Statement of Purpose

Ray Tsai

I am applying to the Ph.D program in Algorithm, Combinatorics, and Optimization (ACO) at Georgia Tech to pursue my interest in combinatorics, especially in extremal graph theory and its applications to adjacent fields like theoretical computer science.

As a mathematics-computer science major at UC San Diego, I dedicated most of my undergraduate studies to mathematics, primarily through honors and graduate-level coursework. While I found beauty across fields in mathematics, I was especially drawn to combinatorics due to its deceiving simplicity and intimate ties to real-world problems. I aspire to become a researcher who not only specializes in combinatorics but also bridges combinatorics with other disciplines. I believe the ACO program at Georgia Tech is the ideal place to achieve this goal.

During my sophomore year, I began my research journey by undertaking independent study under Professor Jacques Verstraete, exploring the extensive literature on extremal graph theory. Through the readings, I was exposed to a variety of powerful techniques for tackling extremal graph problems, such as probabilistic methods, stability, and finite geometric constructions, which laid the groundwork for my future research. Currently, I am working on my honors thesis centering on the Double Turán problem, which asks for the maximum possible number of edges across n subgraphs of a complete graph K_n such that no pairwise intersection of these subgraphs contain a specified forbidden structure. After dedicating my summer to studying the triangle-free case, I established a tight upper bound on the number of edges under the stricter condition that each subgraph is induced. Specifically, I showed that the extremal condition is uniquely achieved when all subgraphs are complete bipartite graphs, by recursively expanding the intersection of all subgraphs. This case serves as a stepping stone for the project, and I am currently working on the general triangle-free case.

As I advanced in my studies, I also grew interested in computational complexity theory, which studies combinatorial problems with a computational lens grounded in real-world problems. After a quarter spent working through Sanjeev Arora and Boaz Barak's *Computational Complexity: A Modern Approach* [1], I joined Professor Russell Impagliazzo's research group, studying Multicalibration to address unintended bias in learning models from the perspective of complexity theory. The project opened my eyes to the unexpected connections between theoretical computer science and combinatorics, as it brought the application of combinatorics to a new level by modeling the fairness of algorithms with the random-like structures yielded by Szemeredi's Regularity Lemma. Through the project, I realized the boundless potential of real-world application of combinatorial tools and it adds another layer of meaning to my interest in combinatorics.

My undergraduate experiences opened my appetite for mathematical research, but

it also humbled me with the immense breadth and depth of the discipline. Graduate school is a crucial first step to further proceed in the realm of combinatorics. I plan to continue my research in extremal combinatorics in graduate school, as well as explore adjacent fields such as discrete geometry, random graphs, and topics that I can bridge combinatorics with.

The ACO program at Georgia Tech seems tailor-made for my goals. With its emphasis on combinatorics and interdisciplinary nature with computer science and industrial engineering, I can build solid foundations in various fields while furthering my studies in combinatorics, which would realize my aspiration to become a researcher who connects combinatorics with other disciplines.

The diverse expertise of the faculty members in combinatorics will provide me with comprehensive training in the field. I am particularly interested in collaborating with Professor Rose McCarty on extremal combinatorics problems. An aspect of her research that I find intriguing is on Thomassen's famous conjecture on girth in graphs. In her recent work [4], she gave a polynomial bound on the average degree of a graph that guarantees the existence of of a induced C_4-free subgraph with a chosen average degree, and I am intrigued collaborate with her to find an analogous result for the non-induced case. In addition to pure combinatorics, she is also active in the realm of theoretical computer science, such as her work on model checking problem on monadically stable graph classes [3], aligns well with my interest in exploring the intersection of combinatorics and computer science under her guidance.

As a Sudoku enthusiast, I am also excited to work with Professor Tom Kelly on problems related to Latin squares. My first exposure to Latin squares came in a graduate combinatorics class assignment, where I as amazed of their behavior exhibit under probabilistic and entropy-based settings. Professor Kelly has done extensive works on random Latin squares, and I am excited to learn more about this area under his mentorship.

Moreover, I want to collaborate with faculty beyond the math department. In particular, Professor Dana Randall's research on the behavior of randomize algorithms in combinatorial problems is particularly appealing to me, especially her work on the stable marriage problem [2] which studies the convergence time of using random walk to generate stable matchings. I would like to leverage my specialization in combinatorics to contribute to her research in this area.

I have only begun to scratch the surface of combinatorics, and there are still a lot of topics to explore. I am excited about the prospect of joining Georgia Tech's Ph.D program in ACO to receive comprehensive training in mathematical research, and I am confident in my ability to evolve into a capable researcher and make meaningful contributions.

References

[1] Sanjeev Arora and Boaz Barak. *Computational complexity: a modern approach*. Cambridge University Press, 2009.

[2] Nayantara Bhatnagar, Sam Greenberg, and Dana Randall. Sampling stable marriages: Why spouse-swapping won't work. pages 1223–1232, 01 2008.

[3] Jan Dreier, Ioannis Eleftheriadis, Nikolas Mählmann, Rose McCarty, Michał Pilipczuk, and Szymon Toruńczyk. First-order model checking on monadically stable graph classes, 2023.

[4] Xiying Du, António Girão, Zach Hunter, Rose McCarty, and Alex Scott. Induced c_4-free subgraphs with large average degree, 2023.