

Factor Oracle for Machine Improvisation

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Septembre 2016



Preliminaries

Preliminaries

Word

A **word** s is a finite sequence $s = s_1 s_2 \dots s_m$ of length $|s| = m$ on a finite alphabet Σ .

$s =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

Factor

A word $x \in \Sigma^*$ is a **factor** of s if and only if s can be written $s = uxv$ with $u, v \in \Sigma^*$. Given integers i, j where $1 \leq i \leq j \leq m$, we denote a *factor* of s as $s[i \dots j] = s_i s_{i+1} \dots s_j$.

$s =$

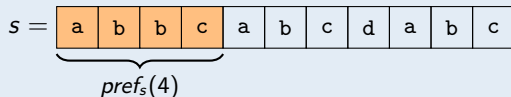
a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

$\underbrace{\hspace{10em}}$
 $s[3, 5]$

Preliminaries

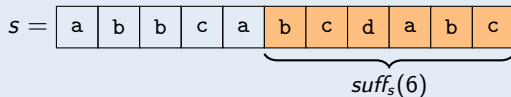
Prefix

A factor x of s is a **prefix** of s if $s = xu$ with $u \in \Sigma^*$. The i th *prefix* of s , denoted $\text{pref}_s(i)$, is the prefix $s[1 \dots i]$.



Suffix

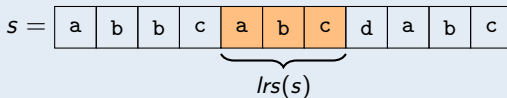
A factor x of s is a **suffix** of s if $s = ux$ with $u \in \Sigma^*$. The i th *suffix* of s , denoted $\text{suff}_s(i)$, is the suffix $s[i \dots m]$.



Preliminaries

Longest Repeated Suffix (LRS)

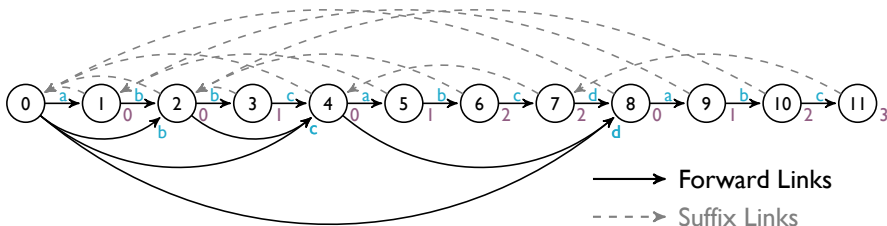
A factor x of s is the **longest repeated suffix** of s if x is a suffix of s and $|x|$ is maximal.



Factor Oracle

Factor Oracle

Overview

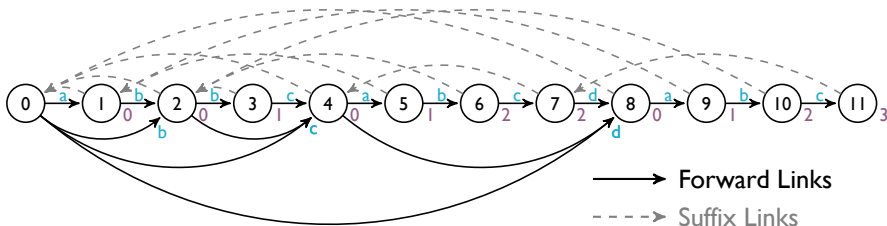


Factor Oracle

The **factor oracle** of a word s of length m is a *deterministic finite automaton* (Q, q_0, F, δ) where $Q = \{0, 1, \dots, m\}$ is the set of states, $q_0 = 0$ is the starting state, $F = Q$ is the set of terminal states and δ is the transition function.

Factor Oracle

Overview

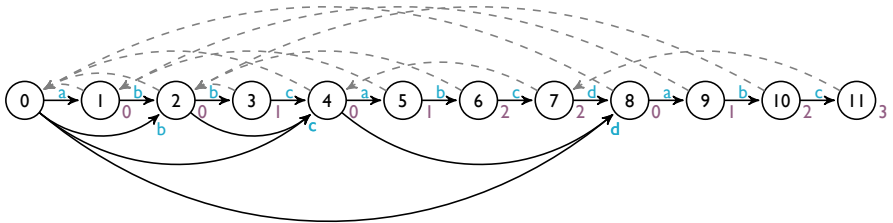


Suffix Link

The **suffix link** of a state i of the factor oracle of a word s , is equal to the state in which the *longest repeated suffix (lrs)* of $s[1 \dots i]$ is recognized.

Factor Oracle

Overview

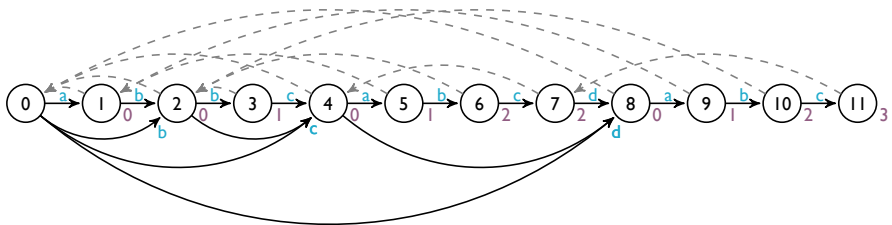


Suffix Links

- $s = \text{abbcababcdabc}$

Factor Oracle

Overview

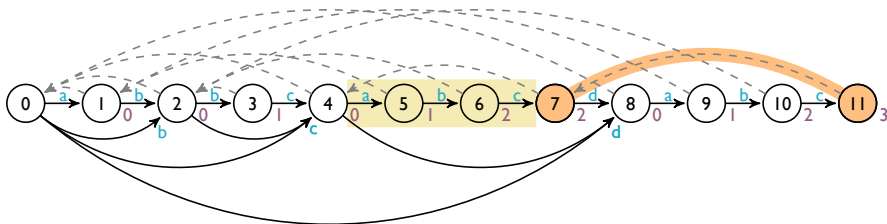


Suffix Links

- $s = \text{abbc}\text{abcd}\text{abc}$
- $\text{lrs}(s) = \text{abc}$

Factor Oracle

Overview



Suffix Links

- $s = \text{abbcababcdabc}$
- $\text{lrs}(s) = \text{abc}$
- $S(11) = 7$

Factor Oracle

Algorithm - Construction

Algorithm 1 Construction of a Factor Oracle

```
1: function FactorOracle( $p = p_1 p_2 \dots p_m$ )
2:   Create a new oracle  $P$  with an initial state 0
3:    $S_P(0) \leftarrow -1$ 
4:   for  $i \leftarrow 1, m$  do
5:     Oracle( $p = p_1 p_2 \dots p_i$ )  $\leftarrow$  AddLetter(Oracle( $p = p_1 p_2 \dots p_{i-1}$ ),  $p_i$ )
6:   end for
7:   return Oracle( $p = p_1 p_2 \dots p_m$ )
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Factor Oracle

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0

Factor Oracle

Algorithm - Construction

Algorithm 2 Incremental update of Factor Oracle

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 $m = 0$

0₀

Factor Oracle

Algorithm - Construction

Algorithm 2 Incremental update of Factor Oracle

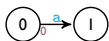
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Factor Oracle

Algorithm - Construction

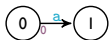
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Factor Oracle

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Factor Oracle

Algorithm - Construction

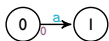
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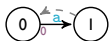
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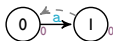
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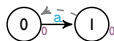
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Factor Oracle

Algorithm - Construction

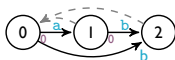
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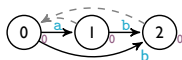
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Factor Oracle

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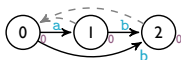
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 $m = 2$



Factor Oracle

Algorithm - Construction

Algorithm 2 Incremental update of Factor Oracle

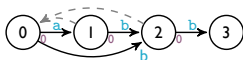
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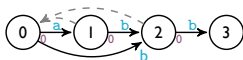
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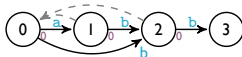
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$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 2$ $\pi_1 = 2$ $k = 0$



Factor Oracle

Algorithm - Construction

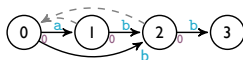
Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
2:   ...
3:   if  $k = -1$  then
4:     ...
5:   else
6:      $S_{p\sigma} \leftarrow$  state that leads the transition from  $k$  by  $\sigma$ 
7:      $lrs_{p\sigma} \leftarrow \text{LengthCommonSuffix}(\pi_1, S(m+1) - 1) + 1$ 
8:   end if
9:   ...
10: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 2$ $\pi_1 = 2$ $k = 0$



Factor Oracle

Algorithm - Construction

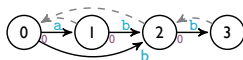
Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
2:   ...
3:   if  $k = -1$  then
4:     ...
5:   else
6:      $S_{p\sigma} \leftarrow$  state that leads the transition from  $k$  by  $\sigma$ 
7:      $lrs_{p\sigma} \leftarrow \text{LengthCommonSuffix}(\pi_1, S(m+1) - 1) + 1$ 
8:   end if
9:   ...
10: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 2$ $\pi_1 = 2$ $k = 0$



Factor Oracle

Algorithm - Construction

Algorithm 2 Incremental update of Factor Oracle

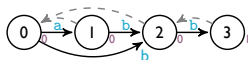
```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
2:   ...
3:   if  $k = -1$  then
4:     ...
5:   else
6:      $S_{p\sigma} \leftarrow$  state that leads the transition from  $k$  by  $\sigma$ 
7:      $lrs_{p\sigma} \leftarrow \text{LengthCommonSuffix}(\pi_1, S(m+1) - 1) + 1$ 
8:   end if
9:   ...
10: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

