
TIME-VARYING UNCERTAINTY AND BUSINESS CYCLES

A SHOCK RESTRICTED STRUCTURAL VECTOR
AUTOREGRESSION APPROACH

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To my Mum.

What I wrote when handing in the diploma thesis for my first degree still holds more than ever:

My family has been my strongest support in all my life. I especially want to express my gratitude to my dear uncle Dr. med. Farshid Mavaddat, my cousins Dr. Paik Saber and Neisan Saber and my aunt Mahshid Mavaddat.

Most of all I want to thank my Mum, Dorrie Mavaddat, who has always been there for me and supported and helped me in all my ventures. It is due to her that I stand where I am today.

Innsbruck, July 2020

Abstract

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Zusammenfassung

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1. Introduction

It can be commonly postulated that being faced with uncertainty is a state inherent to life's existence and in every aspect a natural part of our complex world.

Focussing on *economic* uncertainty, [an investor being concerned about the performance of her assets, an employee being worried about future income streams during retirement or at all still having a job one year from now or a business continuously defending its position on the market](#) are just a few examples of individuals or a collective being exposed to uncertainty in every-day life. Many of these risks/uncertainties¹ have given rise to institutions and entire industries as an integral part of today's modern economies: sophisticated financial markets (with their risk-transformation-function being a pivotal element besides the purpose to raise capital) and the finance and insurance industry would not exist (or albeit in a different form) if it was not for the existence of uncertainty. On the academic front, ever more complex models promise remedy to make exposures to uncertainty and foresight calculable and/or manageable.

Going beyond these exemplary scenarios where economic agents seem to take into account some sort of probabilistic assessment of the future turn of events, on an aggregate (macro) level specific (tail) events can cause (economic or political) uncertainty to ultimately feed into the micro-level (firms and households) in a potentially damaging way, being rather perceived as exogenous shocks (i.e., outside of one's own area of influence) than part of a known probabilistic state space.

[As aptly formulated by Nicholas Bloom](#)², after the Great Recession, the subsequent turmoil in the Eurozone's sovereign debt markets and the Arab Spring being just a few examples, the most recent major political events³ of Brexit, Trump's election in

¹Note that we are currently still using these terms interchangeably but will give a brief overview of the discussion and their subtle differences in the literature below.

²Bloom is part of a larger group of contributors to the most recent macroeconomic literature that has started to systematically disentangle the effects of uncertainty both theoretically and empirically.

³One could potentially also call them 'shocks'.

the US, the trade war with China and a seeming worsening of the relationship between forces of the West and East have intensified concerns about uncertainty. Assessments from the [Federal Open Market Committee \(2009\)](#)⁴ and the International Monetary Fund ([IMF, 2012, 2013](#))⁵, for example, suggest how multilayered uncertainty can be by referring to how factors such as “[...] U.S. and European fiscal, regulatory, and monetary policies [have] contributed to a steep economic decline in 2008-2009 and slow recoveries afterward.” ([Baker et al., 2015](#), p.1594)

On a high level, [Dequech \(2000\)](#) identifies four basic social practices in today’s societies that have a stabilizing effect on the coexistence in our (as he calls it) *economic reality*: (1) legal contracts (that reduce the uncertainty of future values of nominal variables), (2) the ‘State’ which enforces legal contracts in case one party to a contract would decide to not fulfill its obligations, (3) Market-Makers (a very prominent example being central banks in their roles as lenders of last resort), and lastly, (4) informal institutions including conventions like ‘socially shared and/or prescribed standards of thought and behavior’.

Together these features improve subjective and objective uncertainty but can obviously not completely eliminate it.

And whoever analyses *uncertainty* and its underlying meaning in the literature hoping to clearly differentiate it from related concepts such as *risk* or *ambiguity*, will quickly realize that an unambiguous definition is not straightforward. Confining ourselves to economics only, there is a vast array of literature starting at the beginning of the 20th century that refers to these concepts in different schools of thought that at their core do have a philosophical component to them. Different parlance within the profession which [even causes similar concepts to be named differently](#) or the same term being used with a different meaning additionally complicates matters.

While a thorough discussion of every aspect of the related concepts is beyond the scope of this work, we still want to give a brief overview of a few insights by

⁴“Widespread reports from business contacts noted that uncertainties about health-care, tax, and environmental policies were adding to businesses’ reluctance to commit to higher capital spending” ([Federal Open Market Committee, 2009](#)).

⁵“More seems to be at work, however [...] - namely, a general feeling of uncertainty’. Assessing the precise nature and effects of this uncertainty is essential, but [...] not easy. [...] Uncertainty appears more diffused, more Knightian in nature.” ([IMF, 2012](#)). In [IMF \(2013\)](#) the authors dedicated a whole section (pp. 70-67) to the study of the effects of uncertainty, stating that “[a] common view is that high uncertainty in general, and high policy uncertainty more specifically, has held back global investment and output growth in the past two years.” ([IMF, 2013](#), p. 70).

referring to a few selecting writings:

Dow (2016) tries to clarify the difference between the mainstream and Keynesian⁶ understanding of *uncertainty*⁷, its distinction from *risk* and the resulting theoretical implications⁸. According to her assessment, in the traditional mainstream analysis within a Bayesian framework most knowledge is viewed as known or known to be stochastic in some form (called *risk*), meaning that the state space is known and numerical probabilities can be assigned. Under this approach, mainstream traditional macroeconomic models give only very limited room, if any, to *uncertainty* as a state of absence of such probabilities in the form of exogenous sources of shocks⁹.

Read from this dualistic mainstream perspective, Dow (2016) asserts that Keynes' standpoint was also for a long time interpreted in a dualistic way, albeit flipped in the sense that only in special circumstances, knowledge (here referred to as *risk*) could be treated as certain and (fundamental) uncertainty was regarded as the general case (see the introduction in Keynes, 1921). This dualistic interpretation was seemingly reinforced by Keynes's reflections on long-term expectations (Keynes, 1937, p. 214/214): "About [uncertainty] there is no scientific basis on which to form any calculable probability whatever. We simply do not know."

But Dow (2016) clarifies that in the meantime it is well-established that Keynes did not give up on scientific knowledge (i.e., did not advocate nihilism) by giving so much weight to uncertainty but that his view on uncertainty is rather multidimensional and consists of various degrees. As an advocate of this view, Dequech (2000) argues that Keynes referred to both ambiguity in his early writings (Keynes, 1921)¹⁰ and fundamental uncertainty in his later ones (Keynes, 1937) and that simply speaking of Keynesian uncertainty may actually be too vague in most circumstances.

Camerer and Weber (1992, p. 330) describe ambiguity as "uncertainty about probability, created by missing information that is relevant and could be known". Dequech (2011, p. 623) states that while usually the decision-maker under ambiguity cannot

⁶Keynes and Knight are often-times regarded as the two prominent figures that introduced the concept of *fundamental uncertainty* into the economic sciences.

⁷Dow (2016) also makes account to von Mises, Hayek and Shackle who have also dealt with uncertainty in their works but focusses on Keynes's views on fundamental uncertainty arguing for it to be the most developed and philosophically-grounded counterpoint to the mainstream theory.

⁸Note that there are also schools that reject the distinction between *uncertainty* and *risk* altogether (e.g., Savage, 1954).

⁹Dequech (2000, p. 43) refers to some economists partly neglecting fundamental uncertainty following the argument that it might lead to 'theoretical nihilism' (see e.g., Coddington, 1982).

¹⁰According to interpreters, in terms of Keynes (1921), for both ambiguity and fundamental uncertainty there is no basis for the assignment of point, numerical probabilities (Dequech, 2000).

reliably assess the probability of each event, she usually knows all possible events (or it is at least “predetermined or knowable ex ante”) while the case of fundamental uncertainty “[...] is characterized by the possibility of creativity and non-predetermined structural change”. So under this concept and “[w]ithin the bounds imposed by natural laws, the list of possible events is not predetermined or knowable ex ante [...] as *the future is yet to be created*¹¹” (Dequech, 2011, p. 623).¹²

With regard to fundamental uncertainty, Dequech (2000) mentions technological or managerial innovations as examples of human creativity that can disrupt our realities which is closely connected to the process of creative destruction of Schumpeter (1942). Unpredictable structural changes (e.g. historical changes) to our economic reality on the other hand are more typically political, social or cultural disruptions where institutions and technological change play a key-role. Interestingly, the capitalist system as we know it is regarded as endogenously causing uncertainty due to competing economic actors and decision-makers innovating in search for extra profits (see Kregel, 1987)¹³.

Coming back to the mainstream view, while the crisis and subsequent Great Recession has triggered a rethinking of the mainstream approach to uncertainty by emphasizing “institutional impediments to information access (asymmetric information)” (Dow, 2016, p. 8), degrees of uncertainty were only gradually added to the picture: the term ‘ambiguity’ was acquired as well (also following Camerer and Weber, 1992, p. 330) to account for these various degrees of uncertainty while ‘unknown unknowns’ (i.e. unimaginable events or ‘black swans’ as dubbed by Taleb, 2008) would have to be seen as ‘*knowable unknowns*’ to still be consistent with the Bayesian framework. The term ‘ambiguity’ is thereby introduced as falling either into the category of risk or uncertainty depending on the ability to quantify higher-order probabilities.

Trying to establish a typology within the introduced terms, Dequech (2011) sets up three dimensions along which he classifies relevant concepts¹⁴:

1. between substantive and procedural uncertainty:¹⁵

¹¹Italics added.

¹²Dequech (2000) sees the work of Shackle (2017) as advocating this argument.

¹³Despite of the above remarks, Keynes’ notion of uncertainty as described in Keynes (1921) and its connection to Keynes (1937) has been subject to much controversy due to differing interpretations (Dequech, 2000).

¹⁴Apart from this classification we do not discuss any further interrelationships between the various concepts.

¹⁵Proposed by Dosi and Egidi (1991, p. 145) whereby substantive uncertainty results from the “lack of all the information which would be necessary to make decisions with certain outcomes”

2. between weak and strong uncertainty:¹⁶

weak uncertainty is then further subdivided into *Knightian risk* (see [Knight \(1921\)](#); often simply called 'situations of risk' where agents are faced with objective probabilities, either a priori or statistical probabilities (i.e., relative frequencies) and *Savage's uncertainty* (see [Savage \(1954\)](#); who introduced a full theory of 'personal probability' where *personal* belief governs probabilities, based on prior [groundbreaking work of Ramsey \(1926\)](#)¹⁷ and [de Finetti \(1937\)](#));

3. between ambiguity and fundamental uncertainty.¹⁸

Abstracting from the (in parts philosophical) considerations above, the question arises as to whether there is any consent in the literature about whether uncertainty can be quantified and, if so, how this could be achieved? And once having a meaningful measure of uncertainty, what *real* effect, if any, does it exert on business cycles?

Within the past ten years and largely triggered by a seminal work by [Bloom \(2009\)](#)¹⁹, macroeconomists have started studying these questions more intensively than ever before. At a first glance, as summarized by [Jurado et al. \(2015, p. 1177\)](#), partial equilibrium analyses to date suggest that increased levels of uncertainty “[...] can depress hiring, investment, or consumption if agents are subject to fixed costs or partial irreversibilities (a 'real options' effect), if agents are risk averse (a 'precautionary savings' effect), or if financial constraints tighten in response to higher uncertainty (a 'financial frictions' effect).”

But as aptly formulated by [Bontempi et al. \(2016, p. 24\)](#), the results found in the literature to date are not unambiguous and, among others, raise the following *key*²⁰ questions:²¹

and procedural uncertainty from “limitations on the computational cognitive capabilities of the [respective] agents to pursue unambiguously their objectives, given [...] available information” in a complex decision problem.

¹⁶Whereby under weak uncertainty “[...] an agent can form [...] a unique, additive and fully reliable probability distribution” based on relevant and good-quality information and strong uncertainty consequently “[...] by the absence of such a distribution” ([Dequech, 2011, p. 622/623](#)).

¹⁷Written “in opposition” to [Keynes \(1921\)](#).

¹⁸Which are two types of strong and substantive uncertainty ([Dequech, 2000](#)).

¹⁹Bloom’s contribution triggered a wave of studies that challenged the effects that he had reported in his initial publication in [Bloom \(2009\)](#). We will come back to his study in Section 2.2.

²⁰Italics added by me.

²¹We have slightly adapted/extended [Bontempi et al. \(2016\)](#)’s objections.

1. Are uncertainty shocks temporary or more persistent?
2. Is the degree of persistence of identified negative uncertainty effects related to the econometric specification and/or the particular uncertainty measure used?
3. Does the time span over which a model is estimated play any role?

And finally:

4. Are uncertainty shocks an exogenous impulse or an endogenous response to macroeconomic fluctuations (as posed by [Ludvigson et al., 2018](#))? In other words, is uncertainty a decisive factor contributing to business cycles or does the causation go vice versa?

The remainder of this thesis is organized as follows. Section 2 reviews the related literature triggered by the seminal work of [Bloom \(2009\)](#) and thereby elaborates on potential shortcomings of the popularized econometric frameworks, Section 3 hence introduces the econometric framework of [Ludvigson et al. \(2018\)](#) that tries to alleviate the identified shortcomings by shedding light on the question of whether uncertainty shocks are an exogenous impulse (as frequently postulated in the literature) or an endogenous response to macroeconomic shocks by utilizing a novel shock restricted structural vector autoregression (SVAR) approach. Section 4 gives an overview of the data used and Section 5 presents the results of the Baseline-model of [Ludvigson et al. \(2018\)](#). Section 7 finally summarizes and concludes. Additional information and results are deferred to Appendix A and Appendix B, respectively.

2. Related Literature

While uncertainty in theoretical and empirical applications has been dealt with already long before the Great Recession, it is safe to say that research in the field surged in the past decade to understand the relationship between uncertainty and business cycles and to put, among others, arguments that point at increased uncertainty as a possibly compounding factor to the Great Recession and subsequent slow recovery on steadier grounds.

This growing body of literature covers the entire spectrum including studies examining the matter from (i) a solely theoretical, (ii) purely empirical or (iii) both perspectives in both micro- and macroeconomic analyses. In our review below we will refer to works falling into either of the three categories whereby contributions having both an empirical and theoretical component are in a slight majority. Note, however that also purely theoretical or empirical works often pick up previous theoretical or empirical results that have been reported in the literature and thereby all the more establish a theory-empirics-nexus. Further, while the Great Recession is frequently mentioned as a reference, the implications of the results are often-times generalized without specific attribution to the Great Recession. At most, key economic features (e.g. zero lower bounds, etc.) are considered in model extensions.²²

Empirically, various analyses have produced seemingly unambiguous results in various identification schemes predominantly in a VAR-framework thereby deploying different uncertainty proxies in an attempt to measure the underlying latent stochastic process (since uncertainty is an intrinsically unobservable concept as formulated by (Bloom, 2014)). But as we will discuss in the following, these empirical findings result from the econometric specifications which might be problematic. Theoretically, most models stress contractionary effects of uncertainty although diametrical effects might be at work as well. Abstracting from problems revolving around the *measurement* of uncertainty and empirical analyses first, Section 2.1 below starts with an overview

²²An exception is Schaal (2017) that investigates the fit of his model to the data ...

of the theoretical behavioural channels that have been identified in the literature, Section 2.2 subsequently presents a few contributions in greater detail including an analysis of their econometric frameworks.²³ Section 2.3 looks at the robustness of the results in VAR-settings while Section 2.4 takes a brief look at stylized facts of a few selected uncertainty measures that were popularized in the literature. Section 2.5 finally concludes the discussion and presents the main research question which is whether uncertainty can be seen as the source of economic fluctuations or itself endogenously responds to business cycles.

2.1. Theoretical Behavioural Channels, *or*: The Micro-Macro-Nexus

In the microeconomics literature, there is no single uncertainty theory and various mechanisms have been identified through which aggregate uncertainty suggests to play a role in altering economic agent's behavior which in turn adversely (and in some cases positively) affects economic activity. In particular, the literature largely evolves around *real-options*- (on which a large part of the literature has focussed so far), *risk-aversion*-,²⁴ *risk premia*- and *Oi-Hartman-Abel effects* that help forming a better understanding of the forces at work.²⁵ While the mentioned effects are partial equilibrium effects (PE; *ceteris paribus* effects) with generally unambiguous signs in isolation, together in one combination or another in general equilibrium (GE) analyses the net effects on investment, consumption unemployment and the access to finance might be less explicit due to equilibrating price and quantity changes.

The *real-options-effect* primarily applies to firms (investment and hiring decisions) but has also been expanded to households (consumption and thus savings decisions).

In the case of firms the real-options effect gave rise to the analysis of business cycle dynamics within the traditional framework of irreversible investment: when being faced with uncertainty firms become more cautious about investment and hiring if those decisions cannot easily be reversed (so-called 'investment adjustment costs') which is summarized under the *delay effect*.²⁶ More specifically, a company acquiring capital for a purchase price that will differ from the resale price essentially buys a put

²³Besides theoretical approaches, one part of the empirical literature uses micro-data (mostly firm-level data) in an attempt to extract uncertainty effects.

²⁴*Precautionary savings* effects are classified somewhere in-between. See below for details.

²⁵This taxonomy is based on Bloom (2014).

²⁶These option-effects are alleviated, however, if, on the other hand, delays would be so costly as to hamper firm's ability to wait.

option and through an investment today forgoes a call option (i.e., the chance to buy later). In an environment of higher uncertainty the value of a call option increases (an incentive to postpone investment to avoid costly mistakes) while with respect to partial reversibility the value of a put option that is being obtained by investing today increases with uncertainty, leaving the overall effect ambiguous. These (partial) irreversibilities have been introduced as an answer to the *Oi-Hartman-Abeld*-effect (which we introduce below), among others, by [Dixit and Pindyck \(1994\)](#), [Bernanke \(1983\)](#), [Abel and Eberly \(1996\)](#) and [McDonald and Siegel \(1986\)](#).²⁷ For effects on hiring see e.g. [Bentolila and Bertola \(1990\)](#). In the more recent uncertainty literature, [Bloom \(2009\)](#), [Bloom et al. \(2012\)](#) and [Schaal \(2017\)](#) study the real-options effect (see Section 2.2 below for more details).

Besides uncertainty affecting the *levels* of investment and hiring (the *delay effect*), economic agents might also become *less sensitive* to changes in external conditions in an uncertain environment (the *caution effect*; see e.g. [Bloom, 2009](#)).²⁹ In this, [Bloom \(2014\)](#) also sees an explanation for uncertainty stalling productivity growth due to an impediment to the productivity-enhancing reallocation of resources across firms (see also [Bloom et al., 2012](#)).

On the consumption side, equivalent patterns arise (delay and caution effects) and lead to precautionary spending cutbacks: uncertainty causes households to reduce the consumption of durable goods (items with a high real option value under uncertainty) like houses, cars, etc. (see e.g., [Eberly, 1994](#); [Romer, 1990](#)). *Risk aversion* might turn this precautionary spending effect into precautionary *savings* effect which is defined as the “[...] additional saving that results from the knowledge that the future is uncertain” ([Carroll and Kimball, 2006](#), p. 2). Causing economic agents to cut back on consumption and increase labor supply to self-insure against future shocks is associated with a likely aggregate contractionary effect in the short run but an ambiguous effect in the long-run due to differing effects on consumption (-) and investment (+). While in theory (as mentioned by [Bloom, 2014](#)), consumption cutbacks and a higher saving rate may induce investment and growth in the longer

²⁷[Dixit and Pindyck \(1994\)](#), primarily looking at investment, introduce the “option value for better (but never complete) information” when it comes to investment decisions.²⁸ [Bernanke \(1983\)](#) investigates optimal investment decisions under uncertainty in an oil cartel and finds that high uncertainty causes firms to postpone investments and hiring in an environment where investment projects cannot easily be undone and labor market laws cause changes to the labor force (either hires or fires) to be costly.

²⁹This result, among others, increases pressure on authorities to put forth monetary and fiscal interventions strong enough to outweigh these effects.

term, [Fernandez-Villaverde et al. \(2011\)](#) suggest that for a small open economy also potential positive long-run effects are impeded due to money fleeing the country. Likewise, in New Keynesian models they show that for large but more closed economies no positive savings (investment)-effects apply because through interactions between sticky prices (nominal rigidities) and search frictions missing market adjustments of interest rates and output prices might cause uncertainty shocks to translate into 'aggregate demand shocks' (see in particular [Leduc and Liu, 2016](#) and [Basu and Bundick, 2017](#) and our summary of their findings in Section 2.2 below).

