Question 3

In this coding question, you'll implement a classifier with logistic regression

$$F(w) = \frac{1}{N}\sum_{i=1}^{N} \log(1 + e^{-\langle w, x_i \rangle y_i}).$$

In [27]:
```
# import statements
import numpy as np
from matplotlib import pyplot as plt
from sklearn.datasets import fetch_openml
```

Load MNIST Data

In [28]:
```
# !pip3 install scikit-learn
# this cell will take a minute to run depending on your internet connection
X, y = fetch_openml('mnist_784', version=1, return_X_y=True) # getting data
print('X shape:', X.shape, 'y shape:', y.shape)
X, y = fetch_openml('mnist_784', version=1, return_X_y=True) # getting data
print('X shape:', X.shape, 'y shape:', y.shape)
# this cell processes some of the data

# if this returns an error of the form "KeyError: 0", then try running the f
# X = X.values # this converts X from a pandas dataframe to a numpy array

X = X.values
digits = {j:[] for j in range(10)}
for j in range(len(y)): # takes data assigns it into a dictionary
    digits[int(y[j])].append(X[j].reshape(28,28))
digits = {j:np.stack(digits[j]) for j in range(10)} # stack everything to be
for j in range(10):
    print('Shape of data with label', j, ':', digits[j].shape )
```

```
X shape: (70000, 784) y shape: (70000,)
X shape: (70000, 784) y shape: (70000,)
Shape of data with label 0 : (6903, 28, 28)
Shape of data with label 1 : (7877, 28, 28)
Shape of data with label 2 : (6990, 28, 28)
Shape of data with label 3 : (7141, 28, 28)
Shape of data with label 4 : (6824, 28, 28)
Shape of data with label 5 : (6313, 28, 28)
Shape of data with label 6 : (6876, 28, 28)
Shape of data with label 7 : (7293, 28, 28)
Shape of data with label 8 : (6825, 28, 28)
Shape of data with label 9 : (6958, 28, 28)
```

Data PreProcess

In [29]:
```
x_4 = digits[4][:500].reshape(500,-1)
x_9 = digits[9][:500].reshape(500,-1)

x_4_test = digits[4][500:1000].reshape(500,-1)
x_9_test = digits[9][500:1000].reshape(500,-1)

x_train = np.vstack((x_4, x_9))
x_train = x_train.astype('float32') / 255.0

x_test = np.vstack((x_4_test, x_9_test))
x_test = x_test.astype('float32') / 255.0

y_train = np.hstack((-1 * np.ones(500), np.ones(500)))
y_test = np.hstack((-1 * np.ones(500), np.ones(500)))
```

Define $F(w)$ and $\nabla F(w)$

In [30]:
```
def F(w):
    sum = 0
    N = len(x_train)
    for i in range(N):
        sum += np.log(1 + np.exp(-y_train[i] * np.dot(w, x_train[i])))
    return sum / N

def dF(w):
    sum = 0
    N = len(x_train)
    for i in range(N):
        sum += -y_train[i] * np.exp(-y_train[i] * np.dot(w, x_train[i])) * x
    return sum / N
```

