

# Time Series Forecasting

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30 April 2022

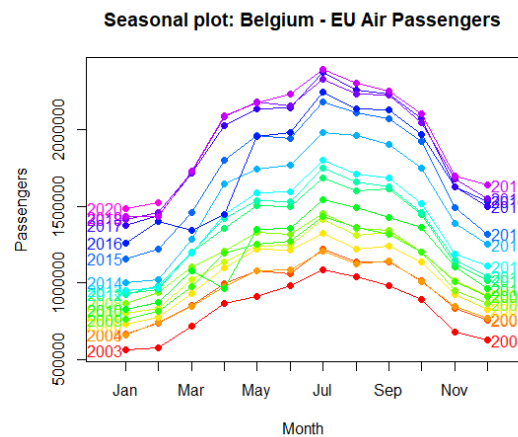
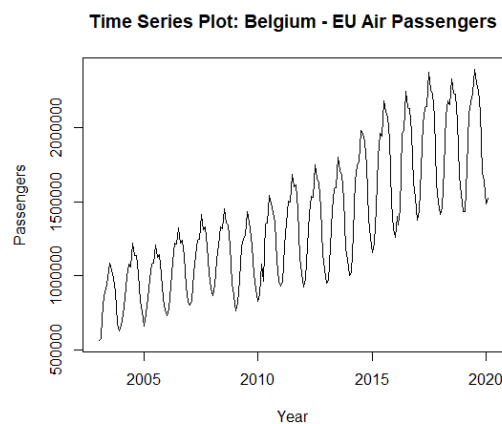
This paper applies time series analysis and forecasting techniques on two different datasets. The aim is to discuss different methods and display the forecasting process from start to end. The programming language used is R and the attached script contains the code used for all the analysis used in this paper.

## Part A : Belgium – EU Air Passenger Time Series

The first part of this paper is based on the air passenger dataset which contains international intra-EU air passenger transport by Belgium and EU partner countries, from January 2003 to October 2021. The dataset is divided into the following sets.

Training Set	Test Set	Remaining
Jan 2003 - Dec 2017	Jan 2018 – Feb 2020	Mar 2020 – Oct 2021

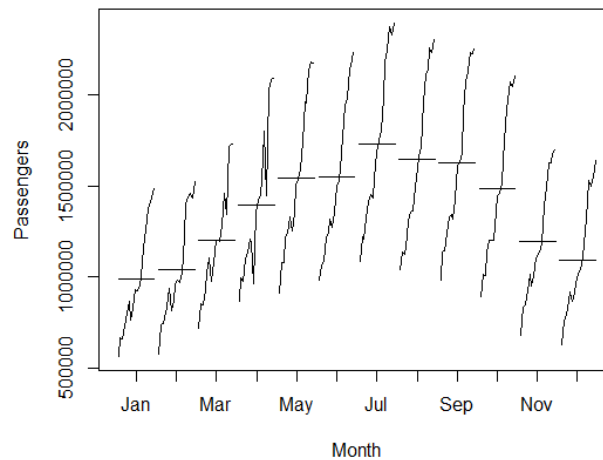
To begin, the first step is to study and explore the data using graphs.



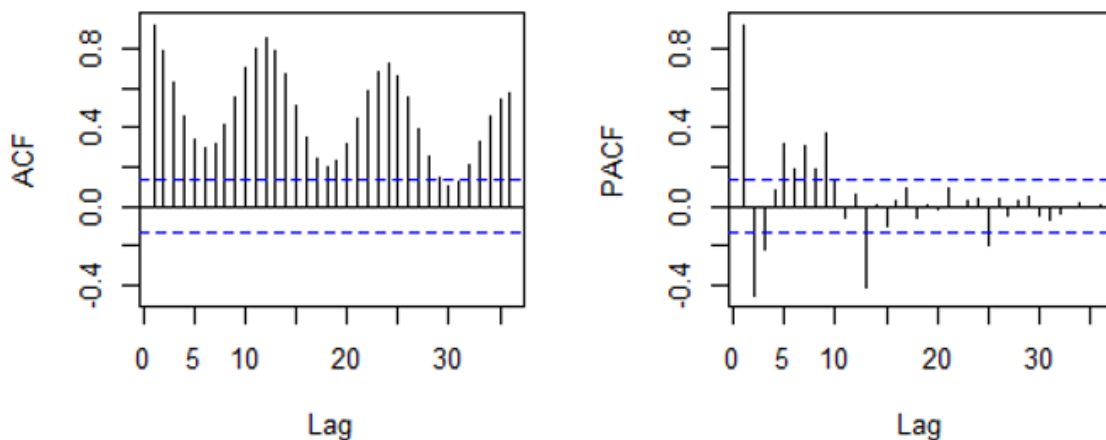
The time series plot shows that the data has a strong seasonal pattern and an upward trend. The spread of the seasonal patterns also increases with time which means that there is an increase in the variance over time. This shows that the forecasting methods to be applied on this dataset will need to take into account seasonality and trend.

The seasonal plot displays the data by season (month in this case). It helps to inspect seasonal patterns and shows that there are peaks in July and troughs in January and December. This could mainly be due to holiday season in the summers where there is more air travel. The fact that the lines move upward each year shows that there is an upward trend in the time series. The size of shift of the lines is also gradually increasing per year which means that the variance in the seasonality increases over time. It can also be seen that the passenger count is unusually low in April 2010 and 2017. This could be due to some unusual circumstances.

**Seasonal subseries plot: Belgium - EU Air Passengers**



A seasonal sub series plot is another way of inspecting the seasons. It creates individual plots for each season. This plot shows that the number of passengers is increasing overtime for each month, indicating the existence of an increasing trend. It also shows that the mean for July is much higher than January/December, indicating seasonality.



Autocorrelation measures the linear relationship between lagged values of a time series. This means correlation between observations that are one period apart, two periods apart and so on. Partial autocorrelation removes the dependency between lags. For example, if observations in a point in time have high autocorrelation with observations in lag and 2, the high autocorrelation with lag 2 could depend on the autocorrelation with lag 1. In this case, the partial autocorrelation will remove the in between dependency and only show the partial autocorrelation with lag 2. If the partial autocorrelation at lag 2 is not significant, it reflects that only lag 1 plays an important role.

The plots above correspond to the autocorrelation function and the partial autocorrelation. The blue dotted lines represent the 95 % confidence bounds and autocorrelations beyond the blue lines can be considered as significantly different from zero.

Seasonality and trend seen in the time series plot are also reflected in the ACF plot. The waves indicate that there is seasonality in the data. The peaks at lags 12, 24, and 36 show seasonality of length 12 i.e. monthly seasonality. This means that an observation correlates most strongly with observations that are 12 months apart e.g., March 2017 will correlate strongly with March 2016. On the other hand, the positivity and gradual decay in the autocorrelation signifies the presence on a trend.

In the PACF plot, it can be seen that recent lags play a much more important role than lags beyond 12. This means that the past year has a much more impact than the other years. The other lags seem to highly correlate in the ACF plot because of the dependency on the seasonal lags (lags 12 months apart).