Turning to financial markets, triggered by asset price volatility and the explosion of credit spreads throughout 2007-2009, another branch of the literature points to financial frictions (i.e., agency and/or moral hazard problems; for both equity and debt) as an additional channel through which uncertainty affects macroeconomic outcomes. First, it puts upward pressures on the cost of finance by increasing risk premia (*risk premia effect*) due to investors' demand for an adequate risk-compensation (equity). In addition, creditors might drive up interest rates (i.e., rising credit spreads) increasing the user cost of capital and retract lending activity overall which further hampers firms' ability to borrow (debt) and in effect decreases investment spending and subsequently output (see e.g., [Gilchrist et al., 2014](#); [Christiano et al., 2014](#); [Arellano et al., 2011](#); [Arellano et al., 2016](#)).³⁰

Besides the above mentioned detrimental effects of uncertainty, as noted by [Bloom \(2014\)](#), within the framework of real options the effects are not universal insofar as they are strongly subject to the condition of (ir)reversibility: if firms can easily adjust their behavior to allow for more flexibility under deteriorating circumstances the expected detrimental effects might be alleviated (see, e.g., [Valletta and Bengali, 2013](#)).

Two potential effects through which uncertainty can potentially have a positive effect on growth in the long run entered the literature as *growth options* and the *Oi-Hartman-Abel* - effect. For *growth options* the argument goes that uncertainty can encourage investment in the race for a potential price (see e.g. [Bar-Ilan and Strange, 1996](#)). In other words, uncertainty may stimulate research and development, meaning that firms appear more willing to innovate when faced with a more uncertain

³⁰ A potential additional precautionary effect related to *risk aversion* studied in the literature is 'managerial risk aversion' whereby under uncertainty investment decreases in particular for firms where the managers themselves hold a considerable amount of shares (see e.g. [Panousi and Papanikolaou, 2012](#)).

future. Although widely neglected empirically, the *Oi-Hartman-Abel* - effect (Oi, 1961; Hartman, 1972; Abel, 1983) is based on the impact of uncertainty on revenue. As outlined in Saltari and Ticchi (2000), the intuition goes as follows: an increase in uncertainty about the output price raises the investment of a risk-neutral competitive firm with constant returns to scale technology due to the convexity of the marginal product of capital with respect to the output price (or, in general, the stochastic variable). By Jensen's inequality, larger uncertainty about these variables implies a higher expected value of the marginal product of capital, and so a higher investment. This result, however, follows under the assumption of labor being a fully flexible input so that after observing a price shock, a firm can adjust labor so that the marginal product of capital increases more than proportionally relative to price. While these effects might typically not be very strong in the short run, they can unfold better in the medium to long run (Bloom, 2014).

In the more detailed discussion of a few selected works below, *search frictions* play a prominent role and are linked to the presented *option-value* channel by producing similar effects. As aptly described by Leduc and Liu (2016), search frictions alone already suffice to cause uncertainty shocks to be contractionary (in comparison to RBC models featuring a spot labor market). This is because they “provide an additional mechanism for uncertainty shocks to generate large increases in unemployment via [...]” the option-value channel. “With search frictions, a job match represents a long-term employment relationship that is irreversible. When times are uncertain, the option value of waiting increases and the match value declines [...]” and firms correspondingly respond by reducing hiring.” According to Leduc and Liu (2016), this option-value effect in their model with search frictions originates for similar reasons that are generally described in the literature about irreversible investment decisions under uncertainty (e.g., Bernanke, 1983; Bloom, 2009; Bloom et al., 2012).

Nominal rigidities/sticky prices are a key ingredient of many uncertainty studies because they help aggravating the effect of uncertainty shocks on unemployment through reductions in aggregate demand. Further, in such model constellations, inflation decreases due to the fall in demand.

2.2. Close-Up: Selected Works in More Detail

Bloom (2009) was among the first to propose uncertainty shocks as a 'new' driver of business cycles and triggered much of the literature in subsequent years by building

his argument particularly around the adverse effect of real-options on investment. Bloom (2009) himself links his work to the work of Bernanke (1983) and Hassler (1996) and in the theoretical part adds uncertainty as a stochastic process (consistent with the empirical stock market volatility index constructed from realized volatility in the S&P 500 and the VXO) into a (large-scale) standard model of the firm (as in e.g., Abel and Eberly, 1996) together with a mix of labor and capital adjustment costs³¹ which, under the assumption of nonconvexity, produce a central region where firms temporarily pause their investment and hiring if business conditions are sufficiently bad (and consequently only hire and invest when the business conditions are sufficiently good). The interaction of the time-varying uncertainty (represented by the time-series constructed from realized volatility in the S&P 500 and the VXO) with the nonconvex adjustment costs feeds into time-varying real-option effects that generate fluctuations in hiring and investment. A simulated large temporary uncertainty shock in the model (in the form of a rise in the variance of productivity and demand growth) moves the hiring/firing and investment/disinvestment thresholds outward accordingly which expands the region of inaction (the real-option value of waiting is worth more than the returns to investment and/or hiring) and leads to a rapid drop in hiring, investment, productivity and output triggered by the high real-option value to waiting. The rapid drop is, however, followed by a corresponding rebound and even an eventual overshoot in the 6-8 months into the shock as firms start to work off their piled up demand for labor and capital once the impact of the uncertainty shock subdued.^{32,33,34}

Bloom (2009) finds these partial equilibrium effects to be entirely consistent with empirical evidence as shown in impulse response functions of vector autoregression (VAR) estimations on actual data. In particular, using 17 identified 'exogenous

³¹ Adjustment costs make it hiring/firing and investment/disinvestment decisions costly.

³² Bloom (2009) finds productivity growth to react similarly due to the temporary halt slowing down the reallocation of labor and capital between low and high productivity firms.

³³ Bloom (2009, p. 646) notes that pure real-options effects should actually only lead to a convergence back to trend in the *levels* without an overshoot. This medium-term overshoot occurs due to a burst of net hiring/investment promoted through more firms clustered around the hiring/investment than the firing/disinvestment threshold and only subdues in the long run (a phenomenon Bloom (2009) calls "volatility overshoot").

³⁴ A more traditional stand-alone first-moment shock conversely produces a long and persistent drop over several quarters.

shocks³⁵, in support of his theoretical model, [Bloom \(2009\)](#) estimates VARs³⁶ showing that an uncertainty shock in the data (the VXO-measure) produces a short-run drop in industrial production of 1% which lasts for about 6 months (a confirmation for the 'wait-and-see approach') and subsequently creates a longer-run overshoot. In his setting, these results are robust to the effect of an uncertainty shock on employment and a range of alternative approaches (including variable ordering, variable inclusion, shock definitions, shock timing, and detrending). The robustness of Bloom's results also hold in a 'pseudo' general equilibrium setting by building the fall in interest rates, prices, and wages after an uncertainty shock into the simulation, the fundamental reason being that uncertainty shocks make firms temporarily insensitive to price changes (the "cautionary effect" which drastically reduces the responsiveness to monetary or fiscal policy following the immediate aftermath of an uncertainty shock).

[Gilchrist et al. \(2014\)](#) question the sole significance of the traditional "wait-and-see" effect to investment caused by capital adjustment frictions (as popularized by e.g. [Bloom, 2009](#)) and rather see the main mechanism through which changes in uncertainty impact the macroeconomy in financial distortions. Backed by empirical micro- and macro-evidence, their quantitative general equilibrium model investigates investment dynamics for firms exposed to time-varying idiosyncratic uncertainty, two forms of adjustment frictions (partial irreversibility and nonconvex fixed capital adjustment costs) and frictions in both the debt and equity markets. Including this set of features allows them to individually address and isolate the adverse real-option effects and financial friction effects. Being able to turn financial distortions on and off in their model, the key variable affected throughout the transmission mechanism

³⁵Applying Bloom's algorithm for shock-classification results in 20 events as shown in Table [A.1](#). Note that in his own contribution, [Bloom \(2009, p. 676\)](#) reports 17 events of maximum stock market volatility in Table A.1 and assigns each of these event dates to actual events categorized as either 'war', 'terror', 'oil', or 'economic'. In our reproduction of Bloom's Table in Table [A.1](#) we record a few discrepancies: we do not pick up November 1978 (OPEC II) and instead pick up October 1966, August 1982, January 1991, August 2007 and November 2007.

³⁶The estimations are run using monthly data from 06/1962-06/2008 (which excludes most of the Credit Crunch) and include: (in that order) log(S&P500 stock market index), an uncertainty measure (either the raw stock market volatility series that Bloom constructs through sticking actual realized stock market volatility and the CBOE's VXO-data which is available as of 1986 together or various indicator variables capturing months with maximum volatility, or months capturing the first time that volatility exceeded a certain threshold; for further modifications see [Bloom, 2009](#)), Federal Funds Rate, log(average hourly earnings), log(consumer price index), hours worked, log(employment), log(industrial production). Contrary to the description in [Bloom \(2009\)](#) the code shows that all variables except the stock market volatility series as a proxy for uncertainty are HP-detrended. Bloom's results are robust to various alternative approaches including variables ordering, variable inclusion, shock definitions, shock timing, and detrending

under financial frictions is the supply of credit: increased uncertainty amplifies the already adverse response of investment to heightened volatility triggered through a sharp and persistent increase in corporate bond credit spreads due to which firms, confronted with higher user cost of capital, radically bring down investment outlays even further and delever. Specifically, the authors attribute more than three-quarters of the adverse effect of an uncertainty shock on investment to the presence of financial distortions.^{37,38}

In their empirical part, [Gilchrist et al. \(2014\)](#) construct a proxy for idiosyncratic uncertainty by making use of high-frequency firm-level stock market data and as part of their micro evidence show that (including a full set of control variables including a firm's leverage, profitability and other indicators of creditworthiness) their uncertainty proxy statistically significant enters 'credit-spread regressions' that model credit spreads for a sample of approximately 1,100 firms that have a significant portion of outstanding liabilities (i.e., an increase in idiosyncratic uncertainty leads to a significant increase of credit spreads). In 'investment regressions', however, after controlling for firm-individual credit spreads, the marginal effect of uncertainty on investment virtually vanishes leaving credit spreads as a statistically significant determinant for capital spending.

As part of their macro-evidence the authors subsume their idiosyncratic uncertainty measure to an aggregate uncertainty measure and use structural vector autoregressions (SVARs) over the period 1963:Q3 - 2012:Q3 consisting of eight variables^{39,40} in two different identification schemes examining the impact of two different shocks: innovations to the idiosyncratic uncertainty measure and to the credit spreads (i.e., a financial shock). The corresponding impulse response functions upon an uncertainty shock show a widening of credit spreads consistent with the micro-estimations. This influence aside, the investment component of aggregate output (business expenditures on fixed capital and consumer spending on durable goods) falls strongest with a maximum 1.5% decline reached six quarters after the shock. A credit-spread shock

³⁷[Gilchrist et al. \(2014\)](#) see the dynamics in their model to be consistent with the empirical evidence developed in [Stock and Watson \(2012\)](#) who see the combination of uncertainty and credit supply shocks as the primary drivers of the slowdown during the Great Recession.

³⁸Through the combination of partial irreversibilities and financial frictions, in their model [Gilchrist et al. \(2014\)](#) also identify "capital liquidity" shocks as a new potential source of shocks: in this scenario adverse shocks to the liquidation value of capital affect the debt capacity of firms (through haircuts on collateral).

³⁹Four lags of each endogenous variable are used.

⁴⁰ $\log(\text{real business fixed investment})$, $\log(\text{real personal consumption expenditure on durable goods})$, $\log(\text{real personal expenditures on nondurable goods and services})$, $\log(\text{real GDP})$, $\log(\text{GDP deflator})$, federal funds rate, the 10-year BBB-Treasury credit spread and the constructed proxy for idiosyncratic uncertainty at the aggregate level.

(an increase of almost 30 basis points in the BBB-Treasury spread) also points at significant and long lasting economic contractions (depressing the investment component of aggregated demand *and* nondurable goods consumption) while the effect on uncertainty is negligible. An alternative identification scheme (ordering credit spreads before the uncertainty proxy), however, shows far less adverse effects of uncertainty shocks on the macroeconomy whereas credit spread shocks (a financial shock) show a protracted and long-lasting effect on economic activity (GDP, business fixed investment and major categories of consumer spending) and an immediate jump in uncertainty. Interestingly, [Gilchrist et al. \(2014, p. 14\)](#) see the immediate response of uncertainty to innovation in credit spreads to suggest that “[...] fluctuations in uncertainty may arise endogenously in response to changes in broader financial conditions [...]” which is an issue which we will pick up again in Section 2.5.

[Bachmann and Bayer \(2013\)](#) add to the real-options literature of e.g. [Bloom \(2009\)](#) or [Bloom et al. \(2012\)](#) (as mentioned by [Schaal, 2017](#)) by

[Bloom et al. \(2012\)](#)deploy a measure of establishment-level volatility constructed from Census data which, using annual input-output data from the Census of Manufactures and the Annual Survey of Manufactures, present the cross-sectional interquartile range (IQR) of innovations to establishment-level TFP, estimated from an AR(1) process; (the estimation controls for time and plant-level fixed effects and four-digit price deflators) → mentioned in [Schaal \(2017\)](#)

[Alexopoulos and Cohen \(2015\)](#)

[Born and Pfeifer \(2014\)](#)

[Fernandez-Villaverde et al. \(2011\)](#)

[Schaal \(2017\)](#) focusses his analysis on the experience of the U.S. labor market over the series of recessions since 1970 by solely looking at the real options effect to hiring (i.e. employment decisions)⁴¹ and explicitly models various forces at work into an equilibrium search-and-matching model⁴² of firm dynamics under time-varying uncertainty⁴³ (and heterogeneity in productivity and size) that, theoretically, might

⁴¹Adding to literature that considers non-convex adjustment costs in labor like [Bloom \(2009\)](#), [Bloom et al. \(2012\)](#) or [Bachmann and Bayer \(2013\)](#).

⁴²Insofar it is related to [Leduc and Liu \(2016\)](#) who extended the New-Keynesian DSGE framework of [Basu and Bundick \(2017\)](#) by search frictions. However, in comparison to these representative agent approaches, [Schaal \(2017\)](#) states his model to be more closely related to the firm dynamics and heterogeneous agent literature of [Hopenhayn \(1992\)](#).

⁴³Schaal deploys time-varying establishment-level volatility in total factor productivity (TFP) as his uncertainty measure arguing for its volatility in the data to be “[...] an important determinant

leave the overall effect on unemployment to be ambiguous:⁴⁴ First, similarly as the adverse real options effect to investment, under high uncertainty the option value of waiting increases and firms are expected to delay hiring resulting in an increase of unemployment. Second, however, separations also tend to be subject to an option value, meaning that with increasing uncertainty, firms should also hesitate to reduce their work-force since searching for new workers would be costly in case of a rise in future productivity. Third, firms might also be hit differently by uncertainty shocks causing a reallocation of the labor force to firms hit by large positive shocks that correspondingly substantially expand.

The introduction of time-varying idiosyncratic shocks (i.e., at the micro-level) into the model improves the overall fit compared to other search-and-matching models measured against the benchmark of a range of business cycle moments. The model's aggregate response to time-varying idiosyncratic and aggregate productivity shocks, however, does not show any strong real options effects resulting from the sheer size of the U.S. labor market flows which leave estimated search costs too low to cause any serious irreversibilities while playing a major role in labor market flows (i.e., reallocation). Overall, the prevailing major effect is a massive increase in layoffs at firms that are unusually negatively hit by the shock accompanied by a modest rise in hiring (at firms hit positively) which is insufficient (dampened by general equilibrium and real option effects) to do counter the rise in layoffs.

Finally, running macro-experiments the model shows a rather successful fit in replicating the output dynamics of the recessions from 1972-2009 with the worst fit in the rise of unemployment (only 40% of the total actual rise in employment) in the 2007-2009 recession with a large part of the magnitude and persistence remaining unexplained.

Based on the empirical evidence of a VAR including the VXO as a measure of uncertainty, gross domestic product (GDP), consumption, investment, hours worked, the GDP deflator, the M2 money stock, and a measure of the monetary policy stance (in that order) over the 1986-2014 sample period,⁴⁵ Basu and Bundick (2017) argue for the resulting co-movement of output, consumption, investment, and hours worked to be a key empirical feature of an economy's (i.e., the data's) response following an uncertainty shock and hence also as a "[...] key minimum condition that business-cycle models driven by uncertainty fluctuations should satisfy" (Basu

of employment decisions and labor market reallocation." (Schaal, 2017)

⁴⁴Note that in comparison to Leduc and Liu (2016) who combine nominal rigidities and search frictions (see below), Schaal (2017)'s search model assumes flexible prices.

⁴⁵All variable apart from the monetary policy measure enter the VAR in log levels.

and Bundick, 2017, p. 937).⁴⁶ Identifying the uncertainty-shock using a Cholesky decomposition with the VXO ordered first, the authors assume that uncertainty shocks can affect output and its components immediately but that other variables' shocks, however, do not influence the VXO on impact. With this ordering, their VAR generates statistically significant co-moving declines in output, consumption, investment, and hours worked with a peak response after approximately one year whereby their key stylized fact is robust to several modifications.⁴⁷ These insights are subsequently brought into a New Keynesian DSGE model (monopolistic, one-sector closed-economy model) where they calibrate their uncertainty shock process using fluctuations in the VXO and add nominal price rigidities (sticky prices) to replicate the transmission mechanism which was identified in the data. According to Basu and Bundick (2017), the resulting dynamics due to the incorporated price rigidity in a model where output is demand-determined are able to support the intuition that an increase in uncertainty which induces precautionary savings, reduces household expenditures, output and labor input and eventually the demand for capital and investment as suggested by their VAR-model.⁴⁸

Complementing the existing literature⁴⁹, Leduc and Liu (2016) add to this literature by finding that uncertainty shocks produce effects similar to aggregate demand shocks.

While Born and Pfeifer (2014) don't find convincing evidence for large real effects of policy uncertainty shocks in a standard DSGE model, to highlight aggregate demand effects (and abstracting from additional features that would also shed light on effects on investment), Leduc and Liu (2016)'s theoretical New-Keynesian DSGE framework that incorporates both search frictions (an extension vis-a-vis Basu and Bundick (2017)) and nominal rigidities as key ingredients where the interplay of the reciprocal amplification of an option-value channel arising from search frictions in the labor market and decreases in aggregate demand stemming from sticky prices in the goods

⁴⁶At the same time, they see this feature as missing from the related work of Bloom (2009), Bachmann and Bayer (2013) or Gilchrist et al. (2014).

⁴⁷These modifications include: inclusions of stock prices in the VAR (as is done e.g., by Bloom (2009)), measurement of uncertainty using the VIX instead of the VXO, a re-ordering of the VAR placing uncertainty last or an adjustment of the sample period to exclude the Great Recession.

⁴⁸Before introducing their model's features, Basu and Bundick (2017) show that uncertainty shocks in standard closed economy real business cycle (RBC) (i.e., competitive, flexible prices) models produce a counterintuitive aggregate expansion. Monopolistic competition under sufficiently sticky prices changes the dynamics.

⁴⁹The empirical results in, among others, Bloom (2009), Bloom et al. (2012), Jurado et al. (2015), Scotti (2016), Bachmann et al. (2013) and the theoretical results in Bloom (2009), Gilchrist et al. (2014), Arellano et al. (2011), Basu and Bundick (2017), Bloom et al. (2012), Born and Pfeifer (2014).

market produces a transmission mechanism of uncertainty on unemployment and inflation that is consistent with their empirical observation.⁵⁰ Specifically, their theoretical model is guided by the empirical evidence derived from a four-variable Bayesian VAR (BVAR) on US data consisting of a measure of uncertainty, the unemployment rate, the CPI year-on-year inflation rate and a short-term interest rate (as represented by the three-month Treasury bills rate). In this setting, the authors declare the observed joint dynamics of sharply rising unemployment and decreasing inflation following an uncertainty shock to resemble features of a negative aggregate demand shock.⁵¹ Contrary to the sole usage of the VIX as an uncertainty measure, [Leduc and Liu \(2016\)](#) report the dynamics to be robust to their alternative uncertainty measure of consumers' perceived uncertainty derived from the Thomson Reuters/University of Michigan Surveys of Consumers.⁵² Their rationale for using a consumer uncertainty measure is that, by construction of the survey, interviewees do not have (complete) information of the current month's macroeconomic data. Hence, it is assumed that survey participants will condition their answers on all previous realizations of macroeconomic indicators except time t .⁵³ The resulting IRFs at impact of an uncertainty shock show a strongly persistent unemployment increase, peaking after approx. 18 months and significantly lasting for about three years. Inflation shoots in the opposite direction with a peak effect after around 20 months and staying significant for almost two years.⁵⁵

⁵⁰Under existing search frictions that reduce the value of a new match and cause firms to reduce hiring which drives up unemployment and ultimately reduces aggregate demand, when additionally prices are sticky, an increase in uncertainty also leads to a decline in aggregate demand which reinforces the option-value channel and generates an increase in unemployment. Under existing nominal rigidities the effect of decreasing aggregate demand is amplified when adding search frictions because the value of a new match decreases even further which ultimately pushes up the unemployment rate which in turn reduces households' income even further and aggravates the reduction in aggregate demand. This interaction considerably amplifies the fall in aggregate demand in response to uncertainty shocks in comparison to standard DSGE models that solely consider nominal rigidities and go without search frictions as in e.g. [Basu and Bundick \(2017\)](#).

