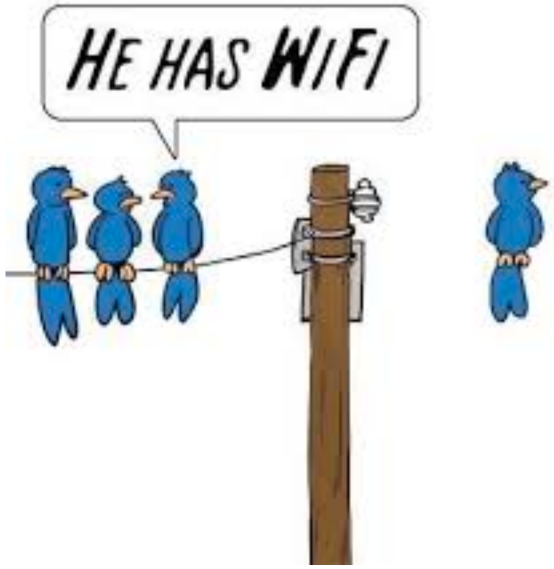
Graph Matchings and Wireless Communication

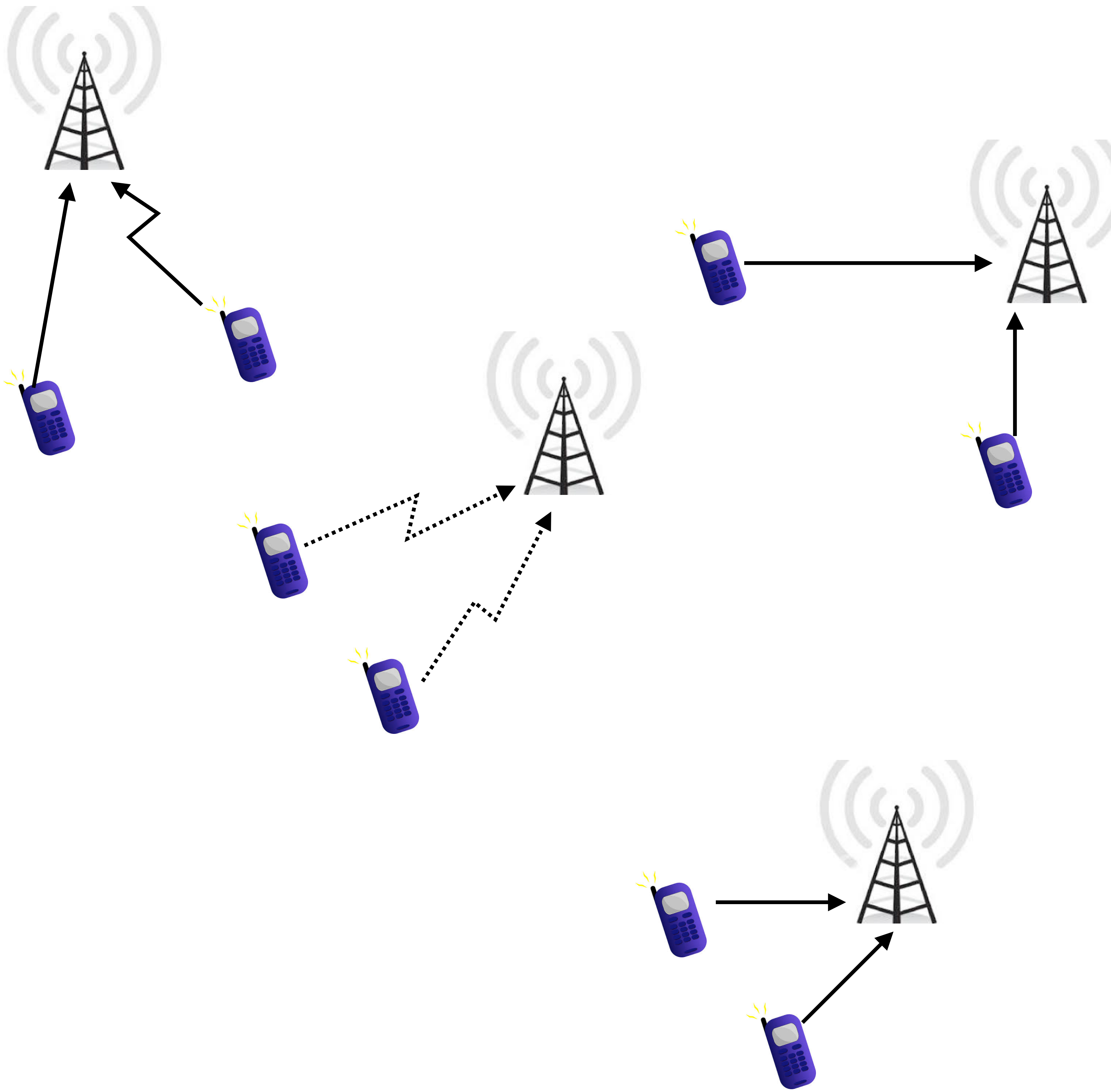
Rahul Vaze

Graph Matchings and Wireless Communication

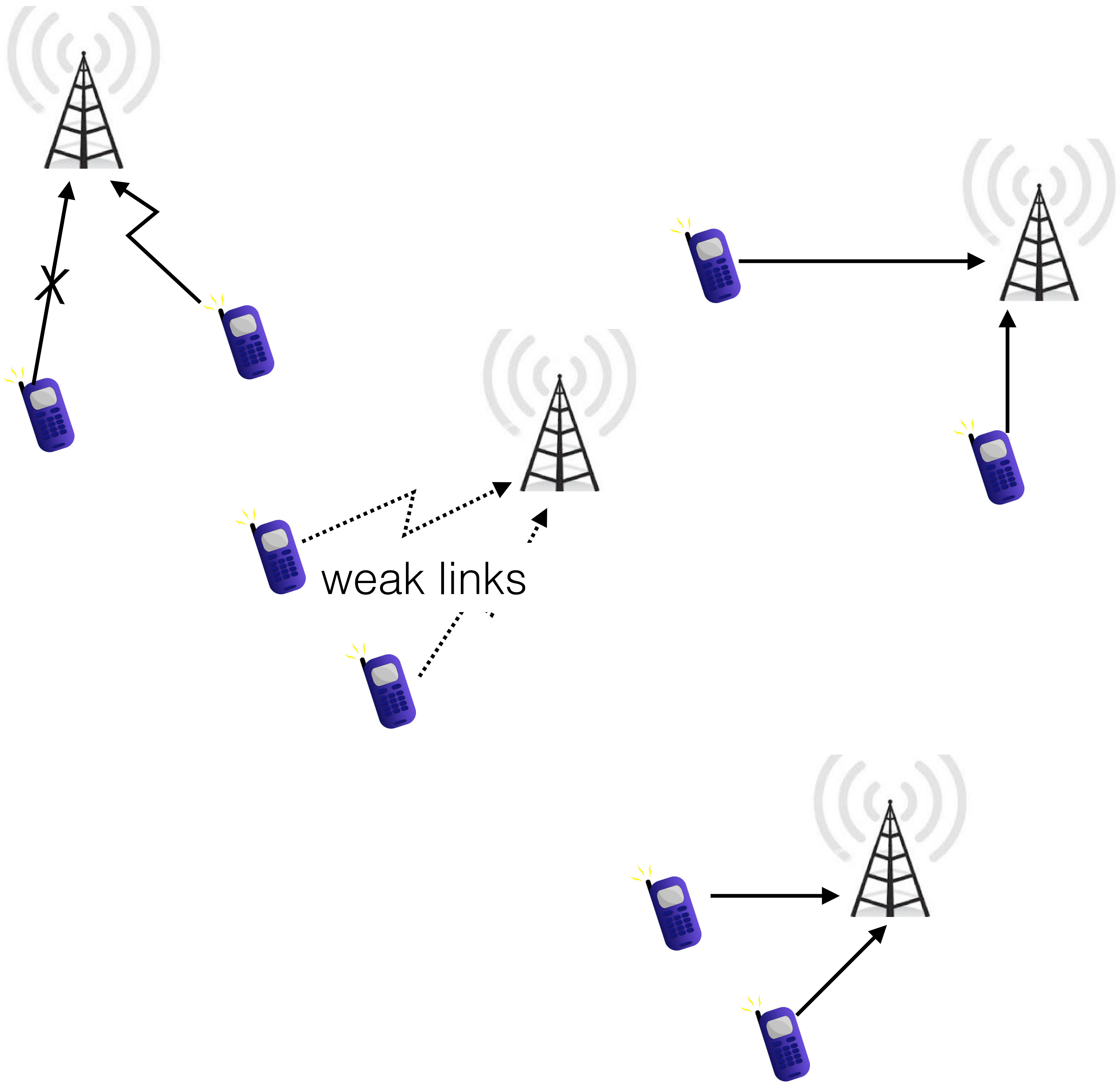


Rahul Vaze

