

Factor Oracle for Machine Improvisation

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Preliminaries

Preliminaries

Word

A **word** s is a finite sequence $s = s_1 s_2 \dots s_m$ of length $|s| = m$ on a finite alphabet Σ .

$s =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

Factor

A word $x \in \Sigma^*$ is a **factor** of s if and only if s can be written $s = uxv$ with $u, v \in \Sigma^*$. Given integers i, j where $1 \leq i \leq j \leq m$, we denote a *factor* of s as $s[i \dots j] = s_i s_{i+1} \dots s_j$.

$s =$

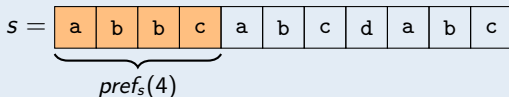
a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

$\underbrace{\hspace{10em}}$
 $s[3, 5]$

Preliminaries

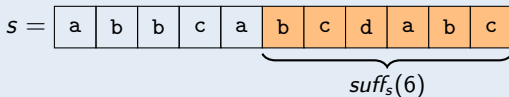
Prefix

A factor x of s is a **prefix** of s if $s = xu$ with $u \in \Sigma^*$. The i th *prefix* of s , denoted $\text{pref}_s(i)$, is the prefix $s[1 \dots i]$.



Suffix

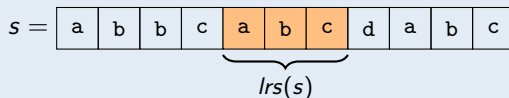
A factor x of s is a **suffix** of s if $s = ux$ with $u \in \Sigma^*$. The i th *suffix* of s , denoted $\text{suff}_s(i)$, is the suffix $s[i \dots m]$.



Preliminaries

Longest Repeated Suffix (LRS)

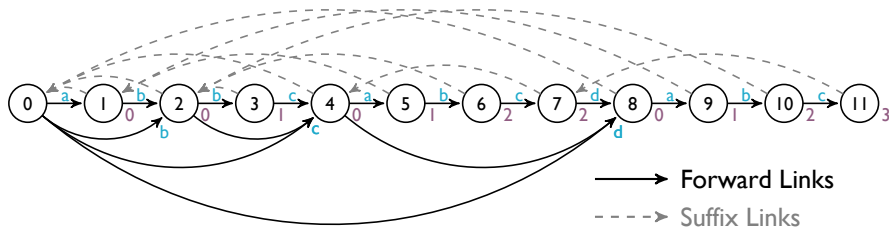
A factor x of s is the **longest repeated suffix** of s if x is a suffix of s and $|x|$ is maximal.



Factor Oracle

Factor Oracle

Overview

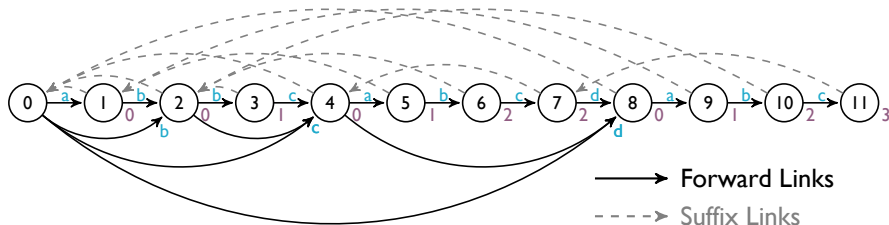


Factor Oracle

The **factor oracle** of a word s of length m is a *deterministic finite automaton* (Q, q_0, F, δ) where $Q = \{0, 1, \dots, m\}$ is the set of states, $q_0 = 0$ is the starting state, $F = Q$ is the set of terminal states and δ is the transition function.

Factor Oracle

Overview

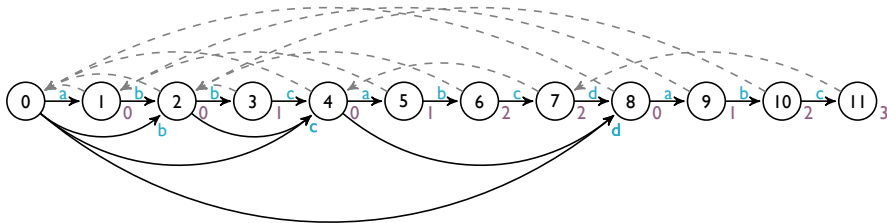


Suffix Link

The **suffix link** of a state i of the factor oracle of a word s , is equal to the state in which the *longest repeated suffix (lrs)* of $s[1 \dots i]$ is recognized.

