Arithmetic Progression

General term : $a_n = a + (n-1)d$

n^{th} **term from end :** $a + (m-n)d$

where n^{th} term from end $= (m-n+1)$ from start

if A,B,C are in AP: $2B = A + C$

$a_1 + a_n = a_2 + a_{n-1}$
$2a_n = a_{n+k} + a_{n-k}$

selecting middle term of AP:

ODD $\Rightarrow -a+$ CD $\rightarrow d$
EVEN $\Rightarrow (a-d)(a+d)$ CD $\rightarrow 2d$

Sum of AP :

with common difference $S_n = \frac{n}{2}\{2a + (n-1)d\}$

with last term $S_n = \frac{n}{2}\{a + l\}$

results of sum of AP :

- Seq is AP $\rightarrow$ if sum of n terms form $An^2 + Bn$
- if ratio of sum is given, then ratio of n^{th} term is
  $\rightarrow$ replace n by $2n-1$
- if ratio of n^{th} term is given, then ratio of sum is
  $\rightarrow$ replace n by $\frac{n+1}{2}$

Geometric Progression

General terms : $a_n = ar^{r-1}$

n^{th} **term from end :** $a_n = ar^{m-n}$

using last term : $a_n = l\left(\frac{1}{r}\right)^{n-1}$

Properties of GP :

- GP divided/multiplied by constant, stays GP
- reciprocal of GP, is GP
- $a_1a_n = a_2a_{n-1} = a_ka_{n-k+1}$
- $b^2 = ac$
- for a GP $a_1, a_2 \ldots a_n$
  $\Rightarrow GM = (a_1a_2\ldots a_n)^{\frac{1}{n}}$
- if $a_1, a_2 \ldots a_n \Rightarrow$ GP
  $\log a_1, \log a_2 \ldots \log a_n \Rightarrow$ AP and vice-versa

results of sum of GP :

$$S_n = \begin{cases} a\left(\dfrac{1-r^n}{1-r}\right), & r \neq 1 \\ na, & r \end{cases}$$

$$S_n = \frac{a - lr}{1-r} = \frac{lr - a}{r-1}$$

$$S_\infty = \begin{cases} \dfrac{a}{1-r} & -1 < r < 1 \end{cases}$$

Harmonic Progression

if $a_1, a_2 \ldots a_n$ are in HP

then $\frac{1}{a_1}, \frac{1}{a_2} \ldots \frac{1}{a_n}$ are in AP

common difference : $d = \frac{1}{a_2} - \frac{1}{a_1}$

general term : $a_n = \dfrac{1}{\frac{1}{a_1} + (n-1)d}$

Harmonic Mean(H)

- $H = \frac{2ab}{a+b}$
- $a, H_1, H_2 \ldots, H_n, b$ are in HP

  then $\frac{1}{a}, \frac{1}{H_1}, \frac{1}{H_2} \ldots, \frac{1}{H_n}, \frac{1}{b}$ are in AP

  $\rightarrow \frac{1}{b} = (n+2)^{th}$

  $\rightarrow \frac{1}{b} = \frac{1}{a} + (n+1)D \quad D = \frac{a-b}{(n+1)ab}$

Arithmetico-geometric Sequence

$a, (a+d)r, (a+2d)r^2, \ldots$ is a A.G. sequence

n^{th} **term :** $a_n = \{a + (n-1)d\}.r^{n-1}$

sum of ∞ term : $S_\infty = \frac{a}{1-r} + \frac{d.r}{(1-r)^2}$

Sum of Some Sequence

first n natural no : $\frac{n(n+1)}{2}$

square of first n no : $\frac{n(n+1)(2n+1)}{6}$

cube of first n no : $\left\{\frac{n(n+1)}{2}\right\}^2$

4^{th} power of n no : $\frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$

Prashant Dhirendra : prashant.dhiru@gmail.com

Relation between AM, GM and HM

- **A,G and H between 2 numbers(a and b):**

$$A = \frac{a+b}{2} \qquad G = \sqrt{AB} \qquad H = \frac{2ab}{a+b}$$

- $A > G > H$
- quadratic eq having a and b as its roots

$$x^2 - 2Ax + G^2 = 0$$

- the two numbers(a,b) are $A \pm \sqrt{A^2 - G^2}$
- if A and G are in the ratio $m : n$, then the number(a,b) are in ratio

$$m + \sqrt{m^2 - n^2} : m - \sqrt{m^2 - n^2}$$

- **A,G and H between 3 numbers(a, b, c):**

$$A = \frac{a+b+c}{3} \quad G = (abc)^{\frac{1}{3}} \quad \frac{1}{H} = \frac{1}{3}\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$$

- cubic equation where a,b,c are the roots

$$x^3 - 3Ax^2 + \frac{3G^3}{H}x - G^3 = 0$$

Tips and Tricks

(black space for tips,tricks and important question)

Prashant Dhirendra : prashant.dhiru@gmail.com