$m = 2 \quad \pi_1 = 2 \quad k = 0$

$lcs(2, 1) = 0$



Factor Oracle

Algorithm - Construction

Algorithm 2 Incremental update of Factor Oracle

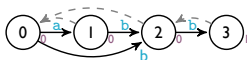
```
1: function AddLetter(Oracle( $p = p_1, p_2 \dots p_m$ ),  $\sigma$ )
2:   Create state  $m + 1$ 
3:   Create a new transition from  $m$  to  $m + 1$  labeled by  $\sigma$ 
4:    $\pi_1 \leftarrow m$ 
5:    $k \leftarrow S_p(m)$ 
6:   ...
7: end function
```

$\triangleright \delta(m, \sigma) = m + 1$

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 3$



Factor Oracle

Algorithm - Construction

Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )  
2:   Create state  $m + 1$   
3:   Create a new transition from  $m$  to  $m + 1$  labeled by  $\sigma$   $\triangleright \delta(m, \sigma) = m + 1$   
4:    $\pi_1 \leftarrow m$   
5:    $k \leftarrow S_p(m)$   
6:   ...  
7: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 3$



Factor Oracle

Algorithm - Construction

Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )  
2:   Create state  $m + 1$   
3:   Create a new transition from  $m$  to  $m + 1$  labeled by  $\sigma$   $\triangleright \delta(m, \sigma) = m + 1$   
4:    $\pi_1 \leftarrow m$   
5:    $k \leftarrow S_p(m)$   
6:   ...  
7: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

$m = 3 \quad \pi_1 = 3 \quad k = 2$



Factor Oracle

Algorithm - Construction

Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
2:   ...
3:   while  $k > -1$  and there is no transition from  $k$  by  $\sigma$  do
4:     Create a new transition from  $k$  to  $m + 1$  by  $\sigma$ 
5:      $\pi_1 \leftarrow k$ 
6:      $k \leftarrow S_p(k)$ 
7:   end while
8:   ...
9: end function
```

$\triangleright \delta(k, \sigma) = m + 1$

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 3$ $\pi_1 = 3$ $k = 2$



Factor Oracle

Algorithm - Construction

Algorithm 2 Incremental update of Factor Oracle

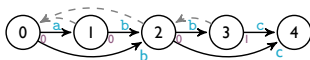
```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
2:   ...
3:   while  $k > -1$  and there is no transition from  $k$  by  $\sigma$  do
4:     Create a new transition from  $k$  to  $m + 1$  by  $\sigma$ 
5:      $\pi_1 \leftarrow k$ 
6:      $k \leftarrow S_p(k)$ 
7:   end while
8:   ...
9: end function
```

$\triangleright \delta(k, \sigma) = m + 1$

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

$m = 3 \quad \pi_1 = 3 \quad k = 2$



Factor Oracle

Algorithm - Construction

Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
2:   ...
3:   while  $k > -1$  and there is no transition from  $k$  by  $\sigma$  do
4:     Create a new transition from  $k$  to  $m + 1$  by  $\sigma$ 
5:      $\pi_1 \leftarrow k$ 
6:      $k \leftarrow S_p(k)$ 
7:   end while
8:   ...
9: end function
```

$\triangleright \delta(k, \sigma) = m + 1$

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

$m = 3 \quad \pi_1 = 2 \quad k = 2$



Factor Oracle

Algorithm - Construction

Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
2:   ...
3:   while  $k > -1$  and there is no transition from  $k$  by  $\sigma$  do
4:     Create a new transition from  $k$  to  $m + 1$  by  $\sigma$ 
5:      $\pi_1 \leftarrow k$ 
6:      $k \leftarrow S_p(k)$ 
7:   end while
8:   ...
9: end function
```

$\triangleright \delta(k, \sigma) = m + 1$

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

$m = 3 \quad \pi_1 = 2 \quad k = 0$



Factor Oracle

Algorithm - Construction

Algorithm 2 Incremental update of Factor Oracle

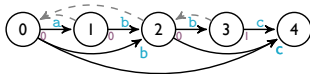
```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
2:   ...
3:   while  $k > -1$  and there is no transition from  $k$  by  $\sigma$  do
4:     Create a new transition from  $k$  to  $m + 1$  by  $\sigma$ 
5:      $\pi_1 \leftarrow k$ 
6:      $k \leftarrow S_p(k)$ 
7:   end while
8:   ...
9: end function
```

$\triangleright \delta(k, \sigma) = m + 1$

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

$m = 3 \quad \pi_1 = 2 \quad k = 0$



Factor Oracle

Algorithm - Construction

Algorithm 2 Incremental update of Factor Oracle

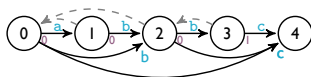
```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
2:   ...
3:   while  $k > -1$  and there is no transition from  $k$  by  $\sigma$  do
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6:      $k \leftarrow S_p(k)$ 
7:   end while
8:   ...
9: end function
```

$\triangleright \delta(k, \sigma) = m + 1$

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

$m = 3 \quad \pi_1 = 0 \quad k = 0$



Factor Oracle

Algorithm - Construction

Algorithm 2 Incremental update of Factor Oracle

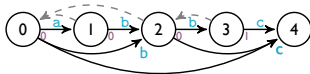
```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
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7:   end while
8:   ...
9: end function
```

$\triangleright \delta(k, \sigma) = m + 1$

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

$m = 3 \quad \pi_1 = 0 \quad k = -1$



Factor Oracle

Algorithm - Construction

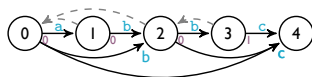
Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
2:   ...
3:   if  $k = -1$  then
4:      $S_{p\sigma} \leftarrow 0$ 
5:      $lrs_{p\sigma} \leftarrow 0$ 
6:   else
7:     ...
8:   end if
9:   ...
10: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

$m = 3 \quad \pi_1 = 0 \quad k = -1$



Factor Oracle

Algorithm - Construction

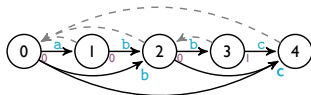
Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter(Oracle( $p = p_1, p_2 \dots p_m$ ),  $\sigma$ )
2:   ...
3:   if  $k = -1$  then
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6:   else
7:     ...
8:   end if
9:   ...
10: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

$m = 3 \quad \pi_1 = 0 \quad k = -1$



Factor Oracle

Algorithm - Construction

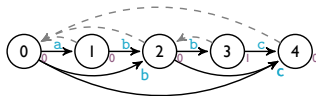
Algorithm 2 Incremental update of Factor Oracle

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1: function AddLetter(Oracle( $p = p_1, p_2 \dots p_m$ ),  $\sigma$ )
2:   ...
3:   if  $k = -1$  then
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6:   else
7:     ...
8:   end if
9:   ...
10: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

$m = 3 \quad \pi_1 = 0 \quad k = -1$



Factor Oracle

Algorithm - Construction

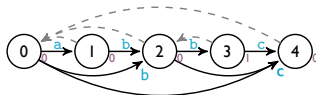
Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )  
2:   Create state  $m + 1$   
3:   Create a new transition from  $m$  to  $m + 1$  labeled by  $\sigma$   $\triangleright \delta(m, \sigma) = m + 1$   
4:    $\pi_1 \leftarrow m$   
5:    $k \leftarrow S_p(m)$   
6:   ...  
7: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 4$



Factor Oracle

Algorithm - Construction

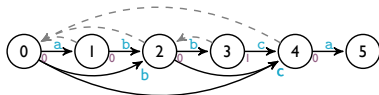
Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )  
2:   Create state  $m + 1$   
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4:    $\pi_1 \leftarrow m$   
5:    $k \leftarrow S_p(m)$   
6:   ...  
7: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 4$



Factor Oracle

Algorithm - Construction

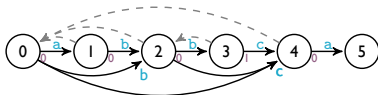
Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter(Oracle( $p = p_1, p_2 \dots p_m$ ),  $\sigma$ )
2:   Create state  $m + 1$ 
3:   Create a new transition from  $m$  to  $m + 1$  labeled by  $\sigma$   $\triangleright \delta(m, \sigma) = m + 1$ 
4:    $\pi_1 \leftarrow m$ 
5:    $k \leftarrow S_p(m)$ 
6:   ...
7: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

$m = 4 \quad \pi_1 = 4 \quad k = 0$



Factor Oracle

Algorithm - Construction

Algorithm 2 Incremental update of Factor Oracle

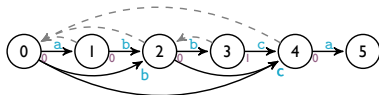
```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
2:   ...
3:   while  $k > -1$  and there is no transition from  $k$  by  $\sigma$  do
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5:      $\pi_1 \leftarrow k$ 
6:      $k \leftarrow S_p(k)$ 
7:   end while
8:   ...
9: end function
```

$\triangleright \delta(k, \sigma) = m + 1$

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

$m = 4 \quad \pi_1 = 4 \quad k = 0$



Factor Oracle

Algorithm - Construction

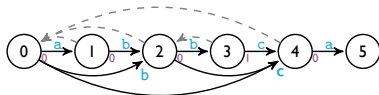
Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )  
2:   ...  
3:   if  $k = -1$  then  
4:     ...  
5:   else  
6:      $S_{p\sigma} \leftarrow$  state that leads the transition from  $k$  by  $\sigma$   
7:      $lrs_{p\sigma} \leftarrow \text{LengthCommonSuffix}(\pi_1, S(m+1) - 1) + 1$   
8:   end if  
9:   ...  
10: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 4$ $\pi_1 = 4$ $k = 0$



Factor Oracle

Algorithm - Construction

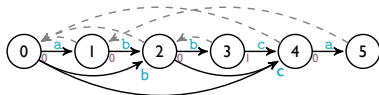
Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )  
2:   ...  
3:   if  $k = -1$  then  
4:     ...  
5:   else  
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8:   end if  
9:   ...  
10: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 4$ $\pi_1 = 4$ $k = 0$



Factor Oracle

Algorithm - Construction

Algorithm 2 Incremental update of Factor Oracle

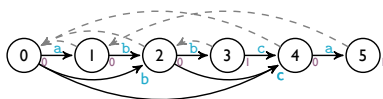
```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
2:   ...
3:   if  $k = -1$  then
4:     ...
5:   else
6:      $S_{p\sigma} \leftarrow$  state that leads the transition from  $k$  by  $\sigma$ 
7:      $lrs_{p\sigma} \leftarrow \text{LengthCommonSuffix}(\pi_1, S(m+1) - 1) + 1$ 
8:   end if
9:   ...
10: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