⁵¹Adding habit formation to their model, their calibrated DSGE model comes closest to the empirical results from the VAR-model.

⁵²Due to data availability of the respective uncertainty measures, their estimation window accordingly covers 01/1978 - 10/2013 for the model containing the consumers' perceived uncertainty and 01/1986 - 10/2013 in case of the VIX.

⁵³Placing the uncertainty measure first in their Choleski ordering hence implies that on impact of a shock only unemployment, inflation and the nominal interest rate are allowed to respond. With their identification strategy [Leduc and Liu \(2016, p. 23\)](#) assume that their "[...] measured uncertainty contains *some*⁵⁴ exogenous component and does not reflect endogenous responses of other macroeconomic variables.

⁵⁵To accomodate the zero lower bound (ZLB) in U.S. monetary policy after 2008, [Leduc and Liu \(2016\)](#) report their VAR to also be robust to the usage of the two-year Treasury bond yield (that did not reach the ZLB) as an alternative indicator of the monetary policy stance. To accomodate the possibility of survey respondents' perceptions of bad economic times instead of uncertainty about the

Commenting on the theoretical results of Section 2.1 in combination with the empirical VARs of the selected contributions described above, Ludvigson et al. (2018, p. 4) argue that while the theoretical literature which proposes uncertainty as a source of lower economic growth “[...] almost always presumes that uncertainty is an exogenous shock to the volatility of some economic fundamental”, the empirical literature, however, typically used uncertainty proxies which are strongly correlated with financial markets. According to Ludvigson et al. (2018) this practice makes it necessary to distinguish between the two in empirical applications to understand the nexus between real economic/financial market uncertainty and recessions. We will come back to this issue in Section 2.5.

2.3. Robustness of Econometric Specifications in VAR-Settings⁵⁶

Various models have been suggested/studied in the literature each of them suggesting a slightly different set of variables to include, a different specification overall and/or ordering with which the variables enter the VAR. We will start with a replication of the baseline VAR(12)-8⁵⁷ that Bloom (2009) estimated as a support of his theoretical model of the firm to disentangle responses of output and employment to heightened uncertainty in Section ??

The growing body of recent empirical literature on the *potential* causal effects of time-varying economic uncertainty on macroeconomic activity (aggregates) qualitatively largely has one thing in common: in a VAR context, uncertainty shocks seem to have a negative effect on output (or proxies thereof such as industrial production or employment). But as summarized by Bontempi et al. (2016, p. 23), “[...] this key finding is only robust in regard to the uncertainty impact in the short run, whereas in the long run different works [using different uncertainty proxies and slight modifications in the econometric specifications] have pointed to somewhat heterogeneous output responses.”

→ ADD HERE SOME PARTS FROM Ludvigson et al. (2018) where they very

future, Leduc and Liu (2016) follow a similar approach like Baker et al. (2015) and add a variable for consumer sentiment as an additional control to their BVAR. The results suggest that uncertainty is indeed forward-looking.

⁵⁶works that combine empirics with theory and establish dsge-models in conjunction with empirical analysis on a micro or macro-level; I have to include this footnote somewhere where it makes sense. A non exhaustive list of selected works includes, for example, Ramey and Ramey (1995), Aghion et al. (2005), Mills (2000) and Imbs (2007) on the relationship of volatility and growth, Leahy and Whited (1996) and Bloom et al. (2007) on the effect of uncertainty on investment, Barlevy (2004) and Gilchrist and Williams (2005) on the effects of uncertainty on business cycles, etc.

⁵⁷Our notation ‘VAR(p) - v ’ denotes the number of lags p and the number of variables included v .

pointedly explain some of the flaws of current structural models.

[Bontempi et al. \(2016\)](#) write: “Findings in the literature suggest that every time uncertainty is modelled within the macroeconomic VAR context, it [largely] displays a significant negative relationship with economic activity, as uncertainty shocks are broadly found to exert a negative impact on output and employment.[...]”

However, this key finding is only robust with regard to the uncertainty impact in the short run, whereas in the long run different works have pointed to somewhat heterogeneous output responses:

For example, the results in [Bloom \(2009\)](#) sustain the over-shooting effect of a VIX uncertainty shock on real activity: following the shock, the economy suffers in the short term, but in the long run the initial level of output is surpassed. The evidence in Figure 6 of [Bachmann et al \(2013\)](#) suggests that Bloom’s over-shooting is more due to the use of the finance-based measure rather than to any genuine uncertainty effect. ([Jurado et al. \(2015\)](#) argue that Bloom’s over-shooting is an artefact in the data mainly due to his HP filtering, since with raw data the over-shooting dynamics vanish.) The latter fact reinforces our caveats about the reliability of measures of macroeconomic uncertainty based solely on financial information, and suggests that researchers need to be careful when proxying uncertainty with these finance-based measures, as they may label certain transitory financial crises as uncertainty shocks. [Jurado et al. \(2015\)](#) and [Bachmann et al. \(2013\)](#) instead utilize forecast-based measures. Their VAR models reveal that the dynamic response of output to uncertainty shocks sharply reduces the level of production with effects that persist well beyond the horizons considered in their exercise (i.e., more than 4-5 years after the shock). [Baker et al. \(2015\)](#) model the economy by slightly reducing the number of variables in Bloom’s VAR (from 8 to 5 macroeconomic variables, uncertainty included), and use their news-based economic policy uncertainty index. They report a negative dynamics response of manufacturing production to a shock. However, unlike [Jurado et al. \(2015\)](#) and [Bachmann et al. \(2013\)](#), these output responses are significantly negative for only the first 15-18 months after the shock, before gradually declining to zero, i.e. without overshooting.”

[Bloom \(2009, p. 651\)](#) writes: “What is striking about Figure 12 is the similarity of the size, duration, and time profile of the simulated response to an uncertainty shock compared to the VAR results on actual data shown in Figure 2.”

as written in the Appendix of [Basu and Bundick \(2017, p. 5\)](#): “As we discuss in the main text, the Federal Reserve hit the zero lower bound on nominal interest rates at

the end of 2008. While we model this outcome rigorously using our theoretical model, it is less clear how to model the stance of monetary policy during our 1986-2014 sample period econometrically. [...] If we use the 1962-2008 sample of Bloom (2009) with the federal funds rate as the measure of monetary policy, our stylized fact remains: Higher uncertainty generates declines in output, consumption, investment, and hours worked.

Contrary to Bloom’s findings, for example, Jurado et al. (2015) and Bachmann et al. (2013) using their own forecast-based measures (see Section 4.0.1) find a rather different response-pattern: their VARs show a sharp reduction in output at impact whose effect is far more persistent lasting for a couple of years after the shock with no signs of the overshooting effect as in Bloom (2009).

Bachmann et al. (2013) take the same line of reasoning as Bloom (2009) by looking at investment adjustment costs.....

Baker et al. (2015) Baker et al. (2015, p. 1595) write: “In Section IV we provide evidence of how firm-level and aggregate outcomes evolve in the wake of policy uncertainty movements. Causal inference is challenging, because policy responds to economic conditions and is likely to be forward looking. To make progress we follow a micro and a macro estimation approach. [...] Our second approach fits vector autoregressive (VAR) models to U.S. data. [...] The U.S. VAR results indicate that a policy uncertainty innovation equivalent to the actual EPU increase from 2005-2006 to 2011-2012 foreshadows declines of about 6% in gross investment, 1.1% in industrial production, and 0.35% in employment.

Jurado et al. (2015) replicate Bloom (2009)’s VARs in their contribution, renounce to detrend the variables however, arguing that the HP filter uses information over the entire sample which makes it difficult to interpret the timing of an observation and report results using Bloom (2009)’s original detrended variables in their Appendix.

2.4. Selected Uncertainty Measures and Some Stylized Facts

Various uncertainty measures exist (and not *the one* measure) because the stochastic process which we commonly call ‘uncertainty’ is unobservable. Jurado et al. (2015, p. 1182) emphasize that “[t]he literature on measuring uncertainty is still in its infancy” and so far existing research has popularized various proxies or indicators of

uncertainty.

The lack of an objective uncertainty measure hence poses a serious challenge to studies trying to quantify its macroeconomic effects: they have to come up with a reasonable uncertainty measure in the first place. [Jurado et al. \(2015, p. 1178\)](#) explain that “[w]hile most of the measures have the advantage of being directly observable, their adequacy as proxies for uncertainty depends on how strongly they are correlated with this latent stochastic process [i.e., with unobservable uncertainty]”.

Besides creating their own measure of economic uncertainty, [Orlik and Veldkamp \(2014\)](#) describe commonly used proxies of uncertainty by means of the following taxonomy and refer to selected works along the way⁵⁸:

- ▷ **Forecast Dispersion:** STILL HAVE TO ADD DETAILED DESCRIPTION!
- ▷ **Mean-Squared Forecast Error:** STILL HAVE TO ADD DETAILED DESCRIPTION!
- ▷ **Volatility and Confidence Measures**, where we find three alternative approaches:
 - ‘finance-based’ - measures, assuming that financial volatility is a suitable indicator for aggregate macroeconomic uncertainty:
 - * the traditional measure of uncertainty is the implied (e.g., the market volatility indices VIX/VXO) or realized volatility of stock market returns; based on these measures, [Bloom \(2009\)](#) constructs his shock-measure (henceforth called ‘Bloom-shock’)
 - ‘forecast-based’⁵⁹ - measures that rely either on an economy’s predictability or the measurement of discrepancies between professional forecasts assuming that impeded predictability or a large dispersion between forecasts, respectively, points at a more uncertain economy:
 - * cross-sectional dispersion measures of subjective business or consumer confidence (direction or point forecasts) based on survey data assuming that “[...] this inter-personal dispersion measure is an acceptable proxy

⁵⁸Note that we have slightly extended the original taxonomy of [Orlik and Veldkamp \(2014\)](#) by mixing it with [Bontempi et al. \(2016\)](#)’s taxonomy. Only the measures that link to a dedicated section will be examined in more detail below.

⁵⁹Note to self: we still have to fit ‘the cross-sectional dispersion of firm profits, stock returns, or productivity’ somewhere into our taxonomy!

for the average dispersion of intra-personal uncertainty” (Ozturk and Sheng, 2017, p. 4): e.g., Bachmann et al. (2013) use survey data in their analysis, Jurado et al. (2015) also mention analysts’ forecasts (as e.g., used by D’Amico and Orphanides); Leduc and Liu (2016) instead use a measure constructed using data from the Thomson Reuters/University of Michigan Surveys of Consumers

- * Jurado et al. (2015) construct a (as Orlik and Veldkamp (2014) call it) ‘state-of-the-art’ macro uncertainty index (henceforth called ‘macro uncertainty index’) that reflects uncertainty around objective statistical forecasts by looking at the common (instead of idiosyncratic) variation in uncertainty across hundreds of economic time series⁶⁰
- besides stock market volatility (VXO), Bloom (2009) uses⁶¹:
 - * cross-sectional standard deviation/spread of firms’ pretax profit growth (taken from the quarterly accounts of public companies; taken from firms with 500+ months of data in the Center for Research in Securities Prices (CRSP) stock-returns file)
 - * standard deviation (cross-sectional dispersion) of annual five-factor Total Factor Productivity (TFP) growth within the NBER manufacturing industry data base (measures the within-year cross-industry spread of SIC 4-digit manufacturing TFP growth rates, calculated using the five-factor TFP growth figures from the NBER data base))
 - * cross-sectional dispersion across macro forecasters over their predictions for future GDP calculated from the Livingstone half-yearly survey of professional forecasters
- ‘news-based’ - approaches that measure the degree of uncertainty by extracting the frequency with which specific words appear in newspaper articles, assuming that journalists are able to assess emerging uncertainty⁶².

⁶⁰We will pick up on this measure when presenting the (S)VAR-framework of Ludvigson et al. (2018) in Section 4.0.1. As noted by Ozturk and Sheng (2017),

⁶¹Gilchrist et al. (2014, p. 1) write that “[m]acroeconomists have made a convincing argument that the well-documented countercyclical behavior of the cross-sectional dispersion of economic variables such as labor income, business profits, productivity, and stock returns reflects fluctuations in the volatility of the underlying economic shocks - that is, swings in economic uncertainty.”

⁶²Note that this approach is similar to the narrative analysis used by Romer and Romer (2004), Romer and Romer (2017) or Ramey (2009).

- * [Alexopoulos and Cohen \(2009\)](#) construct a news-based index derived from the number of New York Times' articles on uncertainty and economic activity,
- * [Baker et al. \(2015\)](#) construct an index of economic policy uncertainty (EPU) for a number of countries (including the United States) by searching for uncertainty-related keywords within a pool of widely read country-specific newspapers (henceforth called 'EPU index'), in a similar vein, [Moore \(2016\)](#) constructs an index of economic uncertainty for Australia,
- * inspired by classical 'news-based' - approaches, [Bontempi et al. \(2016\)](#) and [Castelnuovo and Tran \(2017\)](#) construct 'Google Trends Uncertainty' indices (they call their index GT- and GTU-index, respectively) based on freely accessible, real time Google Trends data; thereby the focus is shifted from journalists drawing the public's attention to specific topics to the public's individual Internet searches

Figures 1 (on different scales) and 6 (normalized to the same scale) exemplarily show three uncertainty proxies including NBER recessions for the U.S that have been used in the literature. These are the financial market volatility index VXO/realized volatility of stock market returns popularized by [Bloom \(2009\)](#), the macro uncertainty index introduced by [Jurado et al. \(2015\)](#) and the economic policy uncertainty index (EPU) following [Baker et al. \(2015\)](#).

A first preliminary visual comparison highlights that similar to [Bachmann et al. \(2013\)](#)'s assessment in their own comparison, almost all recession periods correspond to elevated uncertainty across all measures. Apart from this commonality, the VXO and EPU are affected by much noisier fluctuations over time than e.g., the macro uncertainty index. The VXO and EPU often move together prior to the crisis with distinct variations due to pronounced reactions of the VXO series to events with a connection to financial markets (e.g., the Asian financial crisis, the WorldCOM fraud, the Lehman Brothers failure, etc.). On the other hand, the EPU shows stronger spikes related to e.g., the Gulf Wars, presidential elections, and fiscal uncertainties due to tax or spending debates which causes the EPU to be elevated during most of the recovery period showing more variation including several spikes. This suggests that the EPU, by construction, indeed reacts much stronger to the political turmoil in the past decade than financial markets themselves.⁶³

⁶³Specifically being aware of these differences between the VXO and their EPU measure, [Baker](#)

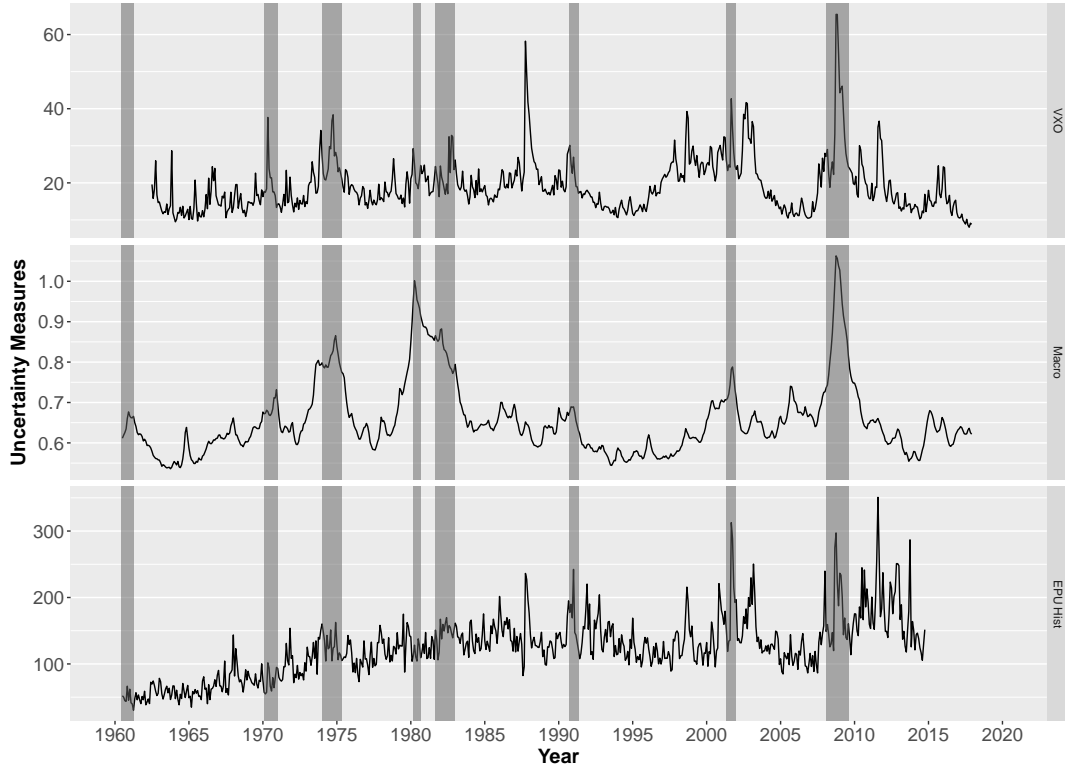


Figure 1.: Comparison of uncertainty measures (faceted). *Note:* Shaded areas denote NBER recession dates. Data frequencies are monthly. The macro uncertainty series starts in July 1960, the EPU in January 1960 and the VXO in July 1962.

The macro uncertainty index overall stands out with an overall smoother trajectory and marked spikes that correspond to recessions but also, as aptly formulated by [Jurado et al. \(2015\)](#) themselves, significant independent variation as compared to the other commonly used uncertainty measures suggesting that “[...] quantitatively important uncertainty episodes occur far more *infrequently*⁶⁴ than what is indicated from common uncertainty proxies, but that when they *do* occur, they display larger and more persistent correlations with real activity” ([Jurado et al., 2015](#), p. 1181). This observation can be confirmed by looking at Figures 1 and 6⁶⁵ where increases in the macro uncertainty estimates are mostly associated with protracted recessions

et al. (2015) construct a variation of their original EPU to be more strongly geared towards uncertainty on equity markets. Naturally, they report this financed-based EPU to correlate more strongly with the VXO/volatility series. For [Baker et al. \(2015\)](#) this exercise serves as a proof-of-concept in so far that a reasonable proxy for a specific *type* of economic uncertainty can indeed be constructed using text-search techniques and that the variations between the VXO/volatility series and the original EPU are important and justified by construction.

⁶⁴Italics added.

⁶⁵And also Table 2 which we will discuss below.

Table 1.: Correlation matrix of uncertainty measures. [check correlation between EPU and Macro! [Baker et al. \(2015\)](#)] report a value of 0.42 while we get 0.15!; also, after re-estimation the correlation between VXO and MACRO decreased from 0.63 to 0.53 -> check again!

	VXO	Macro	EPU
VXO	1	0.490	0.530
Macro	0.490	1	0.300
EPU	0.530	0.300	1

whereas more modest recessions are accompanied with smaller spikes. A period where the macro uncertainty index stands out is the recessionary period from 1980-1982: throughout this episode the index is highly elevated while other measures are comparatively low. The VXO on the other hand also shows spikes outside of recession periods that correspond to events exclusively related to stock-markets (most notably the large spike commonly to as 'Black Monday' (October, 19th 1987) when stock markets experienced their largest single-day percentage decline ever recorded). The macro uncertainty index, however, barely moves around that time. Interestingly, the EPU shows the largest spike surrounding the years 2001-2001 (the dot-com bubble).

Despite an overall co-movement between the series, the identified episodes of substantial variation in the processes' dynamics suggest that "[t]hese [time series] are clearly not measures of the same [latent] stochastic process [...]" ([Orlik and Veldkamp, 2014](#), p. 31).

