Week 3 assignment

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Q.1

$$\begin{array}{c|cc} & L & R \\ \hline T & -1 & -4 \\ B & -3 & 3 \end{array}$$

Game 1 (zero sum)

- No pure NE
- Mixed NE:
  Left, Row : $(P, 1-P)$,
  Column : $(q, 1-q)$

Solve for indifference:

Row:

- T: $-1q - 4(1-q) = -1q \cdot 4 + 4q = 3q - 4$
- B: $-3q + 3(1-q) = -3q + 3 - 3q = (-6q + 3)$

Set equal: $3q - 4 = -6q + 3 \Rightarrow \boxed{q = \frac{7}{9}}$

Column:

- L: $-1p - 3(1-p) = -1p - 3 + 3p = (2p - 3)$
- R: $-4p + 3(1-p) = -4p + 3 - 3p = (-7p + 3)$

$\Rightarrow 2p - 3 = -7p + 3 \Rightarrow p = \frac{6}{9} = \frac{2}{3}$

Value of the game

= Expected payoff using $(\frac{2}{3}, \frac{1}{3})$ and $(\frac{7}{9}, \frac{2}{9})$

$= \left(\frac{2}{3}\right)\left(\frac{7}{9}\right)(-1) + \left(\frac{2}{3}\right)\left(\frac{2}{9}\right)(-4) + \left(\frac{1}{3}\right)\left(\frac{7}{9}\right)(-3)$

$+ \left(\frac{1}{3}\right)\left(\frac{2}{9}\right)(3) = \frac{-2}{3}$

$\Rightarrow$ value $= -\frac{2}{3}$

Game 2 (Zero Sum):

	L	C	R
T	3	7	2
B	-3	1	5

- No pure NE
- Mixed NE :
  using indifference method
  $\Rightarrow$ Row : $(5/9, 4/9)$
  $\Rightarrow$ Column : $(\frac{7}{18}, \frac{5}{18}, \frac{6}{18})$
  $\Rightarrow$ value = 1.11

Game 3 (Non-zero Sum)

	L	R
T	5,16	15,8
B	16,7	8,15

- (T,L) : (5,16) $\rightarrow$ Row prefers B
- (B,R) : (8,15) $\rightarrow$ Row prefers T
- (B,L) : NE (16,7)
- (T,R) : NE (15,8)
$\rightarrow$ Pure NE : (B,L) & (T,R)

- No mixed NE needed
  (since 2 pure NE exist)

Game 4 (Non zero sum)

	L	R
T	15,3	15,10
B	15,4	15,7

- Row always gets 15, indifferent
- Column prefers R in both cases
- Any strategy for Row, Column plays R
- NE : (Any of T or B, R)

Q.2] a) Let x = P1 plays strategy 1
          y = P2 plays strategy 1

a) $U(x,y) = 5xy - 2x + 6y - 2$

(b) $U1 = 3xy - 4x + 5$,

$\qquad U2 = 7xy + 7x - 8y + 12$

→ This is a zero sum

$\qquad$ → use u and $-u$

	$y=1$	$y=0$
$x=1$	$4,18$	$1,24$
$x=0$	$5,4$	$5,12$

	$y=1$	$y=0$
$x=1$	1	-2
$x=0$	4	-2

Check: $U(x,y) = xy \cdot 5 + 2x0 + y6 + $ constant -2 → Matches.

Q.3] Let p = probability of C (clean)

(a) Expected payoff : • clean : $10-g = g$

$\qquad$ • Not clean : $10 \times (1-(1-p)^{\wedge 9})$

→ $g = 10(1-(1-p)^9)$

→ $(1-p)^9 = 0.1$

→ $p = 1-(0.1)^{1/9} = 0.244$

(b) Generalize : $(1-p)^{(n-1)} = 0.1$ → $p = 1-(0.1)^{\frac{1}{(n-1)}}$

c) Limit as $n \to \infty$ → $(1-p)^n \to e^{(-np)}$

$\qquad$ if $p \to 0$, Still tends to 1

$\qquad$ → So $\lim = 1$ → Nobody cleans.

(d) Name of the effect : Diffusion of Responsibility.

Q.4) **(a)** • pure strategy :: subsets of numbers $\{1, 2, \ldots k\}$
  • Payoff :

  If player wins : $M - cost$
  Else : $- cost$
  If no unique winner : $- cost$

(b) Game is finite & mixed strategies allowed
  ⇒ Nash Existance Theorem ⇒ Symmetric NE exists.

c) Every player chooses 1 with prob p_1.
  winning prob. $= p_1 (1-p_1)^{n-1}$
  Expected payoff $= [M \times p_1 (1-p_1)^{n-1}] - p_1$
  NE ⇒ payoff $= 0$:
  $M (1-p_1)^{n-1} = 1$

(d) If prob > 0 for "no purchase"
  ⇒ Payoff ≥ 0
  But payoff $= 0$ in NE ⇒ any positive payoff would
  incentivize buying ⇒ So, expected payoff $= 0$.

(e) If $M < n$, then
  Total expected value $< n \times 1$ ⇒ some must not buy.
  → Not purchasing must be part of equilibrium
  → Then expected payoff $= 0$ for all in symmetric NE.