VCM: The Vector-Based Coloring Method for Grid Wireless Ad Hoc and Sensor Networks

Cédric Adjih, **Ichrak Amdouni**, Pascale Minet

15th ACM International Conference On Modeling, Analysis And Simulation of Wireless And Mobile Systems — MSWiM 2012, Oct. 21-25, 2012

OUTLINE

- Context
- Problem Statement
- Our Approach: VCM Method
- VCM components
- Performance Evaluation
- Conclusion

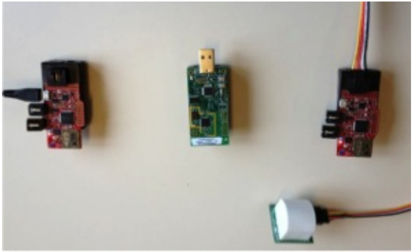
1

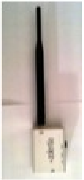
Context & Problem Statement

Context

Wireless Ad Hoc and Sensor Networks

- Energy saving for nodes on batteries
- Signal interference
- $\rightarrow$ Close nodes should not transmit at the same time

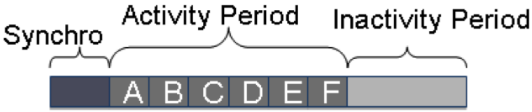




Context

A Common Solution

- TDMA (Time Division Multiple Access)
- Nodes in the same area transmit each in turn
  - No collisions
  - Energy saving

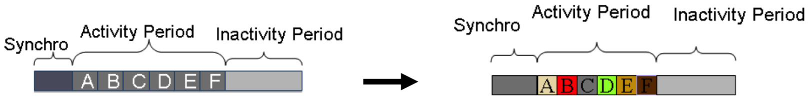


An example of TDMA cycle

Context: Coloring Framework

TDMA with spatial reuse:

- "Coloring" (TDMA-ASAP, TRAMA, ZMAC, OSERENA, . . .)
  1. No two interfering nodes have the same color
  2. A color is mapped to a time slot
  3. A node is awake in
     - its slot if it has a message to transmit
     - the slots of its neighbors if it has messages to receive
  4. And sleeps the remaining time

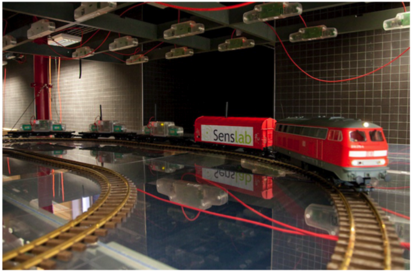


- Objective: minimum number of colors
  - Reduce activity duration and save energy
  - Reduce end to end delays

Context: Grid Topologies

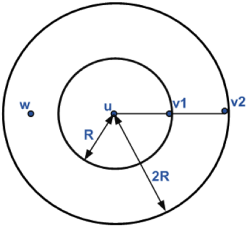
Grid topologies

- Regular topologies easy to deploy
  - No localization required
- Suitable for WSNs (optimal coverage, e.g surveillance application)
- Also: infinite grid = first step towards random topologies



Assumption: network model & neighborhood

- Unit disk model: nodes u and v are 1-hop neighbors if they are at distance $\leq R$
  (Note: R is not obligatory equal to the grid step)
- u and v are h-hop neighbors iff the shortest path between them has a length of h hops

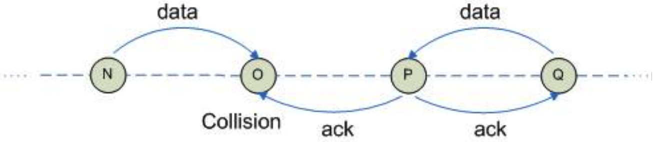


- u and v_2 are 2-hop neighbors
- u and w are not.

- Approximation used for interference area:
  - Distance $h.R \approx h$ hops

Problem Statement

- Interferences are not limited to 2 hops
  - Because of acknowledgements



- 2-hop coloring is not sufficient
- In *h*-hop coloring, no two nodes that are at less than or equal to *h* hops away have the same color.

Problem Statement

- Our objective:
  - Perform efficient h-hop coloring of (infinite) grids
- The problems to solve:
  1. How to color them?
  2. How to obtain the optimum number of colors?
  3. How to generalize these results to any radio range?

