$$P(o/p) = \frac{\text{Number of favourable o/p's}}{\text{Total number of o/p's (all possible)}}$$

$\{1, 2, 3, 4, 5, 6\}$

what is the probability of o/p being even no. ?

$$P(E) = \frac{3}{6} = \frac{1}{2}$$

Frequentist's defn of probability.

$P\{$ of Hitting the bull's eye $\}$.

$$= \frac{\text{Area of Bull's eye}}{\text{Area of Dart Board}}$$

$$\frac{\pi r^2}{\pi R^2} = \left(\frac{r}{R}\right)^2.$$

(1) Limit our search or look out for outcomes to a definite set.

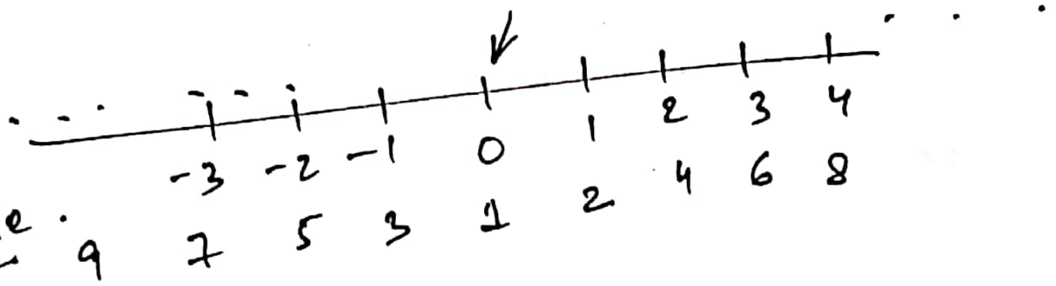
(2) Measure for determining o/c's of interest. w.r.t the all possible o/c's.

(1) Countability & Un-countability :

One-to-one mapping.

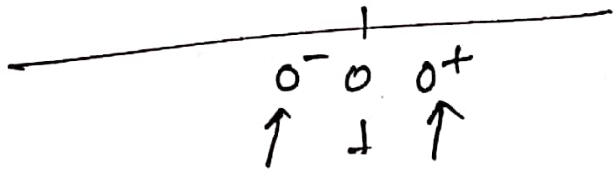
Set of Objects $\longrightarrow \mathbb{N}$

$\mathbb{I} := \{\text{set of integers}\}.$



→ Countable.

$\mathbb{R}$.



Uncountable :

↓

Analog / continuous.

Countable. → Discrete.

(1) Probability : Countable. & Uncountable.

(1)

~~Field~~;

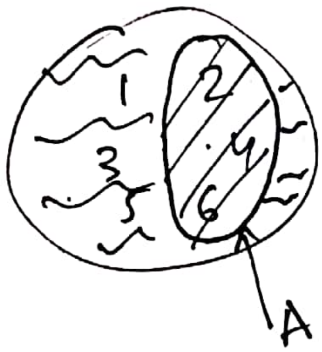
Sample space: Ω.

$= \{$ All possible o/c's of an experiment $\}$

Dice.

$\Omega = \{1, 2, 3, 4, 5, 6\}$.

$A = \{2, 4, 6\}$.



$A \subset \Omega$.

$$P(A) = \frac{\text{No. of } \{A\}}{\text{No. of } \{\Omega\}} = 3/6 = 1/2.$$

Event: Any sub~~space~~ set of the sample-space Ω.

$P\{1, 3\}$. $P\{2, 5\}$.

$A \subset \Omega$.

$A^c \subset \Omega$.

$P\{A\} = P\{$ even values $\}$.

$= 1 - P\{$ odd values $\}$

$= 1 - P(A^c)$.

(1) <u>Field</u> :

It is set of subsets of a universal set. following rules stated:

(a) $\Omega \in \mathcal{F}$.

✓ (b) If $A \in \mathcal{F}$
then, $A^c \in \mathcal{F}$.

✓ (c) If $A_1, A_2, \ldots A_n \in \mathcal{F}$.
$A_1 \cup A_2 \cup A_3 \ldots \cup A_n \in \mathcal{F}$.
$\forall$ finite value of n.

