

MA 02 LINEAR ALGEBRA II
PRACTICE EXAM – FINAL B

July 24 (Wed), 2024

Section: C7.

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★ This is ‘Version B’ of the practice exam. (There is ‘Version A’.)
The actual exam may not be very similar to these practice exams. The purpose of these practice exams is to give you an idea of how the actual exam will look like, in terms of the length and the format.

[I] (20pts) (1) Let $f(x)$ and $g(x)$ be polynomials. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

True or false : $f(A)g(A) = g(A)f(A)$.

Answer : ☐ True. ☐ False. (Check one.)

(2) Let a , b and c be real numbers. Let $A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$ be a symmetric matrix.

True or false :

There exists an orthogonal matrix Q such that $Q^{-1}AQ$ is a diagonal matrix.

Answer : ☐ True. ☐ False. (Check one.)

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[II] (20pts) Find the characteristic polynomial, the eigenvalues, and then eigenvectors associated with each of the eigenvalues of A . Is A diagonalizable? If not, then find the Jordan canonical form of A .

$$A = \begin{bmatrix} 1 & -1 \\ 1 & 3 \end{bmatrix},$$

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[III] (20pts) Find the characteristic polynomial, the eigenvalues, and then eigenvectors associated with each of the eigenvalues of A . Then diagonalize the matrix

$$A = \begin{bmatrix} 6 & 2 \\ 2 & 3 \end{bmatrix},$$

with the condition the matrix Q with which $Q^{-1}AQ$ equals the diagonal matrix is orthogonal .

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[IV] (40pts) Let a , b , c and d be real numbers. Let

$$A = \begin{bmatrix} a & b & c & d \\ b & a & d & c \\ c & d & a & b \\ d & c & b & a \end{bmatrix}, \quad \text{and} \quad Q = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}.$$

(1) $AQ =$

(2) $Q^2 =$

(3) $Q^{-1}AQ =$