Observation: Simulation Results of OSERENA

- **Many heuristics (priority choice that defines the coloring order)**

Radio range	Grid size	Dens.	Optim	Prio1		Prio2	
				line	random	line	random
1	10x10	3.6	8C	8C 58R	13.8C 67.4R	8C 54R	11.8C 77.8R
	20x20	3.8	8C	8C 118R	15.4C 82.4R	8C 114R	14.8C 110R
	30x30	3.87	8C	8C 178R	15.4C 93R	8C 174R	15.4C 116.6R
1.5	10x10	6.84	16C	16C 67R	23.6C 94.8R	16C 65R	19.4C 107.6R
	20x20	7.41	16C	16C 137R	27.6C 144.6R	16C 134R	26.6C 166.6R
	30x30	7.6	16C	16C 207R	28.4C 169.2R	16C 204R	27.8C 195.4R
2	10x10	10.04	25C	30C 85R	33.8C 136.4R	30C 123R	28.2C 121.8R
	20x20	11.01	25C	33C 175R	41.8C 236.6R	33C 174R	36.4C 282.4R
	30x30	11.34	25C	33C 265R	44.4C 278R	34C 264R	42.8C 376.6R
2.5	10x10	15.8	45C	52C 94R	50.8C 176.8R	49C 105R	42.33C 146.17R
	20x20	17.85	45C	54C 194R	66.8C 348.2R	54C 197R	64.8C 359.8R
	30x30	18.56	45C	55C 294R	74C 422.2R	58C 297R	73.6C 440.8R
3	10x10	21.16	68C	70C 112R	68C 193R	71C 116R	67.6C 197.8R
	20x20	24.49	68C	80C 232R	93.4C 449.4R	79C 231R	93C 451.4R
	30x30	25.64	68C	83C 352R	107.8C 601.2R	81C 351R	107.8C 601.4R

Observation: Simulation Results of OSERENA

- **Many heuristics (priority choice that defines the coloring order)**
- **OSERENA: number of colors increases with the grid size.**

Radio range	Grid size	Dens.	Optim	Prio1		Prio2	
				line	random	line	random
1	10x10	3.6	8C	8C 58R	13.8C 67.4R	8C 54R	11.8C 77.8R
	20x20	3.8	8C	8C 118R	15.4C 82.4R	8C 114R	14.8C 110R
	30x30	3.87	8C	8C 178R	15.4C 93R	8C 174R	15.4C 116.6R
1.5	10x10	6.84	16C	16C 67R	23.6C 94.8R	16C 65R	19.4C 107.6R
	20x20	7.41	16C	16C 137R	27.6C 144.6R	16C 134R	26.6C 166.6R
	30x30	7.6	16C	16C 207R	28.4C 169.2R	16C 204R	27.8C 195.4R
2	10x10	10.04	25C	30C 85R	33.8C 136.4R	30C 123R	28.2C 121.8R
	20x20	11.01	25C	33C 175R	41.8C 236.6R	33C 174R	36.4C 282.4R
	30x30	11.34	25C	33C 265R	44.4C 278R	34C 264R	42.8C 376.6R
2.5	10x10	15.8	45C	52C 94R	50.8C 176.8R	49C 105R	42.33C 146.17R
	20x20	17.85	45C	54C 194R	66.8C 348.2R	54C 197R	64.8C 359.8R
	30x30	18.56	45C	55C 294R	74C 422.2R	58C 297R	73.6C 440.8R
3	10x10	21.16	68C	70C 112R	68C 193R	71C 116R	67.6C 197.8R
	20x20	24.49	68C	80C 232R	93.4C 449.4R	79C 231R	93C 451.4R
	30x30	25.64	68C	83C 352R	107.8C 601.2R	81C 351R	107.8C 601.4R

Observation: Simulation Results of OSERENA

- **Many heuristics (priority choice that defines the coloring order)**
- **OSERENA: number of colors increases with the grid size.**
- For random priority assignement, number of colors is 15.4 for 30x30 grid
  - Not always optimal

