Logs, Surds and Indices

1 Indices

- $a^2+b^2+c^2-ab-bc-ca$
  $=\frac{1}{2}\left[(a-b)^2+(b-c)^2+(c-a)^2\right]$
- $(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$
- $(a+b)^3=a^3+b^3+3ab(a+b)$
  $(a-b)^3=a^3-b^3-3ab(a-b)$
- $a^3+b^3=(a+b)(a^2-ab+b^2)$
  $a^3-b^3=(a-b)(a^2+ab+b^2)$
- $a^3+b^3+c^3-3abc$
  $=(a+b+c)\left(a^2+b^2+c^2-ab-bc-ca\right)$
  $=\frac{1}{2}(a+b+c)\left((a-b)^2+(b-c)^2+(c-a)^2\right)$
- $a^n-b^n=(a-b)\left(a^{n-1}+a^{n-2}b+a^{n-3}b^2+\cdots+b^{n-1}\right)$
  $a^n+b^n=(a+b)\left(a^{n-1}-a^{n-2}b+a^{n-2}b^2-\cdots+ab^{n-2}-b^{n-1}\right)$
- If $\frac{a}{b}=\frac{c}{d}$ then,
  $\frac{a}{b}=\frac{c}{d}=\frac{a+c}{b+d}=\frac{\left(a^2+c^2\right)^{\frac{1}{2}}}{\left(b^2+d^2\right)^{\frac{1}{2}}}=\cdots$

2 Surds

- $a+\sqrt{b}=c+\sqrt{d}\Leftrightarrow a=c\,\&\,b=d$
- $\sqrt{a+\sqrt{b}}=\sqrt{x}+\sqrt{y}\Leftrightarrow\sqrt{a-\sqrt{b}}=\sqrt{x}-\sqrt{y}$
- $\sqrt[3]{a+\sqrt{b}}=x+\sqrt{y}\Leftrightarrow\sqrt[3]{a-\sqrt{b}}=x-\sqrt{y}$

3 Logs

$$b^p=n$$
$$p=\log_b n$$

3.1 Cases:

- If $n<0$, $\log_b n$ is imaginary
- If $n=0$, $\log_b n$ doesn't exist
- If $n>0$, $\log_b n$ exists for $b>0\,\&\,b\neq 1$

3.2 Properties:

- $b^{\log_b n}=n$
- $a^{\log_b n}=n^{\log_b a}$
- $\log_b b=1$, $\log_b 1=0$
- $\log_b n=\frac{1}{\log_n b}$
- $\log_b n=\log_a n\log_b a=\frac{\log_a n}{\log_a b}$
- $\log_b(mn)=\log_b m+\log_b n$
- $\log_b\left(\frac{m}{n}\right)=\log_b m-\log_b n$
- $\log_{b^a} n^x=\frac{x}{a}\log_b n$
  $\log_b n^x=x\log_b n$
  $\log_{b^a} n=\frac{1}{a}\log_b n$
- $\log_b n=\frac{\log n}{\log b}$
- $\log n=\log_e n$
- Common logarithms $\Rightarrow b=10$ (base 10)
  Natural logarithms $\Rightarrow b=e$ (base e)

3.3 Useful Results

Result 1 If $b>1$, then

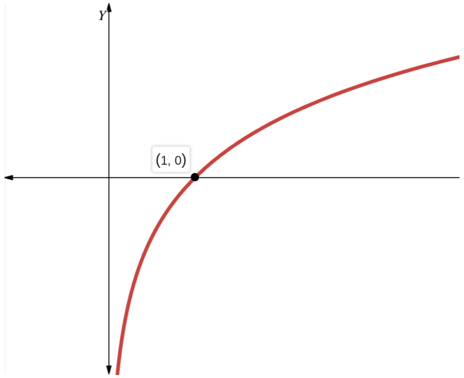

Figure 1: Graph for $y=\log_b n, b>1$

- $\log_b n<0\Rightarrow 0<n<1$
- $\log_b n=0\Rightarrow n=1$
- $\log_b n>0\Rightarrow n>1$
- $x>y\Rightarrow\log_b x>\log_b y$
- $\log_b n$ is an increasing function.

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- $n > b \Rightarrow \log_b n > 1$
- $0 < n < b \Rightarrow \log_b n < 1$
- $n = b \Rightarrow \log_b n = 1$

Result 2 If $0 < b < 1$, then

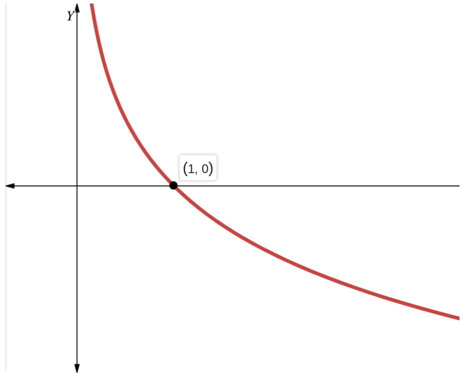

Figure 2: Graph for $y = \log_b n, 0 < b < 1$

- $\log_b n < 0 \Rightarrow n > 1$
- $\log_b n = 0 \Rightarrow n = 1$
- $\log_b n > 0 \Rightarrow 0 < n < 1$
- $x > y \Rightarrow \log_b x < \log_b y$
- $\log_b n$ is an decreasing function.
- $0 < n < b \Rightarrow \log_b n > 1$
- $n = b \Rightarrow \log_b n = 1$
- $n > b \Rightarrow \log_b n < 1$

4 Componento and Dividendo

If $\frac{p}{q} = \frac{a}{b}$, then:

$$\frac{p-q}{p+q} = \frac{a-b}{a+b} \text{ or, } \frac{p+q}{p-q} = \frac{a+b}{a-b}$$

$$\frac{q-p}{q+p} = \frac{b-a}{b+a} \text{ or, } \frac{q+p}{q-p} = \frac{b+a}{b-a}$$

5 Rule of Cross Multiplication

Consider Equations:
$a_1x + b_1y + c_1z = 0$ and $a_2x + b_2y + c_2z = 0$

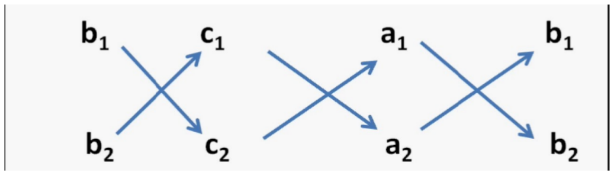

Figure 3: Cross Products for $x, y\, and\, z$

$$\frac{x}{b_1c_2 - c_1b_2} = \frac{y}{c_1a_2 - a_1c_2} = \frac{z}{a_1b_2 - b_1a_2}$$

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