Linear Algebra cheat sheet

Vectors

dot product: $u * v = ||u|| * ||v|| * cos(\phi) = u_x v_x + u_y v_y$

cross product: $u \times v = \begin{pmatrix} u_y v_z - u_z v_y \\ u_z v_x - u_x v_z \\ u_x v_y - u_y v_x \end{pmatrix}$

enclosed angle:

$$cos\phi = \frac{u * v}{||u|| * ||v||}$$

$$||u|| * ||v|| = \sqrt{(u_x^2 + u_y^2)(v_x^2 + v_y^2)}$$

Matrices

operations

transpose: $[A^{\mathrm{T}}]_{ij} = [A]_{ji}$: "mirror over main diagonal"
multiply: $A_{N\times M} * B_{R\times K} = M_{N\times K}$
invert: $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{\det(\mathbf{A})} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

determinants

$det(A \cdot B) = det(A) \cdot det(B)$
TODO: other rules

eigenvalues/vectors

definiteness

common properties

square: $N \times N$
symmetric: $A = A^T$
orthogonal: $A^T = A^{-1}$
diagonal: 0 except $a_k k$
$\Rightarrow$ Eigenvalues on main diagonale

regular = invertible = nonsingular

A is invertible iff:
There is a matrix $B := A^{-1}$ such that $AB = I = BA$.
$det(A) \neq 0$
T$Ax = b$ has exactly one solution for each b.
The columns of A are linearly independent.
$\Rightarrow det(A)^{-1} = det(A^{-1})$
$\Rightarrow (A^{-1})^{-1} = A$
$\Rightarrow (A^T)^{-1} = (A^{-1})^T$

diagonalizable

If A can be diagonalized:

$$P^{-1}AP = \begin{pmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{pmatrix}$$

then:

$$AP = P \begin{pmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{pmatrix}$$

triangular

rank / kernel / trance