## Transformation

The plots above show that the variance of the seasons increases overtime. Sometimes, this can lead to issues in some procedures as most procedures assume constant variance. The assumption of constant variance is made when creating prediction intervals, otherwise, the prediction intervals (the confidence that we have in the predictions) might be under or overestimated. Therefore, transformations can be helpful to stabilize the variance. There are multiple ways to transform data ranging from taking the square root to the log. This paper uses the box-cox family methods to identify which one should be used. The box-cox method realizes the transformations by letting the value of a parameter ( $\lambda$ ) change. If the value of  $\lambda$  is 1 then no transformation or if the value of  $\lambda$  is 0 then the transformation is done by taking the natural log. Box-cox gives the  $\lambda$  value that optimizes the variance around the seasonal component i.e., it will look for the  $\lambda$  value that gives the most constant spread.

After applying the box-cox method to the passengers dataset (train + test), a  $\lambda$  value of 0.0146 was obtained. Since the value is close to zero, the  $\lambda$  value was set to be zero. Such a small change does not impact forecasts and a  $\lambda$  of zero means a log transformation which provides better interpretability as changes in a log value are relative changes on the original scale. Using log transformation also guarantees that there will be no negative forecasts. This is relevant for the passengers data as the number of passengers cannot be negative.

A quick test was run using the ETS framework to see if a transformation adds any value. ETS model was run with  $\lambda$  and bias adjustment and another ETS model was run without any transformation. The MASE obtained for the transformed data was 0.567 which is better than the MASE of 0.589 on non-transformed data. The residuals of the of the transformed data also show better performance with a higher Ljung-Box p value. Therefore, it is decided to proceed with transformation.

In **all** the subsequent steps of the project, the  $\lambda$  value of zero will be specified in the forecast functions so that back transformed results are obtained which are compatible to be compared with the original data. It is important to note that when a forecast is back transformed to the original scale, the forecast is not the average value as it usually is but it is the median value. The aim is to forecast the average value so a bias adjustment is made i.e. re-adjust the forecast to make sure it reflects the average forecasted value. This is especially important when comparing the forecast accuracy of original data with forecast accuracy of transformed data and then back transformed. Therefore, the bias adjustment parameter will also be set to true in **all** subsequent forecasting functions.

This paper will now apply different models to the passengers time series and compare the results.

### Seasonal Naïve Forecast

The first model applied is the seasonal naïve method. Forecasts under this method are equal to the last observed value from the same season. For example, a forecast for January 2021 would be the same as the observed value of January 2020 under this method.

After forecasting, the residual diagnostics were checked. Residuals are the difference between observed values and their forecast based on all previous observations. It is assumed that the residuals are white noise. White noise data are uncorrelated across time with zero mean and constant variance. If this is not the case, there is information left in the residuals that should be used in computing forecasts. Therefore, the first residual diagnostic is to check the ACF of the residuals of the seasonal naïve forecast to see if they indeed look like white noise.

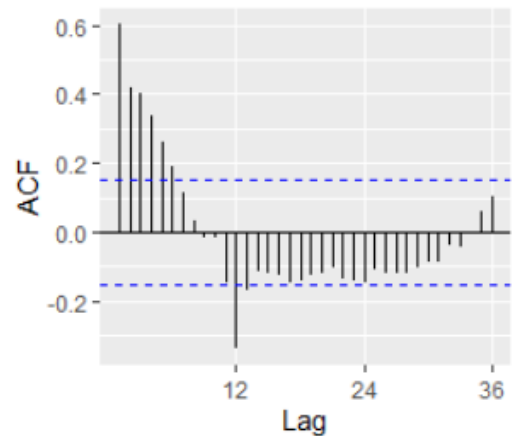
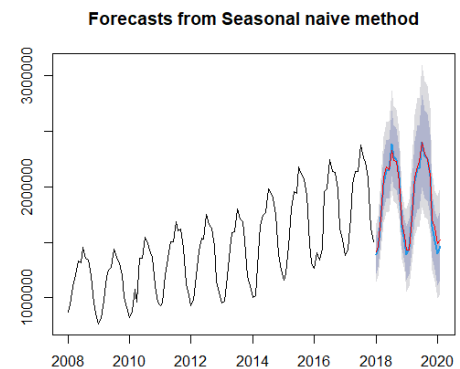
ACF shows significant autocorrelation at multiple lags which means that the series is not a white noise series. A second check is made using the Ljung-Box test. Ljung-Box test is a statistical test to check if the autocorrelations are different from zero. The null hypothesis assumes white noise series. For the seasonal naïve forecast residuals, the p value of the Ljung-Box test is close to zero which means that the null hypothesis is rejected. This signifies that the residuals are not white noise. It can be concluded that the seasonal naïve method was not able to capture all the information from the time series.

Forecasts were made on the test set and then evaluated. The mean absolute scaled error for the forecast is 0.377 which means that the accuracy is good as the value is low. The forecast graph also shows that the forecast is very similar to the observed test data. The value will be later compared with the accuracy of other models for a better comparison.

### STL Decomposition

A time series is a function of three components – seasonal component, trend-cycle component, and the remainder component. The function can be additive (sum of the different components) or multiplicative (multiplication of the components). The additive model is used if the magnitude of the seasonal fluctuations does not vary with the level of the time series. On the other hand, if the variation in the seasonal pattern is proportional to the level of the time series, a multiplicative model is more appropriate.

Seasonal component just shows the changes according to season without the trend and irregularities. Trend cycle is the data after getting rid of the seasonal fluctuations and the irregular fluctuations. It shows the overall movement of the data. A by product of decomposition is seasonally adjusted data. It is the seasonal time series where the seasonal component has been eliminated so what remains is the trend

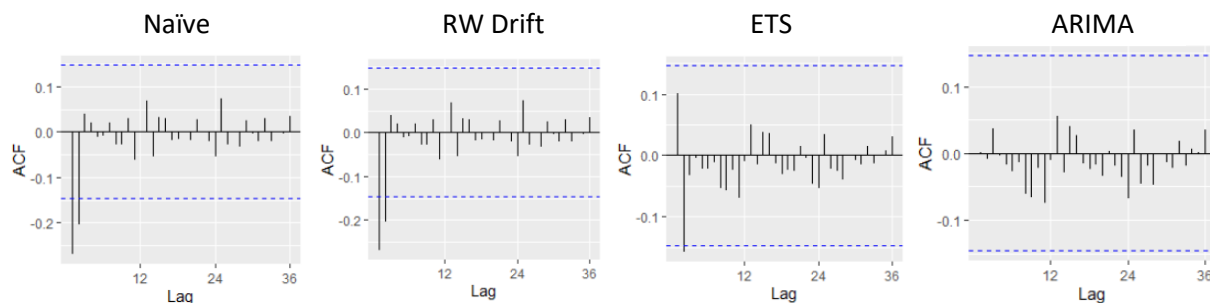


and the irregular component. It is used to see how the time series evolves without the usual fluctuations that happen every season.

The decomposition method used in this section of the paper is called Seasonal and Trend decomposition using Loess (STL). Loess stands for local estimation procedures – these procedures cut the time series into pieces so a local linear estimation of the individual pieces can provide a smooth trend of the whole data that is nonlinear and changing over time. STL is an additive model can work on data with time varying seasonality.

