Your work on these assignments must be your own and follow the guidelines for academic integrity in the course syllabus. Any discussions with other students must be cited in your submission. If you have any questions, please ask the instructor.

Consult your email for the link and instructions on how to submit the assignment via Crowdmark. There is a deadline of **due 4 Apr 2022, 11:59pm**.

Students in MATH 4300 will be graded out of 75 **points**.

Students in MATH 7300 will be graded out of 85 **points**.

There are up to 90 points available.

[10] 1. Find an example of 8 points in the plane that determine only 4 ordinary lines (lines containing exactly 2 points). (See lecture notes Exercise 272).

[15] 2. See Exercise 276: Prove that the (9_3) configuration given in Figure 15.9 (p 396) is not isomorphic to the Pappus configuration (see Figure 1.2). The isomorphism here is as combinatorial configurations.

[10] 3. Show that for $n = 4$, the Bezdek-Connelly theorem is tight: There are 4 unit circles in the plane with each circle intersecting at least one other and not two tangent with exactly 4 intersection points.

[15] 4. (See Exercise 279). Let q be a power of a prime. Show that the projective plane over $\mathbb{F}_q$ contains a complete $(q+1)$-point. See the hint in the lecture notes.

[15] 5. (See Exercise 281) Let P be a set of n points in the plane. Use the Szemerédi-Trotter theorem to prove that the number of triangles with area equal to 1 determined by the points of P is $O(n^{7/3})$. (Hint: Define lines that would contain points that complete a unit triangle with some pair of points in P).

[15] 6. Use the Szemerédi-Trotter theorem to show that, for every k, m, given a set of m points in the plane, the maximum possible number of distinct lines such that each of them contains at least k points of P is at most $O(m^2/k^2 + m/k)$.

[10] 7. Chose one of the following to answer (you need answer only one part for full points):

(a) Show that the crossing number of the Petersen graph (see Figure 17.3) is 2.

(b) Show that the $cr(K_{3,2,2}) = 2$.

Total points: 90