To shed some light on the above statement, we will comparatively discuss a few univariate and multivariate properties of the various uncertainty measures in turn.

Table 2 reports several summary statistics including key figures regarding the *distribution* of the uncertainty measures, their *cyclical* and its nexus to the business cycle, *persistence* and whether we have any evidence for non-*stationarity*. The results point at a few stylized facts:

First, none of the measures passes the Shapiro-Wilk test, meaning that their respective distributions are significantly different from the normal distribution. This is in line with the figures on skewness and kurtosis: the distribution of changes in the measures are positively skewed and show excess kurtosis (fat tails), most pronounced for VXO and EPU indicating that there are more extreme values in these series as compared to Macro1 and Macro12. Together, these observations point at tails on the right side of the distribution being longer than the ones on the left (meaning that the majority

of the density's mass and the median lies to the left of the means). Similarly to the dynamics of business cycles, the above findings suggest that increases in the uncertainty measures tend to be larger than decreases. In other words, uncertainty increases faster than it decreases (Moore, 2016). Further, across all uncertainty measures, the VXO and EPU suggest a higher variability (according to the variation coefficients) which is also visible from the visual inspection of Figures 1 and 6.

Second, uncertainty appears to be countercyclical. The measures largely show means and variances that are higher during recessions versus expansions. This is further supported by negative contemporaneous correlations of the uncertainty measures with key economic indicators like industrial production and employment in manufacturing. The figures are strongest for the macro uncertainty series (Macro1 and Macro12) while VXO and EPU are more weakly associated with the business cycle. In line with Bloom (2014), uncertainty seems to be higher during recessions.⁶⁶

Third, as persistence seems to be a relevant ingredient of the impact of uncertainty on business cycles (as shown by, among others, Schaal (2017) or Jurado et al. (2015)), the estimated half-life shocks (derived from the AR(1)-coefficients) of Macro1 and Macro12 with 57 and 129 months, respectively, show a much higher persistency than the other uncertainty proxies with approx. 4 months which is “[...] a finding relevant for theories where uncertainty is a driving force of economic downturns, including those with more prolonged periods of below-trend economic growth” (Jurado et al., 2015, p. 1193).

In the literature these measures find both proponents and opponents. With respect to Bloom's derivation of the so-called 'Bloom-shocks' off of the VXO/stock return volatility series, Bontempi et al. (2016, p. 3), for example, write that “[o]n the one hand, the pre-selection of directly observable specific events is easy to perceive but somewhat arbitrary.” While the uncertainty measure as suggested by Jurado et al. (2015) is independent of any single observable economic indicator or event, Bontempi et al. (2016, p. 13) conclude that “[...] it is a computationally intensive black box that is not directly linked to the uncertainty perceived by the general public.” In the course of the construction of their own macro uncertainty measure (see Section 4.0.1) and with respect to the differences between the macro uncertainty index and other uncertainty measures, Jurado et al. (2015) criticize that most movements

⁶⁶The unconditional negative correlations are, of course, uninformative about causation between real activity and uncertainty.

⁶⁷KPSS-test with $H_0 = (\text{trend/level})$ stationary.

Table 2.: Summary Statistics on the Dynamics of Uncertainty Proxies.

Note: IP-corr(k) and EMP-corr(k) are the absolute cross-correlation coefficients between a measure of uncertainty and the 12 month moving average of industrial production or employment in manufacturing growth in period $t + k$, i.e., $IP\text{-corr}(k) = |corr(u_t, \Delta\%IP_{t+k})|$ and $EMP\text{-corr}(k) = |corr(u_t, \Delta\%EMP_{t+k})|$. A positive k means uncertainty is correlated with future IP/EMP. Half-lives are based on estimates from a univariate AR(1) model for each series and calculated as $HL = \ln(2)/\ln(AR(1).coef)$. The sample for each uncertainty measure is the largest available, starting in July 1962 in the case of VXO, EPU, Macro1 and Macro12.

	VXO	EPU	Macro1	Macro12
<i>Distribution</i>				
Coeff. of Variation (σ/μ)	0.33	0.37	0.14	0.08
Skewness (Levels)	1.23	0.77	1.67	1.61
Excess Kurtosis (Levels)	-0.98	-1.21	0.23	-0.08
Skewness (Change)	0.69	0.48	0.75	0.90
Excess Kurtosis (Change)	2.06	3.33	0.35	0.60
Shapiro-Wilk (p-value)	0	0	0	0
<i>Cyclicalilty</i>				
Downturn/Upturn μ ratios	1.35	1.14	1.25	1.13
Downturn/Upturn σ ratios	0.98	1.33	1.72	1.58
IP-corr(0)	-0.36	-0.33	-0.64	-0.61
IP-corr(-12)	-0.15	-0.22	-0.37	-0.33
IP-corr(+12)	-0.13	-0.17	-0.21	-0.29
EMP-corr(0)	-0.43	-0.33	-0.63	-0.57
EMP-corr(-12)	-0.21	-0.24	-0.24	-0.20
EMP-corr(+12)	-0.24	-0.21	-0.34	-0.40
<i>Persistence</i>				
AR(1)	0.83	0.81	0.99	0.99
Half Life (in months)	3.66	3.26	57.8	129
<i>Stationarity Tests</i>				
Level Stationary (p-value) ⁶⁷	0.02	0.01	0.09	0.03
Trend Stationary (p-value)	0.10	0.01	0.01	0.01

in widely used uncertainty proxies such as the VXO/stock market volatility (as the most commonly used) or measures of cross-sectional dispersion measures are not associated with a broad-based movement in economic uncertainty (according to their own definition). For them, this result is important as it “[...] suggests that

much of the variation in common uncertainty proxies is not driven by uncertainty” (Jurado et al., 2015, p. 1180). According to their analysis, the “[...] conditions under which common proxies are likely to be tightly linked to the typical theoretical notion of uncertainty may be quite special” (Jurado et al., 2015, p. 1178). The authors attribute this finding to episodes where stock market volatility changes even if there is no change in uncertainty about economic fundamentals. This can happen if leverage changes, or if movements in risk aversion or sentiment are important drivers of asset market fluctuations (see also Bekaert et al. (2013), Moore (2016) or Ozturk and Sheng (2017)). For them, it is difficult to uphold the assumption that the level of economic uncertainty in October 1987 and the Financial Crisis of 2008/09 were similar (which a measure such as the VXO/stock market volatility would suggest). Further, “[c]ross-sectional dispersion in individual stock returns can fluctuate without any change in uncertainty if there is heterogeneity in the loadings on common risk factors. Similarly, cross-sectional dispersion in firm-level profits, sales and productivity can fluctuate over the business cycle merely because there is heterogeneity in the cyclicalities of firms’ business activity” (Jurado et al., 2015, p. 1178). In the same vein, for Orlik and Veldkamp (2014, p. 30) other uncertainty proxies are informative “[...] but have a less clear connection to a conditional variance definition of uncertainty”. With their uncertainty measure, Jurado et al. (2015) also see another shortcoming of other uncertainty measures fulfilled: other empirical proxies (like e.g. the cross-sectional dispersion in firms’ sales growth) have been argued (among others by Schaal (2011)) to not being persistent enough to be seen as a compounding factor to the prolonged levels of unemployment that have happened during or after some recessions. Regarding the EPU-index, Ozturk and Sheng (2017) state that like all similar measures, these indices rely heavily on the attentiveness of reporters and editors.

2.5. Exogenous Shocks vs. Endogenous Response

Already Bloom (2014) mentions that while uncertainty seems to influence business cycles, it also appears to be endogenously increasing during recessions. This issue plays a crucial role in attempts to disentangle the real effects of uncertainty, meaning that uncertainty itself may be a function of the business cycle. In this respect, popularized VARs employed in the literature might not be capable of adequately separating cause and effect.

Ludvigson et al. (2018) identify two issues in the empirical modeling of uncertainty and point at the following shortcomings: First, in a VAR context (results of which in the literature we have reported in Sections 2.2 and 2.3), the deployed models primarily rely on recursive schemes as an identification strategy whose justification in terms of ordering due to contemporaneous movement of the variables seems sometimes arbitrary and not well grounded.⁶⁸ And because of the wide spectrum of uncertainty theories which does not provide identifying restrictions for empirical work, for Ludvigson et al. (2018, p. 5) the “[...] question of cause and effect is fundamentally an empirical” one. Second, picking up the critique mentioned already in Section 2.2, uncertainty stemming from financial markets might play a different role in business cycles than real economic uncertainty, referring to Ng and Wright (2013) who find that all the post-1982 recessions have their origins in financial market disturbances and that these recessions come along with distinctly different features as recessions where financial uncertainty plays a subordinate role.

Despite showing own variation, the rise of uncertainty in recessions as confirmed by a growing body of literature and our own analysis as suggested by Table 2 is robust to the specific uncertainty measure deployed as a proxy to capture the latent stochastic process of uncertainty. But despite both theoretical and empirical advances that confirm for uncertainty playing a role in business cycles, the question about uncertainty being an exogenous source of business cycle fluctuations or an endogenous response thereof is yet to be solved. As pointed out by Ludvigson et al. (2018, p. 1), “[...] the question of causality and the identification of exogenous variation in uncertainty is a long-standing challenge of the uncertainty literature [which] arises in part because there is no single uncertainty model, hence no theoretical consensus on whether the uncertainty that accompanies deep recessions is primarily a cause or effect (or both) of declines in economic activity.” As we have seen in the discussion in previous chapters, theoretically, most models would point at an adverse effect but even positive effects have been suggested. Even within models pointing at adverse effects we find various reasoning approaches so that in effect there is “[...] no single uncertainty theory or all-encompassing structural model” (Ludvigson et al., 2018, p. 5) that could be used to map to the data.

To account for these identified issues, Ludvigson et al. (2018) suggest a novel identifi-

⁶⁸Ludvigson et al. (2018) also discuss other commonly used identification schemes: sign restrictions, long-run restrictions, IV estimations, etc. Because of ambiguous theoretical signs of the relationships between uncertainty and real activity, the authors conclude that sign restrictions are inappropriate. Similarly they reject IV analysis, stating that it is very difficult to find truly exogenous instruments.

cation strategy within an SVAR framework which differentiates between macro and financial uncertainty and which we will present next the context of our empirical analysis in Section 3.

3. Econometric Model

Ludvigson et al. (2018) suggest a novel identification scheme in an SVAR-context in an attempt to empirically solve the endogeneity problem inherent to widely used VAR-analyses in the uncertainty-literature. This chapter hence first sets the stage by discussing the theoretical background regarding (S)VARs and the so-called identification problem in Section 3.1 followed by a presentation of the econometric framework of Ludvigson et al. (2018) in Section 3.2.

3.1. Preliminaries: Theoretical Background on VARs⁶⁹

3.1.1. (S)VARs and the Derivation of Impulse Responses

We consider a stable (which implies stationarity) bivariate⁷⁰ dynamic stochastic simultaneous equations model in a *structural* VAR (SVAR) representation for the two time series y_{1t} and y_{2t} ⁷¹ (i.e., a bivariate SVAR(1)) whereby the time path of the two time series depend on one lag of each variable and also influence each other contemporaneously (i.e., the two variables are endogenous)⁷² so that

$$\begin{aligned}y_{1t} &= a_{01}y_{2t} + a_{11}y_{1t-1} + a_{12}y_{2t-1} + \epsilon_{1t} \\y_{2t} &= a_{02}y_{1t} + a_{21}y_{1t-1} + a_{22}y_{2t-1} + \epsilon_{2t}\end{aligned}\tag{3.1}$$

⁶⁹This entire section draws heavily from various sources including LIST ALL THE BOOKS and other resources I have used!

⁷⁰Note that we start with a bivariate, i.e., two-equations-VAR for ease of exposition. As soon as we switch to matrix notation, the case of $n = 2$ versus $n > 2$ (for a multivariate VAR-system) is negligible.

⁷¹In all our following notation we abstract from deterministic regressors (i.e., trend or constant) and exogenous regressors, meaning that we only consider the stochastic part of a DGP.

⁷²Hence, the example we start with is an SVAR(1).

with

$$\boldsymbol{\epsilon}_t \sim i.i.d \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix} \right), \quad (3.2)$$

meaning that the error terms $\boldsymbol{\epsilon}_t$ are exogenous and independent and identically distributed for $t = 1, \dots, T$, i.e., white noise.⁷³ A slight rearrangement of Equation 3.1 gives

$$\begin{aligned} y_{1t} - a_{01}y_{2t} + &= a_{11}y_{1t-1} + a_{12}y_{2t-1} + \epsilon_{1t} \\ y_{2t} - a_{02}y_{1t} + &= a_{21}y_{1t-1} + a_{22}y_{2t-1} + \epsilon_{2t} \end{aligned} \quad (3.3)$$

or

$$\begin{bmatrix} 1 & -a_{01} \\ -a_{02} & 1 \end{bmatrix} \begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} y_{1t-1} \\ y_{2t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{bmatrix} \quad (3.4)$$

which, in matrix notation, compactly reduces to

$$\mathbf{A}_0 \mathbf{y}_t = \mathbf{A}_1 \mathbf{y}_{t-1} + \boldsymbol{\epsilon}_t \quad (3.5)$$

with \mathbf{A}_0 being called the *impact matrix* containing the contemporaneous effects of an increase of each endogenous variable on the other variable, respectively and $\boldsymbol{\epsilon}_t$ is a white noise process with

$$\begin{aligned} \mathbb{E}(\boldsymbol{\epsilon}_t) &= \mathbf{0} \\ \mathbb{E}[\boldsymbol{\epsilon}_t \boldsymbol{\epsilon}_\tau'] &= \begin{cases} \boldsymbol{\Sigma}_\epsilon, & \text{for } t = \tau \\ \mathbf{0}, & \text{otherwise.} \end{cases} \end{aligned} \quad (3.6)$$

Hence the elements off the main diagonal are zero, i.e., structural shocks are assumed to be uncorrelated. A VAR in such a structural representation can be translated into its *reduced form* representation, i.e., a standard VAR model (in this case a VAR(1) because we are considering only one lag at the moment), by premultiplying 3.5 with the inverse of the impact matrix \mathbf{A}_0 (assuming that it exists) which gives

$$\begin{aligned} \mathbf{A}_0^{-1} \mathbf{A}_0 \mathbf{y}_t &= \mathbf{A}_0^{-1} \mathbf{A}_1 \mathbf{y}_{t-1} + \mathbf{A}_0^{-1} \boldsymbol{\epsilon}_t \iff \\ \iff \mathbf{y}_t &= \mathbf{B}_1 \mathbf{y}_{t-1} + \mathbf{u}_t \end{aligned} \quad (3.7)$$

⁷³Note that for the moment we do not yet impose normality of the errors.

and which eliminates the previous complication due to contemporaneous relationships with $\mathbf{B}_1 = \mathbf{A}_0^{-1}\mathbf{A}_1 \iff \mathbf{A}_1 = \mathbf{A}_0\mathbf{B}_1$ and $\mathbf{u}_t = \mathbf{A}_0^{-1}\boldsymbol{\epsilon}_t \iff \boldsymbol{\epsilon}_t = \mathbf{A}_0\mathbf{u}_t$.⁷⁴

The reduced form errors \mathbf{u}_t are linear combinations of the structural errors $\boldsymbol{\epsilon}_t$ with the following reduced-form variance-covariance matrix⁷⁵

$$\begin{aligned}\boldsymbol{\Sigma}_u &\equiv \mathbb{E}[\mathbf{u}_t\mathbf{u}_t'] = \mathbb{E}[(\mathbf{A}_0^{-1}\boldsymbol{\epsilon}_t)(\mathbf{A}_0^{-1}\boldsymbol{\epsilon}_t)'] = \\ &= \mathbb{E}[\mathbf{A}_0^{-1}\boldsymbol{\epsilon}_t\boldsymbol{\epsilon}_t'\mathbf{A}_0^{-1'}] = \\ &= \mathbf{A}_0^{-1}\mathbb{E}[\boldsymbol{\epsilon}_t\boldsymbol{\epsilon}_t']\mathbf{A}_0^{-1'} = \\ &= \mathbf{A}_0^{-1}\boldsymbol{\Sigma}_\epsilon\mathbf{A}_0^{-1'}\end{aligned}\tag{3.8}$$

The variance-covariance matrix $\boldsymbol{\Sigma}_u$ of the reduced-form VAR is assumed to be a symmetric positive definite matrix which, however, may not be a diagonal matrix due to instantaneous correlations between the error-terms. As a consequence, isolated shocks in the components of \mathbf{u}_t may not be likely. This reduced form VAR (1) is covariance stationary *if and only if* the eigenvalues of \mathbf{A}_1 have modulus less than 1.

If we assume contemporaneous relationships among the endogenous variables, the regressors in the SVAR representation are correlated with the error term and hence would result in biased OLS estimates.⁷⁶ Therefore, in econometric applications the VAR in its reduced form representation is being estimated from the data. While the reduced-form VAR is the one being estimated, it is a purely econometric model without any theoretical component and does not say anything about the structure of the economy and hence the reduced-form error terms \mathbf{u}_t cannot be interpreted as structural shocks.⁷⁷ Rather, the goal is to get back to the structural representation with a diagonal covariance matrix and economic meaning.⁷⁸ Without imposing any further restrictions, however, the parameters in the SVAR, i.e., models with contemporaneous relationships,⁷⁹ are not identified, i.e., given values of the reduced form parameters \mathbf{B}_1 and $\boldsymbol{\Sigma}_u$, it is not possible to uniquely solve for the structural

⁷⁴Note that here we are still deriving all results for a bivariate vector autoregression of order 1, i.e., a VAR(p). For a multivariate system with $n > 2$, accordingly we would write that the relationship between each coefficient matrix in reduced form \mathbf{B}_j and structural form \mathbf{A}_j is governed by $\mathbf{B}_j = \mathbf{A}_0^{-1}\mathbf{A}_j$.

⁷⁵Note that in the bivariate case, $\boldsymbol{\Sigma}_u$ is a diagonal matrix only if $-a_{01} = -a_{02} = 0$, i.e., only if there is no contemporaneous correlation.

⁷⁶If $\mathbf{A}_0 = \mathbf{I}_n$ then this is not the case.

⁷⁷Note that in the estimated covariance matrix for $\hat{\boldsymbol{\Sigma}}_u$ the respective errors are usually correlated so that shocks to one variable are usually accompanied with a response to other variables.

⁷⁸In practice, the error series in reduced-form VARs are usually correlated while in a structural setting correlated reduced-form shocks are broken down into uncorrelated structural shocks that allow a clearer assessment of effects within a model.

⁷⁹In a model without contemporaneous relationship no identification issues would emerge.

parameters \mathbf{A}_0 , \mathbf{A}_1 and Σ_ϵ . In particular, the knowledge of the impact matrix \mathbf{A}_0^{-1} would allow to recover \mathbf{A}_1 via $\mathbf{A}_1 = \mathbf{A}_0\mathbf{B}_1$ and the structural errors via $\epsilon_t = \mathbf{A}_0\mathbf{u}_t$ and subsequently the variance covariance matrix of the structural errors Σ_ϵ .⁸⁰

Abstracting from possible identification schemes for \mathbf{A}_0^{-1} for the moment, under the assumption of covariance stationarity (i.e., if neither the mean nor the autocovariance of the process depend on time t) of the vector process \mathbf{y}_t , \mathbf{y}_t (i.e., the reduced form VAR) can be transformed into its vector moving average (VMA(∞); also called Wold MA) representation according to the Wold decomposition theorem⁸¹ (i.e., there exists a unique mapping) whereby all past values of \mathbf{y}_t are substituted out which translates Equation 3.7 into a linear combination of all \mathbf{u}_t s (reduced-form innovations) over time (whereby the corresponding MA-weights do not depend on time t but only on j , i.e., how long ago the shock \mathbf{u} occurred):

$$\begin{aligned} \mathbf{y}_t &= \boldsymbol{\mu} + \boldsymbol{\Psi}_0\mathbf{u}_t + \boldsymbol{\Psi}_1\mathbf{u}_{t-1} + \boldsymbol{\Psi}_2\mathbf{u}_{t-2} + \boldsymbol{\Psi}_3\mathbf{u}_{t-3} + \cdots = \\ &= (\text{or more compactly written}) = \\ &= \boldsymbol{\mu} + \sum_{k=0}^{\infty} \boldsymbol{\Psi}_k\mathbf{u}_{t-k} \end{aligned} \tag{3.9}$$

with $\boldsymbol{\Psi}_0 = \mathbf{I}_2$ and $\boldsymbol{\Psi}_k$ can be computed recursively via $\boldsymbol{\Psi}_k = \sum_{j=1}^k \boldsymbol{\Psi}_{k-j}\mathbf{B}_j$ for $k = 1, 2, \dots$.^{82,83}

⁸⁰In the bivariate case we consider here, at least 1 restriction on the parameters of the SVAR is required to enable the identification of all structural parameters.

