Game Theory: Week 3 Assignment

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Deadline: 5th July 2025

Problem Set: Nash Equilibria in Strategic Games (8 Marks)

For each of the four games below, find all Nash Equilibria (pure and mixed). For the zero-sum games (Games 1 and 2), the payoffs are shown for the Row player only; you should also state the value of the game.

	L	R
T	-1	-4
B	-3	3

Game 1 (Zero-Sum)

	L	C	R
T	3	-1	2
B	-3	1	5

Game 2 (Zero-Sum)

	L	R
T	5, 16	15, 8
B	16, 7	8, 15

Game 3 (Non-Zero-Sum)

	L	R
T	15, 3	15, 10
B	15, 4	15, 7

Game 4 (Non-Zero-Sum)

Problem 2: Reconstructing Strategic Games (4 Marks)

In each of the following items, find a two-player game in strategic form in which each player has two pure strategies, such that in the mixed extension of the game the payoff functions of the players are the specified functions. [1]

(a) (2 marks) $U(x, y) = 5xy - 2x + 6y - 2$. (This is a zero-sum game).

(b) (2 marks) $U_1(x, y) = 3xy - 4x + 5$, and $U_2(x, y) = 7xy + 7x - 8y + 12$.

Problem 3: The Public Goods Problem (5 Marks)

Recall the following problem from week 2:

A group of ten students must clean a common area.

- If **at least one** student cleans (action C), everyone gets a benefit of 10.

[1] Let x be the probability that Player 1 chooses their first strategy, and y be the probability that Player 2 chooses their first strategy.

- Any student who cleans incurs a personal cost of 1.
- Students who don't clean (action NC) incur no cost.
- If **no one** cleans, all payoffs are 0.

(a) (3 marks) For this, find the symmetric Mixed Strategy Nash Equilibrium. (The probability distribution of every student is identical in a symmetric MSNE)

(b) (1 mark) Generalize your result from part (a) to find the equilibrium with n students.

(c) (1 mark) Find the limit of the probability that no one cleans as the number of students, n, approaches infinity.

(d) (0 marks) What does your answer in part (c) suggest regarding volunteering in large groups? [2]

Problem 4: The Lottery Game (8 Marks)

Consider a lottery game with n participants competing for a prize worth M, where $M > 1$. Every player may purchase as many numbers as they wish in the range $\{1, 2, ..., K\}$, at a cost of \$1 per number. The set of all the numbers that have been purchased by *only one* of the players is then identified, and the winning number is the smallest number in that set. The player who purchased that number is the winner, receiving the full prize. If no number is purchased by only one player, no player receives a prize.

(a) (2 marks) Formally define a player's set of pure strategies and their payoff function.

(b) (1 mark) Briefly justify why a symmetric Nash Equilibrium in mixed strategies must exist in this game.

(c) (2 marks) For $p_1 \in (0, 1)$, consider a symmetric strategy profile where every player purchases only the number 1 with probability p_1, and with probability $1 - p_1$ does not purchase any number. What condition must M, n, and p_1 satisfy for this to be a Nash Equilibrium?

(d) (1 mark) Show that if at a symmetric equilibrium there is a positive probability that a player will not purchase any number, then their expected payoff must be 0.

(e) (2 marks) Show that if $M < n$, then at every symmetric equilibrium the expected payoff of every player is 0. (Hint: First show that not purchasing must be part of the equilibrium strategy).

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[2] Observation based question, not graded, however if you state exactly the name of the effect we are looking for you get 1 mark bonus

Bonus Problem (3 Marks)

The following is a larger zero-sum game where payoffs are for the Row player. For a game of this size, you will need to use a graphing calculator or online optimization software.
Hint: To solve this, consider the Row player's problem. They want to find a mixed strategy (p_1, p_2, p_3) that maximizes their guaranteed minimum expected payoff, which we can call v. This means that for any pure strategy the Column player chooses, the Row player's expected payoff is at least v. This can be set up as a system of inequalities to be optimized. A similar (but minimizing) problem exists for the Column player.

To receive full credit, you must:

1. Formulate the optimization problem (objective function and all constraints) for both the Row and Column players based on the hint above.
2. State the optimal mixed strategies for both players and the value of the game.
3. **Attach a screenshot of your calculator/software output** showing the setup and solution to at least one of the players' optimization problems.

	C1	C2	C3	C4	C5
R1	4	-1	3	-2	1
R2	2	1	-3	0	-1
R3	-1	2	1	-4	3

Bonus Game (Zero-Sum)