B.Sc.(H) Physics Sem 3 **ComputationalLab - MP II**
Sept. 13, 2021: ONLINE **Time Limit : 4 Hrs**

Name ____________________ **Roll No.** ____________________

The following rules apply:

- **Each step carries weight.**
- **Step should be commented.**
- **Answers must be supported.**

Question:	1	2	Total
Points:	9	11	20
Score:			

Experiment 02

User Defined Function : Legendre Function Series Expansion Method

The Legendre differential equation is

$$(1 - x^2)y'' - 2xy' + n(n+1)y = 0$$

where n is a +ve integer. It arises particularly in boundary value problems with spherical symmetry.
The Legendre polynomial of first kind $P_n(x)$, as a series solution in $[-1, 1]$ is

$$P_n(x) = \sum_{i=0}^{m} \frac{(-1)^i \, (2n-2i)!}{2^n \, i! \, (n-i)! \, (n-2i)!} x^{(n-2i)}$$

Here $m = n/2$ if n is even and $m = (n-1)/2$ if n is odd.
In the experiment you will be required to use the above $P_n(x)$ expression to find the differential polynomials, prove the recurrence relations of the Legendre polynomials as well as the orthogonality properties.
The orthogonality property is :

$$\int_{-1}^{1} P_n(x)P_m(x) = \frac{2}{2n+1}\delta_{mn} \quad where \quad \begin{cases} \delta_{mn} = 0 & : m \neq n \\ \delta_{mn} = 1 & : m = n \end{cases}$$

Questions:

- *Where do you find physical application of Legendre Polynomial, name few problems?*
- *What's the importance of recurrence relations?*
- *What's the physical meaning you draw from orthogonality or normalization?*

1. **METHOD/CODE**

   (a) (1 point) **(Factorial Function)**
   The Gamma function $\Gamma(n)$ or the factorial $n! = \Gamma(n+1)$ (when n is a positive integer) follow the recursive relation

   $$\Gamma(n+1) = n\Gamma(n) \quad \text{or} \quad (n+1)! = (n+1)n!$$

   Write a code to generate the Gamma function $\Gamma(n)$ for an arbitrary n.

   (b) (3 points) **(Legendre Function)**
   Write a code to generate the Legendre function as sum of terms of the series expansion.

   (c) (3 points) **(Derivative of Legendre Function)**
   Write a code to generate the derivative of Legendre function as sum of terms of the series expansion.

   (d) (2 points) **(Compare with inbuilt Python command)** The Legendre Polynomials and their differentials.

2. **APPLICATIONS**

   (a) (2 points) **(Legendre Polynomial)**
   Generate the values of $P_0(x)$, $P_1(x)$ and $P_2(x)$ Legendre polynomials in the range $x = (-1, 1)$ using explicit series representation taking 100 data points of x within the given range. Store them in tabulated form as x, $P_0(x)$, $P_1(x)$ and $P_2(x)$ in the data file *leg00.dat*. Retrieve these values and plot these functions. Display the same on the console.

   (b) (2 points) **(Derivative of the Legendre Polynomial)**
   Generate the values of derivative $P_1'(x)$, $P_2'(x)$ and $P_3'(x)$ of the Legendre polynomial $P_1(x)$, $P_2(x)$ and $P_3(x)$ in the range $(-1, 1)$ using explicit series representation taking 100 data points of x within the given range. Append the same in the data file *leg01.dat*. Retrieve these values and plot $P_0'(x)$, $P_2'(x)$ along with $P_1(x)$. Display the same on the console/ screen.

   (c) (4 points) **(Prove the recursion relation for Legendre polynomial)**

      1. $nP_n(x) = xP_n'(x) - P_{n-1}'(x)$
      Take input from **leg01.dat** fot $n = 2$ & verify the RHS & LHS for each of the data points x. Store in **leg02.dat** the data x, n, $(n-1)$, $P_n(x)$, $P_n'(x)$ and $P_{n-1}'(x)$ in column form. Display the results on the console.
      2. $(2n+1)xP_n(x) = (n+1)P_{n+1} + nP_{n-1}(x)$
      Take input from **leg00.dat** fot $n = 2$ & verify the RHS & LHS for each of the data points x. Store in **leg03.dat** the data x, n, $(n-1)$, $(n+1)$, $P_n(x)$, $P_{n-1}(x)$ and $P_{n+1}(x)$ in column form. Display the results on the console.
      3. $nP_n(x) = (2n-1)xP_{n-1}(x) - (n-1)P_{n-2}(x)$ for Legendre polynomial.
      Take input from **leg00.dat** for $n = 3$ & verify the RHS & LHS for each of the data points x. Store in **leg04.dat** the data x, n, $(n-1)$, $(n+1)$, $P_n(x)$, $P_{n-1}(x)$ and $P_{n+1}(x)$ in column form. Display the results on the console.

   (d) (3 points) **(Prove the Orthogonality relation)**

   $$\int_{-1}^{1} P_n(x)P_m(x)dx = \frac{2}{2n+1}\delta_{mn}$$

   Take input from **leg00.dat** for $n = m$ and $n \neq m$ & verify. Display your result for the corresponding values of n and m in matrix form.