To forecast using decomposition, the seasonal component is forecasted using the seasonal naïve method. The seasonally adjusted data is forecasted using one of the non-seasonal time series methods. These are then combined to get a forecast on the original data. This paper uses four methods to forecast the seasonally adjusted data - naïve, random walk with drift, ets, and arima. This is done using a convenient *stlf()* function in R which decomposes and forecasts in one step.

Residuals and forecast accuracy on the test set were then checked for the four models.

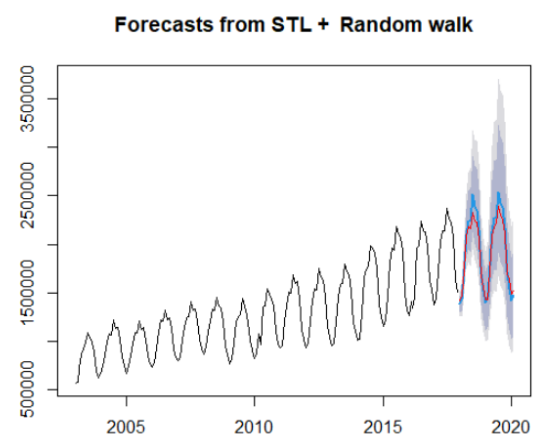


Autocorrelation plot shows that the residuals of the naïve and random walk with drift model had some correlations i.e. might not be white noise. ETS performs better with almost all correlations within the critical bounds but ARIMA seems to perform best based on the autocorrelations as the lines are close to 0 i.e. is white noise. Overall all four models seem to be capturing the time series well.

The result of the Ljung-Box test show that the p values of all the 4 models are above 0.05 which means that the null hypothesis that the residuals are white noise stands. The p value is extremely high for ARIMA and ETS decompositions.

Four different accuracy measures were considered and based on the MASE the naïve method has the highest accuracy (lowest MASE) followed by the ARIMA method.

Giving preference to the simplicity of the model and focusing on high forecast accuracy, the STL decomposition using naïve method for forecasting seasonally adjusted data is selected as the best performing model out of the four. The graph shows the forecast in blue and the observed test data in red.



## ETS Framework

The next concept applied in this paper is of exponential smoothing. In the simple forecasting methods such as the naïve method, all the weight is put on the last observation for forecasting or in the case of the mean method the weight is distributed equally across all observations. In most cases, it is desirable to weigh the recent data more highly but also not completely ignore the previous observations. Exponential smoothing comes into play here as it produces forecasts that are weighted averages of past observations, with the weights decaying exponentially as the observations get older.

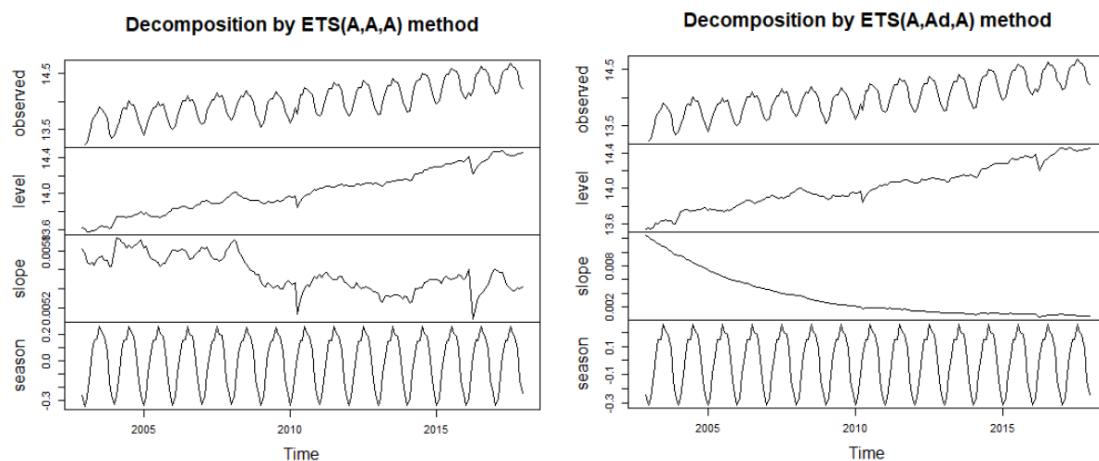
Exponential Smoothing Methods combine error, trend, and seasonal components to produce the smoothing. Each component can be combined either additively, multiplicatively, or be left out of the model. ETS stands for these three components. Since there are different ways of combining these components, there are multiple exponential smoothing methods the ETS framework provides an automatic way of selecting the best method.

The ETS framework estimates the different models using maximum likelihood method. It chooses parameters in such a way that the likelihood that these data have been realized is maximized. Once it obtains the likelihood expression, it calculates the Akaike Information Criterion (AIC). AIC balances the maximum likelihood that can be obtained and the number of parameters that we need to do so. Ideally, we want the likelihood as high as possible and the model as simple as possible with the lowest number of parameters needed.

First, this paper applies two different models using the ETS framework on the passengers dataset. Since transformed data is used (through setting lambda as 0), multiplicative models are not applicable. The transformation has already taken the seasonality variance into account. Therefore, the Additive Holt-Winters' method with additive errors (Error = Additive, Trend = Additive, Seasonality = Additive) is applied twice. Once with damping set to true and the other with damping set to false.

Comparing the two models based on the AIC shows that the model with damping set to true has the best fit with the lowest AIC.

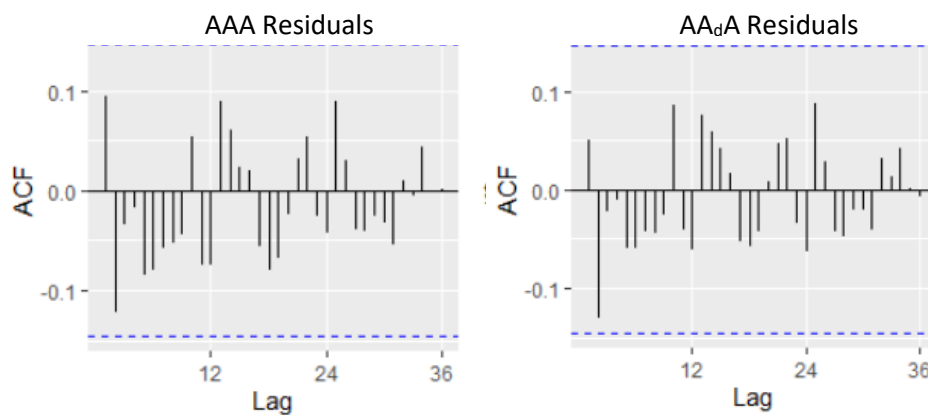
The automated ETS procedure was then applied to get the best model based on a grid search of all possible models. The result shows that the best model is also the Additive Holt-Winters' method with additive errors which was tested before.



The AIC of the AA<sub>d</sub>A model is lower compared to the AAA which means that it provides the more optimal fit. The alpha for both the fits is close to 0.5 (not high) which is why the level series is smoother than the observed series. A low alpha value gives more weight to all the observations and a high alpha value gives more weight to recent observations.

The beta value of 0.0013 is the same for both the fits. It shows how the slope changes overtime and a low value shows that there is constant trend. However, since there is an added element of damping in the AA<sub>d</sub>A model, the graph shows that the slope keeps falling change in the slope becomes stagnant (damped) overtime.