⁸¹In words, the Wold Theorem states that any covariance stationary process has an infinite order, moving-average representation.

⁸²Note that we are still looking at the bivariate case. For the n -dimensional case, we would have $\boldsymbol{\Psi}_0 = \mathbf{I}_n$.

⁸³Note that Equation 3.9 shows the general reduced-form VMA(∞) representation for a VAR(p). In case of our bivariate example with one lag only (see Equation 3.7), the reduced-form VMA-representation would be $\mathbf{y}_t = \boldsymbol{\mu} + \mathbf{I}_2\mathbf{u}_t + \mathbf{B}_1\mathbf{u}_{t-1} + \mathbf{B}_1^2\mathbf{u}_{t-2} + \mathbf{B}_1^3\mathbf{u}_{t-3} + \cdots$. In other words, for a VAR(1)-process, the recursion $\boldsymbol{\Psi}_k = \sum_{j=1}^k \boldsymbol{\Psi}_{k-j}\mathbf{B}_j$ implies that $\boldsymbol{\Psi}_0 = \mathbf{I}_n$, $\boldsymbol{\Psi}_1 = \mathbf{B}_1$, \dots , $\boldsymbol{\Psi}_i = \mathbf{B}_1^i$.

For a VAR(2)-process, for example, the recursion $\boldsymbol{\Psi}_k = \sum_{j=1}^k \boldsymbol{\Psi}_{k-j}\mathbf{B}_j$ results in $\boldsymbol{\Psi}_1 = \mathbf{B}_1$, $\boldsymbol{\Psi}_2 = \boldsymbol{\Psi}_1\mathbf{B}_1 + \mathbf{B}_2 = \mathbf{B}_1^2 + \mathbf{B}_2$, \dots , $\boldsymbol{\Psi}_i = \boldsymbol{\Psi}_{i-1}\mathbf{B}_1 + \boldsymbol{\Psi}_{i-2}\mathbf{B}_2$, \dots . The coefficient matrices $\boldsymbol{\Psi}_i$ approach zero for $i \rightarrow \infty$ which is a consequence of the stability of the respective VAR processes. Correspondingly, Equation 3.10 becomes $\mathbf{y}_t = \boldsymbol{\mu} + \mathbf{I}_2\mathbf{A}_0^{-1}\epsilon_t + \mathbf{B}_1\mathbf{A}_0^{-1}\epsilon_{t-1} + \mathbf{B}_1^2\mathbf{A}_0^{-1}\epsilon_{t-2} + \mathbf{B}_1^3\mathbf{A}_0^{-1}\epsilon_{t-3} + \cdots$. So for the structural VAR(1) model, the impulse responses to the structural shocks from n period are given by $\mathbf{B}_1^n\mathbf{A}_0^{-1}$. The MA representation of a stable (S)VAR(p) process is not necessarily of infinite order since that coefficients may turn zero as of a certain lag.

The corresponding *structural* moving average (SMA(∞); structural Wold MA) representation of \mathbf{y}_t is based on an infinite moving average of the *structural* innovations $\boldsymbol{\epsilon}_t$ and is obtained by substituting the mapping between structural and reduced form shocks $\mathbf{u}_t = \mathbf{A}_0^{-1}\boldsymbol{\epsilon}_t$ for all t into Equation 3.9 so that

$$\begin{aligned} \mathbf{y}_t &= \boldsymbol{\mu} + \boldsymbol{\Psi}_0 \mathbf{A}_0^{-1} \boldsymbol{\epsilon}_t + \boldsymbol{\Psi}_1 \mathbf{A}_0^{-1} \boldsymbol{\epsilon}_{t-1} + \boldsymbol{\Psi}_2 \mathbf{A}_0^{-2} \boldsymbol{\epsilon}_{t-2} + \boldsymbol{\Psi}_3 \mathbf{A}_0^{-1} \boldsymbol{\epsilon}_{t-3} + \cdots = \\ &= (\text{or more compactly written}) = \\ &= \boldsymbol{\mu} + \sum_{k=0}^{\infty} \boldsymbol{\Psi}_k \mathbf{A}_0^{-1} \boldsymbol{\epsilon}_{t-k} \\ &= \boldsymbol{\mu} + \sum_{k=0}^{\infty} \boldsymbol{\Theta}_k \boldsymbol{\epsilon}_{t-k} \end{aligned} \quad (3.10)$$

which means that $\boldsymbol{\Theta}_k = \boldsymbol{\Psi}_k \mathbf{A}_0^{-1}$ for $k = 0, 1, \dots$. In particular, it holds that $\boldsymbol{\Theta}_0 = \mathbf{A}_0^{-1} \neq \mathbf{I}_2$ (in contrast to the VAR-coefficient $\boldsymbol{\Psi}_0$ shown above).

Looking at the SMA(∞) representation of our bivariate system

$$\begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} + \begin{bmatrix} \theta_{11}^{(0)} & \theta_{12}^{(0)} \\ \theta_{21}^{(0)} & \theta_{22}^{(0)} \end{bmatrix} \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{bmatrix} + \begin{bmatrix} \theta_{11}^{(1)} & \theta_{12}^{(1)} \\ \theta_{21}^{(1)} & \theta_{22}^{(1)} \end{bmatrix} \begin{bmatrix} \epsilon_{1t-1} \\ \epsilon_{2t-1} \end{bmatrix} + \cdots \quad (3.11)$$

we see that the elements of the $\boldsymbol{\Theta}_k$ matrices, $\theta_{ij}^{(k)}$, are the dynamic multipliers/impulse responses of y_{1t} and y_{2t} to changes in ϵ_{1t} and ϵ_{2t} .

Generalizing the above SMA(∞) representation to time $t + s$

$$\begin{bmatrix} y_{1t+s} \\ y_{2t+s} \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} + \begin{bmatrix} \theta_{11}^{(0)} & \theta_{12}^{(0)} \\ \theta_{21}^{(0)} & \theta_{22}^{(0)} \end{bmatrix} \begin{bmatrix} \epsilon_{1t+s} \\ \epsilon_{2t+s} \end{bmatrix} + \cdots + \begin{bmatrix} \theta_{11}^{(s)} & \theta_{12}^{(s)} \\ \theta_{21}^{(s)} & \theta_{22}^{(s)} \end{bmatrix} \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{bmatrix} + \cdots \quad (3.12)$$

or more compactly to

$$\mathbf{y}_{t+s} = \boldsymbol{\mu} + \boldsymbol{\Theta}_0 \boldsymbol{\epsilon}_{t+s} + \cdots + \boldsymbol{\Theta}_s \boldsymbol{\epsilon}_t + \cdots \quad (3.13)$$

shows that the $\boldsymbol{\Theta}$ - matrices consisting of the MA-coefficients hold the structural dynamic multipliers (the impulse responses). In our bivariate example, these impulse

responses are

$$\begin{aligned}\frac{\partial y_{1t+s}}{\partial \epsilon_{1t}} &= \theta_{11}^{(s)}, \frac{\partial y_{1t+s}}{\partial \epsilon_{2t}} = \theta_{12}^{(s)} \\ \frac{\partial y_{2t+s}}{\partial \epsilon_{1t}} &= \theta_{21}^{(s)}, \frac{\partial y_{2t+s}}{\partial \epsilon_{2t}} = \theta_{22}^{(s)}\end{aligned}\tag{3.14}$$

or more compactly

$$\frac{\partial \mathbf{y}_{t+s}}{\partial \boldsymbol{\epsilon}_t} = \boldsymbol{\Theta}_s\tag{3.15}$$

meaning that in matrix $\boldsymbol{\Theta}_s$ the entry (i, j) shows the impact of a one-unit increase in the j th variable's innovation at date t (ϵ_{jt}) on the value of the i th variable at time $t + s$ ($y_{i,t+s}$).

Subsequently, a plot of the elements (i, j) of the respective structural MA matrices $\{\boldsymbol{\Theta}_s\}_{i,j}$ as a function of s , i.e.,

$$\frac{\partial y_{t+s}}{\partial \epsilon_t}\tag{3.16}$$

is the *impulse-response function* that describes the response of $y_{i,t+s}$ to a one-time one-unit impulse in y_{jt} with all other variables dated t or earlier held constant.

3.1.2. Structural VARs and the Identification Problem

The above derivations have not yet explained the identification of \mathbf{A}_0^{-1} that is needed to seamlessly connect the estimated reduced-form VAR with the structural VAR that is of actual interest.

The usual strategy is to first estimate the reduced-form system as in Equation 3.7. In this VAR(1)-model this amounts to estimating $n^2 + \frac{n(n+1)}{2}$ parameters (n^2 for the coefficient matrix \mathbf{B}_1 and $\frac{n(n+1)}{2}$ for the variance covariance-matrix of the reduced-form errors).

In our bivariate case above, we concluded that we need at least one restriction on the parameters of Equation 3.5 so that the system of equations becomes solvable. Identification schemes include, among others, zero short-run restrictions (known as Choleski identification), sign restrictions, zero long-run restrictions (also called Blanchard-Quah following [Blanchard and Quah, 1989](#)), etc.

For example, imposing a short-run restriction by assuming that the impact matrix

\mathbf{A}_0 is lower triangular with

$$\mathbf{A}_0 = \begin{bmatrix} 1 & 0 \\ -a_{02} & 1 \end{bmatrix} \quad (3.17)$$

i.e., the restriction that $-a_{01} = 0$, is sufficient to just identify $-a_{02}$ (from the reduced form covariance matrix Σ_u ; shown below) and subsequently results in

$$\mathbf{A}_0^{-1} = \mathbf{\Theta}_0 = \begin{bmatrix} 1 & 0 \\ a_{02} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \theta_{21}^{(0)} & 1 \end{bmatrix} \quad (3.18)$$

for the SMA representation imposing the restriction that the value y_{2t} does not have a contemporaneous effects on y_{1t} while due to $-a_{02} \neq 0$ we would allow for the reverse. The reasoning behind such an identification strategy usually stems from arguments that declare certain variable to be sticky, meaning that they do not respond immediately to certain shocks. Further, we see that the restriction $a_{01} = 0$ in the SVAR representation is equivalent to assuming $\theta_{12}^{(0)} = 0$ in the SMA, meaning that ϵ_{1t} has no contemporaneous impact on y_{2t} .

With \mathbf{A}_0 being a lower triangular matrix of the above form, further the reduced form VAR errors $\mathbf{u}_t = \mathbf{A}_0^{-1} \epsilon_t$ become

$$\mathbf{u}_t = \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ a_{02} & 1 \end{bmatrix} \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{bmatrix} = \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} + a_{01}\epsilon_{1t} \end{bmatrix} \quad (3.19)$$

The SVAR representation derived under above the assumption hence establishes a recursive causal ordering (i.e., a cascading causal chain of shocks) which may be computed using the Choleski factorization of the reduced form covariance matrix Σ_u (the Choleski identification is hence also called recursive identification).⁸⁴

Writing down the Choleski factorization of the positive semi-definite matrix Σ_u gives

$$\Sigma_u = \mathbf{P}\mathbf{P}' \quad \text{with} \quad \mathbf{P} = \begin{bmatrix} p_{11} & 0 \\ p_{21} & p_{22} \end{bmatrix} \quad (3.20)$$

⁸⁴In this identification scheme the order of the variables entering the VAR matters because the variable placed on top is assumed to be the most exogenous and is only affected by a shock to itself. The actual ordering depends on the researcher's own thinking about the most likely propagation chain a shock will adhere to. Further note, that establishing that certain shock have effects only on some variables at a certain point in time is identical to certain *variables* having effects only on some variable at a certain point in time.

with \mathbf{P} being a lower triangular matrix with $p_{ii} \leq 0, i = 1, 2$ ⁸⁵. A closely related variant of the this classical representation of the Cholesky decomposition is the $\mathbf{L}\mathbf{\Lambda}\mathbf{L}'$ decomposition (triangular factorization) in the form of

$$\mathbf{\Sigma}_u = \mathbf{L}\mathbf{\Lambda}\mathbf{L}' \quad (3.21)$$

where \mathbf{L} is a lower triangular matrix with 1s along the diagonal and $\mathbf{\Lambda}$ is a diagonal matrix with non-negative elements

$$\mathbf{L} = \begin{bmatrix} 1 & 0 \\ l_{21} & 1 \end{bmatrix}, \mathbf{\Lambda} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}, \lambda_i \leq 0, i = 1, 2 \quad (3.22)$$

whereby the representation of this decomposition is related to the classical notation of the Cholesky decomposition of the form $\mathbf{P}\mathbf{P}'$ via

$$\begin{aligned} \mathbf{\Sigma}_u &= \mathbf{L}\mathbf{\Lambda}\mathbf{L}' = \mathbf{L}\mathbf{\Lambda}^{1/2}(\mathbf{\Lambda}^{1/2})'\mathbf{L}' = (\mathbf{L}\mathbf{\Lambda})^{1/2} \left((\mathbf{L}\mathbf{\Lambda})^{1/2} \right)' \\ &= \mathbf{P}\mathbf{P}'. \end{aligned} \quad (3.23)$$

In other words, the triangular decomposition $\mathbf{\Sigma}_u = \mathbf{L}\mathbf{\Lambda}\mathbf{L}'$ is obtained from the Choleski decomposition $\mathbf{\Sigma}_u = \mathbf{P}\mathbf{P}'$ by defining a diagonal matrix \mathbf{D} which has the same main diagonal as \mathbf{P} and by specifying $\mathbf{L} = \mathbf{P}\mathbf{D}^{-1}$ and $\mathbf{\Lambda} = \mathbf{D}\mathbf{D}'$.

Starting from a reduced form VAR

$$\begin{aligned} \mathbf{y}_t &= \mathbf{A}_1\mathbf{y}_{t-1} + \mathbf{u}_t \\ \mathbf{\Sigma}_u &= \mathbb{E}[\mathbf{u}_t\mathbf{u}_t'] \end{aligned} \quad (3.24)$$

performing the triangular factorization on the covariance matrix $\mathbf{\Sigma}_u$ and constructing a pseudo SVAR model by premultiplying the reduced form VAR by \mathbf{L}^{-1} gives

$$\mathbf{L}^{-1}\mathbf{y}_t = \mathbf{L}^{-1}\mathbf{A}_1\mathbf{y}_{t-1} + \mathbf{L}^{-1}\mathbf{u}_t \quad (3.25)$$

⁸⁵The distinction between the application of a Choleski decomposition and the triangular factorization, which we will discuss in turn, in the context of VARs is very subtle and is related to whether a (reduced-form) VAR is estimated and the resulting $\mathbf{\Theta}_j$ impulse responses are simply mechanically orthogonalized which results in the orthogonalized impulses having unit variance or whether, in the context of an SVAR, a triangular factorization as described next is established and \mathbf{A}_0 is chosen in a way which implies a Wold causal ordering which, qualitatively, establishes the same orthogonalized impulse responses as well without, however, necessarily having unit variances for the ϵ_{ts} .

which can be rewritten to

$$\mathbf{A}_0 \mathbf{y}_t = \mathbf{A}_1 \mathbf{y}_{t-1} + \boldsymbol{\epsilon}_t \quad (3.26)$$

with $\mathbf{A}_0 = \mathbf{L}^{-1}$, $\mathbf{A}_1 = \mathbf{L}^{-1} \mathbf{B}_1$, $\boldsymbol{\epsilon}_t = \mathbf{L}^{-1} \mathbf{u}_t$.

Hence, with a proper choice of \mathbf{A}_0 , the pseudo structural errors $\boldsymbol{\epsilon}_t$ have a diagonal covariance matrix $\boldsymbol{\Lambda}$ resulting from

$$\begin{aligned} \mathbb{E}[\boldsymbol{\epsilon}_t \boldsymbol{\epsilon}_t'] &= \mathbf{L}^{-1} \mathbb{E}[\mathbf{u}_t \mathbf{u}_t'] \mathbf{L}^{-1'} = \\ &= \mathbf{L}^{-1} \boldsymbol{\Sigma}_u \mathbf{L}^{-1'} = \\ &= \mathbf{L}^{-1} \mathbf{L} \boldsymbol{\Lambda} \mathbf{L}' \mathbf{L}^{-1'} = \\ &= \boldsymbol{\Lambda}, \end{aligned} \quad (3.27)$$

meaning that the matrix \mathbf{L}^{-1} rescales the reduced-form errors to norm λ and accounts for (eliminates) the correlation between the reduced-form errors. The (i, j) element of $\boldsymbol{\Lambda}$ gives the variance of u_{ji} . Ultimately the structural \mathbf{A}_0 - matrix is

$$\mathbf{A}_0 = \begin{bmatrix} 1 & 0 \\ -a_{02} & 1 \end{bmatrix} = \mathbf{L}^{-1} = \begin{bmatrix} 1 & 0 \\ t_{21} & 1 \end{bmatrix} \quad (3.28)$$

The above result can also be seen by the following:

Knowing that one way to orthogonalize impulse-responses in a reduced-form VAR with possibly correlated errors in $\boldsymbol{\Sigma}_u$ is the Choleski decomposition (which gives rise to the Wold causal ordering), when the reduced-form variance-covariance matrix instead is decomposed in its triangular factorization as $\boldsymbol{\Sigma}_u = \mathbf{L} \boldsymbol{\Lambda} \mathbf{L}'$ and knowing that $\boldsymbol{\Sigma}_u \equiv \mathbb{E}[\mathbf{u}_t \mathbf{u}_t'] = \mathbf{A}_0^{-1} \boldsymbol{\Sigma}_\epsilon \mathbf{A}_0^{-1'}$ (which is equivalent to $\boldsymbol{\Sigma}_\epsilon = \mathbf{A}_0 \boldsymbol{\Sigma}_u \mathbf{A}_0'$), establishes that $\mathbf{L} = \mathbf{A}_0^{-1}$ and $\boldsymbol{\Sigma}_\epsilon = \boldsymbol{\Lambda}$.

The Wold causal ordering (as one example of restrictions) implied in \mathbf{A}_0^{-1} being a lower triangular matrix with $\text{diag}(\mathbf{A}_0^{-1}) = 1$ causes the system of equations $\boldsymbol{\Sigma}_\epsilon = \mathbf{A}_0^{-1} \boldsymbol{\Sigma}_u \mathbf{A}_0^{-1'}$ having a unique solution (at least locally).

It is particularly this mechanical standard Wold-causal ordering of the elements of \mathbf{y}_t on which the VAR literature often relies to establish the triangular factorization of the reduced-form variance-covariance matrix $\boldsymbol{\Sigma}_u$ which, for example, [Jordà \(2005\)](#) criticizes. At the same time, [Jordà \(2005, p. 4\)](#) finds that the “statistical-based structural identification of contemporaneous causal links” is a difficult venture. The

framework of [Ludvigson et al. \(2018\)](#) makes a suggestion for this issue where the \mathbf{P} -matrix to orthogonalize the reduced-form disturbances is not mechanically constructed as the Cholesky decomposition of the error covariance matrix (zero short-run restrictions; recursive identification) but rather is obtained from contemporaneous and theory-based restrictions placed on the \mathbf{P} - matrix.

3.2. Econometric Framework of [Ludvigson et al. \(2018\)](#)⁸⁶

As outlined in Section 2.5, [Ludvigson et al. \(2018, p. 2\)](#) ask, first, whether uncertainty is “primarily a source of business cycle fluctuations or a consequence of them” and, second, whether models of uncertainty should distinguish between financial and ‘real’ macroeconomic uncertainty. Their suggested model hence accounts for the distinction between both, types of uncertainty and introduces a novel identification strategy that “allows for simultaneous feedback between uncertainty and real activity” which the authors achieve by means of two types of *shock-based* restrictions consisting of (as they call them) “event constraints” and “correlation constraints” within the class of SVAR models.

3.2.1. Foundation: Observational Equivalence

As formulated in [Rubio-Ramírez et al. \(2010\)](#), the identification of structural vector autoregressions (i.e., drawing inference from the reduced-form representation back to the underlying structural parameters in SVARs) is at the root of the ‘identification problem’ which forces researches to come up with additional ‘*a priori*’ restrictions⁸⁷, so-called “identifying restrictions”⁸⁸.

