THE RESPONSE OF PRESERVICE TEACHERS TO ALGEBRAIC MISCONCEPTIONS OF STUDENTS

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Abstract

The purpose of this study was to analyze preservice teachers’ (PSTs) assistance to middle grade students’ algebraic misconceptions. The most difficult concept to address was about the slope and intercept of a linear equation and the easiest misconception to resolve was related to the independent variable of a linear equation. The knowledge of linear equations in general and the abstraction of the linear equation from a table was positively correlated with misconceptions related to percentages and the slope and intercept of a linear equation, respectively. There was no statistically significant relationship between PSTs’ self-efficacy and the addressing of misconceptions.

*Keywords*: Preservice Teachers’ misconception, problem solving, self efficacy

**Theoretical Framework**

**Middle School Students’ Algebraic Misconception**

Students’ mathematical misconceptions (e.g., the equal sign or negative signs, Booth & Koedinger, 2008) can hinder their abilities of solving and learning relevant problems. Combining nine prerequisite content areas for formal algebra courses (Welder, 2007) and the Common Core State Standards for Mathematics’ (CCSSO, 2010) middle school content domains, Bush and Karp (2013) identified four content areas - ratios and proportional relationships, the number system, expressions and equations, and functions - as prerequisite knowledge and skills critical for middle grades (Grades 6, 7 and 8). Correspondingly, they summarized 30 common misconceptions through the review of 187 manuscripts, such as misunderstanding the connections among proportional relationships, incorrectly or procedurally using fraction algorithms, believing that the equal sign means “the answer is” rather than expressing a relationship, lacking the understanding of and not seeing the connection between multiple representations of functions. Researchers identified some other typical mistakes committed by students in the solving of problems with linear equations (Powell, 2012) and the understanding four symbolic representation concepts such as bracket usage, equality, operational symbols and letter usage (Welder, 2012).

Other findings showed that students lacked accurate knowledge of direct and inverse proportions (Dogan & Cetin, 2009), had problems with ratios and proportions (Kaplan, Isleyen, & Ozturk, 2011), had difficulty understanding the concept of function and equation as well as variables, constants and unknowns (Kocakaya Baysal, 2010; Li, 2006).

**Ability/Effectiveness of Preservice Teachers to Address Students’ Misconceptions**

Shulman (1987) included pedagogical content knowledge (PCK) and knowing students in the categories of teachers’ knowledge base. He identified PCK as a decisive category to distinguish experts from their colleagues. With PCK teachers can tailor content and pedagogy to meet diverse learners’ needs for understanding mathematics content. An, Kulm and Wu (2004) further proposed that the main part of PCK is the knowledge of students’ thinking, which includes their understanding of mathematical conceptions and possible underlying misconceptions. Teachers could assist students to re-conceptualize existing misconceptions to improve algebraic ability through instructional strategies as long as they identified their misconception with diagnostic information (Russell, O’Dwyer & Miranda, 2009).

However, pre-service teachers have difficulty determining the misconceptions of their students (Sandir & Aztekin, 2013). For example, they often do not realize that students have misconceptions about the equal sign (Russell et al., 2009). In-service teachers have similar issues. They could with difficulty detect misconceptions about core algebraic concepts such as the equal sign and variables (Asquith, Stephens, Knuth & Alibali, 2007).

Bush and Karp (2013) suggested that teacher education should equip pre-service teachers with the knowledge of strategies addressing students’ misconceptions. One instructional technique, erroneous examples, was employed by McLaren et al. (2012) to target students’ misconceptions. In this study mathematical mistakes were intentionally imbedded in the detailed solution of the problems to challenge the students. Another proposed strategy by An and Wu (2004) was to examine the homework of the students. Teachers went through a 4-step process: detecting mistakes, analyzing reasons, creating solutions and correcting them. The strategies employed by both studies showed to be effective.

**Teachers’ Self-Efficacy**

In many school settings, especially in primary grades, teachers have an obligation to teach mathematics, and research has shown that the image they have towards their mathematical ability plays a crucial role in their personal success, as well as the success of their students (Swars, Daneer, & Giesen, 2006). Therefore, this obligation requires that the teachers should be fairly confident in their ability to teach the mathematics content.

Swars and colleagues (2006) described self-efficacy as the teacher’s belief that he or she has adequate skills for teaching effectively, and that this effective teaching influences successful outcomes despite external factors that students may face. They use this theory in reference to mathematics justifiably because as Bandura affirmed, “efficacy beliefs shape casual attributions, regardless of nature of the activity” (1999, p. 29). PSTs hold certain beliefs about how well they understand math and how well they are able to teach it. If not addressed prior to entering the classroom, PSTs may unintentionally pass their disfavor towards mathematics on to their students. Factors that affect self-efficacy, such as anxiety and past experiences, help to pre-determine the attitude that teachers will exhibit in the classroom (Gresham, 2008; Swars et. al, 2006). Interestingly enough, in these two studies, teachers who had developed fear towards mathematics and felt that they struggled in it, claimed that they would be great teachers of mathematics, which showed that they had very high levels of self-efficacy (Brown, Westenskow, & Moyer-Packerham, 2011). These teachers displayed a seemingly reversed affect in that they grew more convinced of their abilities to teach math because of their prior shortcomings.