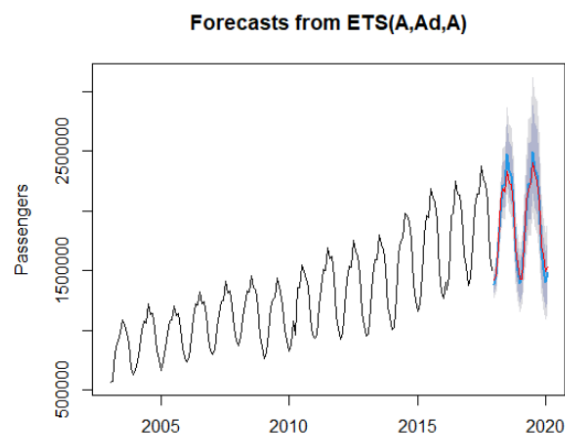
The decision of the best model cannot be made yet without looking at the residual diagnostics and the forecast accuracy.



The correlations of the residuals for both the models are within the critical bounds so it looks like the residuals are white noise. However, the Ljung-Box test shows p values less than 0.05 which means that the residuals have significant correlations and not white noise.

Four different accuracy measures were considered and based on the MASE along with other methods, the AA<sub>d</sub>A model has a considerably higher accuracy. Therefore, it is selected as the best model. The parameters and forecast are as shown below. The graph shows the forecast in blue and the observed test data in red.

Alpha	0.5585
Beta	0.0013
Gamma	1e-04
Phi	0.98



## ARIMA Forecasting

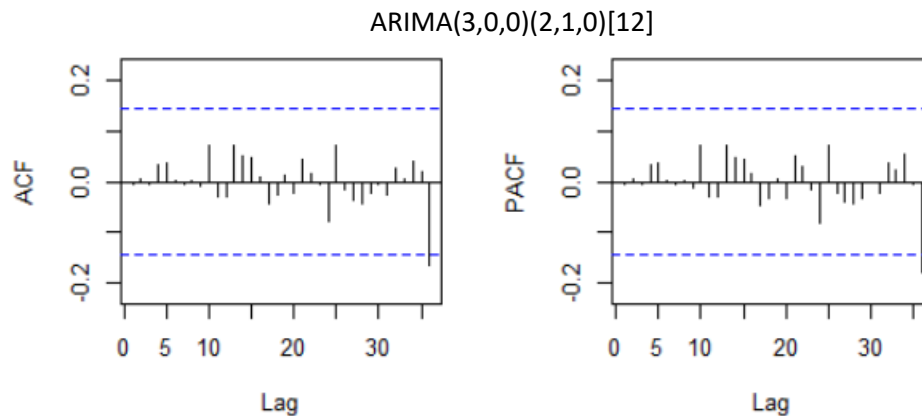
This section applies ARIMA modeling method for forecasting. It stands for Auto Regressive Integrated Moving Average and unlike the ETS it works by getting rid of all non-stationary aspects in a time series. An ARIMA model comprises of three items – p, d, and q. p is the order of the auto regressive (AR) term, q is the order of the moving average (MA) term, and d is the number of differencing required to make the time series stationary. AR means the number of lags to be used as predictors and MA means the number of lagged forecast errors to be used by the model.

If a time series has seasonal patterns, then seasonal terms are also added to the model which are denoted by P, D, and Q.

This paper uses the auto ARIMA procedure to find the optimal model through a grid search. Auto ARIMA works by selecting the number of differences through a unit root test and selects p and q by minimizing AIC. As a first step, the auto ARIMA function is used on the transformed dataset with all the short-cuts turned off to be able to search all possible models. The resulting optimal model is ARIMA(3,0,0)(2,1,0)[12] with drift. This model can be represented using the backward shift operator as follows:

$$(1 - \phi_1 B - \phi_2 B^2 - \phi_3 B^3)(1 - \Phi_1 B^{12} - \Phi_2 B^{24})(1 - B^{12})y_t = e_t$$

The residual diagnostics and forecast accuracy of the model were then checked.

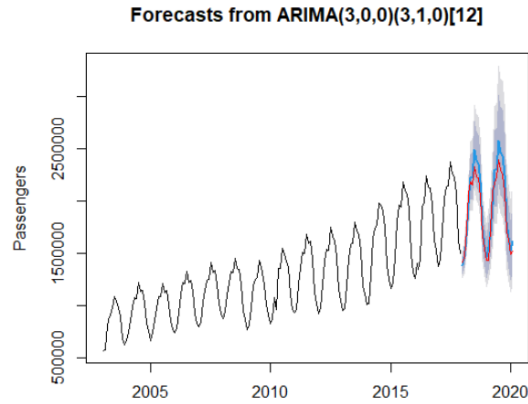


The plots show that most of the autocorrelations are within the critical bound except at lag 36. The spikes for the seasonal lags are increasing. This suggests a seasonal MA(3). Inspecting the PACF using the same logic it suggests a seasonal AR(3). Therefore ARIMA(3,0,0)(2,1,3)[12] and ARIMA(3,0,0)(3,1,0)[12] were also ran. The level of differencing will be kept the same so that the models can be compared by AIC as well.

	<a href="#">ARIMA(3,0,0)(2,1,0)[12]</a>	ARIMA(3,0,0)(2,1,3)[12]	ARIMA(3,0,0)(3,1,0)[12]
Residuals	Spikes at lags 36 in the residual autocorrelation plots.	No spikes beyond the critical boundary	No spikes beyond the critical boundary
Ljung-Box	P = 0.9955	P = 0.6551	P = 0.9629
MASE	1.523	1.437	0.985
AIC	-526.78	-535.42	-519.3

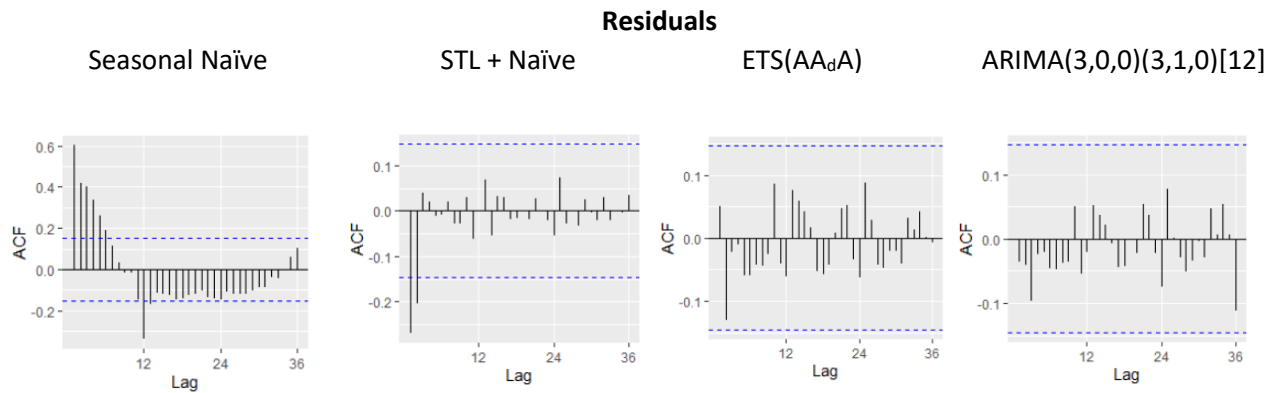


The table above compares the output from the three models. All three pass the Ljung-Box test but the first has an extremely high p value which means that the residuals are white noise only. The difference in AIC is extremely high between the optimal model and the other two. The second model produces the lowest AIC which means it is a simple model with few parameters to obtain the maximum likelihood. However, the forecast accuracy on the test set shows that the third model has the highest accuracy. Since the difference in accuracy between the models is considerable the third model (ARIMA(3,0,0)(3,1,0)[12]) is selected as the best one while compromising on the quality of the residuals and simplicity of the model. The graph below shows the forecast in blue and the observed test data in red.



