B.Sc.(H) Physics Sem 3	Computational Lab - MP II
Sept 06, 2021: ONLINE	Time Limit : 4 Hrs

Name ____________________ **Roll No.** ____________________

The following rules apply:

- **Each step carries weight.**
- **Step should be commented.**
- **Answers must be supported.**

Question:	1	2	3	Total
Points:	0	6	4	10
Score:				

Experiment 1 : Numerical Integration

Title : Newton Cotes Methods for Definite Integrals
Method: Trapezoidal and Simpson Rules

We choose integration as our first application to computer programming as it is well known already from high school mathematics. However, majority of integrals are not tractable by pen and paper, and a computerized solution approach is both very much simpler and much more powerful. In this experiment we probe the reliability of Newton Cotes integration methods.

Composite Trapezoidal Method

While using lower order methods for higher order function, we divide the interval $[a, b]$ *into* N *subintervals or trapezoids. Therefore each trapezoid* $[x_k, x_{k+1}]$ *has width* $h = (b-a)/N$ *and the region under the curve is broken into these trapezoids. Thus the integration of each trapezoid is evaluated to get the complete area under the curve as*

$$\begin{aligned} I &= \int_a^b f(x)dx = \int_{x_0=a}^{x_1} f(x)dx + \int_{x_1}^{x_2} f(x)dx + ... + \int_{x_{n-1}}^{x_n=b} f(x)dx \\ &\approx \frac{h}{2}\left[f(x_0 = a) + f(x_n = b) + 2[f(x_1) + f(x_2) + ... + f(x_{n-1})]\right] \end{aligned}$$

Composite Simpson Method

While using lower order methods for higher order function, we divide the interval $[a, b]$ *into* $2N$ *(even) subintervals. Therefore each subinterval* $[x_k, x_{k+1}]$ *has width* $h = (b-a)/2N$ *and the region under the curve is broken into these subinterval. Thus the integration of each subinterval is evaluated to get the complete area under the curve as*

$$\begin{aligned} I &= \int_a^b f(x)dx = \int_{x_0=a}^{x_2} f(x)dx + \int_{x_2}^{x_4} f(x)dx + ... + \int_{x_{2n-2}}^{x_{2n}=b} f(x)dx \\ &\approx \frac{h}{3}\Big(f(x_0 = a) + f(x_{2n} = b) \\ &+ 4[f(x_1) + f(x_3) + ... + f(x_{2n-1})] \\ &+ 2[f(x_2) + f(x_4) + ... + f(x_{2n-2})]\Big) \end{aligned}$$

1. Concept
   (a) (0 points) **(Newton Cotes Method)**
   Explain the Newton Cotes method to numerically find a definite integral of a function $f(x)$ whose analytic form is unknown but whose values are given only for finite number of x values.
   (b) (0 points) **(Composite Method)**
   Both Trapezoid and Simpson methods are accurate only up to a finite order p polynomial into which the given function $f(x)$ can be expanded. Briefly explain the composite method which can be used in case function of higher order.

2. Method
   (a) (2 points) **(Trapezoid Method for Given Experimental Data)**
   Given the values y_i of an unknown function $f(x)$ for equi-spaced independent variable x_i for $i = 0, 1, ..., n$; the integral I of the function $f(x)$ is given by the formula

$$I \approx \frac{h}{2}\left\{f(x_0) + f(x_n) + 2\left[f(x_1) + f(x_2) + ... + f(x_{n-1})\right]\right\}$$

   (b) (2 points) **(Simpson Method for Given Experimental Data)**
   Given the values y_i of an unknown function $f(x)$ for equi-spaced independent variable x_i for $i = 0, 1, ..., 2n$; the integral I of the function $f(x)$ is given by the formula

$$I \approx \frac{h}{3}\left\{f(x_0) + f(x_{2n}) + 4\left[f(x_1) + f(x_3) + ... + f(x_{2n-1})\right] + 2\left[f(x_2) + f(x_4) + ... + f(x_{2n-2})\right]\right\}$$

   (c) (1 point) **(Error in Integration)**

$$Error = I_{analytic} - I_{numerical}$$

   Also calculate the truncation error and convergence in your numerical result.
   (d) (1 point) **(Variation with step size)**
   The numerical value of the integral I depends on the number of sub-intervals and therefore the step-size h (the domain $[a, b]$ kept same). A log-plot of $I(h)$ vs h is a useful tool to understand the convergence of an integration method.

3. Applications
   (a) (1 point) **(Given Data)**
   The current I through and the voltage V across an electric element is measured as

Voltage (V)	0.0	0.5	2.0	4.05	8.0	12.5	18.0	24.5	32.0	40.5	50.0
Current (mA)	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0

   Estimate the value of power delivered to the element using Trapezoidal and Simpson rule.
   (b) (3 points) **(Given Function)**
   Let $f(x)$ be any function defined in the domain $[a, b]$ (to be given in class but for the purpose of testing take $f(x) = x^2$). Make a function that inputs the function $f(x)$, it's domain $[a, b]$, divides the domain in N sub-intervals $[x_k, x_{k+1}]$, calculates the values of the function $f(x_k)$ at the intervals points x_k and then performs Trapezoidal and Simpson integration. Call this function to evaluate $f(h)$ for given h and plot $[h, f(h)]$.