FU08 - Automata and Languages
Exercise 4

NGUYEN Tuan Dung
s1312004

December 16, 2024

Question 1: Answer the following question

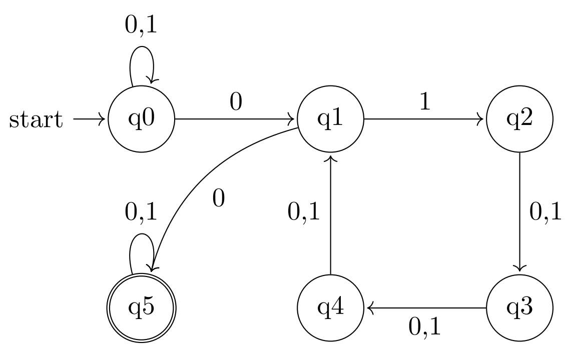
For $\Sigma = \{0, 1\}$, construct an NFA accepting the following language:
The set of all strings such that some two 0's are separated by a string whose length is $4i$, for some $i \geqslant 0$.

Solution:

• **State definition**:

We call the length of the interim string between the two 0's n. We want the following language to be accepted: $(0 \vee 1)^* \ 0 \ (0 \vee 1)^{4\mathrm{i}} \ 0 \ (0 \vee 1)^*$

- q_0: The string does not contain anything.
- q_1: The string contains the first 0, and $n\%4 = 0$.
- q_2: The string contains the first 0, and $n\%4 = 1$.
- q_3: The string contains the first 0, and $n\%4 = 2$.
- q_4: The string contains the first 0, and $n\%4 = 3$.
- q_5: The string contains the first 0, $n = 4i$ (or $n\%4 = 0$), and a second 0. $\mathrm{q}_5 \in \mathbb{F}$.



Question 2: Answer the following question

Construct two DFA's equivalent to the NFA's
(a) $M_1 = (\{p, q, r, s\}, \{0, 1\}, \delta_1, p, \{s\})$
(b) $M_2 = (\{p, q, r, s\}, \{0, 1\}, \delta_2, p, \{q, s\})$
where δ_1 and δ_2 are given by the following tables

δ_1	0	1
p	{p,q}	{p}
q	{r}	{r}
r	{s}	-
s	{s}	{s}

Table 1: Transition Table δ_1

δ_2	0	1
p	{q,s}	{q}
q	{r}	{q,r}
r	{s}	{p}
s	-	{p}

Table 2: Transition Table δ_2

Solution:
a. We define a DFA $N_1 = (Q', \sum, \delta_1', q_1', \mathbb{F}')$:
1. $Q' = \mathscr{P}(Q) = \{\emptyset, \{p\}, \{q\}, \{r\}, \{s\}, \{p, q, r, s\}, \{p, q\}, \{p, r\}, \{p, s\}, \{q, r\}, \{q, s\}, \{r, s\}, \{p, q, r\}, \{p, r, s\}, \{p, q, s\}, \{q, r, s\}\}$.
2. $\sum = \{0, 1\}$.
3. $q_1' = \{p\}$.
4. $\mathbb{F}' = \{\{s\}, \{p, q, r, s\}, \{p, s\}, \{q, s\}, \{r, s\}, \{p, r, s\}, \{p, q, s\}, \{q, r, s\}\}$.
Since we know that $\delta_1'(R, a) = \{q \in Q | q \in \delta(r, a)$ for $r \in R\}$. Hence, we obtain the following state transitions.

