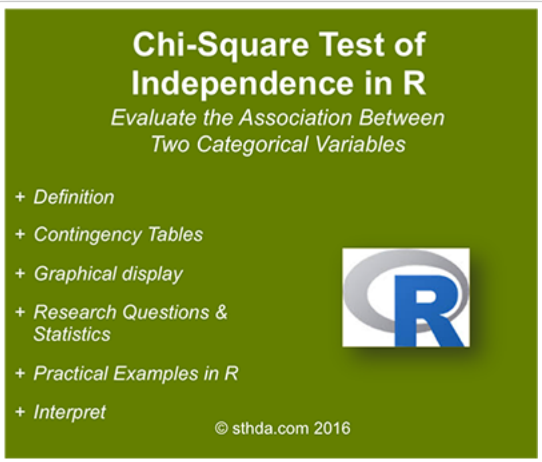
Chi-Square Test of Independence in R

The **chi-square test of independence** is used to analyze the frequency table (i.e. **contengency table**) formed by two categorical variables. The **chi-square test** evaluates whether there is a significant association between the categories of the two variables. This article describes the basics of **chi-square test** and provides practical examples using **R software**.



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Data format: Contingency tables

We'll use *housetasks* data sets from STHDA: http://www.sthda.com/sthda/RDoc/data/housetasks.txt.

```
# Import the data
file_path <- "http://www.sthda.com/sthda/RDoc/data/housetasks.txt"
housetasks <- read.delim(file_path, row.names = 1)
# head(housetasks)
```

An image of the data is displayed below:

	Wife	Alternating	Husband	Jointly
Laundry	156	14	2	4
Main_meal	124	20	5	4
Dinner	77	11	7	13
Breakfeast	82	36	15	7
Tidying	53	11	1	57
Dishes	32	24	4	53
Shopping	33	23	9	55
Official	12	46	23	15
Driving	10	51	75	3
Finances	13	13	21	66
Insurance	8	1	53	77
Repairs	0	3	160	2
Holidays	0	1	6	153

The data is a contingency table containing 13 housetasks and their distribution in the couple:

- rows are the different tasks
- values are the frequencies of the tasks done :
- by the *wife* only
- alternatively
- by the husband only
- or jointly

Graphical display of contengency tables

Contingency table can be visualized using the function **balloonplot()** [in *gplots* package]. This function draws a graphical matrix where each cell contains a dot whose size reflects the relative magnitude of the corresponding component.

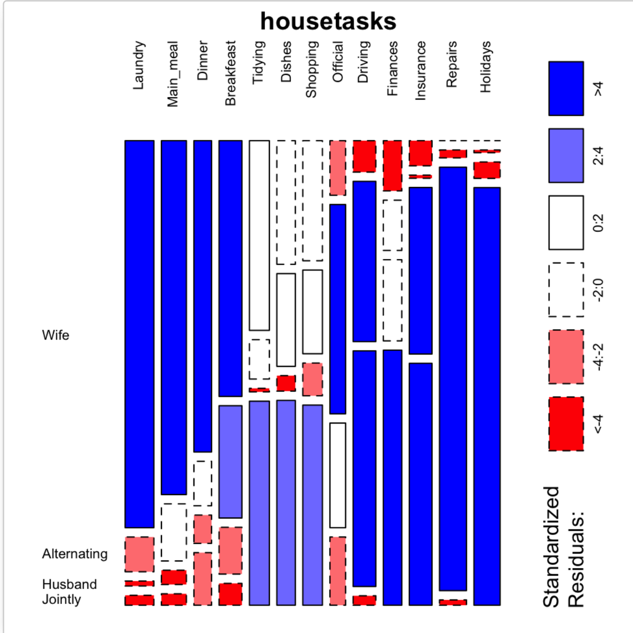
! To execute the R code below, you should install the package **gplots**: **install.packages("gplots")**.

```
library("gplots")
# 1. convert the data as a table
dt <- as.table(as.matrix(housetasks))
# 2. Graph
balloonplot(t(dt), main ="housetasks", xlab ="", ylab="",
            label = FALSE, show.margins = FALSE)
```

Note that, row and column sums are printed by default in the bottom and right margins, respectively. These values can be hidden using the argument *show.margins = FALSE*.

