Statement of Purpose

Ray Tsai

I am applying to the Master's program in Computer Science at Stanford University, specializing in theoretical computer science to translate my mathematical foundation, particularly in combinatorics, to abilities that can address real-world problems. My primary interests lie in combinatorial optimization and cryptography, where the interplay between theory and application is most apparent. Although I don't have a specific industry goal in mind, I aspire to develop innovative algorithms and solutions to address challenges in areas like data privacy and resource optimization.

As a mathematics-computer science major at UC San Diego, I placed most of my attention to mathematics in my undergraduate studies, primarily through honors and graduate-level coursework. While I appreciate the beauty of pure mathematics, I wasn't satisfied pursuing abstract mathematics for its own sake. I instead looked for a field in mathematics with a closer connection to the real world and turned to graph theory, the study of relationships that can model a wide range of practical problems effectively.

During my sophomore year, I began my research journey by directly reading under Professor Jacques Verstraete, exploring the vast literature on extremal graph theory. Through the readings, I was exposed to a variety of powerful techniques for tackling extremal graph problems, such as probabilistic methods, stability, and finite geometric constructions, which laid the groundwork for my honors thesis centered on the Double Turán problem. The problem asks for the maximum possible number of edges across n subgraphs of a complete graph K_n such that no pairwise intersection of these subgraphs contains a specified forbidden structure. After dedicating my summer to studying the triangle-free case, I established a tight upper bound on the number of edges under the stricter condition that each subgraph is induced. Specifically, I showed that the extremal condition is uniquely achieved when all subgraphs are complete bipartite graphs, by recursively expanding the intersection of all subgraphs. This case serves as a stepping stone for the project, and I am currently working on the general triangle-free case.

Expanding my interests to include the computational aspects of combinatorial problems, I had the opportunity to learn about computational complexity theory directly from Professor Russell Impagliazzo, studying Arora and Barak's text, Computational Complexity: A Modern Approach [2]. With this foundation, I then joined Professor Impagliazzo's research group, studying Multicalibration to mitigate unintended biases to certain subpopulations in learning models from the perspective of complexity theory. This research opened my eyes to the surprising connections between abstract mathematical concepts and the practical applications of machine learning. Specifically, the project brought the application of extremal combinatorics and complexity theory to a new level by modeling the fairness of algorithms with random-like structures yielded by Szemeredi's Regularity Lemma, further strengthening my commitment to applying theoretical results to practical problems.

Stanford's Master's program offers the ideal path to translate my mathematical expertise into a career focused on developing practical algorithms and solutions. I aspire to contribute to fields where combinatorial tools are applied extensively, and taking courses like Matching Theory (MS&E 319), Combinatorial Optimization (CS 261), Algebraic Error Correcting Codes (CS 250), and Advanced Topics in Cryptography (CS 355) would provide me with the necessary background of combinatorial applications to computer science.

Moreover, I am looking forward to the research opportunities at Stanford and aim to pursue distinction in research. I am most curious to investigate approximation algorithms. Classic combinatorial optimization problems, such as finding Hamiltonian cycles and matchings in graphs, frequently arise in real-world contexts, where exact solutions are often computationally infeasible and rely on approximation algorithms for pratical solutions. Thus I hope to utilize my specialized knowledge in combinatorics to advance the Pareto frontier of these algorithms, pushing the boundaries of their efficiency and accuracy. Professor Aviad Rubinstein's expertise in approximation algorithms and algorithmic game theory aligns with my research interest. I am especially intrigued by his algorithm for approximating maximum matching size in [3], which beats the $\frac{1}{2}$-approximation barrier without even requiring a full traversal of the graph! Given the ubiquity of combinatorial problems in various applications, I am excited to explore the possibilities of improving these algorithms under his guidance, making them more accessible for practical applications. If possible, I would also like to collaborate with Itai Ashlagi from the Management Science and Engineering department. He has contributed significantly to the area of market design and matching markets, and I am particularly drawn to his work on kidney exchange. He previously used integer programming to find long chains in kidney exchange that maximize kidney transplants [1], and I am curious if approximate algorithms can be used to tackle other NP-hard problems regarding this urgent topic.

Additionally, I am interested in studying both the theoretical and practical aspects of cryptography, particularly in the area of pseudorandomness. Professor Li-Yang Tan's work on the tightness of Impagliazzo's hardcore theorem [4] revealed the limitations in hardness amplification for pseudorandom generators (PRG), and I want to study its implications on the security and efficiency of PRGs. On the other hand, I would also like to join Professor Dan Boneh's applied cryptography group and work on projects related to pseudorandom functions.

I have only begun to scratch the surface of theoretical computer science, and there are still a lot to explore. The brilliant faculty and their collaborative spirit in Stanford's Theory Group offer the ideal mentorship to transform my mathematical knowledge into tangible, real-world solutions. With the mathematical foundation I built during my undergraduate studies, I am confident in my ability to excel in the program and evolve into a capable problem-solver. I look forward to contributing to the vibrant academic community at Stanford and bridging the worlds of mathematics and computer science.

References

[1] Ross Anderson, Itai Ashlagi, David Gamarnik, and Alvin E. Roth. Finding long chains in kidney exchange using the traveling salesman problem. *Proceedings of the National Academy of Sciences*, 112(3):663–668, 2015.

[2] Sanjeev Arora and Boaz Barak. *Computational complexity: a modern approach*. Cambridge University Press, 2009.

[3] Soheil Behnezhad, Mohammad Roghani, Aviad Rubinstein, and Amin Saberi. *Beating Greedy Matching in Sublinear Time*, pages 3900–3945.

[4] Guy Blanc, Alexandre Hayderi, Caleb Koch, and Li-Yang Tan. The sample complexity of smooth boosting and the tightness of the hardcore theorem, 2024.