for a = 0
$\delta_1'(\emptyset, 0) = \emptyset$
$\delta_1'(\{p\}, 0) = \{p, q\}$
$\delta_1'(\{q\}, 0) = \{r\}$
$\delta_1'(\{r\}, 0) = \{s\}$
$\delta_1'(\{s\}, 0) = \{s\}$
$\delta_1'(\{p, q, r, s\}, 0) = \{p, q\} \cup \{r\} \cup \{s\} \cup \{s\} = \{p, q, r, s\}$
$\delta_1'(\{p, q\}, 0) = \{p, q\} \cup \{r\} = \{p, q, r\}$
$\delta_1'(\{p, r\}, 0) = \{p, q\} \cup \{s\} = \{p, q, s\}$
$\delta_1'(\{p, s\}, 0) = \{p, q\} \cup \{s\} = \{p, q, s\}$
$\delta_1'(\{q, r\}, 0) = \{r\} \cup \{s\} = \{r, s\}$
$\delta_1'(\{q, s\}, 0) = \{r\} \cup \{s\} = \{r, s\}$
$\delta_1'(\{r, s\}, 0) = \{s\} \cup \{s\} = \{s\}$
$\delta_1'(\{p, q, r\}, 0) = \{p, q\} \cup \{r\} \cup \{s\} = \{p, q, r, s\}$
$\delta_1'(\{p, r, s\}, 0) = \{p, q\} \cup \{s\} \cup \{s\} = \{p, q, s\}$
$\delta_1'(\{p, q, s\}, 0) = \{p, q\} \cup \{r\} \cup \{s\} = \{p, q, r, s\}$
$\delta_1'(\{q, r, s\}, 0) = \{r\} \cup \{s\} \cup \{s\} = \{r, s\}$

for a = 1
$\delta_1'(\emptyset, 1) = \emptyset$
$\delta_1'(\{p\}, 1) = \{p\}$
$\delta_1'(\{q\}, 1) = \{r\}$
$\delta_1'(\{r\}, 1) = \emptyset$
$\delta_1'(\{s\}, 1) = \{s\}$
$\delta_1'(\{p, q, r, s\}, 1) = \{p\} \cup \{r\} \cup \{s\} \cup \emptyset = \{p, r, s\}$
$\delta_1'(\{p, q\}, 1) = \{p\} \cup \{r\} = \{p, r\}$
$\delta_1'(\{p, r\}, 1) = \{p\} \cup \emptyset = \{p\}$
$\delta_1'(\{p, s\}, 1) = \{p\} \cup \{s\} = \{p, s\}$
$\delta_1'(\{q, r\}, 1) = \{r\} \cup \emptyset = \{r\}$
$\delta_1'(\{q, s\}, 1) = \{r\} \cup \{s\} = \{r, s\}$
$\delta_1'(\{r, s\}, 1) = \emptyset \cup \{s\} = \{s\}$
$\delta_1'(\{p, q, r\}, 1) = \{p\} \cup \{r\} \cup \emptyset = \{p, r\}$
$\delta_1'(\{p, r, s\}, 1) = \{p\} \cup \{s\} \cup \emptyset = \{p, s\}$
$\delta_1'(\{p, q, s\}, 1) = \{p\} \cup \{r\} \cup \{s\} = \{p, r, s\}$
$\delta_1'(\{q, r, s\}, 1) = \{r\} \cup \emptyset \cup \{s\} = \{r, s\}$

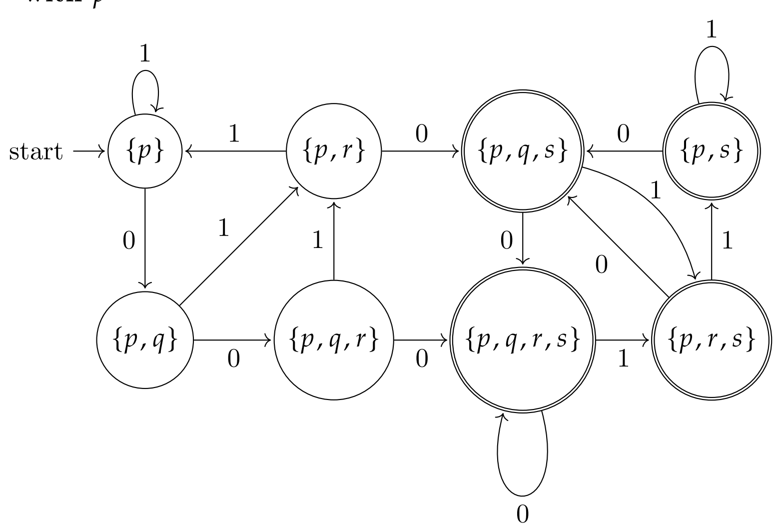
Now, we construct the state transition table. (next page)

