Statement of Purpose

Ray Tsai

I am applying to the Master's program in Computer Science at Stanford University to deepen my understanding of theoretical computer science, particularly in complexity theory and approximation algorithms. I am drawn to the increasing relevance of these areas in addressing real-world challenges, and I aim to apply my mathematical background to develop innovative solutions. Whether in academia or industry, I aim to contribute to advancements that bridge theoretical foundations with practical applications.

As a mathematics-computer science major at UC San Diego, I dedicated most of my attention to mathematics in my undergraduate studies, primarily through honors and graduate-level coursework. While I found beauty in purely theoretical concepts, I faced an existential crisis pursuing mathematics for its own sake. I instead looked for a field in mathematics with an intimate connection to the real world and turned to graph theory, the study of relationships that can model a wide range of practical problems effectively.

During my sophomore year, I began my research journey by undertaking independent study under Professor Jacques Verstraete, exploring the extensive literature on extremal graph theory. Through the readings, I was exposed to a variety of powerful techniques for tackling extremal graph problems, such as probabilistic methods, stability, and finite geometric constructions, which laid the groundwork for my future research. Currently, I am working on my honors thesis centering on the Double Turán problem, which asks for the maximum possible number of edges across n subgraphs of a complete graph K_n such that no pairwise intersection of these subgraphs contain a specified forbidden structure. After dedicating my summer to studying the triangle-free case, I established a tight upper bound on the number of edges under the stricter condition that each subgraph is induced. Specifically, I showed that the extremal condition is uniquely achieved when all subgraphs are complete bipartite graphs, by recursively expanding the intersection of all subgraphs. This case serves as a stepping stone for the project, and I am currently working on the general triangle-free case.

As I advanced in my studies, I also grew interested in computational complexity theory, which studies combinatorial problems with a computational lens grounded in real-world problems. After a quarter spent working through Sanjeev Arora and Boaz Barak's *Computational Complexity: A Modern Approach* [1], I recently joined Professor Russell Impagliazzo's research group, studying Multicalibration to address unintended bias in learning models from the perspective of complexity theory. This research opened my eyes to the surprising connections between abstract mathematical concepts and the practical applications of machine learning. Specifically, the project brought the application of extremal combinatorics and complexity theory to a new level by modeling the fairness of algorithms with the random-like structures yielded by Szemeredi's Regularity

Lemma. Through the project, I realized the boundless transformative power of combinatorial tools, which adds another layer of meaning to my interest in theoretical computer science.

Pivoting from a mathematical background, I aim to fill in the gaps in my computer science knowledge, and Stanford's Master's program offers an ideal path for this, particularly with its Theoretical Computer Science specialization. I look forward to gaining a rigorous grounding in computational complexity through courses like the CS 254 series, and I aim to formalize my knowledge of Multicalibration by taking Algorithmic Fairness (CS 256), a subject I have mostly self-studied. Additionally, I plan to learning more about combinatorial applications to computer science, and taking courses like Matching Theory (MS&E 319), Combinatorial Optimization (CS 261), and Open Problems in Coding Theory (CS 351) would be a great start.

Moreover, I am excited about the research opportunities at Stanford and aim to pursue distinction in research, particularly under the guidance of Aviad Rubinstein on approximate algorithms. His innovative work on algorithms for approximating the size of maximum matchings is especially compelling. I was intrigued by his algorithm in [2] that beats the $\frac{1}{2}$-approximation barrier without even requiring a full traversal of the graph. Professor Rubinstein's research direction is not only exciting but of great practical applications, and I hope to leverage my specialization in combinatorics to develop innovative algorithms with him.

Another faculty member I want to work with is Professor Li-Yang Tan on topics in hardness amplification. During my undergraduate research with Professor Impagliazzo, I encountered Professor's work on the tightness of Impagliazzo's hardcoretheorem [3], which reveals that the circuit size loss in the hardcore theorem is inherently necessary, regardless of the proof technique. I would like to investigate a parallel result for Yao's XOR lemma in collaboration with Professor Tan.

I have only begun to scratch the surface of theoretical computer science, and there are still a lot of topics to explore. I am excited about the prospect of joining Stanford's Master's in Computer Science program to expose me to a broader range of topics in computer science through rigorous coursework and research opportunities. With the mathematical foundation I built during my undergraduate studies, I am confident in my ability to excel in the program and evolve into a capable problem-solver. I look forward to contributing to the vibrant academic community at Stanford and bridging the worlds of mathematics and computer science.

References

[1] Sanjeev Arora and Boaz Barak. *Computational complexity: a modern approach*. Cambridge University Press, 2009.

[2] Soheil Behnezhad, Mohammad Roghani, Aviad Rubinstein, and Amin Saberi. *Beating Greedy Matching in Sublinear Time*, pages 3900–3945.

[3] Guy Blanc, Alexandre Hayderi, Caleb Koch, and Li-Yang Tan. The sample complexity of smooth boosting and the tightness of the hardcore theorem, 2024.