Intro to Quantum Computing

MnP

Week #3 Assignment

Deadline: 3 July, 11:59 PM

1. **Circuits**

   (a) Create a circuit that implements Superdense coding in Qiskit.

   (b) Create a circuit that implements Quantum teleportation in Qiskit.

2. **A Bit of Information Theory**
   (Tip: Try to read up a bit on Holevo Bound from Nielsen and Chuang)
   Just like classical information stored is described by **Shannon entropy**

$$H(X) = -\sum_i p(x_i) \log p(x_i),$$

   where X is a random variable and $p(x_i)$ is the probability (necessarily non-zero) of it being x_i, quantum information (stored in a state) is described by the **von Neumann entropy**

$$S(\rho) = -\operatorname{Tr} \rho \log(\rho)$$

   (The log on a matrix is defined on the spectral decomposition, i.e. if $A = \sum_a a \left|a\right\rangle \left\langle a\right|$, then $\log(A) = \sum_a \log(a) \left|a\right\rangle \left\langle a\right|$, a being all non-zero eigenvalues)
   von Neumann entropy can be treated as Shannon entropy in the case of orthogonal states.
   Now, say Alice has to choose from states $\{\rho_i\}$, each having probability p_i, to send to Bob, who will make a measurement and collapse the state, then the **Holevo Bound** gives the following relation for maximum accessible information:

$$I_{acc} \leq \chi \equiv S\left(\sum_i p_i \rho_i\right) - \sum_i p_i S(\rho_i)$$

Show that Superdense coding achieves this maximum bound in the case of two qubits.

3. **Oracles**

   Consider the Boolean function $f : \{0,1\}^2 \to \{0,1\}$ defined by $f(x_1, x_2) = x_1 \oplus x_2$.

   (a) Suppose there's an Oracle O_f implementing the function f. How does this gate act on the state $|x_1 x_2\rangle \otimes |y\rangle$?

   (b) Explain how this operation can be implemented using basic quantum gates.

   (c) Using Qiskit and the Aer simulator, construct a 3-qubit quantum circuit that implements the oracle O_f. Here, two qubits represent the input bits x_1 and x_2, and the third qubit is an ancilla initialized to $|0\rangle$.

   Simulate the circuit for all four computational basis inputs $|x_1 x_2\rangle$ and verify that the ancilla qubit correctly reflects $f(x_1, x_2)$ in the output.

4. **QFT**

   Use the following code snippet:

```
from qiskit.circuit.library import QFT
qft_circ = QFT(num_qubits=n, inverse=False, do_swaps=True).decompose()
circ = prep.compose(qft_circ)
```

   to add a QFT to your `QuantumCircuit` object (named `circ` here). Now, create a circuit which takes the quantum fourier transform of the states $|000\rangle, |001\rangle, |010\rangle, \ldots, |111\rangle$.

   Plot the final statevectors corresponding to all initial states to try to gain a geometrical intuition behind the QFT, and explain.