B.Sc. (H) Physics Semester 3 — **Computational Lab MP-II**
November 08, 2021 — **Time Limit : 4 Hrs**

Name ____________________ **Roll No.** ____________________

The following rules apply:

- **Each step carries weight.**
- **Step should be commented.**
- **Answers must be supported.**

Question:	1	2	3	Total
Points:	0	8	12	20
Score:				

Experiment 7

Simultaneous Equations : Matrix Method
Gauss-Elimination & Gauss Siedel Method

There are many problems in which we need to simultaneously solve a set of n equations in m variables.

$$a_{11}x_1 + a_{12}x_2 + ... + a_{1m}x_m = b_1$$

$$a_{21}x_1 + a_{22}x_2 + ... + a_{2m}x_m = b_2$$

$$\cdots$$

$$a_{n1}x_1 + a_{n2}x_2 + ... + a_{nm}x_m = b_n$$

A prerequisite of getting a unique solution is that $m = n$ and all n equations must be independent. Writing it in a compact form we get

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} \longrightarrow AX = B \longrightarrow \sum_{j=1}^{m} a_{ij}x_j = b_i \quad \forall \quad i = 1 \textit{ to } n$$

Now there are different approaches to the solution ;
In Gauss Elimination Method the row operations are used to bring matrix A to echelon form E and then Back Substitution is used to find the unique solution X.
In Gauss Seidel Method the unique solution X is obtained iteratively starting with some approximate but arbitrary chosen solution X'.

1. **CONCEPT**
   (a) (0 points) **(Gauss Elimination)**
   Briefly explain the theory of Gauss Elimination method for solving simultaneous equations.
   (b) (0 points) **(Gauss Seidel)**
   Briefly explain the theory of Gauss Seidel method for solving simultaneous equations.

2. **METHOD/CODE**
   (a) (2 points) **(Gauss Elimination)**
   Taking a_{ii} as the pivot element corresponding to i^{th} row, we reduce all the rows R_k below the i^{th} row by applying the row operation

$$R_k \quad \longrightarrow \quad R_k - \left(\frac{a_{ki}}{a_{ii}}\right) R_i$$

   It is to be noted that *pivoting* may be required in cases where some of the diagonal elements may assume zero value.
   (b) (2 points) **(Backward Substitution)**
   The row reduced matrix would assume a triangular form ($n = m$) and the solution will then be obtained as given below

$$\begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ 0 & r_{22} & \cdots & r_{2n} \\ 0 & 0 & \ddots & \vdots \\ 0 & 0 & 0 & r_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix} \longrightarrow \quad rX = c \quad \longrightarrow \quad x_n = \frac{c_n}{r_{nn}} \;\&\; x_i = \frac{1}{r_{ii}}\left[c_i - \sum_{j=i+1}^{n} r_{ij}x_j\right]$$

   (c) (4 points) **(Gauss Seidel)**
   In many cases, the equations can be ordered in such a way the coefficient of x_k in the k^{th} equation is large in magnitude relative to all other coefficients in that equation and we can write

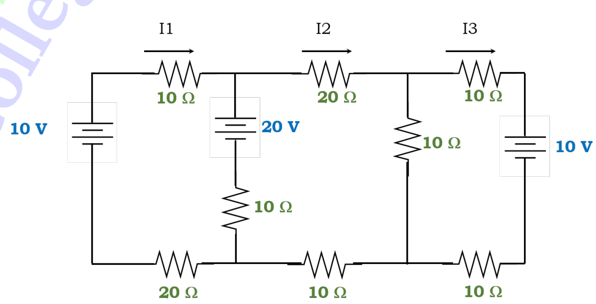
$$x_1 = \frac{1}{a_{11}}\left[b_1 - a_{12}x_2 - ... - a_{1n}x_n\right] \quad , \quad x_2 = \frac{1}{a_{22}}\left[b_2 - a_{21}x_1 - ... - a_{2n}x_n\right]$$

$$\cdots$$

$$x_n = \frac{1}{a_{nn}}\left[b_n - a_{n1}x_1 - ... - a_{n(n-1)}x_{n-1}\right]$$

   The initial approximations may be taken to be $x_i^{(0)} = b_i/a_{ii}$.

3. **APPLICATIONS**
   (a) (6 points) **(Electric Circuits)**
   Consider the following circuit



   Write down the correponding simultaneous equations for currents. Using these simultaneous equations write the coefficient matrix A and the augmented matrix AB for finding currents in the circuit. Solve it using the Gauss elimination method.
   (b) (6 points) **(Electric Circuits)**
   Solve a circuit (to be given on the spot) using the the Gauss Seidel method.
   The problem on Gauss Seidel to be submitted by 22^{nd} **November,** 2021.