Your work on these assignments must be your own and follow the guidelines for academic integrity in the course syllabus. Any discussions with other students must be cited in your submission. If you have any questions, please ask the instructor.

Consult your email for the link and instructions on how to submit the assignment via Crowdmark. There is a deadline of **due 21 Mar 2022, 11:59pm**.

Students in MATH 4300 will be graded out of 70 **points**.
Students in MATH 7300 will be graded out of 80 **points**.
There are up to 95 points available.

[15] 1. (a) Give a configuration of 6 points in general position (with no two x-coordinates the same) with no 4-cup and no 4-cap.

(b) Using the proof of Theorem 7.2.4, find a placement of $\binom{k+\ell-4}{k-2}$ points with no k-cup and no ℓ-cap.

[15] 2. Prove that for each k, there exists $n = n(k)$ so that for every $n(k)$-point set in the plane, there is either a convex k-gon (whose vertices are in the point set) or else k points lying on a common line. (Note that there is no requirement that the points be in general position).

[10] 3. Give an example of a 3-colouring of $\mathbb{R}^2$ so that all colours are used and every line receives exactly two colours.

[15] 4. Show that for every 2-colouring of $\mathbb{R}^2 = \mathbb{E}^2$, one of the colour classes contains pairs of points at every possible distance d, $0 < d < \infty$.

[10] 5. Give a 2-colouring of $\mathbb{E}^2$ to show that if S denotes the four vertices of a unit square, $\mathbb{E}^2 \not\to (S)^{\bullet}_2$.

[15] 6. Finish the details sketched in class: If $C \subseteq \mathbb{R}^d$ is a convex, centrally symmetric set and has finite non-zero volume, then C is bounded. (You can assume, from the non-zero volume condition, that C contains a small 'box' and hence a small ball).

[15] 7. Prove that if $C \subseteq \mathbb{R}^d$ is convex, centrally symmetric and bounded with $vol(C) > k2^d$, then C contains at least $2k$ lattice points.

Total points: 95