Device-2-Device Communication



Device-2-Device Communication



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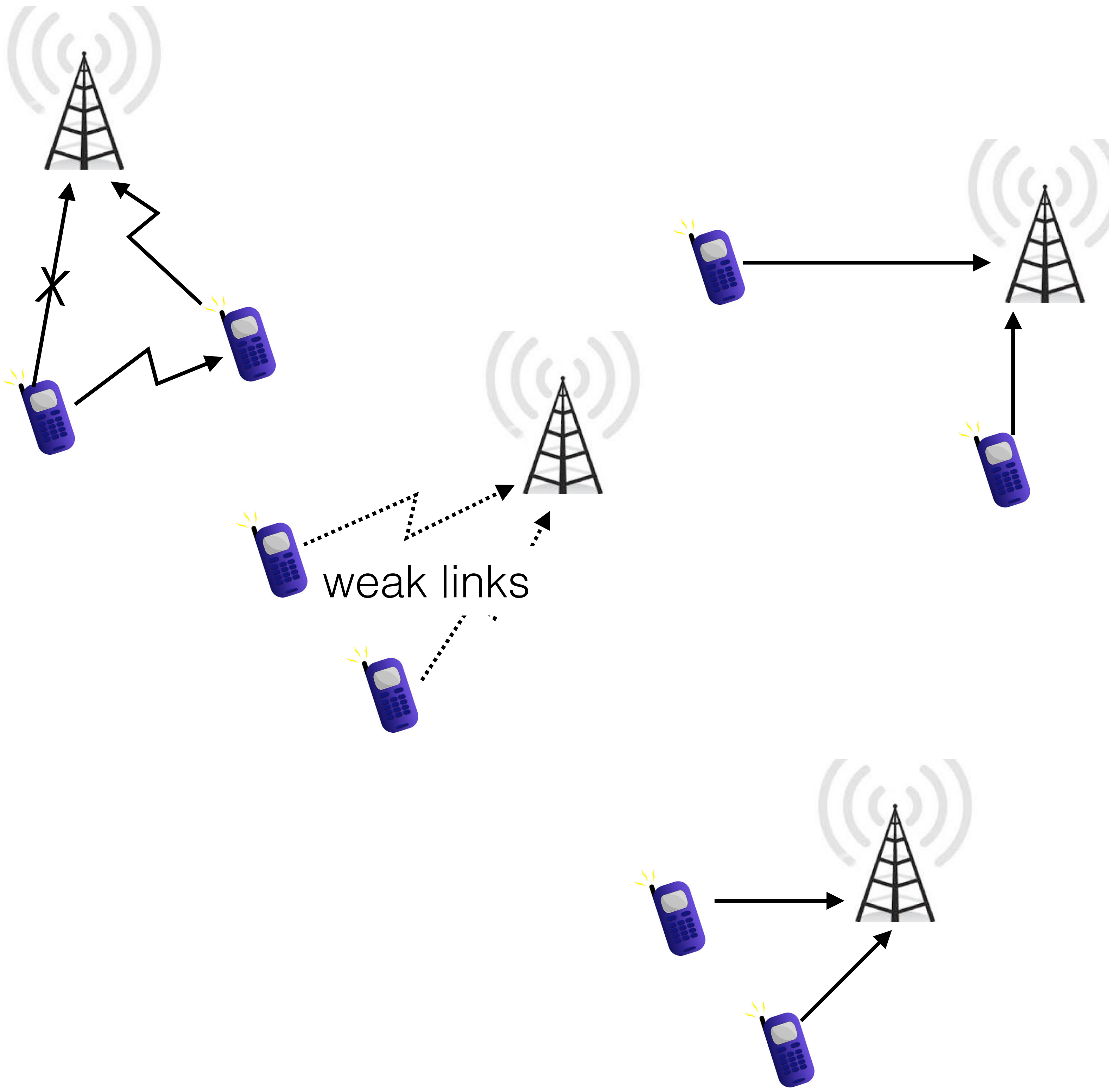

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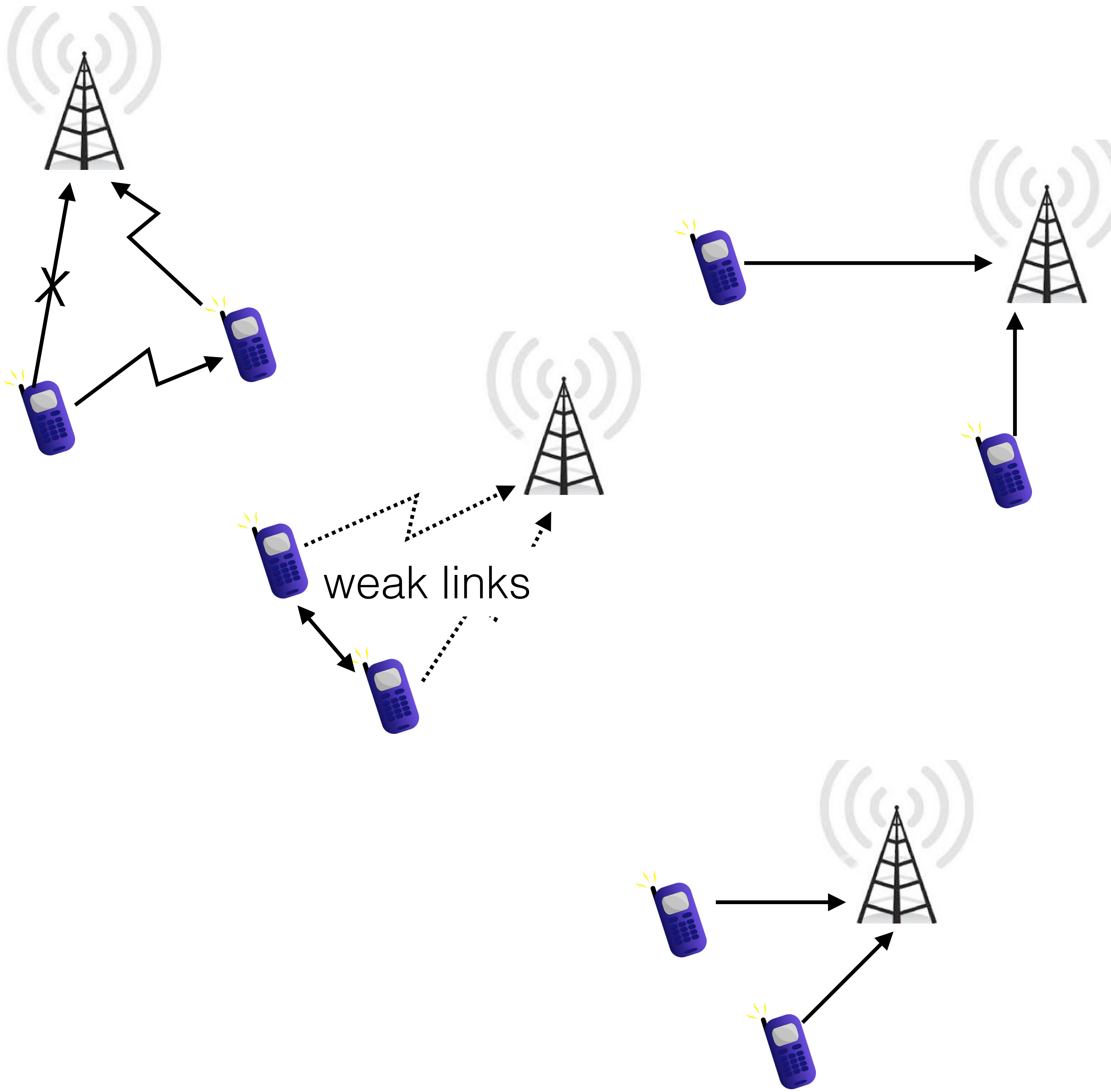