State transition table

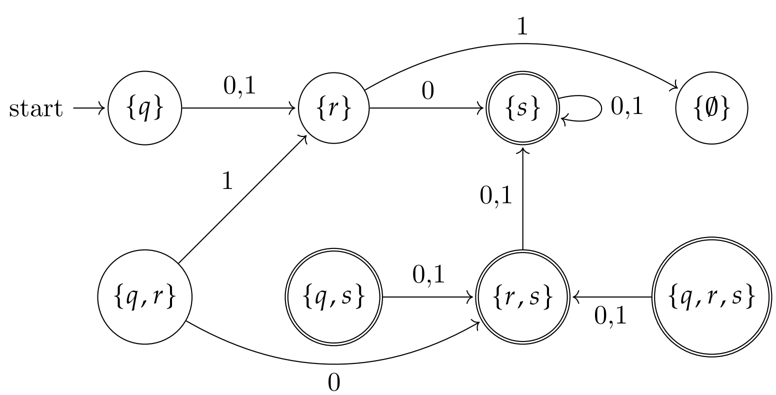
δ_1'	0	1
$\{p\}$	$\{p, q\}$	$\{p\}$
$\{q\}$	$\{r\}$	$\{r\}$
$\{r\}$	$\{s\}$	$\emptyset$
$\{s\}$	$\{s\}$	$\{s\}$
$\{p, q\}$	$\{p, q, r\}$	$\{p, r\}$
$\{p, r\}$	$\{p, q, s\}$	$\{p\}$
$\{p, s\}$	$\{p, q, s\}$	$\{p, s\}$
$\{q, r\}$	$\{r, s\}$	$\{r\}$
$\{q, s\}$	$\{r, s\}$	$\{r, s\}$
$\{r, s\}$	$\{s\}$	$\{s\}$
$\{p, q, r\}$	$\{p, q, r, s\}$	$\{p, r\}$
$\{p, r, s\}$	$\{p, q, s\}$	$\{p, s\}$
$\{p, q, s\}$	$\{p, q, r, s\}$	$\{p, r, s\}$
$\{q, r, s\}$	$\{r, s\}$	$\{r, s\}$
$\{p, q, r, s\}$	$\{p, q, r, s\}$	$\{p, r, s\}$

Now, we construct a DFA for this state transition table.

• **with** p



• **without** p



b. We define a DFA $N_2 = (Q', \sum, \delta'_2, q'_2, \mathbb{F}')$:
1. $Q' = \mathscr{P}(Q) = \{\emptyset, \{p\}, \{q\}, \{r\}, \{s\}, \{p,q,r,s\}, \{p,q\}, \{p,r\}, \{p,s\}, \{q,r\}, \{q,s\}, \{r,s\}, \{p,q,r\}, \{p,r,s\}, \{p,q,s\}, \{q,r,s\}\}$.
2. $\sum = \{0, 1\}$.
3. $q'_2 = \{p\}$.
4. $\mathbb{F}' = \{\{p,q,r,s\}, \{q,s\}, \{p,q,s\}, \{q,r,s\}, \{q\}, \{s\}, \{p,q\}, \{p,s\}, \{q,r\}, \{r,s\}, \{p,q,r\}, \{p,r,s\}\}$.
Since we know that $\delta'_2(R,a) = \{q \in Q | q \in \delta(r,a) \text{ for } r \in R\}$. Hence, we obtain the following state transitions.

for a = 0
$\delta'_2(\emptyset, 0) = \emptyset$
$\delta'_2(\{p\}, 0) = \{q,s\}$
$\delta'_2(\{q\}, 0) = \{r\}$
$\delta'_2(\{r\}, 0) = \{s\}$
$\delta'_2(\{s\}, 0) = \{\emptyset\}$
$\delta'_2(\{p,q,r,s\}, 0) = \{q,s\} \cup \{r\} \cup \{s\} \cup \emptyset = \{q,r,s\}$
$\delta'_2(\{p,q\}, 0) = \{q,s\} \cup \{r\} = \{q,r,s\}$
$\delta'_2(\{p,r\}, 0) = \{q,s\} \cup \{s\} = \{q,s\}$
$\delta'_2(\{p,s\}, 0) = \{q,s\} \cup \emptyset = \{q,s\}$
$\delta'_2(\{q,r\}, 0) = \{r\} \cup \{s\} = \{r,s\}$
$\delta'_2(\{q,s\}, 0) = \{r\} \cup \emptyset = \{r\}$
$\delta'_2(\{r,s\}, 0) = \{s\} \cup \emptyset = \{s\}$
$\delta'_2(\{p,q,r\}, 0) = \{q,s\} \cup \{r\} \cup \{s\} = \{q,r,s\}$
$\delta'_2(\{p,r,s\}, 0) = \{q,s\} \cup \{s\} \cup \emptyset = \{q,s\}$
$\delta'_2(\{p,q,s\}, 0) = \{q,s\} \cup \{r\} \cup \emptyset = \{q,r,s\}$
$\delta'_2(\{q,r,s\}, 0) = \{r\} \cup \{s\} \cup \emptyset = \{r,s\}$

