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Time-Varying Uncertainty and Business Cycles

A Shock-Restricted Structural Vector Autoregression Approach

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To my Mum.

I want to use this opportunity and thank

- ▷ Christian Aichmüller, Judith Santer, Nathan Harrold, Rudolf Hakenschmidt, Alban Zeiger, Katrin Findeisen, Sabrina Ronacher, Caro Poullain, Silvia Hribik, Cécile Herzl and Bettina Lubach for the most amazing time during our Master's programme and beyond.
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Having studied the paper of Ludvigson et al. (2018) (which, together with its updated version in Ludvigson et al. (2019), is a central piece of this work) and after having set up a first preliminary test-version (without the correlation constraints) of their shock-restricted SVAR-algorithm in R, I detected implausible discrepancies in comparison to the author's IRFs and experienced severe bottle-necks with respect to computing power due to the inefficient way I had implemented the recursion for the derivation of the Ψ-matrices to be able to ultimately calculate the IRFs derived off of the Θ-matrices. Thankfully, Sai Ma, PhD (one of the co-authors) kindly provided me with the Matlab-code for the algorithm of their Baseline-scenario as well the data they used which greatly helped me to detect errors in my own implementation.

Lastly, what I wrote when handing in the diploma thesis for my first degree still holds more than ever:

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Abstract

Uncertainty is a latent and generally unobservable stochastic process. Insofar, the uncertainty literature which experienced a renaissance with the publication of Bloom (2009) is largely trying to solve both, questions related to the measurement and *types* of uncertainty and, most prominently, the role of time-varying uncertainty on and in business cycles, respectively. In the meantime a voluminous amount of both theoretical and empirical work has produced various uncertainty measures that, in various empirical VAR-settings, largely suggests that uncertainty exerts a negative influence on the business cycle. Departing from frequently used recursive schemes in VARs which don't allow for the link between uncertainty and business cycles to contemporaneously go vice-versa by construction, Ludvigson et al. (2018, 2019) suggest a novel shock-restricted SVAR-approach whose technicalities we discuss and implement in R for this thesis. Their results indicate that, first, the uncertainty literature has to distinguish between macroeconomic and financial uncertainty and, second, that higher macroeconomic uncertainty is an endogenous *response* to shocks to aggregate output while financial uncertainty can be seen as a likely source for protracted declines in aggregate output.

Zusammenfassung

Unsicherheit ist ein latenter und im Allgemeinen nicht beobachtbarer stochastischer Prozess. Insofern versucht die Literatur, die mit der Veröffentlichung von Bloom (2009) eine Art Renaissance erfuhr, weitgehend sowohl Fragen zur Messung und den *Arten* von Unsicherheit als auch, vor allem, zur Rolle zeitlich variierender Unsicherheit auf bzw. in Konjunkturzyklen zu beantworten. In der Zwischenzeit hat eine große Menge von sowohl theoretischen als auch empirischen Arbeiten verschiedene Unsicherheitsmaße vorgeschlagen, die in verschiedenen empirischen VAR-Anwendungen weitgehend darauf hindeuten, dass Unsicherheit einen negativen Einfluss auf den Konjunkturzyklus ausübt. Abweichend von den üblichen rekursiven Ansätzen in VAR-Modellen, bei denen der Zusammenhang zwischen Unsicherheit und der Konjunktur bereits per Konstruktion nicht gleichzeitig einen umgekehrten Impuls geben kann, schlagen Ludvigson et al. (2018, 2019) einen neuartigen Schock-beschränkten SVAR-Ansatz vor dessen technischen Details wir in dieser Arbeit darstellen und in R implementieren. Ihre Ergebnisse deuten darauf hin, dass, Erstens, in der Unsicherheits-Literatur zwischen makroökonomischer und finanzieller Unsicherheit unterschieden werden muss und, Zweitens, dass eine höhere makroökonomische Unsicherheit eine endogene Reaktion auf Konjunktur-Schocks ist, während finanzielle Unsicherheit eine wahrscheinliche Quelle für langwierige Konjunktureinbrüche darstellt.

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1. Introduction

It can be commonly postulated that being faced with uncertainty is a state inherent to life's existence and in every aspect a natural part of our complex world.

Focussing on *economic* uncertainty, an investor being concerned about the performance of her assets, an employee being worried about future income streams during retirement or at all still having a job one year from now[1] or a business continuously defending its position on the market are just a few examples of individuals or a collective being exposed to uncertainty in every-day life. Many of these risks/uncertainties[2] have given rise to institutions and entire industries as an integral part of today's modern economies: sophisticated financial markets and the finance and insurance industry would not exist (or albeit in a different form) if it was not for the existence of certain risks/uncertainties. On the academic front, ever more complex models promise remedy to make exposures to uncertainty and foresight calculable and/or manageable. On the other hand in many countries government institutions like social security or public health care have been established as collective cushion to cater for unpredictabilities.

Going beyond these exemplary scenarios where economic agents seem to take into account some sort of probabilistic assessment of the future turn of events, on an aggregate (macro) level specific (tail) events seemingly cause (economic or political) uncertainty to ultimately feed into the micro-level (firms and households) in a potentially damaging way. These scenarios are rather being perceived as exogenous shocks (i.e., outside of one's own area of influence) than part of a known probabilistic state space.

After the Great Recession, the subsequent turmoil in the Eurozone's sovereign debt markets and the Arab Spring being just a few examples, the most recent major

[1] Examples taken from Stockhammer and Ramskogler (2007).

[2] Note that we are currently still using these terms interchangeably but will give a brief overview of the discussion and their subtle differences in the literature below.

political events[3] of Brexit, Trump's election in the US, the trade war with China and a seeming worsening of the relationship between forces of the West and East have intensified concerns about uncertainty (as mentioned by Baker et al. (2015) or Baker et al. (2019)). Assessments from the Federal Open Market Committee (2009)[4] and the International Monetary Fund (IMF, 2012, 2013)[5], for example, suggest how multilayered uncertainty can be by referring to how factors such as "[...] U.S. and European fiscal, regulatory, and monetary policies [have] contributed to a steep economic decline in 2008-2009 and slow recoveries afterward" (Baker et al., 2015, p.1594). The most recent emergence of the Covid-19 pandemic as an external shock of unprecedented size (when looking at the post-war period) constitutes a new class of shocks which, ex-ante, is also best regarded as being outside a probabilistic state-space.

On a high level, Dequech (2000) identifies four basic social practices in today's societies that have a stabilizing effect on the coexistence in our (as he calls it) *economic reality*: (1) legal contracts (that reduce the uncertainty of future values of nominal variables), (2) the 'State' which enforces legal contracts in case one party to a contract would decide to not fulfill its obligations, (3) Market-Makers (a very prominent example being central banks in their roles as lenders of last resort), and lastly, (4) informal institutions including conventions like "socially shared and/or prescribed standards of thought and behavior" (Dequech, 2000, p. 54). Together these features improve subjective and objective uncertainty but can obviously not completely eliminate it.

And whoever analyses *uncertainty* and its underlying meaning in the literature hoping to clearly differentiate it from related concepts such as *risk* or *ambiguity*, will quickly realize that an unambiguous definition is not straightforward. Confining ourselves to economics only, there is a vast array of literature starting at the beginning of the 20th century that refers to these concepts in different schools of thought that at their core do have a philosophical component to them. Different parlance within the

[3] One could potentially also call them 'shocks'.

[4] "Widespread reports from business contacts noted that uncertainties about health-care, tax, and environmental policies were adding to businesses' reluctance to commit to higher capital spending" (Federal Open Market Committee, 2009).

[5] "More seems to be at work, however [...] - namely, a general feeling of uncertainty'. Assessing the precise nature and effects of this uncertainty is essential, but [...] not easy. [...] Uncertainty appears more diffus, more Knightian in nature." (IMF, 2012). In IMF (2013) the authors dedicated a whole section (pp. 70-67) to the study of the effects of uncertainty, stating that "[a] common view is that high uncertainty in general, and high policy uncertainty more specifically, has held back global investment and output growth in the past two years." (IMF, 2013, p. 70).

profession which even causes similar concepts to be named differently (Dow, 2016) or the same term being used with a different meaning additionally complicates matters.

While a thorough discussion of every aspect of the related concepts is beyond the scope of this work, we still want to give a brief overview of a few insights by referring to a few selecting writings:

Dow (2016) tries to clarify the difference between the mainstream and Keynesian[6] understanding of *uncertainty*[7], its distinction from *risk* and the resulting theoretical implications[8]. According to her assessment, in the traditional mainstream analysis within a Bayesian framework most knowledge is viewed as known or known to be stochastic in some form (called *risk*), meaning that the state space is known and numerical probabilities can be assigned. Under this approach, mainstream traditional macroeconomic models give only very limited room, if any, to *uncertainty* as a state of absence of such probabilities in the form of exogenous sources of shocks[9].

Read from this dualistic mainstream perspective, Dow (2016) asserts that Keynes' standpoint was also for a long time interpreted in a dualistic way, albeit flipped in the sense that only in special circumstances, knowledge (here referred to as *risk*) could be treated as certain and (fundamental) uncertainty was regarded as the general case (see the introduction in Keynes, 1921). This dualistic interpretation was seemingly reinforced by Keynes's reflections on long-term expectations: "About [uncertainty] there is no scientific basis on which to form any calculable probability whatever. We simply do not know" (Keynes, 1937, p. 214).

But Dow (2016) clarifies that in the meantime it is well-established that Keynes did not give up on scientific knowledge (i.e., did not advocate nihilism) by giving so much weight to uncertainty but that his view on uncertainty is rather multidimensional and consists of various degrees. As an advocate of this view, Dequech (2000) argues that Keynes referred to both ambiguity in his early writings (Keynes, 1921)[10] and

[6]Keynes and Knight are often-times regarded as the two prominent figures that introduced the concept of *fundamental uncertainty* into the economic sciences.

[7]Dow (2016, p. 3) also makes account to von Mises, Hayek and Shackle who have also dealt with uncertainty in their works but focusses on Keynes's views on fundamental uncertainty arguing for it to be the most developed and "philosophically-grounded [...] counterpoint to the mainstream" theory.

[8]Note that there are also schools that reject the distinction between *uncertainty* and *risk* altogether (e.g., Savage, 1954).

[9]Dequech (2000, p. 43) refers to some economists partly neglecting fundamental uncertainty following the argument that it might lead to 'theoretical nihilism' (see e.g., Coddington, 1982).

[10]According to interpreters, in terms of Keynes (1921), for both ambiguity and fundamental uncertainty there is no basis for the assignment of point, numerical probabilities (Dequech, 2000).

fundamental uncertainty in his later ones (Keynes, 1937) and that simply speaking of Keynesian uncertainty may actually be too vague in most circumstances.

Camerer and Weber (1992, p. 330) describe ambiguity as "uncertainty about probability, created by missing information that is relevant and could be known". Dequech (2011, p. 623) states that while usually the decision-maker under ambiguity cannot reliably assess the probability of each event, she usually knows all possible events (or it is at least "predetermined or knowable ex ante") while the case of fundamental uncertainty "[...] is characterized by the possibility of creativity and non-predetermined structural change". So under this concept and "[w]ithin the bounds imposed by natural laws, the list of possible events is not predetermined or knowable ex ante [...] as *the future is yet to be created*[11]" (Dequech, 2011, p. 623).[12]

With regard to fundamental uncertainty, Dequech (2000, p. 48) mentions "technological or managerial innovations" as examples of human creativity that can disrupt our realities which he sees being closely connected to the process of "creative destruction" of Schumpeter (1942). "Unpredictable structural changes" (e.g. historical changes) to our economic reality on the other hand are more typically political, social or cultural disruptions where institutions and technological change play a key-role (Dequech, 2000, p. 49). Interestingly, as outlined by Dequech (2000), the capitalist system as we know it is regarded as endogenously causing uncertainty due to competing economic actors and decision-makers that innovate due to the possibility of additional profits (see Kregel, 1987)[13].

Coming back to the mainstream view, while the crisis and subsequent Great Recession has triggered a rethinking of the mainstream approach to uncertainty by emphasizing "institutional impediments to information access (asymmetric information)" (Dow, 2016, p. 8), degrees of uncertainty were only "gradually" (Dow, 2016, p. 11) added to the picture: the term 'ambiguity' was acquired as well (also following Camerer and Weber, 1992, p. 330) to account for these various degrees of uncertainty while 'unknown unknowns' (i.e. unimaginable events or 'black swans' as dubbed by Taleb, 2008) would have to be seen as *'knowable unknowns'* to still be consistent with the Bayesian framework. The term 'ambiguity' is thereby introduced as falling either into the category of risk or uncertainty depending on the ability to quantify higher-order

[11]Italics added.

[12]Dequech (2000) sees the work of Shackle (2017) as advocating this argument.

[13]Despite of the above remarks, Keynes' notion of uncertainty as described in Keynes (1921) and its connection to Keynes (1937) has been subject to much controversy due to differing interpretations (Dequech, 2000).

probabilities (Dow, 2016).

Trying to establish a typology within the introduced terms, Dequech (2011) sets up three dimensions along which he classifies relevant concepts[14]:

1. between substantive and procedural uncertainty:[15]
2. between weak and strong uncertainty:[16]
   weak uncertainty is then further divided into *Knightian risk* (see Knight (1921); often simply called 'situations of risk' where agents are faced with objective probabilities, either a priori or statistical probabilities (i.e., relative frequencies) and *Savage's uncertainty* (see Savage (1954); who introduced a full theory of 'personal probability' where *personal* belief governs probabilities, based on prior work of Ramsey (1926)[17] and de Finetti (1937));
3. between ambiguity and fundamental uncertainty.[18]

Abstracting from the (in parts philosophical) considerations above, the question arises as to whether there is any consent in the literature about whether uncertainty can actually be quantified and, if so, how this could be achieved? And once having a meaningful measure of uncertainty, what *real* effect, if any, does it exert on business cycles?

Within the past ten years and largely triggered by a seminal work by Bloom (2009)[19], the uncertainty literature experienced a renaissance and macroeconomists have started studying these questions more intensively than ever before. At a first glance, as summarized by Jurado et al. (2015, p. 1177), partial equilibrium analyses to date suggest that increased levels of uncertainty "[...] can depress hiring, investment, or

[14] Apart from this classification we do not discuss any further interrelationships between the various concepts.

[15] Proposed by Dosi and Egidi (1991, p. 145) whereby substantive uncertainty results from the "lack of all the information which would be necessary to make decisions with certain outcomes" and procedural uncertainty from "limitations on the computational cognitive capabilities of the [respective] agents to pursue unambiguously their objectives, given [...] available information" in a complex decision problem.

[16] Whereby under weak uncertainty "[...] an agent can form [...] a unique, additive and fully reliable probability distribution" based on relevant and good-quality information and strong uncertainty consequently "[...] by the absence of such a distribution" (Dequech, 2011, p. 622/623).

[17] Written "in opposition" to Keynes (1921).

[18] Which are two types of strong and substantive uncertainty (Dequech, 2000).

[19] Bloom's contribution triggered a wave of studies that challenged the effects that he had reported in his initial publication in Bloom (2009). We will come back to his study in Section 2.2.

consumption if agents are subject to fixed costs or partial irreversibilities (a 'real options' effect), if agents are risk averse (a 'precautionary savings' effect), or if financial constraints tighten in response to higher uncertainty (a 'financial frictions' effect)."

But as aptly formulated by Bontempi et al. (2016, p. 24), the *empirical*[20] results found in the literature to date are not unambiguous which, based on their assessment, raises, among others, the following *key*[21] questions:[22]

1. Are uncertainty shocks temporary or more persistent?
2. Is the degree of persistence of identified negative uncertainty effects related to the econometric specification and/or the particular (type of) uncertainty measure used?
3. Does the time span over which a model is estimated play any role?
   And finally:
4. Are uncertainty shocks an exogenous impulse or an endogenous response to macroeconomic fluctuations (as posed by Ludvigson et al., 2019)? In other words, is uncertainty a decisive factor contributing to business cycles or does the causation go vice versa?

The remainder of this thesis is organized as follows. Section 2 reviews a selection of the related literature triggered by the seminal work of Bloom (2009) and thereby elaborates on potential shortcomings of the popularized econometric approaches and picks up on Questions 1 & 2 along the way, Section 3 hence introduces the econometric framework of Ludvigson et al. (2019) that tries to alleviate the identified shortcomings by shedding light on the central question of whether uncertainty shocks are an exogenous impulse (as frequently postulated in the literature) or an endogenous response to uncertainty shocks and whether the particular *type* of uncertainty matters by utilizing their novel shock restricted structural vector autoregression (SVAR) approach (Question 4 in combination with the latter part of Question 2 regarding the *types* of uncertainty)[23]. Section 4 gives an overview of the data used and Section 5 presents the results of a Baseline-model which we have implemented in R. Section 6 finally summarizes and concludes. Additional information and results are deferred to Appendix A and Appendix B, respectively.

[20] Italic's added.
[21] Italics added.
[22] We have slightly adapted/extended Bontempi et al. (2016)'s objections.
[23] Note that 'Question 3' will not be elaborated further in this work.

2. Related Literature

While uncertainty has been dealt with in theoretical and empirical applications already long before the Great Recession, it is safe to say that research in the field surged in the past decade to understand the relationship between uncertainty and business cycles and to put, among others, arguments that point at increased uncertainty as a possibly compounding factor to the Great Recession and subsequent slow recovery on steadier grounds.[24]

This growing body of literature covers the entire spectrum including studies examining the matter from (i) a solely theoretical, (ii) purely empirical or (iii) both perspectives in both micro- and macroeconomic analyses. In our review below we will refer to works falling into either of the three categories whereby, with respect to the selection of works presented here, contributions having both an empirical and theoretical component are in a slight majority. Note, however that also purely theoretical or empirical works often pick up previous theoretical or empirical results that have been reported in the literature and thereby all the more establish a theory-empirics-nexus. Further, while the Great Recession is frequently mentioned as a reference, the implications of the results are often-times generalized without specific attribution to the Great Recession. At most, key economic features (e.g. zero lower bounds, etc.) are considered in model extensions.[25]

Empirically, various analyses have produced seemingly unambiguous results in various identification schemes predominantly in a VAR-framework thereby deploying different uncertainty proxies in an attempt to measure the underlying latent stochastic process (since uncertainty is an intrinsically unobservable concept as formulated by Bloom, 2014). But as we will discuss in the following, these empirical findings overall result from econometric specifications which might potentially be problematic.

[24]While this scenario might seem intuitive and is partly also supported by theoretical and empirical work, in the remainder of this thesis we will see that the answers are not clear-cut.

[25]An exception is, e.g., Schaal (2017) that investigates the fit of his model to past recessions.

Theoretically, most models stress contractionary effects of uncertainty although diametrical effects might be at work as well. Abstracting from problems revolving around the *measurement* of uncertainty and empirical analyses and deferring them to Sections 2.4 and 2.2, Section 2.1 below starts with an overview of the theoretical behavioural channels that have been identified in the literature, Section 2.2 subsequently presents a few contributions in greater detail including an analysis of their econometric frameworks in VAR-settings. Section 2.3 looks at the robustness of some of the popularized VAR-results while Section 2.4 takes a brief look at the ambiguous definition of uncertainty in the literature and stylized facts of a few selected and popular uncertainty measures. Section 2.5 finally concludes the discussion and presents the main research question which is whether uncertainty can be seen as a source of economic fluctuations or itself endogenously responds to business cycles.

2.1. Theoretical Behavioural Channels, *or*: The Micro-Macro-Nexus

In the microeconomics literature, there is no single uncertainty theory and various mechanisms have been identified through which aggregate uncertainty suggests to play a role in altering economic agent's behavior which largely adversely and in some cases also suggests to positively affect economic activity. In particular, the literature largely evolves around *real-options-* (on which a large part of the literature has focussed so far), *risk-aversion-*,[26] *risk premia-* and *Oi-Hartman-Abel effects* that help forming a better understanding of the forces at work.[27] While the mentioned effects are partial equilibrium effects (PE; ceteris paribus effects) with generally unambiguous signs in isolation, in general equilibrium (GE) analyses the described effects continue to play a role but might need additional frictions to derive the same results (Jurado et al., 2015). The summary of the various effects below is largely based on Bloom (2014).

The *real-options-effect* primarily applies to firms (investment and hiring decisions) but has also been expanded to households (consumption and thus savings decisions).

In the case of firms the real-options effect gave rise to the analysis of business cycle dynamics within the traditional framework of irreversible investment: when being faced with uncertainty firms become more cautious about investment and hiring if those decisions cannot easily be reversed (so-called 'investment adjustment costs')

[26] *Precautionary savings* effects are classified somewhere in-between. See below for details.
[27] This taxonomy is based on the summary in Bloom (2014).

which is summarized under the *delay effect*.[28] More specifically, a company acquiring capital for a purchase price that will differ from the resale price essentially buys a put option and through an investment today forgoes a call option (i.e., the chance to buy later). In an environment of higher uncertainty the value of a call option increases (an incentive to postpone investment to avoid costly mistakes) while with respect to partial reversibility the value of a put option that is being obtained by investing today increases with uncertainty, leaving the overall effect ambiguous. These (partial) irreversibilities have been introduced as an answer to the *Oi-Hartman-Abel*-effect (which we introduce below), among others, by Dixit and Pindyck (1994), Bernanke (1983), Abel and Eberly (1996) and McDonald and Siegel (1986).[29] For effects on hiring see e.g. Bentolila and Bertola (1990) . In the more recent uncertainty literature, Bloom (2009), Bloom et al. (2012) and Schaal (2017) study the real-options effect (see Section 2.2 below for more details).

Besides uncertainty affecting the *levels* of investment and hiring (the *delay effect*), economic agents might also become *less sensitive* to changes in external conditions in an uncertain environment (the *caution effect*; see e.g. Bloom, 2009).[31] In this, Bloom (2014) also sees an explanation for uncertainty stalling productivity growth due to an impediment to the productivity-enhancing reallocation of resources across firms (see also Bloom et al., 2012).

Besides the above mentioned detrimental effects of uncertainty, as noted by Bloom (2014), within the framework of real options the effects are not universal insofar as they are strongly subject to the condition of (ir)reversibility: if firms can easily adjust their behavior to allow for more flexibility under deteriorating circumstances the expected detrimental effects might be alleviated (see, e.g., Valletta and Bengali, 2013).

On the consumption side, equivalent patterns arise (delay and caution effects) and lead to precautionary spending cutbacks: uncertainty causes households to reduce the

[28]These option-effects are alleviated, however, if, on the other hand, delays would be so costly as to hamper firm's ability to wait.

[29]Dixit and Pindyck (1994), primarily looking at investment, introduce the "option value for better (but never complete) information" when it comes to investment decisions.[30] Bernanke (1983) investigates optimal investment decisions under uncertainty in an oil cartel and finds that high uncertainty causes firms to postpone investments and hiring in an environment where investment projects cannot easily be undone and labor market laws cause changes to the labor force (either hires or fires) to be costly.

[31]This result, among others, increases pressure on authorities to put forth monetary and fiscal interventions strong enough to outweigh these effects.

consumption of durable goods (items with a high real option value under uncertainty) like houses, cars, etc. (see e.g., Eberly, 1994; Romer, 1990). *Risk aversion* might turn this precautionary spending effect into a precautionary *savings* effect which is defined as the "[...] additional saving that results from the knowledge that the future is uncertain" (Carroll and Kimball, 2006, p. 2). Causing economic agents to cut back on consumption and increase labor supply to self-insure against future shocks is associated with a likely aggregate contractionary effect in the short run but an ambiguous effect in the long-run due to differing effects on consumption (-) and investment (+). While in theory (as mentioned by Bloom, 2014), consumption cutbacks and a higher saving rate may induce investment and growth in the longer term, Fernandez-Villaverde et al. (2011) suggest that for a small open economy also potential positive long-run effects are impeded due to money fleeing the country. Likewise, in New Keynesian models they show that for large but more closed economies no positive savings (investment)-effects apply because through interactions between sticky prices (nominal rigidities) and search frictions missing market adjustments of interest rates and output prices might cause uncertainty shocks to translate into 'aggregate demand shocks' (see in particular Leduc and Liu, 2016 and Basu and Bundick, 2017 and our summary of their findings in Section 2.2 below).

Turning to financial markets, triggered by asset price volatility and the explosion of credit spreads throughout 2007-2009, another branch of the literature points to financial frictions (i.e., agency and/or moral hazard problems; for both equity and debt) as an additional channel through which uncertainty affects macroeconomic outcomes. First, it puts upward pressures on the cost of finance by increasing risk premia (*risk premia effect*) due to investors' demand for an adequate risk-compensation (equity). In addition, creditors might drive up interest rates (i.e., rising credit spreads), increasing the user cost of capital and retract lending activity overall which further hampers firms' ability to borrow (debt) and in effect decreases investment spending and subsequently output (see e.g., Christiano et al. (2014), Arellano et al. (2011), Arellano et al. (2016) and our summary of Gilchrist et al. (2014) in Section 2.2 below).[32]

Two potential effects through which uncertainty can potentially have a positive

[32] A potential additional precautionary effect related to *risk aversion* studied in the literature is 'managerial risk aversion' whereby under uncertainty investment decreases in particular for firms where the managers themselves hold a considerable amount of shares (see e.g. Panousi and Papanikolaou, 2012).

effect on growth in the long run entered the literature as *growth options* and the *Oi-Hartman-Abel* - effect. For *growth options* the argument goes that uncertainty can encourage investment in the race for a potential price (see e.g. Bar-Ilhan and Strange, 1996). In other words, uncertainty may stimulate reserach and development, meaning that firms appear more willing to innovate when faced with uncertainty. The *Oi-Hartman-Abel* - effect (Oi, 1961; Hartman, 1972; Abel, 1983) is based on the impact of uncertainty on revenue. As outlined in Saltari and Ticchi (2000), the intuition goes as follows: an increase in uncertainty about the output price raises the investment of a risk-neutral competitive firm with constant returns to scale technology due to the convexity of the marginal product of capital with respect to the output price (or, in general, the stochastic variable). By Jensen's inequality, larger uncertainty about these variables implies a higher expected value of the marginal product of capital, and so a higher investment. This result, however, follows under the assumption of labor being a fully flexible input so that after observing a price shock, a firm can adjust labor so that the marginal product of capital increases more than proportionally relative to price. While these effects might typically not be very strong in the short run, they can unfold better in the medium to long run (Bloom, 2014).

In the more detailed discussion of a few selected works below *search frictions* play a prominent role and are linked to the presented *option-value* channel by producing similar effects. As described by Leduc and Liu (2016), search frictions alone already suffice to cause uncertainty shocks to be contractionary (in comparison to RBC models featuring a spot labor market). This is because they "[p]rovide an additional mechanism for uncertainty shocks to generate large increases in unemployment via an option-value channel. With search frictions, a job match represents a long-term employment relationship that is irreversible. When times are uncertain, the option value of waiting increases and the match value declines [...]" and firms correspondingly respond by reducing hiring (Leduc and Liu, 2016, p. 2). According to Leduc and Liu (2016), this option-value effect in their model with search frictions originates for similar reasons that are generally described in the literature about irreversible investment decisions under uncertainty (e.g., Bernanke, 1983; Bloom, 2009; Bloom et al., 2012).

Nominal rigidities/sticky prices are also a key ingredient of many uncertainty studies because they help aggravating the effect of uncertainty shocks on unemployment through reductions in aggregate demand. Further, in such model constellations, inflation decreases due to the fall in demand.

