Serge Lang - Fundamentals of Differential Geometry
Carter Hinsley's notes

Part I
General differential theory

1 Differential calculus

1.1 Categories

1.2 Topological vector spaces

A topological vector space E over $\mathbb{R}$ is a vector space with a topology where addition and scalar multiplication are continuous. We assume such spaces are Hausdorff and locally convex (local in the sense that every nbhd of 0 contains an open convex nbhd U of 0).

The category TVS has topological vector spaces as its objects and continuous linear maps as its morphisms. Lang denotes the hom-sets $L(\mathbf{E}, \mathbf{F})$. The continuous r-multilinear maps $\psi : \mathbf{E} \times \ldots \times \mathbf{E} \to \mathbf{F}$ are denoted $L^r(\mathbf{E}, \mathbf{F})$. The symmetric and alternating r-multilinear maps are denoted $L^r_s(\mathbf{E}, \mathbf{F})$ and $L^r_{\mathrm{sym}}(\mathbf{E}, \mathbf{F})$ respectively. Isomorphisms in TVS are called toplinear isomorphisms. $\mathrm{Lis}(\mathbf{E}, \mathbf{F})$ denotes the toplinear isomorphisms of $\mathbf{E}$ onto $\mathbf{F}$, while $\mathrm{Laut}(\mathbf{E})$ denotes the toplinear automorphisms of $\mathbf{E}$. If a hom-set is written without a target object, the target is assumed to be $\mathbb{R}$. These are called *forms* (particularly, continuous forms, r-multilinear forms, etc.).

Complete spaces.

Banach(able) spaces. Any complete TVS whose topology can be defined by a norm is called a Banachable space. When equipped with the norm, this is called a Banach space.