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Review

Structure of exotic light nuclei: $Z = 2, 3, 4$

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Abstract. I examine the history and current state of knowledge of the structure of so-called "exotic" light nuclei with $Z = 2$–4, from ^{7}He to ^{16}Be. I review the available experimental information and the models that have been applied to these nuclei. I pay particular attention to the interplay among energies, widths (or strengths), and microscopic structure. Throughout the presentation, I focus on a unified description of these nuclei. I point out contradictions within the data, and I suggest experiments that are still needed.

1 Introduction

I have made no attempt to define the adjective "exotic", nor have I made much use of the term "exotic nuclei". Most of the nuclei to be discussed here have an unusual neutron-proton ratio, so that a label of UNPR nuclei might be more apt. It is a major task to review all the information available for all these nuclei, but I have attempted it for such nuclei with $Z = 2, 3$, and 4.

First I mention the various models that have been used to aid in understanding these nuclei. Then I discuss the situation for many neutron-rich light nuclei. Throughout, I suggest future experiments that might help to answer some outstanding questions.

Many of the nuclei to be discussed here are well described as consisting of a p-shell core plus one or more nucleons in the $2s1d$ shell. In some cases, this structure mixes with a pure p-shell configuration.

In all my research, I have made much use of the compilations *Energy levels of light nuclei*. I was saddened when their publication ceased. Throughout this document, unless indicated otherwise, excitation energies and J^{π} values are taken from one of these compilations: $A = 8$–10 [1], $A = 11$ [2], $A = 12$ [3], $A = 13$–15 [4], $A = 16, 17$ [5], $A = 18$–19 [6], $A = 20$ [7].

2 Models

The simple shell model, with two-body forces, provides an excellent framework for discussing the detailed properties of these nuclei. Near the beginning of the $1p$ shell, LS coupling is a much better approximation than jj [8]. Many of the nuclei to be discussed here are well described as consisting of a $1p$-shell core plus one or more nucleons in the $2s1d$ shell. In some cases, this structure mixes with a pure p-shell configuration. Multi-shell shell-model calculations are fraught with several difficulties. Unless the interactions are renormalized, it frequently turns out that the calculated low-lying states are unrealistically dominated by multi-particle multi-hole configurations. Also, care must be taken to remove contributions of spurious states that arise from translation of the center of mass. For nuclei that are predominantly of the structure having two or three sd-shell nucleons coupled to a p-shell core, a simple shell model has met with surprising success. This model uses local single-particle energies (spe's) and global two-body residual interaction matrix elements (2BME). The latter are taken to be those that arose from an earlier treatment of ^{18}O [9], in which a simple model was used to separate the $(sd)^2$ and core-excited components of the first nine positive-parity states. If present, mixing of these states with p-shell states is handled separately.

Other structure models that I will mention as we go along include multi-channel algebraic scattering (MCAS) [10], the microscopic cluster model (MCM) [11], antisymmetrized molecular dynamics (AMD) [12], adiabatic hyperspherical expansion method three-body cluster model [13].

Occasionally, weak coupling, in the style of Bansal-French/Zamick [14,15], is useful in estimating which core-excited components are important in a given state. Simple one-body potential models are frequently applied to the problems of Coulomb energies and widths.

Occasionally, the calculation of a matter radius is desired. Experimentally, these are usually obtained from a measurement of interaction or reaction cross sections, with the aid of a Glauber model, or some such. Whenever the total density can be modeled as the sum of the density of a core and that of a valence neutron, an exact expression relates the three radii.

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In that case, the three radii are connected by the equation [16,17]

$$A{R_m}^2 = (A-1){R_c}^2 + (A-1){R_v}^2/A, \tag{1}$$

where R_m is the matter radius of nucleus A, R_c is the matter radius of the $A-1$ core, and ${R_v}^2$ is the expectation value of r^2 for the valence neutron. This formula is similar to ones that have been used previously [18–21] in other contexts. And it has been used subsequently by others. It is identical to the equation in refs. [18,19], and is a special case of the general expression in ref. [17].

This approach is exact if two conditions hold: 1) if the total density can be modeled as the sum of the density of a core and that of a valence neutron, and 2) if the configuration of the last neutron is known. Of course, such an expression is useful only if the configuration is dominated by a single term (or a few terms). In such cases, the rms radius of the valence neutron can be evaluated by solving the Schroedinger equation for a neutron in the relevant orbital, and then computing the expectation value of r^2. With multiple configurations, the various values of ${R_v}^2$ are weighted by the wave-function intensities.

For even-N nuclei, we can also use the $2n$ procedure of ref. [19], in which the matter radius is computed from the expression

$${R_m}^2 = ((A-2)/A)({R_c}^2 + 2{R_v}^2/A). \tag{2}$$

This $2n$ procedure was applied to several nuclei in ref. [19]. It was later proposed by Bhagwat *et al.* [21] for use in a much more sophisticated model. The $2n$ procedure and the approximation of $S_n = S_{2n}/2$ has become a common feature of work in this field [22–26]. In order to make a 0^+ state, the last two neutrons must be identical. So, having them share the binding energy equally is reasonable. The $2n$ equation is identical to that of ref. [21], but slightly different from that of ref. [25]. It is a special case of the generalized expression in ref. [17].

This expression is especially useful if the nucleus $A-1$ is unbound. Many have stated that the use of the relation $S_n = S_{2n}/2$ means that $A-1$ is bound by S_n, but a simple argument demonstrates that this is definitely not the case. Consider any partition of the $2n$ energy into S_{n1} and S_{n2}, with $S_{n1} + S_{n2} = S_{2n}$. Because the two neutrons have $J^\pi = 0^+$, the two neutrons must be in the same orbital. Because the two neutrons are identical, symmetry requires an equal term with S_{n1}, S_{n2} reversed, so that the net effect is that the separation energy of each neutron is just its average, which is $S_{2n}/2$. We recently compared the $1n$ and $2n$ procedures for computing matter radii of ^{17}N, for which all the relevant states of ^{16}N are bound. The two results differed by only 0.02 fm [27].

For more than 50 years, the distorted-wave Born approximation (DWBA) [28, 29] has been the procedure of choice for the analysis of direct transfer reactions. The shape of the angular distribution is characteristic of the ℓ value of the transition, and the magnitude of the cross section is proportional to the spectroscopic factor S, which has a dual existence. One is a theoretical construct, namely the square of an overlap integral between a many-body wave function and that of a core + single nucleon. This quantity is well defined, but it is model dependent because both of the wave functions depend on the model used to construct them. The other existence is an experimental one —the ratio of an experimental transfer cross section to one calculated for transfer of a single particle. This S is also model dependent through the parameters used to compute the single-particle transfer cross section. These two definitions are truly independent, but satisfaction is reached when they turn out to be approximately equal for a given nuclear state. Uncertainties plaguing the extraction of an absolute S are greatly reduced for relative spectroscopic factors, especially if the ratio involves two transitions of the same ℓ value.

Transfer of three or more nucleons is usually treated as cluster transfer, whereas two-nucleon transfer is customarily considered microscopically. As we shall see later, a subset of the latter, namely the (t,p) reaction, is an ideal tool for investigating the structure of core + $2n$ states.

If the state reached in single-nucleon transfer is unbound with respect to decay by emission of the transferred particle, a different approach [30] must be used to evaluate the radial integrals that are at the heart of a DWBA calculation. For such states, simultaneous determination of the spectroscopic factor and the decay width provides a powerful tool to identify the ℓ value of the resonance [31]. Or, if the ℓ value is known, determination of the spectroscopic factor allows for a confident calculation of the width. This approach has also been used for alpha transfer [32,33].

3 Neutron-rich Beryllium nuclei

I now discuss exotic nuclei that have a large neutron excess. Perhaps my favorite of all the nuclei being considered is ^{12}Be, so I will start with it.

3.1 ^{12}Be

Perhaps ^{12}Be is the most studied of the nuclei to be discussed here. In the early days, two reactions were used to populate it. By means of the ^{7}Li(^{7}Li$,2p$) ^{12}Be reaction, Howard *et al.* [34] determined a value of 24950(100) keV for the ^{12}Be mass excess and reported an excited state of ^{12}Be at 810(100) keV. The ^{14}C(^{18}O,^{12} Be)^{20}Ne reaction was used by Ball *et al.* [35] to establish a value of 25050(50) keV for the ^{12}Be mass excess, but they found a single excited state in ^{12}Be at 2090(50) keV with no evidence for a state at 810 keV. From the most recent mass evaluation [36], the mass excess is 25078(2) keV. In the intervening years, no other experiment has seen a state near 0.8 MeV.

In the ^{10}Be(t,p) reaction, excited states of ^{12}Be were found at 2089(20), (2712(20), tentative), 4559(25), and 5703(25) keV [37]. These are listed in the first column of table 1. That experiment also observed a weak peak concerning which those authors stated “We do not claim that this belongs to the ^{10}Be$(t,p)^{12}$Be reaction, but if it did

Table 1. Results of the reaction ^{10}Be$(t,p)^{12}$Be.

Excitation energy (MeV)			σ_{rel} (e)	$\sigma_{\max}$ (mb/sr)	J^π
Ref. [37] (a)	Reanal. (b)	Ref. [38] (c)			
0	0	0 (d)	100	2.78	0^+
2.089(20)	2.102(12)	2.111(3)	137	3.79	2^+
(2.712(20))	2.702(17)	2.730(3)	15	0.41	$(0^+, 1^-)$
4.559(25)	4.56(25)	4.580(5)	79	2.20	$(2^+, 3^-)$
5.703(25)	5.70(25)	5.724(6)	41	1.15	(4^+)

(a) Data consisted of one exposure at $E_t = 17$ MeV. Energies are averages of values measured at four forward angles.
(b) Reanalysis of data of ref. [37].
(c) Two exposures at 15 MeV, one at 17 MeV.
(d) Ground-state Q value determined to be $-4.808.3(4.2)$ MeV.
(e) 100 $\sigma_{\text{exc.}}/\sigma_{\text{g.s.}}$ at 3.75 deg.

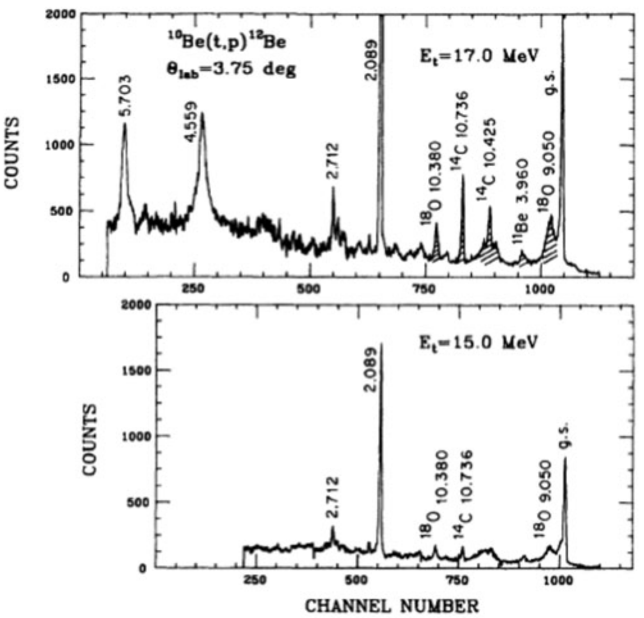

Fig. 1. Spectra of protons from the reaction ^{10}Be$(t,p)^{12}$Be, at a laboratory angle of 3.75 degrees, and $E_t = 17.0$ MeV (top) and 15.0 MeV (bottom). States in ^{12}Be are labeled by their excitation energies. Peaks from impurities in the target are labeled by final nucleus and their excitation energy.

the peak would correspond to an energy level in ^{12}Be at about 2240 keV." Spectra for this reaction are displayed in fig. 1.

The reaction ^{14}C(^{14}C,^{16}O) [39] populated three states of ^{12}Be: at $E_x = 0, 2.10$, and 2.68(3) MeV. The latter was weak, but the authors discussed the possibility that it had $J^\pi = 0^+$. It is likely the same as the state at 2.712(20) MeV that was only tentatively assigned to ^{12}Be in ref. [37].

In a more detailed investigation of the reaction ^{10}Be(t,p) [38], a weak broad state was observed near 5.4 MeV, and a hint of another just below the 5.72 MeV peak. The ground state Q value and excitation energies of other states had smaller uncertainties than in [37] (see table 1). At these negative Q values, angular distributions for 0^+ and 1^- are very similar, as are those for 2^+ and 3^-. Because Bernas *et al.* had suggested 0^+ for the 2.7 MeV state, we indicated a preference for 0^+ over 1^- for it. The weak state at 2.70 MeV was later determined [40] to be 1^-, and an excited 0^+ state was discovered at 2.25 MeV [41]. The strong states at 4.58 and 5.72 MeV are now thought to be 3^- and 4^+, respectively. We had initially preferred 2^+ over 3^- for the 4.58 MeV state, but Millener [42] pointed out that its cross section was too large for a 2^+ state, because most of the 2^+ strength expected in the *psd* space is exhausted by the first 2^+ state at 2.1 MeV. We agreed [43]. It is now clear [44, 45] that this state has all the characteristics —energy, (t,p) cross section, and decay width— to be identified with the 3^- state expected in this energy region.

Because the g.s. of ^{11}Be was known to have $J^\pi = 1/2^+$, it was obvious that excitations into the $2s1d$ shell would be important for low-lying states of ^{12}Be. Barker [46] was the first to perform a cross-shell shell-model calculation for 0^+ states in ^{12}Be. A major question is whether the g.s. is mostly $(sd)^2$ or mostly p shell. Barker's wave functions have the majority (67%) of the s^2 component in the first-excited 0^+ state. Almost simultaneously with [46], different experiments [37, 47] demonstrated the preponderance of $(sd)^2$ in ^{12}Be(g.s.). In the ^{10}Be(t,p) reaction [37,38], the g.s. cross section was about 4–5 times that expected for a pure p-shell g.s. The beta decay half life of 24.4(3.0) ms [47] suggested a reduction in strength of more than a factor of two from that expected for the p-shell ^{12}Be(g.s.), using wave functions from Cohen-Kurath [48]. Later work [38, 49] confirmed this fact. Suzuki and Otsuka [49] concluded that they needed about 65% breaking of the neutron p-shell closure in order to reproduce the experimental $\log ft$ value. These disagree with Barker's calculation, in which the majority of the $(sd)^2$ configuration was in the first excited 0^+ state.

We later performed an $(sd)^2$ shell-model calculation [50], involving only the $s_{1/2}$ and $d_{5/2}$ orbitals, and took the p-shell component from the work of Cohen-Kurath [48]. We did not compute the mixing between p-shell and $(sd)^2$ components, but allowed that mixing to

Table 2. Wave function amplitudes of the lowest two 0^+ states in ^{12}Be [50].

State	s^2	d^2	p-shell
g.s.	0.728	0.387	0.566
exc.	−0.500	−0.265	0.825

be determined from experimental quantities. The lower $(sd)^2$ state was mixed with the p-shell one in a two-state model, with the amplitudes adjusted to fit experimental values —primarily ^{10}Be(t,p) cross sections [38] and the ^{12}Be-^{12}O mass difference [51]. Our wave functions thus had the same s^2/d^2 ratio (0.78/0.22) in the first two 0^+ states, and the same relative sign. In Barker's calculation, the s^2 and d^2 amplitudes had different signs in the first two 0^+ states and different s^2/d^2 ratios.

The wave functions thus determined (table 2) have stood the test of time, as they have been demonstrated to be consistent with a large range of phenomena —including beta decay [47, 49], $E2$'s connecting both 0^+ states to the first 2^+ state [52–54], neutron removal from ^{12}Be to ^{11}Be [55], among others. Except for the $E2$'s, these processes depend only on the g.s. structure, and not the excited 0^+ state. However, a recent experiment [56] measured the Gamow-Teller strengths connecting the g.s. of ^{12}B to these two 0^+ states of ^{12}Be. These B(GT)'s are proportional to the amount of p-shell structure in the two states. Their results were 0.25(5) for the g.s. and 0.60(5) for the excited 0^+ state. Normalizing the sum to unity provides 0.29(6) and 0.71(6), to be compared with the values of 0.32, 0.68 given above. I have summarized these comparisons elsewhere [57].