Radio range	Grid size	Dens.	Optim	Prio1		Prio2	
				line	random	line	random
1	10x10	3.6	8C	8C 58R	13.8C 67.4R	8C 54R	11.8C 77.8R
	20x20	3.8	8C	8C 118R	15.4C 82.4R	8C 114R	14.8C 110R
	30x30	3.87	8C	8C 178R	15.4C 93R	8C 174R	15.4C 116.6R
1.5	10x10	6.84	16C	16C 67R	23.6C 94.8R	16C 65R	19.4C 107.6R
	20x20	7.41	16C	16C 137R	27.6C 144.6R	16C 134R	26.6C 166.6R
	30x30	7.6	16C	16C 207R	28.4C 169.2R	16C 204R	27.8C 195.4R
2	10x10	10.04	25C	30C 85R	33.8C 136.4R	30C 123R	28.2C 121.8R
	20x20	11.01	25C	33C 175R	41.8C 236.6R	33C 174R	36.4C 282.4R
	30x30	11.34	25C	33C 265R	44.4C 278R	34C 264R	42.8C 376.6R
2.5	10x10	15.8	45C	52C 94R	50.8C 176.8R	49C 105R	42.33C 146.17R
	20x20	17.85	45C	54C 194R	66.8C 348.2R	54C 197R	64.8C 359.8R
	30x30	18.56	45C	55C 294R	74C 422.2R	58C 297R	73.6C 440.8R
3	10x10	21.16	68C	70C 112R	68C 193R	71C 116R	67.6C 197.8R
	20x20	24.49	68C	80C 232R	93.4C 449.4R	79C 231R	93C 451.4R
	30x30	25.64	68C	83C 352R	107.8C 601.2R	81C 351R	107.8C 601.4R

Observation: Simulation Results of OSERENA

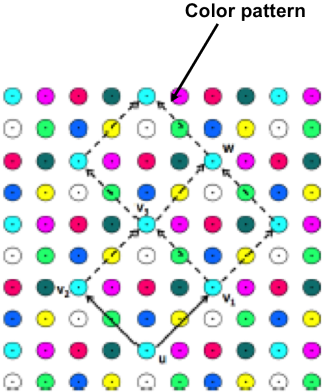
- **Many heuristics (priority choice that defines the coloring order)**
- **OSERENA: number of colors increases with the grid size.**
- For random priority assignement, number of colors is 15.4 for 30x30 grid
  - Not always optimal
- Number of colors is 8 if the priority is given by the line position
  - Regularity of the grid must be taken into account

Radio range	Grid size	Dens.	Optim	Prio1		Prio2	
				line	random	line	random
1	10x10	3.6	8C	8C 58R	13.8C 67.4R	8C 54R	11.8C 77.8R
	20x20	3.8	8C	8C 118R	15.4C 82.4R	8C 114R	14.8C 110R
	30x30	3.87	8C	8C 178R	15.4C 93R	8C 174R	15.4C 116.6R
1.5	10x10	6.84	16C	16C 67R	23.6C 94.8R	16C 65R	19.4C 107.6R
	20x20	7.41	16C	16C 137R	27.6C 144.6R	16C 134R	26.6C 166.6R
	30x30	7.6	16C	16C 207R	28.4C 169.2R	16C 204R	27.8C 195.4R
2	10x10	10.04	25C	30C 85R	33.8C 136.4R	30C 123R	28.2C 121.8R
	20x20	11.01	25C	33C 175R	41.8C 236.6R	33C 174R	36.4C 282.4R
	30x30	11.34	25C	33C 265R	44.4C 278R	34C 264R	42.8C 376.6R
2.5	10x10	15.8	45C	52C 94R	50.8C 176.8R	49C 105R	42.33C 146.17R
	20x20	17.85	45C	54C 194R	66.8C 348.2R	54C 197R	64.8C 359.8R
	30x30	18.56	45C	55C 294R	74C 422.2R	58C 297R	73.6C 440.8R
3	10x10	21.16	68C	70C 112R	68C 193R	71C 116R	67.6C 197.8R
	20x20	24.49	68C	80C 232R	93.4C 449.4R	79C 231R	93C 451.4R
	30x30	25.64	68C	83C 352R	107.8C 601.2R	81C 351R	107.8C 601.4R

2

Our Approach: VCM Method & VCM components

Our Approach



- Profit from the regularity of the grid topology
  - Periodic coloring
  - Color pattern

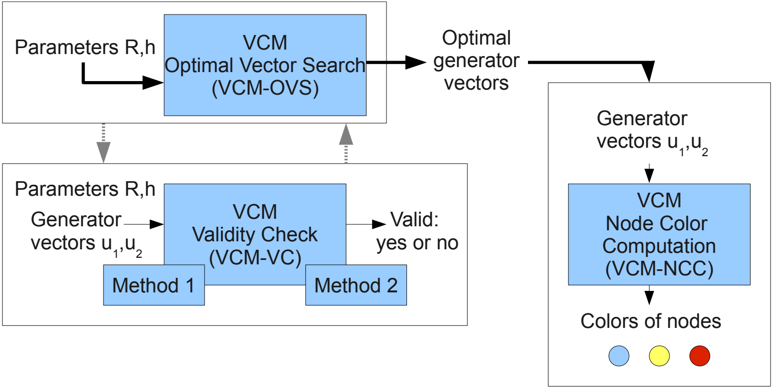
- Periodic Coloring
  - Generator vectors
  - Color unicity inside the parallelogram
  - Lattice color repetition: nodes with the same color form a lattice of generator vectors
  - Coordinate-based color computation