2.2. Close-Up: Selected Works in More Detail

Bloom (2009) was among the first to propose uncertainty shocks as a 'new' driver of business cycles and triggered much of the literature in subsequent years by building his argument particularly around the adverse effect of real-options on investment.[33] In the theoretical model, uncertainty is added as a stochastic process into a (large-scale) standard model of the firm (as in e.g., Abel and Eberly, 1996) together with a mix of labor and capital adjustment costs[34] which, under the assumption of nonconvexity, produce a central region where firms temporarily pause their investment and hiring if business conditions are sufficiently bad (and consequently only hire and invest when the business conditions are sufficiently good). The interaction of the time-varying uncertainty (represented by the time-series constructed from realized volatility in the S&P 500 and the VXO) with the nonconvex adjustment costs feeds into time-varying real-option effects that generate fluctuations in hiring and investment. A simulated large temporary uncertainty shock in the model (in the form of a rise in the variance of productivity and demand growth) moves the hiring/firing and investment/disinvestment thresholds outward accordingly which expands the region of inaction (the real-option value of waiting is worth more than the returns to investment and/or hiring) and leads to a rapid drop in hiring, investment, productivity and output triggered by the high real-option value to waiting. The rapid drop is, however, followed by a corresponding rebound and even an eventual overshoot in the 6-8 months into the shock as firms start to work off their piled up demand for labor and capital once the impact of the uncertainty shock subdued.[35,36,37]

Bloom (2009) finds these partial equilibrium effects to be entirely consistent with empirical evidence as shown in hise impulse response functions of vector autoregressions (VARs) on actual data. In particular, using 17 identified 'exogenous shocks', in support of his theoretical model, Bloom (2009) estimates VARs showing that

[33] Bloom (2009) himself links his work to the work of Bernanke (1983) and Hassler (1996).

[34] Adjustment costs make it hiring/firing and investment/disinvestment decisions costly.

[35] Bloom (2009) finds productivity growth to react similarly due to the temporary halt slowing down the reallocation of labor and capital between low and high productivity firms.

[36] Bloom (2009, p. 646) notes that pure real-options effects should actually only lead to a convergence back to trend in the *levels* without an overshoot. This medium-term overshoot occurs due to a burst of net hiring/investment promoted through more firms clustered around the hiring/investment than the firing/disinvestment threshold and only subdues in the long run (a phenomenon Bloom (2009) calls "volatility overshoot".

[37] A more traditional stand-alone first-moment shock conversely produces a long and persistent drop over several quarters.

an uncertainty shock in the data (the VXO-measure) produces a short-run drop in industrial production of 1% which lasts for about 6 months (a confirmation for the 'wait-and-see approach') and subsequently creates a longer-run overshoot. Details regarding the variables and ordering are summarized in Appendix B.2. In Bloom's setting, these results are robust to the effect of an uncertainty shock on employment and a range of alternative approaches (including variable ordering, variable inclusion, shock definitions, shock timing, and detrending).

Gilchrist et al. (2014) question the sole significance of the traditional "wait-and-see" effect to investment caused by capital adjustment frictions (as popularized by e.g. Bloom, 2009) and rather see the main mechanism through which changes in uncertainty impact the macroeconomy in financial distortions. Backed by empirical micro- and macro-evidence, their quantitative general equilibrium model investigates investment dynamics for firms exposed to time-varying idiosyncratic uncertainty, two forms of adjustment frictions (partial irreversibility and nonconvex fixed capital adjustment costs) and frictions in both the debt and equity markets. Including this set of features allows them to individually address and isolate the adverse real-option effects and financial friction effects.

Being able to turn financial distortions on and off in their model, the key variable affected throughout the transmission mechanism under financial frictions is the supply of credit: increased uncertainty amplifies the already adverse response of investment to heightened volatility triggered through a sharp and persistent increase in corporate bond credit spreads due to which firms, confronted with higher user cost of capital, radically bring down investment outlays even further and delever. Specifically, the authors attribute more than three-quarters of the adverse effect of an uncertainty shock on investment to the presence of financial distortions.[38,39]

As part of their macro-evidence the authors subsume their idiosyncratic uncertainty measure to an aggregate uncertainty measure and use VARs over the period 1963:Q3 - 2012:Q3 examining the impact of two different shocks: innovations to the idiosyncratic uncertainty measure and to the credit spreads (i.e., a financial shock). Details

[38]Gilchrist et al. (2014) see the dynamics in their model to be consistent with the empirical evidence developed in Stock and Watson (2012) who see the combination of uncertainty and credit supply shocks as the primary drivers of the slowdown during the Great Recession.

[39]Through the combination of partial irreversibilities and financial frictions, in their model Gilchrist et al. (2014) also identify "capital liquidity" shocks as a new potential source of shocks: in this scenario adverse shocks to the liquidation value of capital affect the debt capacity of firms (through haircuts on collateral).

regarding the variables and ordering (i.e. their identification schemes I + II) are summarized in Appendix B.2. For their 'identification scheme I', the corresponding impulse response functions upon an uncertainty shock show a widening of credit spreads consistent with their micro-estimations. This influence aside, the investment component of aggregate output (business expenditures on fixed capital and consumer spending on durable goods) falls strongest with a maximum 1.5% decline reached six quarters after the shock. A credit-spread shock (an increase of almost 30 basis points in the BBB-Treasury spread) also points at significant and long lasting economic contractions (depressing the investment component of aggregated demand *and* nondurable goods consumption) while the effect on uncertainty is negligible. Alternatively, 'identification scheme II' (ordering credit spreads before the uncertainty proxy), however, shows far less adverse effects of uncertainty shocks on the macroeconomy whereas credit spread shocks (a financial shock) show a protracted and long-lasting effect on economic activity (GDP, business fixed investment and major categories of consumer spending) and an immediate jump in uncertainty. Interestingly, Gilchrist et al. (2014, p. 14) see the immediate response of uncertainty to innovation in credit spreads to suggest that "[...] fluctuations in uncertainty may arise endogenously in response to changes in broader financial conditions [..]" which touches upon an issue we will pick up again in Section 2.5.

Based on the empirical evidence of a VAR, Basu and Bundick (2017) argue for the resulting co-movement of output, consumption, investment, and hours worked to be a key empirical feature of an economy's (i.e., the data's) response following an uncertainty shock and hence also as a "[...] key minimum condition that business-cycle models driven by uncertainty fluctuations should satisfy" (Basu and Bundick, 2017, p. 937).[40] With their particular ordering (details regarding the variables and ordering are summarized in Appendix B.2), their VAR generates statistically significant co-moving declines in output, consumption, investment, and hours worked with a peak response after approximately one year whereby their key stylized fact is robust to several modifications.[41] These insights are subsequently brought into a New Keynesian DSGE model (monopolistic, one-sector closed-economy model) where they calibrate their uncertainty shock process using fluctuations in the VXO and add nom-

[40]At the same time, they see this feature as missing from the related work of Bloom (2009), Bachmann and Bayer (2013) or Gilchrist et al. (2014).

[41]These modifications include: inclusions of stock prices in the VAR (as is done e.g., by Bloom, 2009), measurement of uncertainty using the VIX instead of the VXO, a re-ordering of the VAR placing uncertainty last or an adjustment of the sample period to exclude the Great Recession.

inal price rigidities (sticky prices) to replicate the transmission mechanism which was identified in the data. According to Basu and Bundick (2017), the resulting dynamics due to the incorporated price rigidity in a model where output is demand-determined are able to support the intuition that an increase in uncertainty which induces precautionary savings, reduces household expenditures, output and labor input and eventually the demand for capital and investment as suggested by their VAR-model.[42]

Complementing the existing literature[43], Leduc and Liu (2016) find that uncertainty shocks produce effects similar to aggregate demand shocks. While Born and Pfeifer (2014) don't find convincing evidence for large real effects of policy uncertainty shocks in a standard DSGE model, to highlight aggregate demand effects (and abstracting from additional features that would also shed light on effects on investment), Leduc and Liu (2016)'s theoretical New-Keynesian DSGE framework that incorporates both search frictions (an extension vis-a-vis Basu and Bundick, 2017) and nominal rigidities as key ingredients where the interplay of the reciprocal amplification of an option-value channel arising from search frictions in the labor market and decreases in aggregate demand stemming from sticky prices in the goods market produces a transmission mechanism of uncertainty on unemployment and inflation that is consistent with their empirical observation.[44]

Specifically, their theoretical model is guided by the empirical evidence derived from a four-variable Bayesian VAR (BVAR) on US data. Details regarding the variables and ordering are summarized in Appendix B.2. In this setting, the authors declare the observed joint dynamics of sharply rising unemployment and decreasing inflation following an uncertainty shock to resemble features of a negative aggregate demand shock.[45] Contrary to the sole usage of the VIX as an uncertainty measure, Leduc and Liu (2016) report the dynamics to be robust to their alternative uncertainty

[42]Before introducing their model's features, Basu and Bundick (2017) show that uncertainty shocks in standard closed economy real business cycle (RBC) (i.e., competitive, flexible prices) models produce a counterintuitive aggregate expansion. Monopolistic competition under sufficiently sticky prices changes the dynamics.

[43]The empirical results in, among others, Bloom (2009), Bloom et al. (2012), Jurado et al. (2015), Scotti (2016), Bachmann et al. (2013) and the theoretical results in Bloom (2009), Gilchrist et al. (2014), Arellano et al. (2011), Basu and Bundick (2017), Bloom et al. (2012), Born and Pfeifer (2014).

[44]This interaction considerably amplifies the fall in aggregate demand in response to uncertainty shocks in comparison to standard DSGE models that solely consider nominal rigidities and go without search frictions as in e.g. Basu and Bundick (2017).

[45]Adding habit formation to their model, their calibrated DSGE model comes closest to the empirical results from the VAR-model.

measure of consumers' perceived uncertainty. The resulting IRFs at impact of an uncertainty shock show a strongly persistent unemployment increase, peaking after approx. 18 months and significantly lasting for about three years. Inflation shoots in the opposite direction with a peak effect after around 20 months and staying significant for almost two years.[46]

Commenting on the theoretical results of Section 2.1 in combination with the empirical VARs of the selected contributions described above, Ludvigson et al. (2019, p. 4) argue that while the theoretical literature which proposes uncertainty as a source of lower economic growth "[...] has focused on uncertainty originating in economic fundamentals", the empirical literature, however, typically used uncertainty proxies which are strongly correlated with financial markets. According to Ludvigson et al. (2019) this practice makes it necessary to distinguish between the two in empirical applications to understand the nexus between real economic/financial market uncertainty and recessions. We will come back to this issue in Section 2.5.

2.3. Robustness of Econometric Specifications in VAR-Settings

The empirical literature on the *potential* causal effects of time-varying economic uncertainty on macroeconomic activity (aggregates) seemingly has one thing in common: in a VAR context, uncertainty shocks (using proxies) largely seem to have a negative effect on output (or proxies thereof such as industrial production or employment). But as summarized by Bontempi et al. (2016, p. 23), "[...] this key finding only seems to be robust in regard to the uncertainty impact in the short run, whereas in the long run different works [using different uncertainty proxies and slight modifications in the econometric specifications] have pointed to somewhat heterogeneous output responses."

With respect to the famous overshooting-effect of Bloom (2009) introduced in Section 2.2 where after an output decline in the short run the initial level of output is surpassed in the long run, Jurado et al. (2015) show that when employing their own forecast-based measure of macro uncertainty instead of stock market volatility/VXO

[46]To accomodate the zero lower bound (ZLB) in U.S. monetary policy after 2008, Leduc and Liu (2016) report their VAR to also be robust to the usage of the two-year Treasury bond yield (that did not reach the ZLB) as an alternative indicator of the monetary policy stance. To accomodate the possibility of survey respondents' perceptions of bad economic times instead of uncertainty about the future, Leduc and Liu (2016) follow a similar approach like Baker et al. (2015) and add a variable for consumer sentiment as an additional control to their BVAR. The results suggest that uncertainty is indeed forward-looking.

(see Section 2.4 for a brief comparison between these measures) in the VAR(12)-8 model[47] popularized by Bloom (2009) (see Section B.2), shocks to macro uncertainty instead lead to a large and protracted decline in real activity for 4-5 years after the shock and the overshooting-effect disappears. In addition, replacing their uncertainty measure by the VXO to replicate the model of Bloom (2009), Jurado et al. (2015) cannot find clear evidence for the overshooting effect reported in Bloom (2009). Jurado et al. (2015) trace this result back to the fact that contrary to a statement in Bloom (2009), the available code shows that all data in the employed VARs is HP-detrended *apart from* the VXO index. Jurado et al. (2015) do not detrend any of the variables for their replication exercise, arguing that the HP filter uses information over the entire sample due to which it is problematic to interpret the timing of an observation. Hence, the overshooting-effect found in Bloom (2009) cannot conclusively be replicated but seems to be sensitive to detrending of the VXO data and disappears when using raw data (i.e. might be an artefact in the data as formulated by Bontempi et al., 2016). In addition, for Bontempi et al. (2016, p. 24) this observation "[...] suggests that researchers need to be careful when proxying uncertainty with [...] finance-based measures, as they may label certain transitory financial crises as uncertainty shocks."

Employing a slightly modified version of the well studied macro VAR popularized by Christiano et al. (2005) (see Section B.2), with the main difference from their VAR being the inclusion of a stock price index and uncertainty resulting in a VAR(12)-11 model, Jurado et al. (2015, p. 1204) find similar protracted responses of uncertainty on industrial production and employment as compared to Bloom's VAR(12)-8 when employing their macro uncertainty measure, while the "[...] response of employment to a VXO disturbance is barely statistically different from zero shortly after the shock and outright insignificant at other horizons". The response of production becomes insignificant after the third month. Jurado et al. (2015) point out that in contrast to the results in Bloom (2009), also for this specification neither of the results generate the overshooting-effect. Similarly, Bachmann et al. (2013) using their own forecast-based measures also cannot confirm the overshooting-effect which they attribute to the usage of the finance-based measure and instead also report prolonged declines in real activity in their specifications.

In conclusion, the above results already give an indication that VAR estimations are

[47]Note that in this notation in $VAR(p) - k$, p stands for the number of lags in the VAR and k for the number of variables.

sensitive to modifications of the data (e.g. detrending), the respective specification and the particular uncertainty measure used. One issue, however, on which the above specifications have been largely silent so far (and which also Jurado et al. (2015) point out) is whether uncertainty can be regarded as a cause of economic contractions or endogenously reacts to declines in real activity itself. This results from the application of recursive schemes which do not allow for uncertainty to endogenously react to declines in real activity.[48] Before coming back to this question, Section 2.4 will continue with a brief overview of popular uncertainty proxies used while Section 2.5 sets the stage for the fundamental issue of endogeneity in the VARs analyzed so far which will be the main concern for the rest of this work.

2.4. Measuring Uncertainty: Overview and Some Stylized Facts

Jurado et al. (2015, p. 1178) argue that "[a] challenge in empirically examining the behavior of uncertainty and its relation to macroeconomic activity, is that no objective measure of uncertainty exists" and that "[w]hile most of the measures have the advantage of being directly observable, their adequacy as proxies for uncertainty depends on how strongly they are correlated with this latent stochastic process [i.e., with unobservable uncertainty]".

While (as formulated by Joëts et al., 2017), in a broad sense uncertainty is being regarded as the *conditional volatility of an unforecastable disturbance*, the empirical literature so far has relied on various measures as proxies for uncertainty, some of which have already been mentioned in Sections 2.2 and 2.3. Among others, these are the implied (e.g., the market volatility indices VIX/VXO) or realized volatility of stock market returns (see e.g. Bloom, 2009), the cross-sectional dispersion of various variables such as profits, stock returns, (total factor) productivity, credit spreads (see e.g. Bloom (2009), Gilchrist et al. (2014), Bloom et al. (2012), etc.)[49], the cross-sectional dispersion of subjective (survey-based) forecasts or business/consumer confidence[50] (see e.g. Bachmann et al. (2013), Leduc and Liu (2016), etc.), or various 'news-based' measures that try to measures the degree of uncertainty by

[48]One exception of the contributions discussed in this Section is Bachmann et al. (2013) which we will pick up again in Section 2.5.

[49]Gilchrist et al. (2014, p. 1) write that "[m]acroeconomists have made a convincing argument that the well-documented countercyclical behavior of the cross-sectional dispersion of economic variables such as labor income, business profits, productivity, and stock returns reflects fluctuations in the volatility of the underlying economic shocks - that is, swings in economic uncertainty."

[50]These measures assume that an impeded predictability or a large dispersion between forecasts, respectively, points at a more uncertain economy. Examples are e.g. future GDP calculated from the Livingstone half-yearly survey of professional forecasters.

extracting the frequency with which specific words appear in newspaper articles (e.g. Alexopoulos and Cohen (2009), Baker et al. (2015), Moore (2016), Bontempi et al. (2016), Castelnuovo and Tran (2017), etc.)[51].

Jurado et al. (2015) see a fundamental issue in this respect arguing that while measures that try to serve as some sort of proxy of a *conditional volatility* are arguably intuitive, uncertainty measures should, following their argumentation, rather capture whether an economy has become more or less *predictable*. In the course of the construction of their own macro uncertainty measure (see Section 4) and its difference in comparison to other uncertainty measures, Jurado et al. (2015) criticize that, in their view, most movements in widely used uncertainty proxies such as the VXO/stock market volatility (as the most commonly used) or measures of cross-sectional dispersion measures are *not* associated with a broad-based movement in economic uncertainty. According to their analysis, the "[...] conditions under which common proxies are likely to be tightly linked to the typical theoretical notion of uncertainty may be quite special" (Jurado et al., 2015, p. 1178) and attribute this argument to episodes where stock market volatility changes even if there is no change in uncertainty about economic fundamentals. This can happen if leverage changes or if movements in risk aversion or sentiment are important drivers of asset market fluctuations (see also Bekaert et al. (2013), Moore (2016) or Ozturk and Sheng (2017)). Further, "[c]ross-sectional dispersion in individual stock returns can fluctuate without any change in uncertainty if there is heterogeneity in the loadings on common risk factors. Similarly, cross-sectional dispersion in firm-level profits, sales and productivity can fluctuate over the business cycle merely because there is heterogeneity in the cyclicality of firms' business activity" (Jurado et al., 2015, p. 1178). Hence, Jurado et al. (2015) argue that predictable fluctuations in frequently used proxies are erroneously attributed to uncertainty and speak for a distinction between *uncertainty* (with respect to *predictability*) of a series and its *conditional volatility*, which is seldomly made in the literature. The construction of their statistical uncertainty measure will be discussed in Section 4.

Figures 1 (on different scales) and 10 (normalized to the same scale) exemplarily show three uncertainty proxies including NBER recessions for the U.S that have been used in the literature. These are the financial market volatility index VXO/realized volatility of stock market returns popularized by Bloom (2009), the macro uncertainty

[51] Note that this approach is similar to the narrative analysis used by Romer and Romer (2004), Romer and Romer (2017) or Ramey (2009).

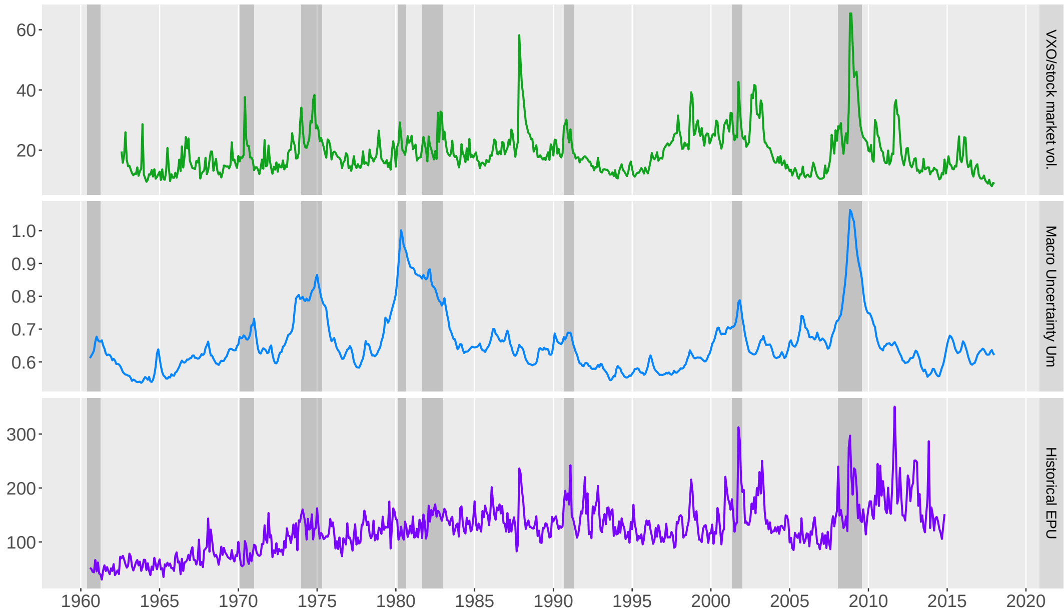

Figure 1.: Comparison of uncertainty measures. *Note:* Shaded areas denote NBER recession dates. Data frequencies are monthly. The macro uncertainty series starts in July 1960, the (Historical) EPU in January 1960 and the VXO in July 1962. (Historical) EPU is only available until 10:2014. See Appendix B.1 for details.

index introduced by Jurado et al. (2015)[52] and the economic policy uncertainty index (EPU) following Baker et al. (2015).

A first preliminary visual comparison highlights that similar to Bachmann et al. (2013)'s assessment in their own comparison, almost all recession periods correspond to elevated uncertainty across all measures. Apart from this commonality, the VXO and EPU are affected by much noisier fluctuations over time than e.g., the macro uncertainty index. As summarized in Baker et al. (2015), the VXO and EPU often move together prior to the financial crisis with distinct variations due to pronounced reactions of the VXO series to events with a connection to financial markets (e.g., the Asian financial crisis, the WorldCOM fraud, the Lehman Brothers failure, etc.). On the other hand, the EPU shows stronger spikes related to e.g., the Gulf Wars, presidential elections, and fiscal uncertainties due to tax or spending debates which causes the EPU to be elevated during most of the recovery period showing more variation including several spikes. This suggests that the EPU, by construction, indeed reacts much stronger to the political turmoil in the past decade than financial markets themselves.[53]

[52]Whose construction will be described in Section 4.

[53]Specifically being aware of these differences between the VXO and their EPU measure, Baker

Table 1.: Correlation matrix of the uncertainty measures from Figure 1.
Note: Correlations are calculated for the period where all three uncertainty measures overlap, i.e. 07:1962 - 10:2014

	VXO	Macro	EPU
VXO	1	0.490	0.530
Macro	0.490	1	0.300
EPU	0.530	0.300	1

The macro uncertainty index overall stands out with an overall smoother trajectory and marked spikes that correspond to recessions but also, as described by Jurado et al. (2015) themselves, significant independent variation as compared to the other commonly used uncertainty measures suggesting that "[...] quantitatively important uncertainty episodes occur far more *in*frequently[54] than what is indicated from common uncertainty proxies, but that when they *do* occur, they display larger and more persistent correlations with real activity" (Jurado et al., 2015, p. 1181). This observation can be confirmed by looking at Figures 1 and 10[55] where increases in the macro uncertainty estimates are mostly associated with protracted recessions whereas more modest recessions are accompanied with smaller spikes. A period where the macro uncertainty index stands out is the recessionary period from 1980-1982: throughout this episode the macro uncertainty index is highly elevated while other measures are low in comparison as pointed out by Jurado et al. (2015). The VXO on the other hand also shows spikes outside of recession periods that correspond to events exclusively related to stock-markets (most notably the large spike commonly referred to as 'Black Monday' (October, 19th 1987) when stock markets experienced their largest single-day percentage decline ever recorded). The macro uncertainty index, however, barely moves around that time (Jurado et al., 2015). Interestingly, the EPU shows the largest spike surrounding the years 2001-2001 (the dot-com bubble). Table 1 shows that while correlations are as expected positive, they are far from 1, indicating the considerable own variation and the observation that they likely stand for different types of uncertainty (Bachmann et al., 2013).

et al. (2015) construct a variation of their original EPU to be more strongly geared towards uncertainty on equity markets. Naturally, they report this financed-based EPU to correlate more strongly with the VXO/volatility series. For Baker et al. (2015) this exercise serves as a proof-of-concept in so far that a reasonable proxy for a specific *type* of economic uncertainty can indeed be constructed using text-search techniques and that the variations between the VXO/volatility series and the original EPU are important and justified by construction.

[54] Italics added.

[55] And also Table 2 which we will discuss below.

Despite an overall co-movement between the series, the identified episodes of substantial variation in the processes' dynamics suggest that "[t]hese [time series] are clearly not measures of the same [latent] stochastic process [...]" (Orlik and Veldkamp, 2014, p. 31).

To shed some light on the above statement, we comparatively discuss a few univariate and multivariate properties of the various uncertainty measures in turn. The selection of summary statistics is a mix of the presentation of summary statistics in Jurado et al. (2015) and Bontempi et al. (2016).

Table 2 reports several summary statistics including key figures regarding the *distribution* of the uncertainty measures, their *cyclicality* and its nexus to the business cycle, *persistence* and whether we have any evidence for non-*stationarity*.[57] The results point at a few stylized facts:

First, none of the measures passes the Shapiro-Wilk test, meaning that their respective distributions are significantly different from the normal distribution. This is in line with the figures on skewness and kurtosis: the distribution of changes in the measures are positively skewed and show excess kurtosis (fat tails), most pronounced for VXO and EPU indicating that there are more extreme values in these series as compared to the macro uncertainty series. Together, these observations point at tails on the right side of the distribution being longer than the ones on the left (meaning that the majority of the density's mass and the median lies to the left of the means as formulated by Bontempi et al., 2016). Similarly to the dynamics of business cycles, the above findings suggest that increases in the uncertainty measures tend to be larger than decreases. In other words, uncertainty increases faster than it decreases (Moore, 2016). Further, across all uncertainty measures, the VXO and EPU suggest a higher variability (according to the variation coefficients) which is also visible from the visual inspection of Figures 1 and 10.

Second, uncertainty appears to be countercyclical. The measures largely show means and variances that are higher during recessions versus expansions (as can be seen from Downturn/Upturn μ/σ ratios). This is further supported by negative contemporaneous correlations of the uncertainty measures with key economic indicators like industrial production (Jurado et al., 2015) and employment in manufacturing. The

[56]KPSS-test with H_0 = (trend/level) stationary.

[57]Note that we have included the financial uncertainty index of Jurado et al. (2015) in the table which will be introduced and discussed in Section 4.

Table 2.: Summary Statistics on the Dynamics of Uncertainty Proxies.
Note: As explained by Jurado et al. (2015) in the caption of their Table 1, IP-corr(k) and EMP-corr(k) are the absolute cross-correlation coefficients between a measure of uncertainty and the 12 month moving average of industrial production or employment in manufacturing growth in period $t + k$, i.e., IP-corr(k) = $|corr(u_t, \Delta\% IP_{t+k}|$ and EMP-corr(k) = $|corr(u_t, \Delta\% EMP_{t+k}|$. A positive k indicates that uncertainty is correlated with future IP/EMP. Half-lifes are based on estimates from a univariate AR(1) model for each series and calculated as $HL = ln(2)/ln(AR(1).coef)$. The sample for each uncertainty measure is the longest available, starting in July 1960 in the case of VXO, EPU, and Macro uncertainty index U_f and July 1962 for the VXO. The summary statistics obtained for U_f will be discussed in Section 4.

	VXO	EPU	Um	Uf
Distribution				
Coeff. of Variation	0.38	0.37	0.14	0.19
Skewness (Levels)	2.02	0.77	1.67	0.69
Excess Kurtosis (Levels)	4.17	−1.21	0.23	−2.57
Skewness (Change)	2.07	0.48	0.75	0.13
Excess Kurtosis (Change)	12.9	3.33	0.35	2.12
Shapiro-Wilk (p-value)	0.00	0.00	0.00	0.00
Cyclicality				
Downturn/Upturn μ ratios	1.5	1.14	1.25	1.25
Downturn/Upturn σ ratios	1.59	1.33	1.72	1.15
IP-corr(0)	−0.42	−0.33	−0.65	−0.51
IP-corr(-12)	−0.17	−0.22	−0.37	−0.23
IP-corr(12)	−0.06	−0.11	−0.20	−0.15
EMP-corr(0)	−0.51	−0.34	−0.64	−0.53
EMP-corr(-12)	−0.23	−0.25	−0.23	−0.17
EMP-corr(12)	−0.24	−0.17	−0.34	−0.31
Persistence				
AR(1)	0.84	0.81	0.99	0.99
Half Life	4.05	3.26	57.83	129.92
Stationarity Tests				
Level Stationary (p-value)[56]	0.1	0.01	0.09	0.03
Trend Stationary	0.1	0.01	0.01	0.01

figures are strongest for the macro uncertainty series while VXO and EPU are more weakly associated with the business cycle. In line with the observation in Bloom

(2014), uncertainty seems to be higher during recessions.[58]

Third, as persistence seems to be a relevant ingredient of the impact of uncertainty on business cycles (as shown by, among others, Schaal (2017) or Jurado et al. (2015)), the estimated half-life shocks (derived from the AR(1)-coefficients) of the macro uncertainty index with 58 months shows a much higher persistency than the other uncertainty proxies with approx. 4 months which is "[...] a finding relevant for theories where uncertainty is a driving force of economic downturns, including those with more prolonged periods of below-trend economic growth" (Jurado et al., 2015, p. 1193).