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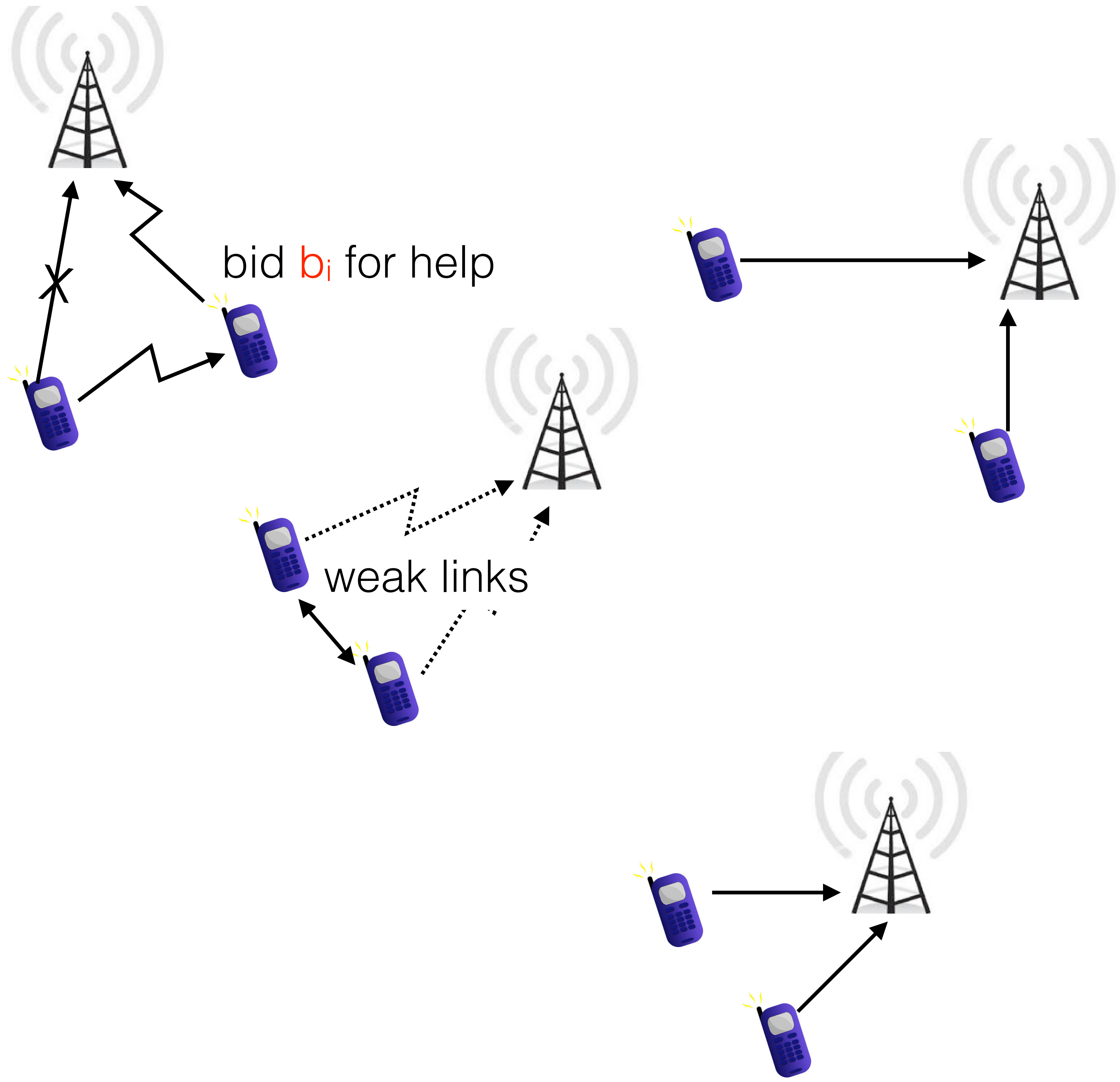

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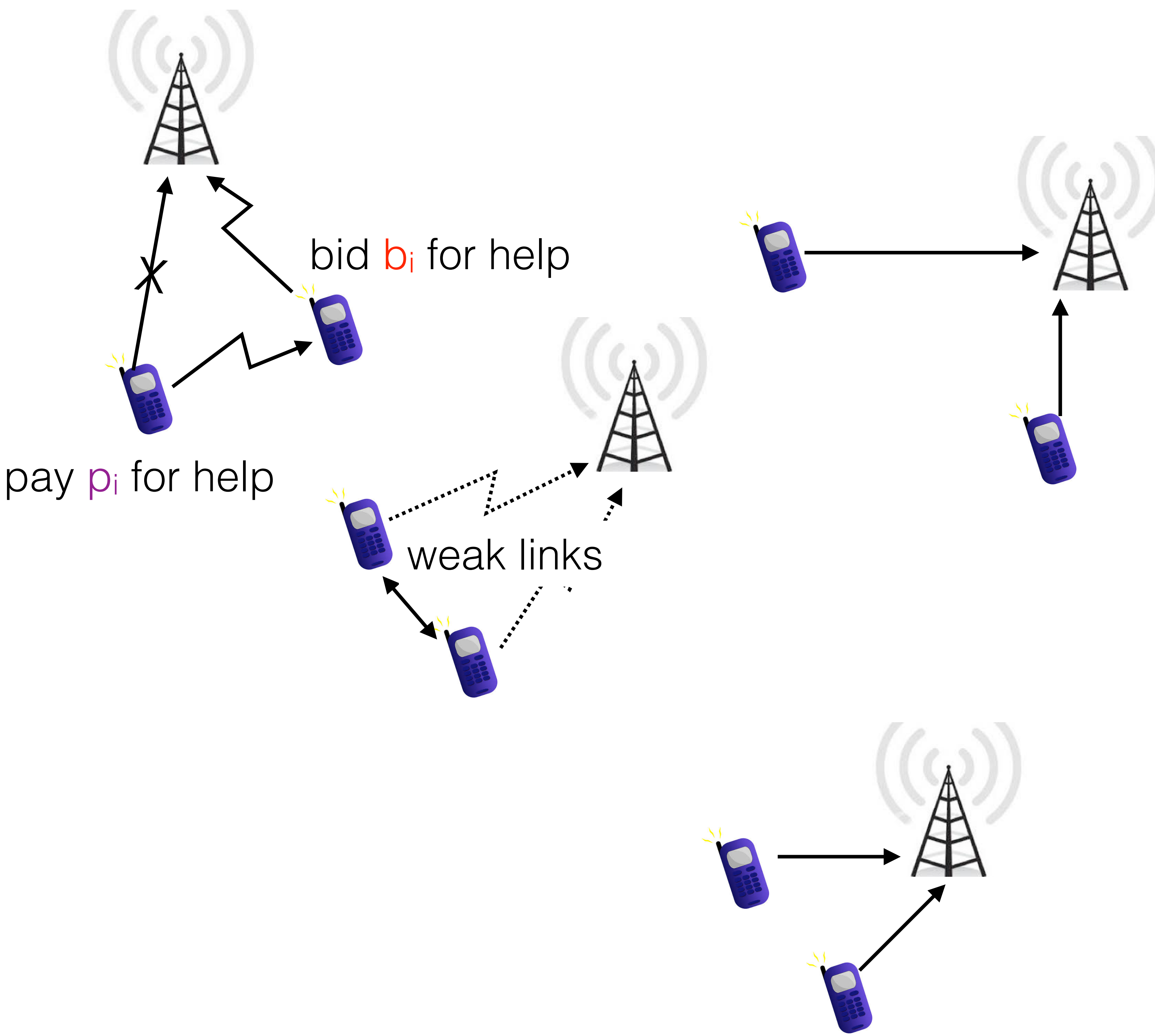

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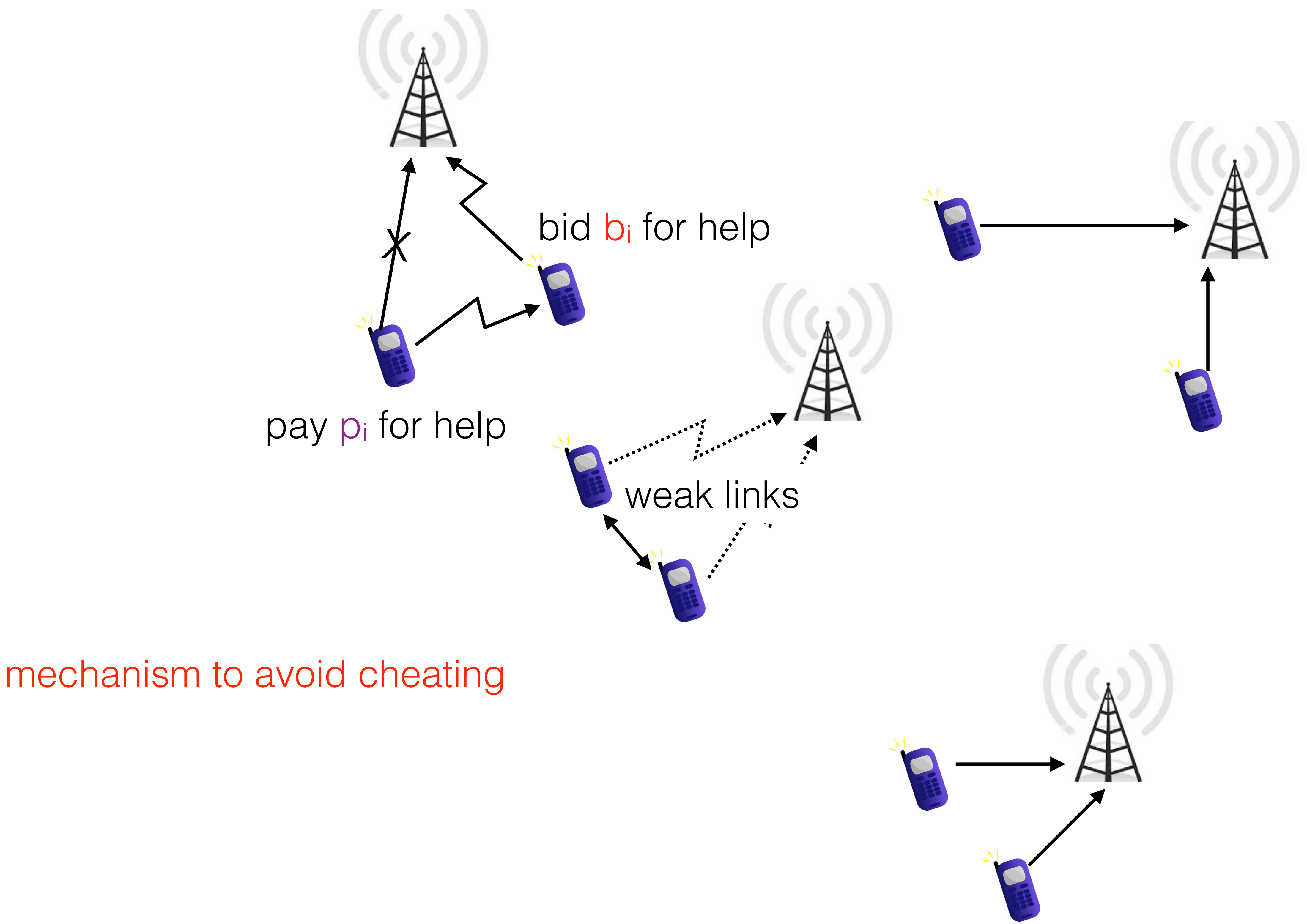