We also computed wave functions for 2^+ and 4^+ states [38, 43]. In the ^{10}Be(t,p) reaction [38], the first-excited 2^+ state was about 20 times as strong as would have been expected for the p-shell 2^+. This observation led to the conclusion that this state had about 80% of the first $(sd)^2$ 2^+ configuration, and thus only about 20% p shell. The state at 5.72 MeV has the energy and (t,p) strength appropriate to the lowest 4^+ state in this nucleus [38,58]. In a simple $(sd)^2$ shell-model calculation, its configuration is predominantly d^2, with a small amount of dd', where d and d' refer to $d_{5/2}$ and $d_{3/2}$, respectively. Thus, its allowed decay is to the $5/2^+$ state at 1.78 MeV in ^{11}Be. The observed width is 86(15) keV [37,38], and the neutron decay energy is 0.77 MeV, for which the computed single-particle width for $\ell = 2$ decay is 54 keV, resulting in a spectroscopic factor of $S = \Gamma_{\rm exp}/\Gamma_{\rm sp} = 1.59(28)$ [45]. The rigorous upper limit on this spectroscopic factor is 2.0, and the $\ell = 2$ S to a pure $d_{5/2}$ state is about 1.95 [58]. However, the 1.78 MeV state of ^{11}Be is not pure $d_{5/2}$.

Several groups have performed theoretical calculations for ^{12}Be. Kanada-En'yo and Horiuchi [12] computed structures of the ground and excited states of ^{12}Be with antisymmetrized molecular dynamics (AMD). They agree that the g.s., first 2^+, and third 0^+ states are mostly $(sd)^2$, and that the second 0^+ and 2^+ states are mostly p shell. They computed the B(GT) from 0^+ states of ^{12}Be to the 1^+ g.s. of ^{12}B, obtaining 0.9 for the g.s. and 2.1 for the excited 0^+ state. Renormalizing these to correspond to ^{12}B $\rightarrow$ ^{12}Be, their numbers are 0.3 and 0.7, compared with 0.25(5) and 0.60(5) experimentally [56]. For the $B(E2;\, 2_1^+$ to g.s.), they predict $14\,e^2$fm^4, compared with the experimental value of 8.0(2.4) [53].

Romero-Redondo *et al.* [13,59] treated ^{12}Be as a three-body system made of an inert ^{10}Be core and two neutrons with the hyperspherical adiabatic expansion method [13]. They missed the energies of the first 2^+ and second 0^+ by about 1 and 0.7 MeV, respectively.

G. Blanchon *et al.* [60] used the particle-particle random-phase approximation (RPA) in calculations of even Be nuclei from ^{8}Be to ^{14}Be. For ^{12}Be, they did not assume an inert ^{10}Be core. Their g.s. wave function had only 25% s^2, whereas the second 0^+ had 74%.

Descouvemont and Baye [11] used the microscopic cluster model (MCM) in the Generator Coordinate Method to calculate natural-parity molecular states in ^{12}Be. Their excitation energy for the first-excited 0^+ state was 6.85 MeV, and their lowest 4^+ state was at 11.7 MeV.

A no-core shell-model calculation [61] considered only the first few states of ^{12}Be. They concluded that "the description of ^{12}Be needs a larger model space" than the one they had used.

Some groups also computed energies of negative-parity states. Kanada-En'yo and Horiuchi obtained the lowest 1^- state at about 6 MeV, compared with 2.70 experimentally. They had a 5^- state near 8 MeV and the first 0^- above that energy. Romero-Redondo *et al.* missed the 1^- energy by only 130 keV, but their 0^- energy is below the 1^-. Blanchon *et al.* calculated 2.59 MeV for the 1^- energy. Descouvemont and Baye [11] computed only natural parity, and thus had no predictions for 0^- or 2^-. Their predictions for the lowest 1^- and 3^- states were 8.36 and 10 MeV, respectively.

The first four negative-parity states of ^{14}C are well understood as having the dominant configuration ^{13}C(g.s.)$\otimes s$ or d neutron [44, 62]. I applied that same model to ^{12}Be, where the core is ^{11}Be$(1/2^-)$ [44]. This simple model predicts a 1^- excitation energy of 2.785 MeV in ^{12}Be, very close to the known energy of 2.702 MeV. (I note that this is an absolute energy prediction.) The predicted 3^- energy is 4.51 MeV, extremely close to the (3^-) candidate discussed above at $E_x = 4.56$ MeV. It should decay exclusively to the $1/2^-$ state with $S = 1$. Decay to the g.s. would require participation of the $1f$ orbital, which we can ignore. Its experimental width is 107(17) keV [38]. The single-particle width for this energy of 1.09 MeV is 118 keV, giving $S = 0.91(14)$ —in good agreement with the model. The calculated results for the 0^- and 2^- states are 3.593 and 5.12 MeV, respectively. Neither of them has been observed. The parentage of the 0^- and 1^- states is ^{11}Be$(1/2^-)\otimes s$, which has a reasonable overlap with ^{11}Be(g.s.) $\otimes\, p_{1/2}$. The latter can be reached in the reaction ^{11}Be(d,p) via $p_{1/2}$ transfer. Two different investigations of that reaction reported spectroscopic factors for the 1^- state of 0.55(20) or 0.50(20) [60] and 0.27(15) [63]. The 0^- and 1^- spectroscopic factors should be approximately equal in this reaction, resulting in a 0^- cross section of about one third of that for the 1^-. My prediction is that it is unbound (the single-neutron separation energy is 3.171 MeV), so that an experiment that looks for it in coincidence with ^{12}Be gammas will not find it.

As mentioned earlier, Romero-Redondo *et al.* [13] used the hyperspherical adiabatic expansion method in the MCM to compute bound states of ^{12}Be. Garrido *et al.* [64] used the same method to compute unbound states. They considered resonances with $J^\pi = 0^+, 0^-, 1^-$, and 2^+. They decided their 0^+ resonance was to be identified with the experimental state at 4.58 MeV [38], and added a three-body force to adjust their calculated energy by about 1 MeV to make it agree with the experimental one. They stated that their 0^+ state should be strong in the ^{10}Be(t,p) reaction. They did not perform any reaction calculations, and they appear to be unaware of the fact that $2n$ transfer amplitudes interfere coherently. Among 0^+ states, only the g.s. will have all amplitudes in phase for this reaction. Thus, their statement that an assignment of 0^+ to this resonance is consistent with its strong population in (t,p) is incorrect. This 0^+ state is predominantly d^2 in both ref. [64] and in our work [65]. With our wave function [65], destructive interference with a smaller s^2 component causes the predicted (t,p) cross section to be very small. The observed cross section [38] for the 4.58 MeV state is about 55 times that expected [65] for this 0^+ state. In any case, this 4.58 MeV state is almost certainly 3^-, as has been demonstrated elsewhere. Some of these points have been addressed in a Comment [66] and a Reply [67].

The only data that are inconsistent with our model are results of the ^{11}Be(d,p) reaction (in reverse kinematics) [63, 68], and I have offered a possible explanation [57] for that disagreement. We all await a good-resolution forward-angle measurement of that reaction.

Another unbound state (or collection of states) has been recently observed in ^{12}Be [69]. It is formed in proton removal from ^{13}B, and decays with a centroid energy of 1.24 MeV to one or both of the first two states of ^{11}Be. Because the extracted width of 634(60) keV is larger than the single-particle width for $\ell = 2$, and because the peak shape is inconsistent with $\ell = 0$, the authors attribute the decay to $\ell = 1$, implying J^π (if a single state) of $0^\pm$, $1^\pm$, or $2^\pm$, or a combination of some of these values if more than one state is decaying. Reference [69] expressed a preference for 2^- (1^-). Preliminary calculations by the present author indicated a preference for 2^+ [70].

Because the identity of the final state could not be determined, the excitation of the ^{12}Be state is not known, but it is in the range 4.4 to 4.8 MeV. The reported width is larger than the width expected for any of the states expected in this region of excitation. The extra width could arise from several different sources, including 1) formation and decay of more than one state, 2) unresolved decays to both ^{11}Be states, and/or 3) an apparent enhancement factor [71] of about 1.6 in many neutron widths extracted from similar decay-in-flight experiments. Reference [69] mentions this latter possibility. Except for 0^-, no un-observed states are predicted below 4 MeV, but in the region 4 to 5.5 MeV, several states are expected — including the third (and perhaps fourth) 0^+, the second (and perhaps third) 2^+, the first 2^-, and perhaps a second 1^- state. Several of these possibilities are listed in table 3. I estimated the expected excitation energies of these states in a model that includes shell-model and weak-coupling considerations.

Table 3. J^π, configurations, and estimated energies (MeV) of states expected in the region 4.0–5.5 MeV in ^{12}Be.

J^π	Dominant configuration	E_x
2^-	^{11}Be$(1/2^-) \otimes d$	5.12
1_2^-	^{11}Be$(3/2^-) \otimes s$	5.09
2_2^+	^{10}Be$(2^+) \otimes (sd)_0^2$	4.03
0_3^+	Second ^{10}Be(g.s.) $\otimes (sd)^2\ 0^+$	4.35
2_3^+	Second ^{10}Be(g.s.) $\otimes (sd)^2\ 2^+$	4.68
0_4^+	^{10}Be$(2^+) \otimes (sd)^2\ 2^+$	5.48
2_4^+	^{12}Be p shell 2^+	5.46

Table 4. Electromagnetic transition strengths in ^{12}Be.

Initial	Final	$E\lambda$	$B(E\lambda)$	$M(E\lambda)$	Ref.
2_1^+	g.s.	$E2$	8.0(2.4) e^2 fm^4	6.32(92) e fm^2	[53]
0_2^+	2_1^+	$E2$	7.0(6) e^2 fm^4	±2.65(11) e fm^2	[41]
g.s.	1_1^-	$E1$	0.051(13) e^2 fm^2	0.226(29) e fm	[40]
0_2^+	g.s.	$E0$	–	0.87(3) e fm^2	[41]

I estimated the spectroscopic factors, and *sp* widths of these states. I then compared the expected widths with the reported width [69] for states(s) decaying by $\ell = 1$ to one or both of the first two states of ^{11}Be with a centroid decay energy of 1.24 MeV. Results eliminate negative parity as the source of the decay, and also rule out two 2^+ states as the decaying state —leaving one or both 0^+ states and one 2^+ as candidates [59]. The model that has been so successful in accounting for results of various experiments involving ^{12}Be can be combined with a simple two-component model of ^{13}B [72, 73] to estimate the expected strengths in proton removal from the latter. The result [74] is that a 2^+ state near 5 MeV should have a strength that is about four times the strength of the lower 2^+ state. I advocate an experiment to test that prediction.

Some information is available concerning electromagnetic transitions in ^{12}Be, as indicated in table 4.

As mentioned earlier, one calculation [12] predicted $B(E2) = 14\,\mathrm{e}^2\mathrm{fm}^4$ for 2^+ to g.s., compared with the experimental value of 8.0(24) above. With conventional effective charges, the shell model discussed in ref. [41] predicts $B(E2) = 25\,\mathrm{e}^2\mathrm{fm}^4$ for 0_2^+ to 2^+, compared to the experimental value of 7.0(6) above —which is already significantly larger than the $B(E2) = 2.9(10)\,\mathrm{e}^2\mathrm{fm}^4$ for this transition in ^{10}Be. I computed both ^{12}Be strengths with the well-established 0^+ wave functions [50] discussed earlier. I found [52] that I could reproduce both $B(E2)$'s, but only if I included a small ($\sim$ 19%) component of ^{10}Be$(2^+) \otimes (sd)^2_0$ in the ^{12}Be 2^+ state.

The $B(E1)$ for 1^- to 0^+ of 0.051(13) e^2 fm^2 in ^{12}Be is large, but it is smaller than the one for $1/2^-$ to $1/2^+$ in ^{11}Be, 0.115(10) e^2 fm^2 (0.36(3) W.u.) [75], which is perhaps the largest known between low-lying states. Because the $B(E1)$ connecting low-lying states contains destructive interference between some of the amplitudes, small changes to the nuclear-structure assumptions can produce large changes in the computed $B(E1)$. But, by far the biggest influence on the calculated value concerns the type of radial wave function used. Millener *et al.* [75] found that the use of wave functions from a Woods-Saxon potential could increase the computed $E1$ amplitudes by a factor of 5–7 (factors of 30–50 in $B(E1)$) compared to the same calculation using harmonic-oscillator wave functions. The reason for this large increase is the large extent of the radial wave functions in a realistic well.

In a two-state model, the magnitude of $M(E0)$ is proportional to the difference in the expectation value of r^2 for the two basis states. Shimoura *et al.* [41] attributed the large $M(E0)$ in ^{12}Be to the large difference between valence radii for $2s_{1/2}$ and $1p_{1/2}$ orbitals. Lack of knowledge of the appropriate neutron effective charge to be used for $E0$ transitions prohibits a precise calculation of the absolute value.

Very little is known about states in ^{12}Be above 6 MeV. Some evidence exists for a special set of states beginning near 10 MeV that have large cluster overlaps with α + ^{8}He and/or ^{6}He + ^{6}He [76–81]. In ^{12}Be + p inelastic scattering, Korsheninnikov *et al.* [78] observed new levels of ^{12}Be at E_x = 8.6, 10 and $\sim$ 14 MeV, and suggested "likely these levels have the cluster structure He + He." In the ^{9}Be(^{15}N,^{12}N) reaction at 240 MeV, Bohlen *et al.* [81] observed several strong states at high excitation. Primarily based on a $J(J+1)$ dependence of energies, they suggested J^π values of (0^+) to (8^+) (even J only) for peaks at 6.7, 7.4, 10.7, 14.6, and 21.7 MeV. Quite recently, Yang *et al.* [80] presented convincing evidence that a state at 10.3 MeV has $J^\pi = 0^+$ and exhibits an enhanced monopole transition matrix element of 7(1) fm^2 to the g.s. They performed an inelastic breakup experiment with 29 MeV/nucleon ^{12}Be incident on a carbon target. The J^π assignment was made on the basis of an angular correlation analysis in the α+^{8}He channel. Two additional states, at 12.1 and 13.6 MeV, were suggested as candidates for 2^+ and 4^+ states, respectively, of the same configuration, but no J^π assignments were made for them.

Earlier, Freer *et al.* [76,77] had suggested higher-J cluster states, specifically (4^+), (6^+), and (8^+) at excitation energies of 13.2, 16.1, and 20.9 MeV in ^{12}Be. These were only suggested J^π's, even though the angular distribution shapes did imply J higher than 0 or 2. A later experiment by Charity *et al.* [82] did not see any of these states. Yang *et al.* [80] postulated that Freer *et al.* did not observe the lower J states because of the lack of forward-angle data.

Such cluster states have been predicted in a variety of theoretical models, including the generalized two-center cluster model [83–85], the generator coordinate method [11, 86], and antisymmetrized molecular dynamics [12]. In many of these calculations, the resonances are described as α-4n–α clusters.

Table 5. Experimental energies (MeV) of supposed cluster states in ^{12}Be and predominantly $(sd)^4$ states in ^{20}O.

^{20}O (exp)		^{12}Be (exp)	
J^π	E_x	$E_x - 10.3$ MeV	J^π
0^+	0	0	0^+
2^+	1.67	1.8	(2^+)
4^+	3.57	3.3	(4^+)

Table 6. Energies (MeV) of cluster states in ^{12}Be and yrast $(sd)^4$ states from a shell-model calculation.

^{12}Be (exp)		^{8}Be $\otimes (sd)^4$ [(a)]			
J^π	E_x	0^+ core		2^+ core	
		J^π	E_x	J^π	E_x
0^+	10.3	0^+	10.3		
(2^+)	12.1	2^+	12.26		
(4^+)	13.6	4^+	14.07		
(6^+)	16.1	6^+	20.23	6^+	17.1
(8^+)	20.9	8^+	none [(b)]	8^+	23.3

(a) A constant has been added to make the 0^+ state come at 10.3 MeV.
(b) The $T = 2$ $(sd)^4$ space has $J_{\max} = 6$.

I demonstrated [87] that the energies of the experimental cluster states are in good agreement with expectations for states of the structure ^{8}Be $\otimes (sd)^4$. Table 5 compares energies of the first three supposed cluster states in ^{12}Be with experimental energies in ^{20}O, which is also predominantly $(sd)^4$.

