which laid the groundwork for my honors thesis centered on the Double Turán problem. The problem asks for the maximum possible number of edges across n subgraphs of a complete graph K_n such that no pairwise intersection of these subgraphs contain a specified forbidden structure. After dedicating my summer to studying the triangle-free case, I established a tight upper bound on the number of edges under the stricter condition that each subgraph is induced. Specifically, I showed that the extremal condition is uniquely achieved when all subgraphs are complete bipartite graphs, by recursively expanding the intersection of all subgraphs. This case serves as a stepping stone for the project, and I am currently working on the general triangle-free case.

Expanding my interests to include the computational aspects of combinatorial problems, I had the opportunity to learn about computational complexity theory directly from Professor Russell Impagliazzo, studying Arora and Barak's text, Computational Complexity: A Modern Approach [1]. With this foundation, I then joined Professor Impagliazzo's research group, studying Multicalibration to mitigate unintended biases to certain subpopulations in learning models from the perspective of complexity theory. This research opened my eyes to the surprising connections between abstract mathematical concepts and the practical applications of machine learning. Specifically, the project brought the application of extremal combinatorics and complexity theory to a new level by modeling the fairness of algorithms with the random-like structures yielded by Szemeredi's Regularity Lemma, further strengthening my commitment to applying theoretical results to practical problems.

Stanford's Master's program offers the ideal path to translate my mathematical expertise into a career focused on developing practical algorithms and solutions. I envision myself contributing to fields like kidney exchange programs and secret sharing schemes, where combinatorial tools are applied extensively, and taking courses like Matching Theory (MS&E 319), Combinatorial Optimization (CS 261), Algebraic Error Correcting Codes (CS 250), and Advanced Topics in Cryptography (CS 355) would provide me the necessary background of combinatorial applications to computer science.

Moreover, I am looking forward to the research opportunities at Stanford and aim to pursue distinction in research. I am particularly excited by the prospect of contributing to the Applied Cryptography Group and collaborating with faculty whose work aligns with my interests in approximation algorithm and pseudorandomness. Professor Aviad Rubinstein is an leading expert in approximation algorithms and algorithmic game theory, and I am interested in working on matching algorithms and its applications with him. I was intrigued by his algorthm for approximating maximum matching size in [2] that beats the $\frac{1}{2}$-approximation barrier without even requiring a full traversal of the graph, and I am curious if a better accuracy can be achieved. My interest in pseudorandomness extends to the work of Professor Li-Yang Tan on the tightness of Impagliazzo's hardcore theorem [3], which revealed limitations in hardness amplification for pseudorandom generators (PRG). This result raised questions about the security and efficiency of PRGs, and I want to study the potential solutions and alternative approaches under his guidance. Furthermore, I would also like to explore the connections between pseudorandomness, multicalibration, and differential privacy under the mentorship of Professor Omer Reingold.

I have only begun to scratch the surface of theoretical computer science, and there are still a lot of topics to explore. I am excited about the prospect of joining Stanford's Master's in Computer Science program to expose me to a broader range of topics in computer science through rigorous coursework and extensive research opportunities. With the mathematical foundation I built during my undergraduate studies, I am confident in my ability to excel in the program and evolve into a capable problem-solver. I look forward to contributing to the vibrant academic community at Stanford and bridging the worlds of mathematics and computer science.

References

[1] Sanjeev Arora and Boaz Barak. *Computational complexity: a modern approach*. Cambridge University Press, 2009.

[2] Soheil Behnezhad, Mohammad Roghani, Aviad Rubinstein, and Amin Saberi. *Beating Greedy Matching in Sublinear Time*, pages 3900–3945.

[3] Guy Blanc, Alexandre Hayderi, Caleb Koch, and Li-Yang Tan. The sample complexity of smooth boosting and the tightness of the hardcore theorem, 2024.