

Week 4 assignment

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Q.1]

Preference profile:

- 3 voters, 3 candidates A, B, C.

Voter 1: Voter 2: Voter 3:

$A > B > C$

$B > C > A$

$C > A > B$

Pairwise comparisons:

- A vs B: A beats B (2 votes to 1)

B vs C: B " C (2 " 1)

C vs A: C " A (2 " 1)

→ NO Condorcet winner (cyclic preferences)

Q.2]

a z b iff

a) $b < a$

b) b divides a

c) ab is a valid English word

d) $a + b = 30$

e) b is descendant of a

f) a likes b

Complete Reflexive

Irreflexive

Transitive

No

Yes

No

Yes

preference relation

No

Yes

No

Yes

Preference relation

No

No

No

No

Neither

No

only if $a=15$

No

No

Neither

No

No

Yes

Yes

Strict preference relation

No

Not always

Not always

Not Always

Neither

Q.3]

(a)

proof Idea:

Each time a is ranked above b by a voter, a earns 1 Borda point from the comparison.

So Summing over all $b \neq a$: $Borda(a) = \sum N_{a,b}$

(b) Borda Score Calculation :

Candidate	Points from 1 st row (23 voters)	2 nd (2)	3 rd (17)	4 th (10)	5 th (8)	Total
A	$2 \times 3 = 6$	$1 \times 2 = 2$	$0 \times 17 = 0$	$1 \times 10 = 10$	$1 \times 8 = 8$	66
B	$1 \times 23 = 23$	$2 \times 2 = 4$	$2 \times 17 = 34$	$0 \times 10 = 0$	$1 \times 8 = 8$	69
C	$0 \times 23 = 0$	$0 \times 2 = 0$	$1 \times 17 = 17$	$2 \times 10 = 20$	$2 \times 8 = 16$	53

Borda winner = B (score 69)

(c) Violation of Arrow's Axioms :

IIA (Independence of Irrelevant Alternatives) is violated.

Because Borda scores depend on full rankings, adding/removing a third candidate can affect the ranking between two others.