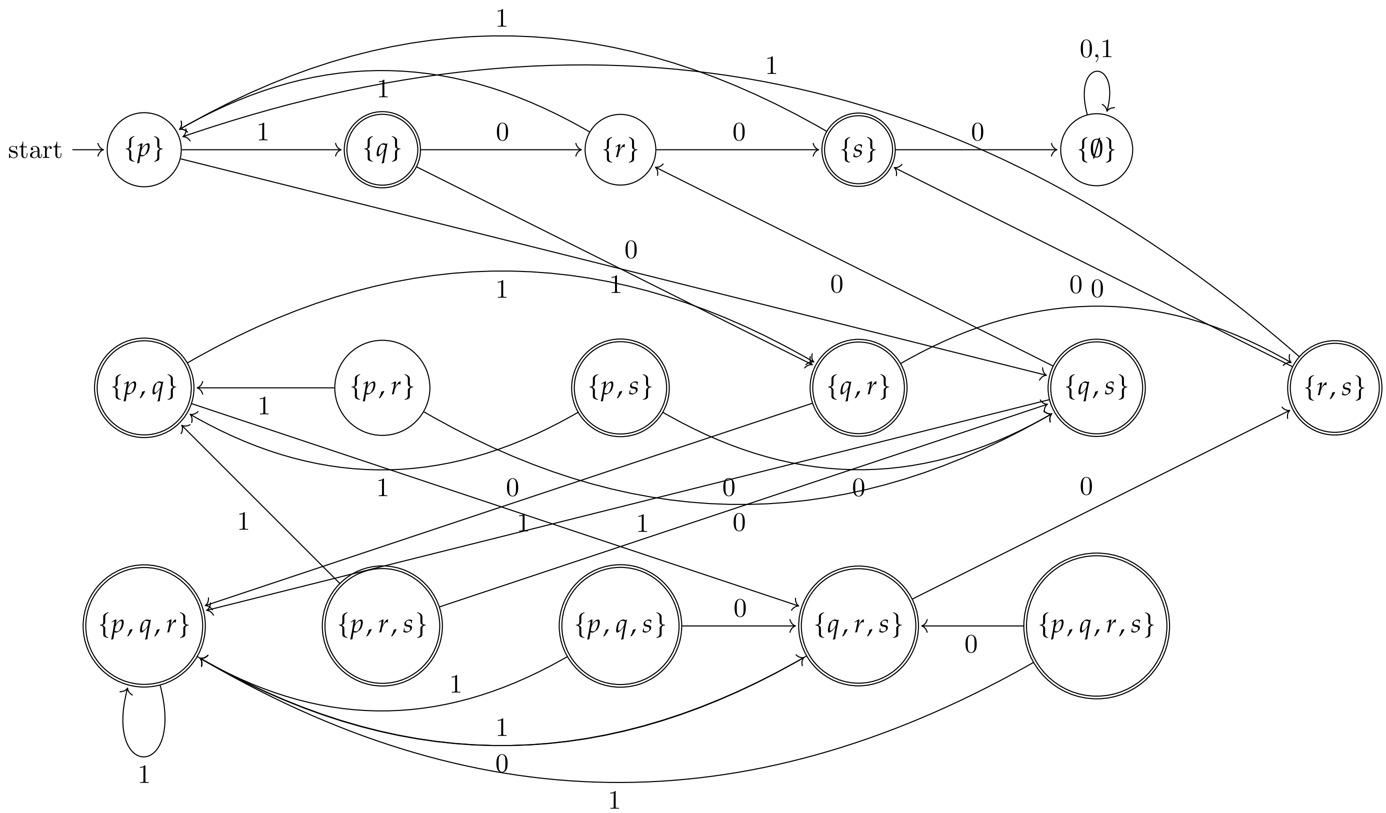
for a = 1
$\delta'_2(\emptyset, 1) = \emptyset$
$\delta'_2(\{p\}, 1) = \{q\}$
$\delta'_2(\{q\}, 1) = \{q,r\}$
$\delta'_2(\{r\}, 1) = \{p\}$
$\delta'_2(\{s\}, 1) = \{p\}$
$\delta'_2(\{p,q,r,s\}, 1) = \{q\} \cup \{q,r\} \cup \{p\} \cup \{p\} = \{p,q,r\}$
$\delta'_2(\{p,q\}, 1) = \{q\} \cup \{q,r\} = \{q,r\}$
$\delta'_2(\{p,r\}, 1) = \{q\} \cup \{p\} = \{p,q\}$
$\delta'_2(\{p,s\}, 1) = \{q\} \cup \{p\} = \{p,q\}$
$\delta'_2(\{q,r\}, 1) = \{q,r\} \cup \{p\} = \{p,q,r\}$
$\delta'_2(\{q,s\}, 1) = \{q,r\} \cup \{p\} = \{p,q,r\}$
$\delta'_2(\{r,s\}, 1) = \{p\} \cup \{p\} = \{p\}$
$\delta'_2(\{p,q,r\}, 1) = \{q\} \cup \{q,r\} \cup \{p\} = \{p,q,r\}$
$\delta'_2(\{p,r,s\}, 1) = \{q\} \cup \{p\} \cup \{p\} = \{p,q\}$
$\delta'_2(\{p,q,s\}, 1) = \{q\} \cup \{q,r\} \cup \{p\} = \{p,q,r\}$
$\delta'_2(\{q,r,s\}, 1) = \{q,r\} \cup \{p\} \cup \{p\} = \{p,q,r\}$