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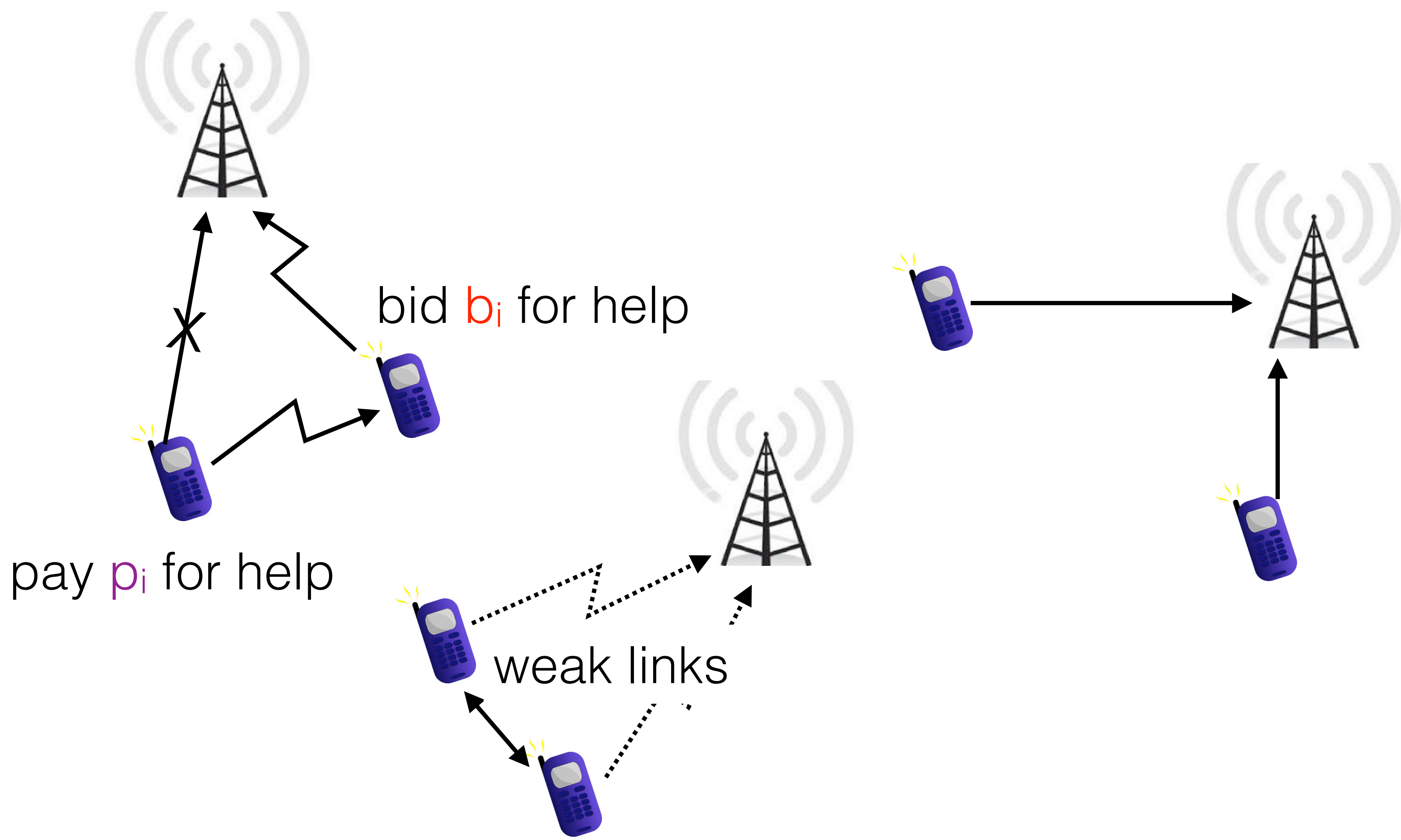
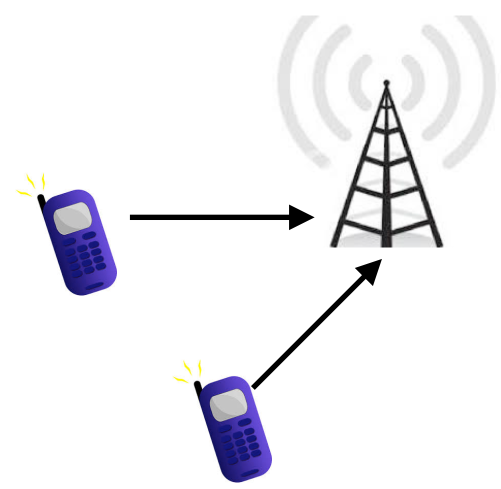
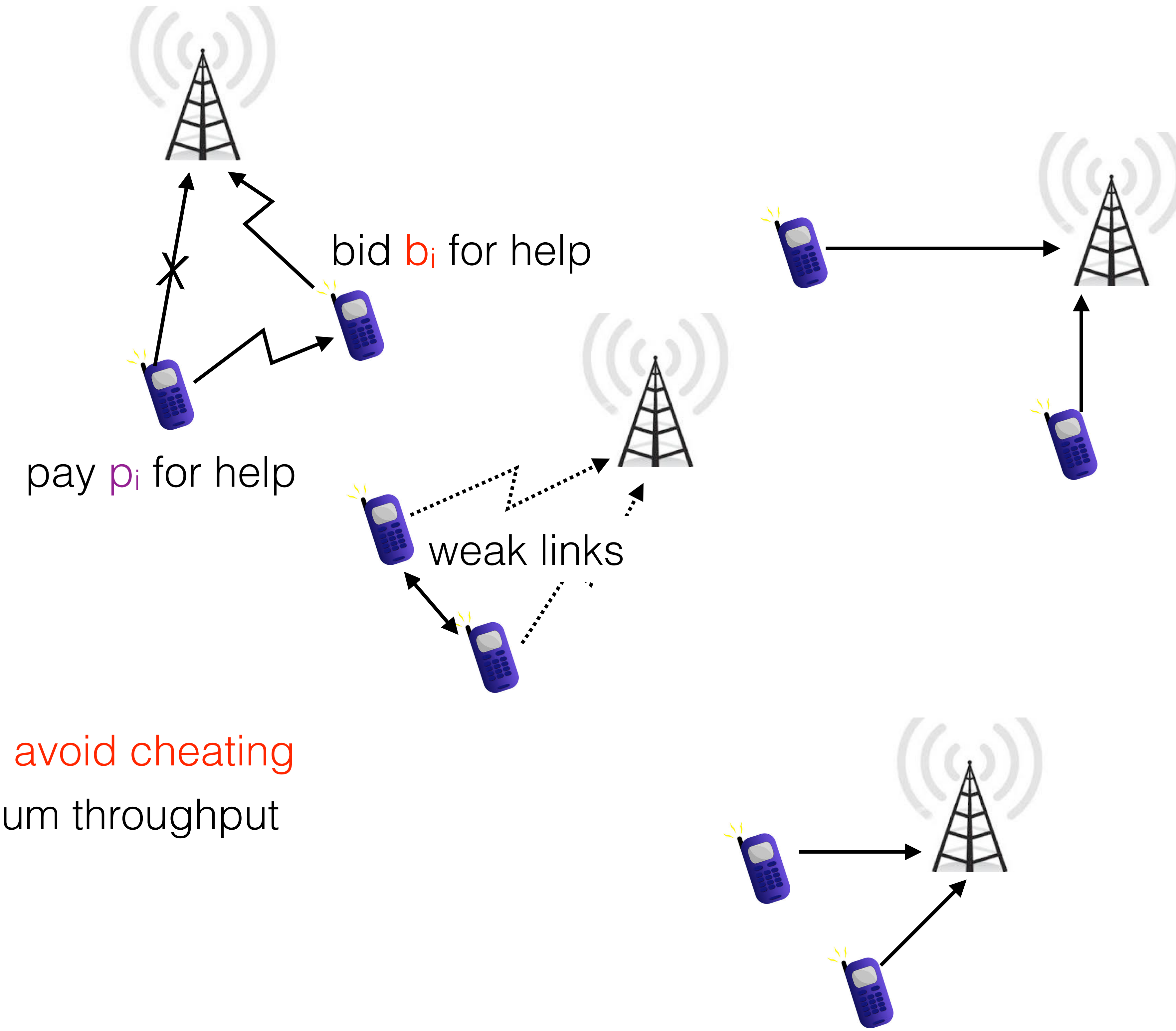