$m = 4 \quad \pi_1 = 4 \quad k = 0$

$lcs(4, 0) = 0$



Factor Oracle

Algorithm - Construction

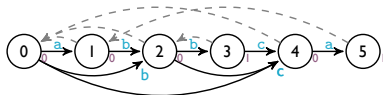
Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )  
2:   Create state  $m + 1$   
3:   Create a new transition from  $m$  to  $m + 1$  labeled by  $\sigma$   $\triangleright \delta(m, \sigma) = m + 1$   
4:    $\pi_1 \leftarrow m$   
5:    $k \leftarrow S_p(m)$   
6:   ...  
7: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 5$



Factor Oracle

Algorithm - Construction

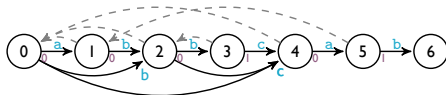
Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )  
2:   Create state  $m + 1$   
3:   Create a new transition from  $m$  to  $m + 1$  labeled by  $\sigma$   $\triangleright \delta(m, \sigma) = m + 1$   
4:    $\pi_1 \leftarrow m$   
5:    $k \leftarrow S_p(m)$   
6:   ...  
7: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 5$



Factor Oracle

Algorithm - Construction

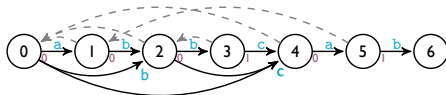
Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )  
2:   Create state  $m + 1$   
3:   Create a new transition from  $m$  to  $m + 1$  labeled by  $\sigma$   $\triangleright \delta(m, \sigma) = m + 1$   
4:    $\pi_1 \leftarrow m$   
5:    $k \leftarrow S_p(m)$   
6:   ...  
7: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

$m = 5 \quad \pi_1 = 5 \quad k = 1$



Factor Oracle

Algorithm - Construction

Algorithm 2 Incremental update of Factor Oracle

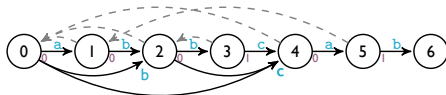
```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
2:   ...
3:   while  $k > -1$  and there is no transition from  $k$  by  $\sigma$  do
4:     Create a new transition from  $k$  to  $m + 1$  by  $\sigma$ 
5:      $\pi_1 \leftarrow k$ 
6:      $k \leftarrow S_p(k)$ 
7:   end while
8:   ...
9: end function
```

$\triangleright \delta(k, \sigma) = m + 1$

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

$m = 5 \quad \pi_1 = 5 \quad k = 1$



Factor Oracle

Algorithm - Construction

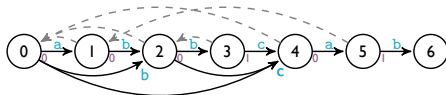
Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )  
2:   ...  
3:   if  $k = -1$  then  
4:     ...  
5:   else  
6:      $S_{p\sigma} \leftarrow$  state that leads the transition from  $k$  by  $\sigma$   
7:      $lrs_{p\sigma} \leftarrow \text{LengthCommonSuffix}(\pi_1, S(m+1) - 1) + 1$   
8:   end if  
9:   ...  
10: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 5$ $\pi_1 = 5$ $k = 1$



Factor Oracle

Algorithm - Construction

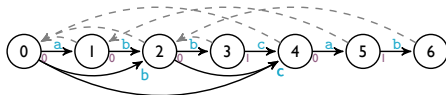
Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
2:   ...
3:   if  $k = -1$  then
4:     ...
5:   else
6:      $S_{p\sigma} \leftarrow$  state that leads the transition from  $k$  by  $\sigma$ 
7:      $lrs_{p\sigma} \leftarrow \text{LengthCommonSuffix}(\pi_1, S(m+1) - 1) + 1$ 
8:   end if
9:   ...
10: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

$m = 5 \quad \pi_1 = 5 \quad k = 1$



Factor Oracle

Algorithm - Construction

Algorithm 2 Incremental update of Factor Oracle

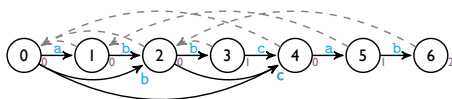
```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
2:   ...
3:   if  $k = -1$  then
4:     ...
5:   else
6:      $S_{p\sigma} \leftarrow$  state that leads the transition from  $k$  by  $\sigma$ 
7:      $lrs_{p\sigma} \leftarrow \text{LengthCommonSuffix}(\pi_1, S(m+1) - 1) + 1$ 
8:   end if
9:   ...
10: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

$m = 5 \quad \pi_1 = 5 \quad k = 1$

$lcs(5, 1) = 1$



Factor Oracle

Algorithm - Construction

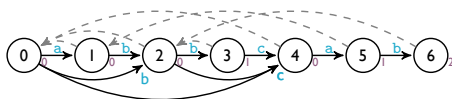
Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )  
2:   Create state  $m + 1$   
3:   Create a new transition from  $m$  to  $m + 1$  labeled by  $\sigma$   $\triangleright \delta(m, \sigma) = m + 1$   
4:    $\pi_1 \leftarrow m$   
5:    $k \leftarrow S_p(m)$   
6:   ...  
7: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 6$



Factor Oracle

Algorithm - Construction

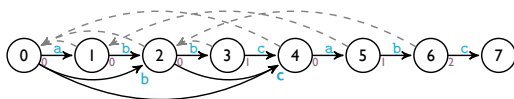
Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )  
2:   Create state  $m + 1$   
3:   Create a new transition from  $m$  to  $m + 1$  labeled by  $\sigma$   $\triangleright \delta(m, \sigma) = m + 1$   
4:    $\pi_1 \leftarrow m$   
5:    $k \leftarrow S_p(m)$   
6:   ...  
7: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 6$



Factor Oracle

Algorithm - Construction

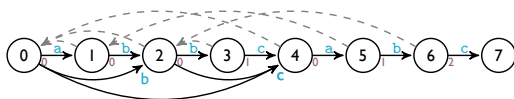
Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
2:   Create state  $m + 1$ 
3:   Create a new transition from  $m$  to  $m + 1$  labeled by  $\sigma$   $\triangleright \delta(m, \sigma) = m + 1$ 
4:    $\pi_1 \leftarrow m$ 
5:    $k \leftarrow S_p(m)$ 
6:   ...
7: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 6$ $\pi_1 = 6$ $k = 2$



Factor Oracle

Algorithm - Construction

Algorithm 2 Incremental update of Factor Oracle

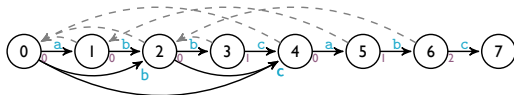
```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
2:   ...
3:   while  $k > -1$  and there is no transition from  $k$  by  $\sigma$  do
4:     Create a new transition from  $k$  to  $m + 1$  by  $\sigma$ 
5:      $\pi_1 \leftarrow k$ 
6:      $k \leftarrow S_p(k)$ 
7:   end while
8:   ...
9: end function
```

$\triangleright \delta(k, \sigma) = m + 1$

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 6$ $\pi_1 = 6$ $k = 2$



Factor Oracle

Algorithm - Construction

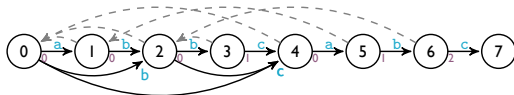
Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
2:   ...
3:   if  $k = -1$  then
4:     ...
5:   else
6:      $S_{p\sigma} \leftarrow$  state that leads the transition from  $k$  by  $\sigma$ 
7:      $lrs_{p\sigma} \leftarrow \text{LengthCommonSuffix}(\pi_1, S(m+1) - 1) + 1$ 
8:   end if
9:   ...
10: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 6$ $\pi_1 = 6$ $k = 2$



Factor Oracle

Algorithm - Construction

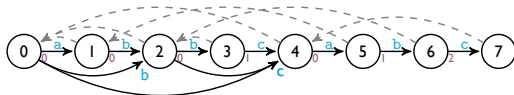
Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
2:   ...
3:   if  $k = -1$  then
4:     ...
5:   else
6:      $S_{p\sigma} \leftarrow$  state that leads the transition from  $k$  by  $\sigma$ 
7:      $lrs_{p\sigma} \leftarrow \text{LengthCommonSuffix}(\pi_1, S(m+1) - 1) + 1$ 
8:   end if
9:   ...
10: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 6$ $\pi_1 = 6$ $k = 2$



Factor Oracle

Algorithm - Construction

Algorithm 2 Incremental update of Factor Oracle

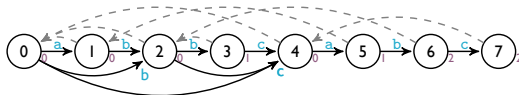
```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
2:   ...
3:   if  $k = -1$  then
4:     ...
5:   else
6:      $S_{p\sigma} \leftarrow$  state that leads the transition from  $k$  by  $\sigma$ 
7:      $lrs_{p\sigma} \leftarrow \text{LengthCommonSuffix}(\pi_1, S(m+1) - 1) + 1$ 
8:   end if
9:   ...
10: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

$m = 6 \quad \pi_1 = 6 \quad k = 2$

$lcs(6, 3) = 1$



Factor Oracle

Algorithm - Construction

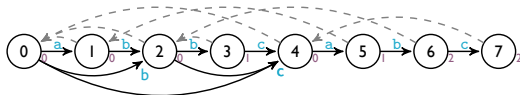
Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )  
2:   Create state  $m + 1$   
3:   Create a new transition from  $m$  to  $m + 1$  labeled by  $\sigma$   $\triangleright \delta(m, \sigma) = m + 1$   
4:    $\pi_1 \leftarrow m$   
5:    $k \leftarrow S_p(m)$   
6:   ...  
7: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 7$