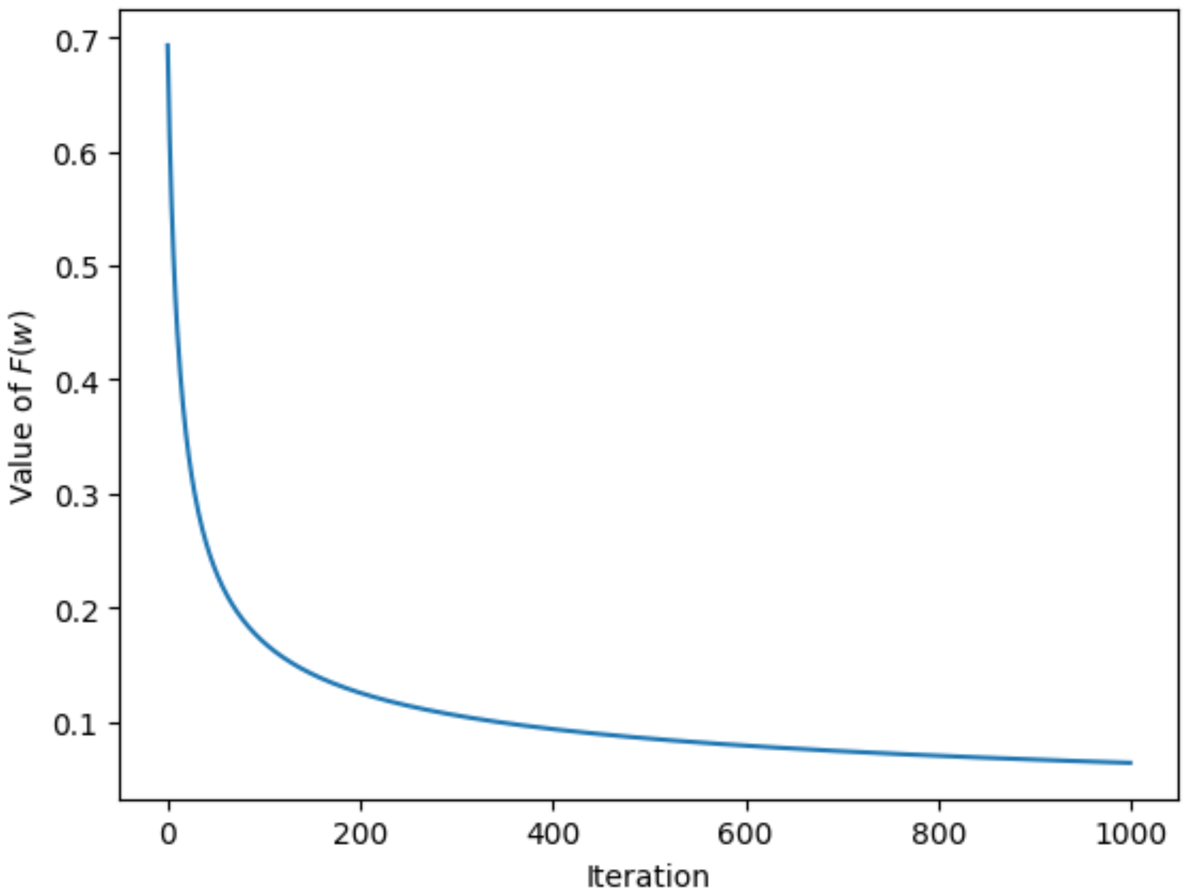
Problem Statement

We will consider the MNIST coding question from HW4. In this question, we run these questions for *differentiating 4's and 9's*. You can reuse the template from the previous homework for loading / formatting MNIST. Implement the following two methods and plot $F(w)$ per iteration for each. You need to submit (i) the code for the algorithms and plots, and (ii) the plots.

Gradient descent with backtracking line search

At each iteration t, initialize the step size $\mu = 10^{-1}$, and use $\gamma = 0.5$ and $\beta = 0.8$ to determine the correct $\mu^{(t)}$. Run your algorithm for at least 10,000 iterations and plot the loss curve $F(w^{(t)})$ as a function of t.

In [31]:
```
T = 1000

w_values = []
iterations = range(T)

def gd(w, mu = 1e-1):
    return w - mu * dF(w)

def backtrack(w, gamma=0.5, beta=0.8):
    mu = 1e-1
    while F(gd(w, mu)) > F(w) - gamma * mu * np.linalg.norm(dF(w))**2:
        mu *= beta
    return mu

w = np.zeros(x_train.shape[1])
for i in iterations:
    w_values.append(F(w))
    mu = backtrack(w)
    w = gd(w, mu)

plt.plot(iterations, w_values)
plt.xlabel("Iteration")
plt.ylabel(r"Value of $F(w)$")
plt.show()
```



Error Rate

In [32]:
```
error = 0

for i in range(1000):
    if np.dot(w, x_test[i]) > 0:
        y_test[i] = 1
    else:
        y_test[i] = -1
    error += (y_test[i] != y_train[i])

print("Error rate:", error / 1000 * 100, "%")
```

