1.

$u = \underset{\Delta\theta}{\text{argmin}}\ g^T\Delta\theta + \frac{1}{\alpha}\|\Delta\theta\|_2^2$

(a) $\nabla_{\Delta\theta} u = g + \frac{2}{\alpha}\Delta\theta = 0 \Rightarrow \boxed{\Delta\theta^* = -\frac{g\alpha}{2}}$

$\theta_{t+1} = \theta_t - \frac{\alpha}{2}\nabla\ell_\theta \Rightarrow$ standard GD with fixed step size $\alpha/2$.

(b) $\cdot$ $\underset{\|\Delta\theta\|_\infty \leq \eta}{\text{argmin}}\ g^T\Delta\theta +$

$\|\Delta\theta\|_\infty = \underset{i}{\max}|\Delta\theta[i]|$

$= \underset{\forall i,\ |\vec{\Delta\theta[i]}|\leq\eta}{\text{argmin}}\ \sum_i^n\left(g[i]\underline{\Delta\theta[i]}\right)$

$\underset{\forall i,\ |\vec{\Delta\theta[i]}|\leq\eta}{\text{argmin}}\ g[i]\Delta\theta[i].$

$\begin{cases} \text{if}\ \text{sign}(g[i]) < 0 \Rightarrow \Delta\theta[j] = +\eta. \\ \qquad\qquad\quad > 0 \Rightarrow \qquad\qquad = -\eta \end{cases}$

$\Delta\theta^* = -\eta\cdot\text{sign}(g).$