Factor Oracle

Algorithm - Construction

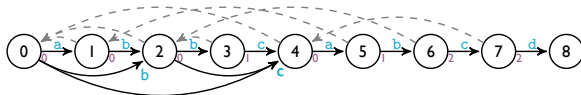
Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )  
2:   Create state  $m + 1$   
3:   Create a new transition from  $m$  to  $m + 1$  labeled by  $\sigma$   $\triangleright \delta(m, \sigma) = m + 1$   
4:    $\pi_1 \leftarrow m$   
5:    $k \leftarrow S_p(m)$   
6:   ...  
7: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 7$



Factor Oracle

Algorithm - Construction

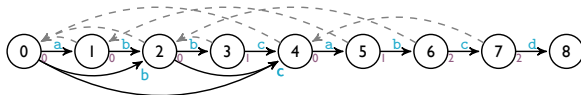
Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )  
2:   Create state  $m + 1$   
3:   Create a new transition from  $m$  to  $m + 1$  labeled by  $\sigma$   $\triangleright \delta(m, \sigma) = m + 1$   
4:    $\pi_1 \leftarrow m$   
5:    $k \leftarrow S_p(m)$   
6:   ...  
7: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 7$ $\pi_1 = 7$ $k = 4$



Factor Oracle

Algorithm - Construction

Algorithm 2 Incremental update of Factor Oracle

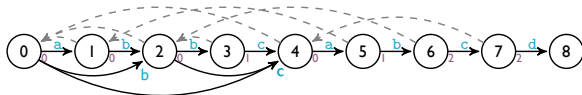
```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
2:   ...
3:   while  $k > -1$  and there is no transition from  $k$  by  $\sigma$  do
4:     Create a new transition from  $k$  to  $m + 1$  by  $\sigma$ 
5:      $\pi_1 \leftarrow k$ 
6:      $k \leftarrow S_p(k)$ 
7:   end while
8:   ...
9: end function
```

$\triangleright \delta(k, \sigma) = m + 1$

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 7$ $\pi_1 = 7$ $k = 4$



Factor Oracle

Algorithm - Construction

Algorithm 2 Incremental update of Factor Oracle

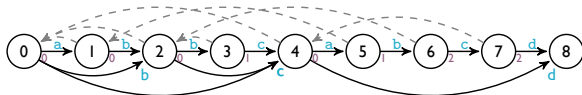
```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
2:   ...
3:   while  $k > -1$  and there is no transition from  $k$  by  $\sigma$  do
4:     Create a new transition from  $k$  to  $m + 1$  by  $\sigma$ 
5:      $\pi_1 \leftarrow k$ 
6:      $k \leftarrow S_p(k)$ 
7:   end while
8:   ...
9: end function
```

$\triangleright \delta(k, \sigma) = m + 1$

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 7$ $\pi_1 = 7$ $k = 4$



Factor Oracle

Algorithm - Construction

Algorithm 2 Incremental update of Factor Oracle

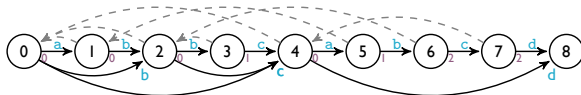
```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
2:   ...
3:   while  $k > -1$  and there is no transition from  $k$  by  $\sigma$  do
4:     Create a new transition from  $k$  to  $m + 1$  by  $\sigma$ 
5:      $\pi_1 \leftarrow k$ 
6:      $k \leftarrow S_p(k)$ 
7:   end while
8:   ...
9: end function
```

$\triangleright \delta(k, \sigma) = m + 1$

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 7$ $\pi_1 = 4$ $k = 4$



Factor Oracle

Algorithm - Construction

Algorithm 2 Incremental update of Factor Oracle

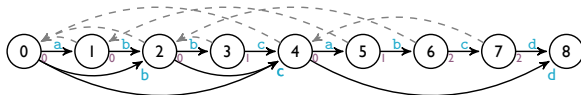
```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
2:   ...
3:   while  $k > -1$  and there is no transition from  $k$  by  $\sigma$  do
4:     Create a new transition from  $k$  to  $m + 1$  by  $\sigma$ 
5:      $\pi_1 \leftarrow k$ 
6:      $k \leftarrow S_p(k)$ 
7:   end while
8:   ...
9: end function
```

$\triangleright \delta(k, \sigma) = m + 1$

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 7$ $\pi_1 = 4$ $k = 0$



Factor Oracle

Algorithm - Construction

Algorithm 2 Incremental update of Factor Oracle

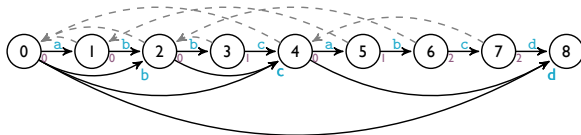
```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
2:   ...
3:   while  $k > -1$  and there is no transition from  $k$  by  $\sigma$  do
4:     Create a new transition from  $k$  to  $m + 1$  by  $\sigma$ 
5:      $\pi_1 \leftarrow k$ 
6:      $k \leftarrow S_p(k)$ 
7:   end while
8:   ...
9: end function
```

$\triangleright \delta(k, \sigma) = m + 1$

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 7$ $\pi_1 = 4$ $k = 0$



Factor Oracle

Algorithm - Construction

Algorithm 2 Incremental update of Factor Oracle

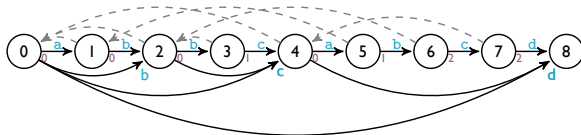
```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
2:   ...
3:   while  $k > -1$  and there is no transition from  $k$  by  $\sigma$  do
4:     Create a new transition from  $k$  to  $m + 1$  by  $\sigma$ 
5:      $\pi_1 \leftarrow k$ 
6:      $k \leftarrow S_p(k)$ 
7:   end while
8:   ...
9: end function
```

$\triangleright \delta(k, \sigma) = m + 1$

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 7$ $\pi_1 = 0$ $k = 0$



Factor Oracle

Algorithm - Construction

Algorithm 2 Incremental update of Factor Oracle

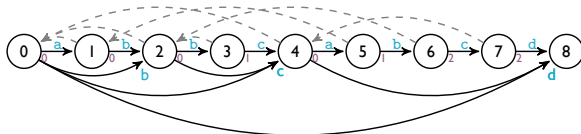
```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
2:   ...
3:   while  $k > -1$  and there is no transition from  $k$  by  $\sigma$  do
4:     Create a new transition from  $k$  to  $m + 1$  by  $\sigma$ 
5:      $\pi_1 \leftarrow k$ 
6:      $k \leftarrow S_p(k)$ 
7:   end while
8:   ...
9: end function
```

$\triangleright \delta(k, \sigma) = m + 1$

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 7$ $\pi_1 = 0$ $k = -1$



Factor Oracle

Algorithm - Construction

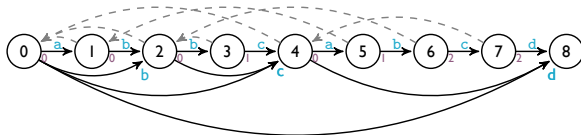
Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
2:   ...
3:   if  $k = -1$  then
4:      $S_{p\sigma} \leftarrow 0$ 
5:      $lrs_{p\sigma} \leftarrow 0$ 
6:   else
7:     ...
8:   end if
9:   ...
10: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 7$ $\pi_1 = 0$ $k = -1$



Factor Oracle

Algorithm - Construction

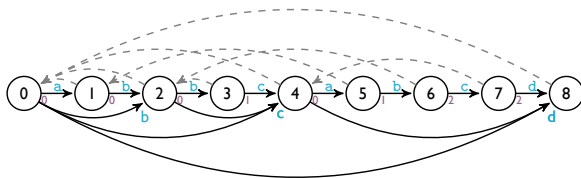
Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
2:   ...
3:   if  $k = -1$  then
4:      $S_{p\sigma} \leftarrow 0$ 
5:      $lrs_{p\sigma} \leftarrow 0$ 
6:   else
7:     ...
8:   end if
9:   ...
10: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

$m = 7 \quad \pi_1 = 0 \quad k = -1$



Factor Oracle

Algorithm - Construction

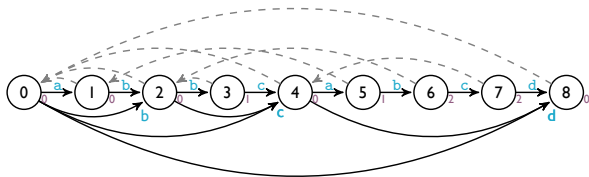
Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
2:   ...
3:   if  $k = -1$  then
4:      $S_{p\sigma} \leftarrow 0$ 
5:      $lrs_{p\sigma} \leftarrow 0$ 
6:   else
7:     ...
8:   end if
9:   ...
10: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 7$ $\pi_1 = 0$ $k = -1$



Factor Oracle

Algorithm - Construction

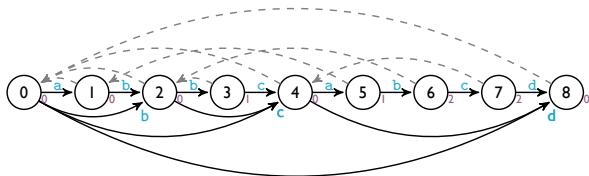
Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )  
2:   Create state  $m + 1$   
3:   Create a new transition from  $m$  to  $m + 1$  labeled by  $\sigma$   $\triangleright \delta(m, \sigma) = m + 1$   
4:    $\pi_1 \leftarrow m$   
5:    $k \leftarrow S_p(m)$   
6:   ...  
7: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 8$



Factor Oracle

Algorithm - Construction

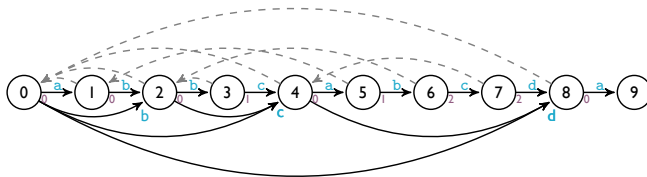
Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )  
2:   Create state  $m + 1$   
3:   Create a new transition from  $m$  to  $m + 1$  labeled by  $\sigma$   $\triangleright \delta(m, \sigma) = m + 1$   
4:    $\pi_1 \leftarrow m$   
5:    $k \leftarrow S_p(m)$   
6:   ...  
7: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 8$