Table 6 compares energies of suggested cluster states having J = 0–8 with calculated energies of yrast states in the $(sd)^4$ space. Note that in this space, the maximum J is 6. From this comparison, it is likely that the 20.9 MeV resonance has $J = 6$, rather than 8. If the 16.1 MeV state has $J = 6$, it is probably of the structure $2 \otimes (sd)^4_4$. There is no contradiction in the fact that these states can be described both ways. Long ago, Wildermuth and co-workers [88] proved the equivalence of shell-model and cluster-model descriptions.

To summarize the situation in ^{12}Be: A quite simple model is consistent with all the data involving all the known positive-parity states of ^{12}Be, except for results of the ^{11}Be(d,p) reaction. That experiment should be repeated, with better resolution and with data at forward angles. If a CD_2 target is used, the absolute cross-section scale should be determined by measuring the D content of the target, not the C content. An even simpler model explains the properties of the known 1^- and 3^- states, but 0^- and 2^- remain unknown. The 0^- should be visible in the ^{11}B(d,p) reaction with a cross section that is about one third of that for the 1^- state. Finally, the nature of the new state, or states, decaying to one or both of the first two states of ^{11}Be with a neutron energy of 1.24 MeV remains to be identified. My strong conviction is that it is the 2^+ state that has most of the p-shell 2^+ configuration, with perhaps some contribution from one or two 0^+ states.

3.2 ^{11}Be

Wilkinson and Alburger [89] were the first to suggest that the g.s. of ^{11}Be might have positive parity, based on their study of beta decay of ^{11}Be to states of ^{11}B. At that time, available information on the ^{11}B levels did not allow a definite conclusion, but the suggestion was made. They stated that if the parity was positive, the spin was likely 1/2. Shortly thereafter, Talmi and Unna [90] considered the effect of the $p_{3/2}$ proton on the neutron interaction, and explained how the ^{11}Be g.s. might have positive parity. They pointed out a fact that seems to be ignored by many even today —the $1/2^+$ state is below the $1/2^-$ in ^{9}Be. They cautioned that the $1/2^+$ and $1/2^-$ energies should be quite close, and they stated "The only definite prediction is thus that the $1/2^+$ and $1/2^-$ levels should be very close in ^{11}Be. It would not be surprising if the ^{11}Be spin is $1/2^-$." Nevertheless, they generally get credit for predicting the $1/2^+$ g.s. Other subsequent evidence [91–93] supported the possibility of positive parity for this state. This conclusion was unambiguously demonstrated by Alburger *et al.* [94] in a detailed experimental study of the levels in ^{11}B which are fed in the beta decay of ^{11}Be. Auton's observation [95] of an $\ell = 0$ angular distribution in the reaction ^{10}Be(d,p) confirmed the $1/2^+$ assignment. Auton also observed the first-excited state, at 0.32 MeV, with an $\ell = 1$ angular distribution. Spectroscopic factors for $1/2^+$ and $1/2^-$ were 0.73(6) and 0.63(15), where the uncertainties are those that arose from ambiguities in optical-model and bound-state parameters.

From the earliest days, an understanding of the structure of the $1/2^+$ ground state (g.s.) of ^{11}Be centered on a dominant configuration of an $s_{1/2}$ neutron from the next major shell coupled to a p shell ^{10}Be(g.s.). The following decades saw a rash of theoretical papers [96–105] concerning this state, espousing widely different spectroscopic factors —from 0.55 [96] to 0.93 [98]. Winfield *et al.* [106] summarized many of these calculations. A later ^{10}Be(d,p) investigation at 25 MeV [107] gave $S = 0.77$. Reanalysis of these two sets of (d,p) data also produced a wide range of S's, some quite small —0.36 or 0.44 in ref. [108] and 0.5 in ref. [109]. Barker [110] quotes a "published" value of 0.19(2), and cites a conference proceeding [111]. Aumann *et al.* [112] studied neutron knockout from ^{11}Be and concluded that its g.s. is dominated by the $s_{1/2}$ single-particle component with a small $2^+ \otimes d_{5/2}$ admixture. Many of the various experimental spectroscopic factors were summarized by Palit [113].

Keeley *et al.* [114] used a continuum discretized coupled channels approach to compute angular distributions for both sets of ^{10}Be(d,p) data. With a g.s. spectroscopic factor of 0.93 [98], their computed forward-angle cross sections at 12 MeV are about 1.22 times the experimental ones, implying $S \sim 0.76$. Quite recently, there appeared a very careful experiment and a detailed analysis of the ^{10}Be$(d,p)^{11}$Be reaction at several energies [115]. The average of their S's was 0.71(5), very close to Auton's value. Some of this excellent agreement may be fortuitous, because the uncertainty in Auton's $S = 0.73(6)$ does not include the uncertainty in absolute cross-section scale arising from normalizing ^{10}Be $+ d$ elastic scattering to calculations of an optical model. Nevertheless, this important spectroscopic factor now appears to have been reliably determined as 0.71(5). It is also consistent with the values of refs. [107, 114].

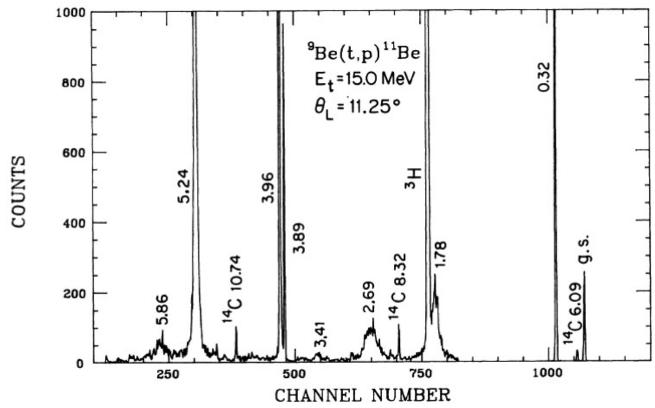

Fig. 2. Spectrum of the reaction ^{9}Be$(t,p)^{11}$Be [116]. States in ^{11}Be are labeled by their excitation energies. Peaks from impurities in the target are labeled by final nucleus and their excitation energy.

In addition to the first two states, Zwieglinski *et al.* [107] also observed a state at 1.78 MeV with an $\ell = 2$ angular distribution —interpreted as $5/2^+$ with $S = 0.50$. Its width of 102(14) keV [116], combined with a single-particle width of 175 keV gives $S = 0.58(8)$ —in good agreement. Presumably the majority of the remaining structure of this state consists of the configuration $2^+ \otimes s$, just as some of the g.s. structure is $2^+ \otimes d$. In a large basis *ab initio* shell model investigation of ^{11}Be [104], the ^{11}Be ground state has a large overlap ($S = 0.82$) with $(^{10}\mathrm{Be(g.s.)} \otimes (s_{1/2}))$ but also with $(^{10}\mathrm{Be}(2^+) \otimes (d_{5/2}))$ ($S = 0.26$). In Teeters and Kurath [105], the $1/2^+$ g.s. has components 0.818 $(0 \otimes s)$ and 0.123 $(2 \otimes d)$; the first $5/2^+$ state is 0.668 $(0 \otimes d)$, 0.179 $(2 \otimes s)$, and 0.088 $(2 \otimes d)$.

The ^{9}Be(t,p) reaction [92, 93, 116–118] has aided considerably in the identification of states in ^{11}Be. A spectrum for this reaction is displayed in fig. 2.

Despite some early confusion [3] and incomplete information [117, 118] concerning J^π values, seven of the first eight known states of ^{11}Be now have unique J^π assignments ([119], and references therein) —the exception is the $J = 3/2$ state at 3.41 MeV, whose parity is uncertain. As indicated in table 7 and fig. 3, these states can be grouped into three separate categories, having zero, one, or two neutrons in the $2s1d$ shell.

Table 7. Properties of the first nine states of ^{11}Be (energies in MeV, widths in keV).

Config.	J^π	Compilation [2]		Other [(a)]		S(g.s.) [(d)]	$S(2^+)$ [(d)]
		E_x	Γ	E_x	Γ		
p shell	$1/2^-$	0.320	–	0.320(2)	–	~ 0.6	–
	$3/2^-$	2.654(10)	206(8)	2.642(9)	228(21)	0.12	–
	$5/2^-$	3.889(1)	< 8	3.888(1)	3.2(8) [(b)]	–	$0.58^{+.20}_{-.26}$
$^{10}\mathrm{Be} \otimes (sd)$	$1/2^+$	0.00	–	0.00	–	0.71(5)	~ 0.2
	$5/2^+$	1.783(4)	100(10)	1.778(4)	104(21)	0.58(8)	~ 0.4
	$3/2(^+)$	3.40(6)	122(8)	3.398(6)	104(17)	0.10	large
$^{9}\mathrm{Be} \otimes (sd)^2$	$3/2^-$	3.955(1)	10(5)	3.955(1)	7.9(7) [(b)]	$\sim 10^{-3}$	0.12–0.17
	$5/2^-$	5.255(3)	45(10)	5.255(3)	29(8) [(c)]	< 0.02	0.024(4)
	$(1/2^-)$	5.849(10)	139(17)	5.849(10)	139(17)		

(a) Reference [116], unless indicated otherwise.
(b) Reference [120].
(c) Reference [119].
(d) See text.

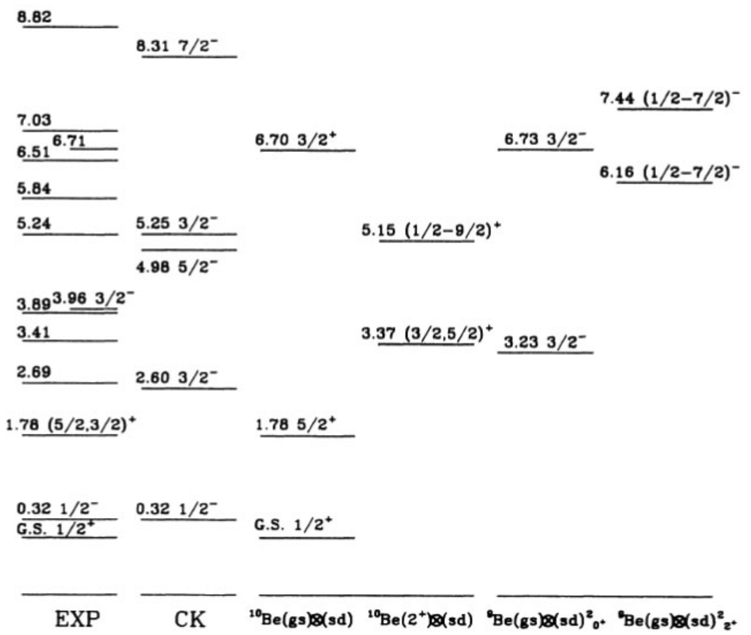

Fig. 3. Experimental and theoretical energy levels of ^{11}Be [116].

The $1/2^-$, $3/2^-$, and $5/2^-$ states at excitation energies of 0.32, 2.69, and 3.89 MeV, respectively, are predominantly of pure p-shell character. The $3/2^-$ and $5/2^-$ levels at 3.96 and 5.24 MeV are dominated by $(sd)^2$ configurations, as evidenced by their large cross sections in the $^9\mathrm{Be}(t,p)$ reaction. Even a small amount of mixing (about 10%) between the $1/2^-$ states was sufficient to double the cross section of the lowest $1/2^-$ state [116].

The g.s. and $5/2^+$ state at 1.78 MeV have been discussed above as having one neutron in the sd shell. If the 3.41 MeV state has positive parity, it would also have this structure, but primarily $2^+ \otimes s$. The $5/2^+$ state of this configuration has presumably mixed with the $0^+ \otimes d$ configuration to form the 1.78 MeV state, but a $5/2^+$ state with approximately the orthogonal linear combination of basis states should exist somewhere in the region. At present, there are no candidates for it. Presumably, it is high enough in energy that it would have a large $\ell = 0$ decay to $^{10}\mathrm{Be}(2^+)$, and would therefore be difficult to observe.

Consideration of neutron decay widths and branching ratios (BR) helped to understand the major components of these ^{11}Be states [119]. The width of the first $3/2^-$ state is 213(15) keV, implying a spectroscopic factor of about 0.12, to be compared with the calculated p-shell value of 0.106 [8] or 0.168 [121]. For the next $3/2^-$ state at 3.96 MeV, the compilation lists a width of 15(5) keV. A careful examination of the (t,p) data [116] provided

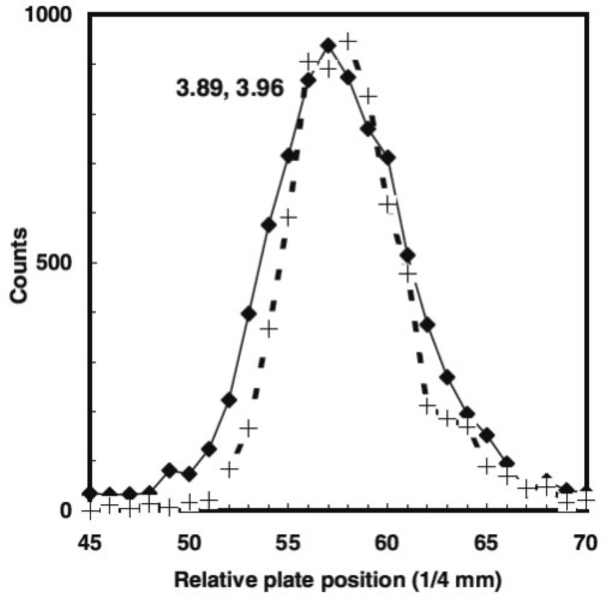

Fig. 4. Data [116] for the 3.96 MeV state (diamonds) and 3.89 MeV state (+ 's) (shifted and re-normalized) superimposed [120].

a width of 7.9(7) keV [120]. For decay to the g.s., the *sp* width is difficult to calculate, but we estimate it to be $\sim$ 2.3 MeV. The g.s. branch for the 3.96 MeV state is 0.22(4) in ref. [122] and 0.48(6) in ref. [123], leading to $S(\text{g.s.}) \sim 0.8 \times 10^{-3}$ or 1.6×10^{-3}. These are small enough to arise from small, neglected components in the wave function. For decay to the 2^+ state, the decay energy is 98 keV, for which Γ_{sp} is 36 keV. Thus, if we use the β decay BR [122], we have $S = 0.17$ for 2^+ decay, quite a large value (though with some uncertainty) for a state thought to be dominated by the configuration $^9\text{Be(g.s.)} \otimes (sd)^2$. The BR from ref. [123] gives $S = 0.12$, still quite large. Millener [8] has $S = 0.864$ for the *p*-shell $3/2^-$ to decay to 2^+, but this is presumably the 2.69 MeV state. These S's would then imply mixing between these two $3/2^-$ states of about 14 to 20%. The strengths of the two states in (t,p) indicate very little mixing ($\sim$ 1%–2%) between them. The small g.s. spectroscopic factor for the 3.96 MeV state is also consistent with very little mixing. It would be very useful to have a better value for the BR of the 3.96 MeV state.

For $5/2^-$, the width in the compilation for the 3.89 MeV state is only a limit, $<$ 10 keV. Analysis in ref. [120] provided 3.2(8) keV. These two states are shifted and superimposed in fig. 4. This comparison demonstrates that the 3.96 MeV state has a larger natural width than does the 3.89 MeV state.

Reference [119] had used the theoretical S [8,121] for the $5/2^-$ state decay to the 2^+, together with the measured 2^+ BR of $0.62^{+.14}_{-.21}$ [122] to predict the width of this state to be $3.1^{+1.5}_{-0.6}$ keV. The result is reasonably consistent with the value of 3.2(8) keV found in ref. [120]. Alternatively, the new measured width, together with the published BR leads to $S = 0.58^{+.20}_{-.26}$ for decay to 2^+, to be compared with theoretical values of 0.57 or 0.66 [8,121]. The state at 5.24 MeV has been assigned $J^\pi = 5/2^-$ and suggested to have the configuration $^9\text{Be}\otimes(sd)^2_{2+}$. Its width is given in the compilation [3] as 45(10) keV. No width is listed for this state in the (t,p) paper, but subsequent analysis [119] of those data yields $\Gamma = 29(8)$ keV, giving an average of 35(6) keV. The only observed n decays [122,123] are to the 2^+. With an energy of 1.38 MeV, the $\ell = 1$ *sp* width is 1.45 MeV, giving $S = 0.024(4)$. This S is small enough that it could easily be acquired by mixing with the *p*-shell $5/2^-$ —which is predicted to have $S = 0.66$. The limit of BR < 0.27 for the g.s. branch corresponds to $S(f_{5/2}) < 0.02$.

