Statement of Purpose

Ray Tsai

I am applying to the Ph.D program in Algorithm, Combinatorics, and Optimization (ACO) at Georgia Tech to pursue my interest in combinatorics, especially in extremal graph theory and its applications to adjacent fields like theoretical computer science.

As a mathematics-computer science major at UC San Diego, I dedicated most of my undergraduate studies to mathematics, primarily through honors and graduate-level coursework. While I found beauty across fields in mathematics, I was especially drawn to combinatorics due to its deceiving simplicity and intimate ties to real-world problems. I aspire to become a researcher who not only specializes in combinatorics but also bridges combinatorics with other disciplines. I believe the ACO program at Georgia Tech is the ideal place to achieve this goal.

During my sophomore year, I began my research journey by directly reading under Professor Jacques Verstraete, exploring the vast literature on extremal graph theory. Through the readings, I was exposed to a variety of powerful techniques for tackling extremal graph problems, from probabilistic methods to algebraic constructions, which laid the groundwork for my honors thesis centered on the Double Turán problem. The problem asks for the maximum possible number of edges across n subgraphs of a complete graph K_n such that no pairwise intersection of these subgraphs contains a specified forbidden structure. After dedicating my summer to studying the case where the forbidden graph is non-bipartite, I established a tight upper bound on the number of edges under the stricter condition that each subgraph is induced. Specifically, I showed that the extremal condition is uniquely achieved when all subgraphs extremal graphs for the forbidden graph, by recursively expanding the intersection of all subgraphs. This case serves as a stepping stone for the project, and I am currently working on the general case.

As I progressed in my research, I also developed an interest in the computational aspects of combinatorial problems. I had the opportunity to learn about computational complexity theory directly from Professor Russell Impagliazzo, studying Arora and Barak's text, Computational Complexity: A Modern Approach [1]. With this foundation, I then joined Professor Impagliazzo's research group, studying Multicalibration to mitigate unintended biases to certain subpopulations in learning models from the perspective of complexity theory. We are trying to connect notions in smooth boosting to multicalibration. By modeling the fairness of algorithms with random-like structures yielded by Szemeredi's Regularity Lemma, this research brought the application of extremal combinatorics and complexity theory to a new level and further strengthened my commitment to utilize my mathematical foundation to solve practical problems.

My undergraduate experiences opened my appetite for mathematical research, but it also humbled me with the immense breadth and depth of the discipline. Graduate

school is a crucial first step to further proceed in the realm of combinatorics. I plan to continue my research in extremal combinatorics in graduate school, as well as explore adjacent fields such as discrete geometry, random graphs, and topics that I can bridge combinatorics with.

The ACO program at Georgia Tech seems tailor-made for my goals. With its emphasis on combinatorics and interdisciplinary nature with computer science and industrial engineering, I can build solid foundations in various fields while furthering my studies in combinatorics, which would realize my aspiration to become a researcher who connects combinatorics with other disciplines.

The diverse expertise of the faculty members in combinatorics will provide me with comprehensive training in the field. I am particularly interested in collaborating with Professor Rose McCarty on extremal combinatorics problems. Thomassen's famous conjecture on girth in graphs is an intriguing aspect of her research. In her recent work [4], she gave a polynomial bound on the average degree of a graph that guarantees the existence of induced C_4-free subgraph with a chosen average degree, and I am intrigued in collaborating with her to find an analogous result for the non-induced case. In addition to pure combinatorics, she is also active in theoretical computer science, such as her work on model-checking problems monadically stable graph classes [3]. This aligns well with my interest in exploring the intersection of combinatorics and computer science, and I believe she can provide me with valuable guidance in this area.

As a Sudoku enthusiast, I am also excited to work with Professor Tom Kelly on problems related to Latin squares. My first exposure to Latin squares came in a graduate combinatorics class assignment, where I was amazed by their behavior exhibited under probabilistic and entropy-based settings. Professor Kelly has done extensive work on random Latin squares, and I am excited to investigate the probabilistic aspect of this topic under his mentorship.

Moreover, I want to collaborate with faculty beyond the math department. In particular, Professor Dana Randall's research on the behavior of randomized algorithms in combinatorial problems is particularly appealing to me, especially her work on the stable marriage problem [2] which studies the convergence time of using random walk to generate stable matchings. I would like to leverage my specialization in combinatorics to learn and contribute to her research in this area.

I have only begun to scratch the surface of combinatorics, and there are still a lot of topics to explore. I am excited about the prospect of joining Georgia Tech's Ph.D program in ACO to receive comprehensive training in mathematical research, and I am confident in my ability to evolve into a capable researcher and make meaningful contributions.

References

[1] Sanjeev Arora and Boaz Barak. *Computational complexity: a modern approach*. Cambridge University Press, 2009.

[2] Nayantara Bhatnagar, Sam Greenberg, and Dana Randall. Sampling stable marriages: Why spouse-swapping won't work. pages 1223–1232, 01 2008.

[3] Jan Dreier, Ioannis Eleftheriadis, Nikolas Mählmann, Rose McCarty, Michał Pilipczuk, and Szymon Toruńczyk. First-order model checking on monadically stable graph classes, 2023.

[4] Xiying Du, António Girão, Zach Hunter, Rose McCarty, and Alex Scott. Induced c_4-free subgraphs with large average degree, 2023.