Factor Oracle

Algorithm - Construction

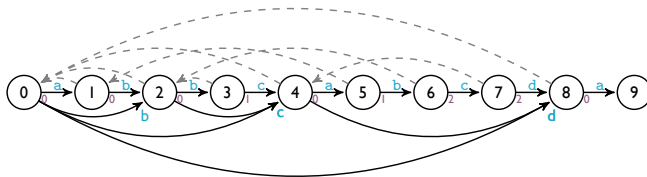
Algorithm 2 Incremental update of Factor Oracle

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1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )  
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4:    $\pi_1 \leftarrow m$   
5:    $k \leftarrow S_p(m)$   
6:   ...  
7: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 8$ $\pi_1 = 8$ $k = 0$



Factor Oracle

Algorithm - Construction

Algorithm 2 Incremental update of Factor Oracle

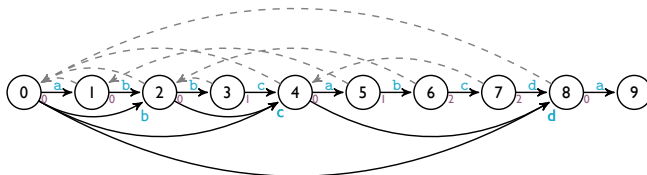
```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
2:   ...
3:   while  $k > -1$  and there is no transition from  $k$  by  $\sigma$  do
4:     Create a new transition from  $k$  to  $m + 1$  by  $\sigma$ 
5:      $\pi_1 \leftarrow k$ 
6:      $k \leftarrow S_p(k)$ 
7:   end while
8:   ...
9: end function
```

$\triangleright \delta(k, \sigma) = m + 1$

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 8$ $\pi_1 = 8$ $k = 0$



Factor Oracle

Algorithm - Construction

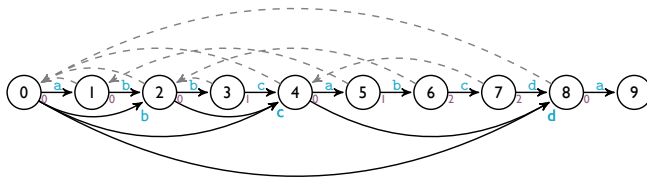
Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
2:   ...
3:   if  $k = -1$  then
4:     ...
5:   else
6:      $S_{p\sigma} \leftarrow$  state that leads the transition from  $k$  by  $\sigma$ 
7:      $lrs_{p\sigma} \leftarrow \text{LengthCommonSuffix}(\pi_1, S(m+1) - 1) + 1$ 
8:   end if
9:   ...
10: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 8$ $\pi_1 = 8$ $k = 0$



Factor Oracle

Algorithm - Construction

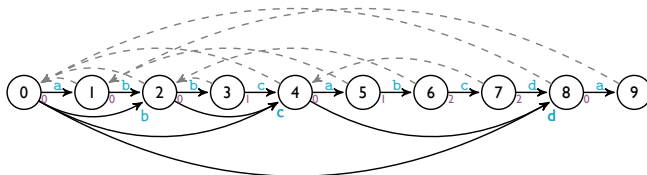
Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
2:   ...
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5:   else
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7:      $lrs_{p\sigma} \leftarrow \text{LengthCommonSuffix}(\pi_1, S(m+1) - 1) + 1$ 
8:   end if
9:   ...
10: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 8$ $\pi_1 = 8$ $k = 0$



Factor Oracle

Algorithm - Construction

Algorithm 2 Incremental update of Factor Oracle

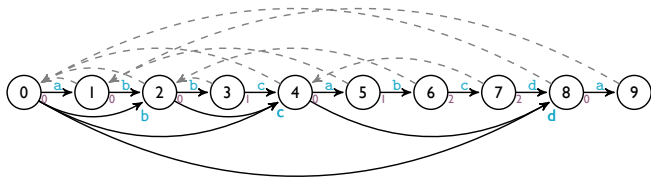
```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
2:   ...
3:   if  $k = -1$  then
4:     ...
5:   else
6:      $S_{p\sigma} \leftarrow$  state that leads the transition from  $k$  by  $\sigma$ 
7:      $lrs_{p\sigma} \leftarrow \text{LengthCommonSuffix}(\pi_1, S(m+1) - 1) + 1$ 
8:   end if
9:   ...
10: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

$m = 8 \quad \pi_1 = 8 \quad k = 0$

$lcs(8, 0) = 0$



Factor Oracle

Algorithm - Construction

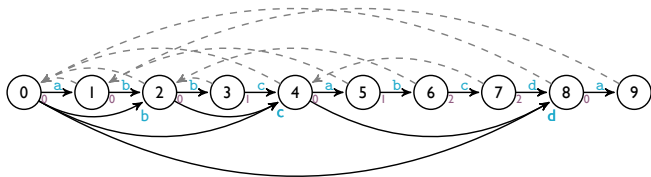
Algorithm 2 Incremental update of Factor Oracle

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1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )  
2:   Create state  $m + 1$   
3:   Create a new transition from  $m$  to  $m + 1$  labeled by  $\sigma$   $\triangleright \delta(m, \sigma) = m + 1$   
4:    $\pi_1 \leftarrow m$   
5:    $k \leftarrow S_p(m)$   
6:   ...  
7: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 9$



Factor Oracle

Algorithm - Construction

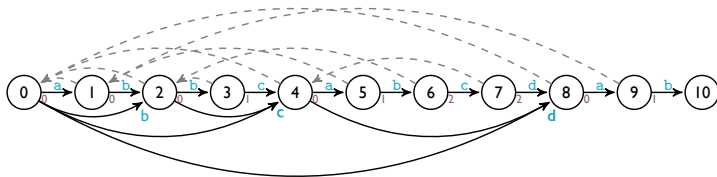
Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )  
2:   Create state  $m + 1$   
3:   Create a new transition from  $m$  to  $m + 1$  labeled by  $\sigma$   $\triangleright \delta(m, \sigma) = m + 1$   
4:    $\pi_1 \leftarrow m$   
5:    $k \leftarrow S_p(m)$   
6:   ...  
7: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 9$



Factor Oracle

Algorithm - Construction

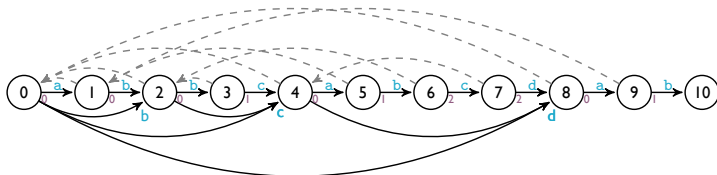
Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )  
2:   Create state  $m + 1$   
3:   Create a new transition from  $m$  to  $m + 1$  labeled by  $\sigma$   $\triangleright \delta(m, \sigma) = m + 1$   
4:    $\pi_1 \leftarrow m$   
5:    $k \leftarrow S_p(m)$   
6:   ...  
7: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 9$ $\pi_1 = 9$ $k = 1$



Factor Oracle

Algorithm - Construction

Algorithm 2 Incremental update of Factor Oracle

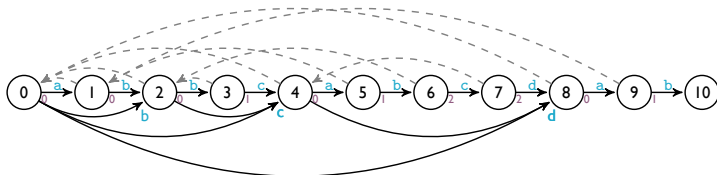
```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
2:   ...
3:   while  $k > -1$  and there is no transition from  $k$  by  $\sigma$  do
4:     Create a new transition from  $k$  to  $m + 1$  by  $\sigma$ 
5:      $\pi_1 \leftarrow k$ 
6:      $k \leftarrow S_p(k)$ 
7:   end while
8:   ...
9: end function
```

$\triangleright \delta(k, \sigma) = m + 1$

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 9$ $\pi_1 = 9$ $k = 1$



Factor Oracle

Algorithm - Construction

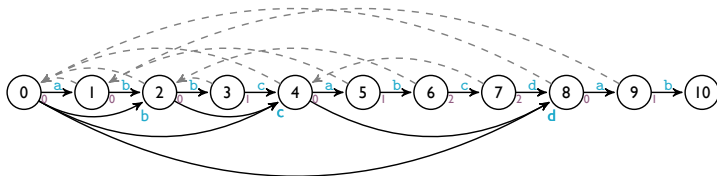
Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
2:   ...
3:   if  $k = -1$  then
4:     ...
5:   else
6:      $S_{p\sigma} \leftarrow$  state that leads the transition from  $k$  by  $\sigma$ 
7:      $lrs_{p\sigma} \leftarrow \text{LengthCommonSuffix}(\pi_1, S(m+1) - 1) + 1$ 
8:   end if
9:   ...
10: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 9$ $\pi_1 = 9$ $k = 1$



Factor Oracle

Algorithm - Construction

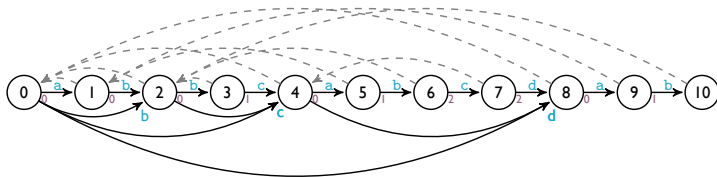
Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
2:   ...
3:   if  $k = -1$  then
4:     ...
5:   else
6:      $S_{p\sigma} \leftarrow$  state that leads the transition from  $k$  by  $\sigma$ 
7:      $lrs_{p\sigma} \leftarrow \text{LengthCommonSuffix}(\pi_1, S(m+1) - 1) + 1$ 
8:   end if
9:   ...
10: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 9$ $\pi_1 = 9$ $k = 1$



Factor Oracle

Algorithm - Construction

Algorithm 2 Incremental update of Factor Oracle

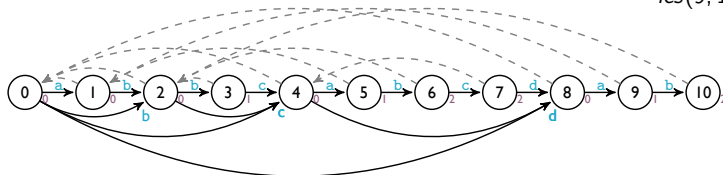
```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
2:   ...
3:   if  $k = -1$  then
4:     ...
5:   else
6:      $S_{p\sigma} \leftarrow$  state that leads the transition from  $k$  by  $\sigma$ 
7:      $lrs_{p\sigma} \leftarrow \text{LengthCommonSuffix}(\pi_1, S(m+1) - 1) + 1$ 
8:   end if
9:   ...
10: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

