Tanget and Normal

1 Tangent And Normal

1.1 Introuction

Let equation of a curve be $y = f(x)$.

And its first derevative be $\dfrac{dy}{dx} = \dfrac{d}{dx}(f(x))$.

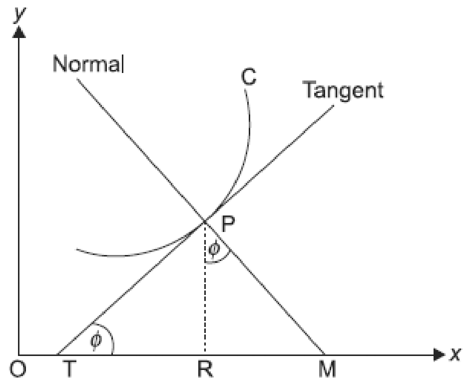

Figure 1: Tangent and Normal

Slope of Tangent:

- at point P: $\dfrac{dy}{dx}$
- parallel to x-axis: $\dfrac{dy}{dx}(x_1, y_1) = 0$
- parallel to y-axis: $\dfrac{dy}{dx}(x_1, y_1) = \infty$

Slope of Normal: at point P $= \dfrac{-1}{\frac{dy}{dx}}$ or $\dfrac{-dx}{dy}$

1.2 Equations

$y - y_1 = m(x - x_1)$ where (x_1, y_1) is the point of tangent and normal.

For Tangent: $y - y_1 = \left(\dfrac{dy}{dx}\right)_P (x - x_1)$

For Normal: $y - y_1 = \dfrac{-1}{\left(\frac{dy}{dx}\right)_P}(x - x_1)$

2 Angle between two curves

2.1 Angle of Intersection

let $C_1 = y = f(x)$ and $C_2 = y = (x)$.

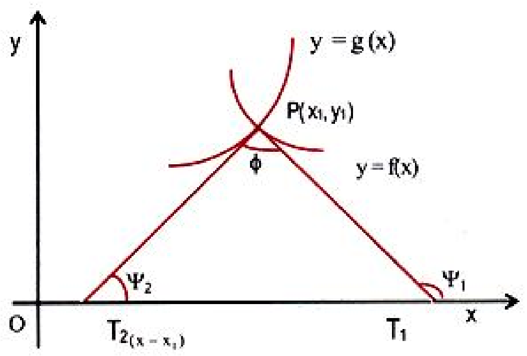

Figure 2: Angle between Curves

PT_1 = tangent to C_1 and PT_2 = tangent to C_2.
Then,

$m_1 = \tan\psi_1$ = slope of tangent of $y = f(x)$ at $P = \left(\dfrac{dy}{dx}\right)_{C_1}$

$m_2 = \tan\psi_2$ = slope of tangent of $y = g(x)$ at $P = \left(\dfrac{dy}{dx}\right)_{C_2}$

$\phi = \psi_1 - \psi_2$

or, $\tan\phi = \dfrac{\tan\psi_1 - \tan\psi_2}{1 - \tan\psi_1 \tan\psi_2}$

or, $\tan\phi = \dfrac{\left(\frac{dy}{dx}\right)_{C_1} - \left(\frac{dy}{dx}\right)_{C_2}}{1 + \left(\frac{dy}{dx}\right)_{C_1}\left(\frac{dy}{dx}\right)_{C_2}}$

Note: The other angle between tangents $= 180^\circ - \theta$

2.2 Orthogonal Curves

If angle of Intersection of two curves is a right angle, the curves are orthogonal curves. **i.e.** $\phi = \dfrac{\pi}{2}$

$\therefore m_1 m_2 = -1 \Rightarrow \left(\dfrac{dy}{dx}\right)_{C_1} \cdot \left(\dfrac{dy}{dx}\right)_{C_2} = -1$

3 Subtangent and Subnormal

Let $y = f(x)$ be a curve. Let $\dfrac{dy}{dx} = f'(x)$ be the slope.

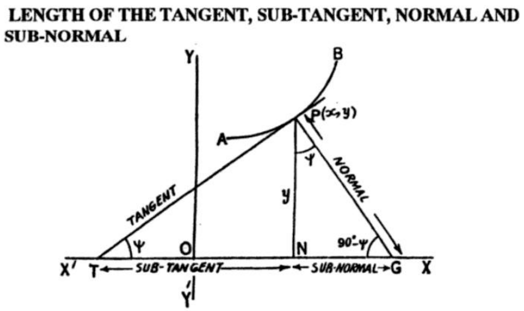

Figure 3: Various lengths

Let the tangent and normal touch x-axis at point T and N respectively. If G is a perpendicular to P, then:

- Length of subtangent (TG) $= |y\cot\psi| = \left|\dfrac{y}{\frac{dy}{dx}}\right|$
- Length of subnormal (NG) $= |y\tan\psi| = \left|y\dfrac{dy}{dx}\right|$
- If PT makes an angle ψ with x-axis, then $\tan\psi = \dfrac{dy}{dx}$
- Length of tangent (PT) $= |y\, cosec\,\psi|$

  $= |y\sqrt{1+\cot^2\psi}|$

  $= \left| y \dfrac{\sqrt{1+\left(\frac{dy}{dx}\right)^2}}{\frac{dy}{dx}} \right|$
- Length of Normal (PN) $= |y\sec\psi|$

  $= |y\sqrt{1+\tan^2\psi}|$

  $= \left| y\sqrt{1+\left(\dfrac{dy}{dx}\right)^2} \right|$