Factor Oracle

Overview

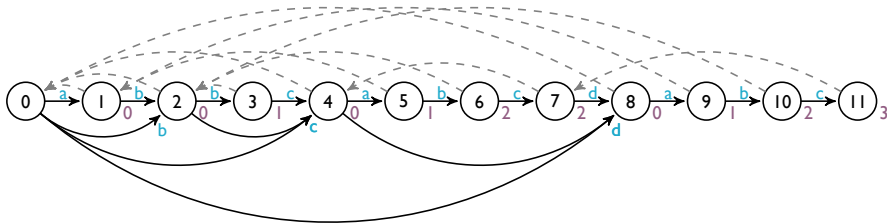


Suffix Links

- $s = \text{abbcababcdabc}$

Factor Oracle

Overview

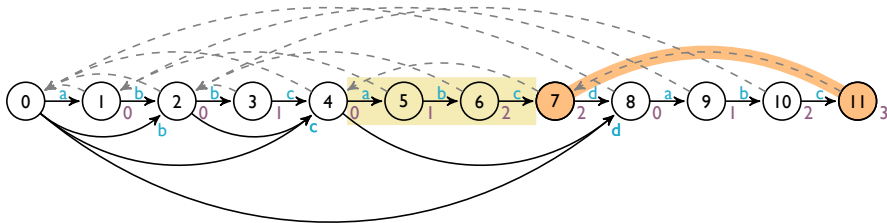


Suffix Links

- $s = \text{abbc}\text{abcdabc}$
- $\text{lrs}(s) = \text{abc}$

Factor Oracle

Overview



Suffix Links

- $s = abbcababcdabc$
- $lrs(s) = abc$
- $S(11) = 7$

Factor Oracle

Algorithm

Algorithm 1 Construction of a Factor Oracle

```
1: function FactorOracle( $p = p_1 p_2 \dots p_m$ )
2:   Create a new oracle  $P$  with an initial state 0
3:    $S_P(0) \leftarrow -1$ 
4:   for  $i \leftarrow 1, m$  do
5:      $\text{Oracle}(p = p_1 p_2 \dots p_i) \leftarrow \text{AddLetter}(\text{Oracle}(p = p_1 p_2 \dots p_{i-1}), p_i)$ 
6:   end for
7:   return  $\text{Oracle}(p = p_1 p_2 \dots p_m)$ 
8: end function
```

Factor Oracle

Algorithm

Algorithm 1 Construction of a Factor Oracle

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6:   end for  
7:   return Oracle( $p = p_1 p_2 \dots p_m$ )  
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```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

Factor Oracle

Algorithm

Algorithm 1 Construction of a Factor Oracle

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$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

(0)

Factor Oracle

Algorithm

Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )  
2:   Create state  $m + 1$   
3:   Create a new transition from  $m$  to  $m + 1$  labeled by  $\sigma$   $\triangleright \delta(m, \sigma) = m + 1$   
4:    $k \leftarrow S_p(m)$   
5:    $\pi_1 \leftarrow m$   
6:   ...  
7: end function
```

slide 1

Factor Oracle

Algorithm

Algorithm 2 Incremental update of Factor Oracle

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```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 0$

0₀

slide 2

Factor Oracle

Algorithm

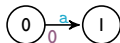
Algorithm 2 Incremental update of Factor Oracle

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```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 0$



slide 3

Factor Oracle

Algorithm

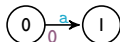
Algorithm 2 Incremental update of Factor Oracle

1: **function** AddLetter($Oracle(p = p_1, p_2 \dots p_m), \sigma$)
2: Create state $m + 1$
3: Create a new transition from m to $m + 1$ labeled by σ $\triangleright \delta(m, \sigma) = m + 1$
4: $k \leftarrow S_p(m)$
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6: ...
7: **end function**

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

$m = 0 \quad k = -1 \quad \pi_1 = 0$



slide 4

Factor Oracle

Algorithm

Algorithm 2 Incremental update of Factor Oracle

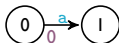
```
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3:   while  $k > -1$  and there is no transition from  $k$  by  $\sigma$  do
4:     Create a new transition from  $k$  to  $m + 1$  by  $\sigma$ 
5:      $\pi_1 \leftarrow k$ 
6:      $k \leftarrow S_p(k)$ 
7:   end while
8:   ...
9: end function
```

$\triangleright \delta(k, \sigma) = m + 1$

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

$m = 0 \quad k = -1 \quad \pi_1 = 0$



slide 5

Factor Oracle

Algorithm

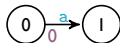
Algorithm 2 Incremental update of Factor Oracle

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5:      $lrs_{p\sigma} \leftarrow 0$ 
6:   else
7:      $S_{p\sigma} \leftarrow$  state that leads the transition from  $k$  by  $\sigma$ 
8:      $lrs_{p\sigma} \leftarrow \text{LengthCommonSuffix}(\pi_1, S(m+1) - 1) + 1$ 
9:   end if
10:  ...
11: end function
```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