mechanism to avoid cheating

ensure maximum throughput

Device-2-Device Communication



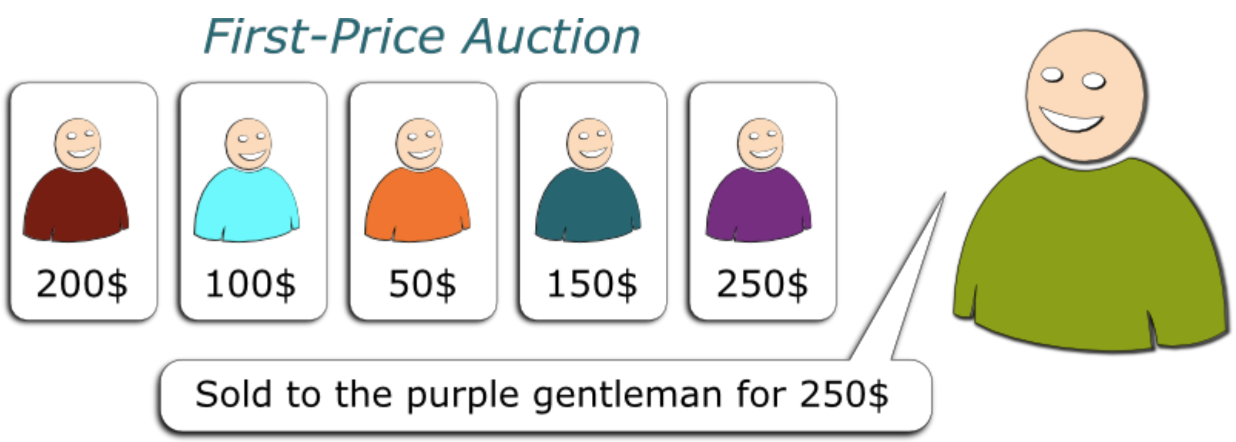
mechanism to avoid cheating

ensure maximum throughput

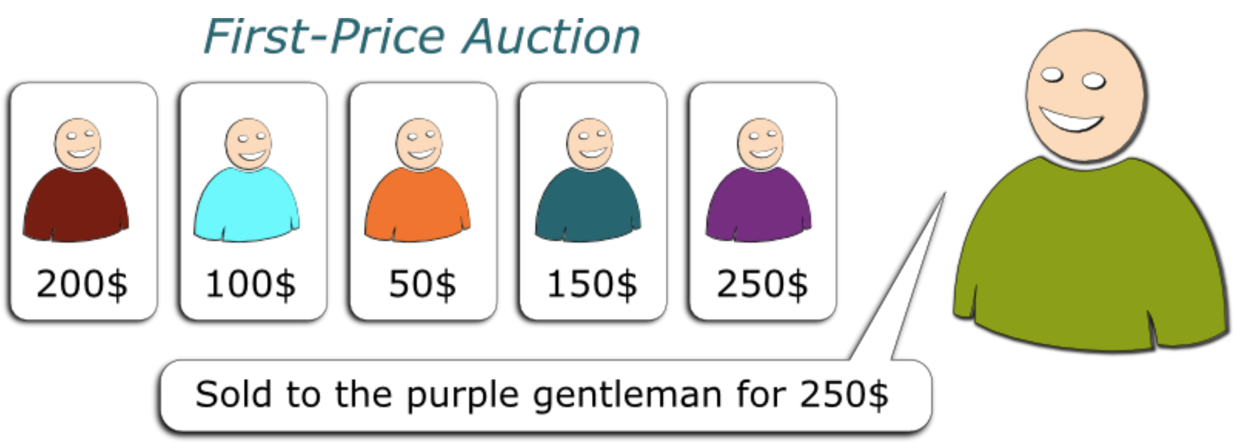
Find ONLINE optimal helper association and incentive rule that is truthful

