Game Theory: Week 4 Assignment

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Deadline: 7th July 2025

Problem 1: Condorcet's Paradox (2 Marks)

The handout introduces the concept of a Condorcet winner. Create an example of a preference profile (i.e., a set of preference orders from a group of voters) for 3 candidates (A, B, C) where no Condorcet winner exists. Show your work by checking the pairwise contests.

Problem 2: Properties of Relations (6 Marks)

For each of the following relations, denoted by $\succsim$, determine whether it is **complete**, **reflexive**, **irreflexive**, or **transitive**. Based on your findings, state whether it qualifies as a preference relation, a strict preference relation, or neither, according to the definitions in the handout. (1 mark each)

(a) A is the set of all subsets of some set S, and $a \succsim b$ if and only if b is a subset of a (i.e., $b \subseteq a$).

(b) A is the set of all natural numbers, and $a \succsim b$ if and only if b is a divisor of a.

(c) A is the set of all 26 letters in the Latin alphabet, and $\alpha \succsim \beta$ if and only if the two-letter string $\alpha\beta$ is a standard word in English.

(d) A is the set of all natural numbers, and $a \succsim b$ if and only if $a + b = 30$.

(e) A is the set of all human beings, past and present, and $a \succsim b$ if and only if b is a descendant of a (e.g., child, grandchild, etc.).

(f) A is the set of people living in a particular neighborhood, and $a \succsim b$ if and only if person a likes person b.

Problem 3: The Borda Method (4.5 Marks)

The Borda Method: The French mathematician Borda proposed the following voting method. Every voter ranks the candidates from most preferred to least preferred. A candidate receives k points (called Borda points) from a voter if that voter ranks the candidate higher than exactly k other candidates. The Borda ranking of a candidate is the total number of Borda points they receive from all voters. The candidate with the most Borda points is the **Borda winner**.

(a) (3 marks) For any pair of distinct candidates a and b, let $N_{a,b}$ be the number of voters who rank a ahead of b. Show that the Borda ranking of a candidate a is equal to the sum of these counts over all other candidates. That is, prove:

$$\text{Borda Ranking}(a) = \sum_{b \in A, b \neq a} N_{a,b}$$

(b) (1 mark) Using the formula from part (a) or the original definition, compute the Borda winner for the following election. Show your calculations for each candidate's Borda score.

No. of Members	**First Choice**	**Second Choice**	**Third Choice**
23	A	B	C
2	B	A	C
17	B	C	A
10	C	A	B
8	C	B	A

(c) (0.5 marks) The Borda method, as a social welfare function, must violate at least one of the three properties from Arrow's Impossibility Theorem (Unanimity, IIA, or Nondictatorship). Which property does it violate? Briefly explain your reasoning.
