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3. $$\frac{d^m y}{dx^m} = f(x, y, y', ----, y^{m-1})$$

Initial conditions are:

$y(x_0) = \alpha_1 \quad y'(x_0) = \alpha_2 ---- \quad y^m(x_0) = \alpha_{m-1}$

To reduce the order of differential Equation, we will consider a vector $s(x)$, which is state of system function as function of x i.e. collection of all dependent variables.

Here $y^i = \frac{d^i y}{dx}$

$$s(x) = \begin{bmatrix} y \\ y' \\ y'' \\ \vdots \\ y^{n-1} \end{bmatrix} = \begin{bmatrix} f(x) \\ f'(x) \\ f''(x) \\ \vdots \\ f^{n-1}(x) \end{bmatrix}$$

Taking derivative of $s(x)$

$$\frac{ds}{dx} = \begin{bmatrix} y' \\ y'' \\ y''' \\ \vdots \\ y^m \end{bmatrix} = \begin{bmatrix} dy/dx \\ d^2y/dx^2 \\ d^3y/dx^3 \\ \vdots \\ d^my/dx^m \end{bmatrix} = \begin{bmatrix} f(x,y) \\ f(x,y,y') \\ f(x,y,y',y'') \\ \vdots \\ f(x,y,y',---,y^{m-1}) \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_m \end{bmatrix}$$

Here, all the elements of $\frac{ds}{dx}$ are related to each other, so higher order derivatives can be written in terms lower order derivatives.

Let's take $y_1 = y' = f(x,y)$

$y_2 = y'' = y_1' = f'(x,y) = f(x,y,y')$

$y_3 = y''' = y_2' = f''(x,y,y') = f(x,y,y',y'')$

$\vdots$

$y_m = y^m = y_{m-1}' = f(x,y,y',---,y^{m-1})$

It's how we've written nth order differential eq^n to n 1st order D.E. in which $y(x_0)$, $y'(x_0)$ --- $y^n(x_0)$ will be 1st elements of $y_1, y_2, ---y_m$ respectively which will be approximated later by numerical methods.

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(b) Show all steps for solving following IVP using RK2 Method

$$y'' - 2y' + 2y = e^{2x}\sin(x)$$

for $0 \le x \le 1$ with $y(0) = -0.4$, $y'(0) = -0.6$ with 5 steps

Ans.

$$y'' - 2y' + 2y - e^{2x}\sin(x) = 0$$

Take $y' = \cancel{y}, z = f(x, y, z)$

$\cancel{y_2} = y'' = \cancel{\ } z' = 2\cancel{y}z - 2y + e^{2x}\sin(x) = g(x, y, z)$

lets divide $[0,1]$ to 5 parts with step size $(h) = \frac{1-0}{5} = 0.2$ as

$$x = [0, 0.2, 0.4, 0.6, 0.8, 1]$$

$$x = [x_0, x_1, x_2, x_3, x_4, x_5]$$

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Step 1: We'll calculate k_1, k_2 for 1st eq^n $f(x,y,z)$ and l_1, l_2 for 2nd eq^n $g(x,y,z)$

$k_1 = h \times f(x_0, y_0, z_0) = 0.2 \times f(0, -0.4, -0.6) = 0.2 \times (-0.6) = -0.12$

$l_1 = h \times g(x_0, y_0, z_0) = 0.2 \times g(0, -0.4, -0.6) = 0.2 \times [\cancel{2x} -1.2 + 0.8 + 0]$

$= 0.2 \times (-0.39999)$

$l_1 = -0.07999$

$k_2 = h \times f(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}, z_0 + \frac{l_1}{2})$

$k_2 = -0.128$

$l_2 = h \times g(x_0 + h/2, y_0 + \frac{k_1}{2}, z_0 + l_1/2)$

$l_2 = 0.2 \times [-1.28 + 0.92 + 0.12193] = 0.2 \times (-0.23806) = -0.04761$

$y_1 = y_0 + \frac{(k_1 + k_2)}{2} = -0.4 + \frac{(-0.12 + (-0.128))}{2} = -0.524$

$z_1 = z_0 + \frac{(l_1 + l_2)}{2} = -0.6 + \frac{(-0.07999 + (-0.04761))}{2} = -0.663806$

Step 2: $x = 0.2$ $\quad y_1 = -0.524$ $\quad z_1 = -0.663806$

$k_1 = h \times f(x_1, y_1, z_1) = 0.2 \times (-0.663806) = -0.1327612$

$l_1 = h \times g(x_1, y_1, z_1) = 0.2 \times (2 \times (-0.663806) - 2 \times (-0.524) + e^{2 \times 0.2} \sin(0.2))$