2.5. Exogenous Impulse vs. Endogenous Response

Already Bloom (2014) mentions that while uncertainty seems to influence business cycles, it also appears to be increasing endogenously during recessions. As indicated at the end of Section 2.3, this issue plays a crucial role in attempts to disentangle the real effects of uncertainty, meaning that uncertainty itself may be a function of the business cycle. In this respect, popularized VARs employed in the literature might not be capable of adequately separating cause and effect.

Ludvigson et al. (2019) identify two issues in the empirical modeling of uncertainty and point at the following shortcomings: First, in a VAR context (results of which in the literature we have reported in Sections 2.2 and 2.3; see also Appendix B.2 for details), the majority of the deployed models relies on recursive schemes as an identification strategy which impacts questions of causality and whose justification in terms of ordering due to contemporaneous movement of the variables seems sometimes arbitrary and not well grounded.[59,60] According to Ludvigson et al. (2019), a possible co-movement between uncertainty and real activity could be due to both, uncertainty exogenously affecting real activity and endogenously reacting to business cycle movements (i.e. first moment shocks). But as we have briefly outlined

[58] The unconditional negative correlations are, of course, uninformative about causation between real activity and uncertainty as pointed out by Jurado et al. (2015).

[59] Ludvigson et al. (2019) also discuss other commonly used identification schemes: sign restrictions, long-run restrictions, IV estimations, etc. Because of partly ambiguous theoretical signs of the relationships between uncertainty and real activity, the authors conclude that sign restrictions are inappropriate. Similarly they reject IV analysis, stating that it is very difficult to find truly exogenous instruments.

[60] E.g. while leaving the possibility that uncertainty fluctuations may be important autonomous economic shocks, Bachmann et al. (2013, p. 28) interpret their findings as reflecting that "[b]ad times breed uncertainty", meaning that uncertainty is a result of negative first moment events in an economy.

in Section 2.3, in the publications discussed that primarily rely on recursive schemes, the ordering between uncertainty and a variable capturing business cycle movements restricts the timing of their relationship (Ludvigson et al., 2019).

Because of the wide spectrum of uncertainty theories and non-existence of an all-encompassing structural model which do not provide identifying restrictions for empirical work, for Ludvigson et al. (2019, p. 5) the "[...] question of cause and effect is fundamentally an empirical" one. Second, picking up the critique mentioned already in Section 2.2, uncertainty stemming from financial markets might play a different role in business cycles than real economic uncertainty, referring to Ng and Wright (2013) who find that all the post-1982 recessions have their origins in financial market disturbances and that these recessions come along with distinctly different features as recessions where financial uncertainty plays a subordinate role.

To account for these identified issues, Ludvigson et al. (2019) suggest a novel identification strategy within an SVAR framework which differentiates between macro and financial uncertainty and which we will present next in Section 3.

3. Econometric Model

Ludvigson et al. (2018, 2019) suggest a novel identification scheme in an SVAR-context in an attempt to empirically solve the ruled out instantaneous simultaneous feedback of variables inherent to widely used VAR-analyses in the uncertainty-literature. Instead of a point identification the authors formulate economically reasoned restrictions on the shocks generated by the SVAR-system to adequately shrink the set of possible solutions. This chapter hence first sets the stage by discussing the theoretical background regarding (S)VARs and the so-called 'identification problem' in Section 3.1 followed by a presentation of the econometric framework of Ludvigson et al. (2019) in Section 3.2 which leverages the global identification approach of SVARs by means of the characteristics of orthogonal matrices following Rubio-Ramírez et al. (2010).[61]

While the authors first considered their shock-restricted structural VAR-approach in Ludvigson et al. (2019) in the context of an application to the analysis of uncertainty and business cycles (with its first draft from March 2015; intermediate draft is Ludvigson et al. (2018); latest draft at the time of writing: August 2019; forthcoming in the American Economic Journal: Macroeconomics), the approach itself is generalized in Ludvigson et al. (2020*b*) (first draft: December 2016; intermediate draft is Ludvigson et al. (2017); latest draft at the time of writing: January 2020).

3.1. Preliminaries: Theoretical Background on (S)VARs

With the seminal work of Sims (1980) VARs became widely adopted in practice as an alternative to large scale macroeconometric models despite initial controversies with regard to their usage and interpretation (Canova, 1995*b*,*a*). Because VARs can be "[...] consistent with many causal structures" (Ludvigson et al., 2020*b*, p. 1),

[61] An approach also used for so-called 'sign-restrictions' in the SVAR-literature (Kilian and Lütkepohl, 2017).

SVARs offer a richer framework by allowing the model to be fully endogenous but need additional assumptions for point identification of the parameters.

3.1.1. (S)VARs and the Derivation of Impulse Responses[62]

As outlined in Zivot (2000)[63], we consider a stable (which implies stationary) bivariate[64] dynamic stochastic simultaneous equations model in a *structural* VAR (SVAR) representation for the two time series y_{1t} and y_{2t}[65] whereby the two time series depend on one lag of each variable and also influence each other contemporaneously (i.e., the two variables are endogenous)[66] so that

$$\begin{aligned} y_{1t} &= a_{01}y_{2t} + a_{11}y_{1t-1} + a_{12}y_{2t-1} + \epsilon_{1t} \\ y_{2t} &= a_{02}y_{1t} + a_{21}y_{1t-1} + a_{22}y_{2t-1} + \epsilon_{2t} \end{aligned} \tag{3.1}$$

with

$$\boldsymbol{\epsilon}_t \sim i.i.d \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix} \right), \tag{3.2}$$

meaning that the error terms $\boldsymbol{\epsilon}_t$ are exogenous and independent and identically distributed for $t = 1, \ldots, T$, i.e., white noise. A slight rearrangement of Equation 3.1 gives

$$\begin{aligned} y_{1t} - a_{01}y_{2t} + &= a_{11}y_{1t-1} + a_{12}y_{2t-1} + \epsilon_{1t} \\ y_{2t} - a_{02}y_{1t} + &= a_{21}y_{1t-1} + a_{22}y_{2t-1} + \epsilon_{2t} \end{aligned} \tag{3.3}$$

[62] In their entirety Sections 3.1.1 and 3.1.3 on the theoretical background are the consolidated result of various sources (books and (lecture/seminar/presentation) materials) including Kilian and Lütkepohl (2017), Lütkepohl (2005), Stock and Watson (2001), Hamilton (1994), Fernández-Villaverde and Rubio-Ramírez (2010), Kunst (2007), Whelan (2016), Zivot (2000), Cesa-Bianchi (n.d.), Foroni (2015), Rossi (2011) and a handout called 'Structural Vector Autoregressive Analysis' prepared by Martin Geiger and Max Breitenlechner at the University of Innsbruck.

[63] Note that the exposition of the results at the beginning of this chapter also strongly benefitted from a handout called 'Structural Vector Autoregressive Analysis' prepared by Martin Geiger and Max Breitenlechner at the University of Innsbruck.

[64] Note that we start with a bivariate, i.e., two-equations-VAR for ease of exposition. As soon as we switch to matrix notation, the case of $n = 2$ versus $n > 2$ (for a multivariate VAR-system) is negligible.

[65] In all our following notation we abstract from deterministic regressors (i.e., trend or constant) and exogenous regressors, meaning that we only consider the stochastic part of a DGP.

[66] Hence, the example we start with is a bivariate SVAR(1) where p in SVAR(p) stands for the number of autoregressive lags.

or

$$\begin{bmatrix} 1 & -a_{01} \\ -a_{02} & 1 \end{bmatrix} \begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} y_{1t-1} \\ y_{2t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{bmatrix} \tag{3.4}$$

which, in matrix notation, compactly reduces to

$$\mathbf{A}_0\mathbf{y}_t = \mathbf{A}_1\mathbf{y}_{t-1} + \boldsymbol{\epsilon}_t \tag{3.5}$$

with $\mathbf{A}_0$ being called the *impact matrix* (also called the *structural impact multiplier matrix*; Kilian and Lütkepohl, 2017) containing the contemporaneous effects of an increase of each endogenous variable on the other variable (Kilian and Lütkepohl, 2017), respectively and $\boldsymbol{\epsilon}_t$ is a white noise process with

$$\begin{aligned} \mathbb{E}(\boldsymbol{\epsilon}_t) &= \mathbf{0} \\ \mathbb{E}(\boldsymbol{\epsilon}_t\boldsymbol{\epsilon}_\tau^{'}) &= \begin{cases} \boldsymbol{\Sigma}_\epsilon, & \text{for } t = \tau \\ \mathbf{0}, & \text{otherwise.} \end{cases} \end{aligned} \tag{3.6}$$

Hence, $\boldsymbol{\Sigma}_\epsilon$ is a diagonal matrix, i.e., structural shocks are assumed to be (mutually) uncorrelated (Kilian and Lütkepohl, 2017). As formulated by Kilian and Lütkepohl (2017, p. 109/p. 217), the elements $\boldsymbol{\epsilon}_t$ being mutually uncorrelated is not sufficient for a model to be considered structural, but "[t]he model is structural in that the elements of $\boldsymbol{\epsilon}_t$ are mutually uncorrelated *and*[67] have clear interpretations in terms of an underlying economic model." A VAR in such a structural representation can be translated into its *reduced form* representation (Kilian and Lütkepohl, 2017), i.e., a standard VAR model (in this case a VAR(1) because we are considering only one lag at the moment), by premultiplying 3.5 with the inverse of the impact matrix $\mathbf{A}_0$ (if it exists, i.e. $\mathbf{A}_0$ is non-singular) which gives

$$\begin{aligned} \mathbf{A}_0^{-1}\mathbf{A}_0\mathbf{y}_t = \mathbf{A}_0^{-1}\mathbf{A}_1\mathbf{y}_{t-1} + \mathbf{A}_0^{-1}\boldsymbol{\epsilon}_t \iff \\ \iff \mathbf{y}_t = \mathbf{B}_1\mathbf{y}_{t-1} + \mathbf{u}_t \end{aligned} \tag{3.7}$$

and which eliminates the previous complication due to contemporaneous relationships with $\mathbf{B}_1 = \mathbf{A}_0^{-1}\mathbf{A}_1 \iff \mathbf{A}_1 = \mathbf{A}_0\mathbf{B}_1$ and $\mathbf{u}_t = \mathbf{A}_0^{-1}\boldsymbol{\epsilon}_t \iff \boldsymbol{\epsilon}_t = \mathbf{A}_0\mathbf{u}_t$ (Kilian and Lütkepohl, 2017).[68]

[67] Italics added.

[68] Note that here we are still deriving all results for a bivariate vector autoregression of order 1, i.e., a VAR(1). For a multivariate system with $n > 2$, accordingly, the relationship between each

With the reduced form errors $\mathbf{u}_t$ being linear combinations of the structural errors $\boldsymbol{\epsilon}_t$ (Kilian and Lütkepohl, 2017; Zivot, 2000), the resulting reduced-form variance-covariance matrix looks as follows:[69]

$$\begin{aligned}\boldsymbol{\Sigma}_u \equiv \mathbb{E}[\mathbf{u}_t\mathbf{u}_t^{'}] &= \mathbb{E}[(\mathbf{A}_0^{-1}\boldsymbol{\epsilon}_t)(\mathbf{A}_0^{-1}\boldsymbol{\epsilon}_t)'] = \\ &= \mathbb{E}[\mathbf{A}_0^{-1}\boldsymbol{\epsilon}_t\boldsymbol{\epsilon}_t^{'}\mathbf{A}_0^{-1'}] = \\ &= \mathbf{A}_0^{-1}\mathbb{E}[\boldsymbol{\epsilon}_t\boldsymbol{\epsilon}_t^{'}]\mathbf{A}_0^{-1'} = \\ &= \mathbf{A}_0^{-1}\boldsymbol{\Sigma}_\epsilon\mathbf{A}_0^{-1'}\end{aligned} \tag{3.8}$$

As outlined in Kilian and Lütkepohl (2017, p. 109), "without loss of generality", the structural errors can be normalized, i.e., $\mathbb{E}(\boldsymbol{\epsilon}_t\boldsymbol{\epsilon}_\tau^{'}) \equiv \boldsymbol{\Sigma}_\epsilon = \mathbf{I}_k$, so that the reduced-form variance-covariance matrix reduces to $\boldsymbol{\Sigma}_u \equiv \mathbf{A}_0^{-1}\mathbf{A}_0^{-1'}$. The variance-covariance matrix $\boldsymbol{\Sigma}_u$ of the reduced-form VAR is assumed to be a symmetric positive definite matrix (Kilian and Lütkepohl, 2017) which, however, may not be a diagonal matrix due to instantaneous correlations between the error-terms (Zivot, 2000). As a consequence, isolated shocks in the components of $\mathbf{u}_t$ may not be likely. This reduced form VAR (1) is covariance stationary if the eigenvalues of $\mathbf{A}_1$ have modulus less than 1 (Kilian and Lütkepohl, 2017; Zivot, 2000).

Abstracting from possible identification schemes for $\mathbf{A}_0^{-1}$ for the moment (to which we will come back in Section 3.1.3), under the assumption of covariance stationarity (Kilian and Lütkepohl, 2017) of the vector process $\mathbf{y}_t$, $\mathbf{y}_t$ (i.e., the reduced form VAR) can be transformed into its vector moving average (VMA(∞); also called Wold MA (Lütkepohl, 2005; Kilian and Lütkepohl, 2017)) representation according to the Wold decomposition theorem[70] (i.e., there exists a unique mapping) whereby all past values of $\mathbf{y}_t$ are successively substituted out which translates Equation 3.7 into a linear combination of all $\mathbf{u}_t$'s (reduced-form errors) over time (whereby the corresponding MA-weights do not depend on time t but only on j, i.e., how long ago the shock $\mathbf{u}$ occurred):

$$\begin{aligned}\mathbf{y}_t &= \boldsymbol{\Psi}_0\mathbf{u}_t + \boldsymbol{\Psi}_1\mathbf{u}_{t-1} + \boldsymbol{\Psi}_2\mathbf{u}_{t-2} + \boldsymbol{\Psi}_3\mathbf{u}_{t-3} + \cdots = \\ &= \sum_{k=0}^{\infty}\boldsymbol{\Psi}_k\mathbf{u}_{t-k}\end{aligned} \tag{3.9}$$

coefficient matrix in reduced form $\mathbf{B}_j$ and structural form $\mathbf{A}_j$ is governed by $\mathbf{B}_j = \mathbf{A}_0^{-1}\mathbf{A}_j$.

[69]Note that in the bivariate case, $\boldsymbol{\Sigma}_u$ is a diagonal matrix only if $-a_{01} = -a_{02} = 0$, i.e., only if there is no contemporaneous correlation.

[70]See Wold (1939). In other words, the Wold Theorem states that any covariance stationary process has an infinite order, moving-average representation (Lütkepohl, 2005).

with $\mathbf{\Psi}_0 = \mathbf{I}_2$ and $\mathbf{\Psi}_k$ can be computed recursively via $\mathbf{\Psi}_k = \sum_{j=1}^{k} \mathbf{\Psi}_{k-j}\mathbf{B}_j$ for $k = 1, 2, \ldots$ (Kilian and Lütkepohl, 2017; Lütkepohl, 2005).[71]

The corresponding *structural* moving average (SMA(∞); structural Wold MA) representation of $\mathbf{y}_t$ is based on an infinite moving average of the *structural* innovations $\boldsymbol{\epsilon}_t$ and is obtained by substituting the mapping between structural and reduced form shocks $\mathbf{u}_t = \mathbf{A}_0^{-1}\boldsymbol{\epsilon}_t$ (Zivot, 2000) for all t into Equation 3.9 so that

$$\begin{aligned}\mathbf{y}_t &= \mathbf{\Psi}_0\mathbf{A}_0^{-1}\boldsymbol{\epsilon}_t + \mathbf{\Psi}_1\mathbf{A}_0^{-1}\boldsymbol{\epsilon}_{t-1} + \mathbf{\Psi}_2\mathbf{A}_0^{-2}\boldsymbol{\epsilon}_{t-2} + \mathbf{\Psi}_3\mathbf{A}_0^{-1}\boldsymbol{\epsilon}_{t-3} + \cdots = \\ &= \sum_{k=0}^{\infty} \mathbf{\Psi}_k\mathbf{A}_0^{-1}\boldsymbol{\epsilon}_{t-k} \\ &= \sum_{k=0}^{\infty} \mathbf{\Theta}_k\boldsymbol{\epsilon}_{t-k}\end{aligned} \tag{3.10}$$

which means that $\mathbf{\Theta}_k = \mathbf{\Psi}_k\mathbf{A}_0^{-1}$ for $k = 0, 1, \ldots$. In particular, it holds that $\mathbf{\Theta}_0 = \mathbf{A}_0^{-1} \neq \mathbf{I}_2$ (in contrast to the VAR-coefficient $\mathbf{\Psi}_0$ shown above).

Looking at the SMA(∞) representation of our bivariate system

$$\begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} = \begin{bmatrix} \theta_{11}^{(0)} & \theta_{12}^{(0)} \\ \theta_{21}^{(0)} & \theta_{22}^{(0)} \end{bmatrix} \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{bmatrix} + \begin{bmatrix} \theta_{11}^{(1)} & \theta_{12}^{(1)} \\ \theta_{21}^{(1)} & \theta_{22}^{(1)} \end{bmatrix} \begin{bmatrix} \epsilon_{1t-1} \\ \epsilon_{2t-1} \end{bmatrix} + \cdots \tag{3.11}$$

shows that the elements of the $\mathbf{\Theta}_k$ matrices, $\theta_{ij}^{(k)}$, are the dynamic multipliers/impulse responses of y_{1t} and y_{2t} to changes in ϵ_{1t} and ϵ_{2t} (Zivot, 2000).

Generalizing the above SMA(∞) representation to time $t+s$ (Zivot, 2000)

$$\begin{bmatrix} y_{1t+s} \\ y_{2t+s} \end{bmatrix} = \begin{bmatrix} \theta_{11}^{(0)} & \theta_{12}^{(0)} \\ \theta_{21}^{(0)} & \theta_{22}^{(0)} \end{bmatrix} \begin{bmatrix} \epsilon_{1t+s} \\ \epsilon_{2t+s} \end{bmatrix} + \cdots + \begin{bmatrix} \theta_{11}^{(s)} & \theta_{12}^{(s)} \\ \theta_{21}^{(s)} & \theta_{22}^{(s)} \end{bmatrix} \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{bmatrix} + \cdots \tag{3.12}$$

or more compactly to

$$\mathbf{y}_{t+s} = \mathbf{\Theta}_0\boldsymbol{\epsilon}_{t+s} + \cdots + \mathbf{\Theta}_s\boldsymbol{\epsilon}_t + \cdots \tag{3.13}$$

[71] The MA representation of a stable (S)VAR(p) process must not be of infinite order since coefficients may turn zero as of a certain lag (i.e. as $i \to \infty$) due to the stability of the time series process (Lütkepohl, 2005).

shows that the $\mathbf{\Theta}$ - matrices consisting of the MA-coefficients hold the structural dynamic multipliers (the impulse responses). In the bivariate example, these impulse responses are

$$\begin{aligned}\frac{\partial y_{1t+s}}{\partial \epsilon_{1t}} &= \theta_{11}^{(s)}, \frac{\partial y_{1t+s}}{\partial \epsilon_{2t}} = \theta_{12}^{(s)} \\ \frac{\partial y_{2t+s}}{\partial \epsilon_{1t}} &= \theta_{21}^{(s)}, \frac{\partial y_{2t+s}}{\partial \epsilon_{2t}} = \theta_{22}^{(s)}\end{aligned} \tag{3.14}$$

or more compactly

$$\frac{\partial \mathbf{y}_{t+s}}{\partial \boldsymbol{\epsilon}_t} = \mathbf{\Theta}_s \tag{3.15}$$

meaning that in matrix $\mathbf{\Theta}_s$ the entry (i, j) shows the impact of a one-unit increase in the jth variable's innovation at date t (ϵ_{jt}) on the value of the ith variable at time $t + s$ $(y_{i,t+s})$ (Zivot, 2000).

Subsequently, a plot of the elements (i, j) of the respective structural MA matrices $\{\mathbf{\Theta}_s\}_{i,j}$ as a function of s, i.e.,

$$\frac{\partial y_{t+s}}{\partial \epsilon_t} \tag{3.16}$$

is the (structural) *impulse-response function* that describes the response of $y_{i,t+s}$ to a one-time one-unit impulse in y_{jt} while all other variables at time t or earlier are held constant (Rossi, 2011). As mentioned by Kilian and Lütkepohl (2017, p. 37), "[...] users of VAR models are [in practice] typically not interested in the slope of the model's parameters themselves, but in smooth nonlinear functions of the VAR model parameters such as impulse responses." In addition, one of the implications of the linearity of VAR-models is that the size of a structural shock is insignificant for the derivation of impulse response functions and simply resembles a rescaling (Kilian and Lütkepohl, 2017).

If we assume contemporaneous relationships among the endogenous variables, the regressors in the SVAR representation are correlated with the error term and hence would result in biased OLS estimates. Therefore, in econometric applications the VAR in its reduced-form representation is being estimated from the data (Kilian and Lütkepohl (2017, p. 19) explain that "[...] [s]tructural VAR analysis is based on the premise that the DGP is well approximated by a reduced-form VAR model"). The reduced-form VAR can be estimated using standard methods (LS and/or ML; see

Kilian and Lütkepohl, 2017). While the reduced-form VAR is the one being estimated, it is solely an econometric model without any theoretical component (Whelan, 2016) and does not say anything about the structure of the economy and hence the reduced-form error terms $\mathbf{u}_t$ cannot be interpreted as structural shocks.[72] Rather, the goal is to get back to the structural representation with a diagonal covariance matrix and economic meaning (Foroni, 2015; Cesa-Bianchi, n.d.).[73] Without imposing any further restrictions, however, the parameters in the SVAR, i.e., models with contemporaneous relationships,[74] are not identified, i.e., based on the reduced form parameters $\mathbf{B}_1$ and $\mathbf{\Sigma}_u$, it is not possible to arrive at a unique solution for the structural parameters $\mathbf{A}_0$, $\mathbf{A}_1$ and $\mathbf{\Sigma}_\epsilon$ (Zivot, 2000). In particular, the knowledge of the impact matrix $\mathbf{A}_0^{-1}$ would allow to recover $\mathbf{A}_1$ via $\mathbf{A}_1 = \mathbf{A}_0\mathbf{B}_1$ and the structural errors via $\boldsymbol{\epsilon}_t = \mathbf{A}_0\mathbf{u}_t$[75] (Kilian and Lütkepohl, 2017) and subsequently the variance covariance matrix of the structural errors $\mathbf{\Sigma}_\epsilon$.[76] After introducing one more element in the toolbox of VAR-analyses in Section 3.1.2 below, Section 3.1.3 will come back to the identification problem.

3.1.2. Forecast Error Variance Decomposition[77]

Besides the impulse response analysis, 'variance decomposition' (more precisely: 'forecast error variance decomposition'; FEVD) is another frequently used tool in the analysis-toolkit of (S)VARs. As summarized by Zivot (2000), the idea behind this decomposition is to derive the proportion of variability of the forecast errors for the h-step-ahead prediction of the (S)VAR variables (based on information at time t) that can be attributed to the respective structural shocks. In other words, the analysis

[72] Note that in the estimated covariance matrix for $\hat{\mathbf{\Sigma}}_u$ the respective errors are usually correlated (Whelan, 2016) so that shocks to one variable are usually accompanied with a response to other variables (Rossi, 2011).

[73] As mentioned by Whelan (2016), in practice, the error series in reduced-form VARs are usually correlated while in a structural setting correlated reduced-form shocks are broken down into uncorrelated structural shocks that allow a clearer assessment of effects within a model. Also, Kilian and Lütkepohl (2017, p. 173) declare that "[f]or an econometric model to be structural it is necessary for the stochastic shocks (or errors) in each equation to be mutually uncorrelated." It is this feature that makes it possible to conduct analyses where one shock is imposed while all else stays equal and the corresponding responses are regarded as "causal effects" of the respective structural shock (Kilian and Lütkepohl, 2017).

[74] In a model without contemporaneous relationship no identification issues would emerge.

[75] As mentioned by Kilian and Lütkepohl (2017, p. 110), given this relationship the "typically mutually correlated reduced-form" errors can be written as "weighted averages of the mutually uncorrelated structural" errors, with the elements of $\mathbf{A}_0^{-1}$ being the weights.

[76] In the bivariate case we consider here, at least 1 restriction on the parameters of the SVAR is required to enable the identification of all structural parameters.

[77] This chapter draws from, among others, Lütkepohl (2005), Kilian and Lütkepohl (2017), Lütkepohl (2010), Zivot (2000) and Sims (2011).

allows to quantify the importance of each shock in a system in explaining the variation in each of the system's variables. The computation of the FEVD can be directly derived from the $\mathbf{\Theta}_k$ matrices, $\theta_{ij}^{(k)}$, which are the dynamic multipliers/impulse responses of the SVARs variables to changes in the respective shocks (and which were derived in Section 3.1.1).

In particular, the following holds (see Lütkepohl (2005) and Kilian and Lütkepohl (2017)):
For an (S)VAR-process as outlined in Section 3.1.1, the error of the optimal h-step-ahead forecast computed at time t for the time $t+h$ for the system is given by

$$\mathbf{y}_{t+h} - \mathbf{y}_t(h) = \sum_{k=0}^{h-1} \mathbf{\Theta}_k \boldsymbol{\epsilon}_{t+h-k} \tag{3.17}$$

With the elements of the $\mathbf{\Theta}_k$ matrices denoted as $\theta_{ij}^{(k)}$, the h-step-ahead forecast error for the n-th component (i.e., the n-th variable) of the system $\mathbf{y}_t$ is given by

$$\begin{aligned} y_{n,t+h} - y_{n,t}(h) &= \sum_{k=0}^{h-1} (\theta_{n1}^{(k)} \epsilon_{1,t+h-k} + \cdots + \theta_{nM}^{(k)} \epsilon_{M,t+h-k}) \\ &= \sum_{m=1}^{M} (\theta_{nm}^{(0)} \epsilon_{m,t+h} + \cdots + \theta_{nm}^{(h-1)} \epsilon_{m,t+1}), \end{aligned} \tag{3.18}$$

meaning that the forecast error of the n-th component might possibly consist of all innovations. With the innovations $\epsilon_{m,t}$ being uncorrelated and having unit variances, the MSE of $y_{n,t}(h)$ can be expressed as

$$\begin{aligned} \mathbb{E}(y_{n,t+h} - y_{n,t}(h))^2 &= \sum_{m=1}^{M} \left((\theta_{nm}^{(0)})^2 + \cdots + (\theta_{nm}^{(h-1)})^2 \right) \\ &= \sum_{k=0}^{h-1} (\mathbf{e}_n^{'} \mathbf{\Theta}_k \mathbf{e}_m)^2 \end{aligned} \tag{3.19}$$

where the last expression is said to be the contribution of innovations in variable m to the forecast error variance (or MSE) of the h-step-ahead forecast of variable n and the vector $\mathbf{e}_m$ is the kth column of the identity matrix.

Defining the h-step-ahead forecast MSE matrix as

$$\begin{aligned}\mathrm{MSE}[\mathbf{y}_t(h)] &= \mathbb{E}[(\mathbf{y}_{t+h} - \mathbf{y}_t(h))(\mathbf{y}_{t+h} - \mathbf{y}_t(h))'] \\ &= \sum_{k=0}^{h-1} \mathbf{\Theta}_k \mathbf{\Sigma}_\epsilon \mathbf{\Theta}_k^{'}, \quad \text{with } [\mathbf{\Sigma}_\epsilon = \mathbf{I}_k] \\ &= \sum_{k=0}^{h-1} \mathbf{\Theta}_k \mathbf{\Theta}_k^{'},\end{aligned} \tag{3.20}$$

the diagonal elements are

$$\mathrm{MSE}[y_{n,t}(h)] = \sum_{k=0}^{h-1} \sum_{m=1}^{M} \left(\theta_{nm}^{(0)}\right)^2 \tag{3.21}$$

The respective proportions $\rho_{nm,h}$ [78] of the h-step-ahead forecast error variance of variable n which is accounted for by the ϵ_{mt} innovations in the variable k is the ratio

$$\rho_{nm,h} = \frac{\sum_{k=0}^{h-1} (\mathbf{e}_n^{'} \mathbf{\Theta}_k \mathbf{e}_m)^2}{\mathrm{MSE}[y_{n,t}(h)]} \tag{3.22}$$

Further, in a stationary model, the limit of the FEVD (i.e., as $h \to \infty$) is the FVED of y_t because the forecast error covariance matrix (or MSPE) converges to the unconditional covariance matrix of y_t (Kilian and Lütkepohl, 2017).

