

# Tanget and Normal

## 1 Tangent And Normal

### 1.1 Introuction

Let equation of a curve be  $y = f(x)$ .

And its first derevative be  $\frac{dy}{dx} = \frac{d}{dx}(f(x))$ .

**Slope of Tangent:**

- at point P:  $\frac{dy}{dx}$
- parallel to  $x$ -axis:  $\frac{dy}{dx}(x_1, y_1) = 0$
- parallel to  $y$ -axis:  $\frac{dy}{dx}(x_1, y_1) = \infty$

**Slope of Normal:** at point P =  $\frac{-1}{\frac{dy}{dx}}$  or  $\frac{-dx}{dy}$

### 1.2 Equations

$y - y_1 = m(x - x_1)$  where  $(x_1, y_1)$  is the point of tangent and normal.

**For Tangent:**  $y - y_1 = \left(\frac{dy}{dx}\right)_P (x - x_1)$

**For Normal:**  $y - y_1 = \frac{-1}{\left(\frac{dy}{dx}\right)_P} (x - x_1)$

## 2 Angle between two curves

### 2.1 Angle of Intersection

let  $C_1 = y = f(x)$  and  $C_2 = y = g(x)$ .

$PT_1$  = tangent to  $C_1$  and  $PT_2$  = tangent to  $C_2$ .

Then,

$$m_1 = \tan \psi_1 = \text{slope of tangent of } y = f(x) \text{ at } P = \left(\frac{dy}{dx}\right)_{C_1}$$

$$m_2 = \tan \psi_2 = \text{slope of tangent of } y = g(x) \text{ at } P = \left(\frac{dy}{dx}\right)_{C_2}$$

$$\phi = \psi_1 - \psi_2$$

$$\text{or, } \tan \phi = \frac{\tan \psi_1 - \tan \psi_2}{1 - \tan \psi_1 \tan \psi_2}$$

$$\text{or, } \tan \phi = \frac{\left(\frac{dy}{dx}\right)_{C_1} - \left(\frac{dy}{dx}\right)_{C_2}}{1 + \left(\frac{dy}{dx}\right)_{C_1} \left(\frac{dy}{dx}\right)_{C_2}}$$

**Note:** The other angle between tangents =  $180^\circ - \theta$

### 2.2 Orthogonal Curves

## 3 Subtangent and Subnormal