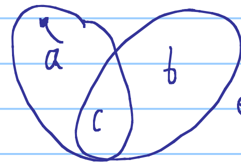


$e(G_1)+e(G_2) \le \binom{a+c}{2} + \binom{b+c}{2} - 2\left(\binom{c}{2} - ex(c,F)\right)$

$= f(a,b,c)$

Show $-f(a,b,c) + f(a,b-1,c+1) > 0$

$\binom{a+c+1}{2} - \binom{a+c}{2} - 2c + 2ex(c+1,F) - 2ex(c,F) > 0$

$a+c-2c+2ex(c+1,F) - \dots > 0$

$a-c+2\left[ex(c+1,F)-ex(c,F)\right] > 0$

$ex(c,F) = ex(c,K_r) + ex(c,\tilde{F})$

If F is color-critical, then

$ex(c,F) = ex(c,K_r)$ $r=\chi(F)$

$ex(c+1,F)-ex(c,F)$

$= c+1-\left\lceil\frac{c+1}{r-1}\right\rceil.$

$\overset{r=3}{=} \left\lfloor\frac{c+1}{2}\right\rfloor$

$a-c+2\Big[\underbrace{(c+1)-\left\lceil\frac{c+1}{r-1}\right\rceil}_{\ge\left\lfloor\frac{c+1}{2}\right\rfloor} + \underbrace{ex(c+1,\tilde{F})-ex(c,\tilde{F})}_{\ge 0}\Big]$

$\ge a$

$m=2$ non-induced ✓

$m=3$ G_1, G_2, G_3

large m:

If a G_i has $\le \left\lfloor\frac{n^2}{m}\right\rfloor$ edges remove it

else, if $e(G_i) > \left\lfloor\frac{n^2}{4}\right\rfloor$

then $\Delta(G_i) \ge \left\lfloor\frac{n}{2}\right\rfloor$

triangles in G_i's is $\ge m\left\lfloor\frac{n}{2}\right\rfloor > \binom{n}{3}$

done if $m > \binom{n}{3}/\left\lfloor\frac{n}{2}\right\rfloor$

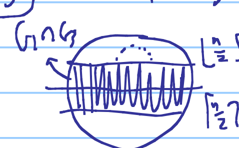
$e(G_1)+e(G_2)+e(G_3)$

$\le e(G_1\cap G_2)+e(G_2\cap G_3)+e(G_1\cap G_3)$

$+ e(G_1) - e(G_1\cap G_2)$

$= e(G_1) + \boxed{e(G_2\cap G_3)} + e(G_1\cap G_3)$

$\le \binom{n}{2} + 2\left\lfloor\frac{n^2}{4}\right\rfloor.$



$G_3 = K_{\lceil\frac{n}{2}\rceil,\lfloor\frac{n}{2}\rfloor}$

$G_2 = K_{\lfloor\frac{n}{2}\rfloor,\lceil\frac{n}{2}\rceil}$