Week 3 assignment

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Q.1)

	L	R
T	-1	-4
B	-3	3

Game 1 (zero sum)

- No pure NE
- Mixed NE:
  Row : $(p, 1-p)$,
  Column : $(q, 1-q)$

Solve for indifference:

Row :

- $T: -1q - 4(1-q) = -1q - 4 + 4q = 3q - 4$
- $B: -3q + 3(1-q) = -3q + 3 - 3q = (-6q + 3)$

Set equal : $3q - 4 = -6q + 3 \Rightarrow q = 7/9$

Column :

- $L : -1p - 3(1-p) = -1p - 3 + 3p = (2p - 3)$
- $R = -4p + 3(1-p) = -4p + 3 - 3p = (-7p + 3)$

$\Rightarrow 2p - 3 = -7p + 3 \Rightarrow p = 6/9 = \frac{2}{3}$

Value of the Game

= Expected payoff using $(2/3, 1/3)$ and $(7/9, 2/9)$

$= \left(\frac{2}{3}\right)\left(\frac{7}{9}\right)(-1) + \left(\frac{2}{3}\right)\left(\frac{2}{9}\right)(-4) + \left(\frac{1}{3}\right)\left(\frac{7}{9}\right)(-3)$
$+ \left(\frac{1}{3}\right)\left(\frac{2}{9}\right)(3) = \frac{-2}{3} \Rightarrow$

value $= -\frac{2}{3}$

Game 2 (zero sum) :

	L	C	R
T	3	7	2
B	-3	1	5

- No pure NE
- Mixed NE :
  using indifference method
  ⇒ Row : (5/9, 4/9)
  ⇒ Column : $\left(\frac{7}{18}, \frac{5}{18}, \frac{6}{18}\right)$
  ⇒ value = 1.11

Game 3 (Non-zero sum)

	L	R
T	5,16	15,8
B	16,7	8,15

- (T, L) : (5, 16) → Row prefers B
- (B, R) : (8, 15) → Row prefers T
- (B, L) : NE (16, 7)
- (T, R) : NE (15, 8)

- No mixed NE needed
  (Since 2 pure NE exist) → Pure NE : (B, L) & (T, R)

Game 4 (Non zero sum)

	L	R
T	15,3	15,10
B	15,4	15,7

- Row always gets 15, indifferent
- Column prefers R in both cases
- Any strategy for Row, Column plays R
- NE : (Any of T or B, R)

Q.2] ⇒ Let x = P1 plays strategy 1
y = P2 plays strategy 1

a) $U(x,y) = 5xy - 2x + 6y - 2$

⇒ This is a zero sum

⇒ use U and $-U$

	$y=1$	$y=0$
$x=1$	1	-2
$x=0$	4	-2

Check : $U(x,y) = xy\,5 + x\,0 + y\,6 +$ constant -2 → Matches.

(b) $U1 = 3xy - 4x + 5$,

$U2 = 7xy + 7x - 8y + 12$

	$y=1$	$y=0$
$x=1$	4, 18	1, 24
$x=0$	5, 4	5, 12

Q.3] Let p = probability of C (clean)

(a) Expected payoff : • clean : $10 - 1 = 9$

• Not clean : $10 \times (1 - (1-p)^9)$

⇒ $9 = 10(1 - (1-p)^9)$

→ $(1-p)^9 = 0.1$

⇒ $p = 1 - (0.1)^{1/9} = 0.244$

(b) Generalize : $(1-p)^{(n-1)} = 0.1$ ⇒ $p = 1 - (0.1)^{\frac{1}{(n-1)}}$

c) Limit as $n \to \infty$ ⇒ $(1-p)^n \to e^{(-np)}$

if $p \to 0$, still tends to 1

→ So Lim $= 1$ ⇒ Nobody clears.

(d) Name of the effect : Diffusion of Responsibility.

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Q.4) (a) • pure strategy : Subsets of numbers $\{1, 2, \ldots k\}$

• Payoff :

If player wins : $M - \text{cost}$

Else : $-\text{cost}$

If no unique winner : $-\text{cost}$

(b) Game is finite & mixed strategies allowed

→ Nash Existance Theorem → Symmetric NE exists.

(c) Every player chooses 1 with prob p_1

winning prob. $= p_1 (1-p_1)^{n-1}$

Expected payoff $= [M \times p_1 (1-p_1)^{n-1}] - p_1$

NE → payoff $= 0$:

$$M(1-p_1)^{n-1} = 1$$

(d) If prob > 0 for "no purchase"

$\Rightarrow$ Payoff ≥ 0

But payoff $= 0$ in NE $\Rightarrow$ any positive payoff would incentivize buying → So, expected payoff $= 0$.

(e) If $M < n$, then

Total expected value $< n \times 1$ → Some must not buy.

→ Not purchasing must be part of equilibrium

→ Then expected payoff $= 0$ for all in Symmetric NE.