BA RICKER

Detecting a Shift in the Ricker-a: A Priori Power Analysis of a Before-After Study Design

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Caveat: This study design tool is an idealized power analysis built upon several simplifying assumptions (Table 1). For a specific study, a more accurate portrayal of power may require changing these assumptions and the underlying equations.

INTRODUCTION

To assess the effect of a management action on salmon population dynamics, spawner-recruit analysis may be employed. Ideally, management would be run as an experiment with treatments interspersed over time and space with some populations receiving treatment and others not (Walters 1986). But in many cases, such an experiment is not feasible because it might be impossible to insulate subpopulations from treatment, or only a single population is available for analysis. In this case, a Before-After (BA) study may be the best study design available, where one examines the Ricker parameters before treatment and after without the benefit of reference or "control" populations (Hinrichsen 2001).

The main goal of this work is to demonstrate the probability of detecting a shift in the density-independent production parameter of a Ricker function using a simple BA design. This is accomplished by describing the study design in a statistically rigorous way, setting up the likelihood function, and then using maximum likelihood theory to estimate power. Power is the probability of rejecting the null hypothesis of "no treatment effect" when the true treatment effect is not zero.

Table 1. — Assumptions used in power analysis[1].

A1	*Ricker production function*. The recruits-per-spawner (R/S) observations follow a Ricker-function with log-normal errors (Hilborn and Walters 1992).
A2	*Independent and identically distributed errors (i.i.d.)*. The errors are independent and identically distributed random normal deviates with mean zero and constant variance.
A3	*One population*. A single population is used in a Before-After (BA) design.
A4	*"Press-type" effect*. At the beginning of the After period when some treatment is applied, the density-independent parameter (*a*), often called the Ricker-a parameter, shifts by a mean value called the treatment effect. This is a "press" type response (Bender et al. 1984).
A5	*Maximum likelihood estimator*. The estimator of the treatment effect is a maximum likelihood estimator (MLE).

1 These are assumptions for an idealized study design. For a specific application, a more accurate study design may require changing these assumptions and the underlying equations. Therefore this analysis should be treated as a rough guide to power.

The R-code for implementing this power analysis may be found in Appendix A (Venerables et al. 2010).

METHODS

The methods are those described in detail in Hinrichsen (2001). The six assumptions used in to derive statistical power are detailed in Table 1. To conduct the power analysis, a model was formulated and maximum likelihood estimators (MLEs) were derived (Mood et al. 1974). These estimators were then used as the basis for testing the null hypothesis of "no treatment" effect.

The Ricker model. —It was assumed that the spawner-recruit (SR) data used in the BA analysis follow a Ricker production function of the following form:

$$\log(R_t / S_t) = a_t - bS_t + \varepsilon_t, \tag{1}$$

where R_t is the number of recruits in year t that were offspring of parents that spawned in year t; S_t is the number of spawners in brood year t; a_t is the density independent part of the production function which takes on the value of a during the Before period and the value of $a + \theta$ during the After period, where θ is the treatment effect; b is the density-dependent parameter, and ε_t is the error that follows a normal distribution with mean zero and variance σ^2.

Maximum likelihood estimator (MLE). —Following the method of (Hinrichsen 2001), the log-likelihood function is developed. Because the errors are assumed to be normally distributed, the MLE may be found by using equation (1) in a least squares estimation procedure. In the equations that follow, n_1 is the number of Before years and n_2 is the number of After years. Using least squares, the Ricker parameters may be estimated as

$$\begin{bmatrix} \hat{a} \\ \hat{\theta} \\ \hat{b} \end{bmatrix} = (\mathbf{X'X})^{-1}\mathbf{X'y}, \tag{2}$$

where

$$\mathbf{X} = \begin{bmatrix} \mathbf{e}_1 & 0 & \mathbf{S}_1 \\ \mathbf{e}_2 & \mathbf{e}_2 & \mathbf{S}_2 \end{bmatrix}, \tag{3}$$

$\mathbf{e}_1$ is a n_1–vector of 1s, $\mathbf{e}_2$ is a n_2–vector of 1s, $\mathbf{S}_1$ is a n_1–vector of spawner observations during the Before period, $\mathbf{S}_2$ is a n_2–vector of spawner observations during the After period, and $\mathbf{y}$ is the vector of log(R/S) all observations. Using equation (2), the MLE of the treatment effect is

$$\hat{\theta} = \bar{y}_2 - \bar{y}_1 + (\bar{S}_1 - \bar{S}_2)\left(\frac{n_1 \operatorname{cov}(S_1, y_1) + n_2 \operatorname{cov}(S_2, y_2)}{n_1 \operatorname{var}(S_1) + n_2 \operatorname{var}(S_2)} \right), \tag{4}$$

where $\bar{y}_2$ and $\bar{y}_1$ are the sample means of log(R/S) during the After and Before years, respectively; $\bar{S}_1$ and $\bar{S}_2$ are the sample means of spawners during the Before and After years, respectively; $\operatorname{cov}(S_1, y_1)$ and $\operatorname{cov}(S_2, y_2)$ are the sample covariances of the spawners and log(R/S) during the Before and After years, respectively; and $\operatorname{var}(S_1)$ and $\operatorname{var}(S_2)$ are the sample variances of spawners during the Before and After years, respectively. The sample variances and covariances in equation (4) use sample size in the denominator, not sample size -1.