$= 0.2 \times (-1.32761 + 1.048 + 0.29637) = 0.2 \times (0.016767)$

$l_1 = 0.0033534$

$k_2 = (z_1 + \frac{l_1}{2}) \times h = -0.132425$

$l_2 = 0.2 \times (-1.324259 + 1.180761 + 0.538472) = 0.2(0.394974)$

$l_2 = 0.078994$

$y_2 = y_1 + \frac{(k_1 + k_2)}{2} = -0.6565935$

$z_2 = z_1 + \frac{(l_1 + l_2)}{2} = -0.6226321$

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Step 3:- $x_2 = 0.4$ $y_2 = -0.656593$ $z_2 = -0.622632$

$k_1 = h \times f(x_2, y_2, z_2)$

$k_1 = 0.2 \times (-0.622632) = -0.124526$

$l_1 = h \times g(x_2, y_2, z_2) = 0.2 \times (-1.2452642 + 1.313187 + 0.86666)$

$= 0.2 \times (0.934589) = 0.186917$

$k_2 = 0.2 \times f(x_2 + h/2, y_2 + \frac{k_1}{2}, z_2 + \frac{l_1}{2})$

$= \cancel{0.2\times} -0.10583$

$l_2 = 0.2 \times g(x_2 + h/2, y_2 + \frac{k_1}{2}, z_2 + \frac{l_1}{2})$

$= 0.2 \times (-1.058346 + 1.43771 + 1.30321) = 0.2 \times (1.68258)$

$l_2 = 0.336516$

$y_3 = y_2 + (k_1 + k_2)/2 = -0.7717740$

$z_3 = z_2 + (l_1 + l_2)/2 = -0.36091509$

Step 4:- $x_3 = 0.6$ $y_3 = -0.7717740$ $z_3 = -0.36091509$

$k_1 = h \times f(x_3, y_3, z_3) = 0.2 \times (-0.36091509) = -0.072183018$

$l_1 = h \times g(x_3, y_3, z_3) = 0.2 \times (-0.72183018 + 1.543548 + 1.87467903)$

$= 0.2 \times (2.696396) = 0.5392793$

$k_2 = -0.01825508$

$l_2 = 0.2 g(x_3 + h/2, y_3 + \frac{k_1}{2}, z_3 + \frac{l_1}{2})$

$= 0.2 \times (-0.182550 + 1.61579 + 2.6124315) = 0.2 \times (4.0451175)$

$l_2 = 0.809122$

$y_4 = y_3 + \frac{(k_1 + k_2)}{2} = -0.81699304$

$z_4 = z_3 + \frac{(l_1 + l_2)}{2} = 0.31328577$

Step 5:- $x_4 = 0.8$ $y_4 = -0.81699304$ $z_4 = 0.31328577$

$k_1 = h \times f(x_4, y_4, z_4) = 0.2 \times (0.31328577) = 0.062657154$

$l_1 = h \times g(x_4, y_4, z_4) = 0.2 \times (0.62657154 + 1.633986 + 3.55830879)$

$= 0.2 \times (5.813645) = 1.16272$

$k_2 = h \times f(x_4 + h/2, y_4 + k_1/2, z_4 + \frac{l_1}{2}) = 0.2 \times (1.7893006 + 1.571328846 + 4.73885)$

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$k_2 = 0.2 \times (8.0994\,8114) = 1.619896$

$y_5 = y_4 + \frac{(k_1 + k_2)}{2} = -0.69619939$

$z_5 = 1.70459843$

x	eqn 1	eqn 2
0	-0.4	-0.6
0.2	-0.524	-0.663806
0.4	-0.6565935	~~-0.6~~ 2 -0.6226321
0.6	-0.7717740	-0.36091509
0.8	-0.81699304	0.31328577
1	-0.69619939	1.7045983

```
(base) hinton@hinton-VirtualBox:~$ /home/hinton/anaconda3/bin/python /home/hinton/Semester_4/MP3/Practical/IVP/2020PHY1140_A8.py
[0.  0.2 0.4 0.6 0.8 1. ]
[[-0.4         -0.6       ]
 [-0.524       -0.66380632]
 [-0.65659359 -0.6226321 ]
 [-0.77177412 -0.36091506]
 [-0.81699316  0.31328587]
 [-0.69619952  1.70459865]]
```