Ω

$\mathcal{F} = \{A_1, A_2, A_3, A_4, A_5 \ldots, \Omega, \phi\}$
$A_1^c, A_2^c, \ldots$

$A_1, A_2 \in \mathcal{F}$.
$A_1 \cup A_2 \in \mathcal{F}$.

$B = \{A_1 \cup A_2 \cup A_3 \ldots \cup A_n\}$

$A_1, A_2 \ldots A_n \in \mathcal{F}$.
$\Rightarrow B \in \mathcal{F}. \Rightarrow B^c \in \mathcal{F}. \Rightarrow \{A_1^c \cap A_2^c \cap A_3^c \ldots \cap A_n^c\} \in \mathcal{F}$.
((c)). (b)

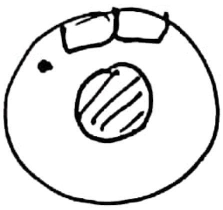
$A_1, A_2, A_3 \ldots \ldots A_n \in \mathcal{F}$.

(b) $A_1^c, A_2^c, \ldots \ldots A_n^c \in \mathcal{F}$.

$B^c \in \mathcal{F}$.

$\Rightarrow \underbrace{A_1^c \cap A_2^c \cap A_3^c \ldots \ldots \cap A_n^c} \in \mathcal{F}$.

$A_1, A_2 \ldots \ldots A_n \in \mathcal{F}$.



$\{A_1 \cup A_2 \cup A_3 \ldots \ldots \cup A_n\} \in \mathcal{F}$

$\{A_1 \cap A_2 \cap A_3 \ldots \ldots \cap A_n\} \in \mathcal{F}$.

(a) $\Omega \in \mathcal{F}$.

(b) $\mathcal{F}$ is closed under compliment.
$A \in \mathcal{F} \Rightarrow A^c \in \mathcal{F}$.

(c) $\mathcal{F}$ is closed under union.
$A_1, A_2 \in \mathcal{F} \Rightarrow A_1 \cup A_2 \in \mathcal{F}$

~~(d)~~ (b) & (c) $\Rightarrow A_1 \cap A_2 \in \mathcal{F}$.

$\mathcal{F} \Rightarrow$ Borel field ($\mathcal{B}$).

$A_1, A_2 \ldots \ldots A_n \in \mathcal{B}$.

$\lim_{n \to \infty} A_1, A_2 \ldots \ldots A_n$

$\lim_{n \to \infty} \bigcup_{i=1}^{n} A_i \in \mathcal{B}$

Axiomatic def^n:

Measurement on a set which follows certain rules:

~~(a) $P \geq 0$, $P(\phi) = 0$~~

(a) $0 \leq P \leq 1$.

(b) $P(\Omega) = 1$.

(c) If $A \cap B = \phi$ where, $A \subset \Omega$, $B \subset \Omega$.

$$P(A \cup B) = P(A) + P(B).$$

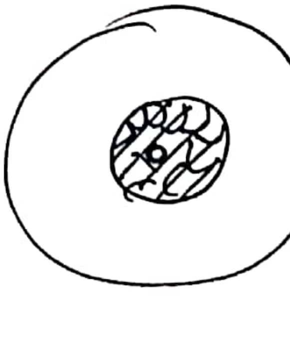
$$\lim_{n\to\infty} A_1, A_2 \ldots A_n$$

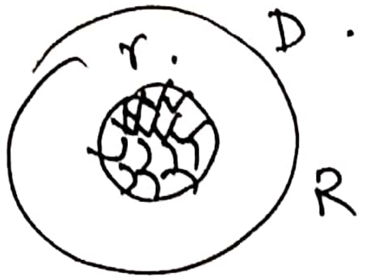
$$\lim_{n\to\infty} P(A_1 \cup A_2 \cup A_3 \ldots \cup A_n) = \lim_{n\to\infty} \sum_{i=1}^{n} P(A_i)$$

Ω.

where, $A_1, A_2 \ldots$ are mutually exclusive.

$\Omega = \{1, 2, 3, 4, 5, 6\}$ $P(\Omega)$





$A_1, A_2, A_3 \ldots\ldots A_\infty$

$\pi r^2 = \sum_{i=1}^{\infty}$ area i^{th} region

Event of hitting the Bull's eye region.