It's also possible to visualize a contingency table as a *mosaic plot*. This is done using the function *mosaicplot*() from the built-in R package *garphics*:

```
library("graphics")
mosaicplot(dt, shade = TRUE, las=2,
           main = "housetasks")
```



- The argument **shade** is used to color the graph
- The argument **las = 2** produces vertical labels

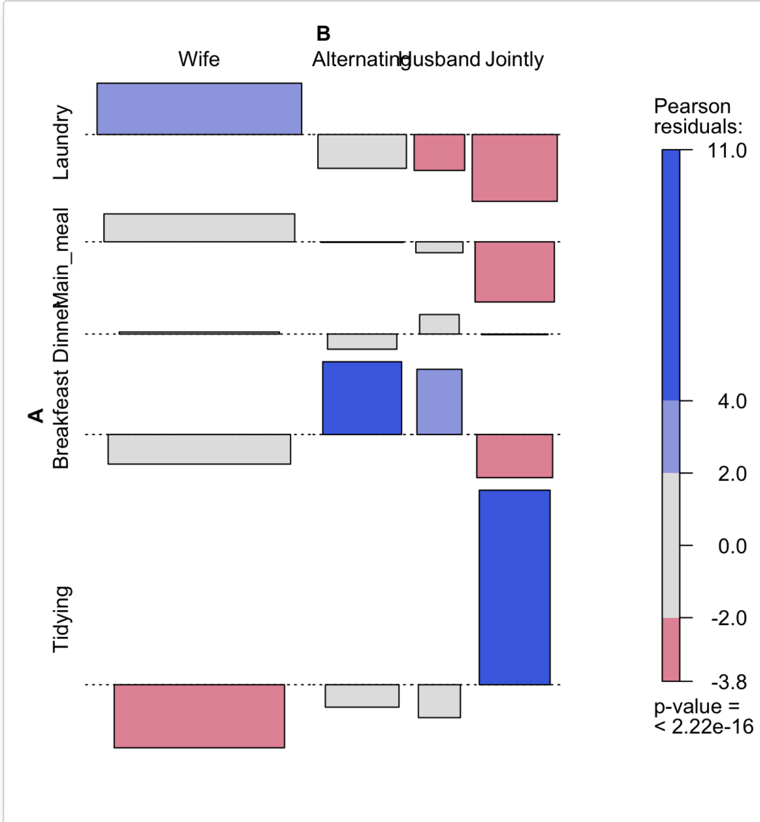
Note that the surface of an element of the mosaic reflects the relative magnitude of its value.

- Blue color indicates that the observed value is higher than the expected value if the data were random
- Red color specifies that the observed value is lower than the expected value if the data were random

From this mosaic plot, it can be seen that the housetasks *Laundry, Main_meal, Dinner and breakfeast* (blue color) are mainly done by the wife in our example.

There is another package named *vcd*, which can be used to make a mosaic plot (function *mosaic*()) or an association plot (function *assoc*()).

```
# install.packages("vcd")
library("vcd")
# plot just a subset of the table
assoc(head(dt, 5), shade = TRUE, las=3)
```



Chi-square test basics

Chi-square test examines whether rows and columns of a contingency table are statistically significantly associated.

- **Null hypothesis (H0)**: the row and the column variables of the contingency table are independent.
- **Alternative hypothesis (H1)**: row and column variables are dependent

For each cell of the table, we have to calculate the expected value under null hypothesis.

For a given cell, the expected value is calculated as follow:

$$e = \frac{row.sum * col.sum}{grand.total}$$

The Chi-square statistic is calculated as follow:

$$\chi^2 = \sum \frac{(o-e)^2}{e}$$

- o is the observed value

- e is the expected value

This calculated Chi-square statistic is compared to the critical value (obtained from statistical tables) with $df = (r-1)(c-1)$ degrees of freedom and p = 0.05.

- r is the number of rows in the contingency table
- c is the number of column in the contingency table

If the calculated Chi-square statistic is greater than the critical value, then we must conclude that the row and the column variables are not independent of each other. This implies that they are significantly associated.

Note that, Chi-square test should only be applied when the expected frequency of any cell is at least 5.

Compute chi-square test in R

Chi-square statistic can be easily computed using the function **chisq.test()** as follow:

```
chisq <- chisq.test(housetasks)
chisq
```

```
	Pearson's Chi-squared test
data:  housetasks
X-squared = 1944.5, df = 36, p-value < 2.2e-16
```

In our example, the row and the column variables are statistically significantly associated (*p-value* = 0).

