Statement of Purpose

Ray Tsai

I am applying to the Master's program in Computer Science at Stanford University to deepen my understanding of theoretical computer science, particularly in complexity theory and the combinatorial side of the study. My goal is to become a researcher working at the intersection of computer science and mathematics, finding ways to bridge the two fields. I intend to pursue a Ph.D. after the master's program to further this objective.

As a mathematics-computer science major at UC San Diego, I dedicated most of my attention to mathematics in my undergraduate studies, primarily through honors and graduate-level coursework. While I found beauty in purely theoretical concepts, I faced an existential crisis pursuing mathematics for its own sake. I instead looked for a field in mathematics with intimate connection to the real world and turned to graph theory, the study of relationships that can model a wide range of practical problems effectively.

During my sophomore year, I began my research journey by undertaking independent study under Professor Jacques Verstraete, exploring the extensive literature on extremal graph theory. Through the readings, I was exposed to a variety of powerful techniques for tackling extremal graph problems, such as probabilistic methods, stability, and finite geometric constructions, which laid the groundwork for my future research. Currently, I am working on my honors thesis centering on the Double Turán problem, which asks for the maximum possible number of edges across n subgraphs of a complete graph K_n such that no pairwise intersection of these subgraphs contain a specified forbidden structure. After dedicating my summer to studying the triangle-free case, I established a tight upper bound on the number of edges under the stricter condition that each subgraph is induced. Specifically, I showed that the extremal condition is uniquely achieved when all subgraphs are complete bipartite graphs, by recursively expanding the intersection of all subgraphs. This case serves as a stepping stone for the project, and I am currently working on the general triangle-free case.

As I advanced in my studies, I also grew interested in computational complexity theory, which studies combinatorial problems with a computational lens grounded in real-world problems. After a quarter spent working through Sanjeev Arora and Boaz Barak's *Computational Complexity: A Modern Approach* [1], I joined Professor Russell Impagliazzo's research group, studying Multicalibration to address unintended bias in learning models from the perspective of complexity theory. The project opened my eyes to the unexpected connections between theoretical computer science and combinatorics, as it brought the application of combinatorics to a new level by modeling the fairness of algorithms with the random-like structures yielded by Szemeredi's Regularity Lemma. Through the project, I realized the boundless potential of real-world application of combinatorial tools and it adds another layer of meaning to my interest in combinatorics.

With my mathematics-heavy background, I aim to build a deeper understanding of computer science concepts to support future Ph.D. work. Stanford's Master's program offers an ideal path for this, particularly with its Theoretical Computer Science specialization. I look forward to gaining a rigorous grounding in computational complexity through courses like the CS 254 series, and I aim to formalize my knowledge of Multicalibration by taking Algorithmic Fairness (CS 256), a subject I have mostly self-studied. Additionally, I plan to learning more about combinatorial applications to computer science, and taking courses like Matching Theory (MS&E 319), Combinatorial Optimization (CS 261), and Open Problems in Coding Theory (CS 351) would be a great start.

Moreover, I am excited about the research opportunities at Stanford and aim to pursue distinction in research, particularly under the guidance of Professor Li-Yang Tan on topics in Hardness Amplification. During my undergraduate research with Professor Impagliazzo, I encountered Professor Tan's work on the tightness of Impagliazzo's hardcore theorem [2], which reveals that the circuit size loss in the hardcore theorem is inherently necessary, regardless of the proof technique. I would like to investigate a parallel result for Yao's XOR lemma in collaboration with Professor Tan.

I have only begun to scratch the surface of theoretical computer science, and there are still a lot of topics to explore. I am excited about the prospect of joining Stanford's Master's in Computer Science program to expose me to a broader range of topics in computer science through rigorous coursework and research opportunities. With the mathematical foundation I built during my undergraduate studies, I am confident in my ability to excel in the program and evolve into a capable researcher. I look forward to contributing to the vibrant academic community at Stanford and bridging the worlds of mathematics and computer science.

References

[1] Sanjeev Arora and Boaz Barak. *Computational complexity: a modern approach*. Cambridge University Press, 2009.

[2] Guy Blanc, Alexandre Hayderi, Caleb Koch, and Li-Yang Tan. The sample complexity of smooth boosting and the tightness of the hardcore theorem, 2024.