$m = 9 \quad \pi_1 = 9 \quad k = 1$

$lcs(9, 1) = 1$



Factor Oracle

Algorithm - Construction

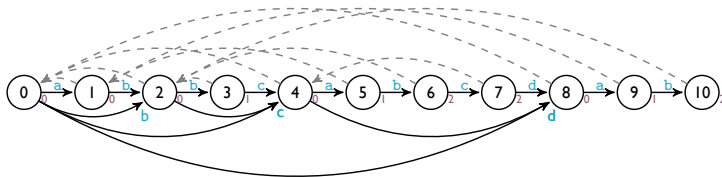
Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )  
2:   Create state  $m + 1$   
3:   Create a new transition from  $m$  to  $m + 1$  labeled by  $\sigma$   $\triangleright \delta(m, \sigma) = m + 1$   
4:    $\pi_1 \leftarrow m$   
5:    $k \leftarrow S_p(m)$   
6:   ...  
7: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 10$



Factor Oracle

Algorithm - Construction

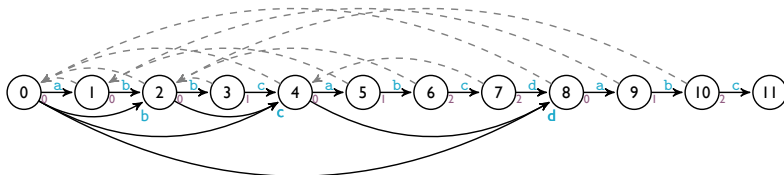
Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )  
2:   Create state  $m + 1$   
3:   Create a new transition from  $m$  to  $m + 1$  labeled by  $\sigma$   $\triangleright \delta(m, \sigma) = m + 1$   
4:    $\pi_1 \leftarrow m$   
5:    $k \leftarrow S_p(m)$   
6:   ...  
7: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 10$



Factor Oracle

Algorithm - Construction

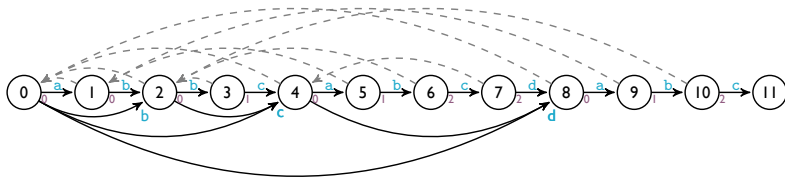
Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )  
2:   Create state  $m + 1$   
3:   Create a new transition from  $m$  to  $m + 1$  labeled by  $\sigma$   $\triangleright \delta(m, \sigma) = m + 1$   
4:    $\pi_1 \leftarrow m$   
5:    $k \leftarrow S_p(m)$   
6:   ...  
7: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 10$ $\pi_1 = 10$ $k = 2$



Factor Oracle

Algorithm - Construction

Algorithm 2 Incremental update of Factor Oracle

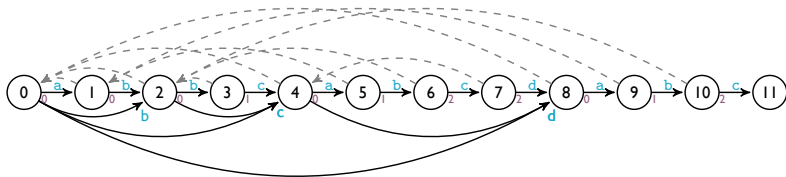
```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
2:   ...
3:   while  $k > -1$  and there is no transition from  $k$  by  $\sigma$  do
4:     Create a new transition from  $k$  to  $m + 1$  by  $\sigma$ 
5:      $\pi_1 \leftarrow k$ 
6:      $k \leftarrow S_p(k)$ 
7:   end while
8:   ...
9: end function
```

$\triangleright \delta(k, \sigma) = m + 1$

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 10 \quad \pi_1 = 10 \quad k = 2$



Factor Oracle

Algorithm - Construction

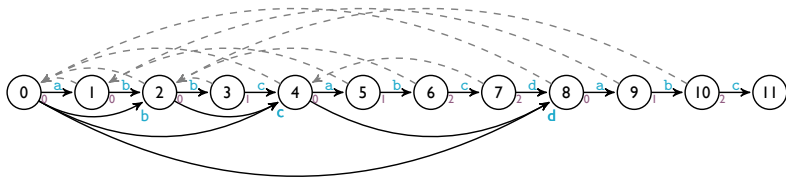
Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
2:   ...
3:   if  $k = -1$  then
4:     ...
5:   else
6:      $S_{p\sigma} \leftarrow$  state that leads the transition from  $k$  by  $\sigma$ 
7:      $lrs_{p\sigma} \leftarrow \text{LengthCommonSuffix}(\pi_1, S(m+1) - 1) + 1$ 
8:   end if
9:   ...
10: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 10$ $\pi_1 = 10$ $k = 2$



Factor Oracle

Algorithm - Construction

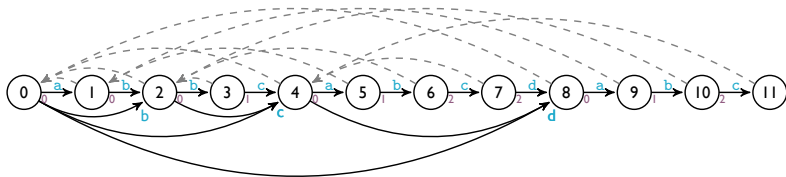
Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
2:   ...
3:   if  $k = -1$  then
4:     ...
5:   else
6:      $S_{p\sigma} \leftarrow$  state that leads the transition from  $k$  by  $\sigma$ 
7:      $lrs_{p\sigma} \leftarrow \text{LengthCommonSuffix}(\pi_1, S(m+1) - 1) + 1$ 
8:   end if
9:   ...
10: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 10$ $\pi_1 = 10$ $k = 2$



Factor Oracle

Algorithm - Construction

Algorithm 2 Incremental update of Factor Oracle

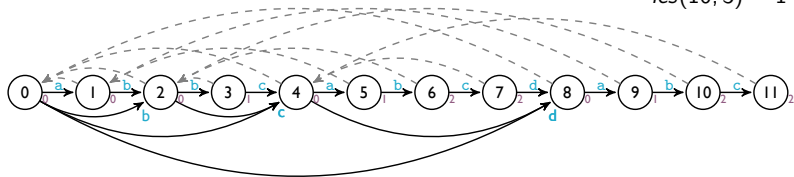
```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )  
2:   ...  
3:   if  $k = -1$  then  
4:     ...  
5:   else  
6:      $S_{p\sigma} \leftarrow$  state that leads the transition from  $k$  by  $\sigma$   
7:      $lrs_{p\sigma} \leftarrow \text{LengthCommonSuffix}(\pi_1, S(m+1) - 1) + 1$   
8:   end if  
9:   ...  
10: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

$m = 10 \quad \pi_1 = 10 \quad k = 2$

$lcs(10, 3) = 1$



Factor Oracle

Algorithm - Construction

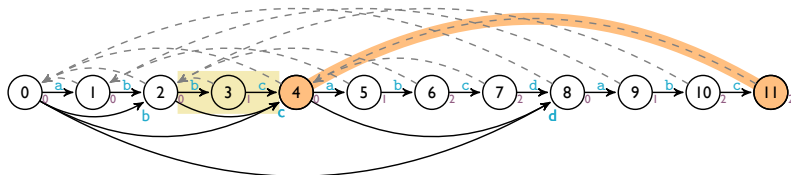
Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
2:   ...
3:   if  $k = -1$  then
4:     ...
5:   else
6:      $S_{p\sigma} \leftarrow$  state that leads the transition from  $k$  by  $\sigma$ 
7:      $lrs_{p\sigma} \leftarrow \text{LengthCommonSuffix}(\pi_1, S(m+1) - 1) + 1$ 
8:   end if
9:   ...
10: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 10$ $\pi_1 = 10$ $k = 2$



Factor Oracle

Algorithm - Construction

Algorithm 2 Incremental update of Factor Oracle

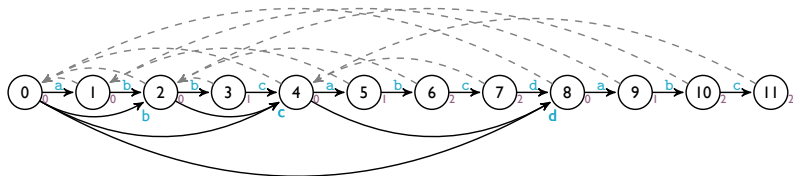
```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
2:   ...
3:    $k \leftarrow \text{FindBetter}(m + 1, p[m + 1 - lrs(m + 1)])$ 
4:   if  $k \neq 0$  then
5:      $lrs_{p\sigma} \leftarrow lrs(m + 1) + 1$ 
6:      $S_{p\sigma} \leftarrow k$ 
7:   end if
8:    $T(S_{p\sigma}) \leftarrow T(S(m + 1)) \cup \{m + 1\}$ 
9:   return  $Oracle(p = p_1 p_2 \dots p_m \sigma)$ 
10: end function
```

$\triangleright T(i) = \{j \mid S(j) = i \wedge i < j \leq m\}$

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 10$ $\pi_1 = 10$ $k = 2$



Factor Oracle

Algorithm - Construction

Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
2:   ...
3:    $k \leftarrow \text{FindBetter}(m + 1, p[m + 1 - lrs(m + 1)])$ 
4:   if  $k \neq 0$  then
5:      $lrs_{p\sigma} \leftarrow lrs(m + 1) + 1$ 
6:      $S_{p\sigma} \leftarrow k$ 
7:   end if
8:    $T(S_{p\sigma}) \leftarrow T(S(m + 1)) \cup \{m + 1\}$ 
9:   return  $Oracle(p = p_1 p_2 \dots p_m \sigma)$ 
10: end function
```