$m = 0 \quad k = -1 \quad \pi_1 = 0$



Factor Oracle

Algorithm

Algorithm 2 Incremental update of Factor Oracle

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$p =$

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---	---	---	---	---	---	---	---	---	---	---

$m = 0 \quad k = -1 \quad \pi_1 = 0$



Factor Oracle

Algorithm

Algorithm 2 Incremental update of Factor Oracle

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```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

$m = 0 \quad k = -1 \quad \pi_1 = 0$



Factor Oracle

Algorithm

Algorithm 2 Incremental update of Factor Oracle

1: **function** AddLetter($Oracle(p = p_1, p_2 \dots p_m), \sigma$)
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7: **end function**

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 1$



slide 9

Factor Oracle

Algorithm

Algorithm 2 Incremental update of Factor Oracle

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$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 1$



slide 10

Factor Oracle

Algorithm

Algorithm 2 Incremental update of Factor Oracle

```
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```

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 1$ $k = 0$ $\pi_1 = 1$



slide 11

Factor Oracle

Algorithm

Algorithm 2 Incremental update of Factor Oracle

```
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$\triangleright \delta(k, \sigma) = m + 1$

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---	---	---	---	---	---	---	---	---	---	---

 $m = 1 \quad k = 0 \quad \pi_1 = 1$



slide 12

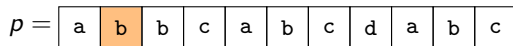
Factor Oracle

Algorithm

Algorithm 2 Incremental update of Factor Oracle

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$\triangleright \delta(k, \sigma) = m + 1$



$$m = 1 \quad k = 0 \quad \pi_1 = 1$$



slide 13

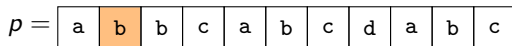
Factor Oracle

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8:   ...
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```

$\triangleright \delta(k, \sigma) = m + 1$



$$m = 1 \quad k = 0 \quad \pi_1 = 0$$



slide 14

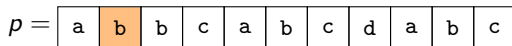
Factor Oracle

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Algorithm 2 Incremental update of Factor Oracle

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$\triangleright \delta(k, \sigma) = m + 1$



$$m = 1 \quad k = -1 \quad \pi_1 = 0$$



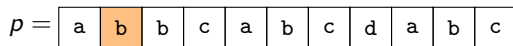
slide 15

Factor Oracle

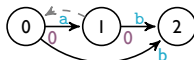
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$$m = 1 \quad k = -1 \quad \pi_1 = 0$$

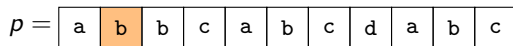


Factor Oracle

Algorithm

Algorithm 2 Incremental update of Factor Oracle

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$$m = 1 \quad k = -1 \quad \pi_1 = 0$$

