MA 02 LINEAR ALGEBRA II

PRACTICE EXAM – FINAL A

July 23 (Tued), 2024

Section: C7.
Instructors: Yasuyuki Kachi (lecture) & Shunji Moriya (recitation)

⋆ **This is 'Version A' of the practice exam. (There is 'Version B'.) The actual exam may not be very similar to these practice exams. The purpose of these practice exams is to give you an idea of how the actual exam will look like, in terms of the length and the format.**

[I] (15pts) Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $B = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$. Suppose $\det A \neq 0$.

(1) <u>True or false</u> :

$$\left(ABA^{-1}\right)^3 = AB^3A^{-1}.$$

[<u>Answer</u>]: ☐ True. ☐ False. (<u>Check one.</u>)

(2) Let

$$f(x) = x^3 + x^2 - 2x - 1.$$

<u>True or false</u> :

$$f\left(ABA^{-1}\right) = Af(B)A^{-1}.$$

[<u>Answer</u>]: ☐ True. ☐ False. (<u>Check one.</u>)

Section: C7.

ID # : ____________________ Name : ____________________________

[II] (15pts) Find the characteristic polynomial, the eigenvalues, and then eigenvectors associated with each of the eigenvalues of A. Then diagonalize A:

$$A = \begin{bmatrix} -3 & -2 \\ 6 & 5 \end{bmatrix}.$$

Give the matrix Q with which $Q^{-1}AQ$ equals the diagonal matrix.

Section: C7.

ID # : ________________ Name : ____________________

[III] (15pts) Find the characteristic polynomial, the eigenvalues, and then eigenvectors associated with each of the eigenvalues of A. Then diagonalize A:

$$A = \begin{bmatrix} 4 & -1 \\ 1 & 1 \end{bmatrix}.$$

Give the matrix Q with which $Q^{-1}AQ$ equals the diagonal matrix.

Section: C7.

ID # : ____________ Name : ____________

[IV] (20pts) State Cayley-Hamilton's theorem for $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Then prove it.

Section: C7.

ID # : ____________________ Name : ____________________________

[V] (40pts) Let b, c, d be real numbers. Let

$$X = \begin{bmatrix} 0 & -d & c \\ d & 0 & -b \\ -c & b & 0 \end{bmatrix}.$$

(1) Calculate X^2. Then calculate

$$A = \left(b^2 + c^2 + d^2\right) I + 2X^2.$$

Show work.

(2) Fill an appropriate scalar in the box $\big($the answer involves b, c and $d\big)$:

$$X^3 = -\left(\boxed{}\right) X.$$

(3) Assume $b^2 + c^2 + d^2 = 1$. Calculate A^2, where A is in (1) above.

Show work.

(4) Find the eigenvalues of A. Identify the one with multiplicity 2, if any.