FU08 - Automata and Languages
Exercise 5

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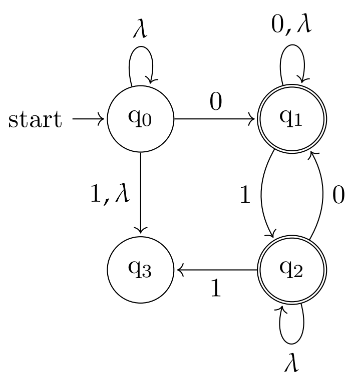
Question 1: Answer the following question

For the input alphabet $\{0, 1\}$, construct a λ-NFA accepts the following language: the set of all strings such that each string should start by 0 and there is no two consecutive 1′s.

Solution:

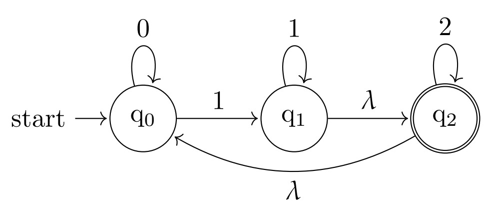
• **State definition**:

- q_0: The string does not contain anything.
- q_1: The string ends in 0. $q_1 \in \mathbb{F}$
- q_2: The string ends in 1. $q_2 \in \mathbb{F}$
- q_3: The string is illegal, the dead state.



Question 2: Answer the following question

For the $\lambda-$NFA



Construct the equaivalent NFA.

Solution:

Let us define the above $\lambda-$NFA as $M = (\{q_0, q_1, q_2\}, \{0, 1, 2\}, \delta, \{q_2\})$.
Moreover, define the NFA as $N = (Q', \sum, \delta', q', \mathbb{F}')$:
1. $Q' = \{q_0, q_1, q_2\}$.
2. $\sum = \{0, 1, 2\}$.
3. $q' = \{q_0\}$.
4. $\mathbb{F}' = \{q_0, q_2\}$.

Since we know that $\delta'(q, a) = \hat{\delta}(q, a)$. Hence, we obtain the following state transitions.
First, we obtain the lambda closure:

- $\lambda-$closure$(q_0) = \{q_0\}$
- $\lambda-$closure$(q_1) = \{q_0, q_1, q_2\}$
- $\lambda-$closure$(q_2) = \{q_0, q_2\}$

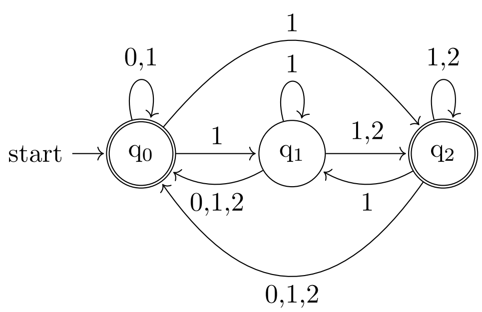
(see next page)

$\delta'(q_0, 0) = \hat{\delta}(q_0, 0) = \delta(\lambda - \text{closure}(q_0), 0) = \delta(\{q_0\}, 0) = \{q_0\}$
$\delta'(q_0, 1) = \hat{\delta}(q_0, 1) = \delta(\lambda - \text{closure}(q_0), 1) = \delta(\{q_0\}, 1) = \{q_0, q_1, q_2\}$
$\delta'(q_0, 2) = \hat{\delta}(q_0, 2) = \delta(\lambda - \text{closure}(q_0), 2) = \delta(\{q_0\}, 2) = \{\emptyset\}$
$\delta'(q_1, 0) = \hat{\delta}(q_1, 0) = \delta(\lambda - \text{closure}(q_1), 0) = \delta(\{q_0, q_1, q_2\}, 0) = \{q_0\}$
$\delta'(q_1, 1) = \hat{\delta}(q_1, 1) = \delta(\lambda - \text{closure}(q_1), 1) = \delta(\{q_0, q_1, q_2\}, 1) = \{q_0, q_1, q_2\}$
$\delta'(q_1, 2) = \hat{\delta}(q_1, 2) = \delta(\lambda - \text{closure}(q_1), 2) = \delta(\{q_0, q_1, q_2\}, 2) = \{q_0, q_2\}$
$\delta'(q_2, 0) = \hat{\delta}(q_2, 0) = \delta(\lambda - \text{closure}(q_2), 0) = \delta(\{q_0, q_2\}, 0) = \{q_0\}$
$\delta'(q_2, 1) = \hat{\delta}(q_2, 1) = \delta(\lambda - \text{closure}(q_2), 1) = \delta(\{q_0, q_2\}, 1) = \{q_0, q_1, q_2\}$
$\delta'(q_2, 2) = \hat{\delta}(q_2, 2) = \delta(\lambda - \text{closure}(q_2), 2) = \delta(\{q_0, q_2\}, 2) = \{q_0, q_2\}$

State transition table:

δ'	0	1	2
q_0	$\{q_0\}$	$\{q_0, q_1, q_2\}$	$\{\emptyset\}$
q_1	$\{q_0\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_2\}$
q_2	$\{q_0\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_2\}$

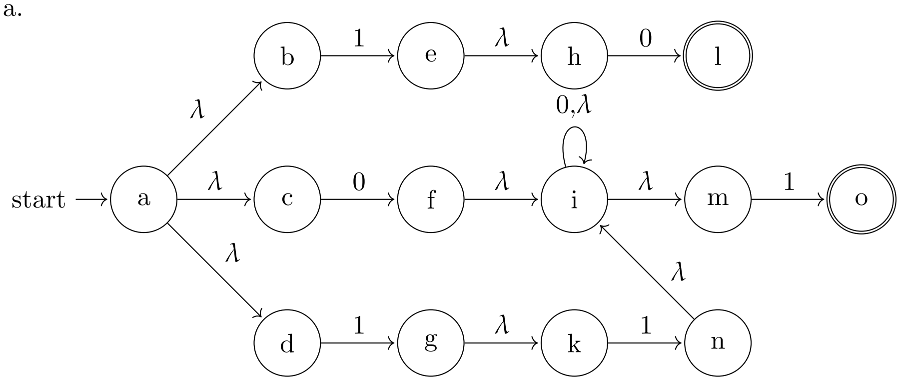
Table 1: NFA's state transition table



Question 3: Answer the following question

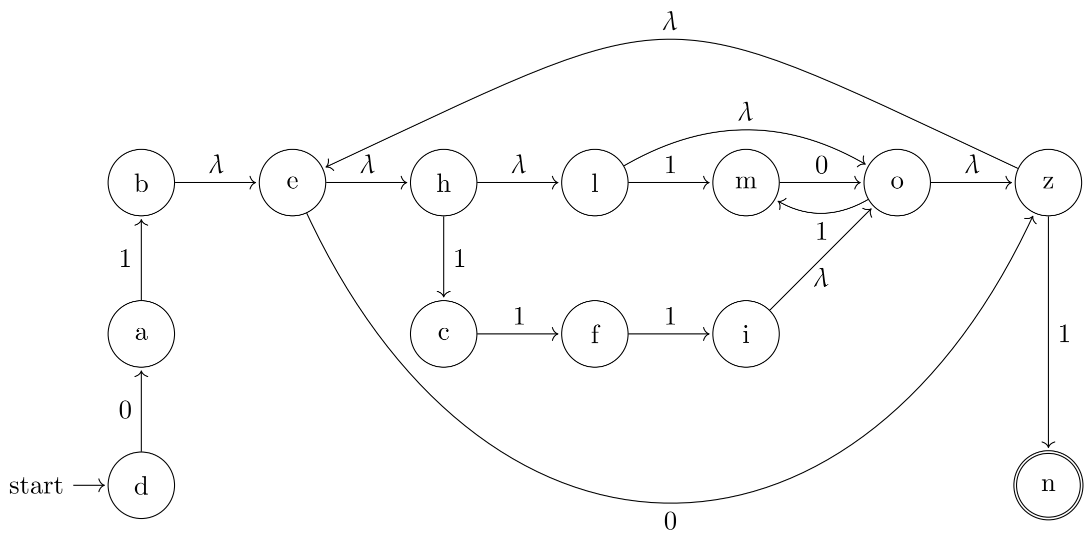
Construct three λ-NFA's equivalent to the following regular expression:
a. $10 + (0 + 11)0^*1$
b. $01\left[((10)^* + 111)^* + 0\right]^* 1$
c. $01^*0 + 01^*10^* + 1^*0^*$

Solution:
a.

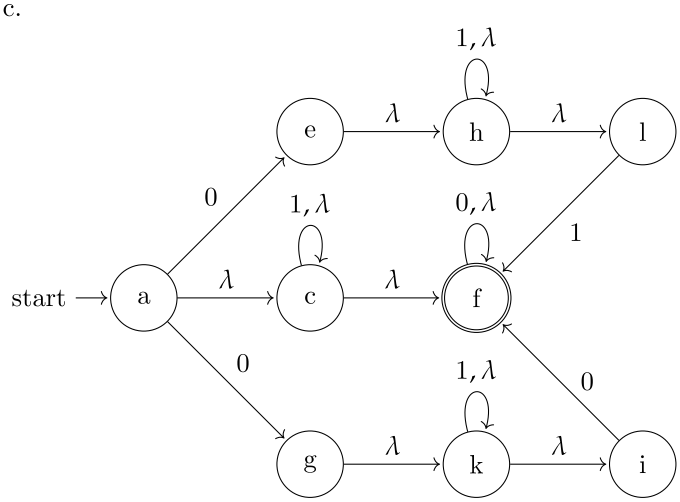


(see next page for b. and c.)

b.



c.



This is the most optimized approach that I could possibly get. :D