3.1.3. SVARs and the Identification Problem

As derived in Section 3.1.1, the identification of $\mathbf{A}_0^{-1}$ is needed to seamlessly connect the estimated reduced-form VAR with the structural VAR that is of actual interest.

The usual strategy is to first estimate the reduced-form system as in Equation 3.7. In the basic setting of a VAR(1)-model this amounts to estimating $n^2 + \frac{n(n+1)}{2}$ parameters (n^2 for the coefficient matrix $\mathbf{B}_1$ and $\frac{n(n+1)}{2}$ for the variance covariance-matrix $\mathbf{\Sigma}_u = \mathbf{A}_0^{-1}\mathbf{A}_0^{-1'}$ of the reduced-form errors; see Kilian and Lütkepohl (2017), Whelan (2016), etc.).

In our bivariate case above, it was argued that at least one restriction on the parameters of Equation 3.5 is necessary (Zivot, 2000) so that the system of equations becomes solvable since the estimated variance-covariance matrix alone is not enough (Ludvigson et al., 2020*b*). As formulated by Kilian and Lütkepohl (2017, p. 212),

[78] Note that we have denoted this ratio ρ following Zivot (2000).

establishing additional restrictions allows to "[...] decompose the reduced-form errors [$\mathbf{u}_t$] into mutually uncorrelated structural shocks, [$\boldsymbol{\epsilon}_t$], with an economic interpretation." Identification schemes include, among others, zero short-run restrictions (known as Choleski identification), sign restrictions, zero long-run restrictions (also called Blanchard-Quah following Blanchard and Quah, 1989), etc. See, among others, Kilian and Lütkepohl (2017) for a detailed exposition of the various approaches in the literature to date.

For example, as shown in Zivot (2000), imposing a short-run restriction by assuming that the impact matrix $\mathbf{A}_0$ is lower triangular with

$$\mathbf{A}_0 = \begin{bmatrix} 1 & 0 \\ -a_{02} & 1 \end{bmatrix} \tag{3.23}$$

i.e., the restriction that $-a_{01} = 0$, is sufficient to just identify $-a_{02}$ (from the reduced form covariance matrix $\boldsymbol{\Sigma}_u$; shown below) and subsequently results in

$$\mathbf{A}_0^{-1} = \boldsymbol{\Theta}_0 = \begin{bmatrix} 1 & 0 \\ a_{02} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \theta_{21}^{(0)} & 1 \end{bmatrix} \tag{3.24}$$

for the SMA representation imposing the restriction that the value y_{2t} has no contemporaneous effects on y_{1t} while due to $-a_{02} \neq 0$ we would allow for the reverse (Zivot, 2000).[79] By means of this "orthogonalization" of the reduced-form errors (meaning, that they become mutually uncorrelated; Kilian and Lütkepohl, 2017), the structural errors can be derived from the reduced-form errors. The reasoning behind such an identification strategy usually stems from arguments that declare certain variables to be sticky, meaning that they do not respond immediately to certain shocks (Whelan, 2016). Further, we see that the restriction $a_{01} = 0$ in the SVAR representation is equivalent to assuming $\theta_{12}^{(0)} = 0$ in the SMA, meaning that ϵ_{1t} has no contemporaneous impact on y_{2t}.

With $\mathbf{A}_0$ being a lower triangular matrix of the above form, further the reduced form VAR errors $\mathbf{u}_t = \mathbf{A}_0^{-1}\boldsymbol{\epsilon}_t$ become (see Zivot, 2000)

$$\mathbf{u}_t = \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ a_{02} & 1 \end{bmatrix} \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{bmatrix} = \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} + a_{01}\epsilon_{1t} \end{bmatrix} \tag{3.25}$$

The SVAR representation derived under the above assumption hence establishes a

[79] Triangulization was also originally proposed by Sims (1980) as an identification strategy.

recursive causal ordering (also called a "Wold causal chain" (Stock and Watson, 2001; Rossi, 2011)) which can be computed using the Choleski factorization of the reduced form covariance matrix $\mathbf{\Sigma}_u$ (Zivot, 2000; the Choleski identification is hence also called recursive identification). In this identification scheme hence the *order* of the variables entering the VAR matters because the variable placed on top is assumed to be the most exogenous. Kilian and Lütkepohl (2017, p. 2020) emphasize that "[...] applying a Cholesky decomposition is appropriate only if the recursive structure [...] can be justified on economic grounds" (as discussed in Section 2.5 above) and, as mentioned by Lütkepohl (2005), largely drives the resulting impulse response functions.

As shown in Zivot (2000), writing down the Choleski factorization of the positive semi-definite matrix $\mathbf{\Sigma}_u$ gives

$$\begin{aligned}\mathbf{\Sigma}_u &= \mathbf{PP'} \quad \text{with} \\ \mathbf{P} &= \begin{bmatrix} p_{11} & 0 \\ p_{21} & p_{22} \end{bmatrix}\end{aligned} \tag{3.26}$$

with $\mathbf{P}$ being a lower triangular matrix with $p_{ii} \leq 0, i = 1, 2$

The Wold causal ordering (as one example of restrictions) implied in $\mathbf{A}_0^{-1}$ being a lower triangular matrix with $diag(\mathbf{A}_0^{-1}) = 1$ causes the system of equations $\mathbf{\Sigma}_\epsilon = \mathbf{A}_0^{-1}\mathbf{\Sigma}_u\mathbf{A}_0^{-1'}$ being solvable.

It is particularly this mechanical standard Wold-causal ordering of the elements of $\mathbf{y}_t$ on which the VAR literature often relies to establish the triangular factorization of the reduced-form variance-covariance matrix $\mathbf{\Sigma}_u$ which, for example, Jordà (2005) criticizes. At the same time, Jordà (2005, p. 4) finds that the "statistical-based structural identification of contemporaneous causal links" is a difficult venture. Kilian and Lütkepohl (2017, p. 10) emphasize that imposing recursive schemes "[...] results in economically meaningless measures of structural shocks, unless this ordering can be economically motivated." The framework of Ludvigson et al. (2018, 2019) makes a suggestion for this issue where the $\mathbf{P}$-matrix to orthogonalize the reduced-form disturbances is not mechanically constructed as the Cholesky decomposition of the error covariance matrix (zero short-run restrictions; recursive identification) but rather obtained from contemporaneous and theory-based restrictions placed on the $\mathbf{P}$-matrix.

3.2. Econometric Framework of (Ludvigson et al., 2018, 2019)[80]

As outlined in Section 2.5, in their empirical exercise Ludvigson et al. (2019, p. 2) ask, first, whether uncertainty is "primarily a source of business cycle fluctuations or a consequence of them" and, second, whether models of uncertainty should distinguish between financial and 'real' macroeconomic uncertainty. Their suggested model hence accounts for the distinction between both types of uncertainty and introduces a novel identification strategy that "[...] allows for simultaneous feedback between uncertainty and real activity [...]" which the authors achieve by means of two types of *shock-based* restrictions consisting of (as they call them) "event constraints" and "correlation constraints" within the class of SVAR models.

Ludvigson et al. (2019) point out that the usage of specific events and/or external variables in an attempt to identify shocks is not new. The rationale goes that constructing shock series from the historical reading of political and economic events (the narrative approach) allows the generation of exogenous and accurately measured shocks. Ramey (2016), however, questions both of these assumptions (exogeneity and accurate measurement). The usage of instruments under the assumption of a zero correlation with some shocks and a non-zero correlation with others promises remedy and allows point identification in SVAR-settings. The approach of Ludvigson et al. (2019), however, gets by without any such exogeneity assumptions and without point identification and rather leverages the properties of the model's shocks to adhere to certain (inequality) conditions to decide whether a solution is permissible or not. This approach leverages findings of Rubio-Ramírez et al. (2010) which we will discuss next.

3.2.1. Identification of SVARs: Rubio-Ramírez et al. (2010)

As formulated in Rubio-Ramírez et al. (2010), the identification of structural vector autoregressions (i.e., drawing inference from the reduced-form representation back to the underlying structural parameters in SVARs) is at the root of the 'identification

[80]Note that the theory on shock-restricted SVARs applied to the concrete case of uncertainty as presented in (Ludvigson et al., 2018, 2019) is generalized in (Ludvigson et al., 2017, 2020*b*). Further, as indicated in the introduction of Section 3, Ludvigson et al. (2018) is the prior version of Ludvigson et al. (2019). While the general approach which was further developed in Ludvigson et al. (2019) is the same as in Ludvigson et al. (2018), the particular restrictions used and the parameterization of the restrictions (their derivation and magnitude) differ across these two versions. For this thesis we have employed a combination/selection of restrictions from both of the two publications by replacing one of the restrictions in Ludvigson et al. (2019) with one from Ludvigson et al. (2018).

problem' which forces researches to come up with additional '*a priori* restrictions'[81], so-called "identifying restrictions"[82].

In this setting due to the symmetry of the variance covariance matrix $\mathbf{\Sigma}_u$ and as outlined in Section 3.1.3, only $\frac{n(n+1)}{2}$ different equations are specified and a necessary condition (as a standard criterion for identification) is that further $\frac{n(n-1)}{2}$ restrictions are needed to identify all n^2 elements of $\mathbf{A}_0^{-1}$ with n being the number of endogenous variables (called the necessary "order condition" by Rothenberg, 1971).[83]

Referring to Rothenberg (1971) and the fact that the "order condition" is only a necessary condition, Rubio-Ramírez et al. (2010) investigate under which conditions such models are globally identified and introduce a novel approach to global identification by exploiting the structure of orthogonal matrices. Central to their paper is the following:

The class of SVARs that Rubio-Ramírez et al. (2010) study[84] have the general form

$$\mathbf{A}_0 \mathbf{y}_t = \sum_{j=1}^{\infty} \mathbf{A}_j \mathbf{y}_{t-j} + \boldsymbol{\epsilon}_t \tag{3.27}$$

where they assume

$$\begin{aligned} \mathbb{E}(\boldsymbol{\epsilon}_t) &= \mathbf{0} \\ \mathbb{E}[\boldsymbol{\epsilon}_t \boldsymbol{\epsilon}_\tau^{'}] &= \mathbf{\Sigma}_\epsilon = \mathbf{I}_n. \end{aligned} \tag{3.28}$$

Compactly writing $\mathbf{A}_+^{'} = [\mathbf{A}_1^{'}, \mathbf{A}_2^{'}, \dots, \mathbf{A}_p^{'}]$ and $\mathbf{x}_t^{'} = [\mathbf{y}_{t-1}^{'}, \mathbf{y}_{t-2}^{'}, \dots, \mathbf{y}_{t-p}^{'}]$, they rewrite their model as

$$\mathbf{A}_0 \mathbf{y}_t = \mathbf{x}_t \mathbf{A}_+ + \boldsymbol{\epsilon}_t \tag{3.29}$$

[81] Author's italics.

[82] Author's quotes.

[83] The case of $n = 3$ illustrates the problem (see Foroni, 2015): $\mathbf{\Sigma}_u = \mathbf{A}_0^{-1} \mathbf{\Sigma}_\epsilon \mathbf{A}_0^{-1'}$ results in

$$\underbrace{\begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}}_{\text{6 values}} = \underbrace{\begin{bmatrix} a_{11}^0 & a_{12}^0 & a_{13}^0 \\ a_{21}^0 & a_{22}^0 & a_{23}^0 \\ a_{31}^0 & a_{32}^0 & a_{33}^0 \end{bmatrix}^{-1}}_{\text{9 unknowns}} \begin{bmatrix} a_{11}^0 & a_{12}^0 & a_{13}^0 \\ a_{21}^0 & a_{22}^0 & a_{23}^0 \\ a_{31}^0 & a_{32}^0 & a_{33}^0 \end{bmatrix}^{-1'}$$

where 6 equations are not sufficient to solve the system of linear equations consisting of 9 unknowns.

[84] Note that we adapt the notation of Rubio-Ramírez et al. (2010) in their original contribution to be consistent with our own notation so far.

with the reduced-form representation implied by the structural model being

$$\mathbf{y}_t = \mathbf{x}_t\mathbf{B} + \mathbf{u}_t \tag{3.30}$$

with $\mathbf{B} = \mathbf{A}_0^{-1}\mathbf{A}_+$ and $\mathbf{u}_t = \mathbf{A}_0^{-1}\boldsymbol{\epsilon}_t$ and

$$\mathbb{E}[\mathbf{u}_t\mathbf{u}_\tau^{'}] = \boldsymbol{\Sigma}_u = (\mathbf{A}_0^{-1}\mathbf{A}_0^{-1'}) \tag{3.31}$$

The parameters of the structural model being $(\mathbf{A}_0, \mathbf{A}_+)$ and of the reduced-form model $(\mathbf{B}, \boldsymbol{\Sigma}_u)$, Rubio-Ramírez et al. (2010) denote the set of all structural parameters by $\mathbb{P}^S$ and the set of all reduced-form parameters by $\mathbb{P}^R$ and define the function $g : \mathbb{P}^S \rightarrow \mathbb{P}^R : g(\mathbf{A}_0, \mathbf{A}_+) = (\mathbf{A}_0^{-1}\mathbf{A}_+, \mathbf{A}_0^{-1}\mathbf{A}_0^{-1'})$ for the relationship between structural and reduced-form parameters.

Equipped with this notation, Rubio-Ramírez et al. (2010) define the important concept of 'observational equivalence' as follows:
Following Rothenberg (1971), two parameter points, $(\mathbf{A}_0, \mathbf{A}_+)$ and $(\widetilde{\mathbf{A}_0}, \widetilde{\mathbf{A}_+})$, are considered *observationally equivalent* iff they both imply the same probability distribution of the data $\mathbf{y}_t$. For linear Gaussian models as studied by Rubio-Ramírez et al. (2010), this turns out to be equivalent to postulating that two parameter points are *observationally equivalent* iff they both have the same reduced-form representation $(\mathbf{B}, \boldsymbol{\Sigma}_u)$. Based on this, Rubio-Ramírez et al. (2010) revert back to their function g and conclude that it shows that two parameter points $(\mathbf{A}_0, \mathbf{A}_+)$ and $(\widetilde{\mathbf{A}_0}, \widetilde{\mathbf{A}_+})$ have the same reduced-form representation iff "there is an orthogonal matrix $\mathbf{Q}$ such that $\mathbf{A}_0 = \widetilde{\mathbf{A_0}}\mathbf{Q}$ and $\mathbf{A}_+ = \widetilde{\mathbf{A}_+}\mathbf{Q}$". In this context, the authors write down two definitions:

Definition 3.1. A parameter point $(\mathbf{A}_0, \mathbf{A}_+)$ is globally identified if and only if there is no other parameter point that is observationally equivalent.

Definition 3.2. A parameter point $(\mathbf{A}_0, \mathbf{A}_+)$ is locally identified if and only if there is an open neighborhood about $(\mathbf{A}_0, \mathbf{A}_+)$ containing no other observationally equivalent parameter point.

Observational equivalence is inherently linked to the identification problem in SVARs which, with respect to the definitions above and Rubio-Ramírez et al. (2010)'s formulation are "neither globally nor locally identified". This situation calls for adequate restrictions on the part of the structural parameters to make a model identifiable. And because observational equivalence is identical to finding an orthogonal matrix $\mathbf{Q}$ as described above, the set of all $(n \times n)$ orthogonal matrices $\mathbb{Q}_n$ plays a central role in the derivation of the econometric framework of Ludvigson et al. (2019) (see Section 3.2.2).

Besides the concept of 'observational equivalence' (and many other definitions and theorems), Rubio-Ramírez et al. (2010) introduce a computationally efficient theorem for the generation of random orthogonal matrices which the authors leverage in the context of the imposition of sign restrictions of an estimated SVAR's impulse response functions. While Ludvigson et al. (2019) do not impose any sign restrictions on the part of the impulse responses themselves (but introduce constraints on the shocks; see the details in Section 3.2.2), they leverage the theorem for the generation of random orthonormal matrices:

Theorem 3.3. Let $\widetilde{\mathbf{M}}$ be an $n \times n$ matrix with each element having an independent standard normal distribution. Let $\widetilde{\mathbf{M}} = \widetilde{\mathbf{Q}}\widetilde{\mathbf{R}}$ be the QR-decomposition of $\widetilde{\mathbf{M}}$ with the diagonal of $\widetilde{\mathbf{R}}$ normalized to be positive. Then $\widetilde{\mathbf{Q}}$ has the uniform (or Haar) distribution.[85]

Ludvigson et al. (2019) make use of this theorem[86] because typically the **QR**-decomposition of a Gaussian Matrix **M** does not guarantee to result in a uniformly distributed orthogonal matrix if the diagonal elements of **R** are not positive (see e.g. Edelman, 2005).

3.2.2. The Model

Ludvigson et al. (2019) assume the following reduced-form finite order VAR representation and corresponding infinite MA representation given by[87]

$$
\begin{aligned}
&\mathbf{y}_t = \sum_{j=1}^{p} \mathbf{B}_j \mathbf{y}_{t-j} + \mathbf{u}_t \\
&\mathbf{y}_t = \sum_{k=0}^{\infty} \mathbf{\Psi_k} \mathbf{u}_{t-k}, \\
&\mathbf{u}_t \sim (\mathbf{0}, \mathbf{\Sigma}_u), \mathbf{\Sigma}_u = \mathbb{E}[\mathbf{u}_t \mathbf{u}_\tau^{'}] = \mathbf{P}\mathbf{\Sigma}_\epsilon \mathbf{P'} = \mathbf{P}\mathbf{I}_n\mathbf{P'} = \mathbf{P}\mathbf{P'}
\end{aligned}
\tag{3.32}
$$

where **P** is the unique lower-triangular Cholesky factor with non-negative diagonal

[85] As outlined in Rubio-Ramírez et al. (2010), the proof results from Stewart (1980).

[86] This is not immediately obvious from the outline in Ludvigson et al. (2019). We became aware of this due to the Matlab-code of their baseline specification (see footnote 107).

[87] Note that we adapt the notation of Ludvigson et al. (2019) in their original contribution to be consistent with our own notation so far.

elements. The authors collect the reduced-form parameters into

$$\phi = \Big(\text{vec}(\mathbf{A}_1)', \text{vec}(\mathbf{A}_2)', \ldots, \text{vec}(\mathbf{A}_\mathbf{p})', \text{vech}(\mathbf{\Sigma}_\mathbf{u})'\Big) \tag{3.33}$$

by making use of the vec[88] and vech[89] operator. Ludvigson et al. (2019) formulate the innovations $\mathbf{u}_t$ to be related to the structural-form SVAR shock $\boldsymbol{\epsilon}_t$ via an invertible $(n \times n)$ - matrix $\mathbf{H}$

$$\mathbf{u}_t = \mathbf{H}\boldsymbol{\Omega}\boldsymbol{\epsilon}_t \equiv \mathbf{A}_0^{-1}\boldsymbol{\epsilon}_t \tag{3.34}$$

with

$$\boldsymbol{\epsilon}_t \sim (\mathbf{0}, \mathbf{\Sigma}_\epsilon), \mathbf{\Sigma}_\epsilon = \mathbb{E}[\boldsymbol{\epsilon}_t\boldsymbol{\epsilon}_\tau^{'}] = \mathbf{I}_n \tag{3.35}$$

meaning that the structural shocks are mean zero with unit variance, serially and mutually uncorrelated (i.e. the variance of the structural innovations are normalized to one; see Lütkepohl (2005), Kilian and Lütkepohl (2017)), and the matrix $\boldsymbol{\Omega}$ is a diagonal matrix with the variance of the shocks in the diagonal entries and the unit effect normalization that $H_{jj} = 1$ for all j (Ludvigson et al., 2019).

$$\text{diag}(\mathbf{H}) = 1, \boldsymbol{\Omega} = \begin{bmatrix} \sigma_{11} & 0 & \ldots & 0 \\ 0 & \sigma_{22} & 0 & 0 \\ 0 & \vdots & \ldots & 0 \\ 0 & 0 & \ldots & \sigma_{kk} \end{bmatrix}, \sigma_{kk} \geq 0 \forall j \tag{3.36}$$

Ultimately, the goal is to study the dynamic effects of the structural shocks (i.e., primarily the impulse response functions; see Section 5.2) as given by

$$\sum_{k=0}^{\infty} \boldsymbol{\Theta}_k = \sum_{k=0}^{\infty} \boldsymbol{\Psi}_k \mathbf{A}_0^{-1} \tag{3.37}$$

on $\mathbf{y}_t$. The authors point out that with autoregressive parameters $\mathbf{B}_j$ being consistently estimated under regularity conditions, the sample residuals $\hat{\mathbf{u}}_\mathbf{t}(\hat{\boldsymbol{\phi}})$ are consistent estimates of the reduced-form errors $\mathbf{u}_t$ and the estimation problem reduces to uniquely identify the impact matrix $\mathbf{A}_0^{-1}$ from $\hat{\boldsymbol{\phi}}$. However, at this stage,

[88] The *vec*-operator takes an $(n \times n)$ matrix and stacks the columns into a single vector of length n^2. See also Hamilton (1994).

[89] The *vech* (vector half) - operator takes a symmetric $(n \times n)$ matrix and stacks the lower triangular half into a single vector of length $\frac{d(d+1)}{2}$.

the model is under-identified because there are more parameters to be estimated in $\mathbf{A}_0^{-1}$ while the (reduced-form) covariance structure of $\hat{\mathbf{u}}_t$ provides less restrictions in the form $FZ(\mathbf{A}_0^{-1}) \equiv \text{vech}(\hat{\mathbf{\Sigma}}_u) - \text{vech}(\mathbf{A}_0^{-1}(\mathbf{A}_0^{-1})') = \mathbf{0}$.[90]

Central to the framework of Ludvigson et al. (2019) is then the following:[91]
By definition they have set $\mathbf{\Sigma}_u = \mathbf{PP}'$ with $\mathbf{P}$ being the unique lower-triangular Cholesky factor. By introducing a set $\mathbb{Q}_n$ of $(n \times n)$ orthonormal matrices,[92] any $\mathbf{A}_0^{-1} = \mathbf{PQ}$ is 'observationally equivalent' (see the explanations in Section 3.2.1) given $\mathbf{\Sigma}_u$ and hence consistent with the reduced form variance-covariance matrix $\mathbf{\Sigma}_u = \mathbf{A}_0^{-1}\mathbf{A}_0^{-1'}$ for any given $\mathbf{Q} = (q_1, q_2, \ldots, q_n) \in \mathbb{Q}_n$[93]. Ludvigson et al. (2019) define this set of observationally equivalent[94] $\mathbf{A}_0^{-1}$ as

$$\mathcal{A}_0 = \{\mathbf{A}_0^{-1} = \mathbf{PQ} : \mathbf{Q} \in \mathbb{Q}_n\} \tag{3.38}$$

whereby according to Ludvigson et al. (2019) the only restriction that can be imposed at this stage follows from combining the unit effect normalization on $\mathbf{H}$ with $\sigma_{jj} \leq 0$ so that a unit change in the structural shock j may be interpreted as a standard deviation increase in variable j (i.e. reduced-form covariance restrictions). Taking this into account, $\mathcal{A}_0$ becomes

$$\boldsymbol{\mathcal{A}}_0 = \{\mathbf{A}_0^{-1} = \mathbf{PQ} : \mathbf{Q} \in \mathbb{Q}_n, \text{diag}(\mathbf{Q}; \phi) \geq 0, FZ(\mathbf{Q}; \phi) = \mathbf{0}\}, \tag{3.39}$$

which is referred to as the *unconstrained set*, while due to the covariance restrictions it is technically not completely unconstrained (Ludvigson et al., 2019).

By collecting the reduced form innovations $\mathbf{u}_t$ into ϕ, 3.39 can be written as

$$\hat{\boldsymbol{\mathcal{A}}}_0(\phi) = \{\mathbf{A}_0^{-1} = \mathbf{PQ} : \mathbf{Q} \in \mathbb{Q}_n, \quad \text{diag}(\mathbf{A}_0^{-1}) \geq 0\}. \tag{3.40}$$

[90]For the case with $n = 3$ variables which we will consider below, there are nine parameters to be estimated in $\mathbf{A_0^{-1}}$ while the covariance structure only provides six restrictions.

[91]Note that the below approach is also leveraged for so-called 'sign-restrictions' in the SVAR-literature (Kilian and Lütkepohl, 2017).

[92]Note that Rubio-Ramírez et al. (2010) were referring only to *orthogonal*, not *orthonormal* matrices. Orthonormality implies orthogonality, i.e., orthonormality is stronger by demanding for each basis vector spanning the respective space to have length 1.

[93]See Section 3.2.1 for the approach that Ludvigson et al. (2019) use for the construction of the $\mathbf{Q}$-matrices following Rubio-Ramírez et al. (2010) (which is called *Householder Transformation* in Kilian and Lütkepohl (2017) based on Stewart (1980) which contains the proof of the approach presented in Rubio-Ramírez et al. (2010)).

[94]Note that Ludvigson et al. (2019) make use of this property of orthonormal matrices but do not specifically mention the term 'observational equivalence' which we have emphasized here solely for explanational purposes. Rather, the authors refer to this term in Ludvigson et al. (2017).

Without any further restrictions, $\hat{\boldsymbol{\mathcal{A}}}_0(\phi)$ contains infinitely many solutions. Ludvigson et al. (2019) hence introduce additional restrictions that go beyond classical sign restrictions, or, more generally, inequality restrictions (which tend to place these restrictions on the impulse response functions and/or $\mathbf{A}_0^{-1}$ itself) in an attempt to yield a smaller solution set denoted as $\overline{\mathcal{A}_0}(\phi)$.

Denoting the collection of zero restrictions imposed on the model as $FZ(\mathbf{Q};\phi)$,[95] the novel restrictions that Ludvigson et al. (2019) define to create a credible identification scheme involve the *identified structural shocks* $\boldsymbol{\epsilon}_t$ either on their own by defining event constraints $FE(\mathbf{Q};\phi;\mathbf{y}_t,\tau^*,\overline{k})$ or in combination with external variables as component correlation constraints denoted $FC(\mathbf{Q};\phi,\mathbf{y}_t,\mathbf{S})$. Thereby the set $\hat{\boldsymbol{\mathcal{A}}}_0(\phi)$ is diminished accordingly.

Ludvigson et al. (2018, 2019) also contrast their non-Bayesian approach to work which has been done on sign-restricted SVARs in a Bayesian context that also draws random orthogonal matrices $\mathbf{Q}$ in the course of the $\mathbf{QR}$-decomposition (see Sections 3.2.3 and 3.2.1) and clarify that these approaches focus on restrictions placed on the sign of IRFs while their approach concentrates on restrictions on the generated shocks. In addition, the authors explain that their approach is frequentist and follows the moment inequality framework of Andrews and Soares (2010) where moment conditions are given by the "[...] inequalities from the event and correlation constraints, and equalities provided by the covariance structure" Ludvigson et al. (2019, p. 14). The $\mathbf{QR}$-decomposition in their approach is rather of technical nature to generate possible candidates for $\mathbf{A}_0$.

After a short description of the SVAR-system being estimated, we will continue with a description of these two set of constraints.

Baseline SVAR(6)-3

Ludvigson et al. (2018, 2019) estimate several VAR systems to identify uncertainty shocks from the VAR residuals using the restrictions which will be explained below. For this work, we will employ their baseline estimation consisting of $\mathbf{y}_t = (U_{Mt}, IPM_t, U_{Ft})$ where U_{Mt} denotes macro uncertainty, IPM_t a measure of real activity (in this case industrial production in manufacturing in log-levels) and U_{Ft} a measure for financial uncertainty. The VAR-System uses $p = 6$ lags. For details on the data we refer to Section 4 and Appendix B.1.

