Math 109 Week 8 Discussion

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1.

Proposition 1. $\sim$ *is equivalent*

Proof. We will show that $\sim$ is reflexive, symmetric, and transitive.

Reflexive: Let $(a, b) \in \mathbb{R}^2$. We will show that $(a, b) \sim (a, b)$. Since $a^2+b^2 = a^2 + b^2$, we have that $(a, b) \sim (a, b)$.

Symmetric: Let $(a, b) \sim (c, d)$, $(a, b), (c, d) \in \mathbb{R}^2$, we will show that $(c, d) \sim (a, b)$. Since $(a, b) \sim (c, d)$, we have $a^2 + b^2 = c^2 + d^2$. This shows that $c^2 + d^2 = a^2 + b^2$, which means that $(c, d) \sim (a, b)$.

Transitive: Let $(a, b) \sim (c, d)$ and $(c, d) \sim (e, f)$, $(a, b), (c, d), (e, f) \in \mathbb{R}^2$, we will show that $(a, b) \sim (e, f)$. Since $(a, b) \sim (c, d)$ and $(c, d) \sim (e, f)$, we have $a^2 + b^2 = c^2 + d^2 = e^2 + f^2$, which shows that $(a, b) \sim (e, f)$. $\square$

2.

Proposition 2. $\sim$ *is not equivalent*

Proof. $\sim$ is not equivalent because it's not transitive. Consider the case $(1, 1), (0, 0), (1, 2) \in \mathbb{R}^2$. $(1, 1) \sim (0, 0)$ because $1 \cdot 0 = 0 = 1 \cdot 0$. $(0, 0) \sim (1, 2)$ because $0 \cdot 1 = 0 = 0 \cdot 2$. However, since $1 \cdot 2 = 2 \neq 1 = 1 \cdot 1$, $(1, 1) \not\sim (0, 0)$. Therefore, $\sim$ is not transitive. $\square$

3.

Proposition 3. $\sim$ *is not equivalent*

Proof. $\sim$ is not equivalent because it's not symmetric. Consider the case $(0, 0), (1, 1) \in \mathbb{R}^2$. $(0, 0) \sim (1, 1)$ because there exist $k = 0 \in \mathbb{R}$ such that $(0, 0) = (k \cdot 1, k \cdot 1)$. However, since there does not exist $k \in \mathbb{R}$ such that $(1, 1) = (k \cdot 0, k \cdot 0)$, $\sim$ is not symmetric. $\square$