Statement of Purpose

Ray Tsai

I am interested in combinatorics, particularly in extremal graph theory and its connections to theoretical computer science.

My curiosity about combinatorics was first sparked in an honors course on graph theory, taught by Professor Jacques Verstraete. Being my first exposure to combinatorics, the course soon intrigued me with the unconventional nature of combinatorial problems. Unlike other fields of mathematics, combinatorial problems often appear as discrete and uncorrelated with each other, but they nevertheless come with surprising relationships beneath the surface.

To deepen my engagement, I proceeded to take the most challenging courses offered at UC San Diego, including other honors and graduate-level combinatorics courses, along with seminars that exposed showcased modern research topics in combinatorics. However, my interest in the field was truly solidified through research experiences under the mentorship of Professor Verstraete, where I explored extensive literature on Turán problems and worked on open problems and conjectures.

The first open problem I attempted was a problem on long paths in Eulerian Digraphs, which asks for the lower bound of the longest path in an Eulerian digraph. This problem is an extension of the famous Erdős-Gallai theorem and is also of interest in theoretical computer science. I attempted the problem with an alternative approach suggested by Professor Verstraete, which involves randomly selecting a cyclic ordering of the vertices and analyzing the expected length of the longest "zig-zag" pattern in the ordering. Although I did not obtain a tangible result due to issues in extending the zig-zag path, the experience gave me a taste of the challenges and obstacles one will face in mathematical research.

My spirit, however, was not dampened by the failure, and I continued to explore other open problems after gaining more mathematical maturity through the honors algebra and analysis sequences. My honors thesis now centers on the Double Turán problem, which asks for the maximum possible number of edges in n subgraphs of a complete graph K_n, with no pairwise intersection of these subgraphs containing a certain forbidden structure. Through this work, I have developed a researcher's mindset and a toolkit that ranges from the probabilistic method to construction techniques, as well as coding skills for searching counterexamples. After dedicating my last summer to studying the triangle-free case, I completed the proof with a tighter condition that each subgraph is induced, which serves as a stepping stone for the general case. Despite the difficulties and frustrations involved in the process, I found myself more motivated than ever and continued to explore the general case.

Alongside my research in extremal combinatorics, I pursued theoretical computer science, drawn by its close ties to combinatorics. After a quarter spent working through Sanjeev Arora and Boaz Barak's *Computational Complexity: A Modern Approach*, I recently joined a research project under Professor Russell Impagliazzo on multicalibration, aiming to reduce unintended bias in learning models. The project opened my eyes to the unexpected connection between algorithm fairness and the Szemerédi regularity lemma through their "random-like" properties. This made me realize the boundless potential of real-world application of combinatorial tools, adding another layer of meaning to my interest

in combinatorics. Recognizing the extensive real-world applications of combinatorial tools added new depth to my interest in combinatorics and its potential impact.

My undergraduate experiences opened my appetite for mathematical research. To feed my curiosity, I hope to dig further into combinatorics, which prompted me to pursue graduate studies in mathematics. In the long term, I aspire to become a researcher in combinatorics, capable of addressing open problems with fresh perspectives and making meaningful contributions to the math community. I see graduate school as a crucial first step toward achieving this goal.

I plan to continue investigating Turán problems and their generalizations in graduate school. An interesting research direction I would like to follow is to build on my undergraduate research. The degenerated case of the Double Turán Problem, for which the forbidden structure is bipartite, is shown to be harder than the non-degenerated case, as seen in the classical Turán problem. Currently, I lack the expertise to tackle this case, but I would like to explore the degenerated double further Turán problems as I acquire more tools and insights in my graduate studies. More specifically, I need to learn more about polynomial equations over finite fields and projective planes, which are crucial tools for constructions in these degenerated cases.

While I have a strong interest in extremal combinatorics, I do not aim to limit myself to a single area early in my research career. I plan to explore areas adjacent to combinatorics such as theoretical computer science to bridge my rather specialized knowledge in extremal combinatorics with other domains and expand my understanding of the mathematical landscape. I believe that cultivating a diverse set of perspectives will lead to more fruitful outcomes in my research career.

[Why this program?]

[Closing Paragraph]