[95] Note that we have the taken the notation for the restrictions from Ludvigson et al. (2017).

Shock-Based Constraints

Ludvigson et al. (2018, 2019) point out that in their setting real activity shocks are considered 'first moment' shocks (which could potentially come from technology, monetary policy, preferences, or fiscal policy innovations). The other two series in the SVAR, financial and macro uncertainty are types of 'second moment' - shocks that could be caused by expected volatility in financial markets or the macro economy, respectively. In this sense, the SVAR-system allows to understand whether shifts on first or second moments (or both) are drivers of business cycles.

Event constraints[96] require that shocks have plausible properties during certain historic episodes. According to Ludvigson et al. (2019, p. 11), "[...] a credible identification scheme should produce shocks that are not grossly at variance with our ex-post understanding of events, at least during episodes of special interest". In particular, event constraints put the bounds $\overline{k}$ on the sign and magnitude of $\boldsymbol{\epsilon}_t = \mathbf{A}_0\mathbf{u}_t$ during particular episodes which are collected into τ^*.

The reason why such special events turn out to be helpful for identification is that, despite the fact that two 'observationally equivalent' (as introduced in Section 3.2.1) structural models $\mathbf{A}_0^{-1}$ and $\widetilde{\mathbf{A}}_0^{-1}$ will produce the corresponding shock-processes $\{\boldsymbol{\epsilon}_t\}_{t=1}^T$ and $\{\widetilde{\boldsymbol{\epsilon}}_t\}_{t=1}^T$ where the first and second moments are equivalent, the individual elements of $\boldsymbol{\epsilon}_t$ and $\widetilde{\boldsymbol{\epsilon}}_t$ are not necessarily equal at each point in time (Ludvigson et al., 2019). For $\widetilde{\mathbf{Q}} \neq \mathbf{Q}$ it can be seen that (Ludvigson et al., 2019)

$$\begin{aligned} \boldsymbol{\epsilon}_t &= \mathbf{A}_0\hat{\mathbf{u}}_t = \\ [\mathbf{A}_0^{-1} &= \mathbf{PQ} \iff \mathbf{A}_0 = \mathbf{Q}^{-1}\mathbf{P}^{-1} \iff \\ \mathbf{A}_0 &= \mathbf{Q}^{'}\mathbf{P}^{-1}] = \\ &\mathbf{Q}^{'}\mathbf{P}^{-1}\hat{\mathbf{u}}_t \end{aligned} \tag{3.41}$$

and

$$\widetilde{\boldsymbol{\epsilon}}_t = \widetilde{\mathbf{A}}_0\hat{\mathbf{u}}_t = \widetilde{\mathbf{Q}}^{'}\mathbf{P}^{-1}\hat{\mathbf{u}}_t = \widetilde{\mathbf{Q}}\mathbf{u}_t \neq \boldsymbol{\epsilon}_t \tag{3.42}$$

at any given t which shows that event constraints could be used to reduce the number of solutions in $\hat{\boldsymbol{\mathcal{A}}}_0(\phi)$ to a smaller set $\overline{\boldsymbol{\mathcal{A}}}_0(\phi)$.

[96] Alternatively also called *event inequality constraints*. As pointed out by Ludvigson et al. (2019), concommitant work is Antolín-Díaz and Rubio-Ramírez (2018) where they also suggest shock-restrictions during historical episodes in a Bayesian setting. Ludvigson et al. (2020*b*) also reference works by e.g. Cieslak and Schrimpf (2019) and Zeev (2018).

To show this, Ludvigson et al. (2019) consider the bivariate case $n = 2$:
Writing down

$$\begin{aligned}
\begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix} &= \begin{bmatrix} a^0_{11} & a^0_{12} \\ a^0_{21} & a^0_{22} \end{bmatrix}^{-1} \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{bmatrix} \iff \\
\begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{bmatrix} &= \begin{bmatrix} a^0_{11} & a^0_{12} \\ a^0_{21} & a^0_{22} \end{bmatrix} \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix} \implies \\
\epsilon_{1t} &= a_{11}u_{1t} + a_{12}u_{2t}
\end{aligned} \tag{3.43}$$

where the values of u_{1t} and u_{2t} are given since data is given for time t in the span $[\tau_1, \tau_2]$. The above example shows that a restriction on the behavior of ϵ_{1t} at a specific time t results in a non-linear restriction on $\mathbf{A}_0^{-1}$ and, correspondingly, on $\mathbf{Q}$.

With the stage set according to the above explanations, Ludvigson et al. (2019) depart from their approach in Ludvigson et al. (2018) where the authors exclusively relied on restrictions formulated based on the ex-post reading of historical events. Instead, in Ludvigson et al. (2019) the authors apply a combination of this approach and add an analyses of the model's uncertainty shocks *before*[97] formulating any restrictions. This approach leverages the algorithm as described in Section 3.2.3 below by generating an *unconstrained* solution set which only fulfills the covariance restrictions and consists of as many elements as the algorithm performs rotations of $\mathbf{P}$ (the unique lower-triangular Cholesky factor of $\hat{\mathbf{\Sigma}}_u$). Subsequently, the generated impact matrices $\mathbf{A}_0^{-1}$ give rise to an equally large number of shock-series $\boldsymbol{\epsilon}_t$. In our case, for 100.000 rotations of the SVAR-system $\mathbf{y}_t = (U_{Mt}, IPM_t, U_{Ft})$ using the original data of Ludvigson et al. (2018, 2019)[98] for the period 1960:07-2015:04, 31.43% of the maxima for the macro uncertainty shocks ϵ_{Mt} fall on September 2008 (when the financial crisis intensified with the failure of Lehman Brothers), while 25.05% are found in December 1970. Placed third, November 1987 follows with 20.89% of the maxima. Looking at the shocks for financial uncertainty, ϵ_{Ft}, also here the majority of the maxima falls on September 2008 (with 30.89%), while as a close second October 1987 follows with 27.27%. Placed third, also here December 1970 indicates heightened uncertainty around that time.

Interestingly, replacing the data for U_{Mt}, IPM_t, U_{Ft} (which using the original data

[97] Author's italics.

[98] As noted at the beginning of this thesis, we have kindly received the data from one of the co-authors.

of Ludvigson et al. (2018, 2019) run from 1960:07 to 2015:04) with their presumed sources as described in Table 6[99] for the same estimation window 1960:07 to 2015:04, produces slightly different results with respect to the order of the biggest shock events for U_{Mt} where following September 2008 placed first, instead of December 1970 (0.21%), November 1987 produces slightly more maxima (0.22%) and hence ranks second. For financial uncertainty the ordering with respect to the biggest shocks stays the same. Using the same replacement data albeit for a longer estimation window from 1960:07 to 2019:04 produces the same ordering for both the largest macro (2nd and 3rd flipped) & financial uncertainty shocks as the replacement data on the estimation window 1960:07 to 2015:04.

In their own implementation, and based on the results of the 'search-algorithm' together with additional a-priori restrictions, Ludvigson et al. (2019) require any $\boldsymbol{\epsilon}_t = \mathbf{A}_0\mathbf{u}_t$ formed from $\mathbf{A}_0^{-1} \in \hat{\boldsymbol{\mathcal{A}}}_0$ to satisfy six event constraints[100] which they parameterize by

$$\begin{aligned} \overline{\mathbf{k}} &= (\overline{k}_1, \overline{k}_2, \overline{k}_3, \overline{k}_4, \overline{k}_5)' \\ \overline{\boldsymbol{\tau}} &= \begin{bmatrix} \overline{\tau}_1 \\ \overline{\tau}_2 \\ \overline{\tau}_3 \\ \overline{\tau}_4 \\ \overline{\tau}_5 \\ \overline{\tau}_6 \end{bmatrix} = \begin{bmatrix} \text{1987:10} \\ \text{2008:09} \\ \text{1970:12} \\ \text{[2007:12, 2009:06]} \\ \text{1979:10} \\ \text{[2011:07, 2011:08]} \end{bmatrix} \end{aligned} \tag{3.44}$$

[99] U_{Mt} should be equal to the macro uncertainty index at horizon $h = 1$, U_{Ft} equally should be the financial uncertainty index at horizon $h = 1$ and IPM_t is replaced with the $log()$ of industrial production as available from FRED. Note, however, that it is not completely clear whether the data for industrial production as used in Ludvigson et al. (2018, 2019) are the same as available on FRED. Also, the data we have received and use for for U_{Mt} and U_{Ft} is slightly different than the series available on the author's website (see Section B.1 for details), which we attribute to revisions of the constructed uncertainty series.

[100] In contrast to Ludvigson et al. (2019), we added one more parameter to the number of parameters in $\overline{\mathbf{k}}$ which we will explain below.

and collect the constraints into a system of inequalities

$$FE(\mathbf{Q};\phi,\mathbf{y}_t,\overline{\tau},\overline{k}) = \left[\begin{array}{c} FE_1(\overline{\tau}_1,\overline{k}_1) \\ FE_2(\overline{\tau}_2,\overline{k}_1,\overline{k}_2), \\ FE_3(\overline{\tau}_3,\overline{k}_4) \\ \hdashline FE_4(\overline{\tau}_4,\overline{k}_5) \\ \hdashline FE_5(\overline{\tau}_5) \\ FE_6(\overline{\tau}_6) \end{array}\right] =$$

$$= \left[\begin{array}{l} \sum_{t=1}^{T} \mathbb{1}_{t=\overline{\tau}_1=1987:10} \cdot \epsilon_{F\overline{\tau}_1} - \overline{k}_1 \geq 0 \\ (\sum_{t=1}^{T} \mathbb{1}_{t=\overline{\tau}_2=2008:09} \cdot \epsilon_{F\overline{\tau}_2} - \overline{k}_2 \geq 0) \vee (\sum_{t=1}^{T} \mathbb{1}_{t=\overline{\tau}_2=2008:09} \cdot \epsilon_{M\overline{\tau}_2} - \overline{k}_3 \geq 0) \\ \sum_{t=1}^{T} \mathbb{1}_{t=\overline{\tau}_3=1970:12} \cdot \epsilon_{M\overline{\tau}_3} - \overline{k}_4 \geq 0 \\ \hdashline \overline{k}_5 - \sum_{t=1}^{T} \mathbb{1}_{t=\overline{\tau}_4 \forall \in [2007:12,2009:06]} \cdot \epsilon_{IPM\overline{\tau}_4} \geq 0 \\ \hdashline (\sum_{t=1}^{T} \mathbb{1}_{t=\overline{\tau}_5=1979:10} \cdot \epsilon_{M\overline{\tau}_5} \geq 0) \wedge (\sum_{t=1}^{T} \mathbb{1}_{t=\overline{\tau}_5=1979:10} \cdot \epsilon_{F\overline{\tau}_5} \geq 0) \\ (\sum_{t=1}^{T} \mathbb{1}_{t=\overline{\tau}_6 \forall \in [2011:07,2011:08]} \cdot \epsilon_{M\overline{\tau}_6} \geq 0) \wedge (\sum_{t=1}^{T} \mathbb{1}_{t=\overline{\tau}_6 \forall \in [2011:07,2011:08]} \cdot \epsilon_{F\overline{\tau}_6} \geq 0) \end{array}\right] \tag{3.45}$$

Together, $FE(\mathbf{Q};\phi,\mathbf{y}_t,\overline{\tau},\overline{k})$ defines inequality constraints based on timing, sign and magnitude that help identification of the underlying structural model.[101] In this set, Ludvigson et al. (2019) refer to the first three constraints as the *big shock events* which are derived off of the search algorithm applied to the unconstrained solution set discussed above and the bottom two constraints as *non-negative events* (visually separated by a horizontal line in 3.45). As the fourth constraint FE_4, which we call *real-activity constraint*, we take a constraint from Ludvigson et al. (2018) and here replace the respective constraint intended by Ludvigson et al. (2019) at this position. While the discussion of the search-algorithm above suggested 1987:11 to be relevant for macro uncertainty with a slightly adjusted data-set, we do no formulate any additional constraints or replace FC_3 (the constraint for 1970:12) at this stage and leave this to subsequent work.

As described in Ludvigson et al. (2019), FE_1 requires that the financial uncertainty shock that occurred in October 1987 be large and exceed the mean by $\overline{k}_1$ standard deviations, FE_2 postulates that either the financial uncertainty shock or the macro

[101] Note that the bounds in $\overline{\mathbf{k}}$ still have to be chosen.

uncertainty shock or both found in September 2008 (the month when Lehman Brothers filed for bankruptcy) must be large and exceed the mean by $\overline{k}_2$ and/or $\overline{k}_3$ standard deviations, respectively, and FC_3 requires the macro uncertainty shock in December 1970 to be large and $\overline{k}_4$ standard deviations above the mean. The non-negative event constraints, FC_5 and FC_6 are introduced by Ludvigson et al. (2019) in addition to the big shock events, arguing that going beyond big shock events, a set of weaker restrictions should be derived off of other major economic events during the sample period. FC_5 and FC_6 demand for financial and macro uncertainty shocks to be positive (i.e., above average) in October 1979 (Ludvigson et al. (2019) refer to this episode as the 'Volcker experiment') and during the months of July 2011 and August 2011 (the debt-ceiling crisis). In a similar spirit, the real activity constraint FC_4 requires that any real activity shocks found during the Great Recession (the NBER dates for the recession coincide with the financial crisis) do not take on any unusually large positive values (as formulated in Ludvigson et al., 2018).

As laid out by Ludvigson et al. (2019), the motivation for the above constraints is that if a potential $\mathbf{Q}$ implies a corresponding shock series that is difficult to hold on to during the key time episodes, it will be removed from the solution set $\hat{\boldsymbol{\mathcal{A}}}_0(\phi)$. Correspondingly, in the postulated inequality constraints the values for the $\overline{k}_i$s have to be meaningful and the timing of events $\overline{\tau}_i$ accurate otherwise solutions that pass these constraints will be meaningless after all.

With regard to the established restrictions for the big shock episodes, Ludvigson et al. (2019, p. 12) point out that they make sure that "*at least some*[102] of the forecast error variance within the system $\mathbf{y}_t$ on the respective dates is attributable to the respective shocks. This holds for the shock ϵ_{Ft} in 1987:10 (FE_1), the shock ϵ_{Mt} in 1970:12 (FE_3) and at least one of the shocks ϵ_{Mt} or ϵ_{Ft} in 2008:09 (FE_3). To clarify, Ludvigson et al. (2019) point out that the constraints, however, do *not*[103] require that all or most of the variability in these episodes to come from shocks declared in the restrictions because they do not rule out other large adverse shocks.

The second set of constraints considered by Ludvigson et al. (2019) are ***correlation constraints***[104] which require that the identified (structural) uncertainty shocks recovered from $\boldsymbol{\epsilon}_t = \mathbf{A}_0\mathbf{u}_t$ show a minimum absolute correlation (co-movement) with certain variables that are external to the VAR estimations which they denote with $\mathbf{S}_t$.

[102] Author's italics.
[103] Author's italics.
[104] Alternatively also called *external variable inequality constraints.*

In their baseline specification they use two external variables $\mathbf{S}_t = (S_{1t}, S_{2t})$, with S_{1t} denoting a measure of aggregate stock market returns and S_{2t} denoting the log difference in the real price of gold (a safe-haven asset), arguing that these variables are likely to contain information about uncertainty shocks.[105]

As a motivating example, Ludvigson et al. (2017) consider the following example of a bivariate reduced-form VAR-model:

$$\begin{aligned}
\mathbf{y}_t &= \sum_{j=1}^{p} \mathbf{B}_j \mathbf{y}_{t-j} + \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix} \iff \\
\mathbf{y}_t &= \sum_{j=1}^{p} \mathbf{B}_j \mathbf{y}_{t-j} + \begin{bmatrix} a_{11}^0 & a_{12}^0 \\ a_{21}^0 & a_{22}^0 \end{bmatrix}^{-1} \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{bmatrix}, \\
\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \end{bmatrix} &\sim \mathcal{N}(\mathbf{0}, \mathbf{I}_n)
\end{aligned} \tag{3.46}$$

As already mentioned before, looking at the variance-covariance matrix $\mathbf{\Sigma}_u$ of the reduced-form residuals $\mathbf{u}_t$ with $\mathbf{\Sigma}_u = \mathbf{A}_0^{-1}\mathbf{A}_0^{-1'}$ which results in

$$\underbrace{\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix}}_{\text{3 values}} = \underbrace{\begin{bmatrix} a_{11}^0 & a_{12}^0 \\ a_{21}^0 & a_{22}^0 \end{bmatrix}^{-1}}_{\text{4 unknowns}} \begin{bmatrix} a_{11}^0 & a_{12}^0 \\ a_{21}^0 & a_{22}^0 \end{bmatrix}^{-1'}$$

Ludvigson et al. (2017) explain that one more restriction would allow a point identification of the impact matrix $\mathbf{A}_0^{-1}$. Instead, here Ludvigson et al. (2017) assume an external variable S_t which must not necessarily be exogenous (like an adequate instrument would be) due to which it would be contemporaneously correlated with at least one of the structural errors ϵ_{1t} and/or ϵ_{2t} (i.e. endogenous). These assumptions translate into S_t being representable by

$$\begin{aligned}
S_t &= \lambda_1 \epsilon_{1t} + \lambda_2 \epsilon_{2t} + \sigma_S \epsilon_{St} \\
&= \lambda_1 \epsilon_{1t} + \lambda_2 \epsilon_{2t} + Z_t
\end{aligned} \tag{3.47}$$

and e_{St} being a shock specific to S which is assumed to be uncorrelated with the structural residuals e_{1t} and e_{2t}, respectively (Ludvigson et al., 2017).

In order to exploit the relationship with S_t external to the VAR in the identifica-

[105]See Table 6 for details on the data sources.

tion of $\mathbf{A}_0^{-1}$, in their example Ludvigson et al. (2017) assume to reject solutions in $\hat{\mathcal{A}}_0(\phi)$ that, e.g., result in an absolute correlation between S_t and e_{2t} below a certain threshold (i.e. too small). While the expression $\frac{\lambda_2}{\sqrt{\lambda_1^2+\lambda_2^2+\sigma_S{}^2}} = c(\mathbf{A_0})$ stands for the correlation between S_t and e_{2t} and depends on the particular $\mathbf{A_0}$ considered, demanding a minimum absolute correlation equals a non-linear constraint between S_t and e_{2t}.

Ludvigson et al. (2019) see their correlation constraint with stock market returns backed by an asset pricing literature that establishes a negative link/correlation between macro/financial uncertainty shocks and (ex post) stock market returns, arguing that these shocks drive the stock market risk premium variation, a leading example being the Capital Asset Pricing Model (see Sharpe, 1964, Lintner, 1965) and variants thereof. With respect to gold, the argument goes that its value is assumed to contain information about uncertainty shocks since “plausibly exogenous increases in uncertainty [...] have typically been associated with increases in returns on quintessential safe-haven assets, the archetypal example being gold.” (Ludvigson et al., 2019, p. 10).

Letting $(c_{M_t,S_{1t}}(\mathbf{A}_0), c_{F_t,S_{1t}}(\mathbf{A}_0), c_{M_t,S_{2t}}(\mathbf{A}_0), c_{F_t,S_{2t}}(\mathbf{A}_0))$ be the sample correlations between the structural shocks $(\epsilon_{Mt}(\mathbf{A}_0), \epsilon_{Ft}(\mathbf{A}_0))$ and the series for the stock market return and the log difference in the real price of gold $\mathbf{S}_t = (S_{1t}, S_{2t})$, respectively, Ludvigson et al. (2019) establish the following system of inequalities as correlation constraints

$$
\begin{aligned}
FC(\mathbf{Q}; \phi, \mathbf{y}_t, \mathbf{S}) = \begin{bmatrix} FC_1(\mathbf{S}) \\ FC_2(\mathbf{S}) \end{bmatrix} = \\
= \begin{bmatrix} (c_{M_t,S_{1t}}(\mathbf{A}_0) \leq 0) \vee (c_{F_t,S_{1t}}(\mathbf{A}_0) \leq 0) \\ (c_{M_t,S_{2t}}(\mathbf{A}_0) \geq 0) \vee (c_{F_t,S_{2t}}(\mathbf{A}_0) \geq 0). \end{bmatrix}
\end{aligned}
\tag{3.48}
$$

$FC_1(\mathbf{S})$ states that uncertainty shocks have to be negatively correlated with the series of the stock market returns while $FC_2(\mathbf{S})$ on the other hand demands them to be positively correlated with the respective series for the log difference in the real price of gold.

Together, the constraints in 3.48 provide cross-equation restrictions on the parameters in a potential $\mathbf{A}_0^{-1}$, whereby the correlations are not invariant to orthonormal rotations (by $\mathbf{Q}$) so that the generated correlations in each draw will be different.

Identified Solution Set and maxG - Solution

Combining default covariance structure restrictions with the introduced event and correlation constraints, Ludvigson et al. (2018, 2019) call the *identifed*[106] solution set

$$\begin{aligned}\overline{\boldsymbol{\mathcal{A}}}_0(\mathbf{Q};\phi,\overline{k},\overline{\tau},\mathbf{S}) = \{\mathbf{A}_0^{-1} = \mathbf{PQ} : \mathbf{Q}\in\mathbb{Q}_n, \quad diag(\mathbf{Q};\phi)\geq 0\}; \\ FZ(\mathbf{Q};\phi) = \mathbf{0}, \\ FE(\mathbf{Q};\phi,\mathbf{y}_t,\overline{\tau},\overline{k}) \geq \mathbf{0}, \\ FC(\mathbf{Q};\phi,\mathbf{y}_t,\mathbf{S}) \geq \mathbf{0}\end{aligned} \tag{3.49}$$

which only contains estimates of $\mathbf{A}_0^{-1}$ that satisfy all constraints. Thereby a particular solution can only be in both $\hat{\boldsymbol{\mathcal{A}}}_0$ (the unconstrained solution set) and $\overline{\boldsymbol{\mathcal{A}}}_0$ if all restrictions are satisfied. And while $\overline{\boldsymbol{\mathcal{A}}}_0$ will be a set as well it should be notably smaller than $\hat{\boldsymbol{\mathcal{A}}}_0$ (without any additional restrictions other than the usual covariance restrictions).

Both constraints together generate moment inequalities (together with the standard reduced-form covariance restrictions) that produce a reduction of the set of possible model parameters consistent with the data that is large enough to ultimately unambiguously derive most dynamic relationship of the SVAR-system (Ludvigson et al., 2019).

The ultimate size of the set $\overline{\boldsymbol{\mathcal{A}}}_0$ depends in particular on the chosen thresholds for the values $\overline{k}$ which we will discuss in Section 3.2.3 below.

And while there is no single solution in $\overline{\boldsymbol{\mathcal{A}}}_0$ that is more likely than another one, as a reference point (and following Ludvigson et al., 2018), we will also refer to the so-called 'maxG' - solution when discussing the results. In particular, the 'maxG' - solution serves as a reference point at which the value of the introduced constraints (inequalities) are maximized simultaneously. Formally, the quadratic norm is chosen

[106] Author's italics.

as the maxG-solution so that

$$\mathbf{A}_0^{maxG} \equiv \underset{\mathbf{A}_0 \in \overline{\mathcal{A}}_0}{\operatorname{argmax}} \sqrt{\overline{f}(\mathbf{A}_0)'\overline{f}(\mathbf{A}_0)} \quad \text{where}$$
$$\overline{f}(\mathbf{Q};\phi,\overline{k},\overline{\tau},\mathbf{S}) = \begin{bmatrix} FZ(\mathbf{Q};\phi)' \\ FE(\mathbf{Q};\phi,\mathbf{y}_t,\overline{\tau},\overline{k})' \\ FC(\mathbf{Q};\phi,\mathbf{y}_t,\mathbf{S})' \end{bmatrix}' \tag{3.50}$$

With these features, Ludvigson et al. (2018) regard the 'maxG' - solution, economically, as the "worst-case" - scenario. While Ludvigson et al. (2019) depart from this definition and identify the maxG-solution as the one for which the inequalities related to the correlation constraints are collectively maximized, we decided to stick with the definition in Ludvigson et al. (2018) and consider the complete set of inequalities in the formulation of the maxG-solution. Hence, with respect to the introduced constraints, the event inequalities will be large for the most extremely positive financial uncertainty shock in October 1987 (FE_1), for the most extreme cumulation of financial and macro uncertainty shocks in September 2008 (FE_2), for the most extreme macro uncertainty shock in December 1970 (FE_3), when the real activity shocks found during the Great Recession are cumulatively most negative (FE_4), for the most extreme cumulation of financial and macro uncertainty in October 1979 (FC_5) and when macro and financial uncertainty shocks in 2011:07 - 2011:08 are cumulatively largest (FC_6). Accordingly, also for the correlation inequalities the maxG -solution demands their absolute value to be largest.

3.2.3. Implementation and Parameter Selection

As indicated above, a decisive part of the solution algorithm is the construction of the unconstrained solution set $\hat{\mathcal{A}}_0$ and the subsequent derivation of the identified set $\overline{\mathcal{A}}_0$.

The constrained solution set $\overline{\mathcal{A}}_0$ is obtained through the following steps as outlined in Ludvigson et al. (2019):

i estimation of the reduced-form model (i.e., the coefficient matrices $\mathbf{B}_i$ and the variance-covariance matrix $\mathbf{\Sigma}_u$); the number of matrices $\mathbf{B}_i$ to be estimated corresponds to the number of lags in the (S)VAR model[107]

[107] In the Matlab-code provided to us by Sai Ma (one of the co-authors of Ludvigson et al. (2018, 2019) we noticed that the authors had computed the variance covariance matrix of the residuals by dividing the residual sum of squares by 652 (the sample size T) while the *var*-package in R applies a degrees of freedom correction via $T - Kp - 1 = 652 - 3 * 6 - 1$ where K stands for the

ii initialization of $\mathbf{A}_0^{-1}$ as the unique lower-triangular Cholesky factor $\hat{\mathbf{P}}$ of $\hat{\mathbf{\Sigma}}_u$ with non-negative diagonal elements

iii rotation of $\hat{\mathbf{P}}$ by K = 1.5 million random orthogonal matrices $\mathbf{Q}$;
each rotation begins by drawing an $(n \times n)$ matrix $\mathbf{M}$ of normally and independently distributed values (i.e., drawn from a normal distribution with $\mu = 0$ and $\sigma = 1$); $\mathbf{Q}$ is then taken to be the orthonormal matrix resulting from a $\mathbf{QR}$ decomposition of $\mathbf{M}$ (see Section 3.2.1)

iv because $\mathbf{A}_0^{-1} = \mathbf{PQ}$, the covariance restrictions are imposed by construction

v every generated $\mathbf{A}_0^{-1}$ resulting from one of the 1.5 million rotations is then confronted with the event and correlation-constraints (for which the parameters $\overline{\mathbf{k}}$ have to be chosen) and stored if successfully passed. [108]

Calculation of IRFs and FEVDs

With the identified set $\overline{\mathcal{A}}_0$ at our disposal, given the $\mathbf{B}_i$-matrices of the reduced-form model, the $\mathbf{\Psi}_k$ - matrices are calculated recursively for $h = 60$ steps ahead via $\mathbf{\Psi}_k = \sum_{j=1}^{k} \mathbf{\Psi}_{k-j}\mathbf{B}_j$. Subsequently, the structural MA coefficient matrices $\mathbf{\Theta}_k$ (as denoted in Kilian and Lütkepohl (2017)) are derived off of $\mathbf{\Theta}_k = \mathbf{\Psi}_k\mathbf{A}_0^{-1}$ for every $\mathbf{A}_0^{-1} \in \overline{\mathcal{A}}_0$ and hold the coefficients for the IRFs at each horizon as explained in Section 3.1.1. The resulting IRFs are presented and discussed in Section 5.2. For the calculation of the FEVDs (as theoretically outlined in Section 3.1.2), similarly the $\mathbf{\Psi}_k$ - matrices are calculated recursively, this time only for $h = 200$ steps ahead (whereby $h = 200$ practically marks ∞ for our purposes). Again, also here the matrices $\mathbf{\Theta}_k$ are derived off of $\mathbf{\Theta}_k = \mathbf{\Psi}_k\mathbf{A}_0^{-1}$ for every $\mathbf{A}_0^{-1} \in \overline{\mathcal{A}}_0$. With the $\mathbf{\Theta}_k$ matrices at our disposal, the MSE matrices are calculated according to $\text{MSE}[\mathbf{y}_t(h)] = \sum_{k=0}^{h-1} \mathbf{\Theta}_k\mathbf{\Theta}_k'$ where the diagonal elements $\text{MSE}[y_{n,t}(h)] = \sum_{k=0}^{h-1}\sum_{m=1}^{M} \left(\theta_{nm}^{(0)}\right)^2$ hold the MSE of the respective variables of the SVAR-system. Ultimately, calculating $\rho_{nm,h} = \sum_{k=0}^{h-1} (\mathbf{e}_n'\mathbf{\Theta}_k\mathbf{e}_m)^2/\text{MSE}[y_{n,t}(h)]$ for every $\mathbf{\Theta}_k$ (derived off of every $\mathbf{A}_0^{-1} \in \overline{\mathcal{A}}_0$) for the respective forecast horizons $h = 1, 2, 3, 4, 5, 12, \infty, h_{max}$ and calculating the respective max and min bounds (since for every $\mathbf{\Theta}_k$ at every forecast horizon we

number of variables and p for the number of lags. The applied estimator for the variance-covariance matrix applied by Ludvigson et al. (2018, 2019) obviously corresponds to the MLE estimator. To be in line with the author's results, we have accordingly applied a correction factor to translate the degrees-of-freedom correction under the OLS-estimation to an MLE-estimator.

