returned to the axially symmetric A-core phase. The supercooling transition, $T_{\rm V}^{*}(p,H)$, was the locus of points where the A-core was globally unstable. We note that results for the supercooling transition require a fine computational grid. For a coarse grid of $h = 0.5\xi$ the supercooling transtion is lower than that shown in Fig. 1 but converges to the reported transition line for $h \lesssim 0.15\xi$. The phase transition lines shown in Fig. 1 were obtained on a $60\xi \times 60\xi$ computational grid with grid spacing $h = 0.1\xi$. Our numerical annealing procedure used to identify the region of metastability of the A-core phase agrees remarkably well with the experimental results for the transition obtained on cooling, both in the magnitude of the supercooling at pressures above p_{c_v}, as well as the rapid cross over in slope of $T_{\rm V}^{*}(p,H)$ with pressure at the lower pressures approaching p_{c_v}. However, our region of metastability does not extend as low in pressure as the experimentally reported transitions on cooling. Our interpretation of the latter is that below p_{c_v} strong-coupling energies are never able to stabilize the A phase in the vortex core, without assistance from the Zeeman energy. This results in the termination of the supercooling line on the equilibrium A-core vortex phase boundary at a pressure near p_{c_v}. We are not able to resolve the origin of the discrepancy in the minimum pressure for the metastable A-core phase within the strong-coupling extension of the GL theory. Such a resolution may require new experiments under rotation with pressure sweeps, or perhaps implementation of the full quasiclassical strong-coupling free energy functional extended to inhomogeneous phases.

VI. SUMMARY AND OUTLOOK

The recent development of a strong-coupling extension of Ginzburg-Landau theory that accounts for the relative stability of the bulk A and B phases has provided the first opportunity to examine the relative stability of the vortex phases discovered in rotating ^{3}He-B and to predict, based on known material properties of superfluid ^{3}He over the full pressure range, the equilibrium and metastable vortex phase transitions. We are able to verify the local and global stability of all the stationary solutions to the extended GL theory over the full (p,T) plane. Only the A-core and D-core phases are global minima anywhere in the (p,T) plane. The results we report provide strong theoretical support for the identification of the experimentally observed phase transitions as the equilibrium and supercooled phase transitions between the high temperature A-core vortex phase with broken time-reversal and mirror symmetries (proposed by Salomaa and Volovik[7]), and the low temperature, low pressure D-core vortex phase with broken axial symmetry (proposed by Thuneberg[8]). Furthermore, both of these transitions are driven by the decrease in strong coupling energies at sufficiently low pressures and temperatures defined by the metastability line $T_{\rm V}(p,H)$. In addition, the broken rotational symmetry of the D-core vortex is identified with the instability of the components within the core with 4π phase winding. Once strong-coupling energies are suppressed by sufficiently low temperature or pressure the doubly quanitized vortices dissociate to gain condensation energy, and as a result break axial symmetry.

We conclude with the two forward looking observations. First, the success of the strong coupling extension of the GL theory, evident by the results for the vortex phase diagram, provides a theoretical tool for studying a wide range of problems involving inhomogeneous phases with complex symmetry breaking and/or novel topological defects, in the strong-coupling limit, that were not previously accessible. A recent example is the analysis of the experimentally measured Bosonic collective mode frequencies ("Higgs masses") of superfluid ^{3}He-B using a time-dependent extension of the strong-coupling GL theory in Ref.[44], which provided consistent experimental results for the strength of the f-wave pairing interaction in superfluid ^{3}He over the full pressure range[45], a material parameter that is important for understanding ground-states and excitations of superfluid ^{3}He at high pressures and high magnetic fields. Secondly, the extended GL theory is supported by the microscopic strong-coupling pairing theory based on leading order corrections to the weak-coupling BCS theory originating from binary collision scattering between fermionic quasiparticles of the normal phase of liquid ^{3}He[14]. Further development of a quantitative microscopic strong-coupling pairing theory to inhomogeneous, non-equilibrium states is well within reach.