The primary evidence for positive parity for the 3.41 MeV state is breakup of ^{11}Be on a carbon target [124], interpreted as $E2$ excitation followed by neutron decay. The angular distribution has a clear $L = 2$ shape, as is also the case for the $5/2^+$ state at 1.78 MeV. However, beta decay favors $3/2^-$. If $J^\pi(3.41)$ is $3/2^+$, its width implies a $d_{3/2}$ spectroscopic factor of $\sim$ 0.10, in agreement with the prediction of [105]. This S is large enough that it should exhibit a clear $\ell = 2$ stripping pattern in $^{10}\text{Be}(d,p)$. Reference [107] states that they investigated the range of excitation energy up to $E_x = 7.0$ MeV with the $^{10}\text{Be}(d,p)$ reaction, but their published spectrum goes only to just above 3 MeV. They state that they did not observe the 3.41 MeV state. However, they also state that the 2.69 MeV state was not excited with measurable strength. The analysis in ref. [119] gave a value of $S \sim 0.12$ for the latter.

Many additional states are known at higher excitation in ^{11}Be. Six other levels were observed in the $^9\text{Be}(t,p)$ reaction at 20 and 23 MeV, but with no J^π information. Six additional states up to 18 MeV were fed in beta decay of ^{11}Li, including two with J^π assignments: $3/2^-$ at 8.02(2) MeV and $5/2^-$ at 10.59(5) MeV [122]. Some high-lying states were selectively excited in heavy-ion induced reactions, including $^9\text{Be}(^{16}\text{O},^{14}\text{O})^{11}\text{Be}$ [123] and $^9\text{Be}(^{13}\text{C},^{11}\text{C})$ [125], but with little structure information. The first $T = 5/2$ state is at 21.16(2) MeV [126]. I will not discuss any of these states further here.

To summarize: the first eight levels of ^{11}Be are reasonably well understood. Outstanding questions include the parity of the 3.41 MeV state and the BR of the 3.96 MeV state. Other positive-parity states of ^{11}Be are now understood as arising from coupling s and d neutrons to the g.s. and first-excited 2^+ state of ^{10}Be. Low-lying negative-parity states consist both of normal *p*-shell states and states with two *sd*-shell neutrons coupled to ^{9}Be. The $^9\text{Be}(t,p)$ reaction aided in this identification. Consideration of neutron decay widths suggested that mixing of *p*-shell and $(sd)^2$ components was small, but even a small amount of mixing (about 10%) was sufficient to double the (t,p) cross section of the lowest $1/2^-$ state [116].

3.3 ^{13,15}Be

Neither ^{13}Be nor ^{15}Be has any bound states. Both the experimental [78,127–141] and theoretical [57,60,142–150]

Table 8. Energies and widths (MeV) of supposed first and second $5/2^+$ resonances.

State	Source	E	Γ	Γ_{calc} (a)	Γ_{calc} (a)(b)
$5/2^+_1$	Randisi *et al.*	$0.85^{+0.15}_{-0.11}$	$0.30^{+0.34}_{-0.15}$	0.041 or 0.057	–
	Marks *et al.*	1.05(10)	0.50(20)	0.067 or 0.094	–
$5/2^+_2$	Randisi *et al.*	2.35(14)	1.50(40)	0.075 or 0.121	0.47 or 0.74
	Marks *et al.*	2.56(13)	2.29(73)	0.102 or 0.166	0.56 or 0.91

(a) Reference [151].
(b) If supposed $5/2^+_2$ is really the first $5/2^+$ state.

situations are extremely confusing in ^{13}Be. Experimentally, the lowest-energy resonance has been reported as $1/2^+$, $1/2^-$, or $5/2^+$. Theoretically, various calculations have advocated all three of these, plus $3/2^+$. I have summarized the experimental and theoretical history elsewhere [152], and I will not repeat that history here, although I will make selected references to it as we go along. Three experiments [153–156] after that summary appeared have not clarified the situation. Two of those papers also contained good summaries [153, 154]. Here, I concentrate on recent developments.

A few items stand out in the saga of ^{13}Be. A plethora of evidence exists for a d-wave resonance (or two) in the energy region 2–3 MeV [78,131–133,137,138,141]. The apparent s-wave structure near threshold [134] is probably not there. Randisi *et al.* concluded that analysis of a very narrow structure at threshold in the ^{12}Be + n channel demonstrated that it resulted from the sequential decay of the unbound ^{14}Be(2^+) state rather than a strongly interacting s-wave virtual state in ^{13}Be. Later suggestions [154, 155] that the lowest d resonance is near 1 MeV are probably incorrect [156]. Events near 1 MeV appear to be clearly present [133,153–155], but they probably do not represent g.s. resonances of that energy. Rather, they are more likely due to decay to excited states of ^{12}Be [151]. Simon *et al.* [137] concluded that the ^{12}Be$+n$ decay scheme is complicated by branches feeding excited states in ^{12}Be.

Reported widths for the two lowest $5/2^+$ resonances suggested by Randisi *et al.* [154] and Marks *et al.* [155] are compared with calculated widths [151] for those resonances in table 8. The calculated widths make use of published spectroscopic factors [151, 154].

The lowest d state should be primarily of the structure $(sd)^3$ [152], even though its $d_{5/2}$ spectroscopic factor to ^{12}Be(g.s.) should be large —0.67 [154] or 0.94 [151]. The second $5/2^+$ state will be mostly $d_{5/2}$ single particle, but with a very small S(g.s.) = 0.01 [154] or 0.0004 [151]. This resonance is an excellent candidate [151] for decay to the excited 0^+ state of ^{12}Be ($S = 0.65$ [154] or 0.85 [151]), or by s wave to the 2^+ ($S = 0.23$ [154] or 0.15 [151]). Its calculated branching ratio is plotted in fig. 5.

It can be noted in table 8 that the experimental widths are too large by about an order of magnitude. However, if the suggested second $5/2^+$ state is actually the first one, then the measured widths are too large by only about a factor of three —even less if a proposed enhancement factor of 1.6 [157] is removed from the experimental widths.

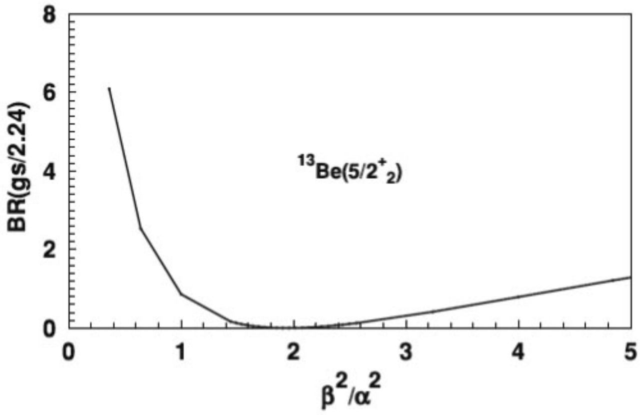

Fig. 5. For the second $5/2^+$ state of ^{13}Be, the ratio of widths for decay to the g.s. and excited 0^+ state is plotted *vs.* β^2/α^2, the ratio of sp to $(sd)^3$ in the $5/2^+$ state.

A peak at 0.51(1) MeV, with a width of 0.45(3) MeV, was clearly observed by Kondo *et al.* [141], and suggested to be $1/2^-$. Their fit to the transverse momentum distribution was about 2/3 $\ell = 1$ and 1/3 $\ell = 0$. We pointed out [158] that its width was too large by about a factor of two, and offered two possible explanations [157,158] for the discrepancy. That peak was not accompanied by any prompt ^{12}Be gamma rays —implying that it represents neutron decay to the g.s. and/or to the excited 0^+ state, whose long lifetime would cause the ^{12}Be nuclei to leave the detection region before decaying. It would be interesting to check whether a mixture of s and d provides as good a fit as $p + s$ for this decay. The reaction they used to produce ^{13}Be was neutron removal from ^{14}Be, which should populate resonances of both parities. Indeed, they reported one or two d-wave resonances just above 2 MeV. Aksyutina *et al.* [153] also used the neutron removal from ^{14}Be to populate resonances in ^{13}Be. However, their width for the 0.45 MeV peak was only 0.11(2) MeV. They found that an $\ell = 0$ resonance alone could not describe the low-energy part of the spectrum. They also needed an $\ell = 1$ component, as previously found by Kondo *et al.* and Simon *et al.*

Unanswered questions that still remain in ^{13}Be are:

1) Is the lowest resonance near 0.5 MeV $1/2^+$ or $1/2^-$, or the two unresolved?
2) Can better evidence be found for decays of ^{13}Be resonances to excited states of ^{12}Be? A recent experi-

ment [159, 160] provided some evidence (though with limited statistics) that events in the 1 MeV region are coincident with 2.11 MeV gammas.

3) Can the spectroscopic factor be measured for the first (and second) $5/2^+$ resonance(s)?
4) Do any of the events near 1 MeV correspond to g.s. resonances?

Turning now to ^{15}Be, Spyrou *et al.* [161] used two-proton knockout reaction from a ^{17}C beam at 55 MeV/u in an attempt to produce this unbound nucleus. No events corresponding to the decays $^{15}\text{Be} \rightarrow {}^{14}\text{Be} + n$ were observed, leading to the conclusion that "any populated states in ^{15}Be decay through three sequential neutron decays into ^{12}Be, via the unbound first excited state of ^{14}Be. Therefore, these states in ^{15}Be must be neutron unbound by more than 1.54 MeV, which is the location of the first excited 2^+ state in ^{14}Be."

Snyder *et al.* [162] were the first to observe ^{15}Be. They populated it using neutron transfer with a 59 MeV/u ^{14}Be beam. Detection of ^{14}Be and neutrons in coincidence produced a reconstructed decay energy spectrum with a resonance at 1.8(1) MeV and a tentative spin-parity assignment of $5/2^+$.

Kuchera *et al.* [163] used two-proton removal from a 55 MeV/u ^{17}C beam in an attempt to populate neutron-unbound states in ^{15}Be and observe decays into $^{12}\text{Be}+3n$. A ^{15}Be component was not needed to describe the data. In the best fit, ^{15}Be was found to be unbound with respect to ^{12}Be by 1.4 MeV (unbound with respect to ^{14}Be by 2.66 MeV).

The g.s. of ^{15}Be is expected to have $J^\pi = 3/2^+$ or $5/2^+$. I investigated the properties of the lowest $5/2^+$ state in a simple $(sd)^3$ shell-model calculation [164] that has previously worked extremely well in predicting absolute energies of the lowest $5/2^+$ state in ^{19}O, ^{17}C, and ^{13}Be [161]. When combined with recent experimental results [162, 163], the analysis produces tight constraints on the s and d single-particle energies in ^{13}Be [164]. For example, if this state is unbound to $^{12}\text{Be}+3n$ by 1.8(1) MeV, the s resonance in ^{13}Be would be at $E_s = 0.48(3)$ MeV.

3.4 ^{14,16}Be

The mass excess of ^{14}Be was first measured in a pion-induced double charge exchange reaction $^{14}\text{C}(\pi^-, \pi^+)^{14}\text{Be}$ [165]. The resulting mass excess was 40.10(16) MeV, which was about 0.6 MeV more bound than expected at the time. The new mass evaluation lists 39.95(13), which corresponds to a $2n$ separation energy of $S_{2n} = 1.27(13)$ MeV to $^{12}\text{Be} + 2n$ [36]. The pion data appeared to contain evidence of weakly-populated excited states, but they were not analyzed. Only the ground state (g.s.) is bound. The first-excited 2^+ state, at $E_x = 1.59(11)$ MeV, $E_{2n} = 0.25(6)$ MeV, was first observed in a heavy-ion reaction [166] and later confirmed by other experiments [167–169]. The current average excitation energy of this state is 1.54 MeV. One experiment [170] reported a state at 4.1 MeV. Simon *et al.* [137] observed a state at $E_x = 2.56(21)$ MeV ($E_{2n} = 1.22(18)$ MeV), with a width of $\Gamma = 1.67(76)$ MeV that had not been observed in other experiments. They suggested it was the second 0^+ state of ^{14}Be, by comparison to the $E_x = 3.03$ MeV 0^+ state in the isotone ^{16}C. J^π and excitation energies in the two nuclei are compared in table 9.

Table 9. J^π and excitation energies (MeV) of the lowest levels in isotones ^{16}C and ^{14}Be.

^{16}C		^{14}Be	
J^π	E_x	J^π	E_x
0^+	0.0	0^+	0.0
2^+	1.766(10)	2^+	1.54
0^+	3.027(12)	(0^+)	2.56(21)
2^+	3.986(7)	(1^-)	3.07(10)
$3^{(+)}$	4.088(7)	2^+	3.55(9)
4^+	4.142(7)	?	(4.1)
2^+	6.109(15)	(3^-)	5.26(14)

An electromagnetic dissociation experiment [170] indicated evidence for a dipole state at $E_{2n} = 1.8(1)$ MeV, with a width $\Gamma = 0.8(4)$ MeV. A more recent experiment [171] used the inelastic scattering of ^{14}Be from hydrogen to investigate low-lying resonances, by detecting $^{12}\text{Be} + n + n$ in coincidence. They observed the extreme tail of the first 2^+ state, and reported two other resonances at $E_{2n} = 2.28(9)$ and 3.99(14) MeV, with widths of 1.5 and 1.0 MeV, respectively. These widths were not obtained by fitting, but rather were held fixed in the analysis. They assigned 2^+ to the first of these and suggested (3^-) for the second.

All the low-lying positive-parity states of ^{14}Be are expected to be well described in terms of two components —a p-shell ^{12}Be(g.s.) coupled to two sd-shell neutrons and a p-shell ^{10}Be(g.s.) plus $\nu(sd)^4$, with the former dominating in the lowest states. In a simple model that has proven quite successful in describing the properties of $(sd)^2$ states in light nuclei, I computed the energy splittings and wave functions of 0^+ and 2^+ $(sd)^2$ states in ^{14}Be. Luckily, the wave functions of the $(sd)^2$ states in ^{14}Be will depend only on the energy difference $E_d - E_s$, and not on the actual energies. I previously estimated this difference to be about 2.3 MeV [152]. The low-energy positive-parity spectrum will contain two 0^+ states, two 2^+ states, and one 3^+. A 4^+ state will lie somewhat (1–2 MeV) higher. (In my space, the 4^+ is pure dd and the 3^+ is pure ds.) The energy splitting between the two 2^+ states will also depend only on $E_d - E_s$. Results of the calculation for 2^+ states are listed in table 10 [172].

Aksyutina *et al.* [171] analyzed fractional energy distributions as functions of ε_{fn} and ε_{nn}. (Here ε_{fn} and ε_{nn} are the relative energies between the heavy fragment and one neutron and between the two neutrons, respectively.) For the first two 2^+ states, they assumed "democratic" decay (no ^{13}Be intermediate state) and did the analysis in

Table 10. Energy splittings and wave functions of the lowest 2^+ $(sd)^2$ states in ^{14}Be.

State	Wave function		ΔE (MeV)
2_1^+	0.92 ds	0.08 d^2	
			2.20
2_2^+	0.08 ds	0.92 d^2	

terms of the ℓ values of the two neutrons, allowing combinations of dd and ds. For the analysis of data for the first 2^+, ref. [171] obtained a dd contribution of 41(7)%, and 9(4)% for the second 2^+ state. They concluded that they had confirmed the $(1d5/2)^2$ nature of the first 2^+ and had determined that the structure $(1d5/2)(2s1/2)$ dominated the second. (They credit refs. [166, 167] for the claim that the first 2^+ state is d^2, but I do not find that statement in either reference.) These results might appear to be in conflict with the wave functions of table 10, but I demonstrated elsewhere [172] that the numerical results do not conflict. I provide a concise summary here.

