Serge Lang - Fundamentals of Differential Geometry
Carter Hinsley's notes

Part I
General differential theory

1 Differential calculus

1.1 Categories

1.2 Topological vector spaces

A topological vector space E over $\mathbb{R}$ is a vector space with a topology where addition and scalar multiplication are continuous. We assume such spaces are Hausdorff and locally convex (local in the sense that every nbhd of 0 contains an open convex nbhd U of 0).

The category TVS has topological vector spaces as its objects and continuous linear maps as its morphisms. Lang denotes the hom-sets $L(\mathbf{E}, \mathbf{F})$. The continuous r-multilinear maps $\psi : \mathbf{E} \times \ldots \times \mathbf{E} \to \mathbf{F}$ are denoted $L^r(\mathbf{E}, \mathbf{F})$. The symmetric and alternating r-multilinear maps are denoted $L^r_s(\mathbf{E}, \mathbf{F})$ and $L^r_{\text{sym}}(\mathbf{E}, \mathbf{F})$ respectively. Isomorphisms in TVS are called toplinear isomorphisms. $\text{Lis}(\mathbf{E}, \mathbf{F})$ denotes the toplinear isomorphisms of $\mathbf{E}$ onto $\mathbf{F}$, while $\text{Laut}(\mathbf{E})$ denotes the toplinear automorphisms of $\mathbf{E}$. If a hom-set is written without a target object, the target is assumed to be $\mathbb{R}$. These are called *forms* (particularly, continuous forms, r-multilinear forms, etc.).

Complete spaces.

Banach(able) spaces. Any complete TVS whose topology can be defined by a norm is called a Banachable space. When equipped with the norm, this is called a Banach space.

1.3 Derivatives and composition of maps

1.4 Integration and Taylor's formula

1.5 The inverse mapping theorem

2 Manifolds

2.1 Atlases, charts, morphisms

2.2 Submanifolds, immersions, submersions

2.3 Partitions of unity

2.4 Manifolds with boundary

3 Vector bundles

3.1 Definition, pull backs

3.2 The tangent bundle

3.3 Exact sequences of bundles

3.4 Operations on vector bundles

3.5 Splitting of vector bundles

4 Vector fields and differential equations

4.1 Existence theorem for differential equations

4.2 Vector fields, curves, and flows

4.3 Sprays

4.4 The flow of a spray and the exponential map

4.5 Existence of tubular neighborhoods

4.6 Uniqueness of tubular neighborhoods

5 Operations on vector fields and differential forms

5.1 Vector fields, differential operators, brackets

5.2 Lie derivative

5.3 Exterior derivative

5.4 The Poincaré lemma

5.5 Contractions and Lie derivative

5.6 Vector fields and 1-forms under self duality

5.7 The canonical 2-form

5.8 Darboux's theorem

6 The theorem of Frobenius

6.1 Statement of the theorem

6.2 Differential equations depending on a parameter

6.3 Proof of the theorem

6.4 The global formulation

6.5 Lie groups and subgroups