### Comparison of the Best Models

The paper will now compare the best models obtained from the four different kind of forecasting models that have been shown so far using residual diagnostics and forecast accuracy.

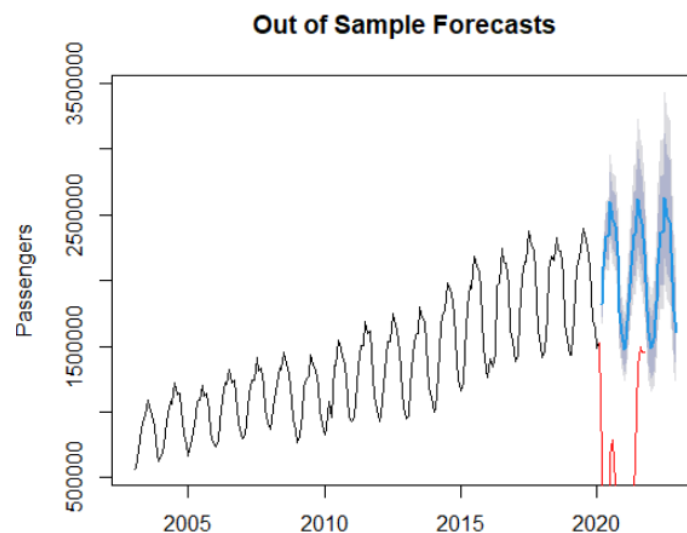


Measures	Seasonal Naïve	STL + Naïve (RW)	ETS(AA <sub>d</sub> A)	ARIMA(3,0,0)(3,1,0)
Ljung-Box Test	p < 2.2e-16; df 24	P = 0.3839; df 24	P = 0.04747; df 24	P = 0.9629; df 24
MASE	0.377	0.747	0.551	0.985
RMSE	46096.78	86601.94	64984.08	109719.52
MAE	35825.84	71013.49	52377.44	93556.98
MAPE	2.017294	3.526007	2.724423	4.657556

From the residual plots it can be seen that the seasonal naïve residuals have information left in them as the correlations are very high. This is also confirmed by the Ljung-Box test. On the other hand the ARIMA model is able to capture the data well and the residuals appear to be white noise. This is also confirmed by the Ljung-Box test. The other accuracy measures unanimously show that the seasonal naïve has the best accuracy on the test set. After this, the ETS(AA<sub>d</sub>A) model has the best accuracy based on all the accuracy measures. **ETS(AA<sub>d</sub>A)** is selected as the final model as it has a good forecast accuracy even though it compromises the quality of the residuals and simplicity of the model.

### Out of Sample Forecasts and Analysis

Based on the final selected model, out of sample forecasts were generated till December 2022 based on the complete timeseries (train + test).



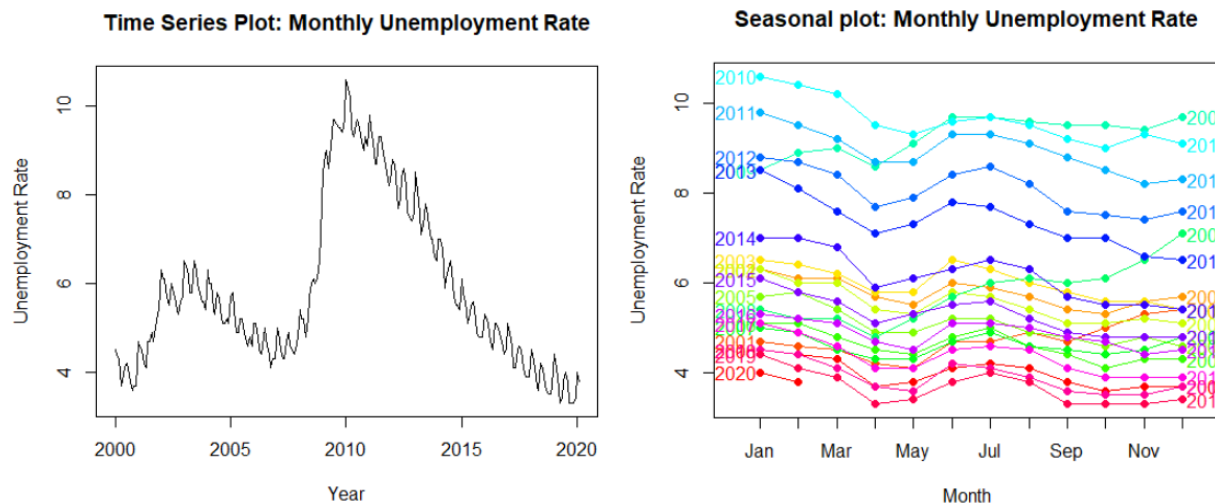
The blue lines show the forecast based on the selected model while the red line shows the observed data during the COVID-19 pandemic. Since these were unforeseen circumstances which were not present in the timeseries used for forecasting, the forecasts are inaccurate. The pandemic can be seen to have a massive impact on the air passenger transport between Belgium and other EU countries. Taking the average of the forecasted values for each month from March 2020 to October 2021 shows that on average 2,150,577 air passengers were forecasted to travel each month. In reality, an average of 539,919 passengers travelled per month in this period. This shows a fall of 75% in the number of passengers from the forecast compared to the reality.

## Part B : U.S. Monthly Unemployment Rate Time Series

The second part of this paper will focus on the monthly unemployment rate time series. The unemployment rate represents the number of unemployed as a percentage of the labor force. This section will first explore the data and then apply different techniques (other than the naïve methods) for forecasting. The results of these techniques will then be compared to obtain the best model and do out of sample forecasting. The series ranges from January 2000 to March 2022. It is divided into the following sets. The forecasting methods will be applied to the training set and the accuracy will be calculated based on the test set.