The variances of the MLEs, are derived by inverting the Fisher information

Matrix:

$$\mathbf{I} = \mathbf{X}'\mathbf{X}/\sigma^2. \tag{5}$$

In particular, the variance of the treatment effect is

$$\mathrm{var}(\hat{\theta}) = \sigma^2 \left(\frac{1}{n_1} + \frac{1}{n_2} + \frac{(\bar{S}_1 - \bar{S}_2)^2}{n_1 \, \mathrm{var}(S_1) + n_2 \, \mathrm{var}(S_2)} \right) \qquad (6)$$

and the standard error is

$$se(\hat{\theta}) = \sqrt{\mathrm{var}(\hat{\theta})}. \qquad (7)$$

The coefficient of variation is

$$CV(\hat{\theta}) = se(\hat{\theta}) / \theta. \qquad (8)$$

Power. —Power is the probability of rejecting the null hypothesis of "no treatment" effect when the actual treatment effect is θ (not equal to zero). Power depends on the true treatment effect, the probability of a type I error (usually called alpha), and the standard error of the estimator. A two-sided alternative hypothesis is used because treatments may result in negative effects. The other necessary inputs for the power analysis are: the number of sites, the number of Before years, the number of After years, the error variance (σ^2), the sample means during the Before and After years, and the sample variances of spawners during the Before and After years. Using the usual least squares approach, we assume that under the null hypothesis $\theta / SE(\hat{\theta})$ follows a central t-distribution with N-3 degrees of freedom, where N is the number of

observations. The critical t-value is then equal to the value of q such that $\Pr\{T_{N-3} >= q\} = \alpha/2$ where T_{N-3} is a random variable that follows a central t-distribution with $N-3$ degrees of freedom. Power is then equal to

$$\Pi = F_{N-3,\mu}(-q) + 1 - F_{N-3,\mu}(q) \tag{9}$$

where $F_{N-3,\mu}(x)$ is the cumulative distribution function of a non-central t distribution with N-3 degrees of freedom and noncentrality parameter $\mu = \theta/SE(\hat{\theta})$ (Appendix A). Experimenters usually chose designs that will deliver power of 0.80 or greater.

Example. — As an example, consider the case where the number of Before and After years are equal, the total number of years varies from 4 to 20, the variance of the spawners during the Before years equals 234,496.5, the variance of the spawners during the After years equals 5,949,591, the difference between the After and Before year spawner sample means is 11,197.67, and the error variance was 0.514. These input sample variances, sample means, and error variance were based on Snake River Fall Chinook SR data. To show the influence of the difference between mean spawners during the Before and After periods, an alternative difference of 0 was used in a sensitivity analysis. The true treatment effect value (θ) took on the alternative values of log(1.5), log(2.0), and log(3.0).

The results of this numerical example showed that power of 0.80 could not be obtained with the inputs described above unless the difference between mean spawners during the Before and After periods was set to zero. This shows that mean spawner differences can have a very large effect on the outcome. Unfortunately, in most cases, the mean spawners numbers will be outside of experimental control. Spawner-recruit data is most often nonexperimental and the investigators carry the hope that nature will, by accident, supply data suitable for testing for a shift in the Ricker-a parameter. One possible way to improve the chance of detecting a

