

Systematic analyses on H- and He-isotopes by using an extended AMD approach

S. Aoyama^a and N. Itagaki^b

^a Information Processing Center, Kitami Institute of Technology,
Kitami 090-8507, Japan

^b Department of physics, University of Tokyo,
Tokyo 113-0033, Japan

We study systematically H- and He-isotopes using an extended AMD (Antisymmetrized Molecular Dynamics) approach. In this approach, AMD combined with the generator coordinate method is extended with the idea of the stochastic variational method. By using this new method, we study many body-cluster states of extremely neutron-rich nuclei such as ${}^7\text{H}$ ($t+n+n+n+n$) or ${}^{10}\text{He}$ (${}^4\text{He}+n+n+n+n+n$).

1. Introduction

Recently, the development of experiments using unstable nuclear beams enables us to study neutron-rich nuclei [1]. It is a very challenging problem to make an extremely neutron-excess system which corresponds to the neutron star. Very neutron-rich nuclei such as ${}^{10}\text{He}$ and ${}^5\text{H}$ have been observed [2,3]. The neutron ratio to the proton is very large as four. Very recently, an experimental evidence of ${}^7\text{H}$ was reported as an enhancement of the cross section above the $t+n+n+n+n$ threshold [4]. Its neutron ratio to the proton is the largest value of six. Then, we have achieved the making of the neutron-excess system on the earth, which corresponds to the surface of the neutron star.

As typical neutron-rich nuclei, which have large root mean squared radii, ${}^{11}\text{Li}$ and ${}^6\text{He}$ have been studied by using the core-nucleus plus the valence neutrons model. For these nuclei, the three-body model (${}^9\text{Li}+n+n$ or ${}^4\text{He}+n+n$) works well explaining the large radii as the neutron halo [5]. ${}^5\text{H}$ can be regard as a similar system if $t+n+n$ picture is valid. And several theoretical works have been carried out with $t+n+n$ model [6,7]. In the extremely neutron-excess system, since the number of the valence neutrons are large, we must solve many-body problem. For example, we need a $t+n+n+n+n$ five-body cluster model for ${}^7\text{H}$ and a ${}^4\text{He}+n+n+n+n+n$ seven-body cluster model for ${}^{10}\text{He}$ at least. However, solving of the many-body cluster problem is not easy task by using the conventional method.

AMD has proved to be a useful method in the systematic studies on neutron-rich nuclei [8]. However, it is known that the simple AMD wavefunction can not reproduce the halo. In order to reproduce the halo, the superposition of AMD wave functions is important [9,8]. On this situation, very recently, we propose a new extended AMD approach [10]. In

this approach, AMD combined with the generator coordinate method [9] is extended with the idea of the stochastic variational method (SVM) [11,12]. We call this new approach AMD triple-S (Superposition of Selected Snapshots) [10]. By using AMD triple-S, we can easily solve many-body cluster states even such as the seven-body cluster state for ^{10}He . Furthermore, we can combine it with analytic continuation in the coupling constant (ACCC) [13–15] to solve many-body resonances, though it needs a larger calculational time. Then, we give only an example of ACCC for ^6He . By using AMD triple-S, in this paper, as a first step of our systematic studies for light neutron-rich nuclei, we give results for H- and He-isotopes with the core-nucleus plus the valence neutrons model.

2. Framework

Here, we describe the present framework in brief [10]. The AMD wave function is fully antisymmetrized as a Slater determinant,

$$\Psi = \mathcal{A}[(\psi_1\chi_1)(\psi_2\chi_2)\cdots(\psi_A\chi_A)], \quad (1)$$

where \mathcal{A} is an antisymmetrization operator, $\psi_i\chi_i$ is corresponding to the wave function of each nucleon. ψ_i has a Gaussian form,

$$\psi_i = \left(\frac{2\nu}{\pi}\right)^{\frac{3}{4}} \exp[-\nu(\vec{r} - \vec{z}_i/\sqrt{\nu})^2 + \vec{z}_i^2/2], \quad (2)$$

where \vec{z}_i is a complex parameter. χ_i represents the spin-isospin eigen function. The oscillator parameter ($b = \frac{1}{\sqrt{2\nu}}$) is common for all nucleons to exactly remove the center-of-mass kinetic energy, $b=1.58$ fm for H-isotopes and $b=1.46$ fm for He-isotopes. When we assume the presence of an cluster such as triton or alpha, it is expressed by assuming a common \vec{z}_i value for nucleons in the cluster.

The values of \vec{z}_i are randomly generated, but we optimize only the imaginary part of these parameters [10] by using the frictional cooling method in AMD [16]:

$$\frac{d\vec{z}_i}{d\tau} = -Im[\frac{\partial E}{\partial \vec{z}_i^*}]i, \quad \frac{d\vec{z}_i^*}{d\tau} = -Im[\frac{\partial E}{\partial \vec{z}_i}]i. \quad (3)$$

During this cooling process, the parity of the Slater determinant is projected.

