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Assignment - 6
(Parameter Estimation)

1) Let $(x_1, x_2, \ldots)$ be a random sample of size n taken from Normal Population with parameters mean $= \theta_1$ and var $= \theta_2$.
Find Maximum Likelihood estimates of these 2 parameters.

→ $\theta_1 = \mu, \quad \theta_2 = \delta^2 \qquad$ MLE = ?

$$f(x) = \frac{1}{\sqrt{2\pi\delta^2}} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\delta^2}} = \frac{1}{\sqrt{2\pi\delta^2}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\delta}\right)^2}$$

$$f(x, \theta_1, \theta_2) = \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{1}{2}\frac{(x-\theta_1)}{\theta_2}}$$

Now, $\theta_1 \in (-\infty, \infty)$
$\theta_2 \in [0, \infty)$

$$L(\theta_1, \theta_2) = \prod_{i=1}^{n} f(x_i, \theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{1}{2}\frac{(x_i-\theta_1)}{\theta_2}}$$

$$= \theta_2^{-\frac{n}{2}} \cdot (2\pi)^{-n/2} \cdot e^{-\frac{1}{2\theta_2}\sum_{i=1}^{n}(x_i-\theta_1)^2}$$

$$\boxed{\log L(\theta_1, \theta_2) = -\frac{n}{2}\log\theta_2 - \frac{n}{2}\log(2\pi) - \frac{\Sigma(x_i-\theta_1)^2}{2\theta_2}}$$

Taking partial derivative (wrt θ_1)

$$\frac{d \log L(\theta_1, \theta_2)}{d\theta_1} = -\frac{2\Sigma(x_i-\theta_1)(-1)}{2\theta_2} = 0$$

$$\hat{\theta}_1 = \mu = \frac{\Sigma x_i}{n} = \bar{x}$$

Wrt θ_2,

$$\frac{d \log L(\theta_1, \theta_2)}{d\theta_2} = -\frac{n}{2\theta_2} + \frac{\Sigma(x-\theta_1)^2}{2\theta_2^2} = 0$$

$$-n\theta_2 + \Sigma(x-\theta_1)^2 = 0$$

$$\hat{\theta}_2 = \hat{\delta}_2 = \frac{\Sigma(x_i - \bar{x})^2}{n}$$

$$\therefore \hat{\mu} = \frac{\Sigma x_i}{n} \;\&\; \hat{\delta}^2 = \frac{\Sigma(x_i - \bar{x})^2}{n}$$

2) Let $X_1, X_2, \ldots, X_n$ be random sample from $B(m, \theta)$ distribution, where $\theta \in \Theta = (0,1)$ is unknown and 'm' is known +ve integer. Compute value of θ using MLE.

→ $${}^mC_{x_i}\, \theta^{x_i} (1-\theta)^{m-x_i} = B(m, \theta)$$

$$L(\theta / x_1, x_2 \ldots x_n) = \prod_{i=1}^{n} \binom{m}{x_i} \theta^{x_i} (1-\theta)^{m-x_i}$$

To compute log likelihood,

$$\log L(\theta | x_1 \ldots x_n) = \sum_{i=1}^{n} \log \binom{m}{x_i} + x_i \log \theta + (m - x_i) \log(1-\theta)$$

$$\frac{dL}{d\theta} = \Sigma \frac{x_i}{\theta} + \frac{m - x_i}{1-\theta} = 0$$

$$\sum_{i=1}^{n} \frac{x_i - \theta(m)}{\theta(1-\theta)} = 0$$

$$\sum_{i=1}^{n} \frac{x_i}{\theta(1-\theta)} - \underbrace{\Sigma \frac{\cancel{\theta}(m)}{\cancel{\theta}(1-\theta)}}_{\frac{m \cdot n}{1-\theta}} = 0$$

$$\sum_{i=1}^{n} \frac{x_i}{\theta\cancel{(1-\theta)}} = \frac{m \cdot n}{\cancel{1-\theta}}$$

$$\boxed{\theta = \frac{\sum_{i=1}^{n} x_i}{m \cdot n}}$$

$\therefore$ MLE

$$B(m, \theta) = \frac{\sum_{i=1}^{n} x_i}{m \cdot n}$$