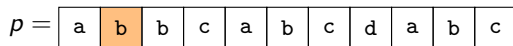


Factor Oracle

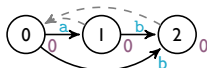
Algorithm

Algorithm 2 Incremental update of Factor Oracle

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1: function AddLetter(Oracle( $p = p_1, p_2 \dots p_m$ ),  $\sigma$ )
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9:   end if
10:  ...
11: end function
```



$$m = 1 \quad k = -1 \quad \pi_1 = 0$$



Factor Oracle

Algorithm

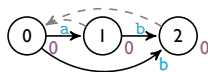
Algorithm 2 Incremental update of Factor Oracle

1: **function** AddLetter($Oracle(p = p_1, p_2 \dots p_m), \sigma$)
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6: ...
7: **end function**

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 2$



slide 19

Factor Oracle

Algorithm

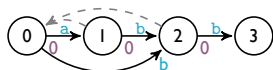
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$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 2$



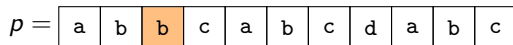
slide 20

Factor Oracle

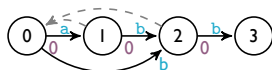
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5: $\pi_1 \leftarrow m$
6: ...
7: **end function**



$$m = 2 \quad k = 0 \quad \pi_1 = 2$$



slide 21

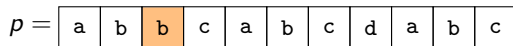
Factor Oracle

Algorithm

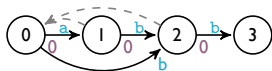
Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
2:   ...
3:   while  $k > -1$  and there is no transition from  $k$  by  $\sigma$  do
4:     Create a new transition from  $k$  to  $m + 1$  by  $\sigma$ 
5:      $\pi_1 \leftarrow k$ 
6:      $k \leftarrow S_p(k)$ 
7:   end while
8:   ...
9: end function
```

$\triangleright \delta(k, \sigma) = m + 1$



$$m = 2 \quad k = 0 \quad \pi_1 = 2$$



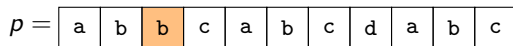
slide 22

Factor Oracle

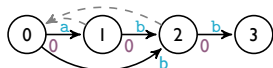
Algorithm

Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
2:   ...
3:   if  $k = -1$  then
4:      $S_{p\sigma} \leftarrow 0$ 
5:      $lrs_{p\sigma} \leftarrow 0$ 
6:   else
7:      $S_{p\sigma} \leftarrow$  state that leads the transition from  $k$  by  $\sigma$ 
8:      $lrs_{p\sigma} \leftarrow \text{LengthCommonSuffix}(\pi_1, S(m+1) - 1) + 1$ 
9:   end if
10:  ...
11: end function
```



$$m = 2 \quad k = 0 \quad \pi_1 = 2$$

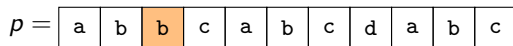


Factor Oracle

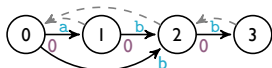
Algorithm

Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
2:   ...
3:   if  $k = -1$  then
4:      $S_{p\sigma} \leftarrow 0$ 
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6:   else
7:      $S_{p\sigma} \leftarrow$  state that leads the transition from  $k$  by  $\sigma$ 
8:      $lrs_{p\sigma} \leftarrow \text{LengthCommonSuffix}(\pi_1, S(m+1) - 1) + 1$ 
9:   end if
10:  ...
11: end function
```



$$m = 2 \quad k = 0 \quad \pi_1 = 2$$

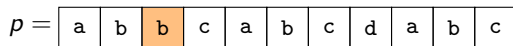


Factor Oracle

Algorithm

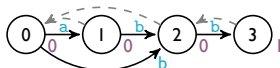
Algorithm 2 Incremental update of Factor Oracle

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7:      $S_{p\sigma} \leftarrow$  state that leads the transition from  $k$  by  $\sigma$ 
8:      $lrs_{p\sigma} \leftarrow \text{LengthCommonSuffix}(\pi_1, S(m+1) - 1) + 1$ 
9:   end if
10:  ...
11: end function
```



$$m = 2 \quad k = 0 \quad \pi_1 = 2$$

$$lcs(2, 1) = 0$$



Factor Oracle

Algorithm

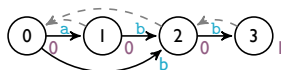
Algorithm 2 Incremental update of Factor Oracle

1: **function** AddLetter($Oracle(p = p_1, p_2 \dots p_m), \sigma$)
2: Create state $m + 1$
3: Create a new transition from m to $m + 1$ labeled by σ $\triangleright \delta(m, \sigma) = m + 1$
4: $k \leftarrow S_p(m)$
5: $\pi_1 \leftarrow m$
6: ...
7: **end function**

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 3$



slide 26

Factor Oracle

Algorithm

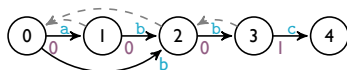
Algorithm 2 Incremental update of Factor Oracle

1: **function** AddLetter($Oracle(p = p_1, p_2 \dots p_m), \sigma$)
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4: $k \leftarrow S_p(m)$
5: $\pi_1 \leftarrow m$
6: ...
7: **end function**

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---

 $m = 3$



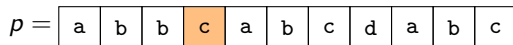
slide 27

Factor Oracle

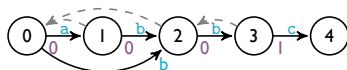
Algorithm

Algorithm 2 Incremental update of Factor Oracle

1: **function** AddLetter($Oracle(p = p_1, p_2 \dots p_m), \sigma$)
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4: $k \leftarrow S_p(m)$
5: $\pi_1 \leftarrow m$
6: ...
7: **end function**



$$m = 3 \quad k = 2 \quad \pi_1 = 3$$



slide 28

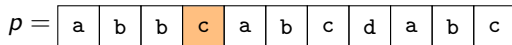
Factor Oracle

Algorithm

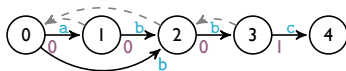
Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
2:   ...
3:   while  $k > -1$  and there is no transition from  $k$  by  $\sigma$  do
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7:   end while
8:   ...
9: end function
```