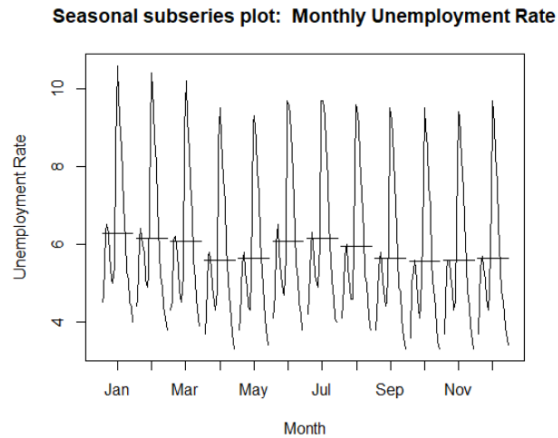
Training Set	Test Set	Remaining
Jan 2000 - Dec 2017	Jan 2018 – Feb 2020	Mar 2020 – Mar 2022

### Data Exploration

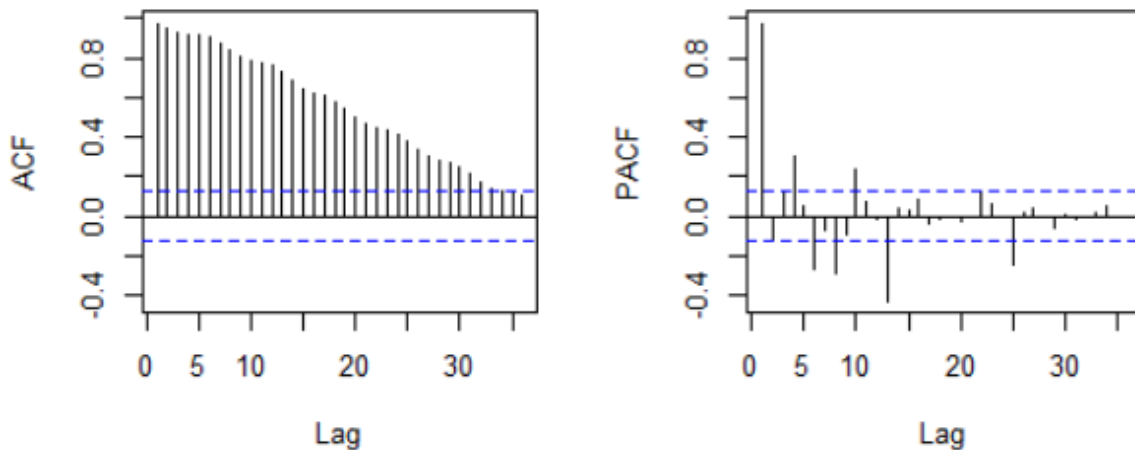


The time series plot shows that the data is cyclical because of the large wave. The smaller waves show that there is a seasonal pattern in the data. A trend cannot be seen in the plot due to its cyclical nature.

The seasonal plot confirms the presence of seasonality. It shows peaks in January, troughs in April, then peaks in July and troughs in September. This shows that the rate rises and falls according to the fiscal quarters. The lines for each year move up each other for a number of years and then move down again which shows the cyclical element of the series.



The seasonal sub series plot shows peaks which represent the extreme cycle in the data. The average of the months seem to be similar to each other which could mean that the seasonality is not strong. However, the scale of the plot is impacted by the outliers as well.

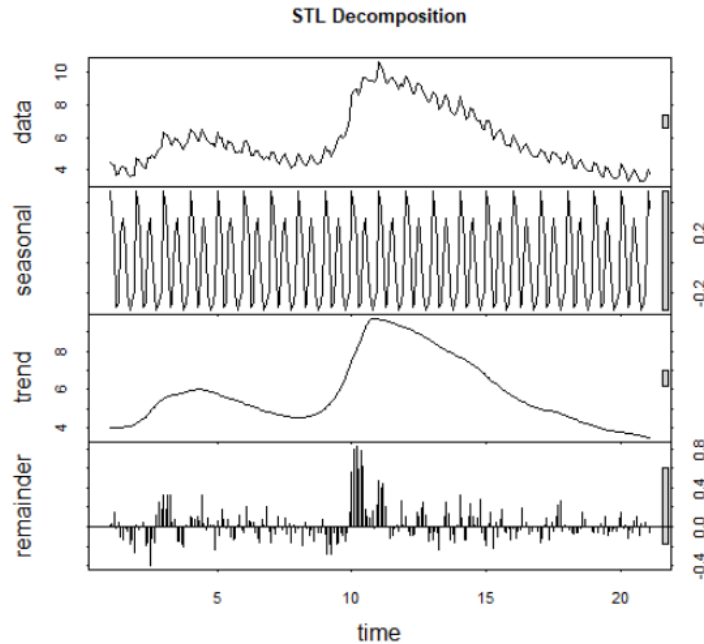


The ACF plot shows significant correlation between the lags. It shows that an observation is correlated with observations till 30 lags way. The PACF plot spikes at 12 and 24 show that there is seasonality in the data. However, observations are also correlated with observations within the 12 lag period. This means that recent lags play an important role.

Since a variance in the spread of the seasons cannot be seen, no transformations are being applied to the dataset. Any special cases have also not been found to warrant the use of any adjustments.

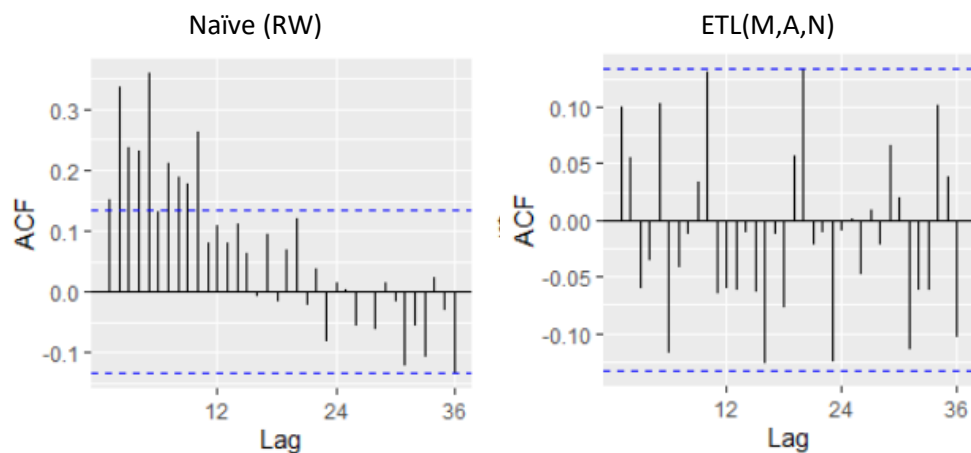
## Decomposition

The data has a cyclical and seasonal component so a decomposition will be used for forecasting. Since the magnitude of the seasonal fluctuations or the variation around the trend-cycle does not vary with the level of the time series, an additive model will be used - STL.



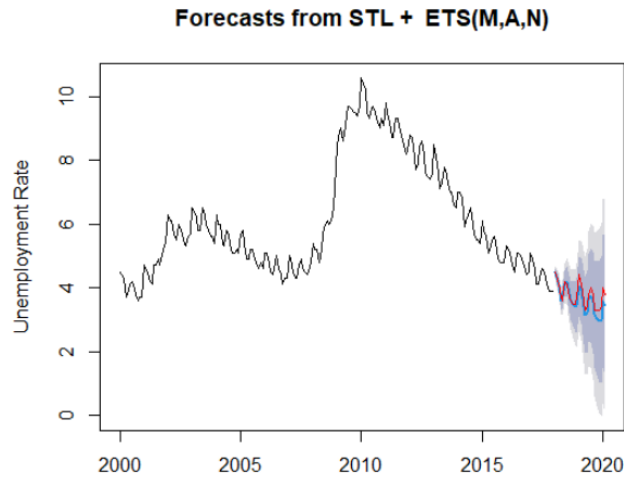
The graph above shows the STL decomposition of the data. The spread of the seasonal component is constant, no linear trend can be identified but rather a cycle. The large positive values in the remainder correspond to the peak in the cycle around 2010. Since the remainder is not constant, a multiplicative ETS model can be applied.