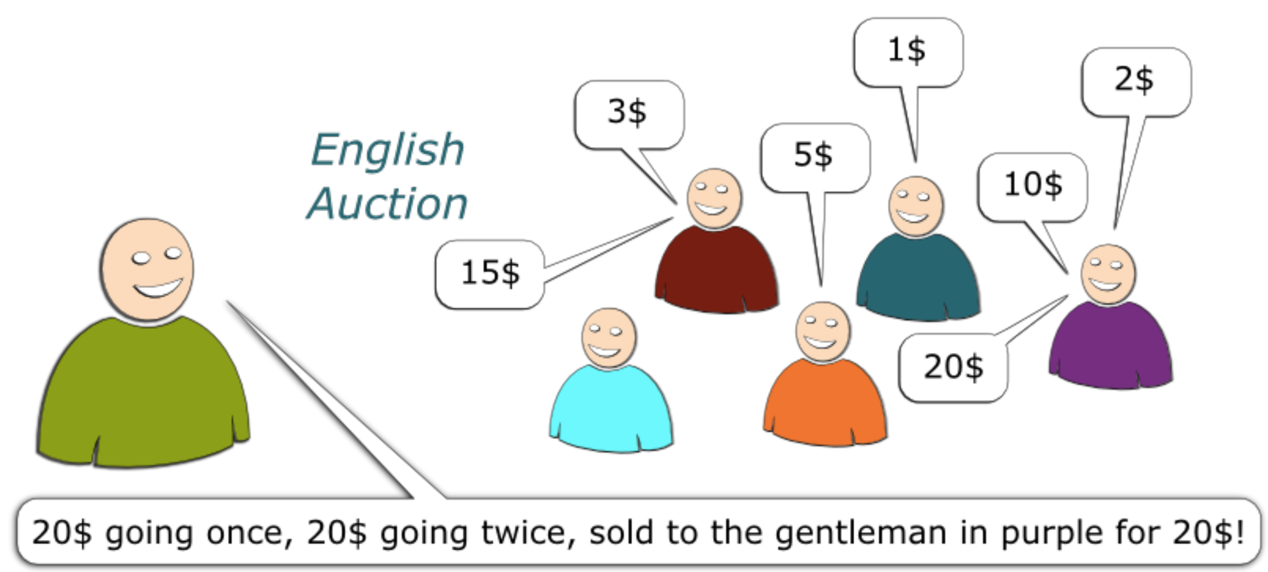
Truthful Auction

Truthful Auction



Truthful Auction



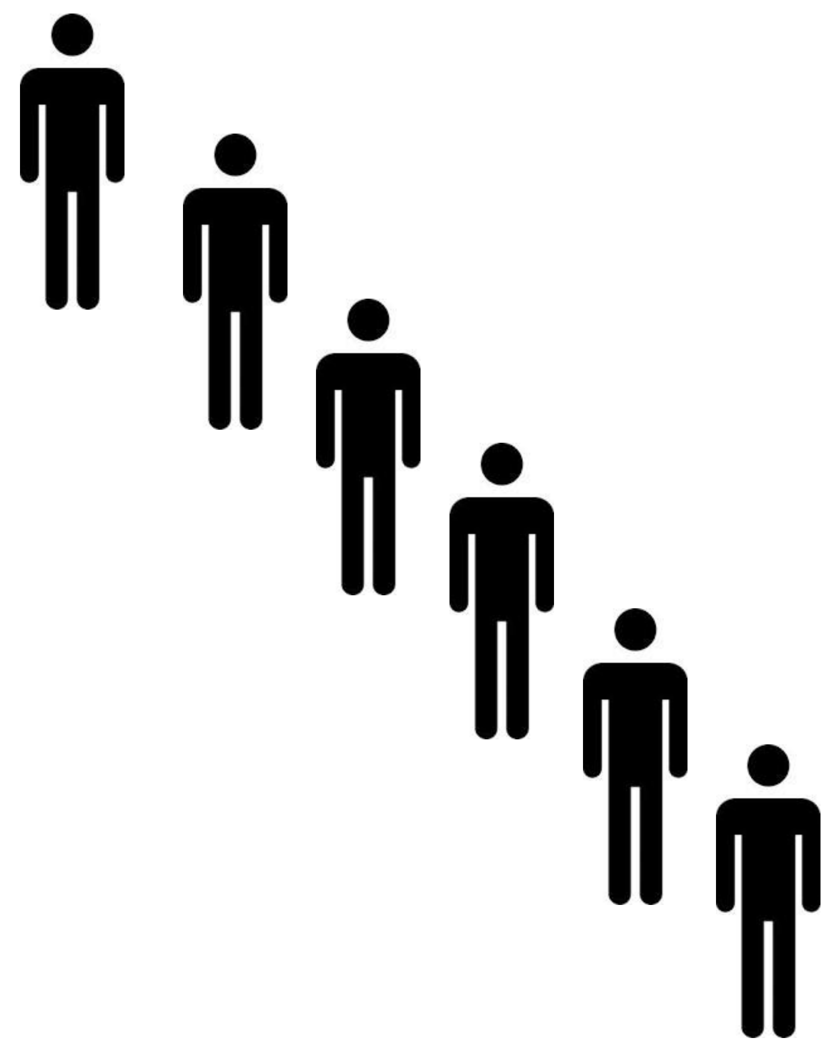


small detour

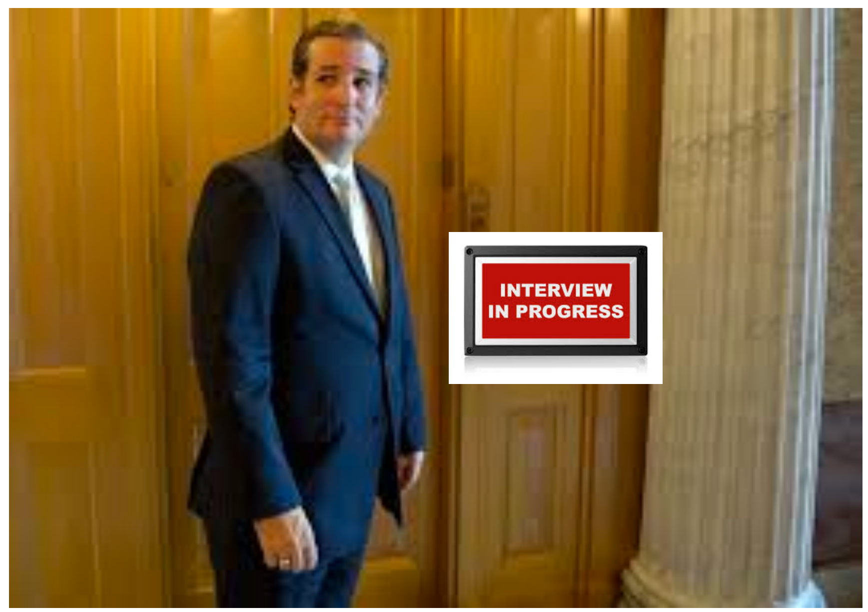

The mathematics behind a perfect match

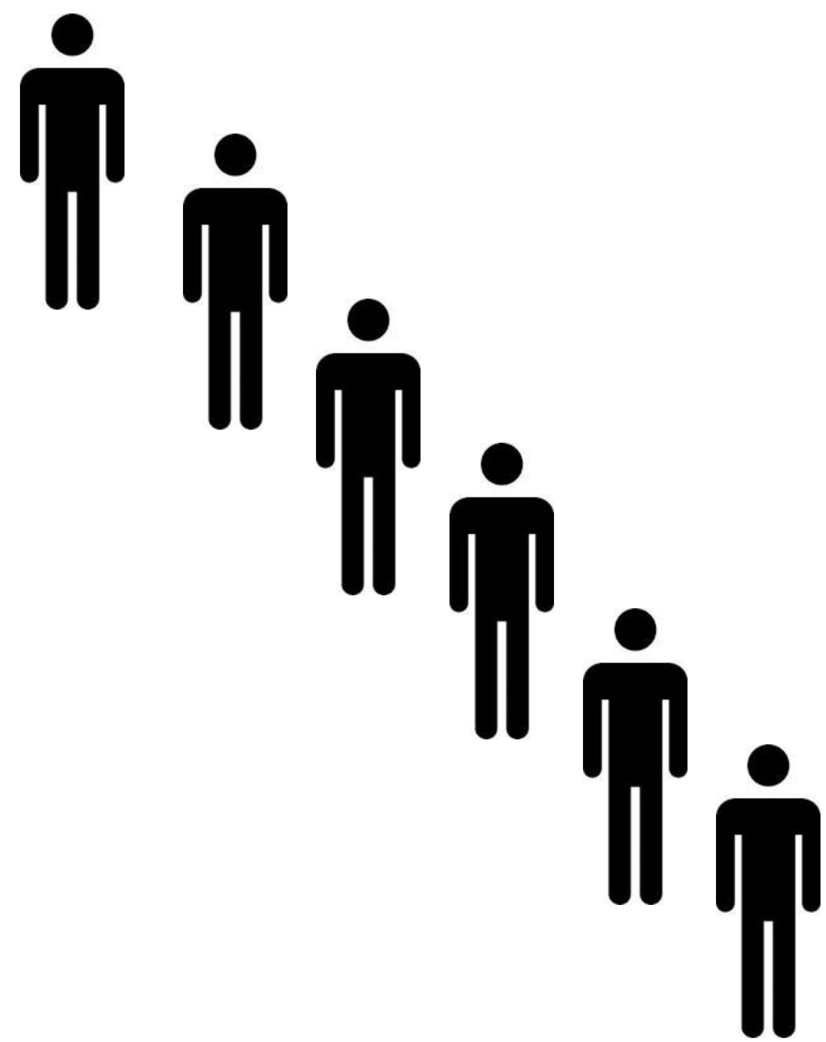
how many to date before committing !