Periodic coloring: challenges

- Generator vectors
  - What are the best vectors?
  - Where/How to find them?
- Color computation
  - How to compute colors?
- Wireless communications constraints
  - Is the coloring valid? (no color conflict)?
  - Color reuse?
  - Optimal number of colors?

VCM (Vector-based Coloring Method) components

- **Optimal h-hop periodic coloring for any h and any radio range R**
  1. VCM-OVS: Optimal Vector search
  2. VCM-NCC: Node Color Computation
  3. VCM-VC: Validity Check

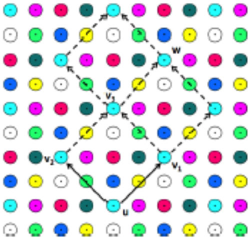


VCM-OVS: Optimal Vector Search

- For VCM, the number of colors is entirely defined by the two generator vectors (u_1, u_2)
  - The number of colors = scalar product of the generator vectors
  - Optimal vectors are those providing the smallest number of colors
- Try all possible vectors (infinite space)
  - Reduce the search space

- Method
  1. Bounds on the optimal number of colors
     - Number of colors $= |\det(u_1, u_2)|$

     Bounds on the optimal number of colors gives
     $\rightarrow$ bounds on the coordinates of the optimal vectors
  2. Usage of reduced vectors



VCM-OVS: Optimal Vector Search

Result 1: A lower bound of the number of colors.

Theorem 1

The number of colors required to color an infinite grid with $R > \sqrt{2}$ is at least $\frac{\sqrt{3}}{2}h^2(R - \sqrt{2})^2$

Result 2: An upper bound of the number of colors.

Theorem 2

The number of colors required to color an infinite grid is at most $\frac{\sqrt{3}}{2}h^2R^2 + 2hR + (2 + hR)\sqrt{2}$

VCM-OVS: Optimal Vector Search

Result 3: Asymptotic number of colors.

Theorem 3

The number of colors $n_c(R)$ of an optimal periodic h-hop coloring for a fixed h verifies: $n_c(R) = \frac{\sqrt{3}}{2} h^2 R^2 (1 + O(\frac{1}{R}))$

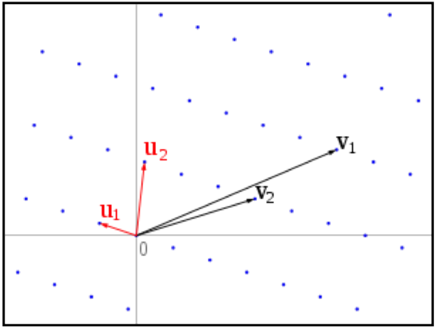
Result 4: Optimality of VCM.

Theorem 4

VCM is asymptotically optimal even considering all possible valid colorings.
In other terms, VCM is a $(1 + g(R))$-approximation of the optimal coloring of the grid with some g verifying $g(R) \to 0$

VCM-OVS: Optimal Vector Search

- Search space reduction:
  - For any couple of vectors, reduced vectors (smaller vectors) generating the same lattice exist (Gauss Lattice Reduction)
  - $\rightarrow$ Search in the space of reduced vectors



$$u_1 \leq u_2$$

$$2u_1.u_2 \leq u_1 u_2$$

VCM-OVS: Optimal Vector Search

- Using bounds and reduction:
  - Possible reduced vector space is finite ($\Theta(R^4)$)
  - Exhaustive search in a reduced space of optimal vectors

For $R > \sqrt{2}$

$l_{1\,min} < |u_1| \leq l_{1\,max}$ and
$l_{2\,min} < |u_2| < l_{2\,max}$ with:

$$\begin{cases} l_{1\,min} = h(R - \sqrt{2}) \\ l_{1\,max} = \sqrt{\frac{2}{\sqrt{3}} S_h} \\ l_{2\,min} = h(R - \sqrt{2}) \\ l_{2\,max} = \frac{2}{\sqrt{3}} \frac{S_h}{h(R-\sqrt{2})} \end{cases}$$