In the present case, $2n$ decay from the first $(sd)^2$ ^{14}Be 2^+ state cannot proceed to the major $(sd)^2$ component of ^{12}Be(g.s.), but only to the $^{12}\mathrm{Be}_{1p}$ component (because the two sd-shell neutrons in ^{14}Be have $J = 2$, but in ^{12}Be, J is 0). But, a small amount of the $(sd)^4$ configuration changes the situation drastically. Let the wave function of the first 2^+ state be

$$^{14}\mathrm{Be}(2_1^+) = A\,^{12}\mathrm{Be}_{1p} \otimes (sd)^2_{2^+} + B\,^{10}\mathrm{Be}_{1p} \otimes (sd)^4_{2^+}, \quad \text{with } A^2 + B^2 = 1.$$

The $(sd)^4$ component has configurations s^2d^2, sd^3, and d^4. The first and third can decay via dd to the s^2 and d^2 terms, respectively, in ^{12}Be(g.s.). The second term will decay via ds to the d^2 term. With a simple $(sd)^4$ wave function, the variation of the calculated dd/ds ratio with A^2 is such that a very small (16(4)%) admixture of the $(sd)^4$ component in the dominantly $(sd)^2$ 2^+ state reproduces agreement with the measured ratio (fig. 6).

The decay data are thus in agreement with a wave function for the first 2^+ state that is predominantly of ds character —and not dd as claimed in ref. [171]. A concise statement of the situation is the reminder that decays (or removal reactions) do not measure the structure of the initial state, but rather the overlap of initial and final states.

For the second 2^+ state, the very small amount of dd decay requires destructive interference between two major amplitudes. As above, let the wave function of the second 2^+ state be

$$^{14}\mathrm{Be}(2_2^+) = A'\,^{12}\mathrm{Be}_{1p} \otimes (sd)^2_{2_2^+} + B'\,^{10}\mathrm{Be}_{1p} \otimes (sd)^4_{2^+}, \quad \text{with } A'^2 + B'^2 = 1.$$

With destructive interference, the dd amplitude will vanish for some value of A', so that to fit the measured dd intensity of 9(4)% would require A'^2 in the vicinity of $A'^2 \sim 0.72$ [172] (fig. 7).

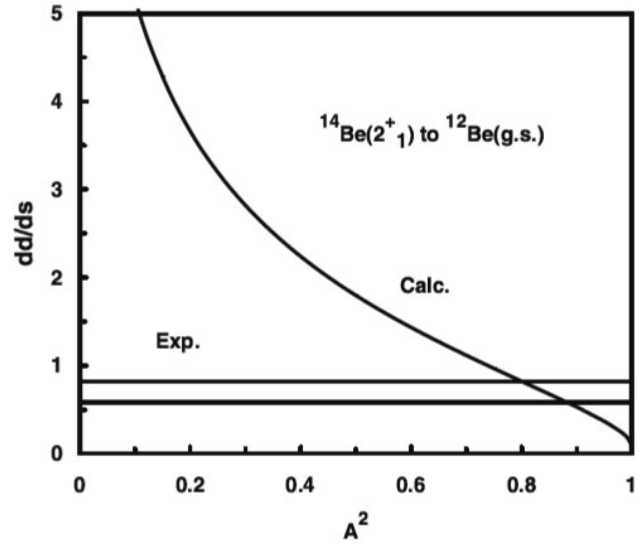

Fig. 6. For $2n$ decay of the first 2^+ state of ^{14}Be, the calculated dd/ds ratio is plotted *vs.* A^2, the amount of $(sd)^2$ in the decaying state. The horizontal lines represent the experimental result [171]. Calculation and experiment agree for $A^2 = 0.84(4)$.

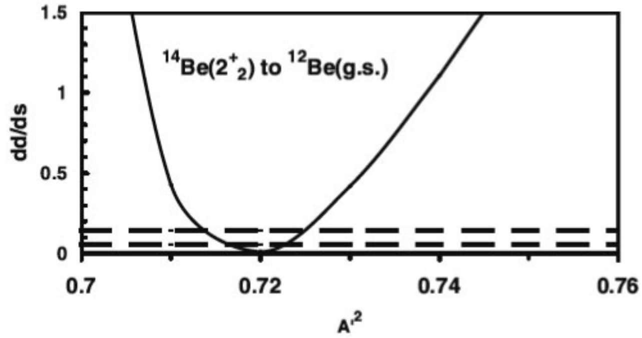

Fig. 7. Same as fig. 6, but for the second 2^+ state of ^{14}Be. The calculated dd/ds ratio is plotted (solid curve) *vs.* A'^2, the amount of $(sd)^2$ in the decaying state. The dashed horizontal lines represent the experimental ratio, implying $A'^2 \sim 0.72$.

Thus, the $(sd)^2$ components of the first two 2^+ states are expected to be 0.84(4) and ~ 0.72, with the remainder being $(sd)^4$.

If the third resonance observed in ref. [171] is neither 3^+ nor 0^+, it is outside the present model. Earlier, I had discussed the interdependence of energies in ^{13}Be and ^{14}Be [173]. One of the conclusions there was that if the first s-wave resonance is below 0.7 MeV in ^{13}Be, then the g.s. of ^{14}Be is greater than 90% $(sd)^2$. These are summarized in table 11.

Moriguchi *et al.* [174] measured the reaction cross sections of ^{14}Be with proton and carbon targets at about 41 and 76 MeV/nucleon. Fitting to the experimental data provided admixtures for the g.s. of ^{14}Be of 72(7)% s^2 and 28(7)% d^2, or 61(9)% s^2 and 39(9)% p^2.

Discussion of configuration mixing in ^{14}Be involves a subtlety that should not be ignored. The only configurations that are important for low-lying states are

Table 11. $(sd)^2$ parentage in selected states in ^{16}C and ^{14}Be.

State	% $(sd)^2$	Ref.
^{16}C(g.s.)	~ 98	[175]
^{14}Be(g.s.)	> 90 $^{(a)}$	[173]
^{14}Be(2_1^+)	84(4)	[172]
^{14}Be(2_2^+)	~ 72	[172]

$^{(a)}$ If $E_s(^{13}\text{Be}) < 0.7\,\text{MeV}$ [173].

$^{12}\text{Be}_{1p} \otimes (sd)^2$ and $^{10}\text{Be}_{1p} \otimes (sd)^4$, where the $1p$ subscript denotes pure p-shell structures, and the sd-shell nucleons are all neutrons. So, in an obvious notation, the g.s. is

$$^{14}\text{Be(g.s.)} = a\,^{12}\text{Be}_{1p} \otimes (sd)^2 + b\,^{10}\text{Be}_{1p} \otimes (sd)^4.$$

The well-established g.s. of ^{12}Be is $\alpha\ ^{10}\text{Be}_{1p}\ (sd)^2 + \beta\ ^{12}\text{Be}_{1p}$, with $\alpha^2 = 0.68$, $\beta^2 = 0.32$ [50]. Thus, for $2n$ removal from ^{14}Be to the physical g.s. of ^{12}Be, the p-shell removal amplitude involves the product $a\alpha$ whereas sd-shell removal has the approximate amplitude $(a\beta + b\alpha)$, and s^2 removal is only about 80–90% of the latter (because some is d^2). Some estimates have $a^2 > 90\%$, so that the p-shell removal strength would be proportional to 0.61 or greater, while s^2 removal strength would be only about 0.54. So, care must be exercised when converting removal strengths to wave-function parentages.

Concerning the splitting between $(sd)^4$ and $(sd)^2$ configurations, one estimate is about 4 MeV [173]. However, a weak-coupling calculation [87] that reproduces the location of the $(sd)^4$ states in ^{12}Be suggests the two are almost degenerate in ^{14}Be.

At least three results have been reported for the matter radius of ^{14}Be: 3.11(38) fm [176], 3.36(19) fm [177], and 3.10(15) fm [178]. Suzuki *et al.* [178] concluded that their value corresponded to a $2s_{1/2}$ occupancy of $P(s^2) = 0.47(25)$. With the same datum, our analysis [179] provided $P(s^2) = 0.55(30)$.

Souza *et al.* [180] computed the core momentum distribution of the weakly-bound n-n-^{12}Be system, and applied it to the data of Zahar *et al.* [181]. They concluded that the s-wave virtual state of ^{13}Be has E_{res} less than 1 MeV.

Adahchour *et al.* [182] applied a Lagange-mesh technique to a three-body description of ^{14}Be and computed a matter radius of 3.16 fm. Their configuration admixture was 75% s^2, 18% d^2.

Ashwood *et al.* [183] measured the helium-cluster breakup and neutron removal cross sections for neutron-rich Be isotopes $^{10-12,14}$Be. They concluded that clustering is important even up to ^{14}Be and that the structure of neutron-rich Be isotopes corresponds to α-Xn-α. With a He + He threshold of 9.09 MeV compared to a $1n$ threshold just above 1 MeV, the He + He cross section was 40(2) mb compared to 750(10) mb for $^{12}\text{Be} + n$.

Unanswered questions in ^{14}Be include

1) Is the state at $E_{2n} = 1.8(1)\,\text{MeV}\ 1^-$?
2) Is the state at $E_{2n} = 1.22(18)\,\text{MeV}\ 0^+$?
3) What is the J^π of the state at $E_{2n} = 3.99(14)\,\text{MeV}$?
4) What is the energy of the lowest $(sd)^4$ state?
5) Where is the 4^+ state?

Little information is available for ^{16}Be. In a first search for it, Baumann *et al.* [184] directed a beam of 140 MeV/nucleon ^{40}Ar onto a beryllium production target and separated the fragments. Many neutron-rich nuclei were identified, but no events of ^{16}Be were recorded.

Spyrou *et al.* [185] were the first to observe ^{16}Be. They used a 53 MeV/u ^{17}B beam and one-proton removal to populate the ground state of ^{16}Be, which they detected as $^{14}\text{Be} + n + n$ coincidences. The two-neutron separation energy was determined to be $-1.35(10)$ MeV. The preponderance of events with a small emission angle between the two neutrons was interpreted as dineutron decay. Marques *et al.* [186] disagreed with the interpretation and stated that "the inclusion of the n-n interaction in the description of direct three-body decay of ^{16}Be generates strong enhancements at low n-n relative energy and angle ... without the need to invoke dineutron decay." It remains to be seen whether the dineutron decay interpretation survives close scrutiny.

3.5 ^{10}Be

I do not consider ^{10}Be to be "exotic" enough for inclusion in the present enterprise, but I do want to remark upon one important fact. A set of suggested $\alpha + {}^6$He "super-cluster" states begin around 6.18 MeV. Several theoretical [187–194] and experimental [76, 195–207] papers have dealt with this supposed band. Sherr and I demonstrated [208] that these states could also be understood as having the structure $^8\text{Be} \otimes (sd)^2$. This is not a contradiction, as Wildermuth and co-workers [88, 209, 210] proved long ago the equivalence of shell-model and cluster-model descriptions.

4 Neutron-rich lithium nuclei

4.1 ^{11}Li

Measurements of the mass of ^{11}Li have had an interesting history, as outlined in table 12. Thibault *et al.* [211] used an on-line mass spectrometer and reported a $2n$ binding energy of 170(80) keV. Wouters *et al.*, [212] with a time-of-flight spectrometer, obtained 320(120) keV. Kobayashi *et al.* [213] used the $^{11}\text{B}(\pi^-, \pi^+)^{11}\text{Li}$ reaction, and measured 340(50) keV. This paper was never published, but the data have appeared several places, including refs. [213–215]. The weighted average of these three values was 295(40) keV. A measurement of the $^{14}\text{C}(^{11}\text{B}, ^{11}\text{Li})^{14}\text{O}$ reaction [216] gave a value of 295(35) keV —agreeing with the previous weighted average to three significant digits. The value stood for about 15 years (the 2003 mass evaluation [51] lists $B_{2n} = 299.6(19.4)$ keV.) New values are 363(22) [217], 378(5) [218], and 369.15(65) [219] keV. As it turned out, only two of the four early measurements agreed with the current value [36] to within 1σ.

Table 12. Results of measurements of separation energy of ^{11}Li.

Year	Reference	Method	S_{2n} (keV)
1975	Thibault *et al.* [211]	on-line mass spectrometer	170(80)
1988	Wouters *et al.*, [212]	time-of-flight spectrometer	320(120)
1991	Kobayashi *et al.* [213–215]	^{11}B$(\pi^-, \pi^+)^{11}$Li	340(50)
1975–1991	first three	weighted average	295(40)
1993	Young *et al.* [216]	^{14}C$(^{11}$B$,^{11}$ Li$)^{14}$O	295(35)
2003	Audi *et al.* [51]	mass evaluation	299.6(19.4)
2009	Roger *et al.* [217]	^{1}H$(^{11}$Li$,^{9}$ Li$)$	363(22)
2009	Gaulard *et al.* [218]	mass spectrometer	378(5)
2008	Smith *et al.* [219]	penning trap	369.15(65)
2012	Wang *et al.* [36]	mass evaluation	369.3(6)

The nucleus ^{11}Li first came to prominence as the first so-called "halo" nucleus. This term was applied to nuclei that had large reaction and/or interaction cross sections. Current understanding is that this phenomenon is associated with simultaneous features —small separation energy and large occupancy of the $2s_{1/2}$ orbital for the last one or two nucleons. I have always thought that the use of "halo" for this effect was incorrect. When I think of a halo in the matter density, I envision a situation involving a region of lower density followed by a higher density at larger radii. As far as I know, all the halo nuclei exhibit monotonically decreasing matter densities —although with slower falloff than for non-halo nuclei. Perhaps ironically, the matter radius of ^{11}Li can be understood with only a modest (about 30%) occupancy of the s orbital.

The magnitude of the $(sd)^2$ component in the ground state (g.s.) of ^{11}Li is still an open question. Knowledge of this quantity is important for calculating several properties of ^{11}Li. It is especially crucial in computing the energy of the mirror nucleus ^{11}O [220]. It might seem reasonable that the majority of the ^{11}Li(g.s.) wave function is just a $p_{3/2}$ proton hole in ^{12}Be(g.s.), for which the $(sd)^2$ component is about 68% [50]. Yet, estimates for this quantity in ^{11}Li range from 0 to 1, with several in the range 0.2–0.5 ([221–227], and references therein).

The customary description of ^{11}Li uses a two-component model, in which the g.s. is a linear combination of two basis states. One of the basis states is the normal p-shell g.s. of ^{11}Li. In a proper treatment, it is not merely two $p_{1/2}$ neutrons coupled to the g.s. of ^{9}Li. Correct treatment of its parentage would involve several core states of ^{9}Li. Antisymmetrization within the p shell requires also the presence of both $p_{1/2}$ and $p_{3/2}$ neutrons coupled to excited-core states having $J^{\pi}_{\text{core}} = 1/2^-$ to $7/2^-$. (With Millener's par4 interaction [8], for $2n$ removal from the p-shell g.s. of ^{11}Li, only about 13% is to the g.s. of ^{9}Li [42].) The other basis state is usually taken to be two sd-shell neutrons coupled to the ^{9}Li g.s. Because the last two neutrons are in a different major shell, antisymmetry does not require the use of excited ^{9}Li cores. Such cores are probably present in this second basis state to some extent, but their contribution is expected to be small, and they are usually neglected.

We had earlier used the sensitivity of a calculation of matter radius to the assumed configuration to estimate the s^2 fraction, $P(s^2)$, in ^{11}Li(g.s.) [179]. Matter radii are usually extracted from measurements of reaction and/or interaction cross sections with the aid of some sort of Glauber-type model. For ^{11}Li, these so-called experimental matter radii [227–230] range from 3.10(16) [228] to 3.71(20) fm [230], with a weighted average of 3.41(8) fm [179]. In our analysis, this R_m produced a $2s_{1/2}$ occupancy of $P(s^2) = 0.33(6)$ [179]. Shulgina *et al.* [224] used 3.29 or 3.31 fm and combined a great deal of other data regarding ^{11}Li to estimate $P(s^2) = 0.368$ [224].

Theoretical estimates of R_m and $P(s^2)$ have differed widely. Myo *et al.* [225] had $R_m = 3.41$ fm and $P(s^2) = 0.469$. Esbensen *et al.* [221–223] had $P(s^2) = 0.231$ or 0.499, with $R_m = 3.29$ or 3.66 fm, respectively. Hagino and Sagawa [226] computed $R_m = 3.53$ and 5.14 fm for pure p^2 and pure s^2, respectively, and used a pairing anti-halo effect to reduce R_m from these values. For comparison, our values for these pure configurations were 2.75 and 4.55 fm [179], respectively.