The observed and the expected counts can be extracted from the result of the test as follow:

```
# Observed counts
chisq$observed
```

```
           Wife Alternating Husband Jointly
Laundry     156          14       2       4
Main_meal   124          20       5       4
Dinner       77          11       7      13
Breakfeast   82          36      15       7
Tidying      53          11       1      57
Dishes       32          24       4      53
Shopping     33          23       9      55
Official     12          46      23      15
Driving      10          51      75       3
Finances     13          13      21      66
Insurance     8           1      53      77
Repairs       0           3     160       2
Holidays      0           1       6     153
```

```
# Expected counts
round(chisq$expected,2)
```

```
            Wife Alternating Husband Jointly
Laundry    60.55       25.63   38.45   51.37
```

```
Main_meal  52.64       22.28   33.42   44.65
Dinner     37.16       15.73   23.59   31.52
Breakfeast 48.17       20.39   30.58   40.86
Tidying    41.97       17.77   26.65   35.61
Dishes     38.88       16.46   24.69   32.98
Shopping   41.28       17.48   26.22   35.02
Official   33.03       13.98   20.97   28.02
Driving    47.82       20.24   30.37   40.57
Finances   38.88       16.46   24.69   32.98
Insurance  47.82       20.24   30.37   40.57
Repairs    56.77       24.03   36.05   48.16
Holidays   55.05       23.30   34.95   46.70
```

Nature of the dependence between the row and the column variables

As mentioned above the total Chi-square statistic is 1944.456196.

If you want to know the most contributing cells to the total Chi-square score, you just have to calculate the Chi-square statistic for each cell:

$$r = \frac{o - e}{\sqrt{e}}$$

The above formula returns the so-called **Pearson residuals (r)** for each cell (or standardized residuals)

Cells with the highest absolute standardized residuals contribute the most to the total Chi-square score.

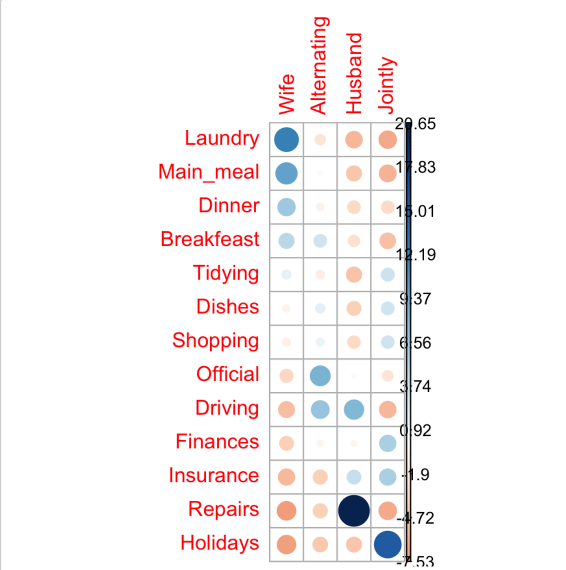
Pearson residuals can be easily extracted from the output of the function **chisq.test()**:

```
round(chisq$residuals, 3)
```

```
             Wife Alternating Husband Jointly
Laundry    12.266      -2.298  -5.878  -6.609
Main_meal   9.836      -0.484  -4.917  -6.084
Dinner      6.537      -1.192  -3.416  -3.299
Breakfeast  4.875       3.457  -2.818  -5.297
Tidying     1.702      -1.606  -4.969   3.585
Dishes     -1.103       1.859  -4.163   3.486
Shopping   -1.289       1.321  -3.362   3.376
Official   -3.659       8.563   0.443  -2.459
Driving    -5.469       6.836   8.100  -5.898
Finances   -4.150      -0.852  -0.742   5.750
Insurance  -5.758      -4.277   4.107   5.720
Repairs    -7.534      -4.290  20.646  -6.651
Holidays   -7.419      -4.620  -4.897  15.556
```

Let's visualize Pearson residuals using the package **corrplot**:

```
library(corrplot)
corrplot(chisq$residuals, is.cor = FALSE)
```



For a given cell, the size of the circle is proportional to the amount of the cell contribution.

The sign of the standardized residuals is also very important to interpret the association between rows and columns as explained in the block below.

1. **Positive residuals** are in blue. Positive values in cells specify an attraction (positive association) between the corresponding row and column variables.