Hiring impatient staff



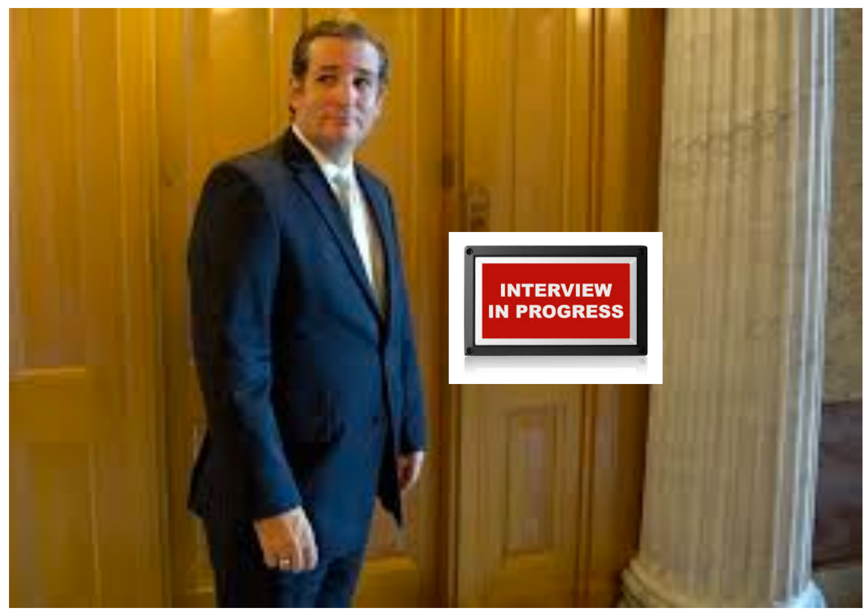
Hiring impatient staff

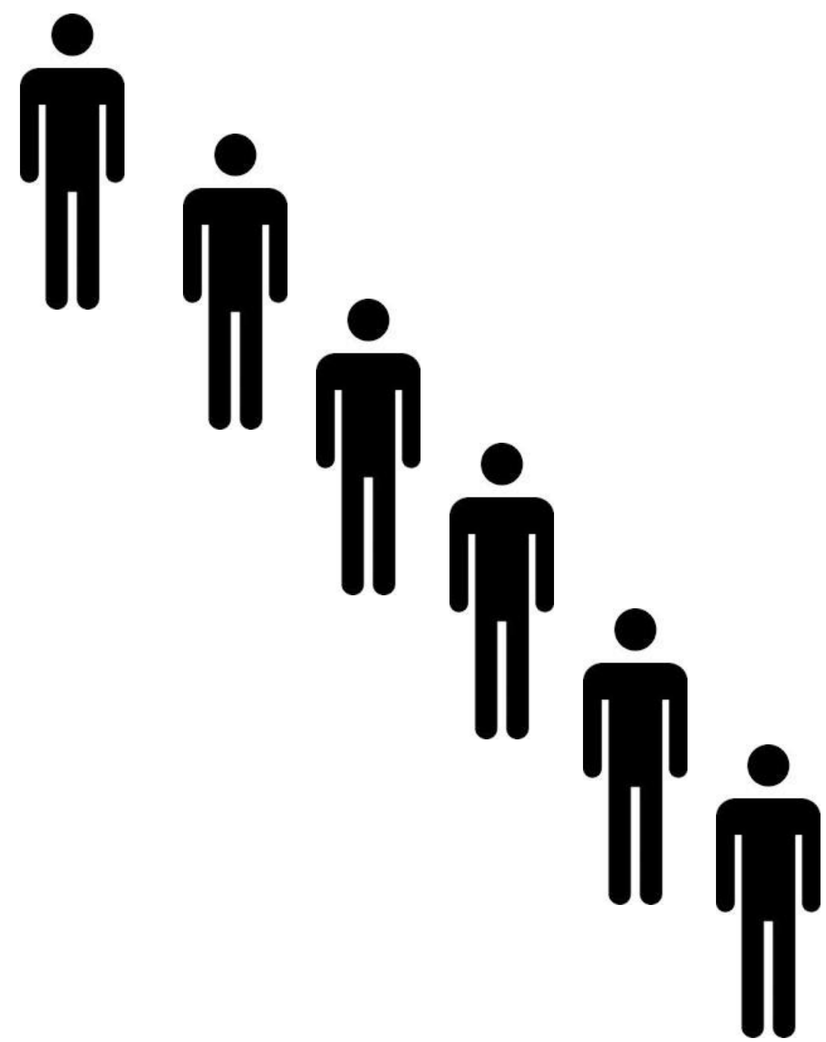




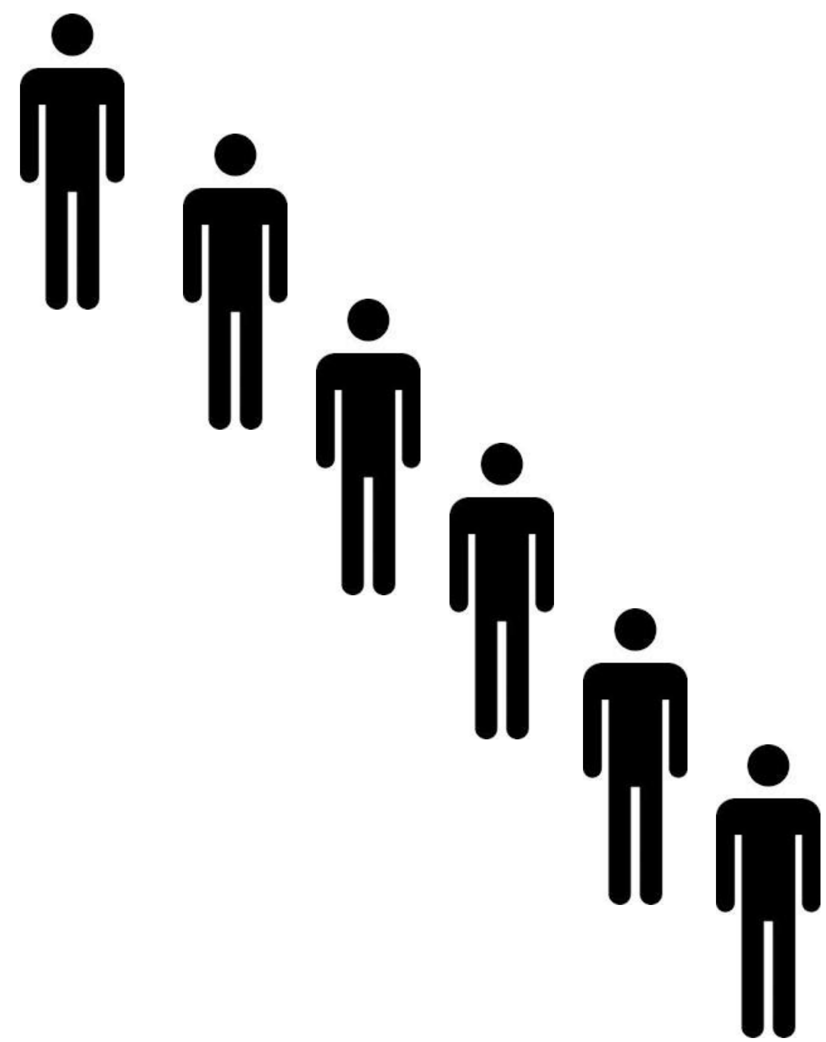
Hiring impatient staff

prob. of choosing best candidate is 1/n





Hiring staff - not adversarial



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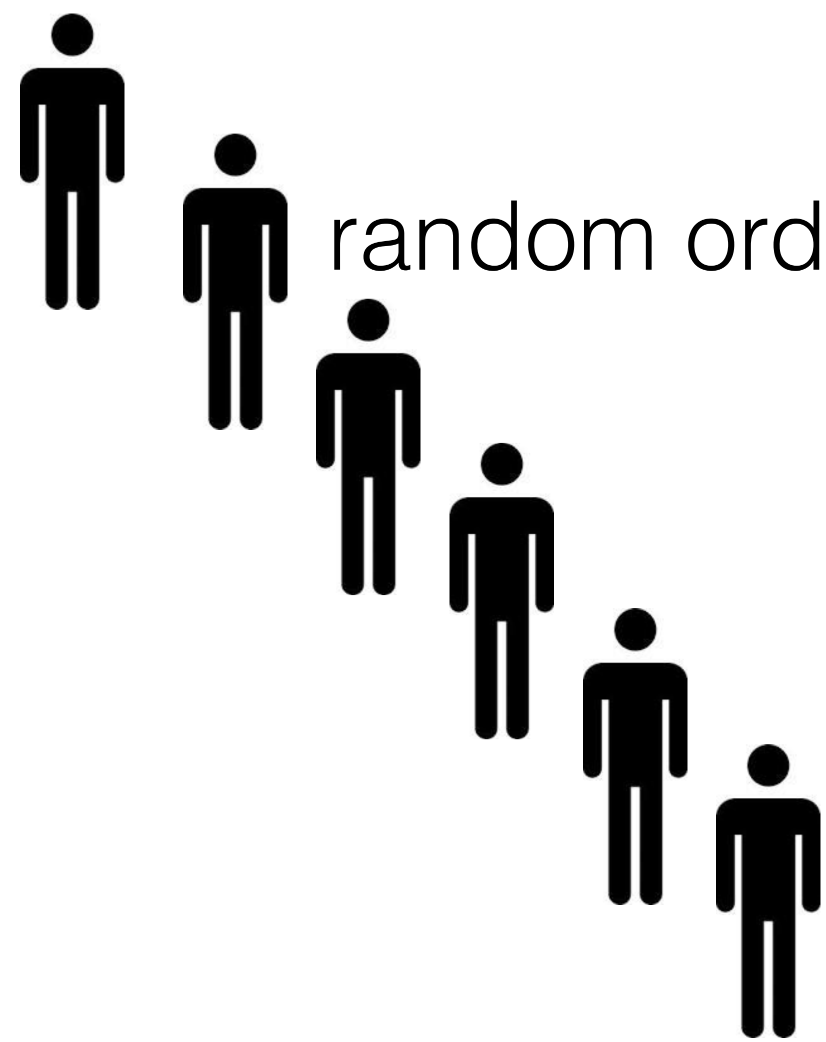
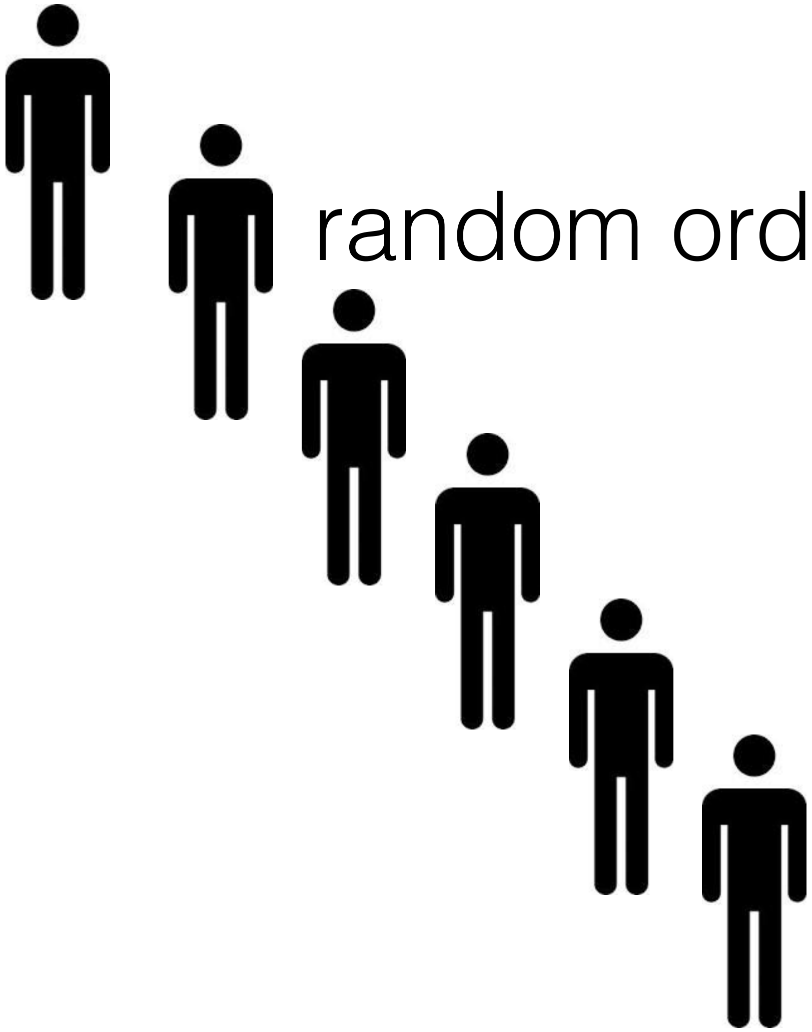