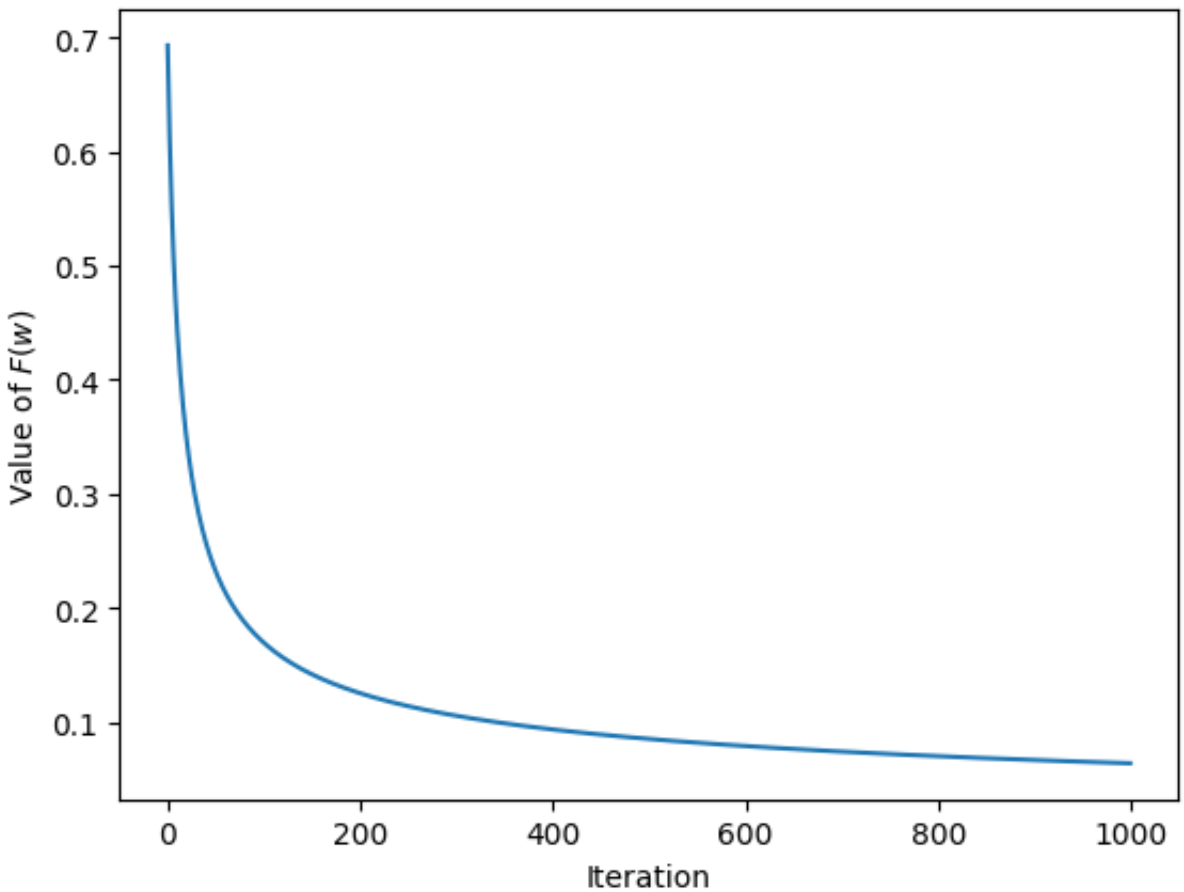
```
Error rate: 4.3 %
```

Gradient Descent with Nesterov acceleration.

You can experiment with the parameters until you find something you like.

In [35]:
```
T = 1000

w_values = []
```

```
iterations = range(T)

def nesterov(w, w_old, mu = 1e-3, beta = 0.9):
    v = w + beta * (w - w_old)
    return v - mu * dF(v)

w = np.zeros(x_train.shape[1])
w_old = w
for i in iterations:
    w_values.append(F(w))
    w_temp = w
    w = nesterov(w, w_old, mu)
    w_old = w

plt.plot(iterations, w_values)
plt.xlabel("Iteration")
plt.ylabel(r"Value of $F(w)$")
plt.show()
```



Error Rate

In [34]:
```
error = 0
```

```
for i in range(1000):
    if np.dot(w, x_test[i]) > 0:
        y_test[i] = 1
    else:
        y_test[i] = -1
    error += (y_test[i] != y_train[i])

print("Error rate:", error / 1000 * 100, "%")
```

```
Error rate: 4.3 %
```