As Esterly (2003) argues, self-efficacy shapes the instructional behavior and academic performance that is used in the classroom. Teachers who are more comfortable with mathematics will step outside of the traditional teaching methods of algorithm and lecture, and this work claimed this alternative type of teaching to be more effective. This identification and minimization of misconceptions at an early stage of their career is essential in developing qualified teachers who will use strategies in the classroom that involve more than rote memorization (Swars et. al., 2006; Hoy 2000). Studying the relationship between teacher self-efficacy and teachers’ misconceptions is important because this will determine the methods used by the preservice teacher (Esterly 20003). Briley (2012) warned that most students will inaccurately judge their performance as too high. Acknowledgement and resolution of these misconceptions may lead to greater gains in self-efficacy. Brand and Wilkins (2007) claimed that efficacy is essential to motivation, and this motivation is directly influenced by performance.

Hence, the research questions of this study were (1) which algebraic misconceptions were the most difficult for middle school mathematics PSTs to address, (2) how was the performance of PSTs in addressing misconceptions related to their algebra problem solving knowledge; and (3) how was PSTs addressing misconception performance related to their teaching self-efficacy?

**Methodology**

**Participants**

There were 29 middle school mathematics PSTs in the study, two males and the rest females. All participants attended the problem-solving course during the Fall 2013 semester.

**Instruments**

Three instruments were used in the study. First, we developed questions about how to assist a middle grade student who had a specific algebraic misconception. The algebra concepts were proportions, percentages, independent variable in a linear equation, the slope and intercept of a linear equation, and the difference between an expression and equation. The completeness of the explanation and the pedagogic approach were scored using a rubric created by researchers. The second instrument was the *Knowledge for Algebra Teaching for Equity* (KATE) test developed by the authors. This test consisted of 11 questions measuring algebra content knowledge, two on identifying misconceptions, three on providing help with misconceptions, and one on pedagogical content knowledge. The last instrument was the *Mathematics Teaching Efficacy Beliefs Instrument* (MTEBI) that included two subscales *Personal Mathematics Teaching Efficacy* (PMTE) and *Mathematics Teaching Outcome Expectancy* (MTOE) (Enochs, Smith, & Huinker, 2000). The participants completed this instrument as a pre and post-survey.

**Findings**

To answer research question one, we ranked the scores given by the PSTs and calculated the mean rank. A higher mean rank indicates a higher capability of resolving the misconception (Table 1). In addition we tested for the uniformity of the ranks using a chi-square test according to Lee and Yu (2013). The chi-square value was 38.67 with four degrees of freedom and an extremely small probability value. Thus, we conclude that there was indeed a difference in the ability of the PSTs to help students resolve misconceptions. As we can see from Table 1, the students were best at helping students with misconceptions about the independent variable of a linear equation and least capable of helping students with a fairly complex problem involving the slope and intercept of a linear equation.

Table 1

*Mean Rank of PSTs’ Addressing Middle School Students Algebra Misconception Score*

|  |  |
| --- | --- |
| Algebraic Misconception Type | Mean Rank |
| Independent variable of a linear equation | 4.33 |
| Percentages | 3.33 |
| Proportions | 3.00 |
| The difference between expression and equation | 2.33 |
| The slope and intercept of a linear equation | 2.00 |

To analyze the relationship between PSTs performance of addressing misconception and their algebra problem solving knowledge, we used rank-biserial correlation. The sample size for this question is 28 because one PST did not answer the KATE instrument. We calculated the correlations between the various misconception scores and the scored responses to algebra questions that were scored as correct/incorrect. Because all these variables are categorical, we calculated the Spearman correlation to explore whether there was any statistical relationship between the types of misconceptions and algebra content knowledge. We found that the misconceptions with percentages are related to a problem where we asked the PSTs to describe a real life situation that represents a certain linear equation. Misconceptions with proportions are negatively related to questions where we showed a line graph and its equation. Also, misconceptions about slope and intercept are related to a problem that required the PSTs to find the linear coefficients from a table (Table 2).

The relationship between proportion misconceptions and line graphs is difficult to explain to due to its negative value. With a closer analysis of the responses to the Line Graph question we realized that the problem statement was ambiguous and created a lot of confusion between students. Indeed, the problem asked whether it was possible to find the equation based on the given information. In reality the information was not sufficient. We hypothesize that students who felt more confident based on their stronger mathematics background would be induced to try to solve the problem and thus give the wrong answer. The positive relationship between misconception ability with percentages and slope/intercept and problems involving linear equations in table or algebraic form is self-explanatory.

Table 2

*Algebra Misconceptions and Content Knowledge*

|  |  |  |  |
| --- | --- | --- | --- |
| Algebraic Misconception Type | KATE | rho | p-value |
| Proportions | Line graph | -0.36 | 0.05 |
| Proportions | Line graph | -0.45 | 0.02 |
| Percentages | Linear eqtn | 0.44 | 0.02 |
| The slope and intercept of a linear equation | Table to eqtn | 0.40 | 0.03 |

The third research question examined the relationship between PSTs’ performance in addressing misconceptions and their self-efficacy beliefs. To analyze this question, we computed the non-parametric correlation (Spearman’s rho) between the types of misconception and the two MTEBI subscales. It was found that there was no statistically significant relationship between either of the subscales and any of the misconception types.

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