A new experiment [231] measured reaction cross sections for scattering of ^{11}Li from targets of H and C, at energies near 31 and 41 MeV/nucleon. With a Glauber model in the optical limit, they extracted density functions for neutron, matter, and proton distributions. Their new value of R_m for ^{11}Li was $3.34^{+.04}_{-.08}$ fm [231]. With this value, and our procedure, I found $P(s^2) = 0.29^{+.02}_{-.04}$ [232], not very different from our earlier estimate [179], but slightly smaller. (In the new calculations, I used 2.32 fm [176], rather than 2.30 as in ref. [179], for the ^{9}Li core radius.)

Moriguchi *et al.* [231] also provided a value for the rms radius of the neutron distribution in ^{11}Li: $R_n = 3.68^{+.07}_{-.10}$ fm. Following the prescription for matter radii, the corresponding equation for R_n is

$$R_n{}^2(11) = (6/8)[R_n{}^2(9) + 2R_v{}^2/8].$$

Because the weighting factors in R_n and R_m are different, this procedure for R_n will (in general) provide a slightly

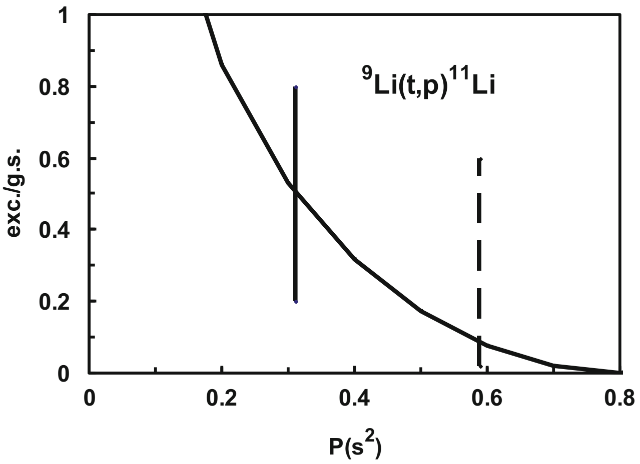

Fig. 8. For the reaction $^9\mathrm{Li}(t,p)^{11}\mathrm{Li}$, the expected cross-section ratio for the (unknown) excited $3/2^-$ state to the g.s. is plotted *vs.* $P(s^2)$, the fractional s^2 intensity in $^{11}\mathrm{Li(g.s.)}$. The solid vertical line just above $P = 0.3$ represents the value of $0.31^{+0.02}_{-0.03}$ extracted from the analysis in [232]. The dashed vertical line near 0.59 is at the lower limit of P for which potential-model and IMME predictions for the $^{11}\mathrm{O(g.s.)}$ mass agree.

different value of $P(s^2)$ than that deduced from the matter radius. With the R_n mentioned above, the resulting $P(s^2)$ is $0.33^{+.03}_{-.05}$. The overall agreement is apparent.

However, some calculations still prefer a higher occupancy. I have suggested another means of determining $P(s^2)$ by using the $^9\mathrm{Li}(t,p)$ reaction [232]. Calculations of relative cross sections for this reaction (in reverse kinematics) suggest a method to observe an expected excited $3/2^-$ state, and to independently determine $P(s^2)$ (see fig. 8).

An early question in ^{11}Li was the possibility of a so-called "soft" dipole. Within the *psd* space, the collective dipole transition contains five components: $p_{1/2} \to s_{1/2}$, $p_{1/2} \to d_{3/2}$, $p_{3/2} \to s_{1/2}$, $p_{3/2} \to d_{3/2}$, and $p_{3/2} \to d_{5/2}$. The isovector dipole giant resonance is a linear combination of these five components, and is usually found at rather high excitation energy. However, if one of the components should have such a different energy from the others that it mixes only weakly, that component might exhibit dipole character at low excitation. In this mass region, that would appear to be the case for the component $p_{1/2} \to s_{1/2}$. An investigation of the $^{11}\mathrm{B}(\pi^-,\pi^+)^{11}\mathrm{Li}$ reaction [213–215] revealed an $L = 0$ angular distribution for the g.s., as expected, and an apparent $L = 1$ angular distribution for a state at an excitation energy of 1.2(1) MeV. This latter state is thus an excellent candidate for the soft dipole.

With a lead target, Sackett *et al.* [233] performed a kinematically complete measurement of the Coulomb dissociation of 28 MeV/nucleon ^{11}Li into ^{9}Li plus two neutrons. The Coulomb dissociation cross section had a peak at $E = 1.0$ MeV with a width $\Gamma = 0.8$ MeV, but the authors could not state definitely that the peak corresponded to a dipole resonance.

Kanungo *et al.* [234] scattered ^{11}Li from a solid deuteron target and reported evidence of a dipole resonance of isoscalar character. The resonance peak had an excitation energy of 1.03(3) MeV with a width of 0.51(11) MeV. The angular distribution was reported to be consistent with a dipole excitation computed in distorted-wave Born approximation. Of course, a pure neutron excitation would have both isoscalar and isovector components.

Smith *et al.* [235] used two-proton removal from a 71 MeV/nucleon ^{13}B beam to populate an unbound excited ^{11}Li resonance at an excitation energy of ~ 1.2 MeV. From measurements of the momentum vectors of the ^{9}Li fragment and neutrons, they concluded that the data could be equally well fit by a simple dineutron-like model or a phase-space model that includes final state interactions. They claimed that sequential decay through the unbound states of ^{10}Li could be excluded.

Korsheninnikov *et al.* [236] studied ^{11}Li with $^{11}\mathrm{Li} + p$ collisions at $75A$ MeV. An excited state was observed at $E_x = 1.25 \pm 0.15$ MeV. It was considered to be a candidate for a neutron halo excitation. Four other ^{11}Li states were reported at ~ 3.0, 4.9, 6.4, and 11.3 MeV.

Very little information is available about other excited states of ^{11}Li.

4.2 ^{10}Li

Neither ^{10}Li nor ^{12}Li has any bound states. Four resonances are expected at low energy in ^{10}Li. A $p_{1/2}$ neutron hole in a pure p-shell ^{11}Li will produce 1^+ and 2^+ states. Most calculations put the 1^+ lower, with a separation of about 0.65 to 1.2 MeV. In ^{12}B, which has the same number of neutrons, the 2^+ is 0.95 MeV above the 1^+. Adding an $s_{1/2}$ neutron to a p-shell ^{9}Li will produce 2^- and 1^- resonances. It is expected that the 2^- will lie lower. Again, in ^{12}B, this expectation is borne out —the 2^- is at 1.67 MeV, 1^- at 2.62 MeV. In ^{10}Li, most calculations put the 2^- below the 1^+. In the latest compilation [1], after reviewing all the evidence available at that time, the compilers suggested that the 2^- g.s. has an energy of 0.025(15) MeV, with a width of 230(60) keV. They concluded that a resonance at $E_r = 0.24(4)$ MeV is probably the 1^+ state.

Several predictions have been made for ^{10}Li. Various calculations [237–240] predict the 1^+ energy in a narrow range, from 0 [238, 239] to 0.25 MeV [240], with the 2^+ higher by 0.3 [240]–1.2 MeV [237]. The same calculations have varying estimates for the 2^- energy —from 0 [237] to 1.7 [239] MeV. Only one [240] has the 1^- below the 2^-. Blanchon *et al.* [241] predicted the $p_{1/2}$ centroid at 0.595 MeV. In Millener's calculations [8, 42], the 2^+ state is 0.366 MeV above the 1^+.

Several experiments have been used to investigate the low-lying structure of ^{10}Li (table 13). Some of them are discussed below.

Santi *et al.* [242] used the $^9\mathrm{Li}(d,p)^{10}\mathrm{Li}$ reaction in inverse kinematics to investigate ^{10}Li. They performed a kinematically complete experiment at an incident ^{9}Li energy of 20 MeV/nucleon. Outgoing ^{9}Li from the breakup

Table 13. Results of various experiments for low-lying states of ^{10}Li (energies and widths in MeV).

Year	Reaction	E_r		Γ	ℓ	Ref.
1997	^{10}Be(^{12}C,^{12}N)	0.24(4)		0.10(7)		[243]
1999	^{9}Be(^{9}Be,^{8}B)	0.50(6)		0.40(6)		[244]
1999	fragmentation	< 0.05			s	[245]
2001	p removal from ^{11}Be				g.s. is s	[246]
2003	^{9}Li(d,p)	0.35(11)		< 0.32		[242]
		or < 0.2		–		
		plus 0.77(24)		< 0.62		
2006	^{9}Li(d,p)	~ 0			s	[247]
		~ 0.38		~ 0.2	p	
2015	$2p$ removal from ^{12}B	0.11(4)		0.2		[248]
		0.50(10)		0.8	both p	
2016	^{11}Li(p,d)	0.62(4)		0.33(7)	p	[249]

of ^{10}Li were detected in coincidence with the recoiling protons from the (d,p) reaction, and the breakup kinematics was reconstructed. They measured a lower limit of $\Delta = 33.10 \pm 0.08$ MeV (*i.e.*, $\Delta > 33.02$ MeV) for the mass excess of ^{10}Li which was consistent with previous measurements. They were able to distinguish the structure of ^{10}Li associated with a ground state ^{9}Li core from the structure associated with a ^{9}Li core in its first excited state. The best fit to their data provided either a single p-wave resonance at $E_n = 0.35(11)$ MeV, or two resonances at $E_n = 0.77(24)$ and < 0.2 MeV.

The same reaction, by Jeppesen *et al.* [247], agreed with the first of these two possibilities and also reported an s-wave structure at threshold. Two-proton removal from ^{12}B observed two low-lying p-shell resonances [248].

A recent ^{11}Li(p,d) [250] experiment observed a single resonance at 0.62(4) MeV, with a width of 0.33(7) MeV, and a spectroscopic factor of 0.67(12). I argued that this experiment had observed only the 2^+ resonance, and not the 1^+ [249].

4.3 ^{12,13}Li

The situation concerning ^{12}Li is somewhat confusing. The (unbound) g.s. is expected to be 2^-, of the structure ^{11}Li(g.s.)$\otimes s$, and an excellent candidate exists just above threshold, with a scattering length of $a_s = -13.7(1.6)$ fm ($E_{res} = 120(15)$ keV) [251] or $a_s > -4$ fm [252]. Two other narrow resonances have been reported at $E_{res} = 250 \pm 20$ ($\Gamma < 15$) and 555 ± 20 ($\Gamma < 80$) keV [253]. Comparison with shell-model calculations suggested (4^-) and (1^-), respectively for them. The latest $A = 12$ compilation [254] provides an excellent description of the existing experimental information, and I will not discuss ^{12}Li further here.

Kohley *et al.* [252] were the first to observe the ^{13}Li (g.s.), using a one-proton removal reaction from ^{14}Be at a beam energy of 53.6 MeV/u. They detected the ^{13}Li ground state with coincidence measurements of ^{11}Li and two neutrons. Their experiment gave a resonance energy of 120^{+60}_{-80} keV.

4.4 ^{9}Li

Despite its large ratio of neutrons to protons, $N/Z = 2$, ^{9}Li is not a very exotic nucleus. All available information is consistent with calculations fully within the $1p$ shell. The g.s. is $3/2^-$, with a $1/2^-$ level at $E_x = 2.69$ MeV, $(5/2^-)$ at 4.31 MeV, and two other states at 5.38 and 6.43 MeV. The ^{7}Li(t,p) reaction [104,255] strongly populates the g.s. with an $L = 0$ angular distribution, and the $(5/2^-)$ state with $L = 2$, as expected from the shell model. The other states are weaker, again as expected. The charge exchange reaction ^{9}Be$(t,{}^{3}$He$)^{9}$Li [256] populates a spin-dipole state at about 6.5 MeV. The g.s. magnetic moment [257] disagrees slightly with expectations for a $(p_{3/2})$ configuration, but is in good agreement with a full p-shell calculation, implying some $p_{1/2}$ occupancy. No hint of positive-parity states has been reported. I will not discuss ^{9}Li further, even though an interesting un-answered question is the location of the state with dominant structure $^{7}\mathrm{Li} \otimes (sd)^2_{0+}$. If its mixing with the g.s. is small, it should be quite strong in ^{7}Li(t,p).

5 Neutron-rich Helium nuclei

5.1 ^{7}He

Within the p-shell space, ^{7}He has only five states —three with $J^\pi = 3/2^-$, one $1/2^-$, and one $5/2^-$. Most calculations predict the g.s. will be $3/2^-$ and will be dominated by the $(p_{3/2})^3$ configuration. The higher two $3/2^-$ states will share the configurations $(p_{3/2})^2_2\, p_{1/2}$ and $p_{3/2}(p_{1/2})^2_0$. The $1/2^-$ is pure $(p_{3/2})^2_0\, p_{1/2}$, and the $5/2^-$ is $(p_{3/2})^2_2$ $p_{1/2}$.

Table 14. Spectroscopic factor for ^{7}He(g.s.) $\rightarrow$ ^{6}He(g.s.) + n.

Year	Process	Value	Ref.
1967	p-shell shell model	0.59	[258, 259]
2002	variational Monte Carlo	0.53	[260]
2005	^{6}He(d,p)	0.37(7)	[261]
2007	^{7}Li$(d,{}^2$He$)^7$He	0.64(9)	[258]
2009	^{8}He neutron knockout	0.61	[262]
2011	Green's function Monte Carlo	0.565	[263]
2012	^{8}He neutron knockout	0.512(18)	[264]
2013	no core shell model	0.56	[265]

Table 15. Experiments and calculations that report a low-lying excited state in ^{7}He.

Year	Process	E_x (MeV)	Γ (MeV)	J^π	Ref.
2001	^{8}He neutron knockout	1.2 (2)	1.0 (2)	$1/2^-$	[266]
2002	^{8}He fragmentation	1.0(1)	0.75(8)	$(1/2^-)$	[267]
2004	RCCSM [(a)]	1.0	0.74	$1/2^-$	[268]
2006	^{8}He(p,d)	0.9(5)	1.0(9)	$1/2^-$	[269]
2006	MCAS [(b)]	1.27	0.03	$7/2^-$	[270]
2006	^{7}Li$(d,{}^2$He$)$	$1.45^{+0.7}_{-0.5}$	~ 2	$1/2^-$	[271]
2007	cluster model	1.1	2.1	$1/2^-$	[225]
2009	cluster model	0.6–1.2	–	$1/2^-$	[272]

(a) Recoil corrected continuum shell model.
(b) Multi-channel algebraic scattering.

The unbound g.s. of ^{7}He was first observed by Stokes and Young [273], with the reaction ^{7}Li$(t,{}^3$He$)$. The resonance energy and width were 0.42(6) MeV and 0.17(4) MeV, respectively. The same two authors later refined their values to 0.44(3) and 0.16(3) MeV [274]. This resonance energy corresponds to a mass excess of 26.11(3) MeV, to be compared with the value of 26.073(8) MeV from the latest mass evaluation [36]. Denby *et al.* [275] used proton knockout from a secondary ^{8}Li beam to populate the g.s. of ^{7}He and extract its energy and width. Results were an energy of 400(10) keV and a width of 125^{+40}_{-15} keV. Lindsay *et al.* [276] obtained proton energy spectra at several angles with the ^{7}Li$(n,p)^7$He reaction and confirmed the ^{7}He energy to be 420(60) keV, with a width less than 200 keV. Beck *et al.* [258] analyzed earlier data from the ^{7}Li$(d,{}^2$He$)$ reaction [271] with an experimental energy resolution of 150 keV (FWHM) to extract parameters of the ^{7}He(g.s.): $E_r = 0.446$ MeV and $\Gamma = 183(22)$ keV (FWHM). In fragmentation of a 227 MeV/nucleon ^{8}He beam on a carbon target, Meister *et al.* [267] obtained an energy of 0.43(2) MeV and a width of $\Gamma = 0.15(8)$ MeV.

