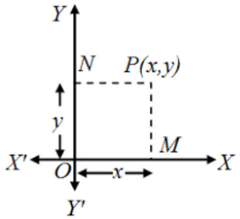
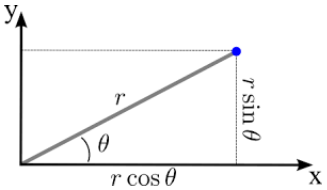
Cartesian System

point co-ordinate:



Note: if both the axis are not $\perp^{ar}$, then they are called **oblique axes**

Polar co-ordinate:



$x = rcos\theta$ $\qquad$ $y = rsin\theta$

$r = \sqrt{x^2+y^2}$ $\qquad$ $\theta = \tan^{-1}\left(\frac{x}{y}\right)$

Distance b/w 2 points $P(x_1, y_2)$ $\quad Q(x_2, y_2)$

$PQ = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

distance from origin : $\sqrt{x^2+y^2}$

polar distance : $\sqrt{r_1^2+r_2^2-2r_1r_2\cos(\theta_1-\theta_2)}$

Application of Cartesian System

For 3 points ABC:

find AB, BC and CA, then if following :

sum of any two equal to third	collinear
$AB = BC = CA$	equilateral $\triangle$
any two equal	isosceles $\triangle$
$AB^2+BC^2 = CA^2$	right angle $\triangle$

For 4 points A,B,C and D :

find AB, BC, CD and DA then if following :

$\underline{AB = BC = CD = DA}$	
$AC = BD$	square
$AC \neq BD$	rhombus
$\underline{AB = CD \text{ and } BD = DA}$	
$AC = BD$	rectangle
$AC \neq BD$	parallelogram

Application of Cartesian System

internal division $\quad \left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$

external division $\quad \left(\frac{m_1x_2-m_2x_1}{m_1-m_2}, \frac{m_1y_2-m_2y_1}{m_1-m_2}\right)$

mid point $\quad \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

division by line : the line $ax+by+c=0$ divides the line PQ in the ratio $\frac{-ax_1+by_1+c}{ax_2+by_2+c}$

co-ordinates of any point on the line segment joining two pointts are $\left(\frac{x_1+\lambda x_2}{1+\lambda}, \frac{y_1+\lambda y_2}{1+\lambda}\right)$

Division by axes

x-axis	y-axis
$\frac{-y_1}{y_2}$	$\frac{-x_1}{x_2}$

Area of Triangle

$$\triangle ABC = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

collinear $\quad \Rightarrow Area = 0$

Area by co-ordinate axes and line is $\frac{c^2}{2ab}$ for Equilateral $\triangle$

$area = \frac{\sqrt{3}}{4}side^2 = \frac{1}{\sqrt{3}}height^2$

Area of Quadrilateral

$$\triangle ABC = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \\ x_4 & y_4 & 1 \end{vmatrix}$$

collinear $\quad \Rightarrow Area = 0$

Two opp. vertices (x_1, y_1) **and** (x_2, y_2) **of rectangle are given**

$$area = |(y_2-y_1) \quad (x_2-x_1)|$$

Area of Triangle using Polar

$$\triangle = \frac{1}{2}\{r_1r_2\sin(\theta_2-\theta_1)+ r_2r_3\sin(\theta_3-\theta_2)+ r_3r_1\sin(\theta_1-\theta_3)\}$$

Area of Triangle with equation of sides

if $a_rx+b_ry+c_r = 0$ is given for $r = \{1,2,3\}$

$$\triangle = \frac{1}{2C_1C_2C_2}\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

where $C_1C_2andC_3$ are co-factors of $c_1c_2andc_3$

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Locus

if A and B are fixed, then the LOCUS P is

Circle	, if $\angle ABC =$ constant
Diameter $= AB$	,if $\angle ABC = \dfrac{\pi}{2}$
Ellipse	,if $PA + PB =$ constant
Hyperbola	if $PA - PB =$ constant