In this setting due to the symmetry of the variance covariance matrix Σ_u , only $\frac{n(n+1)}{2}$ different equations are specified and a necessary condition (as a standard criterion for identification) is that further $\frac{n(n-1)}{2}$ restrictions are needed to identify all n^2 elements of \mathbf{B}_0 ⁸⁹ with n being the number of endogenous variables (called

⁸⁶Note that the exposition in [Ludvigson et al. \(2018\)](#) is based on general work on shock-restricted SVARs in [Ludvigson et al. \(2017\)](#).

⁸⁷Author’s italics.

⁸⁸Author’s quotes.

⁸⁹Note to self: I might have to change the notation here from \mathbf{B}_0 to \mathbf{A}_0 depending on whether or not [Ludvigson et al. \(2018\)](#) actually consider an A or B-model.

the necessary “order condition” by [Rothenberg, 1971](#)).^{90,91}

Referring to [Rothenberg \(1971\)](#) and the fact that the “order condition” is only a necessary condition, [Rubio-Ramírez et al. \(2010\)](#) investigate under which conditions such models are globally identified and introduce a novel approach to global identification by exploiting the structure of orthogonal matrices. Central to their paper is the following:

In line with our own notation, the class of SVARs that [Rubio-Ramírez et al. \(2010\)](#) study have the general form

$$\mathbf{A}_0 \mathbf{y}_t = \sum_{j=0}^{\infty} \mathbf{A}_j \mathbf{y}_{t-j} + \boldsymbol{\epsilon}_t \quad (3.29)$$

where they assume

$$\begin{aligned} \mathbb{E}(\boldsymbol{\epsilon}_t) &= \mathbf{0} \\ \mathbb{E}[\boldsymbol{\epsilon}_t \boldsymbol{\epsilon}_t'] &= \boldsymbol{\Sigma}_\epsilon = \mathbf{I}_n. \end{aligned} \quad (3.30)$$

Compactly writing $\mathbf{A}'_+ = [\mathbf{A}'_1, \mathbf{A}'_2, \dots, \mathbf{A}'_p]$ and $\mathbf{x}'_t = [\mathbf{y}'_{t-1}, \mathbf{y}'_{t-2}, \dots, \mathbf{y}'_{t-p}]$, they rewrite their model as

$$\mathbf{A}_0 \mathbf{y}_t = \mathbf{x}_t \mathbf{A}_+ + \boldsymbol{\epsilon}_t \quad (3.31)$$

with the reduced-form representation implied by the structural model being

$$\mathbf{y}_t = \mathbf{x}_t \mathbf{B} + \mathbf{u}_t \quad (3.32)$$

⁹⁰Similarly to the above A -models, if \mathbf{B}_0 is chosen according to a Choleski decomposition resulting in \mathbf{B}_0 being lower-triangular, enough restrictions make the system solvable.

⁹¹The case of $n = 3$ illustrates the problem: $\boldsymbol{\Sigma}_u = \mathbf{B}_0 \mathbf{B}_0'$ results in

$$\underbrace{\begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}}_{\text{6 values}} = \underbrace{\begin{bmatrix} b_{11}^0 & b_{12}^0 & b_{13}^0 \\ b_{21}^0 & b_{22}^0 & b_{23}^0 \\ b_{31}^0 & b_{32}^0 & b_{33}^0 \end{bmatrix}}_{\text{9 unknowns}}^{-1} \begin{bmatrix} b_{11}^0 & b_{12}^0 & b_{13}^0 \\ b_{21}^0 & b_{22}^0 & b_{23}^0 \\ b_{31}^0 & b_{32}^0 & b_{33}^0 \end{bmatrix}^{-1'}$$

where 6 equations are not sufficient to solve the system of linear equations consisting of 9 unknowns. ACHTUNG! ICH HAB' HIER DIE INVERSE DAZUGESCHRIEBEN IN ANLEHNUNG AN \mathbf{A}_0 ; WENN ES DABEI BLEIBT; SOLLTE ICH \mathbf{A}_0 ANSTATT \mathbf{B} HINSCHREIBEN!

with $\mathbf{B} = \mathbf{A}_0^{-1}\mathbf{A}_+$ and $\mathbf{u}_t = \mathbf{A}_0^{-1}\boldsymbol{\epsilon}_t$ and

$$\mathbb{E}[\mathbf{u}_t \mathbf{u}_t'] = \boldsymbol{\Sigma}_u = (\mathbf{A}_0^{-1} \mathbf{A}_0^{-1'}) \quad (3.33)$$

The parameters of the structural model being $(\mathbf{A}_0, \mathbf{A}_+)$ and of the reduced-form model $(\mathbf{B}, \boldsymbol{\Sigma}_u)$, [Rubio-Ramírez et al. \(2010\)](#) denote the set of all structural parameters by \mathbb{P}^S and the set of all reduced-form parameters by \mathbb{P}^R and define the function $g : \mathbb{P}^S \rightarrow \mathbb{P}^R : g(\mathbf{A}_0, \mathbf{A}_+) = (\mathbf{A}_0^{-1}\mathbf{A}_+, \mathbf{A}_0^{-1}\mathbf{A}_0^{-1'})$ for the relationship between structural and reduced-form parameters.

Equipped with this notation, [Rubio-Ramírez et al. \(2010\)](#) define the important concept of 'observational equivalence' as follows:

Following [Rothenberg \(1971\)](#), two parameter points, $(\mathbf{A}_0, \mathbf{A}_+)$ and $(\widetilde{\mathbf{A}}_0, \widetilde{\mathbf{A}}_+)$, are considered *observationally equivalent* iff they both imply the same probability distribution of the data \mathbf{y}_t . For linear Gaussian models as studied by [Rubio-Ramírez et al. \(2010\)](#), this turns out to be equivalent to postulating that two parameter points are *observationally equivalent* iff they both have the same reduced-form representation $(\mathbf{B}, \boldsymbol{\Sigma}_u)$. Based on this, [Rubio-Ramírez et al. \(2010\)](#) revert back to their function g and conclude that it shows that two parameter points $(\mathbf{A}_0, \mathbf{A}_+)$ and $(\widetilde{\mathbf{A}}_0, \widetilde{\mathbf{A}}_+)$ have the same reduced-form representation iff “there is an orthogonal matrix \mathbf{Q} such that $\mathbf{A}_0 = \widetilde{\mathbf{A}}_0 \mathbf{Q}$ and $\mathbf{A}_+ = \widetilde{\mathbf{A}}_+ \mathbf{Q}$ ”. In this context, [Rubio-Ramírez et al. \(2010\)](#) write down two definitions:

Definition 3.1. A parameter point $(\mathbf{A}_0, \mathbf{A}_+)$ is globally identified if and only if there is no other parameter point that is observationally equivalent.

Definition 3.2. A parameter point $(\mathbf{A}_0, \mathbf{A}_+)$ is locally identified if and only if there is an open neighborhood about $(\mathbf{A}_0, \mathbf{A}_+)$ containing no other observationally equivalent parameter point.

Observational equivalence is inherently linked to the identification problem in SVARs which, with respect to the definitions above and [Rubio-Ramírez et al. \(2010\)](#)'s formulation are “neither globally nor locally identified”. This situation calls for adequate restrictions on the part of the structural parameters to make a model identifiable. And because observational equivalence is identical to finding an orthogonal matrix \mathbf{Q} as described above, the set of all $(n \times n)$ orthogonal matrices \mathbb{Q}_n plays a central role both in their analysis and in the derivation of the econometric framework of

Ludvigson et al. (2018) which will discuss in turn.

3.2.2. The Model Setup

Ludvigson et al. (2018) assume the following reduced-form finite order VAR representation and corresponding infinite MA representation given by⁹²

$$\begin{aligned} \mathbf{y}_t &= \sum_{j=1}^p \mathbf{B}_j \mathbf{y}_{t-j} + \mathbf{u}_t \\ \mathbf{y}_t &= \sum_{k=0}^{\infty} \Psi_k \mathbf{u}_{t-k}, \\ \mathbf{u}_t &\sim (\mathbf{0}, \Sigma_u), \Sigma_u = \mathbb{E}[\mathbf{u}_t \mathbf{u}_t'] = \mathbf{P} \Sigma_\epsilon \mathbf{P}' = \mathbf{P} \mathbf{I}_n \mathbf{P}' = \mathbf{P} \mathbf{P}' \end{aligned} \quad (3.34)$$

where \mathbf{P} is the unique lower-triangular Cholesky factor with non-negative diagonal elements. The authors collect the reduced-form parameters into

$$\phi = \left(\text{vec}(\mathbf{A}_1)', \text{vec}(\mathbf{A}_2)', \dots, \text{vec}(\mathbf{A}_p)', \text{vech}(\Sigma_u)' \right) \quad (3.35)$$

by making use of the vec ⁹³ and vech ⁹⁴ operator. Ludvigson et al. (2018) formulate the innovations \mathbf{u}_t to be related to the structural-form SVAR shock ϵ_t via an invertible $(n \times n)$ - matrix \mathbf{H}

$$\mathbf{u}_t = \mathbf{H} \Omega \epsilon_t \equiv \mathbf{A}_0^{-1} \epsilon_t \quad (3.36)$$

with

$$\epsilon_t \sim (\mathbf{0}, \Sigma_\epsilon), \Sigma_\epsilon = \mathbb{E}[\epsilon_t \epsilon_t'] = \mathbf{I}_n \quad (3.37)$$

and the matrix Ω being a diagonal matrix with the variance of the shocks in the

⁹²Note that we adapt the notation of Ludvigson et al. (2018) in their original contribution to be consistent with our own notation so far.

⁹³The *vec*-operator takes an $(n \times n)$ matrix and stacks the columns into a single vector of length n^2 .

⁹⁴The *vech* (vector half) - operator takes a symmetric $(n \times n)$ matrix and stacks the lower triangular half into a single vector of length $\frac{d(d+1)}{2}$.

diagonal entries and the unit effect normalization that $H_{jj} = 1$ for all j .

$$\text{diag}(\mathbf{H}) = \mathbf{1}, \mathbf{\Omega} = \begin{bmatrix} \sigma_{11} & 0 & \dots & 0 \\ 0 & \sigma_{22} & 0 & 0 \\ 0 & \vdots & \dots & 0 \\ 0 & 0 & \dots & \sigma_{kk} \end{bmatrix}, \sigma_{kk} \geq 0 \forall j \quad (3.38)$$

Ultimately, the goal is to study the dynamic effects of the structural shocks (the impulse response functions) as given by

$$\sum_{k=0}^{\infty} \mathbf{\Theta}_k = \sum_{k=0}^{\infty} \mathbf{\Psi}_k \mathbf{A}_0^{-1} \quad (3.39)$$

on \mathbf{y}_t . With autoregressive parameters \mathbf{B}_j being consistently estimated under regularity conditions, the sample residuals $\hat{\mathbf{u}}_t(\hat{\phi})$ are consistent estimates of the reduced-form errors \mathbf{u}_t and the estimation problem reduces to uniquely identify the impact matrix \mathbf{A}_0^{-1} from $\hat{\phi}$. However, at this stage, the model is under-identified because there are nine parameters to be estimated in \mathbf{A}_0^{-1} while the covariance structure only provides six restrictions.

Central to the framework of [Ludvigson et al. \(2018\)](#) is then the following:

By definition they have set $\mathbf{\Sigma}_u = \mathbf{P}\mathbf{P}'$ with \mathbf{P} being the unique lower-triangular Cholesky factor. By introducing a set \mathbb{Q}_n of $(n \times n)$ orthonormal matrices,⁹⁶ any $\mathbf{A}_0^{-1} = \mathbf{P}\mathbf{Q}$ is 'observationally equivalent' given $\mathbf{\Sigma}_u$ and hence consistent with the reduced form variance-covariance matrix $\mathbf{\Sigma}_u = \mathbf{A}_0^{-1}\mathbf{A}_0^{-1'}$ for any given $\mathbf{Q} = (q_1, q_2, \dots, q_n) \in \mathbb{Q}_n$. [Ludvigson et al. \(2018\)](#) define this set of observationally equivalent \mathbf{A}_0^{-1} as

$$\mathcal{A}_0 = \{\mathbf{A}_0^{-1} = \mathbf{P}\mathbf{Q} : \mathbf{Q} \in \mathbb{Q}_n\} \quad (3.40)$$

whereby the only restriction that can be imposed at this stage follows from combining the unit effect normalization on \mathbf{H} with $\sigma_{jj} \leq 0$ so that a unit change in the structural shock j may be interpreted as a standard deviation increase in variable j . Taking this into account, \mathcal{A}_0 becomes

$$\mathcal{A}_0 = \{\mathbf{A}_0^{-1} = \mathbf{P}\mathbf{Q} : \mathbf{Q} \in \mathbb{Q}_n, \quad \text{diag}(\mathbf{A}_0^{-1}) \geq 0\}. \quad (3.41)$$

⁹⁶Note that [Rubio-Ramírez et al. \(2010\)](#) were referring only to *orthogonal*, not *orthonormal* matrices. Orthonormality implies orthogonality, i.e., orthonormality is stronger by demanding for each basis vector spanning the respective space to have length 1.

By collecting the reduced form innovations \mathbf{u}_t into ϕ so that $\phi = (\text{vec}(\mathbf{A}_1)', \text{vec}(\mathbf{A}_2)', \dots, \text{vec}(\mathbf{A}_p)', \text{vech}(\boldsymbol{\Sigma}_u)')$, 3.41 can be written as

$$\hat{\mathcal{A}}_0(\phi) = \{\mathbf{A}_0^{-1} = \mathbf{P}\mathbf{Q} : \mathbf{Q} \in \mathbb{Q}_n, \quad \text{diag}(\mathbf{A}_0^{-1}) \geq 0\}. \quad (3.42)$$

Without any further restrictions, $\hat{\mathcal{A}}_0(\phi)$ contains infinitely many solutions. Ludvigson et al. (2018) hence introduce additional restrictions that go beyond classical sign restrictions, or more generally inequality restrictions (which tend to place these restrictions on the impulse response functions and/or \mathbf{A}_0^{-1} itself) in an attempt to yield a smaller solution set denoted as $\overline{\mathcal{A}}_0(\phi)$.

Denoting the collection of zero restrictions imposed on the model as $FZ(\mathbf{Q}; \phi)$ as in Rubio-Ramírez et al. (2010), the novel restrictions that Ludvigson et al. (2018) define to create a credible identification scheme involve the *identified structural shocks* ϵ_t either on their own by defining event constraints $FE(\mathbf{Q}; \phi; \mathbf{y}_t, \tau^*, \bar{k})$ or in combination with external variables as component correlation constraints denoted $FC(\mathbf{Q}; \phi; \mathbf{y}_t, \mathbf{S}, \bar{\lambda})$. Thereby the set $\hat{\mathcal{A}}_0(\phi)$ is diminished accordingly. We will outline these two set of constraints in turn.

Shock-Based Constraints

Event constraints require that identified financial uncertainty shocks have plausible properties during two episodes of heightened financial uncertainty: the 1987 stock market crash and the 2007-09 financial crisis. Ludvigson et al. (2018) decide for this constraint because “a credible identification scheme should produce estimates of ϵ_t with features that accord with our ex-post understanding of historical events, at least during episodes of special interest” (Ludvigson et al., 2018, p. 7). In particular, event constraints put the bounds \bar{k} on the sign and magnitude of $\epsilon_t = \mathbf{A}_0 \mathbf{u}_t$ during particular episodes which are collected into τ^* .

The reason why such special events turn out to be helpful for identification is that, despite the fact that two observationally equivalent structural models \mathbf{A}_0^{-1} and $\tilde{\mathbf{A}}_0^{-1}$ will produce the corresponding shock-processes $\{\epsilon_t\}_{t=1}^T$ and $\{\tilde{\epsilon}_t\}_{t=1}^T$ where the first and second moments are equivalent, the individual elements of ϵ_t and $\tilde{\epsilon}_t$ are not necessarily equal at each point in time. This can be easily seen because for $\tilde{\mathbf{Q}} \neq \mathbf{Q}$

we get

$$\begin{aligned}
\epsilon_t &= \mathbf{A}_0 \hat{\mathbf{u}}_t = \\
[\mathbf{A}_0^{-1} &= \mathbf{P}\mathbf{Q} \iff \mathbf{A}_0 = \mathbf{Q}^{-1}\mathbf{P}^{-1} \iff \\
\mathbf{A}_0 &= \mathbf{Q}'\mathbf{P}^{-1}] = \\
&\mathbf{Q}'\mathbf{P}^{-1}\hat{\mathbf{u}}_t
\end{aligned} \tag{3.43}$$

and

$$\tilde{\epsilon}_t = \tilde{\mathbf{A}}_0 \hat{\mathbf{u}}_t = \tilde{\mathbf{Q}}'\mathbf{P}^{-1}\hat{\mathbf{u}}_t = \tilde{\mathbf{Q}}\mathbf{u}_t \neq \epsilon_t \tag{3.44}$$

at any given t which shows that event constraints could be used to reduce the number of solutions in $\hat{\mathcal{A}}_0(\phi)$ to a smaller set $\overline{\mathcal{A}}_0(\phi)$.

To show this, [Ludvigson et al. \(2018\)](#) consider the bivariate case $n = 2$:

Writing down

$$\begin{aligned}
\begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix} &= \begin{bmatrix} a_{11}^0 & a_{12}^0 \\ a_{21}^0 & a_{22}^0 \end{bmatrix}^{-1} \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{bmatrix} \iff \\
\begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{bmatrix} &= \begin{bmatrix} a_{11}^0 & a_{12}^0 \\ a_{21}^0 & a_{22}^0 \end{bmatrix} \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix} \implies \\
\epsilon_{1t} &= a_{11}u_{1t} + a_{12}u_{2t}
\end{aligned} \tag{3.45}$$

where the values of u_{1t} and u_{2t} are given since data is given for time t in the span $[\tau_1, \tau_2]$. The above example shows that a restriction on the behavior of ϵ_{1t} at a specific time t results in a non-linear restriction on \mathbf{A}_0^{-1} and, equivalently, on \mathbf{Q} .

With the stage set according to the above explanations, [Ludvigson et al. \(2018\)](#) require ϵ_t to satisfy three event constraints which they parameterize by $\bar{\mathbf{k}} = (\bar{k}_1, \bar{k}_2, \bar{k}_3)'$ ⁹⁷, $\bar{\tau} = (\bar{\tau}_1, \bar{\tau}_2, \bar{\tau}_3)' = (1987 : 10, [2007 : 12, 2009 : 06], [2007 : 12, 2009 : 06])$ and $\mathbf{y}_t = (U_{Mt}, IPM_t, U_{Ft})$ where U_{Mt} denotes macro uncertainty, IPM_t a measure of real activity (in this case industrial production in manufacturing) and U_{Ft} a measure

⁹⁷The bounds will be set below.

for financial uncertainty and collect the constraints into a system of inequalities

$$\begin{aligned}
 FE(\mathbf{Q}; \phi, \mathbf{y}_t, \bar{\tau}, \bar{k}) &= \begin{pmatrix} FE_1(\bar{\tau}_1, \bar{k}_1) \\ FE_2(\bar{\tau}_2, \bar{k}_2) \\ FE_3(\bar{\tau}_3, \bar{k}_3) \end{pmatrix} = \\
 &= \begin{pmatrix} \sum_{t=1}^T \mathbb{1}_{t=\bar{\tau}_1=1987:10} \cdot \epsilon_{F\bar{\tau}_1} \\ \sum_{t=1}^T \mathbb{1}_{t=\bar{\tau}_2 \in [2007:12, 2009:06]} \cdot \epsilon_{F\bar{\tau}_2} \\ \bar{k}_3 \end{pmatrix} - \begin{pmatrix} \bar{k}_1 \\ \bar{k}_2 \\ \sum_{t=1}^T \mathbb{1}_{t=\bar{\tau}_3 \forall \in [2007:12, 2009:06]} \cdot \epsilon_{IPM\bar{\tau}_3} \end{pmatrix} \geq \mathbf{0}
 \end{aligned} \tag{3.46}$$

Each $FE_i(\bar{\tau}_i, \bar{k}_i)$ represents a vector of constraints of magnitude \bar{k}_i on ϵ_{it} for corresponding $t \in \bar{\tau}_i$. Together, $FE(\mathbf{Q}; \phi, \mathbf{y}_t, \bar{\tau}, \bar{k})$ defines inequality constraints based on timing, sign and magnitude that help identification of the underlying structural model. FE_1 requires that the financial uncertainty shock that occurred in October 1987 be a large one; FE_2 postulates that there must be at least one month during the financial crises 2007-2009 for which the financial uncertainty shock was large and positive and FE_3 requires that any real activity shocks found during the Great Recession (the NBER dates for the recession coincide with the financial crisis) do not take any unusually large positive values.

