

Statement of Purpose

Ray Tsai

My interest lies in combinatorics, particularly in extremal graph theory and its connections to theoretical computer science.

My curiosity about combinatorics was first sparked in an honors undergraduate course on graph theory, taught by Professor Jacques Verstraete. Being my first exposure to combinatorics, the course soon intrigued me with the unconventional nature of combinatorial problems. Unlike other fields of mathematics, combinatorial problems often appear as discrete and uncorrelated with each other, but they nevertheless come with surprising relationships beneath the surface.

However, what truly solidified my interest were the subsequent research experiences that the course has led me to. Researching under Professor Verstraete, I was not only introduced to the vast literature on Turán problems but also to the joy of interacting with theoretical objects and revealing their hidden properties. The deep structure properties graphs exhibit under extremal conditions amaze me. Simple conditions can force an extremal graph to behave in certain ways. Would the graph lose its structure if we slightly relax the conditions? Does the same structure remain with a slightly different forbidden graph? These questions drive me to explore extremal graphs under varying conditions to uncover further information.

On the other hand, research under Professor Russell Impagliazzo revealed the more “applied” side of combinatorics. Working on problems on multicalibration, which aims to avoid inadvertent discrimination in learning models, the unexpected connection between Algorithm Fairness and the Szemerédi’s regularity lemma broadened my view on the possible applications of combinatorial tools in other disciplines. Despite the theoretical nature of combinatorics, its boundless potential for real-world applications adds another layer of meaning to my interest.

To feed my curiosity, I hope to dig further into the world of combinatorics, which prompts me to pursue graduate studies in mathematics. In the long term, I aspire to become a researcher in combinatorics who is capable of tackling open problems with original ideas and making meaningful contributions, and graduate school in mathematics will be the first step toward this goal.

I plan to continue investigating Turán problems and their generalizations in graduate school. A possible research direction I would like to follow is to build on my undergraduate research. The problem I’m currently working on is the Double Turán problem, which extends the classical Turán problem by investigating the maximum number of edges in n subgraphs of a complete graph K_n , where no pairwise intersection of these subgraphs contains a specific forbidden structure. The degenerated case of this problem, for which the forbidden structure is bipartite, is shown to be harder than the non-degenerated case, as seen in the classical Turán problem. Since I am not capable enough to tackle them in my current state of knowledge, I would like to further explore the degenerated double Turán problems as I acquire more tools and insights in my graduate studies.

Despite my strong interest in the Turán problem, I am not looking to focus exclusively on a single area early in my research career. I intend to explore adjacent fields, both to connect my rather specialized knowledge in extremal combinatorics with other areas and to broaden my understanding of the

mathematical landscape. I believe that cultivating a diverse set of perspectives will lead to more fruitful outcomes in my research career.

Why this program

Why I'm qualified

Closing paragraph