Different Form of Eq of St. Line

General Equation	$Ax + By + C = 0$
Slope Intercept form	$y = mx + c$ m = slope , c= y intercept
Point Slope form	$(y - y_1) = m(x - x_1)$
Two Point form	$(y - y_1) = \dfrac{(y_2 - y_1)}{x_2 - x_1}(x - x_1)$
Intercept form	$\dfrac{x}{a} + \dfrac{y}{b} = 1$
Normal or $\perp^{ar}$ form	$x \cos\alpha + y \sin\alpha = p$
Parametric form	$\dfrac{x - x_1}{\cos\theta} = \dfrac{y - y_1}{\sin\theta} = r$

Position of a Point Relative to a Line

- $P(x_1, y_1), Q(x_2, , y_2)$ are on <u>same side of line</u> $ax + bx + c = 0$, if $ax_1 + by_1 + c$ and $ax_2 + by + c$ have same sign or vice versa
- (x_1, y_1) is on the origin side of the line$ax + bx + c = 0$, if $ax_1 + by_1 + c$ and$c$ have same sign or vice versa

Angle Between Two Lines

when slope is given

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$$

$\| \Rightarrow m_1 = m_2 \qquad \perp^{ar} \Rightarrow m_1 m_2 = -1$

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when eq. of line is given

suppose $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are given

- angle : $\tan\theta = \dfrac{a_2b_1 - a_1b_2}{a_1a_2 + b_1b_2}$
- if lines are $\|$, $\Rightarrow \dfrac{a_1}{a_2} = \dfrac{b_1}{b_2}$
- if lines are $\perp^{ar}$, $\Rightarrow a_1a_2 + b_1b_2 = 0$
- if lines coincides $\Rightarrow \dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$

Equation of a Line Making Angle with Another Line

the eq of a line passing through P(x_1, y_1) and making an angle α with the line $y = mx + c$

$$y - y_1 = \frac{m \mp \tan\alpha}{1 \pm m\tan\alpha}(x - x_1)$$

Concurrency of 3 lines

all the 3 line meet at a point of intersection

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

Distance Between Two $\|$ lines

$$d = \frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$$

Line Passing Through Intersection of Two Lines

$L_1 \equiv a_1x + b_1y + c_1 \qquad L_2 \equiv a_2x + b_2y + c_2$

$$\text{Line} \Rightarrow L1 + \lambda L2 = 0$$

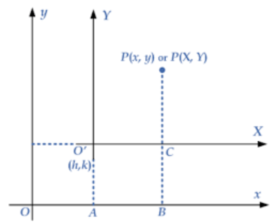
Area of Quadrilateral and Parallelogram

Area of Quadrilateral $= 4\dfrac{c^2}{2ab}$

Area of parallelogram $= \left|\dfrac{(c_1 - c_2)(d_1 - d_2)}{m_1 - m_2}\right|$

$y = m_1x + c_1, y = m_1x + c_2, y = m_2x + d_1$ and $y = m_2x + d_2$ are the eq of sides $\|^{gram}$

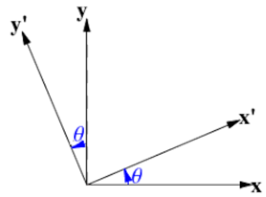
Transformation of Axes



Parallel transformation

then the co-ordinates of point P with respect to new axis O' will be

$$(x - h, y - k)$$



Rotational transformation

New/Old	x↓	y↓
$x' \rightarrow$	$\cos\theta$	$\sin\theta$
$y' \rightarrow$	$-\sin\theta$	$\cos\theta$

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General Eq of Second Degree

General Eq. of second degree in x and y is
$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

it represents a straight line, if

$$\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$$

or, $\quad abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

Eq of Lines Joining the Intersection Points of a Line and Curves to the Origin

Let $lx + my + n = 0$ be the line
and $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ be 2^{nd} degree curve

$$ax^2 + 2hxy + by^2 + 2gx\left(\frac{lx+my}{-n}\right) + 2fy\left(\frac{lx+my}{-n}\right) + c\left(\frac{lx+my}{-n}\right)^2 = 0$$

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