$\left\{\bigcup_{i=1}^{\infty} A_i\right\} =$

$A_i = \{$event of hitting the A_i area$\}$

$P\left\{\bigcup_{i=1}^{\infty} A_i\right\}$

$$\sum_{i=1}^{\infty} P\{A_i\} = \sum_{i=1}^{\infty} \frac{\text{area}_i}{\pi R^2} = \frac{\pi r^2}{\pi R^2}.$$

$= P\{\text{Bull's eye}\}$

$= \lim_{n\to\infty} P\left\{\bigcup_{i=1}^{n} A_i\right\}$

(a) $\Omega \in \mathcal{B}$.

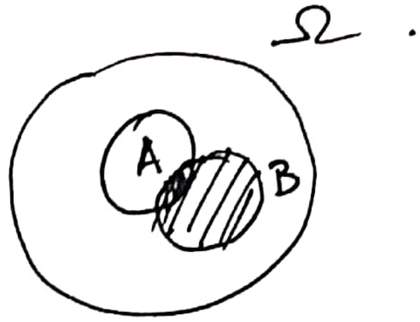
$\mathcal{B}$. =

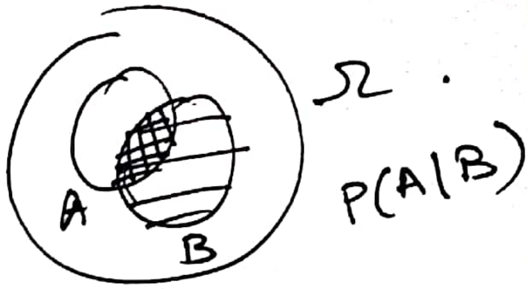
(b) ~~clos~~ $A \in \mathcal{B}$.

$A^c \in \mathcal{B}$.

(c) A_i $i \to \infty$ $\in \mathcal{B}$. $\Rightarrow \bigcup_{i=1}^{\infty} A_i \in \mathcal{B}$.

Baye's theorem:





$P(A) \rightarrow$ Probability of A.

$P(A|B) \rightarrow$ Probability of A. given that ~~B~~. B. has already occured.

~~$P(A)$~~

~~Ae~~

$\Omega = \{1, 2, 3, 4, 5, 6\}$.

$A = \{2\}$. $B = \underbrace{\{2, 4, \cancel{6}\}}$

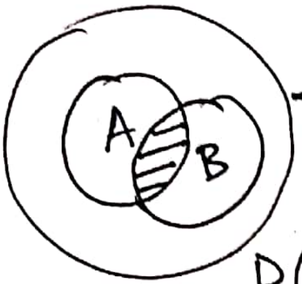
$P(A) = 1/6$. $P(A|B) = 1/2$.

~~$P(A \cap B)$~~

~~$P(A|B) =$~~

$$P(A|B) = \frac{\text{No. of favourable outcomes}}{\text{No. of o/c's in}}$$

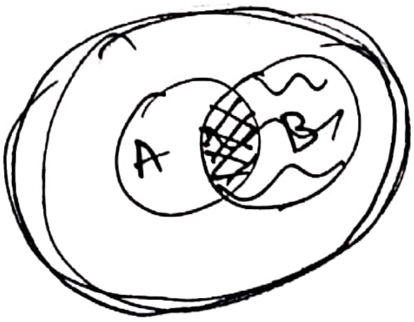
$$\frac{\text{of } (A \cap B)}{B.}$$



$$P(A) = \frac{\text{measurement}(A)}{\text{measurement}(\Omega)}$$

$P(A|B) \rightarrow$ B has occured.

$$P(A|B) = \frac{\text{measurement}(A \cap B)}{\text{measurement}(B)}.$$



$$= \frac{\checkmark \left\{ \dfrac{\text{measurement}(A \cap B)}{\text{meassurement}(\Omega)} \right\}}{\dfrac{m(B)}{m(\Omega)}}$$

$$= \frac{P(A \cap B)}{P(B)}.$$

$$\boxed{P(A|B) = \frac{P(A \cap B)}{P(B)}.}$$

Baye's Theorem.