Name : Himanshu Gupta

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Assignment - 2

Deep Learning : Theory and Practice

1. Input Image = $X(U,V)$

Weight filters = $W(S,T)$

There are k filters.

$$Y^k = X \circledast W^k$$

$$Y^k_{(p,q)} = \sum_{i=0}^{S-1} \sum_{j=0}^{T-1} x(p+i, q+j)\, W^k(i,j)$$

Size of output Image : $Y^k = (U-S+1, V-T+1)$

Cost Function = J

To proove : $\frac{\partial J}{\partial W^k} = X * \frac{\partial J}{\partial Y^k}$

After convolving

$$Y_{11} = W_{11}X_{11} + W_{12}X_{12} + \cdots + W_{1T}X_{1T} \\ \vdots \quad + \quad \vdots \\ W_{R1}X_{R1} + \cdots \cdots + W_{RS}X_{RS} \quad — (1)$$

Similarly

$$Y_{12} = W_{11}X_{12} + W_{12}X_{13} + \cdots + W_{1T}X_{1(T+1)} \\ \vdots \quad + \quad + \\ W_{R1}X_{(R+1)2} + \cdots - - - + W_{RT}X_{R(T+1)} \quad — (2)$$

Scanned by CamScanner

$$\frac{\partial J}{\partial Y} = \begin{bmatrix} \frac{\partial E}{\partial Y_{11}} & \cdots & \frac{\partial E}{\partial Y_{1,V-T+1}} \\ \vdots & & \\ \frac{\partial E}{\partial Y_{V-S+1,1}} & \cdots & \frac{\partial E}{\partial Y_{V-S+1,V-T+1}} \end{bmatrix}$$

Now as we see that changing even one value of w matrix is gonna change every value of Y matrix.

$$\frac{\partial J}{\partial W_{11}} = \frac{\partial J}{\partial Y_{11}} \cdot \left(\frac{\partial Y_{11}}{\partial W_{11}}\right) + \frac{\partial J}{\partial Y_{12}} \cdot \left(\frac{\partial Y_{12}}{\partial W_{11}}\right) + \cdots \qquad (3)$$

$$\cdots + \frac{\partial J}{\partial Y_{V-S+1,V-T+1}} \cdot \frac{\partial Y_{V-S+1,V-T+1}}{\partial W_{11}}$$

Similarly

$$\cancel{\frac{\partial E}{\partial W_{12}} = \frac{\partial E}{}} \qquad (4)$$

$$\frac{\partial J}{\partial W_{12}} = \frac{\partial J}{\partial Y_{11}} \cdot \frac{\partial Y_{11}}{\partial W_{12}} + \cdots + \frac{\partial J}{\partial Y_{V-S+1,V-T+1}} \cdot \frac{\partial Y_{V-S+1,V-T+1}}{\partial W_{12}}$$

From (1) and (3), we can say that.

$$\frac{\partial J}{\partial W_{11}} = \frac{\partial J}{\partial Y_{11}} \cdot X_{11} + \frac{\partial J}{\partial Y_{12}} \cdot X_{12} + \cdots \qquad (5)$$

From (2) and (4), we get

$$\frac{\partial J}{\partial W_{12}} = \frac{\partial J}{\partial Y_{11}} \cdot X_{12} + \cdots \qquad (6)$$

From (5) and (6), we can say that

$$\cancel{\frac{\partial J}{\partial W} = X \circledast \frac{\partial E}{\partial Z}}$$

$$\boxed{\frac{\partial J}{\partial W} = X \circledast \frac{\partial J}{\partial Y}}$$

Hence proved