[108] Note that we wrote 'and/or' here because in Section 5.2 we present solutions where the constraints are applied in various combinations.

would get a slightly different FEVD) generates the results which will be discussed in Section 5.4.

Parameter Selection

The implementation of the event and correlation constraints still needs a reasonable choice of the parameters $\overline{\mathbf{k}}$, which we will discuss below.

As previously mentioned, with respect to the parameter selection, in the previous version of their paper, Ludvigson et al. (2018) had set up their shock-restricted SVAR-approach by exclusively selecting a priori restrictions (as opposed to the combination of restrictions derived directly from the data itself together with a priori restrictions introduced in Ludvigson et al., 2019). In addition, in Ludvigson et al. (2018) they used a mix of theory, empirical analysis and economic reasoning to come up with the specific values for the parameters for $\overline{\boldsymbol{\lambda}}$ and $\overline{\mathbf{k}}$ and thereby used/tested various parameterizations.[109]

In Ludvigson et al. (2019), the authors slightly adjusted their approach as compared to Ludvigson et al. (2018) and introduce the so-called "identification uncertainty" as the variation which is produced in the model's output (primarily for the impulse response analysis) by alternative combinations of (i) the constraints in combination with (ii) alternative specifications/parameterizations for the big shock events $\overline{\mathbf{k}}$. First, to show how the variation of alternative combinations of constraints influences the dynamic causal effects we refer to Section 5.2 where we follow Ludvigson et al. (2019) and present the impulse response analysis showing how starting from the solutions of the unconstrained set $\boldsymbol{\mathcal{A}}_0$, the subsequent addition of restrictions and shrinkage of permissible solutions stepwise delivers more conclusive results. Second, to come up with sensible parameterizations of the big shock events, Ludvigson et al. (2019) analyze the 1.5 million values of $\mathbf{A}_0^{-1} \in \hat{\boldsymbol{\mathcal{A}}}_0$ (the unconstrained set) that give rise to the time series of the system's shocks in $\boldsymbol{\epsilon}_t$ to come up with the parameters $\overline{\mathbf{k}}$. Their analyses of the sample's largest shocks show that they are typically above four standard deviations for U_{Ft} and U_{Mt}. Repeating their analysis for the selected dates of the big shock events (with 100.000 rotations instead of 1.5 Mio), our results are practically identical to the one's of Ludvigson et al. (2019): the 75th-percentile values of e_{Ft} in 1987:10 and 2008:09 are 4.16 ($\overline{k}_1$) and 4.58 ($\overline{k}_2$), respectively, while the 75th-percentile value of e_{Mt} in 1970:12 and 2008:09 is 4.05 ($\overline{k}_4$) and 4.74 ($\overline{k}_3$),

[109] In Ludvigson et al. (2018) the authors had used $\overline{\boldsymbol{\lambda}}$ to parameterize the correlation constraints. In Ludvigson et al. (2019) this is not necessary anymore since the correlation constraints are set to be either larger or smaller (or equal) to 0. Hence, only the choice of a proper $\overline{\mathbf{k}}$ is left.

respectively, which is another indication of the excess skewness and leptokurtosis of the shocks. The 50th-percentile (i.e. the median) of the respective shock-dates are 2.76 and 1.95 for e_{Ft} in 1987:10 and 2008:09 and 1.93 and 2.32 for e_{Mt} in 1970:12 and 2008:09, respectively. While Ludvigson et al. (2019) present results for different values of $\overline{\mathbf{k}}$ (85th-percentile, 75th-percentile, median), the main results under all of these different parameterizations are qualitatively identical. Hence, in our own implementation of the algorithm we limit ourselves to the case of setting the parameters $\overline{\mathbf{k}}$ to their 75th-percentiles. And for the *real activity* constraint ($\overline{k}_5$) we refer to Ludvigson et al. (2018) that have set the parameter to 2 to denounce any solutions where the real activity shocks would be greater than two standard deviations above the mean for the entire period of the Great Recession.

3.2.4. Set-Identification and Inference

One issue we have not elaborated on yet is that the class of SVARs as suggested by Ludvigson et al. (2019) belong to so-called *set-identified* SVARs, derived off the fact that the resulting parameters of the impact matrices $\mathbf{A}_0$ are set identified (i.e., as explained by Kilian and Lütkepohl (2017, p. 437), "[...] there is not a unique point estimate of the structural impulse response functions"). This is a direct result of the imposed restrictions being *inequality* restrictions (Kilian and Lütkepohl, 2017). As clarified by Kilian and Lütkepohl (2017, p. 422), for set-identified SVARs it is only possible to "bound the parameters of interest" and, in particular, inference is much more involved than for point identified SVARs, in particular, because for set-identified any confidence band would revert to both, identification and estimation uncertainty (see also the explanations in Section 3.2.3). Hence, according to Kilian and Lütkepohl (2017, p. 437), when being confronted with inequality restrictions, "[...] [s]tandard asymptotic and bootstrap methods of inference [...] are invalid [...]". Ludvigson et al. (2019) themselves point this out by declaring that their shock-restricted approach in the first place deals with the issue of identification while the issue of inference is yet to be solved. However, to get a sense of the sampling distribution, Ludvigson et al. (2019) perform a Monte Carlo simulation by bootstrapping from the set of shocks $\boldsymbol{\epsilon}_{\mathbf{t}}(\mathbf{A_0})$ in the *identified set* and plot the resulting IRFs including the derived bootstrapped confidence bands. For the presentation of the results in Section 5, especially in Sections 5.2 and 5.4 it should hence be kept in mind that here we are *not* presenting any confidence bands but report the results for the identified set of $\mathbf{A}_0^{-1} \in \overline{\boldsymbol{\mathcal{A}}}_0$ that passed the respective shock restrictions. We will briefly come back to this issue in Section 6.

4. Data

Ludvigson et al. (2018, 2019) consider $\mathbf{y}_t = (U_{Mt}, IPM_t, U_{Ft})$ as the baseline-SVAR-system where U_{Mt} was already mentioned in Section 2.4. As a measure of Y_t the log of real industrial production, ip_t is, used. As external variables, in their baseline specification they use two external variables $\mathbf{S}_t = (S_{1t}, S_{2t})$, with S_{1t} denoting a measure of aggregate stock market returns and S_{2t} denoting the log difference in the real price of gold.

The construction of the uncertainty measures U_{Mt} and U_{Ft} follows the framework Jurado et al. (2015). Especially in contrast to stock market volatility and/or cross-sectional dispersion measures frequently deployed in uncertainty studies (see also Section 2.4), Jurado et al. (2015) have designed a computationally and data-intensive approach by making use of a rich set of time series to measure a common component in the time-varying volatilities of forecast errors across a large number of macroeconomic time series.[110]

To set their approach apart from other commonly used uncertainty measures, Jurado et al. (2015, p. 1178) argue that "[...] what matters for economic decision making is not whether particular economic indicators have become more or less variable or disperse *per se*, but rather whether the economy has become more or less *predictable*; that is, less ore more uncertain" (see Section 2.4).

To formalize the rationale behind their approach, they define the h-period ahead uncertainty in the variable $y_{jt} \in Y_t = (y_{1t}, \ldots, y_{N_y t})'$, denoted by $U^t_{jt}(h)$, as the conditional volatility of the purely unforecastable component of the future value of the series, i.e.,

$$U^t_{jt}(h) \equiv \sqrt{\mathbb{E}\Big[(y_{jt+h} - \mathbb{E}[y_{jt+h}|I_t])^2|I_t\Big]}, \tag{4.1}$$

[110]Note that, apart from practical issues, Jurado et al. (2015, p. 1191) themselves explain the reliance on (most likely revised) "historical" data (instead of "real time" data) with their goal of forming the "[...] most historically accurate estimates of uncertainty at any given point in time in [their] sample".

Table 3.: Periods of high uncertainty according to macro uncertainty index h=1. *Note:* Periods of elevated uncertainty are calculated as months where the value of the respective uncertainty time series is above 1.65 standard deviations of the unconditional mean.

Start	End	Duration	Maximum Year/Month	Maximum Value
Aug 1974	Feb 1975	7 months	Dec 1974	0.87
Dec 1979	Jul 1982	31 months	Apr 1980	1.00
Jun 2008	Aug 2009	14 months	Oct 2008	1.06

where the expectation $\mathbb{E}(\cdot|I_t)$ is taken with respect to information I_t available to economic agents at time t. Hence, if the expectation today (conditional on all available information) of the squared error in forecasting y_{jt+h} (i.e. $\mathbb{E}\Big[(y_{jt+h} - \mathbb{E}[y_{jt+h}|I_t])^2|I_t\Big]$) rises, uncertainty in the variable increases. The individual uncertainties at each date are then aggregated into a *macroeconomic uncertainty* index by using aggregation weights w_j as follows:

$$U^t_{jt}(h) \equiv plim_{N_y \to \infty} \sum_{j=1}^{N_y} w_j U^y_{jt}(h) \equiv \mathbb{E}_w \Big[U^y_{jt}(h)\Big] \tag{4.2}$$

Based on the above definitions, Jurado et al. (2015) emphasize two features that are central to their approach:

1. they distinguish between *uncertainty* in a series y_{jt} and its *conditional volatility*[111] and argue that the proper measurement of uncertainty requires the removal of the entire forecastable component $\mathbb{E}[y_{jt+h}|I_t]$ before the computation of the conditional volatility; without accounting for this, the argument goes, forecastable variation is erroneously classified as "uncertainty". According to Jurado et al. (2015), this is taken into account only in very few occasions in the literature.

2. they claim that macroeconomic uncertainty is "a measure of the common variation in uncertainty across many series" and not equal to the uncertainty in any single series y_{jt}; Jurado et al. (2015) see this as an important argument because of uncertainty-based theories of the business cycle that typically require the existence of common (often countercyclical) variation in uncertainty across large numbers of series; Jurado et al. (2015) hence put their econometric estimation of uncertainty insofar to test, as they expect to find evidence of such an aggregate uncertainty factor common to many series, if the above assumption turns out to be correct.

[111] See also Section 2.4.

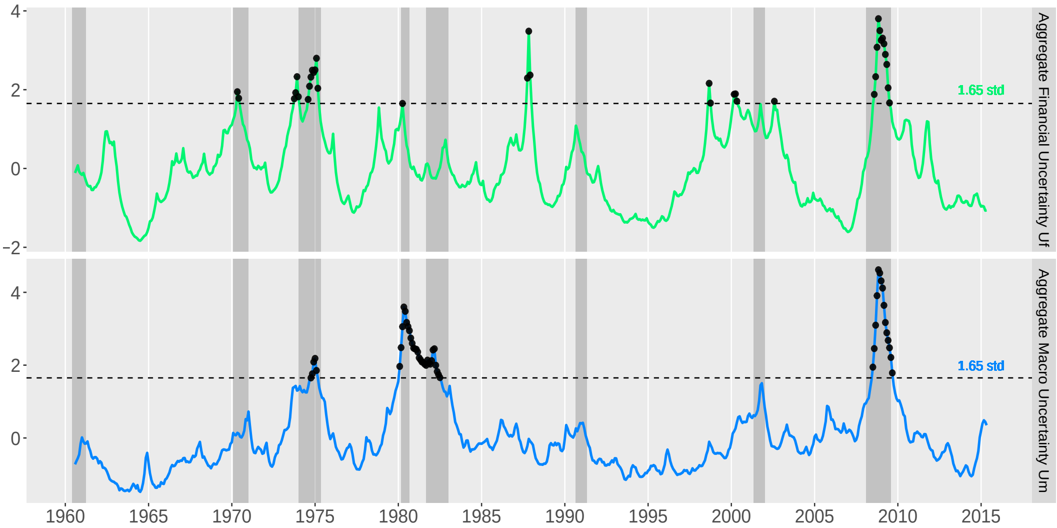

Figure 2.: Macro and Financial Uncertainty Over Time. *Note:* As mentioned in Ludvigson et al. (2019), the plots show the time series of macro uncertainty U_m and financial uncertainty U_f expressed in standardized units. The dashed horizontal lines amount to 1.65 standard deviations above the unconditional mean of each series (here: normalized to zero). Data are monthly and cover the period 1960:7-2015:04. Grey shaded areas denote NBER recession dates in the US. The black dots denote months where the respective uncertainty measure was $>=$ 1.65 standard deviations above the mean.

The econometric approach then consists of three parts:

i estimation of the forecastable component (conditional expectation) $\mathbb{E}[y_{jt+h}|I_t]$: this is achieved by, first, running a factor analysis on a large set of predictors (134 macro and 148 financial time series, respectively) $\{X_{it}\}, i = 1, 2, \ldots, N$ whose linear span/hull comes as close as possible to I_t; second, making use of the formed factors, $\mathbb{E}[y_{jt+h}|I_t]$ is then approximated by means of a diffusion index forecasting model ideal for data-rich environments (the diffusion indices can then be treated as known in the subsequent steps)

ii with the definition of the h-step-ahead forecast error as $V^y_{jt+h} \equiv y_{jt+h} - \mathbb{E}[y_{jt+h}|I_t]$, Jurado et al. (2015) estimate the conditional volatility of this forecast error, i.e., $\mathbb{E}[(y_{jt+h} - \mathbb{E}[y_{jt+h}|I_t])^2|I_t]$ at time t by specifying a parametric stochastic volatility model for both the one-step-ahead prediction errors in y_{jt} and the analogous forecast errors for the factors (from step i above);
these volatility estimates are then used to recursively compute the values of $\mathbb{E}[(V^y_{t+h}|I_t]$ for $h > 1$.

iii in the last step, the 'macroeconomic uncertainty index' $U^y_t(h)$ is constructed from the individual uncertainty measures $U^y_{jt}(h)$ where the base-case is the

Table 4.: Periods of high uncertainty according to financial uncertainty index. *Note:* Periods of elevated uncertainty are calculated as months where the value of the respective uncertainty time series is above 1.65 standard deviations of the unconditional mean.

Start	End	Duration	Maximum Year/Month	Maximum Value
Apr 1970	May 1970	1 months	Apr 1970	1.23
Sep 1973	Dec 1973	4 months	Nov 1973	1.30
Jul 1974	Feb 1975	7 months	Jan 1975	1.38
Mar 1980	Mar 1980	1 months	Mar 1980	1.18
Sep 1987	Nov 1987	2 months	Oct 1987	1.49
Aug 1998	Sep 1998	2 months	Aug 1998	1.27
Feb 2000	Apr 2000	3 months	Mar 2000	1.22
Jul 2002	Jul 2002	1 months	Jul 2002	1.19
Jul 2008	Jun 2009	12 months	Oct 2008	1.55

equally-weighted average of individual uncertainties

The above approach is applied to 134 mostly macroeconomic time series (taken from McCracken and Ng, 2016) for U_{Mt} and to an updated monthly version of 148 financial time series for U_{Ft}. Both datasets have been previously used in Ludvigson and Ng (2007) and Jurado et al. (2015). The 134 macro series cover a broad set of macroeconomic time series where the majority are real activity measures like real output and income, employment and hours, etc including a few bond and stock market indices and foreign exchange measures. The 148 purely financial time series include data from the bond market, stock market portfolio returns and commodity markets. Further details about the exact time-series that are used for the algorithm can be found in Ludvigson et al. (2019).

Their resulting estimated measures of time-varying macro- und financial uncertainty are plotted in Figure 2. As explained in Ludvigson et al. (2019), the macro financial index U_{Mt} reveals 'only' three big episodes of uncertainty: the months during the recessions from 1973-1974 and 1981-1982 and the months during the Great Recession of 2007-2009 (being the most striking episode of heightened uncertainty, followed by the 1981-1982 recession as a close second).[112,113] In comparison to U_{Mt}, the measure for U_{Ft} shows spikes in times where also U_{Mt} is high but overall is more volatile and more frequently shows peaks outside of recessions (the strongest spike being

[112]See Table 3 for the exact dates including the maximum value for the index for each identified episode of high uncertainty.

[113]Jurado et al. (2015) report that a closer look at the estimates reveals that the three series with the highest uncertainty are a producer price index for intermediate materials, a commodity spot price index, and employment in mining between 1973:11 and 1975:3, the Fed funds rate, employment in mining, and the three months commercial paper rate for the 1980:1 and 1982:11 episode and the monetary base, non-borrowed reserves and total reserves between 2007:12 and 2009:6. Jurado et al. (2015) conclude these results to be consistent with the historical account.

in October 1987). The exact dates of elevated uncertainty including the maximum value for the index for each identified episode of high uncertainty are reported in Table 4 where the most extreme values are reported in 1987 and 2008.

As shown in Table 2 in Section 2.4, the two measures are countercyclical with respect to business cycles (i.e. ip_t) with U_{Mt} showing a stronger counter-cyclicality. Overall, as Ludvigson et al. (2019) note, the two uncertainty measures show co-movements but also independent variation (with a correlation coefficient of 0.58).

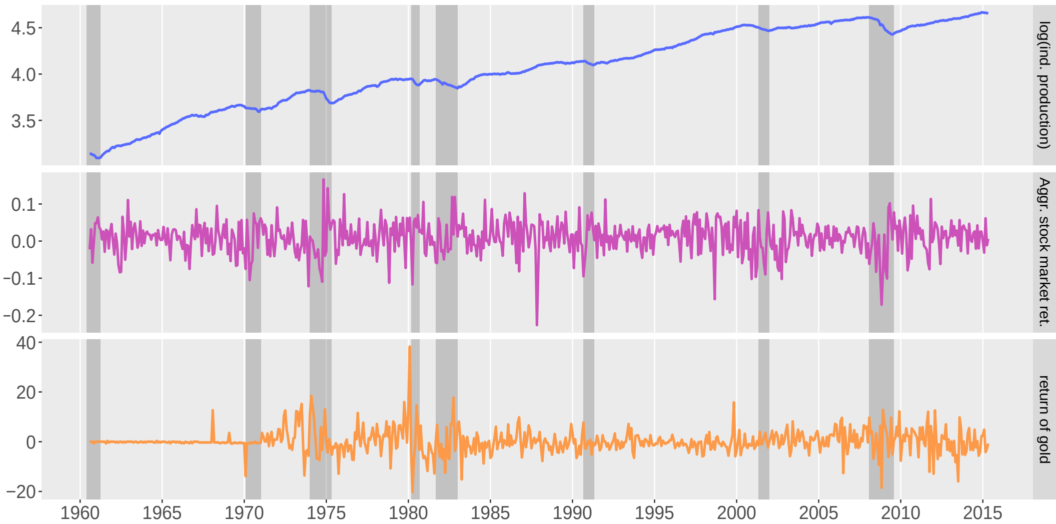

Figure 3.: log(ip), S_1 and S_2 over time. *Note:* The plots show the time series of the log(.) of industrial production ip, the aggregate stock-market return S_1 and the return of the price of gold S_2. Data are monthly and span the period 1960:7-2015:04. Grey shaded areas denote NBER recession dates in the US.

For sake of completeness, Figure 3 shows data other than the two uncertainty measures discussed above which are also part of the estimations. These are the $log(.)$ of the levels of industrial production ip, a variable capturing the aggregate stock-market return (S_1) and a variable capturing the return for the price of gold (S_2) (implemented via the log-difference). The data sources are described in Appendix B.1. S_1 shows higher volatility during most of the recession periods but also significant spikes outside of recessions (most notably the stock-market crash in 10:1987). The series for the real price of gold clearly shows the Bretton-Woods era up until the beginning of the 1970s, after which the series experiences unprecedented volatility in the period 1972-1983 with extreme spikes around 1980. Between 1985 and 2005 the time series shows a distinctive moderation with a visual peak around the turn of the centuries, only to increase again in volatility in the wake of the financial crisis as of

2007.

Stationarity-tests (we used the (augmented) Dickey-Fuller test) indicate that, as expected, the time series for U_m, U_f, $S1$, $S2$ are to be considered as stationary, while the series $log(ip)$ seems to be an $I(1)$-process (i.e. integrated of order 1). With respect to pre-testing for unit roots or co-integration, Kilian and Lütkepohl (2017) (i) point out that these tests have their limitations (as statistical tests generally have) and that (ii) unit root tests have been shown to not being valid in cases of a so-called dominant 'local-to-unity' root (i.e. time-varying unit roots) as shown by, among others, Elliott and Stock (1994) and Cavanagh et al. (1995). In practice, even under the existence of unit roots or cointegration in the data, the result of Stock et al. (1990) is leveraged which shows that "[...] for higher-order autoregressive models in some cases standard methods of inference remain asymptotically valid even in the possible presence of unit roots and cointegration" (Kilian and Lütkepohl, 2017, p. 107). Stock et al. (1990) themselves write that their results show that "[...] the common practice of attempting to transform models to stationary form by difference or cointegration operators whenever it appears likely that the data are integrated is in many cases unnecessary." Hence, we also refrain from differencing $log(ip)$ as in the original contribution of Ludvigson et al. (2019) where ip enters the SVAR in log-levels.

5. Results

The complete set of analyses and computations/algorithms for the generation of the results presented in Section 5 were conducted in R and re-perform a selection of the analyses that Ludvigson et al. (2019) conducted in their own publication.[114] Section 5.1 starts with a preliminary analyses of the behavior of the shocks that are left in the identified set (i.e., passed all shock-restrictions) both for the entire set and, particularly, for the *maxG*-solution and Section 5.2 displays and discusses the impulse response analysis including a closer look at the impact matrices that reveal the information content of various identifying restrictions (i.e., sets of constraints). Section 5.3 shows that the results suggest that recursive schemes (which are not excluded a priori by the inequality constraints) are incompatible with the recovered estimations and, lastly, Section 5.4 builds on the IFR-analysis by reporting the results for the forecast error variance decomposition computed for the solutions in the identified set.

5.1. Analysis of Recovered Shocks

Before jumping to the resulting impulse responses of the baseline system $\mathbf{y}_t = (U_{Mt}, IPM_t, U_{Ft})$ in Section 5.2, Section 5.1 discusses the SVAR's recovered shocks which are rarely analysed although they are a central objective of SVAR analysis (as formulated by Ludvigson et al., 2020*b*) and show a few interesting features.

Figure 4, showing the time series of the recovered shocks for the *maxG*-solution, and similar to the results in Ludvigson et al. (2019), shows that the extracted shocks are not compatible with a normal distribution (Ludvigson et al., 2019). As outlined in Kilian and Lütkepohl (2017), there are many tests for nonnormality of the errors while tests on the basis of the third and fourth moments are the most popular. Here,

[114]The entire R-code is available upon request.

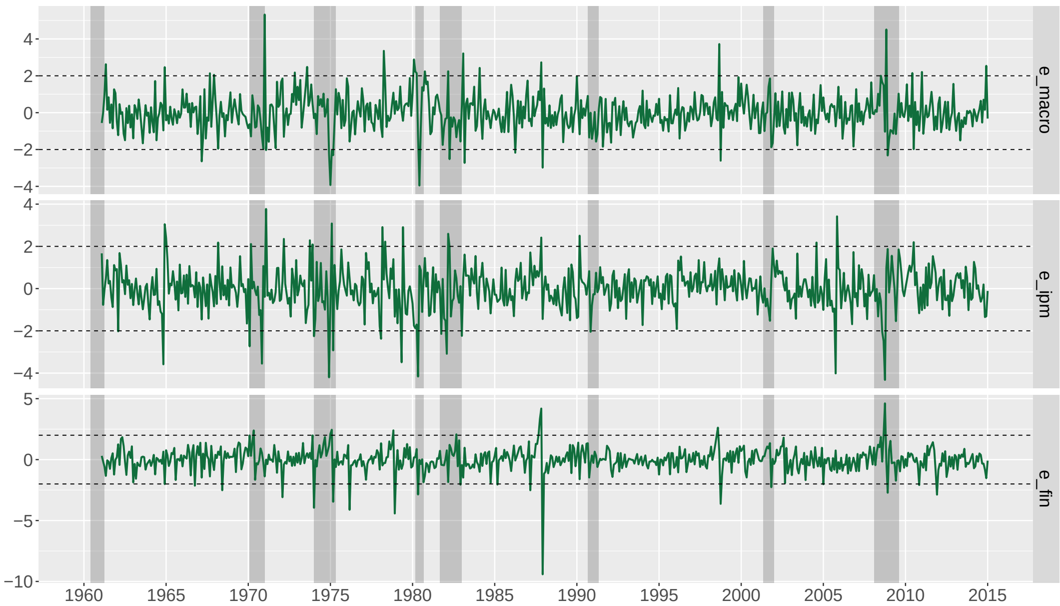

Figure 4.: Standardized Structural Shocks from SVAR (U_M, ip, U_F).
Note: Following Ludvigson et al. (2018), the time series show the maxG solution for the shocks $\boldsymbol{\epsilon}_t = \mathbf{A}_0\mathbf{u}_t$ with $\mathbf{u}_t$ being the residuals derived from the estimated VAR(6) of (U_M, ip, U_F). The horizontal lines amount to 3 standard deviations above and below the unconditional mean of the time series, respectively. The selected bounds are $\overline{k}_1 = 4.16$, $\overline{k}_2 = 4.58$, $\overline{k}_3 = 4.74$, $\overline{k}_4 = 4.05$ and $\overline{k}_5 = 2$. Grey shaded areas denote NBER recession dates in the US.

we have performed the Jarque-Bera Test which, for all three shock series e_{lip}^{maxG}, e_M^{maxG} and e_F^{maxG}, rejects the null hypothesis of the samples having skewness and kurtosis that would match the one of a normal distribution.

For e_{lip}^{maxG} the largest positive shocks are recorded in 1971:01 and 2005:10, while the largest negative shocks appear in 2008:09 and 1974:11. As mentioned by Ludvigson et al. (2019), also our results show a moderation in the volatility of shocks for the post-1983-period with an exception being the stock-market crash in 10:1987 which can be seen for e_F^{maxG}. The two largest positive shocks of e_M^{maxG} are in 1970:12 (around the collapse of the Bretton-Woods system) and 2008:10, while for e_F they occur in 2008:09 and 1987:10. Identical to Ludvigson et al. (2019), also our time series for e_F^{maxG} shows the 1987 stock market crash with a very large positive spike in 1987:10, followed by an even larger downward-spike showing the market's quick recovery. Lastly, for Ludvigson et al. (2019), it is notable that several spikes for both of the two types of uncertainty do not coincide with elevated values for e_{lip}^{maxG} and vice versa (most notably the large spike for e_{lip}^{maxG} in 2005:10 which is also visible in our results).

Because we are dealing with set-identified SVARs (to which the estimated SVAR-system belongs to), the recovered residuals of the *maxG*-solution are *one* representative of possible residuals (out of the pool of admissible solutions). Its features (i.e. non-normality), however, can certainly be extended to all possible residuals in the identified set. And while the distributional features of the recovered residuals would be an aspect to consider with respect to inference in SVAR-systems, due to the explanations in Section 3.2.4 and the fact that we are not dealing with the general case of point-identification, they are not further elaborated on.

