1.
(a)
Expected: 0.1973753273487091, got: 0.1973753273487091, max error: 0.0
Max error all_h: 4.999339580535889e-05
Max error last_h: 2.498924732208252e-05
(b)
Max error all_h: 4.699826240539551e-05
Max error last_h: 4.3138861656188965e-05
(c)
Max error loss_all: 3.314018249511719e-05
Max error loss_last: 2.384185791015625e-07
(d)

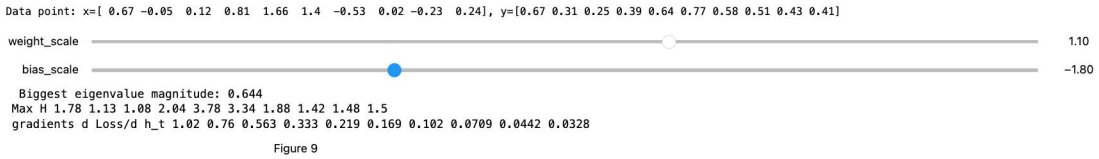
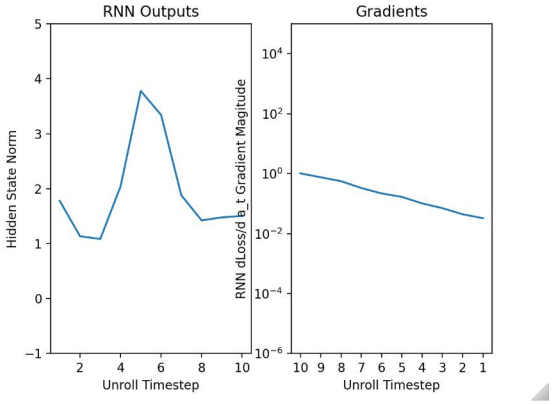


Figure 9



(e)
With loss on the last step only, the backpropagated Jacobian at each step is W^T,,
so gradients contain powers of W.

- If the largest singular value σmax(W) < 1, gradients vanish exponentially with distance from the last step.
- If σmax(W) > 1, gradients explode exponentially.
- If σmax(W) ≈ 1, gradients stay roughly constant.

(f)
More vanishing: tanh

Hidden states are bounded in [-1, 1] and saturate; derivatives are ≤ 1 and typically much < 1 away from 0

More exploding: ReLU

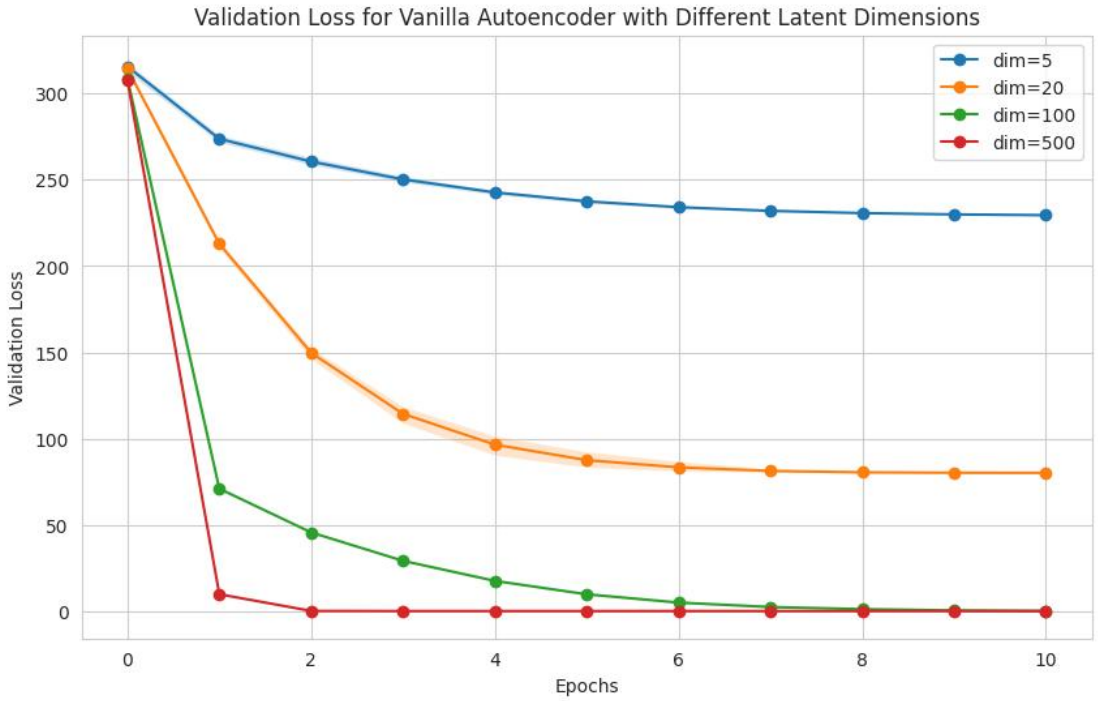
Active paths keep multiplying by W with no saturating shrinkage

(g)
With last_target_only = False, you add a loss at every time step.
This injects gradient at each time, so earlier steps don't rely on long chains of W from the last step, and thus vanishing is much less severe.