Naïve and ets non-seasonal methods will be used for the seasonally adjusted data. The robust parameter is set to true to treat any outliers. The following results are obtained.



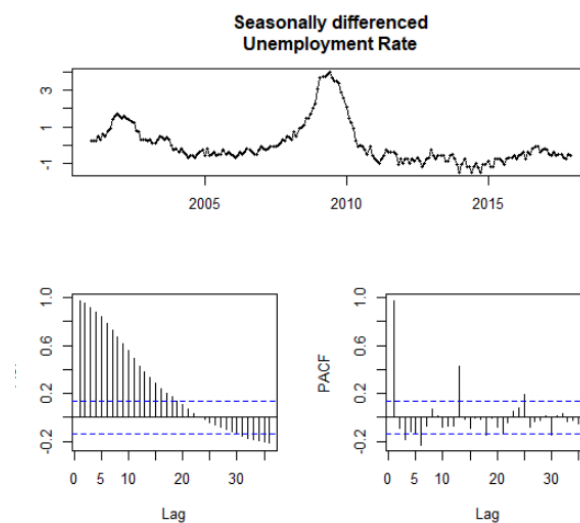
Residual diagnostics look much better for ETL as expected as it has been able to capture the multiplicative element. High autocorrelation spikes for Naïve mean that a lot of information has been left and the residual are not white noise. This is confirmed by the Ljung-Box test which has a very low p value. P value for ETL is 0.043. it performs better than the naïve but it also has spikes. Therefore, it can be said that both models are not able to take all information from the series. Accuracy measures show that ETS performs better with a MASE of 0.213 as compared to MASE of 0.426 for the naïve method. Therefore, STL+ETS is

selected as the better performing model. The forecast is as shown below. The blue lines show the forecast while the red line shows the observed test data.

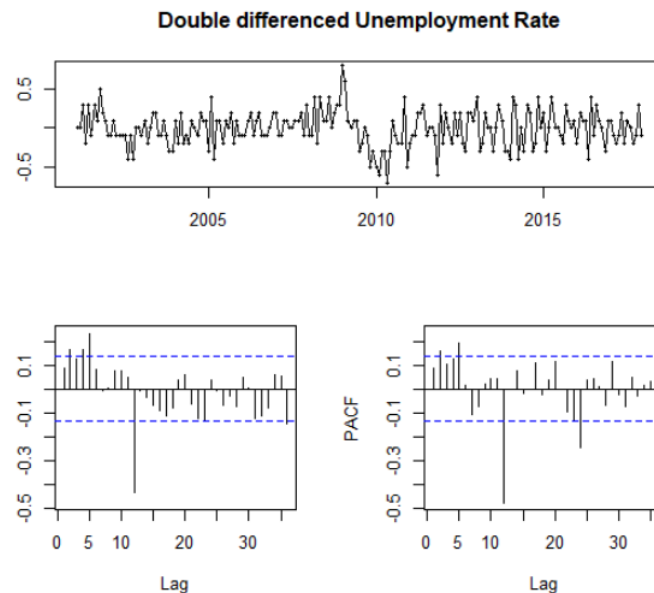


### ARIMA Forecasting

Seasonal Arima will be used for this dataset. The ACF plot of this whole dataset shows non stationary behavior displayed by the spikes. Looking at the spike at lag 12 in the PACF function of the whole dataset which shows seasonality, it is decided to take a seasonal difference.

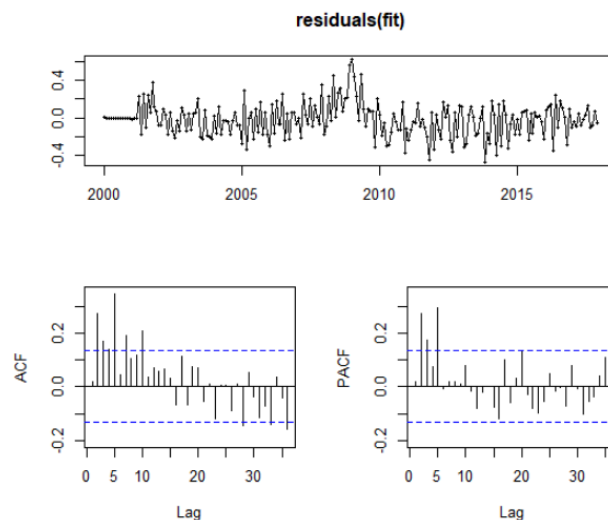


The seasonally differenced data is not very satisfactory as the residuals are not spread around zero. Therefore, we will double difference the data so apart from seasonal differences we also take one period differences.

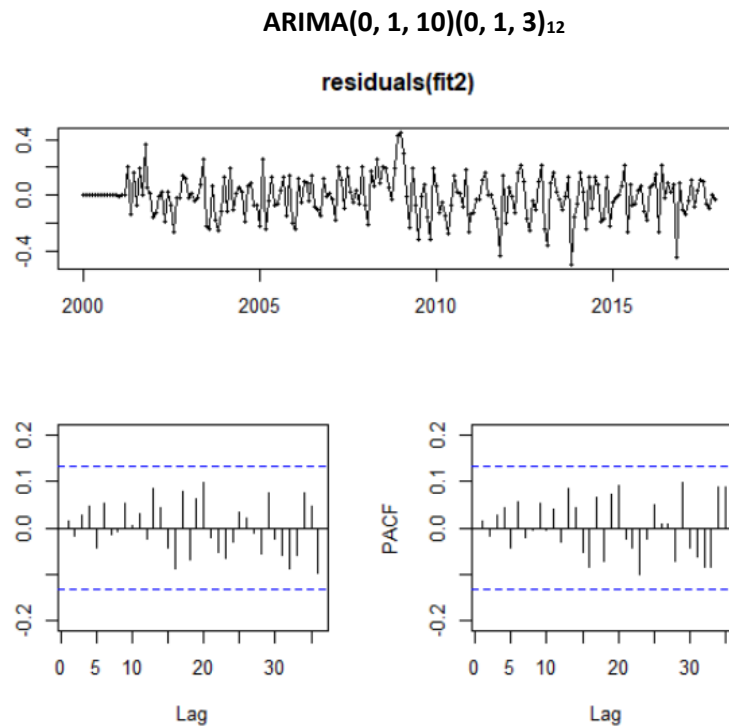


After the double differencing, the residuals are more spread around zero. The ACF and PACF plots show that there might be something at the non-seasonal lags (shown by peaks in the first few lags). The spikes at lag 12 in the ACF indicate seasonal moving average and the spike at lag 12 in the PACF indicates seasonal auto regressive.

First, the moving averages will be taken into account i.e., the spikes at the left of the ACF plot and the spike at lag 12. These spikes indicate seasonal and non-seasonal moving average should be included. Therefore, the model implements is the  $ARIMA(0, 1, 1)(0, 1, 1)_{12}$ . This is the first step and the result will be inspected to see if something is left in the residuals. The following result is obtained.



