Trigonomartric Basics

$\pi^r = 180^\circ$

$$\sin\theta = \frac{1}{\csc\theta} = \frac{P}{H}$$
$$\cos\theta = \frac{1}{\sec\theta} = \frac{B}{H}$$
$$\tan\theta = \frac{1}{\cot\theta} = \frac{P}{B}$$

Identities

$\sin^2\theta + cos^2\theta = 1$
$1 + \tan^2\theta = \sec^2\theta$
$1 + \cot^2\theta = \csc^2\theta$

$$\sec\theta + \tan\theta = \frac{1}{\sec\theta - \tan\theta}$$
$$\csc\theta + \cot\theta = \frac{1}{\csc\theta - \cot\theta}$$

Angle
for $< 360 \quad 580 = (580 - 360) = 220$
Aaja Sanam Teri Cosam(ASTC)
Allied Angle

$180 \pm \theta$ $260 \pm \theta$	$90 \pm \theta$ $270 \pm \theta$
$\sin \to \sin$ $\cos \to \cos$ $\tan \to \tan$	$\sin \leftrightarrow \cos$ $\tan \leftrightarrow \cot$ $\sec \leftrightarrow \csc$

$+/-$ depents on original angle

tan and cot $90^\circ - \theta$
$\tan\theta\tan(90-\theta) = 1$
$\cot\theta\cot(90-\theta) = 1$

Min/Max Value
$-1 \leqslant \sin\theta/\cos\theta \leqslant 1$
$-\infty \leq \tan\theta/\cot\theta \leq \infty$
$-1 \geqslant \sec\theta/\csc\theta \geqslant 1$

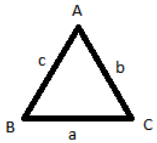
$0 \leqslant \sin^2\theta/\cos^2\theta \leqslant 1$

For eq : $a\sin\theta + b\cos\theta$
min/ max : $\pm\sqrt{a^2+b^2}$

-ve angles
$\sin(-\theta) = -\sin\theta$
$\cos(-\theta) = \cos\theta$
$\tan(-\theta) = -\tan\theta$
NOTE : same for reciprocal

Trigonometry in Geometry

Area of Triangle



$$\text{area} = \frac{1}{2}bc\sin A$$
$$\text{area} = \frac{1}{2}ac\sin B$$
$$\text{area} = \frac{1}{2}ab\sin C$$

Sin Rules

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

Cos Rules : {Similar for A B C }

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Length of Segment

$$\frac{2\pi r\theta}{360^\circ} + 2r\sin(\theta/2)$$

Area of parlleogram : $ab\sin\theta$

Sum and Difference of Angles
$\sin(A+B) = \sin A\cos B + \cos A\sin B$
$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$
$$\cot(A+B) = \frac{\cot A\cot B - 1}{\cot B + \cot A}$$

Double Angle

$$\begin{aligned}\sin(2A) &= 2\sin A\cos A \\ &= \frac{2\tan A}{1+\tan^2 A}\end{aligned}$$

$$\begin{aligned}\cos(2A) &= \cos^2 A + \sin^2 A \\ &= 1 - 2\sin^2 A \\ &= 2\cos^2 A - 1 \\ &= \frac{1-\tan^2 A}{1+\tan^2 A}\end{aligned}$$

$$\tan(2A) = \frac{2\tan A}{1-\tan^2 A}$$

Triple Angle

$$\sin(3x) = 3\sin x - 4\sin^3 x$$
$$\cos(3x) = 4\cos^3 x - 3\cos x$$
$$\tan(3A) = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$$

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Value of some angle

	0 or 2π	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
	0	30	45	60	90
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	$N.D$

Sum into Product

$$\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$$

$$\sin x - \sin y = 2\sin\frac{x-y}{2}\cos\frac{x+y}{2}$$

$$\cos x + \cos y = + 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$$

$$\cos x - \cos y = - 2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$$

$$\tan x \pm \tan y = \frac{\sin(x \pm y)}{\cos x \cos y}$$

Product into Sum

$$2\sin x\cos y = \sin(x+y) + \sin(x-y)$$
$$2\cos x\sin y = \sin(x+y) - \sin(x-y)$$
$$2\cos x\cos y = \cos(x+y) + \cos(x+y)$$
$$2\sin x\sin y = \cos(x-y) - \cos(x+y)$$

60° **Formula**

$$\sin x\sin(60-x)\sin(60+x) = \frac{1}{4}\sin 3x$$

$$\cos x\cos(60-x)\cos(60+x) = \frac{1}{4}\cos 3x$$

$$\tan x\tan(60-x)\tan(60+x) = \tan 3x$$

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