Now, we construct the state transition table.

- **State transition table**

δ'_2	0	1
$\{p\}$	$\{q,s\}$	$\{q\}$
$\{q\}$	$\{r\}$	$\{q,r\}$
$\{r\}$	$\{s\}$	$\{p\}$
$\{s\}$	$\emptyset$	$\{p\}$
$\{p,q\}$	$\{q,r,s\}$	$\{q,r\}$
$\{p,r\}$	$\{q,s\}$	$\{p,q\}$
$\{p,s\}$	$\{q,s\}$	$\{p,q\}$
$\{q,r\}$	$\{r,s\}$	$\{p,q,r\}$
$\{q,s\}$	$\{r\}$	$\{p,q,r\}$
$\{r,s\}$	$\{s\}$	$\{p\}$
$\{p,q,r\}$	$\{q,r,s\}$	$\{p,q,r\}$
$\{p,r,s\}$	$\{q,s\}$	$\{p,q\}$
$\{p,q,s\}$	$\{q,r,s\}$	$\{p,q,r\}$
$\{q,r,s\}$	$\{r,s\}$	$\{p,q,r\}$
$\{p,q,r,s\}$	$\{q,r,s\}$	$\{p,q,r\}$

Now, we construct a DFA for this transition table. (next page)



Since the graph is messy and I could not find a way to draw it efficiently. Please refer to the state transition table to check.

• **State transition table**

δ_2'	0	1
$\{p\}$	$\{q,s\}$	$\{q\}$
$\{q\}$	$\{r\}$	$\{q,r\}$
$\{r\}$	$\{s\}$	$\{p\}$
$\{s\}$	$\emptyset$	$\{p\}$
$\{p,q\}$	$\{q,r,s\}$	$\{q,r\}$
$\{p,r\}$	$\{q,s\}$	$\{p,q\}$
$\{p,s\}$	$\{q,s\}$	$\{p,q\}$
$\{q,r\}$	$\{r,s\}$	$\{p,q,r\}$
$\{q,s\}$	$\{r\}$	$\{p,q,r\}$
$\{r,s\}$	$\{s\}$	$\{p\}$
$\{p,q,r\}$	$\{q,r,s\}$	$\{p,q,r\}$
$\{p,r,s\}$	$\{q,s\}$	$\{p,q\}$
$\{p,q,s\}$	$\{q,r,s\}$	$\{p,q,r\}$
$\{q,r,s\}$	$\{r,s\}$	$\{p,q,r\}$
$\{p,q,r,s\}$	$\{q,r,s\}$	$\{p,q,r\}$