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1]

a)

i) Two-player deterministic setting: No chance/randomness involved.
ii) Perfect information: Both players see all moves and game state.
iii) Alternating moves: Players move one at a time.
iv) Finite game: All plays terminate in finite time (finite number of positions or rules that enforce end).
v) Zero-sum: One player's win is the other's loss (or a draw).

b)

i) examples:
1) Chess (standard rules)
2) Tic-tac-toe
3) Checkers

ii) non- examples
1) Rock-Paper-Scissors: Not sequential; it's simultaneous.
2) Poker: Involves chance (card dealing), imperfect information.
3) Infinite version of Chess (no 50-move or repetition rule): May have infinite plays, violating finiteness.

2] No, the two statements are not logically equivalent. Statement A refers to the existence of strategies — a player might have a deterministic plan to win or draw, regardless of the opponent's moves. Statement B only refers to outcomes — any single completed game must end in one of three results (win/loss/draw).
Key difference: B describes what can happen in a game after it's played. A describes what can be guaranteed before the game is played, assuming optimal strategies.
Conclusion: A implies B, but B does not imply A.

3] a)

1. The number of distinct board positions in chess is finite (bounded by piece positions, castling, en passant rights).
2. So even without the 50-move or repetition rule, if a play continues infinitely, by the Pigeonhole Principle, it must revisit a position.
3. By the definition of Long Chess, any infinite play is a draw.
4. So, despite the infinite length possibility, the game still yields one of: (i) White wins, (ii) Black wins, or (iii) Draw (due to infinite repetition or forced draws).
5. Hence, we can still define a strategy tree, and Zermelo's argument (with some modifications) still applies.

b)

- A winning strategy is defined by the player's move choices based on current game state (finite description).
- Since standard chess includes additional draw conditions (50-move, 3-fold repetition), some Long Chess winning strategies may be "cut off early" in standard chess.
- However, if a player can win in Long Chess (even with infinite sequences), they must be able to win in some finite prefix — otherwise, the game is a draw by definition.
- Thus, the same strategy (possibly shortened) works in standard chess.

4]

Goal: Prove that each position falls into exactly one of the four classes: L, R, N, P.

Proof by induction on game positions (well-founded induction):
Base case:

- If a position is terminal (no moves left):
  - Whoever's turn it is loses (by Axiom v).
  - So this position is in Class P (second player wins).

Inductive step:
Assume all positions with fewer moves remaining are classified.
Let GGG be a non-terminal position:

- Let L(G)L(G)L(G) = positions reachable by Left.
- Let R(G)R(G)R(G) = positions reachable by Right.

Now analyze:

- If Left has a move to a position in P, then Left can force a win if she moves first.
  → So, from G, the next player (Left) can win ⇒ G ∈ N.
- If all of Left's moves lead to R or P, and Right has a move to P, then Right wins as first player ⇒ G ∈ N.
- If all Left moves go to R or P, and all Right moves go to L or P, then the second player can always force P ⇒ G ∈ P.
- If both players have only moves to positions in L (for Right) or R (for Left), then:
  - If Left moves first, all moves go to R ⇒ Right wins ⇒ G ∈ R.
  - If Right moves first, all moves go to L ⇒ Left wins ⇒ G ∈ L.
  - Hence, one of them has a winning strategy irrespective of order.

By checking all combinations of successor classes, every position falls into exactly one of L, R, N, or P.
5]
Let n = number of blue-eyed people.
Base case: n = 1

- Blue-eyed person sees no one else with blue eyes.
- From the visitor's statement (“at least one”), concludes: “It must be me.”
- Leaves on night of Day 1.

n = 2

- Each sees one blue-eyed person.
- Thinks: "If I don’t have blue eyes, the other will see no blue-eyed people and leave on Day 1."
- But no one leaves on Day 1.
- Hence, both realize: “I must also have blue eyes.”
- Both leave on Day 2.

n = 3

- Each sees two others with blue eyes.
- Thinks: "If I don’t have blue eyes, the other two will see only one blue-eyed person each → leave on Day 2."
- But no one leaves on Day 2.
- So, all three deduce: “I must be the third blue-eyed person.”
- All three leave on Day 3.

General Pattern:

- If there are nnn blue-eyed people, all of them leave on Day n, using recursive deduction based on previous days' inaction.

Conclusion:
Yes, people will leave.
If there are 3 blue-eyed people, all 3 leave on Day 3, after deducing via perfect logic and common knowledge.