Game Theory: Week 2 Assignment

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Deadline: 27th June 2025

Problem 1: Foundational Proofs (6 Marks)

(a) (2 marks) Prove that if s_i^* is a **weakly dominant** strategy for each player i, then the strategy profile $s^* = (s_1^*, \ldots, s_n^*)$ is a Nash Equilibrium.

(b) (2 marks) Prove that if the dominance is **strict** for all players, this Nash Equilibrium is **unique**. (This is part of Theorem 6.2 from the notes).

(c) (2 marks) Prove that a strategy that dominates all of a player's other strategies is also a **maxmin strategy** for that player. (This is Theorem 6.1 from the notes).

Problem 2: The Game Show (4 Marks)

William and Henry are participants in a televised game show. Each is asked to submit one of two requests:

- **A:** "Give me Rs. 10,000."
- **B:** "Give the other participant Rs. 40,000."

(a) (2 marks) Model this as a normal-form game by constructing the payoff matrix.

(b) (2 marks) What is the predicted outcome (Assuming both players are rational). Explain your reasoning.

Problem 3: The Public Goods Problem (5 Marks)

A group of ten students must clean a common area.

- If **at least one** student cleans (action C), everyone gets a benefit of 10.
- Any student who cleans incurs a personal cost of 1.
- Students who don't clean (action NC) incur no cost.
- If **no one** cleans, all payoffs are 0.

(a) (2 marks) Define the utility function for an arbitrary student, i.

(b) (3 marks) Find all Pure Strategy Nash Equilibria (PSNE) of this game and justify your answer.

Problem 4: Order-Dependence of IEWDS (5 Marks)

Your task is to construct a **single** 2-player game to demonstrate that the outcome of Iterated Elimination of Weakly Dominated Strategies (IEWDS) can depend on the order of elimination.

(a) (2 marks) Draw the payoff matrix for your game. Your game must have **at least two** Pure Strategy Nash Equilibria. Clearly state the coordinates of both equilibria (e.g., (Up, Left)).

(b) (1.5 marks) Using the game you created in part (a), show a **first sequence** of eliminations that isolates one of the Nash Equilibria. Clearly state which strategy is eliminated at each step and why it is weakly dominated.

(c) (1.5 marks) Starting again with your original game from part (a), show a **different sequence** of eliminations that isolates the *other* Nash Equilibrium.

Problem 5: The Second-Price Auction (5 Marks)

Consider a second-price sealed-bid auction for a single object with n players.

- Each player i has a true valuation v_i and submits a bid b_i.
- The highest bidder wins and pays the **second-highest bid**.
- The winner's utility is $v_i -$ (price paid). The loser's utility is 0.

Prove that for any player i, bidding their true value ($b_i = v_i$) is a **weakly dominant strategy**.