$\triangleright \delta(k, \sigma) = m + 1$



$$m = 3 \quad k = 2 \quad \pi_1 = 3$$



slide 29

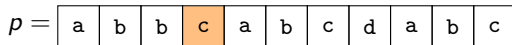
Factor Oracle

Algorithm

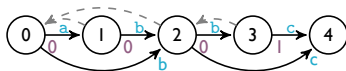
Algorithm 2 Incremental update of Factor Oracle

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8:   ...
9: end function
```

$\triangleright \delta(k, \sigma) = m + 1$



$$m = 3 \quad k = 2 \quad \pi_1 = 3$$



slide 30

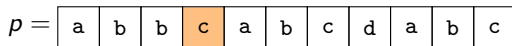
Factor Oracle

Algorithm

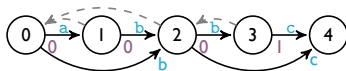
Algorithm 2 Incremental update of Factor Oracle

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```

$\triangleright \delta(k, \sigma) = m + 1$



$$m = 3 \quad k = 2 \quad \pi_1 = 2$$



slide 31

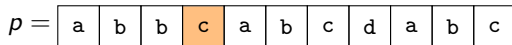
Factor Oracle

Algorithm

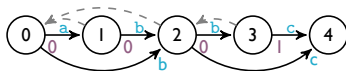
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5:      $\pi_1 \leftarrow k$ 
6:      $k \leftarrow S_p(k)$ 
7:   end while
8:   ...
9: end function
```

$\triangleright \delta(k, \sigma) = m + 1$



$$m = 3 \quad k = 0 \quad \pi_1 = 2$$



slide 32

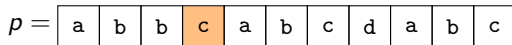
Factor Oracle

Algorithm

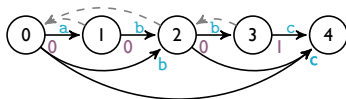
Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
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6:      $k \leftarrow S_p(k)$ 
7:   end while
8:   ...
9: end function
```

$\triangleright \delta(k, \sigma) = m + 1$



$$m = 3 \quad k = 0 \quad \pi_1 = 2$$



slide 33

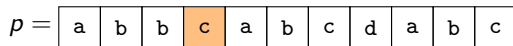
Factor Oracle

Algorithm

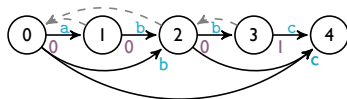
Algorithm 2 Incremental update of Factor Oracle

```
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2:   ...
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7:   end while
8:   ...
9: end function
```

$\triangleright \delta(k, \sigma) = m + 1$



$$m = 3 \quad k = 0 \quad \pi_1 = 0$$



slide 34

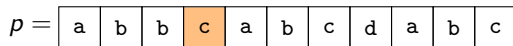
Factor Oracle

Algorithm

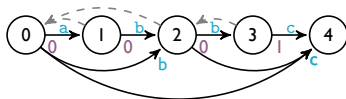
Algorithm 2 Incremental update of Factor Oracle

```
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3:   while  $k > -1$  and there is no transition from  $k$  by  $\sigma$  do
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```