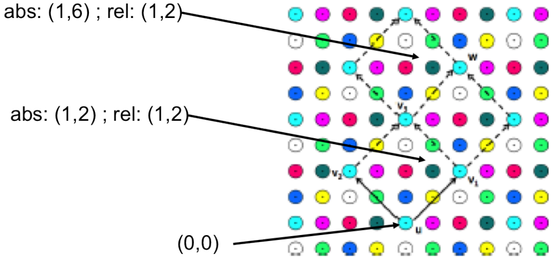
For $R \leq \sqrt{2}$

$$\begin{cases} 0 < |u_1| \leq hR + 1 \\ |u_1| \leq |u_2| \leq \frac{2\sqrt{3}(hR+1)^2}{3|u_1|} \end{cases}$$

VCM-NCC: Node Color Computation

Periodic coloring

- A color is unique inside the parallelogram
- All nodes having the same relative coordinates in their parallelogram share the same color
  - Coordinates-based colors
  - Bijection between relatives coordinates in the parallelogram and colors



VCM-NCC: Node Color Computation

Method

- The coordinates of any node W of coordinates (x, y) in the basis $(u_1 = (x_1, y_1), u_2 = (x_2, y_2))$ in the parallelogram generated by the two generator vectors are given by:

$$\begin{cases} c_1(W) = (xy_2 - yx_2) \text{ modulo } d \\ c_2(W) = (-xy_1 + yx_1) \text{ modulo } d \end{cases}$$

- Construct a bijection between $(c_1(W), c_2(W))$ and the colors in $[0, |d| - 1]$
  - Example:
    1. sort these couples
    2. the colors of the correspondent node is its index-1

VCM-VC: Validity check

- *h*-hop coloring
  - Validity: any two nodes $\leq h$ hops have different colors
    - $\rightarrow$ Check the validity of the coloring
- Finite check (periodic)
- Method
  - Reduction on the number of verifications based on:
    Theorem 4: *If none of the nodes inside the h-hop neighborhood of the origin node* $O(0,0)$ *has the same color as* O *itself, then the coloring is valid*
  - $\rightarrow$ Check colors of all h-hop neighbors of O
- Further optimizations (geometry, lattice reduction) when R sufficiently large:
  - Checking colors of a few points
  - Checking colors of 4 points

VCM: how to apply it?

- Finding the optimal vectors using VCM-OVS
- Filtering of these vectors to keep only valid vectors
- The optimal vectors are those with the smallest absolute value of their determinant
- Color computing
  - Computation of the couple c_1, c_2
  - Applying a bijection to compute the color

VCM complexity

- Theorem: VCM complexity is $\Theta(R^4)$

3
Performance Evaluation & Conclusion

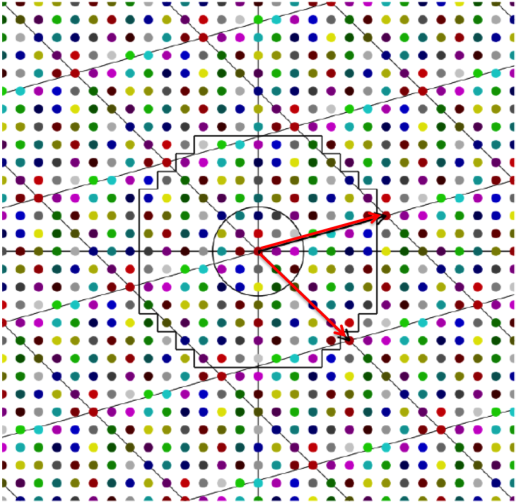
VCM: Results

Radio range	2-hop coloring			3-hop coloring		
	vector1	vector2	colors	vector1	vector2	colors
1	(2,1)	(-1,2)	5*	(2,2)	(-2,2)	8*
1.5	(-3,0)	(0,3)	9*	(4,0)	(0,4)	16*
2	(3,2)	(-2,3)	13*	(4,3)	(-3,4)	25*
2.5	(4,3)	(-1,5)	23*	(5,5)	(-7,2)	45*
3	(5,3)	(-1,6)	33*	(7,5)	(-8,4)	68*
3.5	(5,4)	(-6,3)	39*	(8,5)	(-8,5)	80*
4	(7,3)	(-6,5)	53*	(8,8)	(-11,3)	112*
4.5	(9,2)	(-6,7)	75*	(13,3)	(-9,10)	157*
5	(9,4)	(-1,10)	94*	(14,4)	(3,15)	198*
5.5	(9,6)	(-1,11)	105*	(16,0)	(8,14)	224*
6	(11,4)	(-9,8)	124*	(17,4)	(-12,13)	269*
6.5	(13,1)	(-7,11)	150*	(-19,0)	(9,17)	323*
7	(10,9)	(-4,13)	166*	(15,13)	(-19, 7)	352*