- In the image above, it's evident that there are an association between the column **Wife** and the rows **Laundry, Main_meal**.
- There is a strong positive association between the column **Husband** and the row **Repair**

2. **Negative residuals** are in red. This implies a repulsion (negative association) between the corresponding row and column variables. For example the column Wife are negatively associated (~ "not associated") with the row **Repairs**. There is a repulsion between the column *Husband* and, the rows **Laundry** and **Main_meal**

The contribution (in %) of a given cell to the total Chi-square score is calculated as follow:

$$contrib = \frac{r^2}{\chi^2}$$

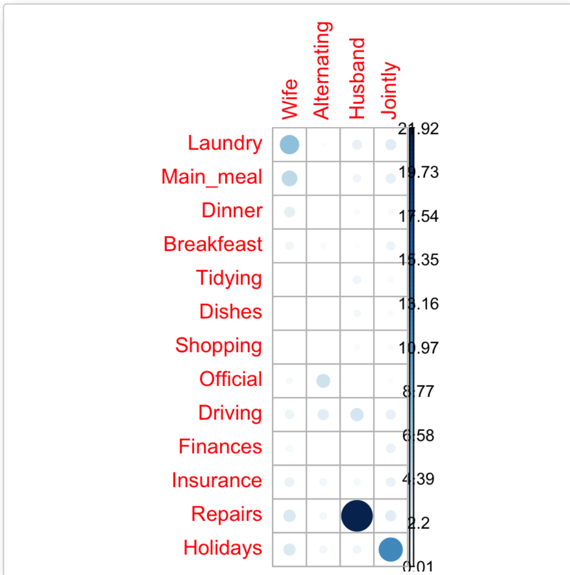
- **r** is the residual of the cell

```
# Contibution in percentage (%)
contrib <- 100*chisq$residuals^2/chisq$statistic
round(contrib, 3)
```

```
           Wife Alternating Husband Jointly
Laundry    7.738       0.272   1.777   2.246
```

```
Main_meal  4.976       0.012   1.243   1.903
Dinner     2.197       0.073   0.600   0.560
Breakfeast 1.222       0.615   0.408   1.443
Tidying    0.149       0.133   1.270   0.661
Dishes     0.063       0.178   0.891   0.625
Shopping   0.085       0.090   0.581   0.586
Official   0.688       3.771   0.010   0.311
Driving    1.538       2.403   3.374   1.789
Finances   0.886       0.037   0.028   1.700
Insurance  1.705       0.941   0.868   1.683
Repairs    2.919       0.947  21.921   2.275
Holidays   2.831       1.098   1.233  12.445
```

```r
# Visualize the contribution
corrplot(contrib, is.cor = FALSE)
```



The relative contribution of each cell to the total Chi-square score give some indication of the nature of the dependency between rows and columns of the contingency table.

It can be seen that:

1. The column “Wife” is strongly associated with Laundry, Main_meal, Dinner
2. The column “Husband” is strongly associated with the row Repairs
3. The column jointly is frequently associated with the row Holidays

From the image above, it can be seen that the most contributing cells to the Chi-square are Wife/Laundry (7.74%), Wife/Main_meal (4.98%), Husband/Repairs (21.9%), Jointly/Holidays (12.44%).

These cells contribute about 47.06% to the total Chi-square score and thus account for most of the difference between expected and observed values.

This confirms the earlier visual interpretation of the data. As stated earlier, visual interpretation may be complex when the contingency table is very large. In this case, the contribution of one cell to the total Chi-square score becomes a useful way of establishing the nature of dependency.

Access to the values returned by chisq.test() function

The result of **chisq.test()** function is a list containing the following components:

- **statistic**: the value the chi-squared test statistic.
- **parameter**: the degrees of freedom
- **p.value**: the **p-value** of the test
- **observed**: the observed count
- **expected**: the expected count

The format of the **R** code to use for getting these values is as follow:

```
# printing the p-value
chisq$p.value
# printing the mean
chisq$estimate
```

See also

- One Proportion Z-Test in R: Compare an Observed Proportion to an Expected One
- Two Proportions Z-Test in R: Compare Two Observed Proportions
- Chi-Square Goodness of Fit Test in R: Compare Multiple Observed Proportions to Expected Probabilities

Infos

This analysis has been performed using **R software** (ver. 3.2.4).

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