Math 109 Week 9 Discussion

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1. There is a surjective function β such that
$$\beta : \mathbb{R} \times \mathbb{Z} \to \mathbb{R} \tag{1}$$
$$\beta(r, z) = r. \tag{2}$$
Since $\mathbb{R}$ is uncountable, $\mathbb{R} \times \mathbb{Z}$ is uncountable.

2. There is a surjective function f such that
$$f : \mathbb{Z}^2 \to \mathbb{Z}[i] \tag{3}$$
$$f(a, b) = a + bi. \tag{4}$$
Since $\mathbb{Z}^2$ is countable, $\mathbb{Z}[i]$ is countable.

3. Let P be the set of all prime numbers. There is a surjective function g such that
$$g : P \to \{\frac{p}{7} \,|\, p \in P\} \tag{5}$$
$$f(x) = \frac{x}{7}. \tag{6}$$
Since $P \subseteq \mathbb{N}$, $\{\frac{p}{7} \,|\, p \in P\}$ is countable.

4. Let $A = \{x \in \mathbb{R} \,|\, x = \frac{n\pi}{2} \text{ or } \frac{ne}{3}\ n \in \mathbb{Z}\}$. There is a surjective function h such that
$$h : \mathbb{Z} \to A$$
$$h(x) = \begin{cases} \frac{n\pi}{4}, & \text{if } n \text{ is even} \\ \frac{(n-1)e}{6}, & \text{if } n \text{ is odd.} \end{cases}$$
Since $\mathbb{Z}$ is countable, A is countable.

5. There exists a bijective function α such that
$$\alpha : \mathbb{R}^4 \to M_2(\mathbb{R}) \tag{7}$$
$$\alpha(a, b, c, d) = \begin{pmatrix} a & b \\ c & d \end{pmatrix}. \tag{8}$$
Since $\mathbb{R}^4$ is uncountable, $M_2(\mathbb{R})$ is uncountable.

6. Let $S = \{ax + b,\ a, b \in \mathbb{R}\}$. There exists a bijective function r such that

$$r : \mathbb{R}^2 \to S \tag{9}$$

$$r(a, b) = ax + b. \tag{10}$$

Since $\mathbb{R}^2$ is uncountable, S is uncountable.