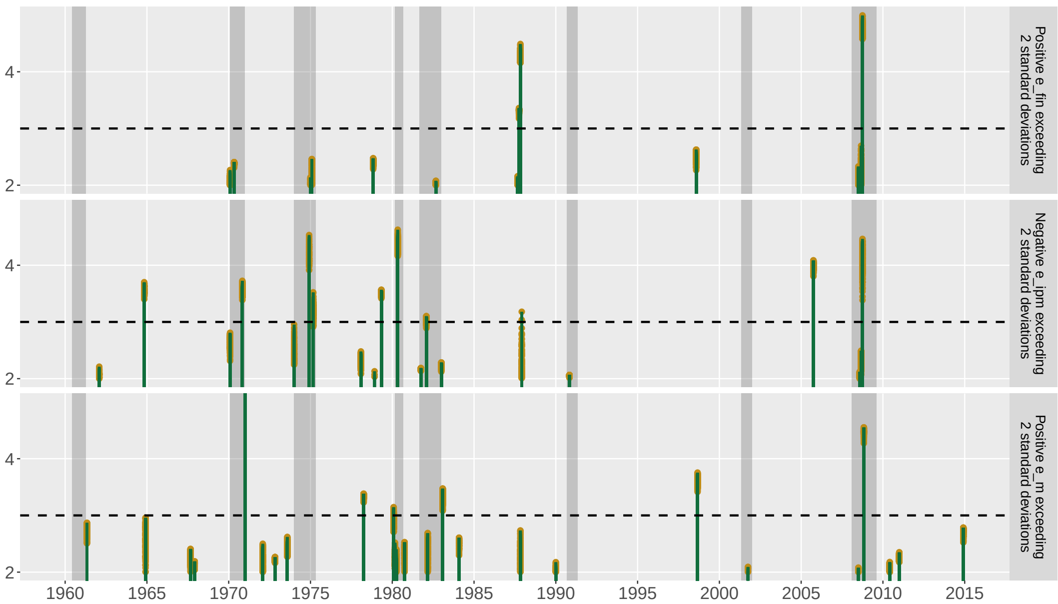

Figure 5.: Large Shocks derived from SVAR (U_M, ip, U_F).
Note: Following Ludvigson et al. (2018), the figure shows all shocks in the identified set that are at least 2 standard deviations above the unconditional mean for the shocks ϵ_M and ϵ_F and at least 2 standard deviations below the mean for ϵ_{ip}. For the middle pane (ϵ_{ipm}) the sign of the shocks were flipped (so that negative shocks exceeding 2 standard deviations also point upwards). The horizontal line amounts to 3 standard deviations. The selected bounds are $\overline{k}_1 = 4.16$, $\overline{k}_2 = 4.58$, $\overline{k}_3 = 4.74$, $\overline{k}_4 = 4.05$ and $\overline{k}_5 = 2$. The sample period is 1960:07-2015:04. Grey shaded areas denote NBER recession dates in the US.

Switching to large "adverse" shocks (large positive uncertainty shocks and large negative real activity shocks recovered by the SVAR-model) and as analyzed by Ludvigson et al. (2019), Figure 5 shows the date and size of the model's recovered shocks to e_M and e_F that are at least two standard deviations above the mean and negative shocks to e_{lip} that exceed two standard deviations and produces very similar results to the one's in Ludvigson et al. (2019). Contrary to Figure 4 that only showed the time series for the maxG-solution, Figure 5 shows the resulting large shocks for all solutions that ended up in the identified set. As pointed out be Ludvigson et al.

(2019), the top panel which shows large financial uncertainty shocks clearly exhibits the largest spikes in 1987 and 2008 (which is demanded by the event constraints). An inspection of large negative shocks for e_{lip} shows that the majority of negative real activity shocks correspond with recessions or are associated with either financial or macro uncertainty. Consequently also e_{lip} shows big shocks in October 2008. The negative real activity shock in 2005 slightly stands out but according to Ludvigson et al. (2019) could be interpreted as a looming sign of the Great Recession. In their analysis of the 10 housing series that constitute the 134 macro time series, Ludvigson et al. (2019) report an onsetting sharp decline as of September 2005. Lastly, the bottom panel of Figure 5 shows that the uncertainty shocks for e_M are a bit more scattered and either coincide or appear with a slight delay after big shocks to real activity or financial uncertainty. The picture for e_M also confirms the period of the Great Moderation as of 1983. Interestingly, around/before the turn of the millennium the spikes for e_M stick out.

Overall, the analysis of the model's residuals confirms the identification scheme to having selected reasonable solutions. And while the solutions clearly adhere to e.g. the event constraints for the 1987 crash or the 2007-09 financial crisis, other large shocks are not ruled which can clearly be seen in Figure 5 where also for e_M and e_{lip}, many large adverse shocks fall into these periods.

5.2. Structural Impulse Response Analysis

Having run the algorithm as described in Section 3.2.3, Figure 7 shows the set of trajectories of the impulse response functions produced by the impact matrices $\mathbf{A}_0$ that passed the complete set of constraints. Hence, the set of trajectories present the solutions in the identified set that shows the response of each variable to a 1 standard deviation increase for each of the respective structural shocks and how they are transmitted through the SVAR-system. Overall, the combination of the full set of event and correlation constraints eliminate more than 99% of all possible resulting $\mathbf{A}_0$-matrices. In particular, from 1.5 Mio rotations of the $\mathbf{P}$ matrix by random orthogonal matrices $\mathbf{Q}$, in our estimations 177 solutions ended up in the identified set.[115] Following Ludvigson et al. (2019), the impulse responses are computed for 60

[115] Note that (although in the context of set-identified sign-restricted SVARs which can be equally extended to shock-restricted SVARs), Kilian and Lütkepohl (2017) make the case that trying to interpret the relative number of solutions that passed the restrictions should not be regarded as an indication about the quality of the imposed restrictions.

steps ahead. With respect to the combination of labels in Figure 7 (and Figure 6), the top label always stands for the respective shock, while the second label at the bottom stands for the response of the respective variable *to* the respective shock. Hence, the IRF in the second row on the left shows the reaction of macro uncertainty to a shock in e_{lip}. The thick dashed line represents the maxG-solution which was defined in Section 3.2.2. Our resulting trajectories match the ones' of Ludvigson et al. (2019) pretty closely. In addition, we want to point out that the shown results are not to be seen as an inferential analysis (in a frequentist perspective) due to the reasons mentioned in Section 3.2.4.

For sake of comparison and to get a better understanding of how the variation of alternative combinations of the constraints work together (Ludvigson et al. (2019) call this "identification uncertainty"; see Section 3.2.3), we first refer to Figure 6. As formulated by Ludvigson et al. (2019), Figure 6 shows that the covariance restrictions alone (i.e. the case for 'No Constraints') without event and correlation-constraints produce ambiguous results (blue shaded areas) which is due to the problem of under-identification which was discussed in Section 3.1.3.[116] The addition of the correlation constraints (external variable constraints; i.e. the case for 'FC Only') gives rise to the salmon-pink IRFs while the cumulative addition of the set of "non-negative" together with the "real activity" event constraints (i.e. the case for 'FC + Non-Negative + RA FE') are the green trajectories. At this stage the big shock events have not yet been introduced. Similar to the analyses in Ludvigson et al. (2019), the identified sets for the 'FC Only' and 'FC + Non-Negative + RA FE' - cases are still largely inconclusive, apart from one observation: as noted by Ludvigson et al. (2019) the plot in the second row, first column (for the effects of a production shock on macro uncertainty) already shows that even without the big shock event constraints, macro uncertainty falls in response to a positive shock to industrial production which, when the argument is flipped, means that a *negative ip*-shock *increases* macro uncertainty.

Adding the big shock event constraints to the picture, Figure 7 results in more conclusive evidence with respect to the dynamics in the SVAR-system. Our impulse response functions exhibit the same features as the one's reported in Ludvigson et al. (2019): Starting with the bottom row, the results suggest that positive shocks to financial uncertainty e_F lead to a lasting decline in real activity (second column),

[116] Note that for the production of Figure 6 we did not run the algorithm for 1.5 rotations for each of the three scenarios ('No Constraints', 'FC Only' and 'FC + Non-Negative + RA FE') otherwise the resulting picture would be too cluttered. Without loss of information content, even for a considerably lower amount of rotations the results are indicative of the underlying behaviour of the restrictions.

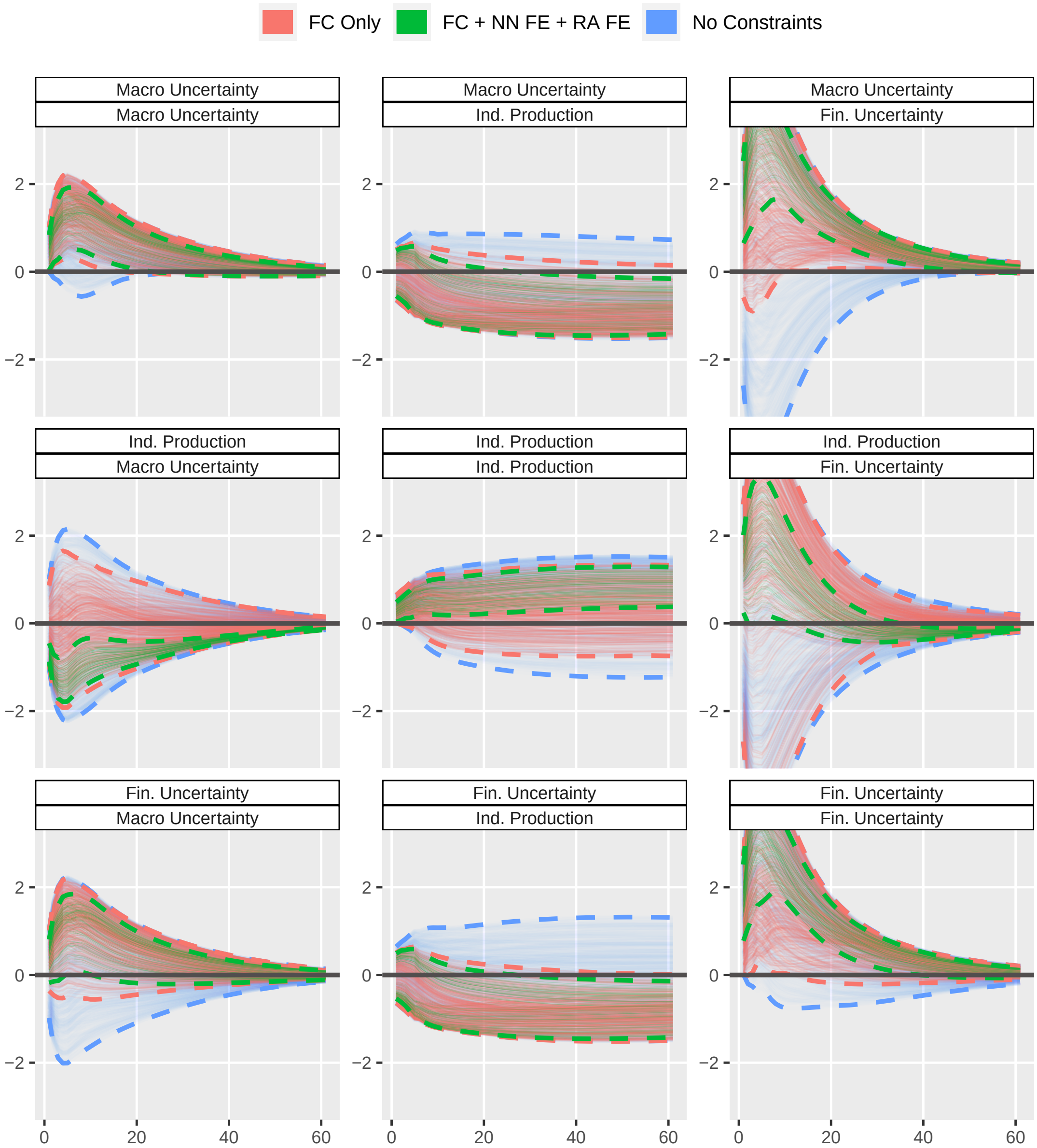

Figure 6.: Impulse Respones from SVAR (U_M, ip, U_F) for various combinations of constraints.
Note: The set of solutions under 'No Constraints' (500 rotations) are derived from switching off all constraints following Ludvigson et al. (2019). 'FC only' correspondingly shows the set of solutions when only the correlation constraints (external variables constraints) are turned on (3.000 rotations) while under the regime 'FC + NN FE + RA FE' the "non-negative event" and "real activity" - restrictions are added (1000 rotations) to the case of 'FC only' (see Section 3.2.2 for the labelling of the respective constraints). The IRFs under the full set of constraints (i.e. FC + all FE (non-negative constraints + big-shock-constraints) is shown in Figure 7. The selected bounds are $\overline{k}_1 = 4.16$, $\overline{k}_2 = 4.58$, $\overline{k}_3 = 4.74$, $\overline{k}_4 = 4.05$ and $\overline{k}_5 = 2$. With respect to the labeling, in each panel the bottom label stands for the response of the respective variable to the shock in the top label (the impulse). The calculations are performed as outlined in Section 3.2.3. The sample spans the period 1960:07-2015:04.

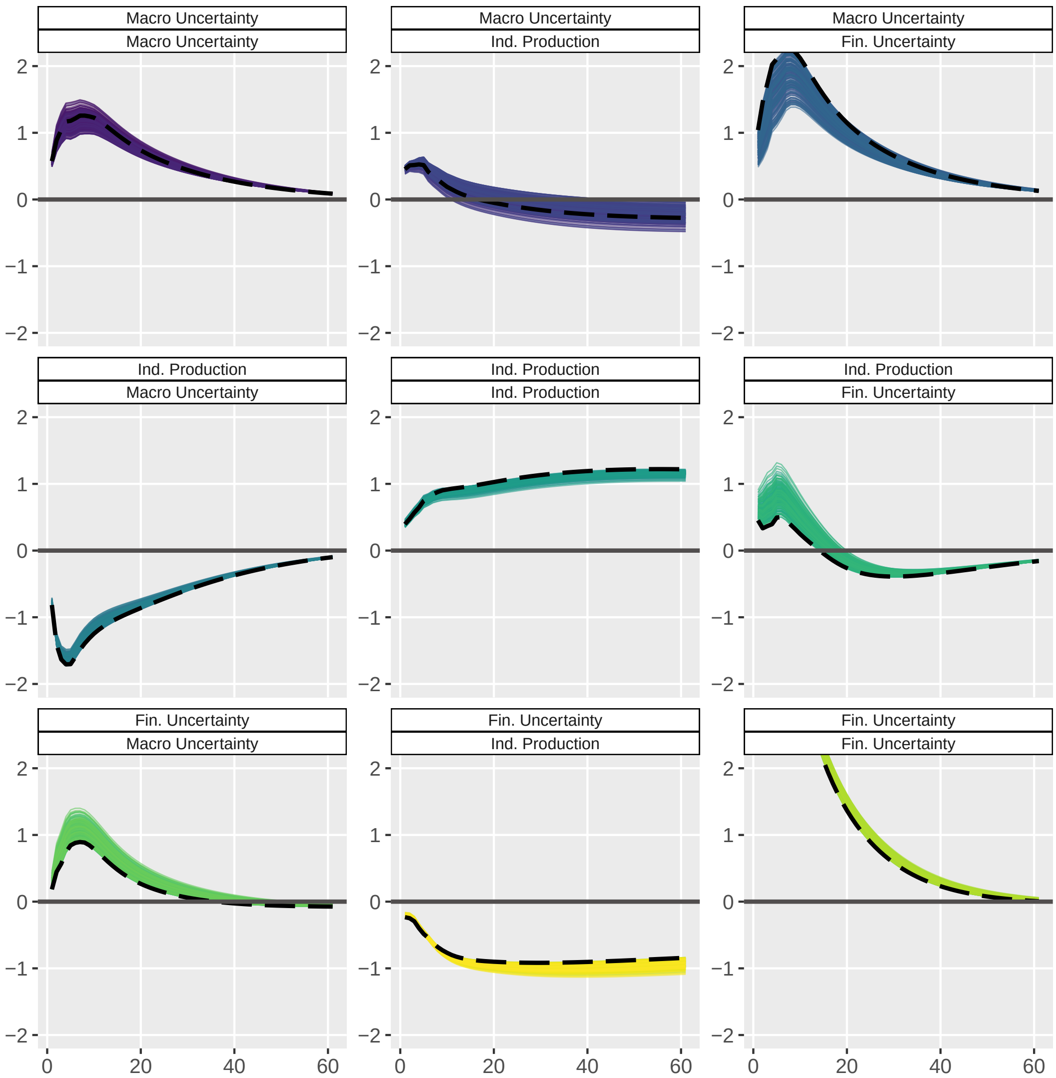

Figure 7.: Impulse Responses from SVAR (U_M, ip, U_F) for full set of constraints.
Note: The thick dashed lines are the maxG solutions. The shaded areas represent the set of solutions that satisfy the full set of constraints, consisting of correlation and event constraints. Of 1.5 Mio rotations, 177 solutions ended up in the identified solution set. The selected bounds are $\overline{k}_1 = 4.16$, $\overline{k}_2 = 4.58$, $\overline{k}_3 = 4.74$, $\overline{k}_4 = 4.05$ and $\overline{k}_5 = 2$. With respect to the labeling, in each panel the bottom label stands for the response of the respective variable to the shock in the top label (the impulse). The calculations are performed as outlined in Section 3.2.3. The sample spans the period 1960:07-2015:04.

while they also cause a sharp increase in U_{Mt} (first column). For the former case (reaction of industrial production to a e_F-shock), the solution-set is "[...] bounded well away from zero as the horizon increases" (Ludvigson et al., 2019, p. 22). With these results, the authors find support for the hypothesis of financial uncertainty being an exogenous trigger leading to a decline in real activity. Contrary to this conclusion, the model gives little support for the impulse going vice versa, i.e., that negative shocks to the real economy induce heightened and protracted financial uncertainty. Instead, the panel in the second row, third column shows an initial increase of financial uncertainty to a *positive* production-shock. Put differently, a negative production shock *decreases* financial uncertainty at least initially (Ludvigson et al., 2019) which is clearly an 'interesting' result.[117]

Looking at macro uncertainty and identical to the results in Ludvigson et al. (2019), positive shocks to real activity induce a sharp decline in macro uncertainty U_{Mt}. Stated differently, a *negative* shock to industrial production would induce a pronounced and protracted *increase* in macro uncertainty as confirmed by the entirety of solutions in the identified set.

Lastly, interestingly, the SVAR-system (at least for the majority of the solution set as well as the maxG-solution) results in an *increase* in production in response to a macro uncertainty shock e_M which Ludvigson et al. (2019) see to be consistent with the growth options theory (first row, second column). For some solutions this effect even persists for the long run, while for others the positive effect at impact turns into a negative effect for longer horizons. Independently of the duration of this effect, the results do not support the empirical evidence of many other empirical applications that point at shocks to macro uncertainty reducing real activity. Instead, as the discussion in the previous paragraph showed, the effect seems to go from real activity shocks to an increase in macro uncertainty and hence weakens the often stated causal nexus between macro uncertainty being a compounding factor in recessions (Ludvigson et al., 2019).

In summary, the model and subsequent results of Ludvigson et al. (2019) suggests that higher macroeconomic uncertainty is an endogenous response to shocks to aggregate output while *financial uncertainty* can be seen as a likely source for protracted declines in aggregate output.[118,119]

[117] As outlined in Kilian and Lütkepohl (2017, p. 112), "[a] direct implication of the linearity of [a] VAR model is that responses to negative shocks are the mirror image of responses to positive shocks." Hence, in the impulse-response analysis above the resulting IRF to a *positive* shock can be flipped in favor of a *negative* shock.

[118] Ludvigson et al. (2018) also run their algorithms for a pre-crisis sample for which they report less conclusive answers. In their conclusion this result shows that the 2007-2009 financial crisis was an "[...] important rare event that can help distinguish the transmission of financial versus real uncertainty shocks" Ludvigson et al. (2018, p. 25). We have not repeated this exercise here.

[119] Here, we also want to point out a potential caveat as outlined by Kilian and Lütkepohl (2017): the analysed impulse response functions show the traditional case of the system's dynamic reaction to one-time (positive/negative) shocks. For the business cycle literature, Kilian and Lütkepohl (2017, p. 120) advocate to also look at historical decompositions which allow to better judge cumulative

As analysed by Ludvigson et al. (2019), the opposite effects of macro and financial uncertainty on industrial production at impact can be read off the off-diagonal elements of the estimated $\mathbf{A}_0^{-1}$-matrices that are part of the identified set. In particular, the elements $A_{0_{YM}}^{-1}$ (which is the response of *ip* to a macro-uncertainty shock) and $A_{0_{YF}}^{-1}$ (which consequently is the response of *ip* to a financial uncertainty shock) drive these responses. In particular, as seen in Figure 7, for all solutions it holds that $A_{0_{YM}}^{-1} > 0$ and $A_{0_{YF}}^{-1} < 0$ as mentioned by Ludvigson et al. (2019).

To better understand the effects of various restrictions and following Ludvigson et al. (2019), Figures 8 and 9 show the distribution of $A_{0_{YM}}^{-1}$ and $A_{0_{YF}}^{-1}$ for various identifying restrictions.

A closer look at Figure 8 shows that under 'No Constraints' (which includes the covariance constraints) the distribution of $A_{0_{YM}}^{-1}$ and $A_{0_{YF}}^{-1}$ include positive and negative values and is hence inconclusive. Cumulatively adding the full set of event constraints, the solutions for $A_{0_{YM}}^{-1}$ are all positive and for $A_{0_{YF}}^{-1}$ all negative, indicating that the event constraints alone are enough to derive the sign of the result at impact. As noted by Ludvigson et al. (2019), adding the correlation constraints (external variables constraints) to arrive at the full set of constraints only slightly changes their magnitudes. In contrast to the results of Ludvigson et al. (2019), however, in our case the distribution of $A_{0_{YM}}^{-1}$ under the full set of event constraints is well away from zero while the results of Ludvigson et al. (2019) shows that there are also solutions in the identified set that get very close to zero.

Figure 9 contrasts 'No constraints' with 'All constraints' and 'All constraints *without* the Lehman event' (i.e. FE_2 as described in Section 3.2.2) tests whether the Lehman event plays a crucial role in the demarcation of the effects for $A_{0_{YM}}^{-1}$ and $A_{0_{YF}}^{-1}$. Interestingly, we do not find any evidence that removing the Lehman event produces any inconclusive results for both factors and almost give the same results as under the 'All constraints' - setting. This is in contrast to the results of Ludvigson et al. (2019) that report that removing the Lehman event while keeping all other restrictions does not allow to derive a conclusive answer of the response of industrial production to a financial uncertainty shock (i.e., $A_{0_{YF}}^{-1}$) since the distribution of values contains both positive and negative values. Based on their own results, Ludvigson et al. (2019) conclude that the Lehman bankruptcy and the 2007-09 financial crisis is a decisive

effects of shocks, arguing that real output's variation "[...] is driven by a sequence of shocks of different magnitude and signs" (not shown here).

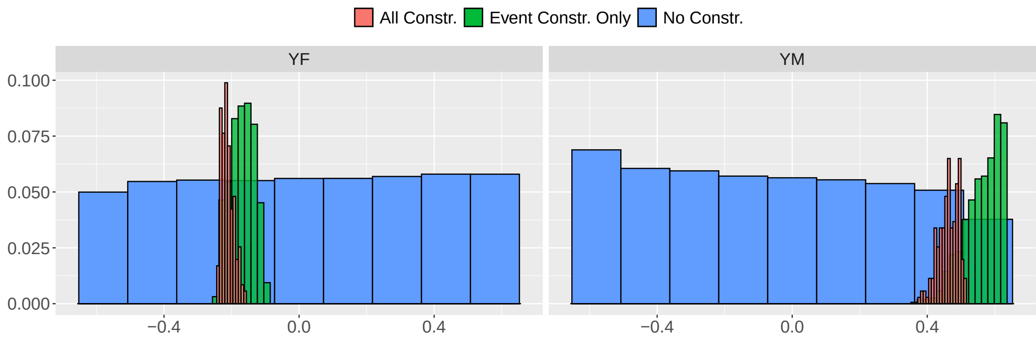

Figure 8.: Distribution of $A^{-1}_{0_{YM}}$ and $A^{-1}_{0_{YF}}$ for various combinations of constraints (Version 1).
Note: Histograms for all values of $A^{-1}_{0_{YM}}$ (left panel) and $A^{-1}_{0_{YF}}$ (right panel) in an identified set under the following different restriction-regimes: 'No Constraints' (i.e. only covariance restrictions), 'Event Constraints Only' and 'All Constraints' (i.e. the full set of event- and correlation constraints simultaneously). $\overline{k}_1 = 4.16$, $\overline{k}_2 = 4.57$, $\overline{k}_3 = 4.73$, $\overline{k}_4 = 4.05$ and $\overline{k}_5 = 2$. Number of rotations: 1.5 Mio ('All Constraints'), 1 Mio ('Event Constraints Only'), 100k ('No Constraints'). The sample spans the period 1960:07-2015:04.

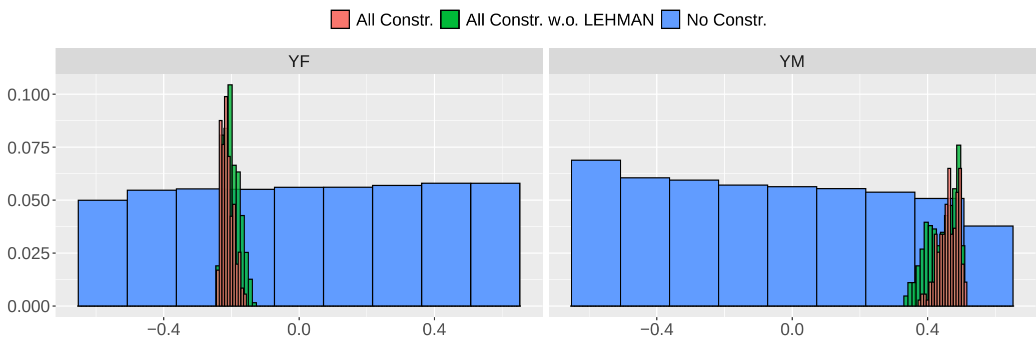

Figure 9.: Distribution of $A^{-1}_{0_{YM}}$ and $A^{-1}_{0_{YF}}$ for various combinations of constraints (Version 2).
Note: Histograms for all values of $A^{-1}_{0_{YM}}$ (left panel) and $A^{-1}_{0_{YF}}$ (right panel) in an identified set under the following different restriction-regimes: 'No Constraints' (i.e. only covariance restrictions), 'All Constraints **without Lehman Event**' and 'All Constraints' (i.e. the full set of event- and correlation constraints simultaneously). $\overline{k}_1 = 4.16$, $\overline{k}_2 = 4.57$, $\overline{k}_3 = 4.73$, $\overline{k}_4 = 4.05$ and $\overline{k}_5 = 2$. Number of rotations: 1.5 Mio ('All Constraints'), 1 Mio ('All Constraints w.o. Lehman'), 100k ('No Constraints').The sample spans the period 1960:07-2015:04.

event for the distinction of financial and macro uncertainty shocks. However, we cannot confirm this based on our results.

5.3. Invalidity of Recursive Identification Schemes

Ludvigson et al. (2019) argue that recursive schemes in VAR-settings are ill-equipped to properly study the dynamic effects. Accordingly, their suggested and estimated model from Section 3.2.2 can be tested to see whether it supports a recursive structure because a priori it does not rule out such a structure.

