B.Sc.(Hons.) Physics
PHC401-32221401
Teacher: Mamta

S.G.T.B. Khalsa College
Mathematical Physics III (2021-22)
Lab Assignment # 2b
Applications of Integration

Due Date and Time :
19.02.2021, 11:59PM
Max. Marks : 20

Note:
This Assignment has to be done individually.

1. (7 marks) **Theory**
   (a) What do you mean by orthogonal polynomials in an interval?
   (b) Write down the orthogonality condition for Legendre Polynomials. Starting from the basis $\{1, x, x^2, \ldots\}$, use Gram-Schmidt procedure to obtain the first three polynomials in the basis orthogonal in the interval [-1:1] with weight function $W(x) = 1$ and show that you get the Legendre Polynomials.
   (c) **Represesntation of a function as a Linear Combination of Legendre Polynomials**
   The Legendre Polynomials form a complete set of orthogonal functions in the range [-1, 1] and hence any piecewise continuous function $f(x)$ can be writen as linear combination of Legendre Polynomials. The series converges to the value of $f(x)$ at a point of continuity and to average of left and right limit at a point of discontinuity. Thus, we may write

$$f(x) = \sum_{n=0}^{\infty} C_n \mathcal{P}_n(x)$$

   i. Determine the coefficients C_n using the orthogonality relation of Legendre Polynomials.
   ii. How many terms will a polynomial of order n have in series of Legendre Polynomials?
   iii. Determine analytically
   (α) All terms in the expansion of the function $f(x) = 2 + 3x + 2x^4$
   (β) First five terms in the expansion of $f(x) = \cos(x)\sin(x)$

2. (10 marks) **Programming**
   (a) Write a python function to find the first n terms in the expansion of a given function. Use the appropriate methods in your integration module for it.
   (b) Write a python program *"A2b-2020PHYxxxx.py"* (*xxxx* being the last four digits of your roll no.) that uses this function to
      i. Display all non-zero coefficients in the expansion of the function of $f(x) = 2 + 3x + 2x^4$
      ii. First 10 coefficients in the expansion of $f(x) = \cos(x)\sin(x)$. Display the number of terms required to get at least an accuracy of 6 significand digits $\forall x \in [-1 : 1]$
   (c) Make a plot in the range $[-2 : 2]$ containing two subplots (with one subplot for each of the functions) displaying the series functions as well as the given function. Use $n = 1, 2, 3, \ldots$ for $f(x) = 2+3x+2x^4$ and $n = 2, 4, 6, 8, 10$ for $f(x) = \cos(x)\sin(x)$.

   Use inbuilt function *scipy.special.legendre(n)* or *scipy.special.eval_legendre* for evaluating the Legendre polynomials.

3. (3 marks) **Discussion**
   Interpret and discuss your results and graphs.