

FU08 - Automata and Languages

Exercise 9

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Question 1: Answer the following questions

For the grammar:

$$\begin{aligned} S &\rightarrow A1B \\ A &\rightarrow 0A \mid \lambda \\ B &\rightarrow 0B \mid 1B \mid \lambda \end{aligned}$$

And the strings 00101 and 1001, give:

- The leftmost derivation
- The rightmost derivation
- The parse tree

Solution:

a. **For the string 00101:**

$$S \rightarrow A1B \rightarrow 0A1B \rightarrow 00A1B \rightarrow 001B \rightarrow 0010B \rightarrow 00101B \rightarrow 00101\lambda$$

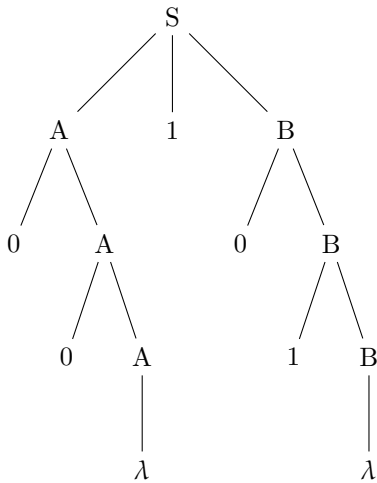
We have successfully derived the input string 00101.

b. **For the string 00101:**

$$S \rightarrow A1B \rightarrow A10B \rightarrow A101B \rightarrow A101 \rightarrow 0A101 \rightarrow 00A101 \rightarrow 00\lambda101$$

We have successfully derived the input string 00101.

c. **For the string 00101:**



For the string 1001:

$$S \rightarrow A1B \rightarrow 1B \rightarrow 10B \rightarrow 100B \rightarrow 1001B \rightarrow 1001\lambda$$

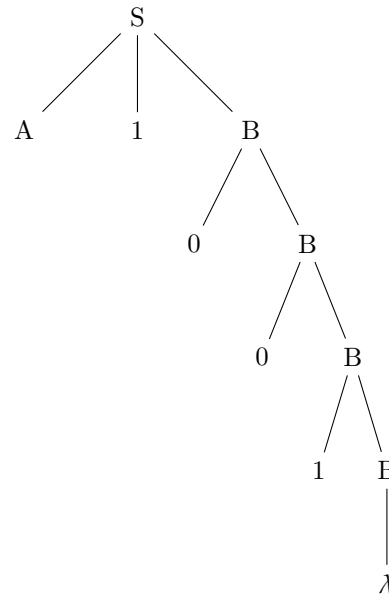
We have successfully derived the input string 1001.

For the string 1001:

$$S \rightarrow A1B \rightarrow A10B \rightarrow A100B \rightarrow A1001B \rightarrow A1001 \rightarrow \lambda1001$$

We have successfully derived the input string 1001.

For the string 1001:



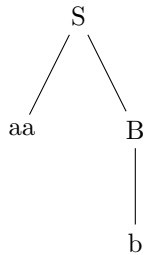
Question 2: Show that the following grammar is ambiguous

$$\begin{aligned} S &\rightarrow AB \mid aaB \\ A &\rightarrow a \mid Aa \\ B &\rightarrow b \end{aligned}$$

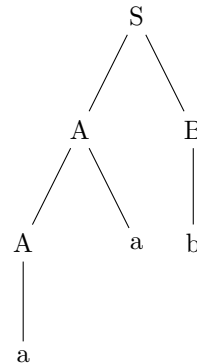
Solution:

Take a random input string $w = aab$. Let us construct the parsing tree of the grammar and see if the derivation tree is unique.

For the string aab:



For the string aab:



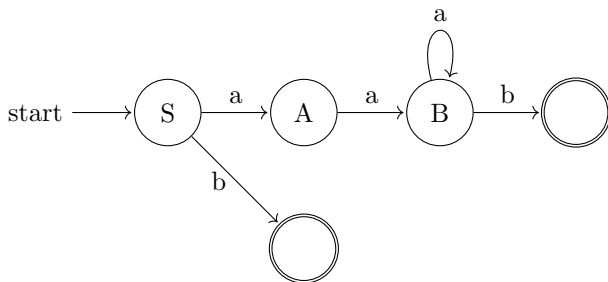
Acknowledge that the same input string $w = aab$ have two different parsing tree representation. Hence, the grammar is ambiguous.

Question 3: Find s-grammar for the following languages

- $L(r)$ where $r = aaa^*b + b$
- $L = \{a^n b^n : n \geq 1\}$

Solution:

a.

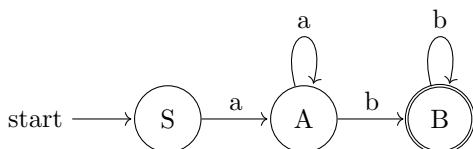


From the automaton, we construct our simple grammar.

$$\begin{aligned} S &\rightarrow aA \mid b \\ A &\rightarrow aB \\ B &\rightarrow aB \mid b \end{aligned}$$

b.

Construct a simple grammar for $L(aa^*bb^*)$.



From the automaton, we construct our simple grammar.

$$\begin{aligned} S &\rightarrow aA \\ A &\rightarrow aAB \mid b \\ B &\rightarrow b \end{aligned}$$