We regard this AMD wavefunction whose imaginary part for \vec{z} -parameter is optimized as the basis function of GCM. In GCM, we superpose the basis functions (Ψ_k) as,

$$\Phi = \sum_k c_k P_{MK}^J \Psi_k. \quad (4)$$

Projection onto a good angular momentum is carried out by the projection operator P_{MK}^J , and the coefficients c_k are determined by diagonalizing the Hamiltonian matrix after this projection. Whether the obtained AMD wavefunction is employed or not as a basis function is determined by the procedure of SVM. Then, the imaginary part is determined by AMD (the frictional cooling) and the real part is stochastically determined by SVM.

As a n - n interaction, for the central part, we use the Volkov No.2 effective $N - N$ potential [17]. For the spin-orbit term, we introduce the G3RS potential [18] ($V_{ts} = 2000$ MeV).

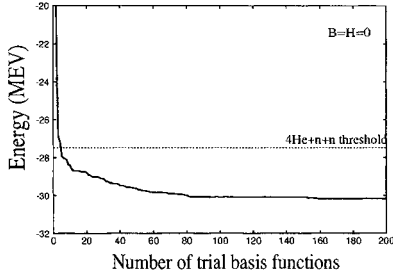


Figure 1. Energy convergence of the ground 0^+ state of ${}^6\text{He}$. The horizontal axis is the number of trial basis functions. The dotted line shows the energy of the ${}^4\text{He}+n+n$ threshold.

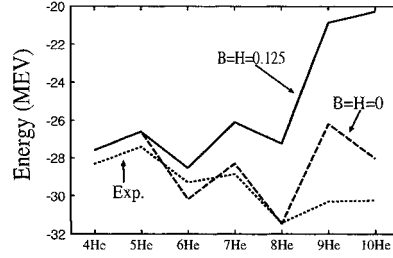


Figure 2. The systematic binding energies for He-isotopes. The solid line is that for $B=H=0.125$, the dashed line is that for $B=H=0$, and the dotted line is that for experiments.

3. Application to He- and H-isotopes

In Figure 1, the 0^+ energy convergence of ${}^6\text{He}$ is shown. The potential parameters for Volkov No.2 are $M=0.6$ and $B=H=0$. Here, the values of z_i are common for four nucleons (α -cluster). The horizontal axis corresponds to the number of trial basis functions. We can clearly see that the energy converge (-30.2 MeV). In order to check the convergency, we carried out the trial several times.

In Figure 2, we show the binding energy for He-isotopes systematically. The solid line is that for $B=H=0.125$. The dashed line is that for $B=H=0$. The dotted line is that for experiments. As seen from figure, for the binding energies of ${}^7\text{He}$ and ${}^8\text{He}$, $B=H=0$ is good. On the other hand, for the binding energies of ${}^5\text{He}$ and ${}^6\text{He}$, $B=H=0.125$ is good. However, for ${}^9\text{He}$ and ${}^{10}\text{He}$, both parameters can not reproduce the binding energies. Therefore, the Volkov No.2 can not reproduce the systematic behavior of the binding energies for He-isotopes. Then, we had better use more realistic interactions rather than using the simple two-body effective interaction.

For ${}^6\text{He}$, the ground state energy becomes overbound for $B=H=0$. In the case of $B=H=0.125$, ${}^4\text{He}+n$ phase-shift is reproduced and the di-neutron is not binding. Then, the binding energy for ${}^6\text{He}$ is more reliable for $B=H=0.125$ than $B=H=0$. As given in Table I, the obtained binding energy is 0.99 MeV, which is very near to the experimental binding energy (0.98 MeV). All the calculation in this paper was carried out within a bound state approximation. Here, we check the applicability of ACCC to the AMD triple-S. The obtained excitation energy of 2^+ state is 1.86 MeV and the decay width is 115 keV. On the other hand, the experimental energy is 1.80 MeV and the decay width is 113 keV. Then the correspondence is very good. It should be mentioned that the calculation has a numerical error, which is estimated as about 0.05 MeV.

In Figure 3, we show the binding energy for H-isotopes systematically. The solid line

Table 1

The calculated separation energy of two neutrons S_{2n} (MeV), the excitation energy and the decay width of 2^+ state (MeV), and r.m.s. radius (fm) of ${}^6\text{He}$ are listed. The references are a: [19], b: [20], c: [21], d: [22]

	S_{2n} (MeV)	$E_x(2^+)$ (MeV)	Γ (MeV)	r.m.s. radius (fm)
AMD triple-S	+0.99	1.89		2.37
AMD triple-S +ACCC	(+0.99)	1.86	115 keV	(2.37)
Exp.	+0.98	1.80	113 keV	2.33 ± 0.04^b 2.48 ± 0.03^c 2.57 ± 0.1^d

is that for $B=H=0$. The dotted line is that for experiments. In the case of $B=H=0$, the potential seems to be sufficiently strong from results for ${}^5\text{H}$. However, the energy of ${}^7\text{H}$ is higher than the experimentally suggested energy region. If the experimental suggestion is right, the additional correlation and/or the modification of the present interaction might be needed. For example, we should develop the present model from a simple $t+n+n+n+n$ model and we should use such a n effective interaction which has the tensor part, because it is known that the tensor force is important for the $t+n$ system.