significant shift is to use a reference population that receives no treatment for comparison in a Before-After-control-impact (BACI) study design (Osenberg and Schmidt 1996).
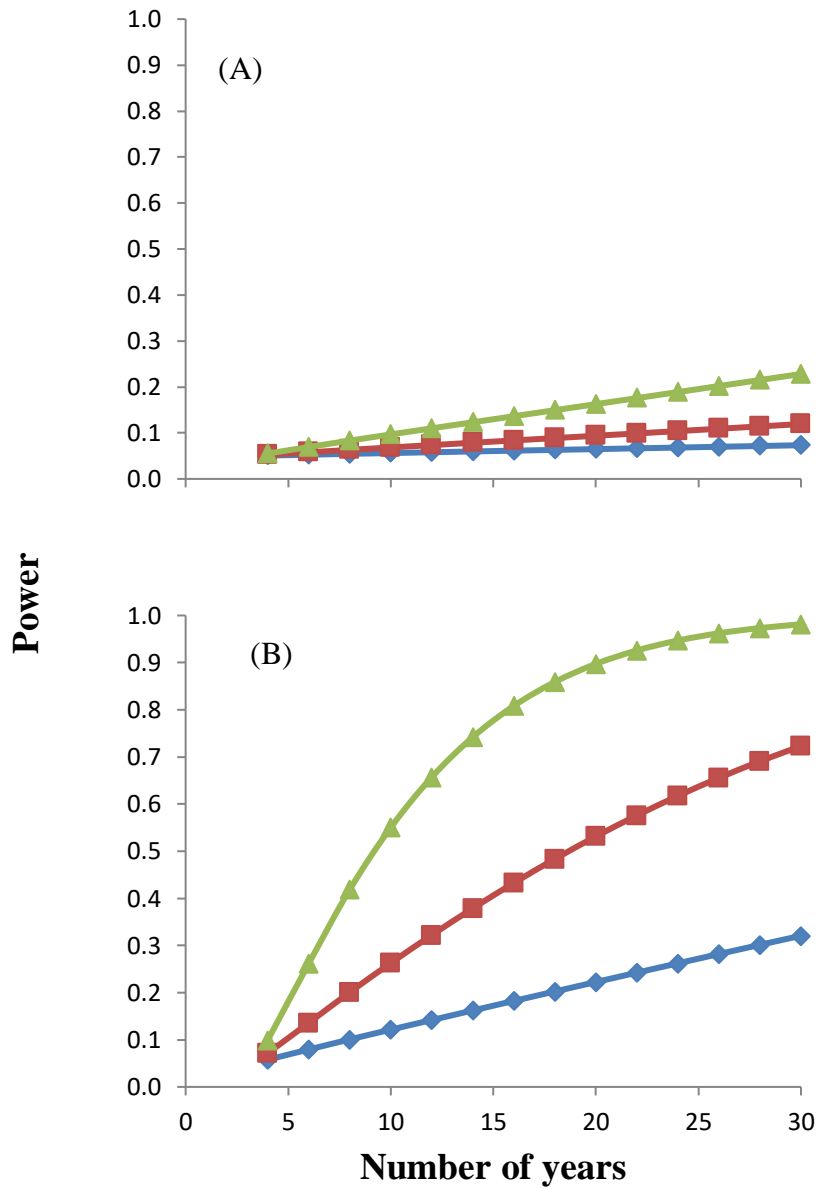
Figure 1.—Power of Before-After design in example where (A) mean spawners differed by 11,197.67 between the Before and After periods and (B) mean spawners were identical between the Before and After periods (see *Example*). Alternative values of the true treatment effect were log(1.5) (diamonds), log(2.0) (squares), and log(3.0) triangles.

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REFERENCES

Bender, E.A., T.J. Case, and M.E. Gilpin. 1984. Perturbation experiments in community ecology: theory and practice. Ecology 65:1–13.

Hilborn, R., and C.J. Walters. 1992. Quantitative fisheries stock assessment: choice, dynamics, and uncertainty. Chapman and Hall, New York.

Hinrichsen, R.A. 2001. High variability in spawner-recruitment data hampers learning. Canadian Journal of Fisheries and Aquatic Sciences 58: 769–776.

Mood, A.M, Graybill, F.A., and D.C. Boes. 1974. Introduction to the theory of statistics, Third Edition. McGraw-Hill, New York, New York.

Osenberg, C.W. and R.J. Schmidt. 1996. Detecting ecological impacts caused by human activities. In Detecting Ecological Impacts: Concepts and Applications in Coastal Habitats, R.J Schmitt and C.W. Osenberg, Editors. Academic Press, New York, New York.

Venerables, W.N., Smith, D.M., and R Development Core Team. 2010. An Introduction to R. Notes on R: A Programming Environment for Data Analysis and Graphics Version 2.11.1 (2010-05-31). http://www.r-project.org

Walters, C.J. 1986. Adaptive management of renewable resources. Macmillan, New York.

APPENDIX A. R CODE

```
# Program to implement a power of a BA study design aimed
# at detecting a shift in the Ricker-a as in Hinrichsen (2001).


#Input definitions
#var is the error variance
#S1 is the sample mean of the spawner counts during the Before period
#S2 is the sample mean of the spawner counts during the After period
#varS1 is the sample variance of the spawner counts during the Before period
#varS2 is the sample variance of the spawner counts during the After period
#n1 is the number of Before years
#n2 is the number of After years
#theta is the true effect size
#alpha is the probability of a Type I error (assuming a two-sided alternative hypothesis)
#
baricker<-
function(var=0.25,S1=100,S2=100,varS1=10,varS2=10,n1=10,n2=10,theta=log(2.0),alpha=0.05){
se<-var*(1/n1+1/n2+(S1-S2)*(S1-S2)/(n1*varS1+n2*varS2))
se<-sqrt(se)
cv<-se/theta
delta<-theta
N<-n1+n2
q<-qt(p=1-alpha/2,df=N-3)
power<-1-pt(q,ncp=delta/se,df=N-3)+pt(-q,ncp=delta/se,df=N-3)


return(list(var=var,S1=S1,S2=S2,varS1=varS1,varS2=varS2,n1=n1,n2=n2,theta=theta,
alpha=alpha,se=se,cv=cv,power=power))
}
#outputs
#se -- standard error
#cv -- coefficient of variation
#power -- probability of rejecting the null hypothesis of no effect
```