Statement of Purpose

Ray Tsai

I am interested in combinatorics, particularly in extremal graph theory and its connections to theoretical computer science.

My curiosity about combinatorics was first sparked in an honors course on graph theory, taught by Professor Jacques Verstraete. Being my first exposure to combinatorics, the course soon intrigued me with the unconventional nature of combinatorial problems. Unlike other fields of mathematics, combinatorial problems often appear as discrete and uncorrelated with each other, but they nevertheless come with surprising relationships beneath the surface.

I deepened my engagement with combinatorics by taking advanced courses, including a graduate-level combinatorics sequence as well as seminars that exposed me to current research topics. However, my interest in the field was truly solidified through research experiences under the mentorship of Professor Verstraete, where I explored extensive literature on Turán problems and worked on open problems and conjectures. My honors thesis now centers on the Double Turán problem, which asks for the maximum possible number of edges in n subgraphs of a complete graph K_n, with no pairwise intersection of these subgraphs contains a certain forbidden structure. Through this work, I have developed a researcher's mindset and a toolkit that ranges from the probabilistic method to construction techniques, as well as coding skills for searching counterexamples. After dedicating my last summer to studying the triangle-free case, I completed the proof with a tighter condition that each subgraph is induced, which serves as a stepping stone for the general case. Despite the challenges and obstacles involed in the process, I found myself more motivated than ever and continued to explore the general case.

Alongside my research in extremal combinatorics, I pursued theoretical computer science, drawn by its close ties to combinatorics. After a quarter spent working through Sanjeev Arora and Boaz Barak's *Computational Complexity: A Modern Approach*, I joined a research project under Professor Russell Impagliazzo on multicalibration, aiming to reduce unintended bias in learning models. The project opened my eyes to the unexpected connection between algorithm fairness and the Szemerédi regularity lemma through their "random-like" properties. This made me realize the boundless potential of real-world application of combinatorial tools, adding another layer of meaning to my interest in combinatorics. Recognizing the extensive real-world applications of combinatorial tools added new depth to my interest in combinatorics and its potential impact.

My undergraduate experiences opened my apetite to mathematical research. To feed my curiosity, I want to dig further into combinatorics, which prompted me to pursue graduate studies in mathematics. In the long term, I aspire to become a researcher in combinatorics, capable of addressing open problems with fresh perspectives and making

meaningful contributions to the math community. I see graduate school as a crucial first step toward achieving this goal.

I plan to continue investigating Turán problems and their generalizations in graduate school. An interesting research direction I would like to follow is to build on my undergraduate research. The degenerated case of the Double Turán Problem, for which the forbidden structure is bipartite, is shown to be harder than the non-degenerated case, as seen in the classical Turán problem. Currently, I lack the expertise to tackle this case, but I would like to further explore the degenerated double Turán problems as I acquire more tools and insights in my graduate studies. More specifically, I need to learn more about polynomial equations over finite fields and projective planes, which are useful tool for constructions in these degenerated cases.

While I have a strong interest in the extremal combinatorics, I do not aim to limit myself to a single area early in my research career. I plan to explore areas adjacent to combinatorics such as theoretical computer science, both to bridge my rather specialized knowledge in extremal combinatorics with other domains and to expand my understanding of the mathematical landscape. I believe that cultivating a diverse set of perspectives will lead to more fruitful outcomes in my research career.

[Why this program?]

[Closing Paragraph]