The motivation for the above constraints is that if a potential \mathbf{Q} implies a corresponding shock series that is difficult to hold on to during our key time episodes, it will be removed from the solution set $\hat{\mathcal{A}}_0(\phi)$. Correspondingly, in the postulated inequality constraints the values for the \bar{k}_i s have to be meaningful and timing of events $\bar{\tau}_i$ accurate otherwise solutions that pass these constraints will be meaningless after all.

With regard to the first two constraints on financial uncertainty for the stock market crash on Black Monday and the recent financial crisis, [Ludvigson et al. \(2018\)](#) declare that the established constraints want to make sure that “*at least some*”⁹⁸ of the forecast error variance of U_F in these episodes of most extreme financial uncertainty is attributable to large shocks that originated in financial markets” which “is a maintained assumption [...] that we argue is grounded in a broad historical reading of the times” ([Ludvigson et al., 2018](#), p. 8) and here is captured by ϵ_F . To clarify, [Ludvigson et al. \(2018\)](#) point out that the constraints, however, do not require that

⁹⁸ Author’s italics.

all or most of the variability in these episodes should come from shock originating in financial markets because they do not rule out large adverse shocks in ϵ_M and ϵ_{IPM} .

The second set of constraints considered by [Ludvigson et al. \(2018\)](#) are **correlation constraints** which require that the identified (structural) uncertainty shocks recovered from $\epsilon_t = \mathbf{A}_0 \mathbf{u}_t$ show a minimum absolute correlation with certain variables that are external to the VAR estimations which they encode with \mathbf{S}_t . In their baseline specification they use aggregate stock market return and impose restrictions on its correlation with uncertainty shocks.

As a motivating example, [Ludvigson et al. \(2017\)](#) consider the following example of a bivariate model:

$$\begin{aligned} \mathbf{y}_t &= \sum_{j=1}^p \mathbf{B}_j \mathbf{y}_{t-j} + \begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix} \iff \\ \mathbf{y}_t &= \sum_{j=1}^p \mathbf{B}_j \mathbf{y}_{t-j} + \begin{pmatrix} a_{11}^0 & a_{12}^0 \\ a_{21}^0 & a_{22}^0 \end{pmatrix}^{-1} \begin{pmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{pmatrix}, \\ \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix} &\sim \mathcal{N}(\mathbf{0}, \mathbf{I}_n) \end{aligned} \tag{3.47}$$

EXAMPLE NOT FINISHED! (if time is left, I can add more explanations to this!)

[Ludvigson et al. \(2018\)](#) → HIER NOCH DIE ECONOMIC INTUITION HINTER DEN CORRELATION CONSTRAINTS hinschreiben!

To formalize this and further letting \hat{u}_{St} be the first order autoregressive residual for S_t , \hat{u}_{St} , being a reduced-form residual, results as a combination of primitive shocks from various sources including, among others, the three shocks modeled within the VAR system. In particular, letting $(c_M(\mathbf{A}_0), c_{IMP}(\mathbf{A}_0), c_F(\mathbf{A}_0))$ be the sample correlations between \hat{u}_{St} and the structural shocks $(\epsilon_{Mt}(\mathbf{A}_0), \epsilon_{IPMt}(\mathbf{A}_0), \epsilon_{Ft}(\mathbf{A}_0))$

and $\bar{\lambda} = (\bar{\lambda}_1, \bar{\lambda}_2, \bar{\lambda}_3)$, the system of inequalities for correlation constraints is

$$\begin{aligned} FC(\mathbf{Q}; \phi, \mathbf{y}_t, \mathbf{S}, \bar{\lambda}) &= \begin{pmatrix} FC_1(\bar{\lambda}_1 < 0, \mathbf{S}) \\ FC_2(\bar{\lambda}_2 \geq 1, \mathbf{S}) \\ FC_3(\bar{\lambda}_3, \mathbf{S}) \end{pmatrix} = \\ &= \begin{pmatrix} \begin{pmatrix} \bar{\lambda}_1 - c_M(\mathbf{A}_0) \\ \bar{\lambda}_1 - c_F(\mathbf{A}_0) \end{pmatrix} \geq \mathbf{0} \\ |c_F(\mathbf{A}_0)| - \bar{\lambda}_2 |c_M(\mathbf{A}_0)| \geq 0 \\ c_{MF} - \bar{\lambda}_3 \geq 0; c_{MF}^2 = c_M(\mathbf{A}_0)^2 + c_F(\mathbf{A}_0)^2 \end{pmatrix} \end{aligned} \quad (3.48)$$

The first and third constraint require that ϵ_M and ϵ_F are negatively correlated with the external variable S_t . In particular, constraint (1) requires that each individual correlation exceed the threshold $\bar{\lambda}_1$ in absolute terms and collectively exceed $\bar{\lambda}_3$. The second constraint requires financial uncertainty shocks to be more highly correlated with the autoregressive residual e_{St} than 'real' (macro) uncertainty shocks bounded by the lower bound $\bar{\lambda}_2$.

Together, the constraints in 3.48 provide cross-equation restrictions on the parameters in a potential \mathbf{A}_0^{-1} , whereby the correlations are not invariant to orthonormal rotations (by \mathbf{Q}) so that the generated correlations in each draw will be different.

Identified Solution Set and maxG - Solution

Combining default covariance structure restrictions with the introduced event and correlation constraints, Ludvigson et al. (2018) call the *identified*⁹⁹ solution set

$$\begin{aligned} \bar{\mathcal{A}}_0(\mathbf{Q}; \phi, \bar{k}, \bar{\tau}, \bar{\lambda}, \mathbf{S}) &= \{\mathbf{A}_0^{-1} = \mathbf{P}\mathbf{Q} : \mathbf{Q} \in \mathbb{Q}_n, \quad \text{diag}(\mathbf{A}_0^{-1}) \geq 0\}; \\ FZ(\mathbf{Q}; \phi) &= \mathbf{0}, \\ FC(\mathbf{Q}; \phi, \mathbf{y}_t, \mathbf{S}, \bar{\lambda}) &\geq \mathbf{0}, \\ FE(\mathbf{Q}; \phi, \mathbf{y}_t, \bar{\tau}, \bar{k}) &\geq \mathbf{0} \end{aligned} \quad (3.49)$$

which only contains estimates of \mathbf{A}_0^{-1} that satisfy all constraints. Thereby a particular solution can only be in both $\hat{\mathcal{A}}_0$ (the unconstrained solution set) and $\bar{\mathcal{A}}_0$ if all restrictions are satisfied. And while $\bar{\mathcal{A}}_0$ will be a set as well it should be notably smaller than $\hat{\mathcal{A}}_0$ (without any additional restrictions other than the usual covariance restrictions).

Both constraints together generate moment inequalities (together with the standard

⁹⁹ Author's italics.

reduced-form covariance restrictions) that produce a reduction of the set of possible model parameters consistent with the data that is large enough to ultimately unambiguously derive most dynamic relationship in the system.

The ultimate size of the set $\overline{\mathcal{A}}_0$ depends in particular on the chosen thresholds for the values $\bar{k}, \bar{\tau}$ and $\bar{\lambda}$ which we will discuss below.

And while there is no single solution in $\overline{\mathcal{A}}_0$ that is more likely than another one, as a reference point (and following [Ludvigson et al. \(2018\)](#)), we will also refer to the so-called 'maxG' - solution when discussing the results. In particular, the 'maxG' - solution serves as a reference point at which the value of the introduced constraints (inequalities) are jointly maximized. Formally, as the maxG-solution the quadratic norm is chosen so that

$$\mathbf{A}_0^{maxG} \equiv \underset{\mathbf{A}_0 \in \overline{\mathcal{A}}_0}{\operatorname{argmax}} \sqrt{\bar{f}(\mathbf{A}_0)' \bar{f}(\mathbf{A}_0)} \quad \text{where} \quad \bar{f}(\mathbf{Q}; \phi, \bar{k}, \bar{\tau}, \bar{\lambda}, \mathbf{S}) = \begin{pmatrix} FZ(\mathbf{Q}; \phi)' \\ FC(\mathbf{Q}; \phi, \mathbf{y}_t, \mathbf{S}, \bar{\lambda})' \\ FE(\mathbf{Q}; \phi, \mathbf{y}_t, \bar{\tau}, \bar{k})' \end{pmatrix}' \quad (3.50)$$

With respect to the introduced constraints, the inequalities will be large for the most extremely positive financial uncertainty shocks in 1987 and during the Great Recession, for the most negative real activity shocks during the Great Recession and when uncertainty shocks have the highest absolute correlation with the variable external to the VAR(in the baseline case: aggregate stock market returns), both jointly and collectively. With these features, the 'maxG' - solution can, economically, be regarded as the “worst-case” - scenario.

Baseline SVAR(6)-3

[Ludvigson et al. \(2018\)](#) estimate several VAR systems to identify uncertainty shocks from the VAR residuals using the restrictions from above. Here we will reproduce their baseline estimation consisting of $\mathbf{y}_t = (U_{Mt}, IPM_t, U_{Ft})$ where U_{Mt} denotes macro uncertainty, IPM_t a measure of real activity (in this case industrial production in manufacturing) and U_{Ft} a measure for financial uncertainty. The VAR-System uses $p = 6$ lags (and will report the results for $p = 12$ in the Appendix).

Note to self: Here I could maybe write down the (S)VAR being estimated in the style of the Bloom-models we had included as well!

3.2.3. Solution Algorithm

As indicated above, a decisive part of the solution algorithm is the construction of the unconstrained solution set $\hat{\mathbf{A}}_0$ and the derivation of the identified set $\overline{\mathbf{A}}_0$.

The unconstrained solution set $\hat{\mathbf{A}}_0$ is obtained through the following steps:

- i estimation of the reduced-form model and initialization of \mathbf{A}_0^{-1} as the unique lower-triangular Cholesky factor $\hat{\mathbf{P}}$ of $\hat{\Sigma}_u$ with non-negative diagonal elements
- ii rotation of $\hat{\mathbf{P}}$ by $K = 1.5$ million random orthogonal matrices \mathbf{Q} ;
each rotation begins by drawing an $(n \times n)$ matrix \mathbf{M} of normally and independently distributed values (i.e., drawn from a normal distribution with $\mu = 0$ and $\sigma = 1$); \mathbf{Q} is then taken to be the orthonormal matrix resulting from a \mathbf{QR} decomposition of \mathbf{M}
- iii because $\mathbf{A}_0^{-1} = \mathbf{PQ}$, the covariance restrictions are imposed by construction

Every generated \mathbf{A}_0^{-1} resulting from one of the 1.5 million rotations is then confronted with the event and correlation-constraints for further processing. This step, however, needs a reasonable choice of the parameters $\bar{\lambda}, \bar{\tau}$ and \bar{k} . Using a mix of theory, empirical analysis and economic reasoning, [Ludvigson et al. \(2018\)](#) set the parameters to $\bar{\lambda}_1 = -0.05$, $\bar{\lambda}_2 = 2$ and the threshold for the collective correlation to $\bar{\lambda}_3 = 0.18$. The choices for \bar{k}_1 and \bar{k}_2 , are, according to [Ludvigson et al. \(2018\)](#) partly guided by [Bloom \(2009\)](#) who chooses to study the dynamic effects of four standard deviation shocks to uncertainty in his impulse response analyses. Accordingly, [Ludvigson et al. \(2018\)](#) set both $\bar{k}_1 = 4$ and $\bar{k}_2 = 4$.¹⁰⁰ Further, \bar{k}_3 is set to 2 to dismiss shocks in real activity that go beyond two standard deviations above their sample mean during 2007-2009.

At each draw of $\mathbf{A}_0^{-1} = \hat{\mathbf{P}}\mathbf{Q}$ and the subsequent generation of $\epsilon_t = \mathbf{A}_0\hat{\mathbf{u}}_t$, the algorithm will only store those models that fulfill all constraints. The algorithm's code itself is available in Appendix [B.2.1](#).¹⁰¹

¹⁰⁰This threshold is chosen considering that the shocks are shown to be non-Gaussian with exhibiting excess skewness and kurtosis.

¹⁰¹We had initially implemented the novel identification strategy of [Ludvigson et al. \(2018\)](#) in R by solely following the descriptions in the paper. Due to some small subtleties in the translation of the algorithm's description into code, our resulting trajectories of the impulse response functions as produced by the identified solution set $\overline{\mathbf{A}}_0$ were slightly off in this initial attempts as compared to the IRFs reported in the paper. We are hence grateful to Sai Mia, one of the co-authors of [Ludvigson et al. \(2018\)](#), who kindly provided us with their Matlab-code of the baseline-cases to identify the errors.

4. Data

4.0.1. A Forecast-Based Measure: Jurado et al. (2015)

Note to self:

- ▷ Gilchrist et al. (2014, p. 1) write: “[...] the common variation in the unforecastable component of a large number of economic indicators (Jurado et al., 2015).”
- ▷ Orlik und Vedkamp schreiben: Jurado Ludvigson and Ng (2015) offer a state-of-the-art macro volatility measure. It uses a rich set of time series, computes conditional volatility of the unforecastable component of the future value of each of these series, and then aggregates these individual conditional volatilities into a macro uncertainty index.

Especially in contrast to stock market volatility and/or cross-sectional dispersion measures frequently deployed in uncertainty studies, Jurado et al. (2015) have designed a computationally and data-intensive approach by making use of a rich set of time series, computing the conditional volatility of the unforecastable component of the future value of each of these series and finally aggregating these conditional volatilities into one composite index.¹⁰²

To set their approach apart from other commonly used uncertainty measures, Jurado et al. (2015, p. 1178) argue that “[...] what matters for economic decision making is not whether particular economic indicators have become more or less *variable* or disperse *per se*, but rather whether the economy has become more or less *predictable*; that is, less or more uncertain.”

To formalize the rationale behind their approach, they define the h -period ahead uncertainty in the variable $y_{jt} \in Y_t = (y_{1t}, \dots, y_{N_y t})'$, denoted by $U_{jt}^t(h)$, as the

¹⁰²Note that, apart from practical issues, Jurado et al. (2015, p. 1191) themselves explain the reliance on (most likely revised) “historical” data (instead of “real time” data) with their goal of forming the “[...] most historically accurate estimates of uncertainty at any given point in time in [their] sample”.

conditional volatility of the purely unforecastable component of the future value of the series, i.e.,

$$U_{jt}^t(h) \equiv \sqrt{\mathbb{E}[(y_{jt+h} - \mathbb{E}[y_{jt+h}|I_t])^2|I_t]}, \quad (4.1)$$

where the expectation $\mathbb{E}(\cdot|I_t)$ is taken with respect to information I_t available to economic agents at time t . Hence, if the expectation today (conditional on all available information) of the squared error in forecasting y_{jt+h} rises, uncertainty in the variable increases. The individual uncertainties at each date are then aggregated into a *macroeconomic uncertainty* index by using aggregation weights w_j as follows:

$$U_{jt}^t(h) \equiv \text{plim}_{N_y \rightarrow \infty} \sum_{j=1}^{N_y} w_j U_{jt}^y(h) \equiv \mathbb{E}_w[U_{jt}^y(h)] \quad (4.2)$$

Based on the above definitions, [Jurado et al. \(2015\)](#) emphasize two features that are central to their approach:

1. they distinguish between *uncertainty* in a series y_{jt} and its *conditional volatility* and argue that the proper measurement of uncertainty requires the removal of the entire forecastable component $\mathbb{E}[y_{jt+h}|I_t]$ before the computation of the conditional volatility; without accounting for this, the argument goes, forecastable variation is erroneously classified as “uncertainty”. According to [Jurado et al. \(2015\)](#), this is taken into account only in very few occasions in the literature.
2. they claim that macroeconomic uncertainty is “a measure of the common variation in uncertainty across many series” and not equal to the uncertainty in any single series y_{jt} ; [Jurado et al. \(2015\)](#) see this as an important argument because of uncertainty-based theories of the business cycle that typically require the existence of common (often countercyclical) variation in uncertainty across large numbers of series; [Jurado et al. \(2015\)](#) hence put their econometric estimation of uncertainty insofar to test, as they expect to find evidence of such an aggregate uncertainty factor common to many series, if the above assumption turns out to be correct.

[Jurado et al. \(2015\)](#)’s econometric approach then consists of three parts:

- i estimation of the forecastable component $\mathbb{E}[y_{jt+h}|I_t]$: this is achieved by, first, running a factor analysis on a large set of predictors (132 macro time series) $\{X_{it}\}, i = 1, 2, \dots, N$ whose linear span/hull comes as close as possible to I_t ;

Table 3.: Periods of high uncertainty according to macro uncertainty index $h=1$.

Start	End	Duration	Maximum Year/Month	Maximum Value
Sep 1974	Mar 1975	7 months	Jan 1975	0.87
Jan 1980	Aug 1982	31 months	May 1980	1.00
Jul 2008	Sep 2009	15 months	Nov 2008	1.06

second, making use of the formed factors, $\mathbb{E}[y_{jt+h}|I_t]$ is then approximated by means of a diffusion index forecasting model ideal for data-rich environments (the diffusion indices can then be treated as known in the subsequent steps)

- ii with the definition of the h -step-ahead forecast error as $V_{jt+h}^y \equiv y_{jt+h} - \mathbb{E}[y_{jt+h}|I_t]$, [Jurado et al. \(2015\)](#) estimate the conditional volatility of this forecast error, i.e., $\mathbb{E}[(V_{t+h}^y)^2|I_t]$ at time t by specifying a parametric stochastic volatility model for both the one-step-ahead prediction errors in y_{jt} and the analogous forecast errors for the factors (from step i above); these volatility estimates are then used to recursively compute the values of $\mathbb{E}[(V_{t+h}^y|I_t]$ for $h > 1$.
- iii in the last step, the *macroeconomic uncertainty* index $U_t^y(h)$ is constructed from the individual uncertainty measures $U_{jt}^y(h)$ where the base-case is the equally-weighted average of individual uncertainties

While [Jurado et al. \(2015\)](#) apply their approach to two large dataset with economic time-series (one monthly dataset containing hundreds of macroeconomic and financial indicators and one quarterly dataset containing 155 firm-level observations on profit growth normalized by sales), here we only confine ourselves to the former, i.e., their *common macro uncertainty* index.

Their resulting estimated measures of time-varying uncertainty are plotted in Figure 2 and reveals 'only' three big episodes of uncertainty: the months during the recessions from 1973-1974 and 1981-1982 and the months during the Great Recession of 2007-2009 (being the most striking episode of heightened uncertainty, followed by the 1981-1982 recession as a close second).^{103,104}

¹⁰³See Table 3 below for the exact dates including the maximum value for the index for each identified episode of high uncertainty.

¹⁰⁴[Jurado et al. \(2015\)](#) report that a closer look at the estimates reveals that the three series with the highest uncertainty are a producer price index for intermediate materials, a commodity spot price index, and employment in mining between 1973:11 and 1975:3, the Fed funds rate, employment in mining, and the three months commercial paper rate for the 1980:1 and 1982:11 episode and the monetary base, non-borrowed reserves and total reserves between 2007:12 and 2009:6. [Jurado et al. \(2015\)](#) conclude these results to be consistent with the historical account.

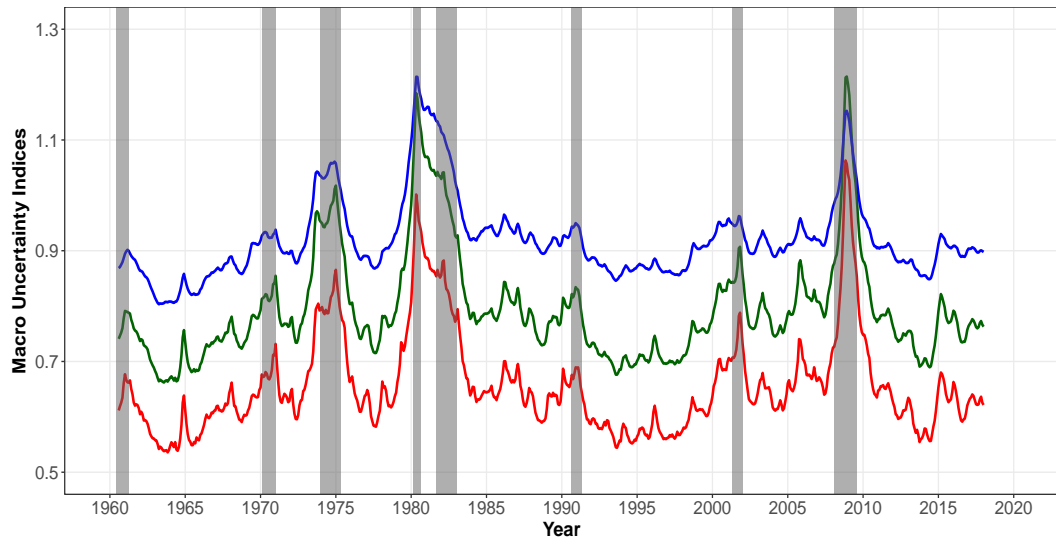


Figure 2.: Aggregate Uncertainty: $h = 1, 3, 12$. *Note:* Horizontal lines indicate 1.65 standard deviations above the mean of each series. Data are monthly and span the period 1960:7-2017:12. Red shaded areas correspond to dates where volatility series exceeds 1.65 standard deviation above the mean of the $h1$ -series. Grey shaded areas denote NBER recession dates in the US. Original Figure 1 in [Jurado et al. \(2015\)](#) spans until 2011. We have taken all available values until December 2017, inclusive.

