A Quick Introduction to Knots and Jones Polynomials

Junyu Lu

University of Manitoba

Nov 2023

DEFINITIONS

Knot = A piece-wise (or smooth) linear embedding of S^1 into $\mathbb{R}^3$ or S^3

DEFINITIONS

Knot = A piece-wise (or smooth) linear embedding of S^1 into $\mathbb{R}^3$ or S^3

Link = A p.l. (or smooth) embedding of disjoint circles into $\mathbb{R}^3$ or S^3

DEFINITIONS

Knot = A piece-wise (or smooth) linear embedding of S^1 into $\mathbb{R}^3$ or S^3

Link = A p.l. (or smooth) embedding of disjoint circles into $\mathbb{R}^3$ or S^3

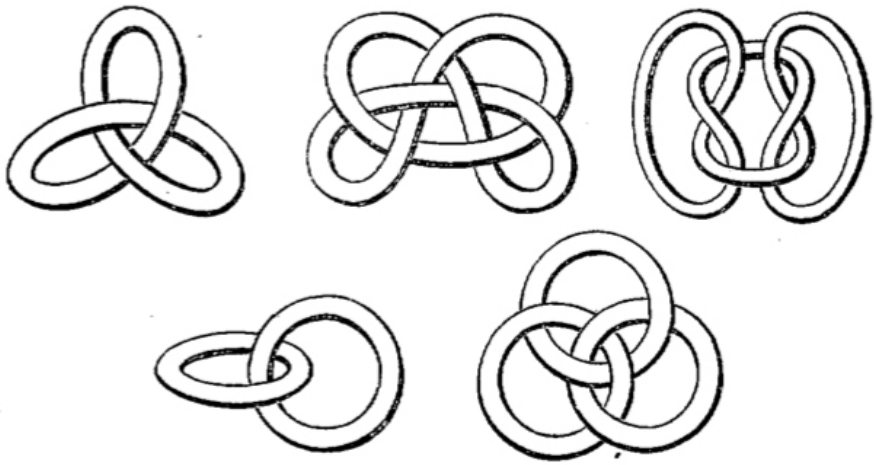
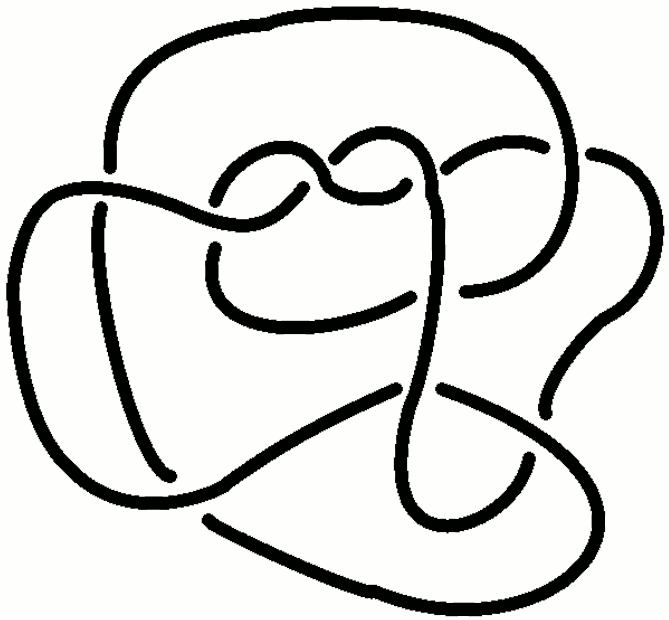
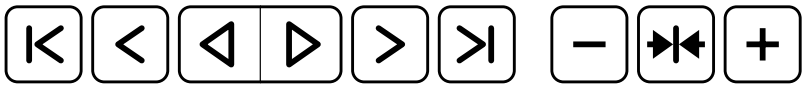


Figure: Illustrations of knots and links, including a trefoil knot, top left, in an 1869 paper by Lord Kelvin on his knotted vortex theory of atoms.

KNOT EQUIVALENCE

Two knots are equivalent if one knot can be pushed about smoothly, without intersecting itself, to coincide with another knot.





Or more rigorously, defined by ambient isotopy or equivalently by an orientation-preserving homeomorphism of S^3 to itself

Reidemeister Moves

Knot Complements

A theorem Gorden and Luecke

Diagrammatic Invariant

Kauffman Bracket

Kauffman Bracket: An Example

Writhe of a Knot/Link

Hopf Link

Left Trefoil Knot

Right Trefoil Knot

Mirror Image

Theorem

The Jones polynomial of the mirror image $\bar{L}$ of an oriented link L is the conjugate under $t \leftrightarrow t^{-1}$ of the polynomial of L.

Mirror Image

Theorem

The Jones polynomial of the mirror image $\bar{L}$ of an oriented link L is the conjugate under $t \leftrightarrow t^{-1}$ of the polynomial of L.

Proof.

asd

Fail for palindromes

Connected Sum

Fail for palindromes

Yet Another Approach

Skein Relation

Yet Another Another Approach

Skein Relation

Yet More Approaches

Skein Relation

Conjecture

Coloured Jones Polynomials

Whatever ite means

CONJECTURES

AJ Conjecture
Volume Conjecture