Statement of Purpose

Ray Tsai

I am applying to the Ph.D program in Algorithm, Combinatorics, and Optimization (ACO) at Carnegie Mellon University to pursue my interest in combinatorics, especially in extremal graph theory and its connections to theoretical computer science. My goal is to become a capable researcher in combinatorics.

As a mathematics-computer science major at UC San Diego, I dedicated most of my attention to mathematics in my undergraduate studies, primarily through honors and graduate-level coursework. While I found beauty across various fields in mathematics, I was especially drawn to combinatorics due to its deceiving simplicity. Combinatorial problems are often stated in friendly ways that a middle schooler can grasp, but yet they are grounded in complex structures and reveal profound implications once digging in.

During my sophomore year, I began research with Professor Verstraete in extremal graph theory, exploring a range of open problems and conjectures. My first focus was on an open problem involving long paths in Eulerian digraphs. The Erdős-Gallai theorem guarantees a path of length k in a graph of average degree k, but it remains unclear if the analogous result holds for directed paths in Eulerian digraphs. Being my first research experience in mathematics, I realized the stark contrast between coursework and research and struggled significantly in navigating through dead ends and setbacks. Nevertheless, this challenges and difficulities I faced introduced perseverance essential to the research process, marking a formative step in my academic journey.

My spirit, however, was not dampened, and I continued to explore other open problems after gaining more mathematical maturity through my coursework. My honors thesis now centers on the Double Turán problem, which asks for the maximum possible number of edges in n subgraphs of a complete graph K_n, with no pairwise intersection of these subgraphs containing a certain forbidden structure. Through this work, I have developed a toolkit that ranges from the probabilistic method to construction techniques. After dedicating my last summer to studying the triangle-free case, I completed the proof with a tighter condition that each subgraph is induced, which serves as a stepping stone for the general case.

As I advanced in my studies, I grew interest in computational complexity theory, which studies combinatorial problems with a computational lens grounded in real-world problems. After a quarter spent working through Sanjeev Arora and Boaz Barak's *Computational Complexity: A Modern Approach*, I joined Professor Impagliazzo's research group, studying Multicalibration to address unintended bias in learning models from the perspective of complexity theory. The project opened my eyes to the unexpected connection between theoretical computer science and combinatorics. Apart from the already commonly used combinatorial tools like the probabilistics method, the project brought the

application of combinatorics to a new level by modeling the fairness of algorithms with the random-like structures yielded by the Szemeredi's Regularity Lemma. This made me realize the boundless potential of real-world application of combinatorial tools, adding another layer of meaning to my interest in combinatorics.

My undergraduate experiences opened my appetite for mathematical research, but it also humbled me with the immense breadth and depth of the field. To further proceed in the realm of combinatorics, graduate school is a crucial first step for me to achieve this goal. I plan to continue my research in extremal combinatorics in graduate school, as well as exploring adjacent fields such as discrete geometry, random graphs, and topics that I can bridge combinatorics with.

The ACO program at Carnegie Mellon University seems tailor made for my goals. The program's emphasis on combinatorics and its interdisciplinary nature with computer science and operation research perfectly matches my intended research direction. The wide range of faculty members with expertise that covers most of combinatorics will provide me with well-rounded training and guidance in the field.

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