# FU08 - Automata and Languages
# Exercise 8

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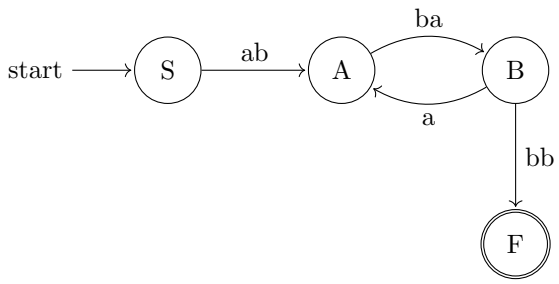
## Question 1: Construct a DFA that accepts the language generated by the grammar

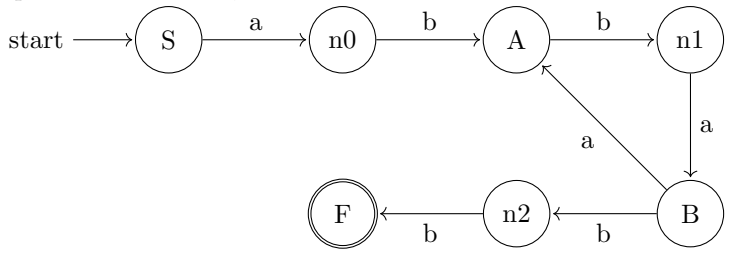
S ⟶ abA,
A ⟶ baB,
B ⟶ aA|bb

***Solution:***

We sketch the DFA out, with the start state S, final state F.




Split the transitions, we obtain the final DFA.




Fast conversion from DFA to regular expression, we obtain abba(aba)*bb.

## Question 2: Construct right- and left-linear grammars for the language

$L = \{a^n b^m : n \geqslant 2, m \geqslant 3\}$

***Solution:***

Fast conversion to regular expression, we obtain aaa*bbbb* = aaa*b*bbb.

• Right-linear grammar: start from left to right
S ⟶ aaA
A ⟶ aA|B
B ⟶ bbbC
C ⟶ bC|$\lambda$

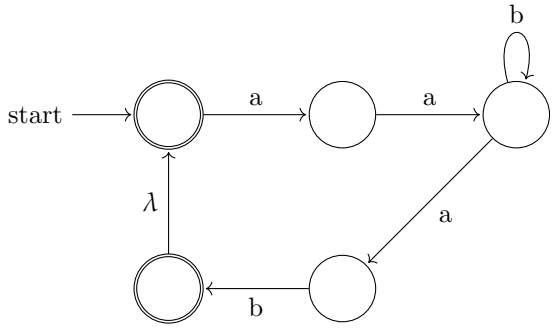
• Left-linear grammar: start from right to left
S ⟶ Abbb
A ⟶ Ab|B
B ⟶ Caa
C ⟶ $\lambda$

## Question 3: Answer the following question

Construct right- and left-linear grammars for the language generated by the following regular expression.
r = (aab*ab)*

***Solution:***

• Right-linear grammar: start from left to right
S ⟶ $\lambda$|aaA
A ⟶ bA|abS




• Left-linear grammar: start from right to left
S ⟶ $\lambda$|Aab
A ⟶ Ab|Saa

**Question 4: Construct a context-free grammar for the language**

$\{a^i b^j c^k : i \neq j \text{ or } j \neq k\}$
that is the language of strings of a's followed by b's followed by c's, such that there are either a different number of a's and b's or a different number of b's and c's, or both.

***Solution:***

Since $i \neq j$ or $j \neq k \Leftrightarrow i < j$ or $i > j$ or $j > k$ or $j < k$.

$S \longrightarrow X_{i<j}C \mid X_{i>j}C \mid AY_{j<k} \mid AY_{j>k}$

$A \longrightarrow aA \mid \lambda$
$B \longrightarrow bB \mid \lambda$
$C \longrightarrow cC \mid \lambda$

$X_{i>j} \longrightarrow aX_{i>j}b \mid aA$

$X_{i<j} \longrightarrow aX_{i<j}b \mid bB$

$Y_{j<k} \longrightarrow bY_{j<k}c \mid cC$

$Y_{j>k} \longrightarrow bY_{j<k}c \mid bB$