B.Sc.(Hons.) Physics	**Lab Assignment # 10**	Due Date and Time :
PHC401-32221401	**Boundary Value Problem**	18.04.2022, 11:59PM
Teacher: Mamta	**Non-Linear Shooting**	Max. Marks : 30

Note:

1. ***This assignment may be done in groups of two. Please pair up with the same student with whom you did the linear BVP assignment.***
2. Please write appropriate comments.
3. Include a brief documentation of your functions
4. Mention the name and roll no of your partner in your report.
5. Use the same method (secant or Newton Raphson) in all parts (Theory, algorithm and program) for solving the nonlinear equation.
6. Cite the references followed by you.

Questions:

1. **Theory** [5]
   Explain the shooting method to solve the Non linear two-point BVP of the form

   $$y'' = f(x, y, y') \quad ; \quad a < x < b \tag{1}$$

   with the Robin BC

   $$\begin{aligned} \alpha_1 y(a) + \alpha_2 y'(a) &= \alpha_3 \\ \beta_1 y(b) + \beta_2 y'(b) &= \beta_3 \end{aligned} \tag{2}$$

   using RK4 for solving the corresponding IVP. Discuss the Neumann and Dirichlet conditions as a special case of this. You may use Newton-Raphson or secant method to find the roots of objective function.

2. **Algorithm or Pseudocode and Numerical calculation** [8]
   (a) Write the algorithm/pseudocode for solving the second order Non linear BVP given in equation (1).
   (b) **??** Show the numerical computation for a few iterations to solve the BVP

   $$y'' = 2y^3 \; ; \quad 0 \le x \le 1 \tag{3}$$

   using shooting method along with RK4 for solving the corresponding IVP and Newton Raphson or secant method for finding roots of the objective function with the following BC :
   i. $y(0) = \frac{1}{3}$, $y(1) = \frac{1}{4}$
   ii. $y'(0) = -\frac{1}{9}$, $y'(1) = -\frac{1}{16}$
   iii. $3y(0) - 9y'(0) = 2$, $y(1) = \frac{1}{4}$
   iv. $y(0) = \frac{1}{3}$, $2y(1) + 2y'(1) = \frac{3}{8}$

   Verify that the the exact solution of the problem is

   $$y_{\text{exact}} = \frac{1}{x+3} \tag{4}$$

3. **Programming** [12]

(a) Make a python function *my_secant or my_NR* (depending upon which method you want to use) to solve a non linear equation

$$f(s) = 0$$

with a given tolerance.

(b) Write a Python code that

i. that solves the problem given in equation (1) satisfying Robin BC using shooting method along with RK4 (in your my_IVP module) for solving the corresponding IVP and Newton Raphson or secant method for finding roots of the objective function. The output should be the functions $y(x)$ and $y'(x)$ as an array.

ii. Plot the final numerical solution y and y' for various step sizes $h = (b-a)/N$, N being the number of steps from a to b in the corresponding IVP along with the exact solution.

(c) Validate your code by solving the BVP given in equation (3) for all the four BC in question 2**??**. Verify the in between computation as done by you in question 2**??**.

(d) Print a table with the column heads x_i, y_{num_i}, y_{exact_i}, $E_i = |y_{\text{exact}_i} - y_{\text{num}_i}|$ for $N = 4$ and $N = 8$.

(e) Now extend your program to solve BVP for $N = 2^k$ with $k = 1, 2, \ldots, 6$. Determine max absolute error (from the exact solution) and the root mean square error for each N. Perform the error analysis and study convergence by finding the ratios of errors E_N/E_{2N} for both max and rms errors. For this print the relevant data in tabulated form.

(f) Plot the solution for each N as points of different style and the exact solution as a continuous curve.

(g) Further extend your program to plot $ln(E_{\max})$ as a function of $ln(N)$ where E is the array of error (absolute or rms) from exact solution at each x_i. Use inbuilt linear regression function to determine the slope of this line.

4. **Discussion** [5]

Interpret and discuss your results and graphs.