# FU08 - Automata and Languages Exercise 6

 $\begin{array}{c} {\rm NGUYEN~Tuan~Dung} \\ {\rm s}1312004 \end{array}$ 

December 27, 2024

## Question 1: Convert the following finite automata into equivalent regular expressions

$$M=(Q, \sum, \delta, q_0, F)$$
 with

$$Q = \{q_0, q_1, q_2, q_3, q_4\}$$

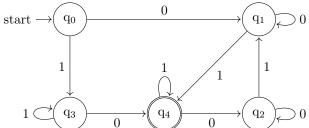
$$\sum = \{0, 1\}$$

 $Q = \{q_0, q_1, q_2, q_3, q_4\}$   $\sum = \{0, 1\}$   $F = \{q_4\}, \text{ and } \delta \text{ is defined by}$ 

δ	0	1
$q_0$	$q_1$	$q_3$
$q_1$	$q_1$	$q_4$
$q_2$	$q_2$	$q_1$
$q_3$	$q_4$	$q_3$
$q_4$	$q_2$	$q_4$

#### Solution:

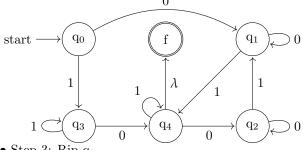
From the state transition table, we construct the DFA.

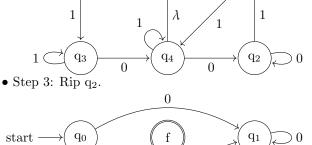


• Step 1: Insert a new end state.

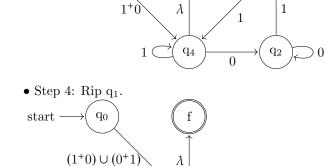
• Step 2: Rip  $q_3$ .

start





 $\operatorname{start}$  $0^{+}1$  $1^{+}0$ 



0

 $q_1$ 

 $1 \cup (0^+10^*1)$ 

• Step 5: Rip q<sub>4</sub>.  $(1^+0) \cup (0^+1)[1 \cup (0^+10^*1)]^*$ start -

 $\implies$  Hence, the regular expression for the finite automata is:  $(1^+0) \cup (0^+1)[1 \cup (0^+10^*1)]^*$ .

### Question 2: Convert the following finite automata into equivalent regular expressions

$$M = (Q, \Sigma, \delta, q_0, F)$$
 with

$$Q = \{q_0, q_1, q_2, q_3\}$$

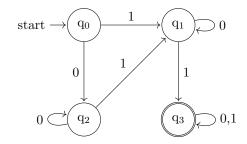
$$\sum = \{0, 1\}$$

$$\begin{split} M &= (Q, \sum, \delta, q_0, F) \text{ with } \\ Q &= \{q_0, q_1, q_2, q_3\} \\ \sum &= \{0, 1\} \\ F &= \{q_3\} \text{ , and delta is defined by } \end{split}$$

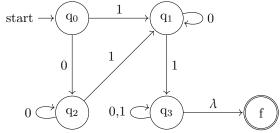
δ	0	1
$q_0$	$q_2$	$q_1$
$q_1$	$q_1$	$q_3$
$q_2$	$q_2$	$q_1$
$q_3$	$q_3$	$q_3$

#### Solution:

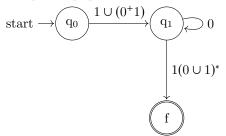
From the state transition table, we construct the NFA.



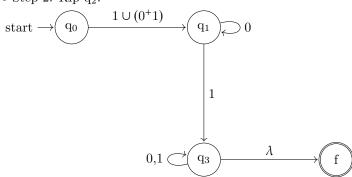
• Step 1: Adding new end state.



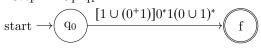
• Step 3: Rip q<sub>3</sub>.



• Step 2: Rip  $q_2$ .



• Step 4: Rip  $q_1$ .



 $\implies$  Hence, the regular expression for the finite automata is:  $[1 \cup (0^+1)]0^*1(0 \cup 1)^*$ .

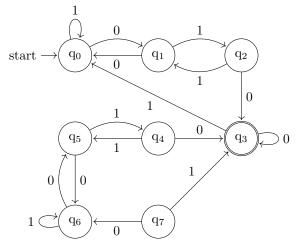
# Question 3: Convert the following finite automata into equivalent regular expressions

$$\begin{split} M &= (Q, \sum, \delta, q_0, F) \text{ with } \\ Q &= \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7\} \\ \sum &= \{0, 1\} \\ F &= \{q_3\} \text{ , and } \delta \text{ is defined by } \end{split}$$

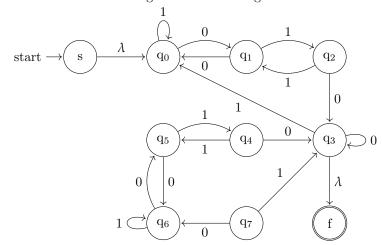
δ	0	1
$q_0$	$q_1$	$q_0$
$q_1$	$q_0$	$q_2$
$q_2$	$q_3$	$q_1$
$q_3$	$q_3$	$q_0$
$q_4$	$q_3$	$q_5$
$q_5$	$q_6$	$q_4$
$q_6$	$q_5$	$q_6$
97	$q_6$	$q_3$

#### Solution:

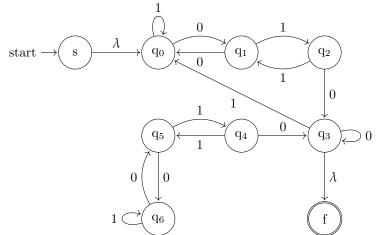
From the above information, let us construct the corresponding NFA.



Let us insert new ending state and starting state.

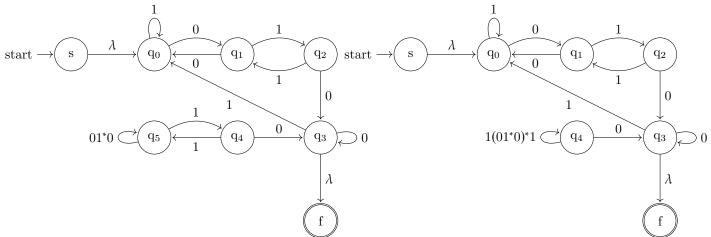


ullet Since  $q_7$  has no path leading to it, we can remove it without affecting the automaton.

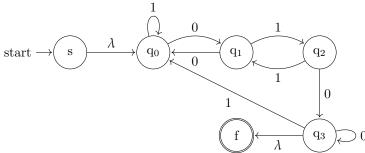


• Step 1: Rip out q<sub>6</sub>.

• Step 2: Rip out  $q_5$ .

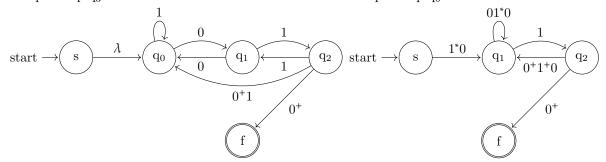


• Since q<sub>4</sub> has no path leading to it, we can remove it without affecting the automaton.

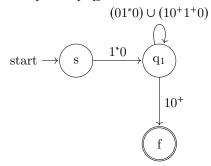


• Step 3: Rip  $q_3$ .

• Step 4: Rip  $q_0$ .



• Step 5: Rip q<sub>2</sub>.



• Step 6: Rip  $q_1$ .

