MATH 8510 GALOIS THEORY

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1. Week 1

Let us agree on some facts from elementary abstract algebra before we dig into Galois theory.

Theorem 1.1. *Let F be a commutative ring. Then $F[X]$ is a PID if and only if F is a field.*

Hence or therefore $\mathbb{Z}[X]$ is not a PID. Indeed, $\langle 2, X\rangle$ is an example of an ideal that cannot be generated by a single polynomial. $K[X,Y]$ is not a PID as $\langle X, Y\rangle$ is not principal.

Theorem 1.2. *An ideal in a PID is prime if and only if it is maximal.*

Theorem 1.3 (Gauss's Lemma)**.** *A polynomial $f(X) \in \mathbb{Z}[X]$ is irreducible if and only if it is irreducible over $\mathbb{Q}[X]$.*

Theorem 1.4 (Eisenstein Criterion)**.** *Let $f(X) = a_0 + a_1X + ... + a_nX^n \in \mathbb{Z}[X]$ be a polynomial over integers where $a_n \neq 0$. Suppose that there exists a prime p such that*

(1) $p \nmid a_n$,
(2) $p \mid a_i$ *for* $i = 0, 1, ..., n-1$,
(3) $p^2 \nmid a_0$.

Then $f(X)$ is irreducible over $\mathbb{Z}[X]$.

References