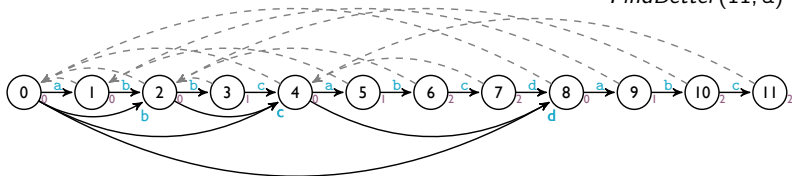
$\triangleright T(i) = \{j \mid S(j) = i \wedge i < j \leq m\}$

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

$m = 10 \quad \pi_1 = 10 \quad k = 7$

$\text{FindBetter}(11, a) = 7$



Factor Oracle

Algorithm - Construction

Algorithm 2 Incremental update of Factor Oracle

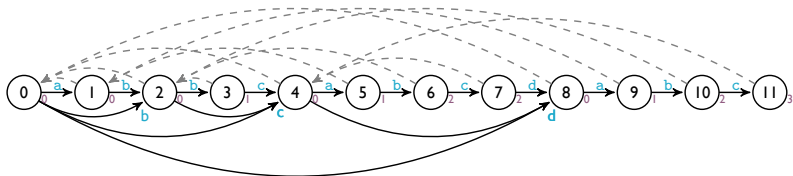
```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
2:   ...
3:    $k \leftarrow \text{FindBetter}(m + 1, p[m + 1 - lrs(m + 1)])$ 
4:   if  $k \neq 0$  then
5:      $lrs_{p\sigma} \leftarrow lrs(m + 1) + 1$ 
6:      $S_{p\sigma} \leftarrow k$ 
7:   end if
8:    $T(S_{p\sigma}) \leftarrow T(S(m + 1)) \cup \{m + 1\}$ 
9:   return  $Oracle(p = p_1 p_2 \dots p_m \sigma)$ 
10: end function
```

$\triangleright T(i) = \{j \mid S(j) = i \wedge i < j \leq m\}$

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 10$ $\pi_1 = 10$ $k = 7$



Factor Oracle

Algorithm - Construction

Algorithm 2 Incremental update of Factor Oracle

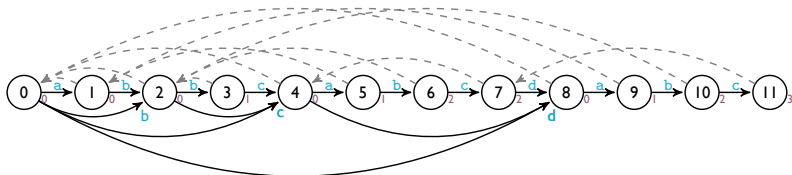
```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
2:   ...
3:    $k \leftarrow \text{FindBetter}(m + 1, p[m + 1 - lrs(m + 1)])$ 
4:   if  $k \neq 0$  then
5:      $lrs_{p\sigma} \leftarrow lrs(m + 1) + 1$ 
6:      $S_{p\sigma} \leftarrow k$ 
7:   end if
8:    $T(S_{p\sigma}) \leftarrow T(S(m + 1)) \cup \{m + 1\}$ 
9:   return  $Oracle(p = p_1 p_2 \dots p_m \sigma)$ 
10: end function
```

$\triangleright T(i) = \{j \mid S(j) = i \wedge i < j \leq m\}$

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 10$ $\pi_1 = 10$ $k = 7$



Factor Oracle

Algorithm - Construction

Algorithm 3 Length Common Suffix Algorithm

```
1: function LengthCommonSuffix( $\pi_1, \pi_2$ )
2:   if  $S(\pi_1) = \pi_2$  then
3:     return  $lrs(\pi_1)$ 
4:   else
5:     while  $S(\pi_1) \neq S(\pi_2)$  do
6:        $\pi_2 \leftarrow S(\pi_2)$ 
7:     end while
8:   end if
9:   return  $\min(lrs(\pi_1), lrs(\pi_2))$ 
10: end function
```

Factor Oracle

Algorithm - Construction

Algorithm 3 Length Common Suffix Algorithm

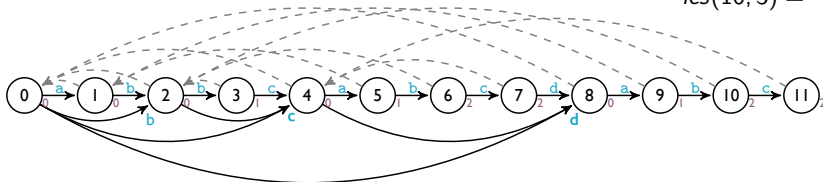
```
1: function LengthCommonSuffix( $\pi_1, \pi_2$ )
2:   if  $S(\pi_1) = \pi_2$  then
3:     return  $lrs(\pi_1)$ 
4:   else
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6:        $\pi_2 \leftarrow S(\pi_2)$ 
7:     end while
8:   end if
9:   return  $\min(lrs(\pi_1), lrs(\pi_2))$ 
10: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

$m = 10 \quad \pi_1 = 10 \quad \pi_2 = 3$

$lcs(10, 3) =$



Factor Oracle

Algorithm - Construction

Algorithm 3 Length Common Suffix Algorithm

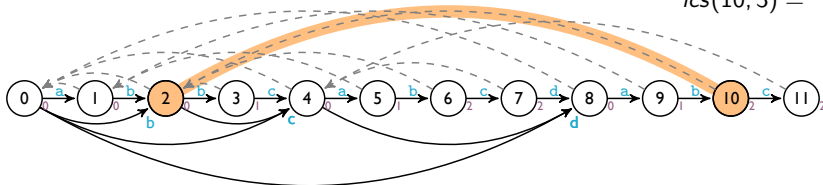
```
1: function LengthCommonSuffix( $\pi_1, \pi_2$ )
2:   if  $S(\pi_1) = \pi_2$  then
3:     return  $lrs(\pi_1)$ 
4:   else
5:     while  $S(\pi_1) \neq S(\pi_2)$  do
6:        $\pi_2 \leftarrow S(\pi_2)$ 
7:     end while
8:   end if
9:   return  $\min(lrs(\pi_1), lrs(\pi_2))$ 
10: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

$m = 10$ $\pi_1 = 10$ $\pi_2 = 3$

$lcs(10, 3) =$



Factor Oracle

Algorithm - Construction

Algorithm 3 Length Common Suffix Algorithm

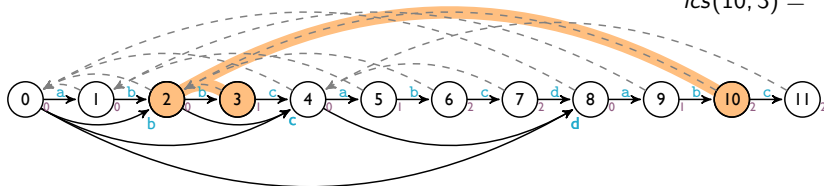
```
1: function LengthCommonSuffix( $\pi_1, \pi_2$ )
2:   if  $S(\pi_1) = \pi_2$  then
3:     return  $lrs(\pi_1)$ 
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8:   end if
9:   return  $\min(lrs(\pi_1), lrs(\pi_2))$ 
10: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

$m = 10 \quad \pi_1 = 10 \quad \pi_2 = 3$

$lcs(10, 3) =$



Factor Oracle

Algorithm - Construction

Algorithm 3 Length Common Suffix Algorithm

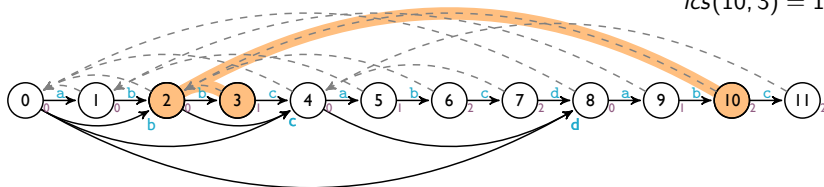
```
1: function LengthCommonSuffix( $\pi_1, \pi_2$ )
2:   if  $S(\pi_1) = \pi_2$  then
3:     return  $lrs(\pi_1)$ 
4:   else
5:     while  $S(\pi_1) \neq S(\pi_2)$  do
6:        $\pi_2 \leftarrow S(\pi_2)$ 
7:     end while
8:   end if
9:   return  $\min(lrs(\pi_1), lrs(\pi_2))$ 
10: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

$m = 10 \quad \pi_1 = 10 \quad \pi_2 = 3$

$lcs(10, 3) = 1$



Factor Oracle

Algorithm - Construction

Algorithm 4 Find Better Algorithm

```
1: function FindBetter( $i, \sigma$ )
2:   for all the elements  $j$  of  $T(i)$  in increasing order do
3:     if  $lrs(j) = lrs(i)$  and  $p[j - lrs(i)] = \sigma$  then
4:       return  $j$ 
5:     end if
6:   end for
7:   return 0
8: end function
```

Factor Oracle

Algorithm - Construction

Algorithm 4 Find Better Algorithm

```
1: function FindBetter( $i, \sigma$ )
2:   for all the elements  $j$  of  $T(S(i))$  in increasing order do
3:     if  $lrs(j) = lrs(i)$  and  $p[j - lrs(i)] = \sigma$  then
4:       return  $j$ 
5:     end if
6:   end for
7:   return 0
8: end function
```

