FU08 - Automata and Languages
Exercise 10

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Question 1: Put the following grammars into Chomsky normal form

1.
$$S \longrightarrow A1B$$
$$A \longrightarrow 0A \mid \lambda$$
$$B \longrightarrow 0B \mid 1B \mid \lambda$$

2.
$$S \longrightarrow AB \mid aaB$$
$$A \longrightarrow a \mid Aa$$
$$B \longrightarrow b$$

3.
$$S \longrightarrow ASB \mid \lambda$$
$$A \longrightarrow aAS \mid a$$
$$B \longrightarrow SbS \mid A \mid bb$$

Solution:
1.
• First, we remove $A \longrightarrow \lambda$:
$$S \longrightarrow A1B \mid 1B$$
$$A \longrightarrow 0A \mid 0$$
$$B \longrightarrow 0B \mid 1B \mid \lambda$$

• Secondly, we remove $B \longrightarrow \lambda$:
$$S \longrightarrow A1B \mid 1B \mid A1 \mid 1$$
$$A \longrightarrow 0A \mid 0$$
$$B \longrightarrow 0B \mid 1B \mid 0 \mid 1$$

• Next, we construct our CNF. Notice that we have to decompose $\{A1B; 1B; A1; 0A; 0B; 1B\}$. Starting easy with $X \longrightarrow 0$:
$$S \longrightarrow A1B \mid 1B \mid A1 \mid 1$$
$$A \longrightarrow XA \mid 0$$
$$X \longrightarrow 0$$
$$B \longrightarrow XB \mid 1B \mid 0 \mid 1$$

• Continue with $Y \longrightarrow 1$:
$$S \longrightarrow AYB \mid YB \mid AY \mid 1$$
$$A \longrightarrow XA \mid 0$$
$$X \longrightarrow 0$$
$$Y \longrightarrow 1$$
$$B \longrightarrow XB \mid YB \mid 0 \mid 1$$

• What's left is only $S \longrightarrow AYB$. We will create $P \longrightarrow AY$.
$$S \longrightarrow PB \mid YB \mid AY \mid 1$$
$$A \longrightarrow XA \mid 0$$
$$B \longrightarrow XB \mid YB \mid 0 \mid 1$$
$$X \longrightarrow 0$$
$$Y \longrightarrow 1$$
$$P \longrightarrow AY$$
The grammar is now in Chomsky Normal Form.

2.

• The grammar does not have any lambda transition, also starting state S does not appear in the RHS. Hence, we notice the terms for decomposition: {aaB; Aa}. Starting with X $\longrightarrow$ a

$$\begin{aligned}
&S \longrightarrow AB \mid XXB\\
&A \longrightarrow a \mid AX\\
&B \longrightarrow b\\
&X \longrightarrow a
\end{aligned}$$

• Next, we replace Y $\longrightarrow$ XB.

$$\begin{aligned}
&S \longrightarrow AB \mid XY\\
&A \longrightarrow a \mid AX\\
&B \longrightarrow b\\
&X \longrightarrow a\\
&Y \longrightarrow XB
\end{aligned}$$

For the grammar above, it is in Chomsky Normal Form.

3.

• The grammar has the starting state S at the RHS. Hence, we create a new state S'.

$$\begin{aligned}
&S' \longrightarrow S\\
&S \longrightarrow ASB \mid \lambda\\
&A \longrightarrow aAS \mid a\\
&B \longrightarrow SbS \mid A \mid bb
\end{aligned}$$

• Next, we ommit S $\longrightarrow \lambda$.

$$\begin{aligned}
&S' \longrightarrow ASB \mid AB \mid \lambda\\
&S \longrightarrow ASB \mid AB\\
&A \longrightarrow aAS \mid a \mid aA\\
&B \longrightarrow SbS \mid bS \mid Sb \mid A \mid bb \mid b
\end{aligned}$$

• Ommit B $\longrightarrow$ A since it violates the rule of CNF.

$$\begin{aligned}
&S' \longrightarrow ASB \mid AB \mid \lambda\\
&S \longrightarrow ASB \mid AB\\
&A \longrightarrow aAS \mid a \mid aA\\
&B \longrightarrow SbS \mid bS \mid Sb \mid bb \mid b \mid aAs \mid a \mid aA
\end{aligned}$$

• Create X $\longrightarrow$ AS.

$$\begin{aligned}
&S' \longrightarrow XB \mid AB \mid \lambda\\
&S \longrightarrow XB \mid AB\\
&A \longrightarrow aX \mid a \mid aA\\
&B \longrightarrow SbS \mid bS \mid Sb \mid bb \mid b \mid aX \mid a \mid aA\\
&X \longrightarrow AS
\end{aligned}$$

• Ommit Y $\longrightarrow$ a since it violates the rule of CNF.

$$\begin{aligned}
&S' \longrightarrow XB \mid AB \mid \lambda\\
&S \longrightarrow XB \mid AB\\
&A \longrightarrow YX \mid a \mid YA\\
&B \longrightarrow SbS \mid bS \mid Sb \mid bb \mid b \mid YX \mid a \mid YA\\
&X \longrightarrow AS\\
&Y \longrightarrow a
\end{aligned}$$

• Create Z $\longrightarrow$ b.

$$\begin{aligned}
&S' \longrightarrow XB \mid AB \mid \lambda\\
&S \longrightarrow XB \mid AB\\
&A \longrightarrow YX \mid a \mid YA\\
&B \longrightarrow SZS \mid ZS \mid SZ \mid ZZ \mid b \mid YX \mid a \mid YA\\
&X \longrightarrow AS\\
&Y \longrightarrow a\\
&Z \longrightarrow b
\end{aligned}$$

• Lastly, create C $\longrightarrow$ SZ.

$$\begin{aligned}
&S' \longrightarrow XB \mid AB \mid \lambda\\
&S \longrightarrow XB \mid AB\\
&A \longrightarrow YX \mid a \mid YA\\
&B \longrightarrow CS \mid ZS \mid SZ \mid ZZ \mid b \mid YX \mid a \mid YA\\
&X \longrightarrow AS\\
&Y \longrightarrow a\\
&Z \longrightarrow b\\
&C \longrightarrow SZ
\end{aligned}$$

The above grammar is in Chomsky Normal Form.