Proposition 1. *Let G_1, G_2, G_3 be graphs such that no K_4 is contained in two graphs. Then, $\delta(G_i) \leq \frac{7}{9}n$ for some i and this bound is tight.*

Proof. Suppose $\delta(G_i) > \frac{7}{9}n$ for all i. Since $|E(G_1)| + |E(G_2)| + |E(G_3)| > \binom{n}{2} + \frac{n^2}{2}$, there exists a triangle T that is contained in two G_i's, say G_1 and G_2. Let $N_i(v)$ denote the neighborhood of vertex v in G_i, and let v_1, v_2, v_3 be the vertices of T. Notice

$$\begin{aligned}
|N_i(v_1) \cap N_i(v_2) \cap N_i(v_3)| &\geq |N_i(v_1) \cap N_i(v_2)| + |N_i(v_3)| - n \\
&\geq |N_i(v_1)| + |N_i(v_2)| + |N_i(v_3)| - 2n \\
&> \frac{7}{3}n - 2n = \frac{n}{3}.
\end{aligned}$$

For simplicity, denote $C_i(T) = N_i(v_1) \cap N_i(v_2) \cap N_i(v_3)$. Since $|C_1(T)| + |C_2(T)| + |C_3(T)| > n$, there exists a vertex u in two $C_i(T)$'s. If $u \in C_1(T) \cap C_2(T)$ then we are done. Assume without loss of generality that $u \in C_1(T) \cap C_3(T)$. It seems like we need T to be a fat triangle to make this work.

We now show that the bound is tight. Fix a Turán graph $T_3(n)$ in complete graph K_n, with parts V_1, V_2, V_3. Equally partition the edges in each V_i into three sets and label the edges in each set 1, 2, 3, respectively. Let $G_i = T_3(n)$ plus the edges labelled i. Since no edge is labelled twice in each V_i, no K_4 is contained in two G_i's. The minimum degree of each G_i is at least $\frac{7}{9}n$. □