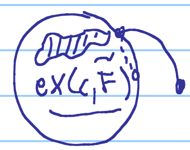
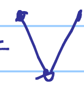
Note Title 6/21/2024

$\boxed{ex(c+1,\tilde{F}) > ex(c,\tilde{F})}$ $\boxed{\chi(F)=3}$

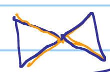
If not



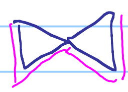
$F =$ 

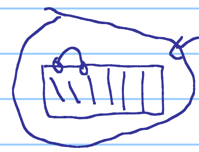
$$ex(n,F) = \underline{ex(n,K_r)} + \underline{ex(n,\tilde{F})} \leftarrow$$

$r = \chi(F)$

e.g. $F =$ 

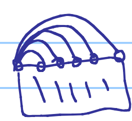
$\chi(F) = r = 3$

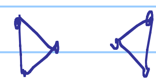




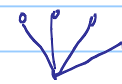
$\tilde{F} = \{\, | \; | \,,\; V \,\} \leftarrow$

$ex(n,\tilde{F}) = 1$



e.g. $F =$ 

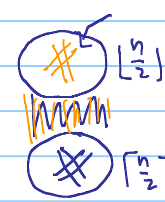
$\tilde{F} = \{\, | \; | \,\} \leftarrow$



$ex(n,\tilde{F}) = n-1$

Non-induced $G_1, G_2, \dots, G_m \subseteq G$ $E(G_i) \cap E(G_j)$ is triangle-free

$$\sum_{i=1}^{m} e(G_i) \leq \binom{n}{2} + (m-1)\left\lfloor \frac{n^2}{4} \right\rfloor$$



$m=2$: complicated

$$|E(G_1 \cap G_2)| \leq \left\lfloor \frac{n^2}{4} \right\rfloor$$

$E(G_1 \Delta G_2)$ ✓

$$e(G_1)+e(G_2) = 2e(G_1\cap G_2) + e(G_1 \Delta G_2)$$
$$\leq 2\left\lfloor \frac{n^2}{4} \right\rfloor + \binom{n}{2} - \left\lfloor \frac{n^2}{4} \right\rfloor$$
$$= \binom{n}{2} + \left\lfloor \frac{n^2}{4} \right\rfloor$$

$m = 3$

$2 \cdot 3 (n/3)^2$

$$3\binom{n/3}{2} + 2\left[\binom{n}{2} - 3\binom{n/3}{2}\right]$$
$$= n(n-1) - 3\binom{n/3}{2} \quad \Big| \quad \binom{n}{2} + 2\lfloor n^2/4 \rfloor$$