

### 1. Theory

1. What is Taylor series representation of a function? What do you mean by radius of convergence of series? What is MacLaurin Series?
2. Write down the Taylor series for a function of two variables.
3. Write the Maclaurin series representation for trigonometric functions  $\sin x$ ,  $\cos x$  and  $\exp(x)$ . Discuss the radius of convergence for each of them.

### 2. Algorithm

Write the algorithm for a function that generates the Taylor Series expansion for

1.  $e^x$
2.  $\sin x$
3.  $\cos x$

### 3. Programming

1. Make two python functions  $\text{MySinSeries}(x, n)$  and  $\text{MyCosSeries}(x, n)$  for finding series representation for  $\sin x$  and  $\cos x$  using  $n$  terms. Here  $x$  is a vector of values and  $n$  is the number of terms you want to sum. The vector  $y$  containing these values should be returned. Take  $x_0$  and  $m$  as input and use these python functions to show that with inclusion of more and more terms of the series the value obtained by series representation approaches the actual values. For this
  - (a) plot  $\sin(x)$  using the inbuilt python function as well as  $\sin(x)$  obtained by your function with  $m$  terms,  $m$  being equal to 1, 2, 5, 10, 20 terms. The plot should be in the range  $[-2\pi, 2\pi]$ .
  - (b) Create an array  $y_0$  containing values of  $\sin(x_0)$  obtained by your series for  $m = 2, 4, \dots, 20$ .
  - (c) plot  $y_0$  as function of  $n$  and on the same graph, plot  $\sin(x_0)$  (a straight horizontal line). Show your output for  $x_0 = \pi/4$ .
  - (d) Both above plots should be plotted as subplots of one plot.
  - (e) Repeat all above for  $\cos(x)$ .
2. Now prove that as the value of  $x$  increases, more terms are required to get a value of  $\sin x$  with a given precision. For this make one more python function that returns the value of  $\sin(x)$  for a given vector  $x$  accurate to a given number of significant digits (should be taken as input from user). It should also return the number of terms required to get this accuracy for each  $x$  in the range  $[0, \pi]$  with a step size of  $\pi/8$ . Plot the value of  $\sin(x)$  obtained this way correct to three significant digits (as points) along with the actual one (as continuous curve). Print the output in a tabulated format as below (Make use of Panda data frame):

Table 1: Values of  $\sin(x)$  accurate to 6 significant digits evaluated using series representation along with  $n$ , the number of terms required to get this accuracy.

$x$	$\sin(x)_{\text{calc}}$	$n$	$\sin(x)_{\text{inbuilt}}$
0	...	...	...
$\pi/8$			
$\vdots$			
$\pi$			

### 3. Discussion

Analyse and discuss your results