Statement of Purpose

Ray Tsai

I am applying to the Master's program in Computer Science at Stanford University to deepen my understanding of theoretical computer science, particularly in complexity theory and the combinatorial side of the study. My goal is to become a researcher working at the intersection of computer science and mathematics, finding ways to bridge the two fields. I intend to pursue a Ph.D. after the master's program to further this objective.

As a mathematics-computer science major at UC San Diego, I dedicated most of my attention to mathematics in my undergraduate studies, primarily through honors and graduate-level coursework. While I found beauty in purely theoretical concepts, I faced an existential crisis pursuing mathematics for its own sake. I instead looked for a field in mathematics with intimate connection to the real world and turned to graph theory, the study of relationships that can model a wide range of practical problems effectively.

I began conducting research with Professor Verstraete in extremal graph theory in my sophomore year, where I explored various open problems and conjectures. The first open problem I attempted was a problem on long paths in Eulerian Digraphs, which asks for the lower bound of the longest directed path in an Eulerian digraph with respect to the average degree. This problem is an extension of the famous Erdős-Gallai theorem and is also of interest in the study of algorithms. I attempted the problem with an alternative approach suggested by Professor Verstraete, which involves randomly selecting a cyclic ordering of the vertices and analyzing the expected length of the longest "zig-zag" pattern in the ordering. Although my work encountered barriers with extending the zig-zag paths, it introduced me to the challenges and perseverance inherent in research.

My spirit, however, was not dampened by the failure, and I continued to explore other open problems after gaining more mathematical maturity through my coursework. My honors thesis now centers on the Double Turán problem, which asks for the maximum possible number of edges in n subgraphs of a complete graph K_n, with no pairwise intersection of these subgraphs containing a certain forbidden structure. Through this work, I have developed a toolkit that ranges from the probabilistic method to construction techniques. After dedicating my last summer to studying the triangle-free case, I completed the proof with a tighter condition that each subgraph is induced, which serves as a stepping stone for the general case.

As I advanced in my studies, I grew interest in computational complexity theory, which studies combinatorial problems with a computational lens grounded in real-world problems. After a quarter spent working through Sanjeev Arora and Boaz Barak's *Computational Complexity: A Modern Approach*, I joined Professor Impagliazzo's research group, studying Multicalibration to address unintended bias in learning models from the perspective of complexity theory. The project opened my eyes to the unexpected connection between theoretical computer science and combinatorics. Apart from the already commonly used combinatorial tools like the probabilistics method, the project brought the application of combinatorics to a new level by modeling the fairness of algorithms with the random-like structures yielded by the Szemeredi's Regularity Lemma. This experience reinforced my interest in the interplay between the two discplines and prompted me to pursue further studies in theoretical computer science.

With my mathematics-heavy background, I aim to build a deeper understanding of computer science concepts to support future Ph.D. work. Stanford's Master's program offers an ideal path for this, particularly with its Theoretical Computer Science specialization. I look forward to gaining a rigorous grounding in computational complexity through courses like the CS 254 series, and I aim to formalize my knowledge of Multicalibration by taking Algorithmic Fairness (CS 256), a subject I have mostly self-studied. Additionally, I plan to learning more about combinatorial applications to computer science, and taking courses like Matching Theory (MS&E 319), Combinatorial Optimization (CS 261), and Open Problems in Coding Theory (CS 351) would be a great start.

Moreover, I am excited about the research opportunities at Stanford and aim to pursue distinction in research, particularly under the guidance of Professor Li-Yang Tan on topics in Hardness Amplification. During my undergraduate research with Professor Impagliazzo, I encountered Professor Tan's work, *"The Sample Complexity of Smooth Boosting and the Tightness of the Hardcore Theorem,"* which reveals that the circuit size loss in Impagliazzo's Hardcore Theorem is inherently necessary, regardless of the proof technique. I would like to investigate a parallel result for Yao's XOR lemma in collaboration with Professor Tan.

I have only begun to scratch the surface of theoretical computer science, and there are still a lot of topics to explore. I am excited about the prospect of joining Stanford's Master's in Computer Science program to expose me to a broader range of topics in computer science through rigorous coursework and research opportunities. With the mathematical foundation I built during my undergraduate studies, I am confident in my ability to excel in the program and evolve into a capable researcher. I look forward to contributing to the vibrant academic community at Stanford and bridging the worlds of mathematics and computer science.