$\triangleright \delta(k, \sigma) = m + 1$



$$m = 3 \quad k = -1 \quad \pi_1 = 0$$



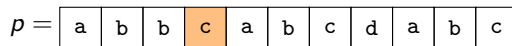
slide 35

Factor Oracle

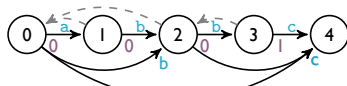
Algorithm

Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
2:   ...
3:   if  $k = -1$  then
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5:      $lrs_{p\sigma} \leftarrow 0$ 
6:   else
7:      $S_{p\sigma} \leftarrow$  state that leads the transition from  $k$  by  $\sigma$ 
8:      $lrs_{p\sigma} \leftarrow \text{LengthCommonSuffix}(\pi_1, S(m+1) - 1) + 1$ 
9:   end if
10:  ...
11: end function
```



$$m = 3 \quad k = -1 \quad \pi_1 = 0$$

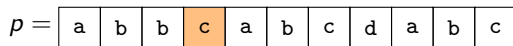


Factor Oracle

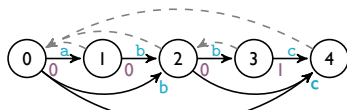
Algorithm

Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
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3:   if  $k = -1$  then
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9:   end if
10:  ...
11: end function
```



$$m = 3 \quad k = -1 \quad \pi_1 = 0$$

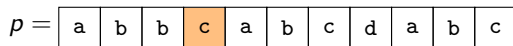


Factor Oracle

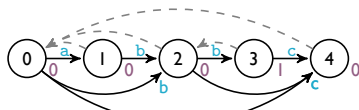
Algorithm

Algorithm 2 Incremental update of Factor Oracle

```
1: function AddLetter( $Oracle(p = p_1, p_2 \dots p_m), \sigma$ )
2:   ...
3:   if  $k = -1$  then
4:      $S_{p\sigma} \leftarrow 0$ 
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9:   end if
10:  ...
11: end function
```



$$m = 3 \quad k = -1 \quad \pi_1 = 0$$



Factor Oracle

Algorithm

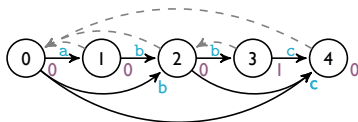
Algorithm 2 Incremental update of Factor Oracle

1: **function** AddLetter($Oracle(p = p_1, p_2 \dots p_m), \sigma$)

2: **end function**

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---



slide 39

Factor Oracle

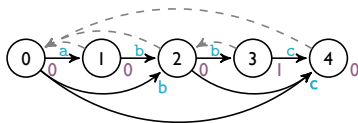
Algorithm

Algorithm 2 Incremental update of Factor Oracle

1: **function** AddLetter($Oracle(p = p_1, p_2 \dots p_m), \sigma$)
2: **end function**

$p =$

a	b	b	c	a	b	c	d	a	b	c
---	---	---	---	---	---	---	---	---	---	---



slide 40

Factor Oracle

Algorithm

Algorithm 3 Find Better Algorithm

```
1: function FindBetter( $i, a$ )
2:   for all the elements  $j$  of  $T(i)$  in increasing order do
3:     if  $lrs(j) = lrs(i)$  and  $p[j - lrs(i)] = a$  then
4:       return  $j$ 
5:     end if
6:   end for
7:   return 0
8: end function
```

Factor Oracle

Algorithm

Algorithm 3 Find Better Algorithm

```
1: function FindBetter( $i, a$ )
2:   for all the elements  $j$  of  $T(\textcolor{red}{S}(i))$  in increasing order do
3:     if  $\text{lrs}(j) = \text{lrs}(i)$  and  $p[j - \text{lrs}(i)] = a$  then
4:       return  $j$ 
5:     end if
6:   end for
7:   return 0
8: end function
```

Factor Oracle

Algorithm

Algorithm 4 Length Common Suffix Algorithm

```
1: function LengthCommonSuffix( $\pi_1, \pi_2$ )
2:   if  $S(\pi_1) = \pi_2$  then
3:     return  $lrs(\pi_1)$ 
4:   else
5:     while  $S(\pi_1) \neq S(\pi_2)$  do
6:        $\pi_2 \leftarrow S(\pi_2)$ 
7:     end while
8:   end if
9:   return  $\min(lrs(\pi_1), lrs(\pi_2))$ 
10: end function
```

Thank you for your attention! 😊

Factor Oracle for Machine Improvisation

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Université de Bordeaux, LaBRI, UMR 5800
Inria - Bordeaux Sud-Ouest

August 2016

