S.G.T.B. Khalsa College

Mathematical Physics III (2021-22)

B.Sc.(Hons.) Physics	Lab Assignment # 9	Due Date and Time :
PHC401-32221401	Boundary Value Problem	18.04.2022, 11:59PM
Teacher: Mamta	Linear Shooting	Max. Marks : 30

Note:

1. ***This assignment may be done in groups of two***
2. Please write appropriate comments.
3. Include a brief documentation of your functions
4. Mention the name and roll no of your partner in your report.
5. Cite the references followed by you.

Questions:

1. **Theory** [8]
   (a) What is the difference between an initial value and boundary value problem? What is a two point BVP?
   (b) i. Write down the general form of the second order boundary value problem (BVP).
   ii. Discuss the three types of Boundary conditions.
   iii. What are homogeneous and non-homogeneous BVP?
   (c) Explain the concept of Shooting method
   (d) Explain Linear shooting method to solve the BVP

$$y''(x) + p(x)y'(x) + q(x)y(x) + r(x) = 0 \quad ; \quad a < x < b \tag{1}$$

   with the Robin boundary conditions

$$\begin{aligned} \alpha_1 y(a) + \alpha_2 y'(a) &= \alpha_3 \\ \beta_1 y(b) + \beta_2 y'(b) &= \beta_3 \end{aligned} \tag{2}$$

   using RK4 for solving the corresponding IVP. Discuss the Neumann and Dirichlet conditions as a special case of this.
   (e) Under what conditions, will the above boundary-value problem have a unique solution?

2. **Algorithm or Pseudocode and Numerical calculation** [8]
   (a) Explain the algorithm/pseudocode for solving a second order linear differential equation of the type given in equation (1).
   (b) Show the numerical computation to solve the BVP

$$y'' + y = \sin(3x) \; ; \quad 0 \le x \le \frac{\pi}{2} \tag{3}$$

   with the BC

$$\begin{aligned} y(0) + y'(0) &= -1 \\ y'(\pi/2) &= 1 \end{aligned} \tag{4}$$

   using Linear Shooting technique with $N = 4$. Verify that the the exact solution of the problem is

$$y_{\text{exact}} = \frac{3}{8}\sin(x) - \cos(x) - \frac{1}{8}\sin(3x) \tag{5}$$

3. **Programming** [10]

(a) Make a python function *Lin_shooting* that solves the problem given in equation (1). Use all necessary arguments in the function. The output should be the functions $y(x)$ and y' as arrays.

(b) Write a Python code that

i. uses the function *Lin_shooting* to solve the BVP (3) with the given Robin BC.

ii. Plot the final numerical solutions y and y' (as points) for various step sizes $h = (b-a)/N$, N being the number of steps from a to b in the corresponding IVP along with the exact solutions as continuous curve.

(c) Validate your code by solving the BVP given in equation (3) along with the BC (4). Verify the in-between computation as done by you on paper for $N = 4$.

(d) Print a table with the column heads x_i, y_{num_i}, y_{exact_i}, $E_i = |y_{\text{exact}_i} - y_{\text{num}_i}|$ for $N = 4$ and $N = 8$.

(e) Now extend your program to solve BVP for $N = 2^k$ with $k = 1, 2, \ldots, 8$. Determine max absolute error (from the exact solution) and the root mean square error for each N. Perform the error analysis and study convergence as discussed in the class. For this print the relevant data in tabulated form.

(f) Plot the solution for each N as points of different style and the exact solution as a continuous curve.

(g) Further extend your program to plot $ln(E_{\max})$ as a function of $ln(N)$ where E is the array of error (absolute or rms) from exact solution at each x_i. Use inbuilt linear regression function to determine the slope of this line.

4. **Discussion** [4]

Interpret and discuss your results and graphs.