Factor Oracle

Algorithm - Construction

Algorithm 4 Find Better Algorithm

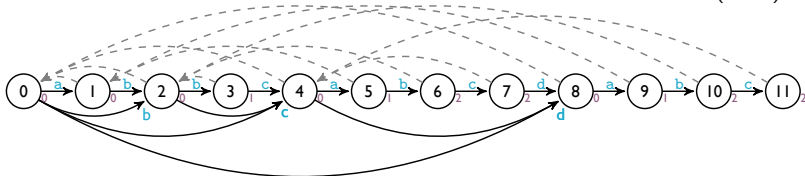
```
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4:       return  $j$   
5:     end if  
6:   end for  
7:   return 0  
8: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

$m = 10 \quad i = 11 \quad \sigma = a$

$\text{FindBetter}(11, a) =$



Factor Oracle

Algorithm - Construction

Algorithm 4 Find Better Algorithm

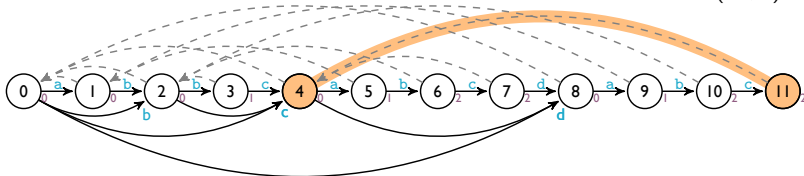
```
1: function FindBetter( $i, \sigma$ )  
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4:       return  $j$   
5:     end if  
6:   end for  
7:   return 0  
8: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

$m = 10 \quad i = 11 \quad \sigma = a$

$\text{FindBetter}(11, a) =$



Factor Oracle

Algorithm - Construction

Algorithm 4 Find Better Algorithm

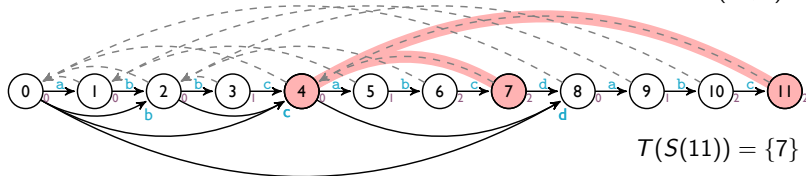
```
1: function FindBetter( $i, \sigma$ )
2:   for all the elements  $j$  of  $T(S(i))$  in increasing order do
3:     if  $lrs(j) = lrs(i)$  and  $p[j - lrs(i)] = \sigma$  then
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6:   end for
7:   return 0
8: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

$m = 10 \quad i = 11 \quad \sigma = a$

$\text{FindBetter}(11, a) =$



Factor Oracle

Algorithm - Construction

Algorithm 4 Find Better Algorithm

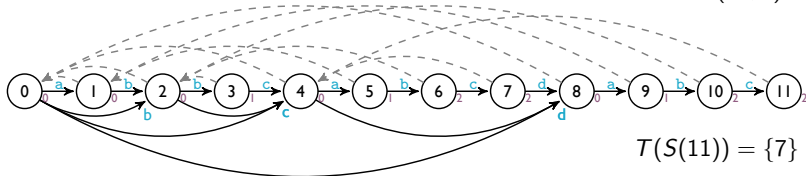
```
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2:   for all the elements  $j$  of  $T(S(i))$  in increasing order do
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5:     end if
6:   end for
7:   return 0
8: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

$m = 10 \quad i = 11 \quad \sigma = a$

$\text{FindBetter}(11, a) =$



$T(S(11)) = \{7\}$

$j = 7$

Factor Oracle

Algorithm - Construction

Algorithm 4 Find Better Algorithm

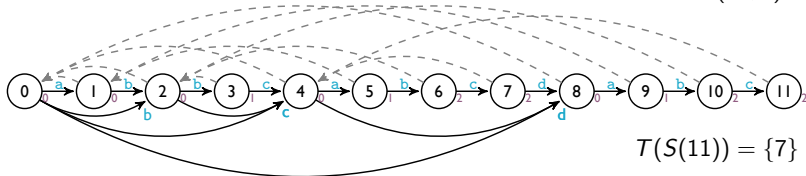
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5:     end if  
6:   end for  
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```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

$m = 10 \quad i = 11 \quad \sigma = a$

$\text{FindBetter}(11, a) =$



$j = 7$

Factor Oracle

Algorithm - Construction

Algorithm 4 Find Better Algorithm

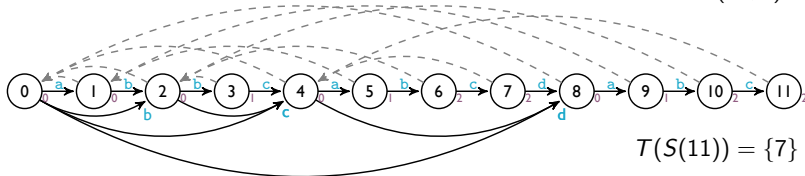
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1: function FindBetter( $i, \sigma$ )
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6:   end for
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8: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

$m = 10 \quad i = 11 \quad \sigma = a$

$\text{FindBetter}(11, a) = 7$



$j = 7$

Factor Oracle

Algorithm - Improvisation

Algorithm 5 FO-Generate function

Require: Oracle $P = p_1, p_2 \dots p_m$ in active state i , a generated sequence v , and a continuation parameter $0 \leq q \leq 1$.

```
1: Generate uniformly distribute random number  $u$ 
2: if  $u < q$  then
3:    $i \leftarrow i + 1$     $v \leftarrow vp_i$ 
4: else
5:   Choose at random a symbol  $\sigma \in \{\sigma_j \mid \delta(S(i), \sigma_j) \neq \perp\}$ 
6:    $i \leftarrow \delta(S(i), \sigma)$     $v \leftarrow v\sigma$ 
7: end if
8: return Sequence  $v$ 
```

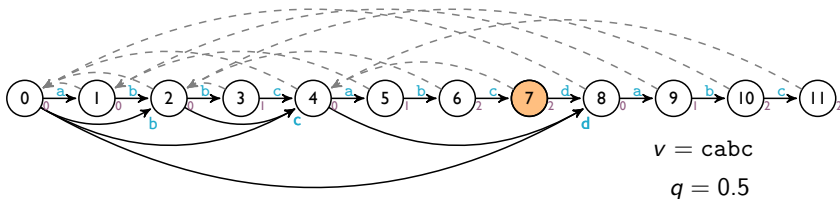
Factor Oracle

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Require: Oracle $P = p_1, p_2 \dots p_m$ in active state i , a generated sequence v , and a continuation parameter $0 \leq q \leq 1$.

- 1: Generate uniformly distribute random number u
 - 2: **if** $u < q$ **then**
 - 3: $i \leftarrow i + 1$ $v \leftarrow vp_i$
 - 4: **else**
 - 5: Choose at random a symbol $\sigma \in \{\sigma_j \mid \delta(S(i), \sigma_j) \neq \perp\}$
 - 6: $i \leftarrow \delta(S(i), \sigma)$ $v \leftarrow v\sigma$
 - 7: **end if**
 - 8: **return** Sequence v
-



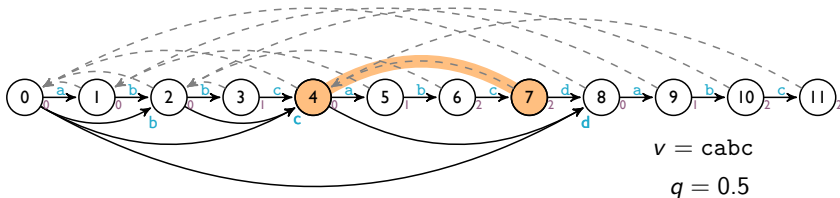
Factor Oracle

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- 7: **end if**
- 8: **return** Sequence v



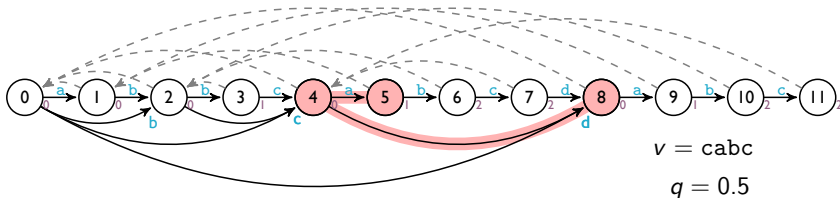
Factor Oracle

Algorithm - Improvisation

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- 1: Generate uniformly distribute random number u
- 2: **if** $u < q$ **then**
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- 4: **else**
- 5: Choose at random a symbol $\sigma \in \{\sigma_j \mid \delta(S(i), \sigma_j) \neq \perp\}$
- 6: $i \leftarrow \delta(S(i), \sigma)$ $v \leftarrow v\sigma$
- 7: **end if**
- 8: **return** Sequence v



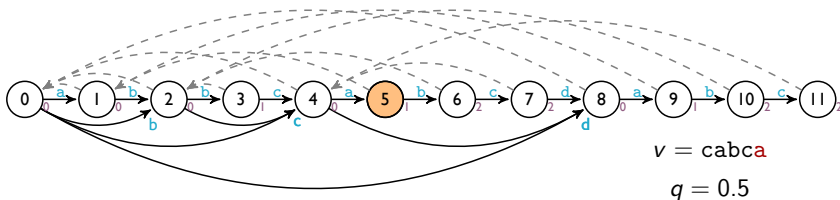
Factor Oracle

Algorithm - Improvisation

Algorithm 5 FO-Generate function

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- 4: **else**
- 5: Choose at random a symbol $\sigma \in \{\sigma_j \mid \delta(S(i), \sigma_j) \neq \perp\}$
- 6: $i \leftarrow \delta(S(i), \sigma)$ $v \leftarrow v\sigma$
- 7: **end if**
- 8: **return** Sequence v



Thank you for your attention! 😊

Factor Oracle for Machine Improvisation

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Université de Bordeaux, LaBRI, UMR 5800
Inria - Bordeaux Sud-Ouest

Septembre 2016



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