- When only the last step has a loss, gradients to early timesteps must pass through many recurrent multiplications, shrinking or blowing up exponentially.
- When every step has a loss, each step's parameters receive a "local" gradient that traveled only a few steps, so those gradients are larger and more stable.
- But if a target at time T truly depends on information from a far earlier time t, the gradient for that dependency still traverses T−t steps and will still vanish/explode the same way.

2. The discrimination and bias in decision-making in employment or education.

3.
(a)
(i)



(ii)

- Reconstruction: Increases monotonically with larger latent size and then plateaus; the elbow appears around the intrinsic dimensionality. It improves rapidly up to ≈20 and shows diminishing returns beyond that.
- Linear probe accuracy: Rises sharply as latent size grows from very small to around ≈20, then plateaus or can dip slightly beyond that.

Why:

- The data are 100D with roughly 20 high-variance, label-relevant dimensions.
- With a very small bottleneck, the autoencoder can't capture all salient structure → higher reconstruction error and poor linear separability.
- As latent size approaches the intrinsic task-relevant dimension (~20), the model can encode the key factors, and thus reconstruction error drops and linear separability peaks.
- Increasing latent size past ~20 mostly captures low-variance/nuisance variation that doesn't help classification (and can dilute separability), so reconstruction keeps improving slightly

(b)

(b) (i) Let $C := XX^T$.

$$L(W_1, W_2) = \|X - W_2 W_1 X\|_F^2$$
$$= tr(C) - 2tr(W_2 W_1 C) + tr(W_2 W_1 C W_1^T W_2^T)$$

$$\rightarrow \underset{m \times k}{\frac{\partial L}{\partial W_2}} = -2CW_1^T + 2W_2 W_1 C W_1^T = 0 \Leftrightarrow CW_1^T = W_2 W_1 C W_1^T$$

$$\rightarrow \underset{k \times m}{\frac{\partial L}{\partial W_1}} = -2W_2^T C + 2W_2^T W_2 W_1 C = 0 \Leftrightarrow W_2^T C = W_2^T W_2 W_1 C.$$

(ii) Let $U_k \in \mathbb{R}^{m \times k}$ be the top-k eigenvectors of C such that $CU_k = U_k S$ and $U_k^T C U_k = S = diag(\sigma_1^2, \ldots, \sigma_k^2)$.

Let $W_2 = U_k$, $W_1 = U_k^T$.

Plugging into the two conditions:

$$\rightarrow -2CU_k + 2U_k U_k^T C U_k = -2CU_k + 2CU_k = 0.$$

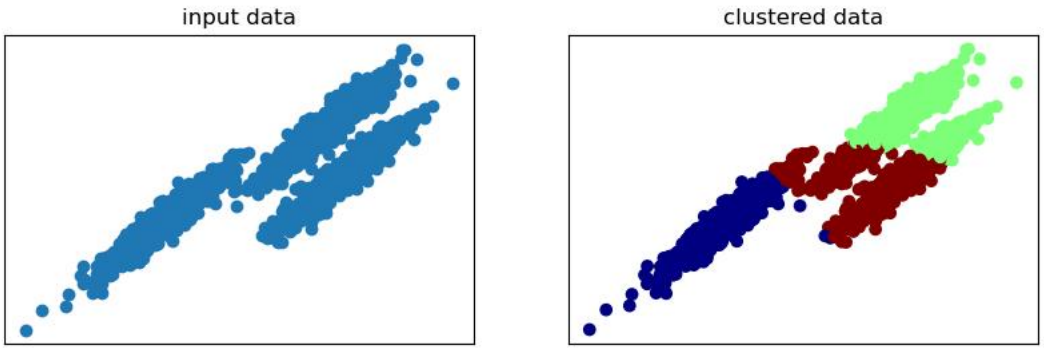
$$\rightarrow -2U_k^T C + 2U_k^T U_k U_k^T C = -2U_k^T C + 2U_k^T C = 0.$$

Hence $(W_2, W_1) = (U_k, U_k^T)$ is a stationary point.

4.

5.

(a)



- The K-means algorithm did not work effectively for this dataset. Looking at the clustered data, K-means has divided the data into three clusters, but these clusters don't properly capture the natural structure of the data(three elongated, curved branches).
- K-means assumes clusters are spherical and similar in size, uses Euclidean distance as its similarity metric.
- In this dataset, points that belong to the same natural cluster might be far apart in Euclidean distance.

(f)

Observation: After normalizing the three feature vectors per point, the scatter shows the capture the natural structure of the data(three elongated, curved branches).

Why normalization works:

- Normalizing each vector to unit L2-norm makes similarity depend on direction (cosine) rather than magnitude. On the unit sphere, Euclidean distance is monotonic with 1 – cosine similarity, so k-means groups by angle. This removes scale/degree effects that previously pulled points with larger norms together even if their directions differed.

(g)