The model does a good job at capturing the seasonal lags as the spikes at 12, 24, 36 have fairly reduced. However, for further improvement, seasonal MA(3) will be used because of the spike at lag 36 and non-seasonal MA(5) because of the spike at lag 10 in the ACF plot. Therefore,  $ARIMA(0, 1, 10)(0, 1, 3)_{12}$ . The output is shown below



The output shows that the  $ARIMA(0, 1, 10)(0, 1, 3)_{12}$  has been able to capture the seasonal variances and non-seasonal lags as all the spikes are well within the bounds.

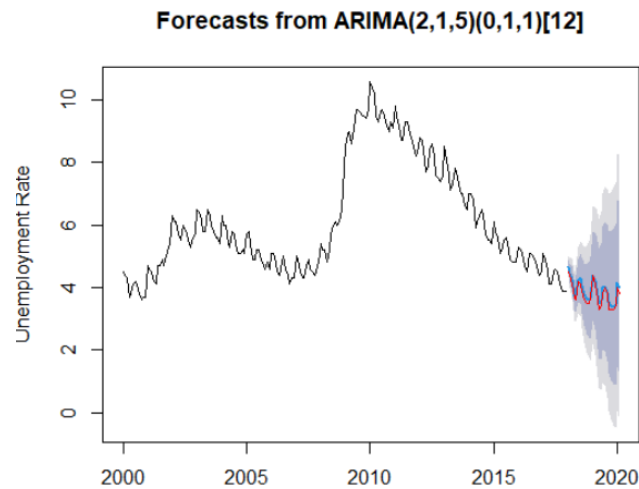
Since the residuals look good, residual diagnostics will be run on the model. The result has 10 moving average components and 3 seasonal moving average components. The Ljung-Box test was also run with lag set as 24 (twice the seasonal period) and fit degrees of freedom set as 13 because there are 13 parameters (coefficients). The p value turns out to be 0.182 which is higher than 0.05. This means that the null hypothesis holds and the residuals cannot be distinguished from white noise. In conclusion,  $ARIMA(0, 1, 10)(0, 1, 3)_{12}$  is a good model.

For the purposes of comparison, auto ARIMA is also run to obtain the optimal model with all the short-cuts turned off and max order set to 10 to include more models. The model produced by auto ARIMA is  $ARIMA(2,1,5)(0,1,1)_{12}$ . This has the same level of differencing as the model used above and has 7 non-seasonal components and 1 seasonal component. The AIC obtained from this model is -118 which is lower than the AIC of  $ARIMA(0, 1, 10)(0, 1, 3)_{12}$  which is -114. Ljung-Box test p value of the  $ARIMA(2,1,5)(0,1,1)_{12}$  calculated with 24 lags and 8 degrees of freedom is 0.15. This means that the null hypothesis holds and the residuals cannot be distinguished from white noise.

The two models will now be compared based on forecast accuracy based on the test set. The MASE of  $ARIMA(0, 1, 10)(0, 1, 3)_{12}$  is 0.32 and the MASE of  $ARIMA(2,1,5)(0,1,1)_{12}$  is 0.148. The overall comparison shows that the optimal model  $ARIMA(2,1,5)(0,1,1)_{12}$  provided by the auto ARIMA procedure is the better

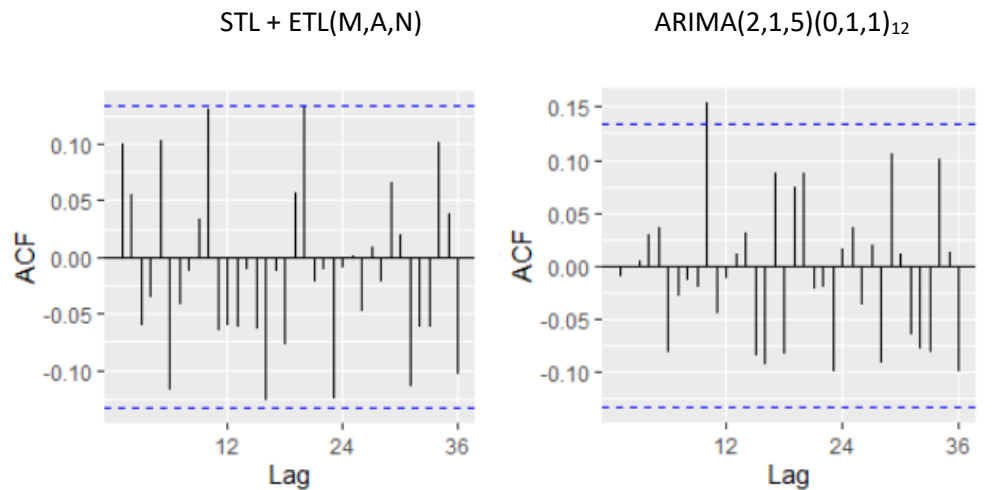


model. The forecast is as shown below. The blue lines show the forecast while the red line shows the observed test data.



### Comparison of the Best Models

The paper will now compare the best models obtained from the STL and Arima forecasting techniques using residual diagnostics and forecast accuracy.

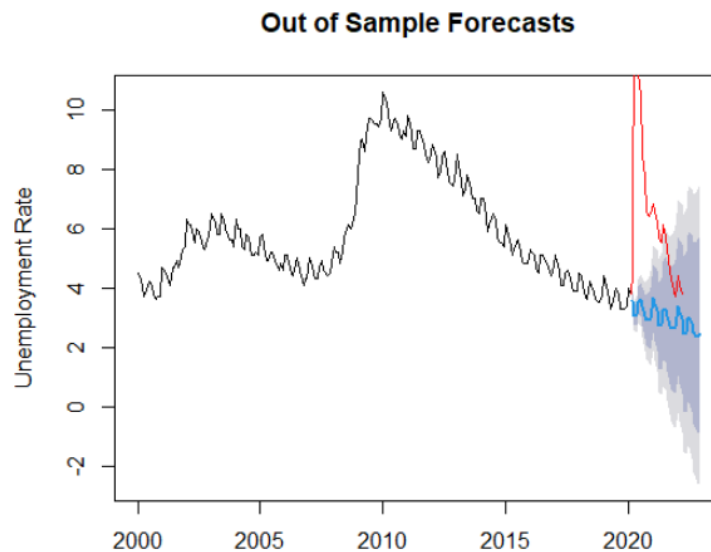


Measures	STL + ETL(M,A,N)	ARIMA(2,1,5)(0,1,1) <sub>12</sub>
Ljung-Box Test	0.043	0.15
MASE	0.2130829	0.1485746
RMSE	0.2160983	0.1400449
MAE	0.1774646	0.1237394
MAPE	4.771581	3.324725

Bases on the residual diagnostics and the forecast accuracy on the test set, **ARIMA(2,1,5)(0,1,1)<sub>12</sub>** is selected as the best model.

### Out of Sample Forecasts and Analysis

Based on the final selected model, out of sample forecasts were generated till December 2022 based on the complete timeseries (train + test).



The blue lines show the forecast based on the selected model while the red line shows the observed data during the COVID-19 pandemic. Since these were unforeseen circumstances which were not present in the timeseries used for forecasting, the forecasts are inaccurate. The pandemic can be seen to have a massive impact on the unemployment rate as many workplaces especially in the service industry faced closure. In April 2020, the unemployment rate rose to 14.4% while the model forecasted a rate of 3.06%. This shows that the pandemic caused the unemployment rate to rise by 4.7 times.