As outlined by Ludvigson et al. (2019), in the considered three-variable SVAR-system $(U_{Macro}, IPM, U_{Fin})$ six possible recursive orderings are possible which translates into three elements of $\mathbf{A}_0^{-1}$ having to be jointly zero for each of the recursive orderings. Following Ludvigson et al. (2019), the analysis of the distribution of the elements $A_{0_{YM}}^{-1}$, $A_{0_{YF}}^{-1}$, $A_{0_{MY}}^{-1}$ and $A_{0_{FM}}^{-1}$ of the matrix $\mathbf{A}_0^{-1}$ under the full set of both event and correlation constraints show that none of the distributions contains values close to zero. The results for $A_{0_{YM}}^{-1}$ and $A_{0_{YF}}^{-1}$, respectively, were discussed in the context of the impulse response analysis in Section 5.2 above, while the distributions for $A_{0_{FM}}^{-1}$ and $A_{0_{MY}}^{-1}$ are deferred to Figure 11 in Appendix A.2. The minimum absolute values in the identified set for $A_{0_{YM}}^{-1}$, $A_{0_{YF}}^{-1}$, $A_{0_{FM}}^{-1}$ and $A_{0_{MY}}^{-1}$ are (rounded) 0.004, 0.002, 0.005 and 0.009, respectively, which (as mentioned by Ludvigson et al., 2019) correspond to the smallest (absolute) impact responses in the identified set displayed in Figure 7. Hence, altogether Ludvigson et al. (2019) conclude that the results reject recursive schemes.

5.4. Forecast Error Variance Decomposition

As formulated in Kilian and Lütkepohl (2017), another frequently studied property of (S)VAR models is how much of the forecast error variance (alternatively also called prediction mean squared error; MSPE) of a time series variable is accounted for by each structural shock. Based on the derivations in Section 3.1.2 and based on Ludvigson et al. (2019), Table 5 below reports the computed fraction of the respective h-step-ahead forecast error variances which for each of the system's variable can be attributed to the respective shocks ϵ_{Mt}, ϵ_{ip} and ϵ_{Ft} for the forecast horizons $h = 1$, $h = 12$ and $h = \infty$ as well as h_{max} which denotes the forecast horizon where the respective fraction is maximized[120].

Following Ludvigson et al. (2019), the results in Table 5 report *ranges* for the proportions of the forecast error variances because the computations are performed for every solution in the identified set. Similar to the results of Ludvigson et al. (2019), shocks in real activity have pronounced (negative) effects on macroeconomic uncertainty (top panel) while they exert minimal effects on financial uncertainty (bottom panel). As observed by Ludvigson et al. (2019), shocks to macro uncertainty (middle panel) explain a large proportion of industrial production (and in Section 5.2

[120]Note that the forecast horizon $h = \infty$ technically means $h = 200$, which is the longest forecast horizon we have considered in the computations

Table 5.: Forecast Error Variance Decomposition SVAR (U_{Mt}, IPM_t, U_{Ft}) where U_{Mt}.
Note: Following the presentation of the results in Ludvigson et al. (2019), each panel shows the fraction of the h-step-ahead forecast-error variance of the variable given in the panel title that is explained by the shock named in the column heading. The row denoted "$h = h_{max}$" reports the maximum fraction of forecast error variance explained across all VAR forecast horizons h whereby the index reports the respective forecast horizon for which the FEVD is maximized. ∞ computationally represents the forecast horizon $h = 200$. The numbers in brackets represent the ranges for these numbers across all solutions in the identified set. The calculations are performed as outlined in Section 3.2.3. The sample spans the period 1960:07-2015:04.

Fraction variation in U_m			
h	U_m Shock	ip Shock	U_F Shock
1	$[0.23, 0.49]$	$[0.48, 0.67]$	$[0.03, 0.20]$
2	$[0.19, 0.44]$	$[0.51, 0.68]$	$[0.06, 0.24]$
3	$[0.18, 0.43]$	$[0.50, 0.68]$	$[0.07, 0.26]$
4	$[0.18, 0.43]$	$[0.49, 0.66]$	$[0.08, 0.29]$
5	$[0.17, 0.43]$	$[0.47, 0.64]$	$[0.10, 0.31]$
12	$[0.21, 0.48]$	$[0.38, 0.54]$	$[0.14, 0.38]$
∞	$[0.24, 0.50]$	$[0.38, 0.55]$	$[0.11, 0.33]$
h_{max}	$[0.23, 0.50]_{19}$	$[0.48, 0.67]_{1}$	$[0.14, 0.38]_{13}$

Fraction variation in ip			
h	U_m Shock	ip Shock	U_F Shock
1	$[0.34, 0.62]$	$[0.28, 0.53]$	$[0.06, 0.14]$
2	$[0.33, 0.61]$	$[0.31, 0.55]$	$[0.06, 0.13]$
3	$[0.30, 0.57]$	$[0.33, 0.58]$	$[0.06, 0.13]$
4	$[0.26, 0.53]$	$[0.35, 0.60]$	$[0.08, 0.15]$
5	$[0.22, 0.49]$	$[0.36, 0.61]$	$[0.11, 0.18]$
12	$[0.07, 0.23]$	$[0.39, 0.62]$	$[0.30, 0.38]$
∞	$[0.01, 0.10]$	$[0.47, 0.65]$	$[0.31, 0.51]$
h_{max}	$[0.34, 0.62]_{1}$	$[0.36, 0.61]_{5}$	$[0.31, 0.51]_{201}$

Fraction variation in U_f			
h	U_m Shock	ip Shock	U_F Shock
1	$[0.03, 0.15]$	$[0.03, 0.11]$	$[0.82, 0.91]$
2	$[0.01, 0.09]$	$[0.01, 0.08]$	$[0.85, 0.94]$
3	$[0.01, 0.07]$	$[0.01, 0.07]$	$[0.85, 0.94]$
4	$[0.01, 0.06]$	$[0.01, 0.07]$	$[0.84, 0.94]$
5	$[0.01, 0.05]$	$[0.01, 0.07]$	$[0.83, 0.93]$
12	$[0.01, 0.02]$	$[0.01, 0.06]$	$[0.77, 0.90]$
∞	$[0.00, 0.00]$	$[0.02, 0.06]$	$[0.71, 0.86]$
h_{max}	$[0.03, 0.15]_{1}$	$[0.03, 0.11]_{1}$	$[0.72, 0.86]_{44}$

suggested that they increase real activity in the short run) for smaller forecast horizons, while the impact of industrial production on itself becomes more important at more distant horizons.

Contrary to macro uncertainty or real activity, shocks to financial uncertainty exhibit the largest impact on themselves while they also shock a large negative effect on production for longer horizons. From this observation Ludvigson et al. (2019, p. 28) conclude that U_{Ft} suggest to be “[...] the most important exogenous impulse in the system.”

6. Concluding Remarks

Frequently used VARs with recursive schemes in the uncertainty literature might empirically not be well grounded to capture the underlying data's DGP and the system's dynamics by ruling out simultaneous feedback between the variables. In addition, those models usually do not distinguish *types* of uncertainty and using individual proxies one at a time suggest that uncertainty largely has a depressing effect on business cycles.

Within the class of SVAR models that also allow for contemporaneous relationships, economically grounded assumptions for point identification often seem too restrictive to defend which is why alternative identification schemes that *set-identify* (Grenziera et al., 2018) SVARs seem like a promising approach. Besides sign-restrictions one strand of the literature which is being pushed by Ludvigson et al. (2020*b*), among others, goes beyond classical sign restrictions and concentrates on the model's recovered *shocks* that might reveal valuable information for sensible inequality restrictions to shrink an SVAR's infinitely many solutions to a manageable solution set even without point identification.

While in the presented application of the shock-restricted SVAR-approach of Ludvigson et al. (2019) the authors have formulated both, a priori shock-restrictions and restrictions derived directly from the data, Ludvigson et al. (2020*b*) formulate that writing an SVAR first as a VAR which provides $n(n+1)/2$ restrictions from the covariance structure and subsequent analysis of the unrestricted model's residuals might prove useful to support evidence for the confirmation of historically relevant episodes even *before* any identifying restrictions are imposed.

With respect to the particular application and the central question related to the role of uncertainty in business cycles, the novel approach of Ludvigson et al. (2018, 2019) questions frequently cited results of the uncertainty-literature to date: the results of the baseline 3-variable SVAR(6)-system indicate that, first, the uncertainty literature should distinguish between macroeconomic and financial uncertainty and their different roles in business cycles and, second, that higher macroeconomic uncertainty is an endogenous *response* to shocks to aggregate output while financial uncertainty can be seen as a likely source for protracted declines in aggregate output. While these initial results are certainly a first start, the above conclusions should serve as an indication that empirical applications in the uncertainty literature have to be cautious with overly restrictive modeling assumptions that might result in precipitous conclusions.

From a statistical perspective, Ludvigson et al. (2018, 2019) themselves point at the challenge of proper inference within the class of *set-identified* models in a frequentist view. While they mention approaches that have been suggested in the literature (Grenziera et al. (2018) for sign-restricted SVARs that only impose restrictions on one set of impulse response function using the Bonferroni approach or Gafarov et al. (2015) that collect the reduced-form model in a $1 - \alpha$ Wald ellipsoid which is conservative), they conclude that "[i]t is fair to say that there exists [yet] no generally agreed upon method for conducting inference in set-identified SVARs, let alone one proven to have correct frequentist coverage properties for shock-restricted SVARs of the type considered" Ludvigson et al. (2020*b*, p. 10).[121] As pointed out by Kilian and Lütkepohl (2017), there have also been similar advances for forecast error variance decompositions. When also considering Bayesian approaches, many more alternative solution approaches have been made so far (Kilian and Lütkepohl, 2017).

With regard to the particular *type* it seems generally agreed that uncertainty as a latent and generally unobservable stochastic process seems to be multilayered/manifold. Hence, distinguishing between financial and macroeconomic uncertainty in empirical models seems to be the minimum amount of sophistication and relates to the question of *when which* type of uncertainty plays *which* role throughout the business cycle. One aspect that we haven't touched upon, however, are aspects related to *policy-induced* uncertainty. This relates particularly to questions of how policies can help overcoming a recession and set counter-cyclical measures once an economy has arrived at the bottom. E.g. Bloom et al. (2013, p. 41) and IMF (2012) write that while "[i]t is difficult for policymakers to overcome the intrinsic uncertainty economies typically face over the business cycle", uncertainty about economic policy seems like an exacerbating factor. Hence, we propose to include economic policy

[121] Kilian and Lütkepohl (2017) report the same approaches in the discussion of the class of sign-restricted SVARs (which are also *set identified* like the class of shock-restricted SVARs that have been discussed here).

uncertainty as a third layer of uncertainty into future structural models since it seems to play another role which should be distinctively separated from macroeconomic and financial uncertainty.

In addition to the above, the structural VAR model as suggested by Ludvigson et al. (2019) might suffer from an omitted variables bias. As summarized by Stock and Watson (2001), shocks in a VAR-system are largely made up of factors that are omitted from the model. Due to this, the VAR estimates will be biased ('omitted variable bias') if any omitted factors turn out to be correlated with the variables in the VAR-system and thwart the results derived from IRF and FEVD-analyses and, as formulated by Lütkepohl (2005, p. 62), "[...] make[...] them worthless for structural interpretations". Hence, the simple three-variable SVAR-system as suggested by Ludvigson et al. (2019) and analyzed in this thesis might also still have to stand this test as well.

Lastly, at the time of writing this thesis the world is halting in the wake of the Covid-19 pandemic which from a global health crisis simultaneously turned into (most likely) the most severe economic decline since the Great Depression due to the necessary lockdown measures applied by governments throughout the world. In context of our uncertainty-taxonomy introduced in Section 1 it is difficult to decide for a classification since on the one hand this external shock might be described as the 'highly improbable' in the spirit of Taleb (2008) or 'strong uncertainty' (due to the absence of the ability to form "[...] a unique, additive and reliable probability distribution" of future events (Dequech, 2011, p. 622/623)). On the other hand, there are also opinions arguing that the pandemic was just a trigger for developments in a system where imbalances and uncertainty was already looming. Irrespective of this classification, Ludvigson et al. (2020*a*) speak of a *costly disaster* and argue that while in typical economic models there is some room for "conventional" disasters (Ludvigson et al., 2020*a*, p. 2) which are short-lived and local in nature, the impact of Covid-19 is regarded as a "multi-period shock that simultaneously disrupts supply, demand, and productivity, [and] is almost perfectly synchronized within and across countries, and wherein health, social, and economic consequences are cataclysmic not just for the foreseeable few weeks after the crisis, but potentially for a long time period" Ludvigson et al. (2020*a*, p. 2). Because the likely impact of the pandemic will outshine every costly disaster in the post-war period, we expect that its aftermath will certainly profoundly influence the landscape of uncertainty-modeling in the future.

A. Appendix

A.1. Additional Tables

A.2. Additional Figures

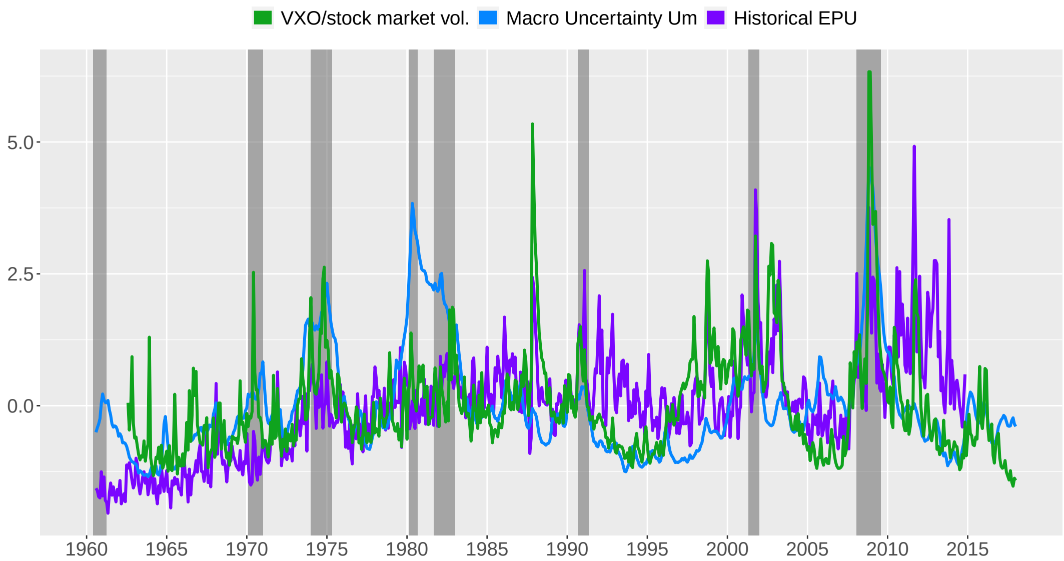

Figure 10.: Comparison of uncertainty measures (facetted). *Note:* Shaded areas denote NBER recession dates. Data frequencies are monthly. The macro uncertainty series starts in July 1960, the EPU in January 1960 and the VXO in July 1962.

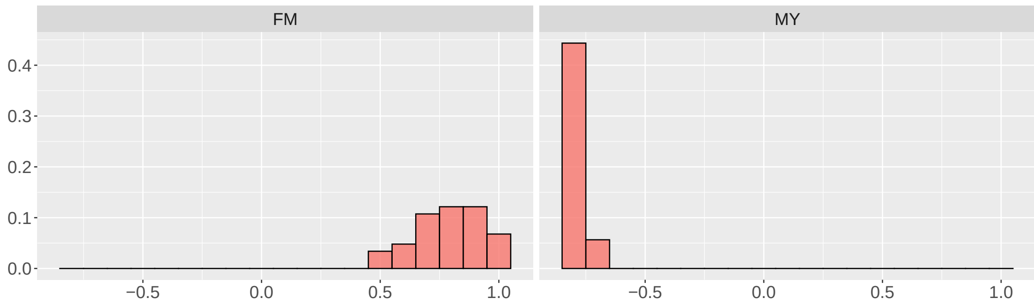

Figure 11.: Distribution of $A^{-1}_{0_{FM}}$ and $A^{-1}_{0_{MY}}$.
Note: Histograms for all values of $A^{-1}_{0_{FM}}$ (left panel) and $A^{-1}_{0_{MY}}$ (right panel) under the full set of event- and correlation constraints. The selected bounds are $\overline{k}_1 = 4.16$, $\overline{k}_2 = 4.58$, $\overline{k}_3 = 4.74$, $\overline{k}_4 = 4.05$ and $\overline{k}_5 = 2$. Plot for 177 solutions out of 1.5 Mio rotations.

B. Appendix

B.1. Data: Sources and Description

Table 6 lists all data and their sources that appear in the main text.

Table 6.: Data Sources.
Note: The data used for the SVAR-system $\mathbf{y}_t = (U_{Mt}, IPM_t, U_{Ft})$ was kindly provided to us by Sai Ma, PhD (one of the co-authors of Ludvigson et al. (2018, 2019)) which included the data for the stock market variable S_{1t} as used by the authors in Ludvigson et al. (2018). Ludvigson et al. (2019) made use of one more external variable S_{2t} which we retrieved from `https://www.macrotrends.net/`. The uncertainty data for U_{Mt} and U_{Ft} is readily available on Sydney Ludvigson's homepage (see `https://www.sydneyludvigson.com/macro-and-financial-uncertainty-indexes`. According to the description in Ludvigson et al. (2018, 2019), respectively, the data for U_{Ft} and U_{Mt} used in their SVAR-estimations corresponds to the respective macro and financial uncertainty data for the forecast horizon $h = 1$. Comparing the most up to date data online to our data used in the estimations we noticed slight discrepancies which we attribute to data revisions. For IPM_t we noticed discrepancies to e.g. the IPM-index available from FRED (see `https://fred.stlouisfed.org/series/INDPRO`), hence we cannot exactly see which IPM data was used. Chicago Board Options Exchange (CBOE).

Variable	Name	Source	Period
IP_t	Industrial Production Index	n.a. (see Ludvigson et al. (2019) for details)	1960M07-
$\text{S\&P500}_t$	S&P's Common Stock Price Index: Composite (Monthly)	YAHOO Finance; see Bloom (2009) for further details	1960M07-
VXO_t	Cboe S&P 100 Volatility Index - VXO	CBOE; see Bloom (2009) for further details	1986M01-
S_{1t}	Center for Research in Securities Prices (CRSP) value-weighted stock-market return of all stocks in NYSQ, AMEX and NASDAQ	CRSP; received from authors; see Ludvigson et al. (2019) for further details	1960M07-
S_{2t}	CPI-deflated gold price	Macrotrends (derived from London Bullion Market Association (LBMA) and the Bureau of Labor Statistics; see Ludvigson et al. (2019) for further details; /purchased/downloaded from `https://www.macrotrends.net/`	1960M07-
U_{Mt}	Macro Uncertainty Index	Jurado et al. (2015); see Ludvigson et al. (2019) for details; received from authors	1960M07-
U_{Ft}	Financial Uncertainty Index	; see Ludvigson et al. (2019) for details; received from authors	1960M07-
EPU_t	Economic Policy Uncertainty Index	Baker et al. (2015); downloaded from `http://www.policyuncertainty.com/`.	1960M07-
EPU Historical_t	Economic Policy Uncertainty Index	Baker et al. (2015); downloaded from `http://www.policyuncertainty.com/`.	1960M07-

B.2. VAR-Equations and Identification Schemes[122]

Monthly VAR(12)-8 following Bloom (2009)

The estimations include 12 lags and use monthly data from 06/1962-06/2008 (which excludes most of the Credit Crunch) and include (in that order; see also top-panel of B.1): log(S&P500 stock market index), an uncertainty measure (either the raw

[122] Our notation 'VAR(p) - v' denotes the number of lags p and the number of variables included v.

stock market volatility series that Bloom constructs through sticking actual realized stock market volatility and the CBOE's VXO-data, available as of 1986, together or various indicator variables like the 'Bloom-shock' or indicators capturing months with maximum volatility, or months capturing the first time that volatility exceeded a certain threshold; for further modifications see Bloom, 2009), Federal Funds Rate, log(average hourly earnings), log(consumer price index), hours worked, log(employment), log(industrial production). Contrary to the description in Bloom (2009), the STATA-code of Bloom shows that all variables except the stock market volatility series as a proxy for uncertainty are HP-detrended. Bloom's results are robust to various alternative approaches including variables ordering, variable inclusion, shock definitions, shock timing, and detrending.

The VARs are estimated via a recursive scheme (i.e, cholesky decomposition) based on the assumption that shocks instantaneously influence the stock market (levels and volatility), then move on to prices (wages, the consumer price index (CPI), and interest rates), and finally eventually affect quantities (hours, employment and output). In particular, Bloom (2009)'s rationale for including the stock-market levels as the first variable in the ordering is to ensure that the impact of stock-market levels is already controlled for when looking at the impact of volatility shocks (see Bloom, 2009, p. 630). Further, Bloom (2009) detrends all variables that enter the VARs using the HP-filter (Hodrick and Prescott, 1997) apart from the uncertainty measure(s).[123]

Jurado et al. (2015) replicate Bloom (2009)'s VARs in their contribution, renounce to detrend the variables however, arguing that the HP filter uses information over the entire sample which makes it difficult to interpret the timing of an observation (hence, their specification is equal to the lower part of B.1) and report results using

[123] In vector B.1 all variables that enter the VAR detrended are marked with a *c*-prefix to indicate that the variable denotes the remaining cycle-component after removal of the trend.

Bloom (2009)'s original detrended variables in their Appendix.

$$\mathbf{y}_t = \begin{bmatrix} y_{1t} \\ y_{2t} \\ y_{3t} \\ y_{4t} \\ y_{5t} \\ y_{6t} \\ y_{7t} \\ y_{8t} \end{bmatrix} = \begin{bmatrix} \text{c-log(S\&P500 Index)} \\ \textit{uncertainty measure} \\ \text{c-federal funds rate} \\ \text{c-log(wages)} \\ \text{c-log(CPI)} \\ \text{c-hours} \\ \text{c-log(employment)} \\ \text{c-log(industrial production)} \end{bmatrix} \tag{B.1}$$

$$\mathbf{y}_t = \begin{bmatrix} y_{1t} \\ y_{2t} \\ y_{3t} \\ y_{4t} \\ y_{5t} \\ y_{6t} \\ y_{7t} \\ y_{8t} \end{bmatrix} = \begin{bmatrix} \text{log(S\&P500 Index)} \\ \textit{uncertainty measure} \\ \text{federal funds rate} \\ \text{log(wages)} \\ \text{log(CPI)} \\ \text{hours} \\ \text{log(employment)} \\ \text{log(industrial production)} \end{bmatrix}$$

Monthly VAR(12)-11 following Jurado et al. (2015) (similar to the macro VAR studied in Christiano et al., 2005)

$$\mathbf{y}_t = \begin{bmatrix} y_{1t} \\ y_{2t} \\ y_{3t} \\ y_{4t} \\ y_{5t} \\ y_{6t} \\ y_{7t} \\ y_{8t} \\ y_{9t} \\ y_{10t} \\ y_{11t} \end{bmatrix} = \begin{bmatrix} \log(\text{IP}_t) \\ \log(\text{EMPM}_t) \\ \log(\text{real consumption}) \\ \log(\text{PCE deflator}_t) \\ \log(\text{NO}_t = \text{NO capital}_t + \text{NO cons}_t) \\ \log(\text{WAGE}_t) \\ \log(\text{HOURS}_t) \\ \text{FFR}_t \\ \log(\text{S\&P500}_t) \\ \text{growth rate of M2}_t \\ \textit{uncertainty (various measures)} \end{bmatrix} \tag{B.2}$$

Quarterly VAR(4)-8 following Gilchrist et al. (2014)

The VARs are estimated via a standard recursive ordering technique over the 1963:Q3-2012:Q3 period using four lags of each endogenous variable. Both identification schemes employ a standard recursive ordering technique.

Identification Scheme I: Innovations in uncertainty are assumed to have an immediate impact on credit spreads and short-term interest rates, but affect economic activity and prices with a lag. Both, shocks to uncertainty and credit spreads (financial shock) are analyzed.

$$\mathbf{y}_t = \begin{bmatrix} y_{1t} \\ y_{2t} \\ y_{3t} \\ y_{4t} \\ y_{5t} \\ y_{6t} \\ y_{7t} \\ y_{8t} \end{bmatrix} = \begin{bmatrix} \text{log(real business fixed investment)} \\ \text{log(real personal consumption expenditure on durable goods)} \\ \text{log(real PCE on nondurable goods and services)} \\ \text{log(real GDP)} \\ \text{log(GDP deflator)} \\ \textit{proxy for uncertainty at aggregate level} \\ \text{10-year BBB-Treasury credit spread} \\ \text{nominal effective federal funds rate} \end{bmatrix} \tag{B.3}$$

Identification Scheme II: Identification Scheme II reverses the causal ordering of the uncertainty proxy and of the variable for the 10-year BBB-Treasury credit spread to analyze the implications of uncertainty shocks conditional on the information contained in the current level of credit spreads.

$$\mathbf{y}_t = \begin{bmatrix} y_{1t} \\ y_{2t} \\ y_{3t} \\ y_{4t} \\ y_{5t} \\ y_{6t} \\ y_{7t} \\ y_{8t} \end{bmatrix} = \begin{bmatrix} \text{log(real business fixed investment)} \\ \text{log(real personal consumption expenditure on durable goods)} \\ \text{log(real PCE on nondurable goods and services)} \\ \text{log(real GDP)} \\ \text{log(GDP deflator)} \\ \text{10-year BBB-Treasury credit spread} \\ \textit{proxy for uncertainty at aggregate level} \\ \text{nominal effective federal funds rate} \end{bmatrix} \tag{B.4}$$

Quarterly VAR(4)-8 following Basu and Bundick (2017)

Basu and Bundick (2017) include the VXO as a measure of uncertainty, gross domestic product (GDP), consumption, investment, hours worked, the GDP deflator, the M2 money stock, and a measure of the monetary policy stance (in that order) over the 1986-2014 sample period including four lags. All variable apart from the monetary policy measure enter the VAR in log levels. Identifying the uncertainty-shock using a Cholesky decomposition with the VXO ordered first, the authors assume that uncertainty shocks can affect output and its components immediately but that other variables' shocks, however, do not influence the VXO on impact.

$$\mathbf{y}_t = \begin{bmatrix} y_{1t} \\ y_{2t} \\ y_{3t} \\ y_{4t} \\ y_{5t} \\ y_{6t} \\ y_{7t} \\ y_{8t} \end{bmatrix} = \begin{bmatrix} \textit{log(uncertainty (VXO))} \\ \text{log(GDP)} \\ \text{log(consumption)} \\ \text{log(investment)} \\ \text{log(jours worked} \\ \text{log(GDP deflator)} \\ \text{log(M2 money stock)} \\ \text{measure of monetary policy stance} \end{bmatrix} \tag{B.5}$$

Monthly VAR(?)-4 following Leduc and Liu (2016)

The four-variable BVAR on US data consists of a measure of uncertainty, the unemployment rate, the CPI year-on-year inflation rate and a short-term interest rate (as represented by the three-month Treasury bills rate). As a measure of uncertainty, Leduc and Liu (2016) use both, the VIX/VXO and data from the Michigan survey (consumers' perceived uncertainty constructed from the Thomson Reuters/University of Michigan Surveys of Consumers). Their rationale for using a consumer uncertainty measure is that, by construction of the survey, interviewees do not have (complete) information of the current month's macroeconomic data. Hence, it is assumed that survey participants will condition their answers on all previous realizations of macroeconomic indicators except time t. With their identification strategy Leduc and Liu (2016, p. 23) assume that their "[...] measured uncertainty contains *some*[124] exogenous component and does not reflect endogenous responses of other macroeconomic variables. In the model including the VIX/VXO, the sample ranges from 01:1986-10:2013, in the models with the consumer uncertainty measure

[124] Author's italics.

from 01:1978-10:2013. Further, placing the uncertainty measure first in their Choleski ordering hence implies that on impact of a shock only unemployment, inflation and the nominal interest rate are allowed to respond.

$$\mathbf{y}_t = \begin{bmatrix} y_{1t} \\ y_{2t} \\ y_{3t} \\ y_{4t} \end{bmatrix} = \begin{bmatrix} \textit{measure of uncertainty} \\ \text{unemployment rate} \\ \text{inflation rate (CPI)} \\ \text{three-month Treasury bills rate} \end{bmatrix} \tag{B.6}$$

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Eidesstattliche Erklärung

Ich erkläre hiermit an Eides Statt, dass ich dir vorliegende Masterarbeit selbständig angefertig habe. Die aus fremden Quellen direkt oder indirekt übernommenen Gedanken sind als solche kenntlich gemacht.

Die Arbeit wurde bisher weder in gleicher noch in ähnlicher Form einer anderen Prüfungsbehörde vorgelegt und auch noch nicht veröffentlicht.

Innsbruck, ____________________ ______________________________

(Unterschrift: Marcel Kropp)