

$$E(X) = \frac{1}{2} \quad E(Y) = \frac{1}{2}$$

解:

(1)

$$X \sim U(0, 1), Y \sim U(0, 1)$$

$$f(x) = \begin{cases} 1 & 0 \leq x \leq 1 \\ 0 & \text{其它} \end{cases}$$

$$f(y) = \begin{cases} 1 & 0 \leq y \leq 1 \\ 0 & \text{其它} \end{cases}$$

$X, Y$  相互独立

$$\therefore f(x, y) = f(x)f(y)$$

$$\therefore f(x, y) = \begin{cases} 1 & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & \text{其它} \end{cases}$$

$$P(X \geq Y) = \iint_{x \geq y} 1 \, dx \, dy = 1 - \frac{1}{2} = \frac{1}{2}$$

(2)

$$E[|X - Y|] = \iint |x - y| \, dx \, dy$$

$$= \int_0^1 dx \int_0^1 |x - y| \, dy$$

$$= \int_0^1 dx \int_0^x (x - y) \, dy + \int_0^1 dy \int_y^1 (y - x) \, dx$$

$$= \frac{1}{6} + \frac{1}{6}$$

$$= \frac{1}{3}$$