Figures see: http://hipercom.inria.fr/SensorNet/VCM/

VCM: Example of grid coloring

- $r = 2.5$
- $h = 3$
- $col = 45$



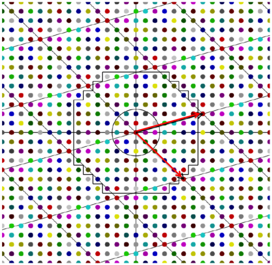
http://hipercom.inria.fr/SensorNet/VCM/

VCM: Comparison with OSERENA

Radio range	Grid size	Colors	
		VCM	line/column
1	10x10	8*	8*
	20x20	8*	8*
	30x30	8*	8*
1.5	10x10	16*	16*
	20x20	16*	16*
	30x30	16*	16*
2	10x10	25*	30
	20x20	25*	33
	30x30	25*	33
3	20x20	68*	80
	30x30	68*	83
3.5	20x20	80*	91
	30x30	80*	91

Conclusion and future works

- A method for grid h-hop coloring for any h and any radio range R.
- Optimal number of colors
  - Does not depend on the size of the grid
- Property on number of colors:
  - The optimal number of colors in h-hop coloring for a fixed h and when R grows to infinity is $\frac{\sqrt{3}}{2}h^2R^2$
- Apply this method to random topologies by mapping random topologies to the grid



http://hipercom.inria.fr/SensorNet/VCM/

Thank You!

ichrak.amdouni@inria.fr

Annex

VCM and real wireless networks

- A neighbor may be at a distance $\leq R$
  - Should start by selecting the right value of R:
  - No two nodes that are at distance greater than R are neighbors.
- Radio links disappearance
  - Coloring remains valid
- Radio links appearance
  - The coloring still be acceptable by the appplication if the percentage of the additional links is below specific threshold.

Problem Statement

- Grids with various transmission ranges? Can we simplify?
  - Is 1-hop coloring of a grid with R = 6 the same as 3-hop coloring of the grid with R = 2, is it the same as 2-hop coloring of the grid with R = 3?

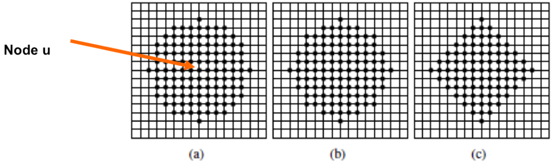
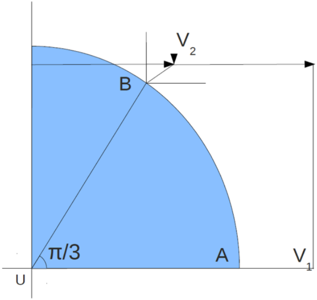


Fig. 2. Nodes having a color different from u: (a) 1-hop coloring $R = 6$; (b) 2-hop coloring $R = 3$ and (c) 3-hop coloring $R = 2$.

- The answer is NO!
  - h and k integers such that $hR_1 = kR_2$
  - h-hop coloring of G with $R = R_1 \neq k$-hop coloring of G with $R = R_2$

VCM-OVS: Optimal Vector Search

Examples of valid vectors: Near hexagonal vectors (upper bound)



$$
\begin{aligned}
|AV_1| &= 2x_2 - hR \\
&= 2(hR\cos(\frac{\Pi}{3}) + \gamma) - hR, \text{with } \gamma \leq 1 \\
&\leq 2
\end{aligned}
$$

$$
\begin{aligned}
n_c &= \det(\overrightarrow{UV_1}, \overrightarrow{UV_2}), \\
&= \det(\overrightarrow{UA}, \overrightarrow{UB}) + \det(\overrightarrow{AV}_1, \overrightarrow{UB}) + \det(\overrightarrow{UV}_1, \overrightarrow{BV_2}) \\
&\leq \det(\overrightarrow{UA}, \overrightarrow{UB}) + |\overrightarrow{AV}_1|\,|\overrightarrow{UB}| + |\overrightarrow{UV_1}|\,|\overrightarrow{BV_2}| \\
&\leq \frac{\sqrt{3}}{2} h^2 R^2 + 2hR + (hR+2)\sqrt{2};
\end{aligned}
$$