Aksyutina *et al.* [262] produced ^{7}He in neutron-knockout with a 240 MeV/u ^{8}He beam on a liquid hydrogen target. An R-matrix analysis of the ^{7}He $\rightarrow$ ^{6}He$(0^+) + n$ peak shape allowed determination of that spectroscopic factor. They reported $S = 0.61$, and they stated that "the spectroscopic factor is 0.61 confirming that ^{7}He is not a pure single-particle state." Of course, if ^{6}He(g.s.) is pure $(p_{3/2})^2$ and ^{7}He is pure ^{6}He(g.s.)$\otimes p_{3/2}$, S is only 0.50. (In a single j shell, for stripping onto a 0^+ target, the sum of S for states with $J = j$ is just the fractional emptiness of that orbital in the target. Here, the sum involves only one state.) So, instead of confirming that ^{7}He is not a pure sp state, their S proves that ^{6}He(g.s.) is not pure $(p_{3/2})^2$, but contains a $(p_{1/2})^2$ component.

Several experiments [258, 261, 262, 264] have measured the ^{7}He(g.s.) $\rightarrow$ ^{6}He(g.s.) + n spectroscopic factor, and many theoretical groups [260, 263, 265, 277] have calculated it. A sampling is listed in table 14.

Mittig *et al.* [282] studied the ^{8}He(p,d) reaction (in reverse kinematics) with a secondary beam of ^{8}He incident on a C_4H_{10} gas target. They extracted a pickup spectroscopic factor of $C^2S = 3(1)$. Skaza *et al.* [269] studied the same reaction and analyzed the results with the coupled-channels Born approximation. A spectroscopic factor $C^2S = 4.4(1.3)$ was deduced for neutron pickup to the ^{7}He(g.s.). They claimed that "this value is consistent with a full $p_{3/2}$ subshell for ^{8}He." Of course, 50% of this structure and 50% of $(p_{3/2})^2$ $(p_{1/2})^2$ would also be consistent within the quoted uncertainty.

I turn now to excited states of ^{7}He. Several experiments [266, 267, 269, 271] and some calculations [268, 270] have reported a low-lying excited state near 1 MeV, as indicated in table 15.

Table 16. Experiments and calculations that do not report a low-lying excited state in ^{7}He.

Year	Process	E_x (MeV)	Γ (MeV)	J^π	Ref.
1985	shell model	4.27	–	$1/2^-$	[239]
1990	shell model	~ 2	–	$1/2^-$	[278]
1997	resonating group	2.3–3.8	–	$1/2^-$	[279]
1998	shell model	2.3	–	$1/2^-$	[280]
2001	^{9}Be(^{15}N,^{17}F)	2.95(10)	1.9(3)	$(1/2^-)$	[281]
2005	^{6}He(d,p)	2–3	Broad	$(1/2^-)$	[261] [(a)]
2005	^{8}He(p,d)	no evidence for a low-lying excited state			[282]
2009	^{8}He neutron knockout	low-lying resonance not observed			[262]
2008	Green function Monte Carlo	2.8	Broad	$1/2^-$	[260]
2013	no core shell model	>2	Broad	$1/2^-$	[265]

[(a)] No evidence was found for a lower-lying, first-excited state reported recently.

The MCAS procedure deserves special mention. Canton *et al.* [270] computed ^{7}He as a neutron coupled to the g.s. and two 2^+ states of ^{6}He, without sufficient regard for the Pauli principle. Their first-excited state has $J^\pi = 7/2^-$, which is a state that does not exist in the p-shell space. It requires participation of the $1f_{7/2}$ or $2p_{3/2}$ orbitals in the 2^+ structure, or for the last neutron. Such a state will exist, but at much higher energy —about $2\hbar\omega \sim 20$ MeV away. The MCAS method has exhibited this failing for several nuclei —high-J states at low excitation energy, nowhere near where they actually exist.

Just as many experiments [261,262,281,282] and even more calculations [239,260,265,278–280] claim the $1/2^-$ resonance is much higher. These are listed in table 16.

Some evidence exists for other excited states of ^{7}He. Bohlen *et al.* [281] used the ^{9}Be(^{15}N,^{17}F) reaction to populate ^{7}He. They reported a g.s. width of 0.14(2) MeV. They observed excited states at $E_x = 2.95(10)$ MeV with $\Gamma = 1.9(3)$ MeV, and $E_x = 5.8(3)$ MeV with $\Gamma = 4(1)$ MeV. As noted above, they suggested the first one was a good candidate for the expected $1/2^-$ state. Korsheninnikov *et al.* [283], in their investigation of the transfer reaction $p(^8\mathrm{He},d)^7\mathrm{He}$, reported an excited state of ^{7}He at $E_x = 2.9(3)$ MeV, with $\Gamma = 2.2(3)$ MeV, that decays primarily into $3n+{}^4$He. They concluded that its structure is a neutron coupled to a ^{6}He core which is in the excited 2^+ state. This is, of course, the expected configuration of the $5/2^-$ state, as they suggested. Its cross section was less than 10% of that of the g.s., indicative of a second-order process, again as expected for the $5/2^-$ state.

Myo *et al.* [284] predicted that the $5/2^-$ and second $3/2^-$ states would be degenerate, with about the same decay widths. Their second $3/2^-$ state has about 88% of the $(p_{3/2})^2(p_{1/2})$ configuration. In their calculations, the first and third $3/2^-$ states are about 92% pure —$(p_{3/2})^3$ and $(p_{3/2})(p_{1/2})^2$, respectively.

Wuosmaa *et al.* [285] used the ^{8}Li$(d,{}^3$He) reaction and reported an energy of 2.9(3) MeV for the $5/2^-$ state, with a proton pickup strength of $C^2S = 0.29(15)$, consistent with the VMC prediction of 0.17 [260]. However, the experimental g.s. strength was 0.36(7) compared to a prediction of 0.58.

Attempts have been made to learn about the structure of ^{7}He by looking at its analogs in ^{7}Li. Rogachev *et al.* [286] used the ^{6}He(p,n) reaction and detected the resonant yield of neutrons in coincidence with γ rays from the decay of the $(0^+, T=1)$ state in ^{6}Li. They observed the analog of the ^{7}He g.s. and claimed "it is conclusively shown that the analog of the recently observed low-lying spin-orbit partner of the ^{7}He ground state does not exist." Boutachkov *et al.* [287] presented evidence for the analog of a broad $1/2^-$ state at a higher excitation energy.

5.2 ^{8}He

Measurements [288–294] of the mass excess of ^{8}He are summarized in table 17, along with values from the two most recent mass evaluations [36,51]. Of the first five measurements, four agreed with the current, extremely precise, value to better than 1.3 σ. The exception is the paper of Tribble *et al.* [292], which differs by 2.1 σ. A similar remark was made by Ryjkov *et al.* [293]. The 2003 mass evaluation [51] missed the current value by 1.7 σ, primarily because of the influence of Tribble's value on their average.

A major question for ^{8}He is the amount of configuration mixing in the g.s. Many workers have treated ^{8}He as a closed $(p_{3/2})^4$ orbital, with no contribution from $p_{1/2}$. This assumption is extremely unlikely to be correct, because jj coupling is not a very good approximation in the $1p$ shell (especially in the lower half of the shell). It is well known that mixing of the $j = \ell \pm 1/2$ orbitals is essential for an understanding of the structure of p-shell nuclei. Near the beginning of the shell, LS coupling is a much better approximation than is jj. Some groups have included $p_{1/2}$, and results of several experiments discussed below appear to require it. Very few researchers have considered the need for excitations into the sd shell. As we shall see later, neither the matter radius of ^{8}He nor the mirror energy difference with ^{8}C suggest any sd-shell occupancy.

Table 17. Measurements of ^{8}He mass excess (MeV).

Year	Process	Mass excess	Ref.	Difference [(a)]
1966	^{26}Mg(α, ^{8}He)^{22}Mg	31.65(12)	[288]	0.33 σ
1974	^{26}Mg(α, ^{8}He)^{22}Mg	31.57(3)	[289]	−1.3 σ
1975	^{18}O(α, ^{8}He)^{14}O	31 600(25)	[290]	−.04 σ
1975	^{64}Ni(α, ^{8}He)^{60}Ni	31.613(16)	[291]	0.19 σ
1977	^{64}Ni(α, ^{8}He)^{60}Ni	31.593(8)	[292]	−2.1 σ
2003	mass evaluation	31 598(7)	[51]	−1.7 σ
2008	Penning trap	31 610:77(69)	[293]	1.6 σ
2012	Penning trap	31 609.72(11)	[294]	
2012	mass evaluation	31 609.68(9)	[36]	

[(a)] Difference between the listed energy and the current value [36], as a multiple of the stated uncertainty.

An early calculation of the structure of ^{8}He was performed by Zhukov *et al.* [295] in a simplified $\alpha+4n$ model. They assumed the $p_{3/2}$ neutron orbital was full and the $p_{1/2}$ empty, and justified this assumption by referring to earlier works [296–298]. They computed the transverse momentum distribution for breakup of ^{8}He into ^{6}He + $2n$ and compared it to experimental data [299] for fragmentation on a carbon target. The calculated distribution was only slightly wider than the experimental one.

Korsheninnikov *et al.* [299], with ^{8}He + p inelastic scattering, reported an excited state at an energy of 3.57(12) MeV, $\Gamma = 0.50(35)$ MeV, whose inelastic angular distribution was consistent with $J^\pi = 2^+$. A 2^+ state at 3.59 MeV with a width of 0.80 MeV was identified by von Oertzen *et al.* [300], who compared it to the 2^+ states in isotones ^{12}C and ^{10}Be. Very early shell-model calculations [239] predicted the first excited 2^+ state to be at 5.83 MeV, and the next two states to have negative parity.

Iwata *et al.* [301] studied dissociation of E/A = 24 MeV ^{8}He into ^{6}He + $2n$, on targets of tin and lead. They concluded that the measured neutron momentum distributions were in favor of the ^{4}He + $4n$ structure, and that ^{8}He does not have a ^{6}He + $2n$ structure.

Alkhazov *et al.* [302] scattered ^{6}He and ^{8}He of about 700 MeV/nucleon from a hydrogen target. Nuclear matter densities were deduced from the data at small momentum transfers and used to extract rms radii of the nuclear matter distributions: $R_m(^6\text{He}) = 2.30(7)$ fm and R_m (^{8}He) = 2.45(7) fm. These values are consistent with structures totally within the $1p$ shell [19]. They concluded that "the data are consistent with the concept that ^{6}He and ^{8}He nuclei have an α-like core and a significant neutron skin."

Weppner *et al.* [303] investigated whether p-nucleus elastic scattering experiments with secondary beams of ^{6}He and ^{8}He could determine the physical structure of these nuclei. The sensitivity was examined by using four different nuclear structure models. They concluded that elastic scattering at intermediate energies (< 100 MeV/nucleon) is not useful in putting constraints on the g.s. wave function of the ^{6}He and ^{8}He nuclei.

Skaza *et al.* [269] studied the ^{8}He(p,d) reaction (in reverse kinematics) and analyzed the results with the coupled-channels Born approximation. A spectroscopic factor $C^2S = 4.4(1.3)$ was deduced for neutron pickup to the ^{7}He g.s. They claimed that "this value is consistent with a full $p_{3/2}$ subshell for ^{8}He." Of course, 50% of this structure and 50% of $(p_{3/2})^2$ $(p_{1/2})^2$ would also be consistent within the quoted uncertainty.

Hagino *et al.* [304] used a core + $4n$ model and solved the five-body Hamiltonian in the Hartree-Fock-Bogoliubov approximation. They concluded "the ground state wave function of ^{8}He deviates significantly from the pure $(p_{3/2})^4$ structure", with 34.9% of the $(p_{3/2})^4$ configuration and 23.7% of the $(p_{3/2})^2$ $(p_{1/2})^2$ configuration. An investigation of the two-neutron transfer reaction ^{8}He(p,t) ^{6}He, by Keeley *et al.* [305], also indicated an appreciable mixture of these two configurations. Itagaki *et al.* [306], with antisymmetrized molecular dynamics, also concluded that the ^{8}He(g.s.) contains configuration mixing. Hagino *et al.* predict additional components of 10.7% $(p_{3/2})^2$ $(d_{5/2})^2$ and 7.8% $(s_{1/2})^2$ $(p_{3/2})^2$.

Chulkov *et al.* [307] studied breakup of ^{6}He and ^{8}He on a hydrogen target and extracted spectroscopic factors for formation of the fragments. They concluded that the ^{8}He structure can be considered as a ^{6}He core plus two valence neutrons, but that the assumption of negligible contribution from the $p_{1/2}$ orbital is incompatible with the experimental results. They needed about 50% contribution of the $(p_{3/2})^2$ $(p_{1/2})^2$ configuration to explain the alpha yield. A similar conclusion was reached by Mackenroth *et al.* [266] in an investigation of ^{8}He fragmentation on a carbon target. Of course, such configuration mixing is needed to form a good $2n$ cluster structure [308].

Skaza *et al.* [309], with a ^{8}He beam on a hydrogen target, investigated (p,p') inelastic scattering. They observed the known 2^+ state at 3.6 MeV and another state at about

5.4 MeV. They concluded that the ^{8}He ground state has a more complex structure than a pure $(p_{3/2})^4$, in agreement with their analysis of results of the ^{8}He(p,t) reaction [305], which suggested mixing of the $(p_{3/2})^4$ and $(p_{3/2})^2$ $(p_{1/2})^2$ configurations.

The low-lying spectrum of ^{8}He was investigated by Fomichev *et al.* [310] and Golovkov *et al.* [311] with the ^{3}H(^{6}He$,p)^8$He $2n$-transfer reactions. States that were populated included the 0^+ ground state (g.s.), 2^+ at 3.6–3.9 MeV and a possible (1^+) at 5.3–5.5 MeV Some evidence was found for a ^{8}He state at about 7.5 MeV. An anomaly near the ^{6}He + $2n$ threshold was discussed in terms of a 1^- continuum.

In one-neutron knockout from ^{8}He, Aksyutina *et al.* [156] claim that the ^{6}He + n "profile function analysis shows a presence of s^2 component in the ^{8}He ground-state wave function." As far as I know, this is the only experiment that makes such a claim. This important question clearly deserves further scrutiny. Earlier, they had stated that "the analysis of the ^{6}He+n relative-energy spectrum did not show any presence of s-wave scattering." [261] Sherr and I [312] found that the ^{8}He/^{8}C mirror energy difference could be reproduced without any sd-shell admixture.

A recent review [313] has summarized a great deal of information concerning ^{6}He and ^{8}He. I have not repeated those discussions here. I do not consider ^{6}He to be "exotic" enough to fit within the scope of the present review.

Unresolved questions in ^{8}He include:

1) What is the configuration admixture in the g.s.? Does evidence exist for an $(sd)^2$ component?
2) Does a 1^- state exist below the first 2^+? If so, what is its structure?
3) Where is the first excited 0^+ state? Is it a p-shell state, or does it involve excitations out of the ^{4}He core, or promotion of neutrons from $1p$ to sd?

5.3 ^{9,10}He

These two nuclei have no bound states. Energies of resonances in the two are intimately connected. Several workers [314–319] have reported an s-wave resonance just above threshold in ^{9}He, whereas other experiments [132, 320, 321] did not see it. Available information for ^{9}He is listed in table 18.

Rogachev *et al.* [322] investigated the resonances in ^{9}He indirectly by examining the isobaric analog states in ^{9}Li. They measured an excitation function for ^{8}He + p elastic scattering in the center-of-momentum energy range from 1.6 to 5.8 MeV. Three $T = 5/2$ states in ^{9}Li (isobaric analogs of ^{9}He) were observed. None were $1/2^+$. The lowest was $1/2^-(3/2^-)$ at 16.1(1) MeV in ^{9}Li, which the authors stated corresponds to 1.1 MeV in ^{9}He. Instead, they found evidence for a broad $J^\pi = 1/2^+$ state in ^{9}He located approximately 3 MeV above the neutron decay threshold.

In a simple model [323, 324], energies in ^{9}He and ^{10}He are related. The energy of the apparent g.s. in ^{10}He has varied widely for different experiments. Results for ^{10}He from various reactions are summarized in table 19.