Finally, as a first step for the future extension, we analyse ${}^7\text{H}$ with a $(t+n+n+n+n)+(p+2n+2n+2n)$ coupled channel model. As shown in Figure 4, ${}^7\text{H}$ is regarded as a system of one proton in six neutrons. If one proton is added in six neutrons, mean-field originates around the proton seed. Then, a triton-like cluster plus four valence neutrons structure can be expected (the upper right in Figure 4). As another possibility, three di-neutron-like cluster originates due to the existence of the proton (the lower right in Figure 4). In other word, three gas-like di-neutron might be condensed. For ${}^5\text{H}$, we obtain the di-neutron gas like state as $E_x=4.1$ MeV. For ${}^7\text{H}$, we obtain the di-neutron gas like state as $E_x=5.6$ MeV. These state should be solved by using the method which can treat many body resonance such as ACCC.

4. Summary

In summary, we study systematically H- and He-isotopes by using AMD triple-S. As a first calculation of the new method, we restrict it within the simple N - N interaction and the simple core-nucleus plus the valence neutrons model, though we will develop them in the near future.

For He-isotope, we can reproduce ${}^6\text{He}$ such as the binding energy of the ground state, the excitation energy and the decay width of 2^+ state. However, we can not reproduce the binding energy for the more neutron-rich He-isotopes. Then, we should change the n - n interaction from the present simple two-body one, which only include the central part and the spin-orbit part.

For H-isotopes, by using the N - N interaction, which almost reproduce the binding energy of ${}^5\text{H}$ within the present model, the binding energy of ${}^7\text{H}$ is smaller than experimentally suggested one. If the experimental suggestion is right, the additional correlation

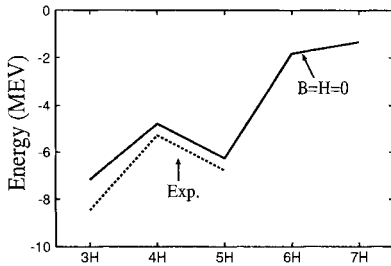


Figure 3. The systematic binding energies for H-isotopes. The solid line is that for $B=H=0$ and the dotted line is that for experiments.

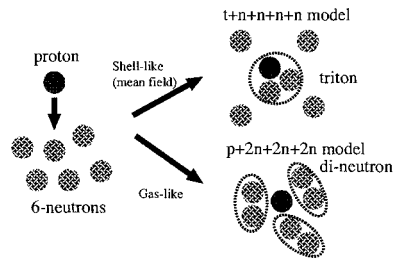


Figure 4. What does it happen if one proton is added in the six neutron system?

and/or the modification of the present interaction might be needed.

As far as AMD triple-S is concerned, we confirmed the usefulness of it. Then, we will develop it in order to discuss the breaking of the core and the effective interaction.

REFERENCES

1. I. Tanihata, J. Phys. **G22**, 157 (1996).
2. A.A. Korshennikov *et al.*, Phys. Lett. **B326**, 31 (1994).
3. A.A. Korshennikov *et al.*, Phys. Rev. Lett. **87**, 092501 (2001).
4. A.A. Korshennikov *et al.*, Phys. Rev. Lett. **90**, 082501 (2003).
5. S. Aoyama, K. Katō and K. Ikeda, Prog. Theor. Phys. Suppl. **No.142**, 35 (2001).
6. K. Descouvemont and A. Kharbach, Phys. Rev. **C68**, 034303 (2003).
7. K. Arai, Phys. Rev. **C68**, 034303 (2003).
8. Y. Kanada-En'yo, H. Horiuchi and A. Ono, Phys. Rev. **C52**, 647 (1995).
9. N. Itagaki and S. Aoyama, Phys. Rev. **C62**, 024303 (2000).
10. N. Itagaki, A. Kobayakawa and S. Aoyama, Phys. Rev. **C68**, 054302 (2003).
11. V.I. Kukulin and V. M. Krasnopol'sky, J. Phys. **G3**, 795 (1977).
12. K. Varga, Y. Suzuki and R.G. Lovas, Nucl. Phys. **A571**, 447 (1994).
13. V.I. Kukulin and V.M. Krasnopol'sky, J. Phys. A **10**, 33 (1977).
14. S. Aoyama, Phys. Rev. Lett. **89**, 052501 (2002).
15. N. Tanaka, Y. Suzuki, K. Varga, and R.G. Lovas, Phys. Rev. C **59**, 1391 (1999).
16. A. Ono *et al.*, Prog. Theor. Phys. **87**, 1185 (1992).
17. A.B. Volkov, Nucl. Phys. **74**, 33 (1965).
18. N. Yamaguchi *et al.*, Prog. Theor. Phys. **62**, 1018 (1979).
19. F. Ajzenberg-Selove, Nucl. Phys. **A490**, 1 (1988).
20. I. Tanihata *et al.*, Phys. Lett. **B289**, 261 (1992).
21. I. Tanihata *et al.*, Phys. Lett. **B160**, 380 (1985).
22. L. V. Chulkov *et al.*, Europhys Lett. **8**, 245 (1989).