VII. ACKNOWLEDGEMENTS

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APPENDIX: MATERIAL PARAMETERS

The following tables summarize the pressure dependent material parameters that determine the properties of the superfluid phases in strong-coupling theory.

p[bar]	n [nm^{-3}]	m^*/m	F_0^a	T_c[mK]	v_f[m/s]	ξ_0[nm]	β_1^{sc}	β_2^{sc}	β_3^{sc}	β_4^{sc}	β_5^{sc}
0.0	16.28	2.80	-0.7226	0.929	59.03	77.21	-0.0098	-0.0419	-0.0132	-0.0047	-0.0899
2.0	17.41	3.05	-0.7317	1.181	55.41	57.04	-0.0127	-0.0490	-0.0161	-0.0276	-0.1277
4.0	18.21	3.27	-0.7392	1.388	52.36	45.85	-0.0155	-0.0562	-0.0184	-0.0514	-0.1602
6.0	18.85	3.48	-0.7453	1.560	49.77	38.77	-0.0181	-0.0636	-0.0202	-0.0760	-0.1880
8.0	19.34	3.68	-0.7503	1.705	47.56	33.91	-0.0207	-0.0711	-0.0216	-0.1010	-0.2119
10.0	19.75	3.86	-0.7544	1.828	45.66	30.37	-0.0231	-0.0786	-0.0226	-0.1260	-0.2324
12.0	20.16	4.03	-0.7580	1.934	44.00	27.66	-0.0254	-0.0861	-0.0233	-0.1508	-0.2503
14.0	20.60	4.20	-0.7610	2.026	42.51	25.51	-0.0275	-0.0936	-0.0239	-0.1751	-0.2660
16.0	21.01	4.37	-0.7637	2.106	41.17	23.76	-0.0295	-0.1011	-0.0243	-0.1985	-0.2801
18.0	21.44	4.53	-0.7661	2.177	39.92	22.29	-0.0314	-0.1086	-0.0247	-0.2208	-0.2930
20.0	21.79	4.70	-0.7684	2.239	38.74	21.03	-0.0330	-0.1160	-0.0249	-0.2419	-0.3051
22.0	22.96	4.86	-0.7705	2.293	37.61	19.94	-0.0345	-0.1233	-0.0252	-0.2614	-0.3167
24.0	22.36	5.02	-0.7725	2.339	36.53	18.99	-0.0358	-0.1306	-0.0255	-0.2795	-0.3280
26.0	22.54	5.18	-0.7743	2.378	35.50	18.15	-0.0370	-0.1378	-0.0258	-0.2961	-0.3392
28.0	22.71	5.34	-0.7758	2.411	34.53	17.41	-0.0381	-0.1448	-0.0262	-0.3114	-0.3502
30.0	22.90	5.50	-0.7769	2.438	33.63	16.77	-0.0391	-0.1517	-0.0265	-0.3255	-0.3611
32.0	23.22	5.66	-0.7775	2.463	32.85	16.22	-0.0402	-0.1583	-0.0267	-0.3388	-0.3717
34.0	23.87	5.82	-0.7775	2.486	32.23	15.76	-0.0413	-0.1645	-0.0268	-0.3518	-0.3815

TABLE I. Material parameters for ^{3}He vs. pressure, with the particle density $n = k_f^3/3\pi^2$ from Ref.[46], the effective mass, m^*, and T_c from Ref.[43], the exchange interaction, F_0^a, is from Ref.[47], the Fermi velocity, $v_f = \hbar k_f/m^*$, calculated from the Fermi wavelength, k_f, and the coherence length is $\xi_0 = \hbar v_f/2\pi k_B T_c$. The strong-coupling parameters, $\beta_i^{\rm sc}$, in units of $|\beta_1^{\rm wc}|$, are from Ref.[14].