

Maxima and Minima

1 In Open Interval

Let $y = f(x)$. The algorithm of finding maxima or minima is:

- I Find $\frac{dy}{dx} = f'(x)$
- II Put $f'(x) = 0$ and solve for $x = c_1, c_2, c_3 \dots c_n$. These are possible points for maxima or minima.
- III Find $f''(x)$ or second derivative. Consider $x = c_1$.
 - If $f''(x) < 0 \Rightarrow$ Maxima
 - If $f''(x) > 0 \Rightarrow$ Minima
 - If $f''(x) = 0 \Rightarrow$ Find $f'''(x)$ or third derivative.
- IV If $f'''(x) \neq 0$, neither maxima nor minima. Check for $x = c_2, c_3 \dots c_n$
- V If $f'''(x) = 0$, find f^{iv} or fourth derivative.
- VI Consider $x = c_2$.
 - If $f^{iv}(x) < 0 \Rightarrow$ Maxima
 - If $f^{iv}(x) > 0 \Rightarrow$ Minima
 - If $f^{iv}(x) = 0 \Rightarrow$ Find $f^v(x)$ or fifth derivative.
- VII This process goes on.

2 In Closed Interval (Local Maxima and Minima)

Let $y = f(x)$ defined on $[a, b]$. The Algorithm of finding Local Maxima or Minima is:

- I Find $\frac{dy}{dx} = f'(x)$
- II Put $f'(x) = 0$ and solve for $x = c_1, c_2, c_3 \dots c_n$
- III Put Values in $f(x)$ and obtain values $f(a), f(c_1), f(c_2), \dots f(c_n), f(b)$
- IV The maximum and minimum values in the above list are Local maxima and minima for the function $y = f(x)$ in the range $[a, b]$ and the corresponding value $a \dots c_n \dots b$ is the point of local maxima or local minima.