5. Results

5.1. Uncertainty Shocks

Note to self:

- ▷ [Ludvigson et al. \(2018, p. 25\)](#) run their algorithm also for a pre-crisis sample which yields less conclusive answers; therefore they write: “A premise of this paper is that the 2007-2009 financial crisis was an important rare event that can help distinguish the transmission of financial versus real uncertainty shocks. This maintained assumption appears supported by the subsample analysis.”

Before jumping to the resulting impulse responses of the baseline system $\mathbf{y}_t = (U_{Mt}, IPM_t, U_{Ft})$ in Figure 5, we first want to briefly discuss the recovered uncertainty shocks which exhibit a few noteworthy features.

5.2. Impulse Response Analysis

5.3. Variance Decomposition

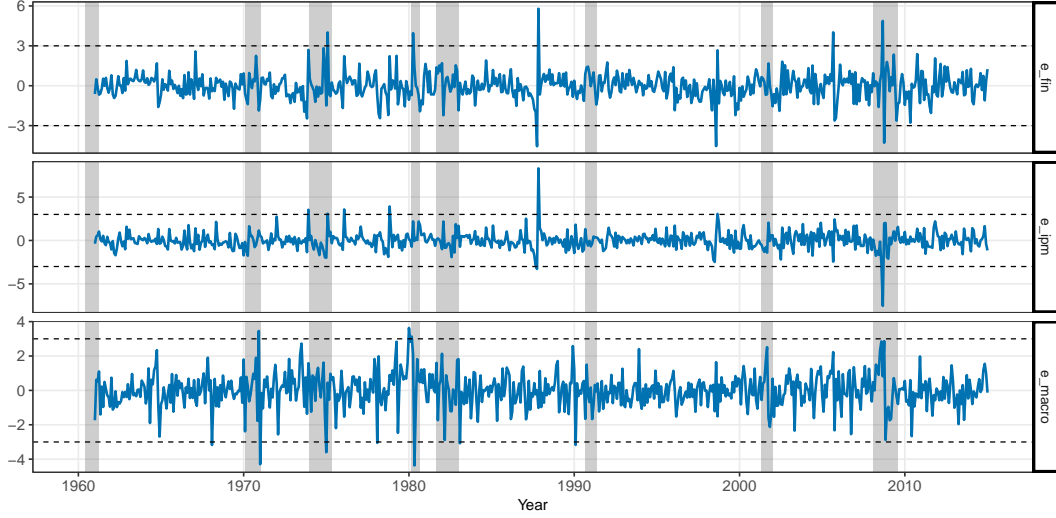


Figure 3.: Time Series of standardized structural shocks from SVAR (U_{Macro}, IPM, U_{Fin}).

Note: The horizontal line corresponds to 3 standard deviations above/below the unconditional mean of each series. The shocks $\epsilon_t = A_0 u_t$ for maxG solution are reported, where u_t holds the residuals from VAR(6) of (U_{Macro}, IPM, U_{Fin}). The selected bounds are $\bar{\lambda}_1 = -0.05$, $\bar{\lambda}_2 = 2$, $\bar{\lambda}_3 = 0.18$, $\bar{k}_1 = 4$, $\bar{k}_2 = 4$, $\bar{k}_3 = 2$.

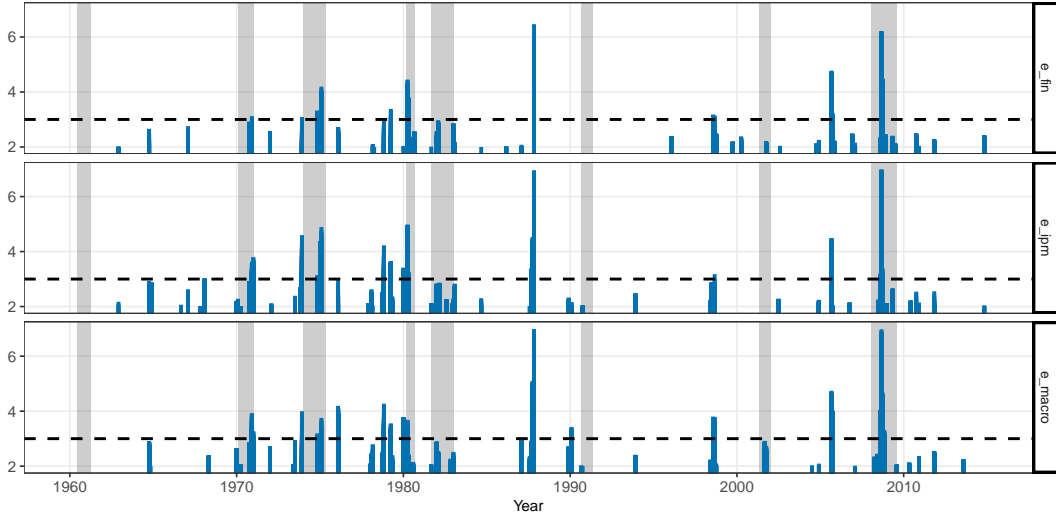


Figure 4.: Large Shocks derived from SVAR (U_{Macro}, IPM, U_{Fin}).

Note: The figure shows all shocks in the identified set that are at least 2 standard deviations above the unconditional mean for ϵ_{Macro} and ϵ_{Fin} and at least 2 standard deviations below the mean for ϵ_{ipm} . For the middle pane (ϵ_{ipm}) the sign of the shocks were flipped (so that negative shocks exceeding 2 standard deviations also point upwards). The horizontal line corresponds to 3 standard deviations. The selected bounds are $\bar{\lambda}_1 = -0.05$, $\bar{\lambda}_2 = 2$, $\bar{\lambda}_3 = 0.18$, $\bar{k}_1 = 4$, $\bar{k}_2 = 4$, $\bar{k}_3 = 2$.

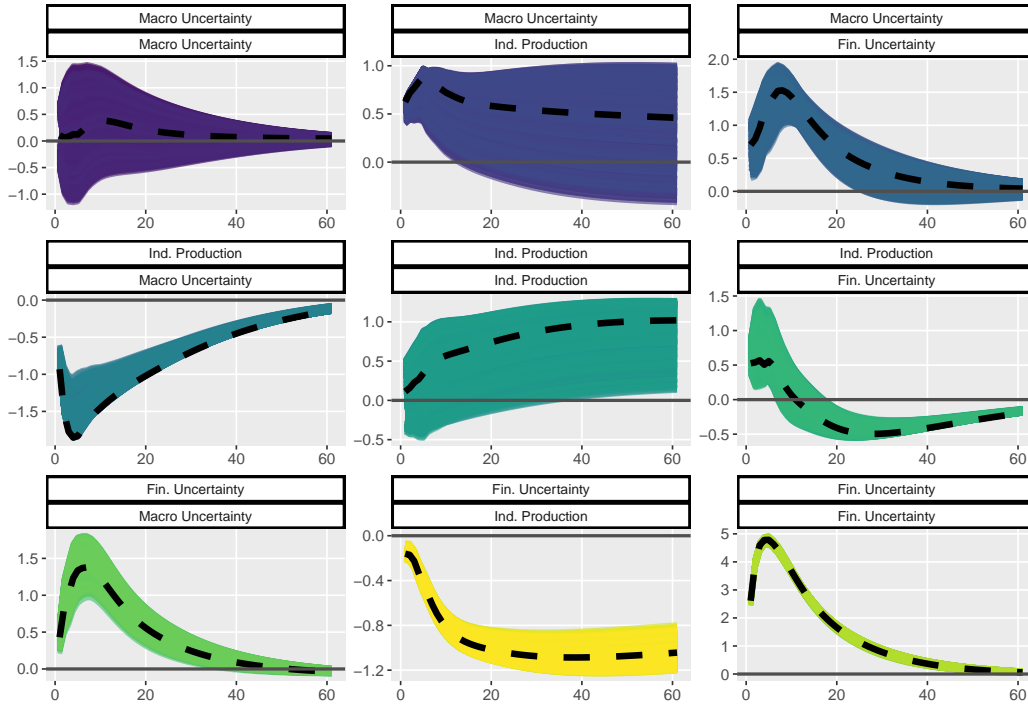


Figure 5.: Impulse Responses from SVAR (U_{Macro}, IPM, U_{Fin}).

Note: The dashed lines are the maxG solutions. The shaded areas represent sets of solutions that satisfy the correlation and event constraints. The selected bounds are $\bar{\lambda}_1 = -0.05$, $\bar{\lambda}_2 = 2$, $\bar{\lambda}_3 = 0.18$, $\bar{k}_1 = 4$, $\bar{k}_2 = 4$, $\bar{k}_3 = 2$.

6. Model Extension/Alternative Estimations

Robustness Checks: Alternative Uncertainty Measures

This part provides results from a large number of robustness exercises designed to check the sensitivity of our results to various assumptions. (this sentence is formulated like this by [Jurado et al. \(2015\)](#)).

7. Conclusion

In the article “Held Back by Uncertainty” Bloom and his Colleagues write: ” Policymakers can help.” -> I should build the conclusion up along these lines!

In Kose and Terrones, 2012 (How Does Uncertainty Affect Economic Performance”), p. 5, the authors write: “Policymakers can do little to alleviate the intrinsic uncertainty economies typically face over the business cycle. However, policy uncertainty is unusually high, and it contributes significantly to macroeconomic uncertainty. By implementing bold and timely measures, policymakers can reduce policy-induced uncertainty and help kick-start economic growth. What precisely policymakers need to do is discussed in the main text of Chapter 1:

There, the authors suggest:

Another problem for VAR-system are omitted variables (which are assumed to be in the innovations). If important variables are omitted from the system, this may lead to major distortions in the impulse responses and makes them worthless for structural interpretations (see Lütkepohl!)

A. Appendix

A.1. Additional Tables

Table 4.: Major Stock-Market Volatility Shocks. Replication of Table A.1 in [Bloom \(2009\)](#).

ID	Date Begin	Date End	Duration	Max Vol	First Vol	Date Max Vol	Date First Vol
1	Oct 1962	Oct 1962	1	26	26	Oct 1962	Oct 1962
2	Nov 1963	Nov 1963	1	28.7	28.7	Nov 1963	Nov 1963
3	Aug 1966	Aug 1966	1	24.4	24.4	Aug 1966	Aug 1966
4	Oct 1966	Oct 1966	1	23.9	23.9	Oct 1966	Oct 1966
5	May 1970	May 1970	1	37.6	37.6	May 1970	May 1970
6	Nov 1973	Dec 1973	2	34.1	28.8	Dec 1973	Nov 1973
7	Jul 1974	Oct 1974	4	38.4	29.8	Oct 1974	Jul 1974
8	Mar 1980	Mar 1980	1	29.2	29.2	Mar 1980	Mar 1980
9	Aug 1982	Aug 1982	1	32.4	32.4	Aug 1982	Aug 1982
10	Oct 1982	Nov 1982	2	32.8	32.8	Oct 1982	Oct 1982
11	Oct 1987	Feb 1988	5	49.4	40.8	Nov 1987	Oct 1987
12	Aug 1990	Oct 1990	3	30.1	27.8	Oct 1990	Aug 1990
13	Jan 1991	Jan 1991	1	26.9	26.9	Jan 1991	Jan 1991
14	Nov 1997	Nov 1997	1	31.6	31.6	Nov 1997	Nov 1997
15	Sep 1998	Oct 1998	2	39.2	39.2	Sep 1998	Sep 1998
16	Sep 2001	Oct 2001	2	42.6	42.6	Sep 2001	Sep 2001
17	Jul 2002	Oct 2002	4	41.7	38.5	Sep 2002	Jul 2002
18	Feb 2003	Mar 2003	2	36.5	36.5	Feb 2003	Feb 2003
19	Aug 2007	Aug 2007	1	25.1	25.1	Aug 2007	Aug 2007
20	Nov 2007	Nov 2007	1	26.7	26.7	Nov 2007	Nov 2007
21	Jan 2008	Mar 2008	3	29	28.1	Mar 2008	Jan 2008

A.2. Additional Figures

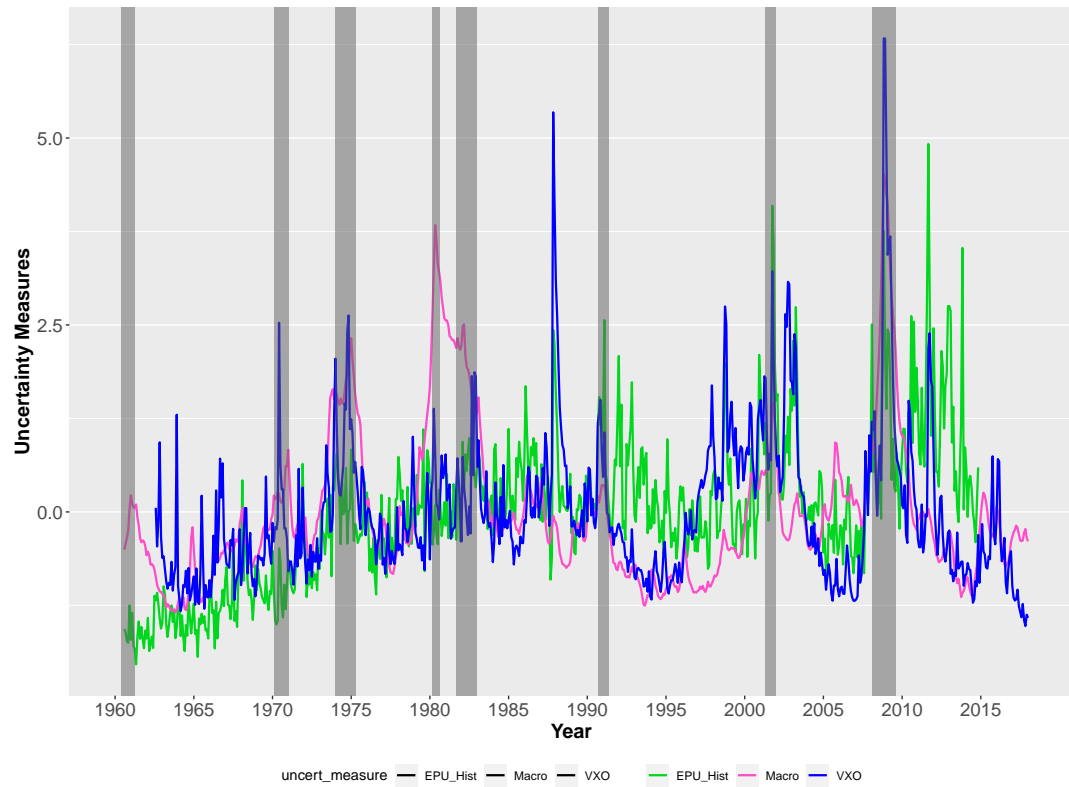


Figure 6.: Comparison of uncertainty measures (faceted). *Note:* Shaded areas denote NBER recession dates. Data frequencies are monthly. The macro uncertainty series starts in July 1960, the EPU in January 1962 and the VXO in July 1962.

A.3. Additional VAR Results

B. Appendix

B.1. Data: Sources and Description

Table 5 lists all data and their sources that appear in the main text. Variables that enter the VAR and/or Local Projection estimations in Section ?? are listed below.

Monthly VAR-8 following Bloom (2009)

Endogenous variables, in order:

Then we create separate explanations about which variables were used for which model:

1. $\log(\text{S\&P500}_t)$
2. uncertainty (various measures)
3. FFR_t
4. $\log(\text{WAGE}_t)$
5. $\log(\text{CPI}_t)$
6. $\log(\text{HOURS}_t)$
7. $\log(\text{EMPM}_t)$
8. $\log(\text{IP}_t)$

Monthly VAR-11 following Jurado et al. (2015)

Endogenous variables, in order:

Then we create separate explanations about which variables were used for which model:

1. $\log(\text{IP}_t)$
2. $\log(\text{EMPM}_t)$
3. $\log(\text{real consumption} \rightarrow \text{still to be added! download from IHS!})$

-
4. $\log(\text{PCE deflator}_t)$ -> still to be added! download from IHS!
 5. $\log(\text{NO}_t = \text{NO capital}_t + \text{NO cons}_t)$ -> two components to be added! download from IHS!
 6. $\log(\text{WAGE}_t)$
 7. $\log(\text{HOURS}_t)$
 8. FFR_t
 9. $\log(\text{S\&P500}_t)$
 10. growth rate of M2_t
 11. uncertainty (various measures)

Table 5.: Data Sources.

Note: Federal Reserve Economic Data (FRED); Chicago Board Options Exchange (CBOE); Michigan Survey of Consumers (MSoC);

Variable	Name	Source	Code	Period
IP_t	Industrial Production Index (Monthly, Seasonally Adjusted)	FRED	INDPRO	1962M07-
$EMPM_t$	All Employees: Manufacturing (Monthly, Seasonally Adjusted)	FRED	MANEMP	1962M07-
$HOURS_t$	Average Weekly Hours of Production and Nonsupervisory Employees: Manufacturing (Monthly, Seasonally Adjusted)	FRED	AWHMAN	1962M07-
CPI_t	Consumer Price Index for All Urban Consumers: All Items (Monthly, Seasonally Adjusted)	FRED	CPIAUSCSL	1962M07-
$NO\ capital_t$	Value of Manufacturers' New Orders for Capital Goods: Nondefense Capital Goods Industries (Monthly, Seasonally Adjusted)	IHS	M_178554409	1962M07-
$NO\ cons_t$	Value of Manufacturers' New Orders for Capital Goods: Nondefense Capital Goods Industries (Monthly, Seasonally Adjusted)	IHS	M_14385863	1962M07-
$WAGE_t$	Average Hourly Earnings of Production and Nonsupervisory Employees: Manufacturing (Monthly, Seasonally Adjusted)	FRED	AHEMAN	1962M07-
$M2_t$	M* Money Stock (Monthly, Seasonally Adjusted)	FRED	M2SL	1962M07-
FFR_t	Effective Federal Funds Rate	FRED	FEDFUNDS	1962M07-
$S\&P500_t$	S&P's Common Stock Price Index: Composite (Monthly)	YAHOO Finance	S&P 500 (^GSPC)	1962M07-
VXO_t	Cboe S&P 100 Volatility Index - VXO	CBOE	VXO	1986M01-
$Michigan_t$	Consumer Uncertainty (Michigan Survey of Consumers)	MSoC	veh_fb_unc	1978M03-
$Macro1/3/12_t$	Macro Uncertainty Index	Jurado et al. (2015) ¹⁰⁵		1962M07-
EPU_t	Economic Policy Uncertainty Index	Baker et al. (2015) ¹⁰⁶	News Based Policy Uncert Index	1962M07-
$EPU\ Historical_t$	Economic Policy Uncertainty Index	Baker et al. (2015) ¹⁰⁷	News-Based Historical Economic Policy Uncertainty	1962M07-

B.2. Code

B.2.1. SVAR-Framework of Ludvigson et al. (2018)

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##The Code will go here.
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Declaration of Authorship

I hereby declare that I prepared this master's thesis independently and that the thoughts taken directly or indirectly from other sources are acknowledged as references accordingly.

The work contained in this thesis has neither been previously submitted to any other examination authority nor published in any other form which has led to the award of a degree.

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Eidesstattliche Erklärung

Ich erkläre hiermit an Eides Statt, dass ich dir vorliegende Masterarbeit selbständig angefertigt habe. Die aus fremden Quellen direkt oder indirekt übernommenen Gedanken sind als solche kenntlich gemacht.

Die Arbeit wurde bisher weder in gleicher noch in ähnlicher Form einer anderen Prüfungsbehörde vorgelegt und auch noch nicht veröffentlicht.

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