A number of explanations have been offered for the different results. Earlier, Grigorenko and Zhukov [325] suggested that the energy measured in proton removal from ^{11}Li might be lower than in other reactions because of initial-state interactions in ^{11}Li. I suggested [326] the differences might arise from the presence of two overlapping 0^+ states, populated with different strengths in different reactions. I estimated the relative strengths to be expected in the different reactions. Results were presented in terms of the mixing between the two basis states —the p-shell one and one with the structure ^{8}He $\otimes$ $(sd)^2$. Recently, Sharov *et al.* [327] made a similar suggestion, but with overlapping 0^+, 1^-, and 2^+ states. The (t,p) experiment [328] found separate energies for 0^+, 1^-, and 2^+ resonances, with (naturally) the 1^- and 2^+ above the 0^+. That 0^+ energy was about 0.7 MeV higher than the average of the "g.s." energies in all other reactions [329–332]. If the variation in the latter were the result of overlapping resonances of different J^π, they should have been larger than the 0^+ energy. Thus, it is unlikely that the observed variation in energy is the result of overlapping resonances of different J^π.

As pointed out above, Grigorenko and Zhukov [325] have discussed the problems with the ^{10}He(g.s.). They suggested a p-shell 0^+ state in the range 2.0–2.3 MeV, and an s^2 "alternative" g.s. at $E < 0.25$ MeV. They theorized that the p-shell 0^+ state might appear as a peak in the cross section near 1.2 MeV (rather than 2.0–2.3 MeV) in reactions involving ^{11}Li. The fact that this model is no longer valid has been discussed elsewhere [327, 328, 333]. Theoretical calculations of Sharov *et al.* [334] indicated that the peak near 1.2 to 1.5 MeV that is populated in the proton knockout reaction with a ^{11}Li beam is likely to be a superposition of 1^-, 0^+, and 2^+ excitations with very similar shapes. As stated above, I pointed out [333] that it is unlikely that the observed variation in energy in different experiments is the result of overlapping resonances of different J^π.

A very simple model [323, 324] has been extremely successful in predicting the absolute energies of $(sd)^2$ 0^+ states in $A + 2n$ light nuclei, where A is a p-shell nucleus and the two neutrons are in the sd shell. For nucleus A+$2n$, the model takes energies of $1/2^+$ and $5/2^+$ states in nucleus $A + 1n$ as single-particle energies (spe's) for s and d, respectively. The two-body residual matrix elements (2BME's) are assumed to be the same in the various nuclei and are taken from work on ^{18}O [9], where experimental data were used to separate the $(sd)^2$ and collective components of the lowest nine positive-parity states in ^{18}O. I have used this simple model and the relationship between computed energies in ^{10}He and single-particle energies in ^{9}He to provide limits on the $s_{1/2}$ energy. Results are displayed in fig. 9. The fact that ^{10}He has no bound states puts a limit on the $2s_{1/2}$ energy in ^{9}He as $E_s > 1$ MeV, contradicting all the experiments that have reported an s state near threshold. Dependence of E_s on the energy of the $(sd)^2$ 0^+ resonance in ^{10}He are as given in the figure.

Table 18. Energies (relative to ^{8}He + n) and widths (both in MeV) of resonances in ^{9}He, from the reactions indicated.

Reaction	E_n	Width	J^π	Ref.
^{9}Be(π^-, π^+)	1.13(10)	0.42(10)	$1/2^-$	[322]
	2.33(10)	0.42(10)	$1/2^+$	
	4.93(10)	0.50(10)	$5/2^+$ or $3/2^-$	
^{9}Be(^{13}C,^{13}O) and ^{9}Be(^{14}C,^{14}O)	1.13	~ 0.30	$1/2^-$	[132]
	2.28	~ 0.85	$1/2^+$ or $3/2^-$	
	4.93			
^{9}Be(^{14}C,^{14}O)	1.27	0.10(6)	$1/2^-$	[320]
	2.37(10)	0.7(2)	$(3/2^-)$	
	4.30(10)	narrow	$(5/2^+)$	
	5.25(10)	narrow		
$2p$ ko from ^{11}Be	(< 0.2)		$1/2^+$	[314]
C(^{11}Be, ^{8}He + n)	< 0.2		$1/2^+$	[315]
C(^{14}B, ^{8}He + n)	$\sim$0		$1/2^+$	
	~ 1.3	~ 1	–	
^{2}H(^{11}Li, ^{8}He + n)	(~ 0)	maybe not a true state		[316]
	1.33(8)	0.10 fixed	$1/2^-$	
	2.42(10)	0.70 fixed	$3/2^-$	
^{2}H(^{8}He, p)	~ 0		$(1/2^+)$	[317]
	~ 1.3		$(1/2^-)$	
	~ 2.3		–	
^{2}H(^{8}He, p)	~ 0		$1/2^+$	[318]
	2.0(2)	~ 2	$1/2^-$	
	> 4.2	> 0.5	$5/2^+$	
^{2}H(^{8}He, p)	0.180(85)	0.18(16)	$1/2^+$	[319]
	1.235(115)	0.13(17)	$(1/2^-)$	
	3.42(78)	2.90(39)	$5/2^+$ or $3/2^+$	

Table 19. Energy and width (both in MeV) of ^{10}He(g.s.) from various reactions.

Label	Reaction	E_{2n}	Γ	Ref.
1	H(^{11}Li, $2p$)	1.7(3)(3)	–	[329]
2	^{2}H(^{11}Li,^{3}He)	1.2(3)	< 1.2	[330]
3	^{10}Be(^{14}C,^{14}O)	1.07(7)	0.3(2)	[331]
4	^{14}Be – $2p2n$	1.60(25)	1.8(4)	[335]
5	^{3}H(^{8}He, p)	2.1(2)	~ 2	[328]
6	^{3}H(^{8}He, p)	~ 3		[311]
7	p knockout from ^{11}Li	1.42(10)	1.11(76)	[332]
8		1.54(11)	1.91(41)	
	^{2}H(^{11}Li,^{3}He)	1.4(3)	1.4(2)	[336]
non (t, p) wt. ave.		1.22(6)	–	
non (t, p) un-wt. ave.		1.40(23)	–	

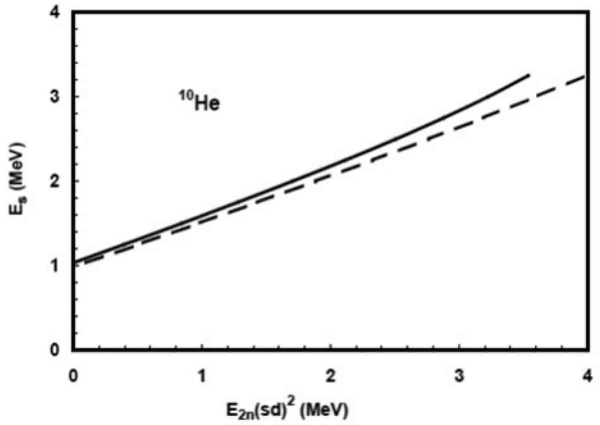

Fig. 9. Relationship between the assumed energy of $1/2^+$ resonance in ^{9}He and the computed energy of $(sd)^2$ 0^+ state in ^{10}He, for $E_d = 4.20$ (solid) and 4.93 MeV (dashed).

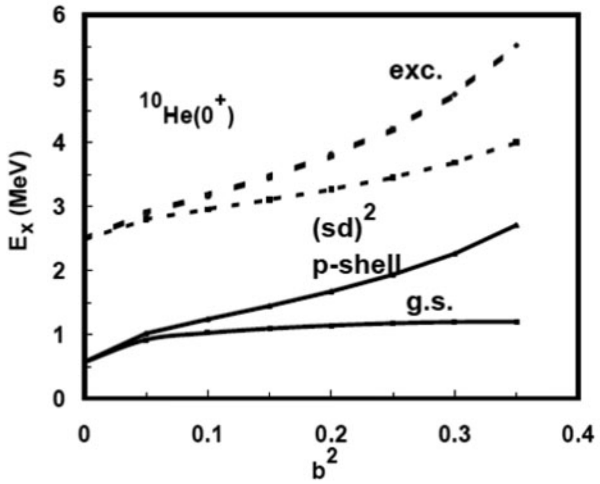

Fig. 10. Plotted *vs.* b^2 (the amount of $(sd)^2$ in ^{10}He(g.s.)) are the energies of the g.s. (lower solid) and excited 0^+ state (upper dashed) that are required to fit results of energy measurements in proton knockout from ^{11}Li and in the reaction ^{8}He(t,p) —assuming the experiments measure the centroid of two overlapping 0^+ resonances. The upper solid curve is the resulting energy of the p-shell basis state and the lower dashed curve is the energy of the $(sd)^2$ 0^+ basis state. This fit makes no assumption about the mixing, but only that a two-state model suffices.

Of course, this is the energy of the pure $(sd)^2$ 0^+ resonance. My analysis supports the view that the variation of ^{10}He "ground-state" energies determined in various reactions is caused by the presence of two overlapping 0^+ resonances. Results of the two simplest reactions —proton knockout and (t,p)— have been used to extract the g.s. and excited 0^+ energies as a function of the mixing parameter b^2 between the p-shell and $(sd)^2$ basis states. With two-state mixing, the results depend on the mixing intensity as indicated in fig. 10.

Combining the last two figures provides the relationship between the $2s_{1/2}$ energy in ^{9}He and the mixing parameter necessary to reproduce the experimental energies in ^{10}He, as depicted in fig. 11.

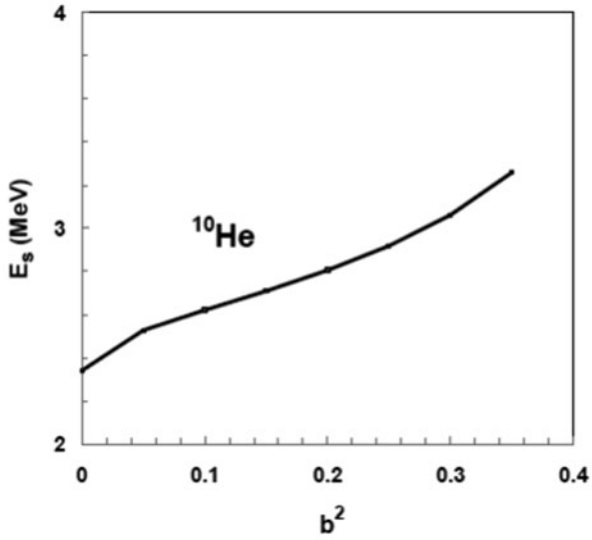

Fig. 11. Combining the dependence of $E(sd)^2$ on b^2 from fig. 10 with the dependence of E_s on $E(sd)^2$ from fig. 9, this plot displays *vs.* b^2 the $1/2^+$ energy in ^{9}He that will fit the ^{10}He "g.s." energies measured in p knockout and (t,p).

Self-consistency of all data requires $b^2 < 0.075$ [333]. These results disagree with many (but not all) earlier theoretical predictions. The theoretical situation was always confusing. Aoyama *et al.* [337], in an ^{8}He-n-n model, calculated the ^{10}He g.s. to be near 1.8 MeV. Later, Aoyama suggested the g.s. was near threshold, and was mostly of s^2 configuration [338]. Korsheninnikov *et al.* [339] deduced the ^{10}He g.s. was near 1 MeV and was mostly p-shell. This prediction is in very good agreement with my current analysis. Kamada *et al.* [340] predicted a 0^+ g.s. at 0.803 MeV and a 1^- at 1.25 MeV, although they acknowledged that the latter might be a simple spurious state. It is to be hoped that future experiments with better statistics and better resolution will be able to test the validity of the present conclusions.

6 Matter radii

Matter radii for several neutron-rich light nuclei are listed in table 20 and plotted in fig. 12. Solid points are R_m values extracted from measurements of reaction and/or interaction cross sections [176–178, 228, 302, 341, 342]. Open circles are results of calculations within the p shell, using the procedures outlined in sect. 2 [19, 179]. Of course, no p-shell calculation is appropriate for ^{11}Be or ^{14}Be, because ^{11}Be(g.s.) is $1/2^+$ and ^{14}Be has at least two neutrons outside the $1p$ shell. In the calculations presented here, the core for ^{12}Be is the $1/2^-$ first-excited state of ^{11}Be; for all the others, the core is the g.s. of the relevant core. The large radius for ^{11}Li is the reason this nucleus was the first to be designated a "halo nucleus". We now know that this phenomenon is associated with the presence of one or two

Table 20. Matter radii (fm) for several $Z = 2, 3, 4$ nuclei.

Nucleus	Core	Valence	$R_{m\,\mathrm{calc}}$ (a)	$R_{m\,\mathrm{exp}}$	Ref.
^{6}He	^{4}He	p^2	2.30	2.48(3)	[341]
				2.33(4)	[342]
				2.30(7)	[302]
^{8}He	^{6}He	p^2	2.69	2.52(3)	[176]
				2.49(4)	[342]
				2.45(7)	[302]
^{7}Li	^{6}Li	$1p$	2.33	2.33(2)	[176]
^{9}Li	^{8}Li	$1p$	2.45	2.32(2)	[176]
				2.41(2)	[177]
^{11}Li	^{9}Li	p^2	2.75	3.41(8) (b)	[179]
				3.12(16)	[228]
				3.16(11)	[341]
				$3.34^{+0.04}_{-0.08}$	[231]
^{10}Be	^{9}Be	$1p$	2.30	2.30(2)	[176]
				2.46(3)	[177]
^{12}Be	^{11}Be$(1/2^-)$	$1p$	2.62	2.59(6)	[176]
^{14}Be	^{12}Be	$(sd)^2$	3.10 (c)	3.10(15)	[178]

(a) References [19, 179], present work.
(b) Weighted average of three earlier values.
(c) For $P(s^2) = 0.55(30)$ [179].

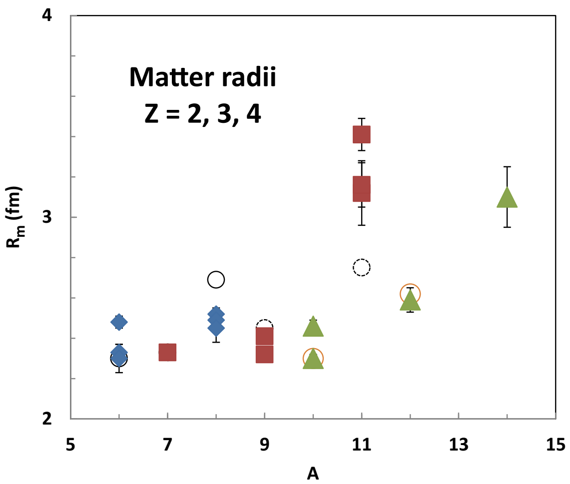

Fig. 12. Closed points are matter radii extracted from experiments for isotopes of He (diamonds), Li (squares), and Be (triangles). Open circles are results of theoretical p-shell calculations.

loosely bound nucleons in the $2s_{1/2}$ orbital. The $2n$ separation energy for ^{11}Li is only 0.369 MeV [36]. We note that the p-shell calculations under-predict the ^{11}Li matter radius. Analysis has demonstrated [179, 232] that an $s_{1/2}$ occupancy of $P(s^2) \sim 0.33$ reproduces the experimental value, and indeed many other properties of ^{11}Li.

It might appear strange that the p-shell calculations reproduce the ^{12}Be radius, since it is well-known (sect. 3.1) to contain appreciable sd-shell occupancy. Two different (but connected) effects are operating here. First, the $2n$ separation energy of ^{12}Be is 3.672 MeV, and the one-neutron separation energy is 3.171 MeV —so that the valence neutrons are not loosely bound. The difference in calculated radii for different orbitals decreases as separation energy increases. Secondly, for any given result for a p-shell calculation, there is a combination of s and d orbitals that gives the same R_m [179]. Nevertheless, a new measurement for ^{12}Be might be worthwhile.

The overall agreement with p-shell calculations for the other nuclei is good. An additional consideration is that the use of excited core states decreases the computed radius of the valence neutron(s), and all the results displayed here used only the g.s. of the relevant core (low-lying $1/2^-$ for the ^{11}Be core of ^{12}Be). This might explain the slight over-prediction for ^{8}He. Unfortunately, the excited core states are all unbound, so that the radius of the core state cannot be computed with the standard procedure.

Sherr and I demonstrated that the experimental value of $R_m = 3.10(15)$ fm for ^{14}Be required $P(s^2) = 0.55(30)$ [179]. A decrease in the uncertainty for this nucleus would be useful.

My understanding of these nuclei has been improved by conversations and extensive correspondence with John Millener. Millener's review on the structure of unstable light nuclei [8] remains an interesting read.

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