random order

Hiring staff - not adversarial

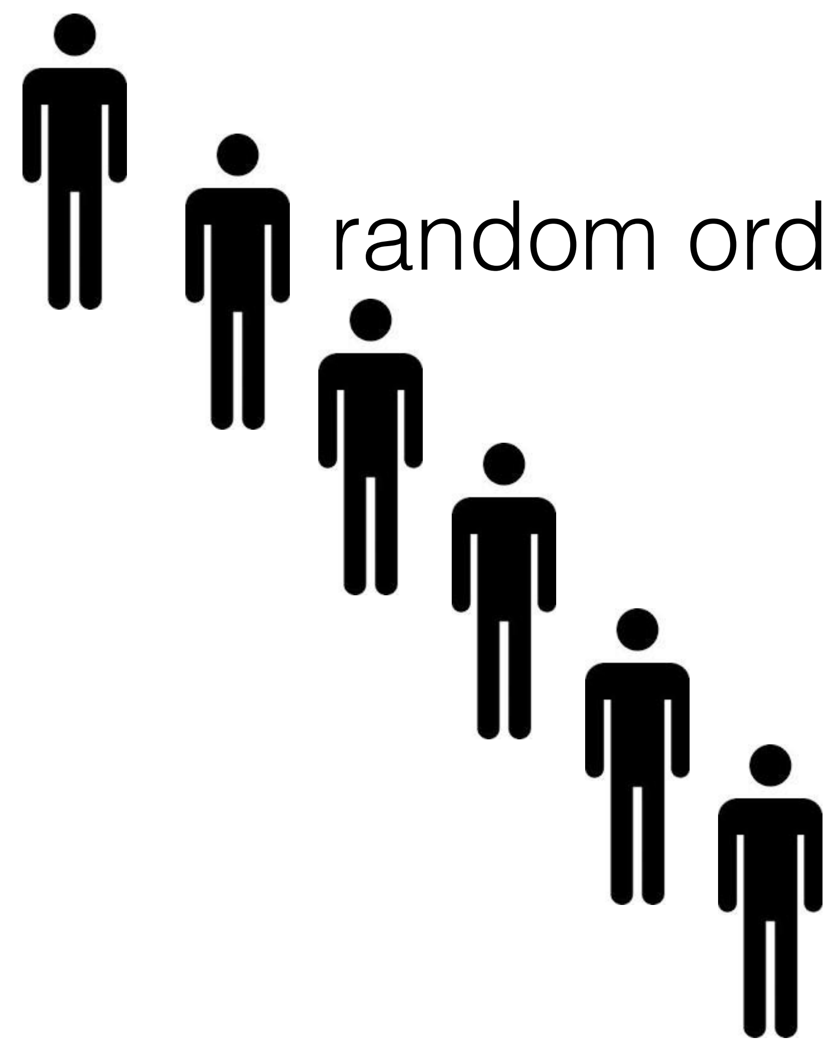


random order

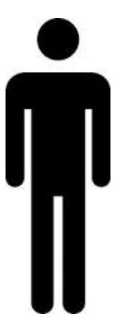
sampling phase

first half

Hiring staff - not adversarial



random order



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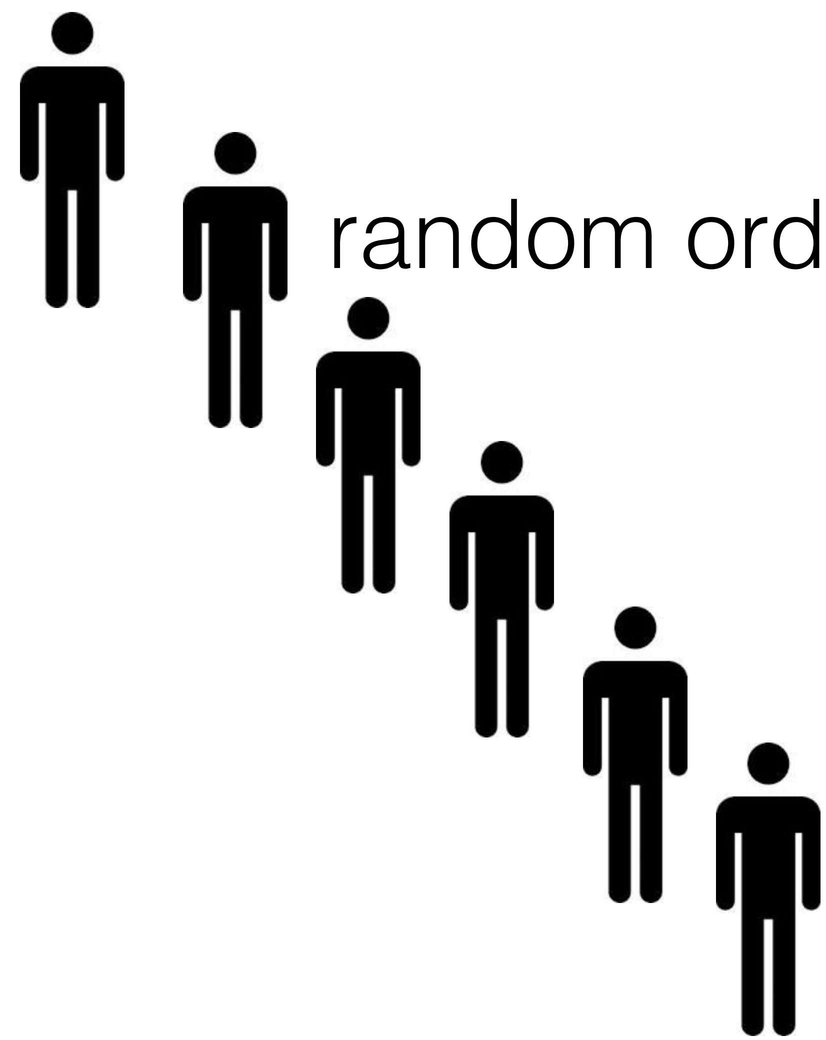
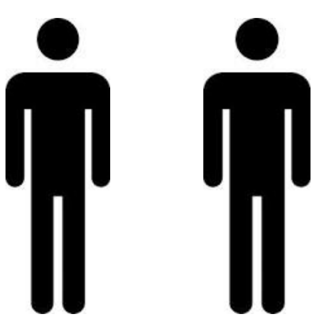


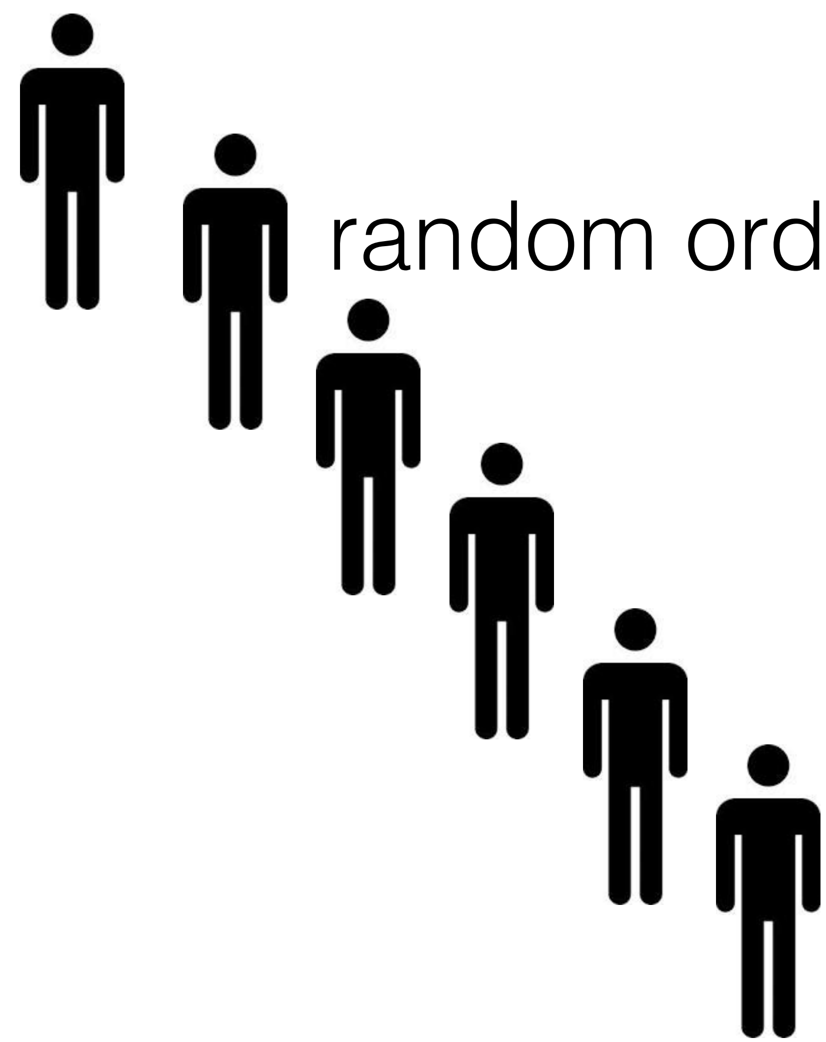
random order



sampling phase

first half

Hiring staff - not adversarial



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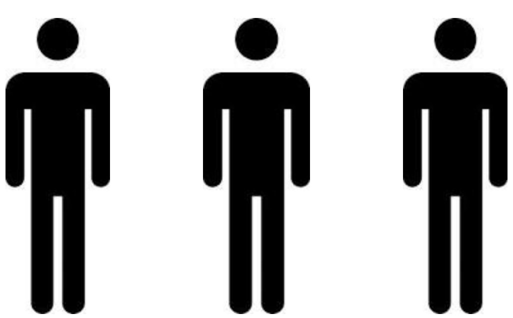

sampling phase

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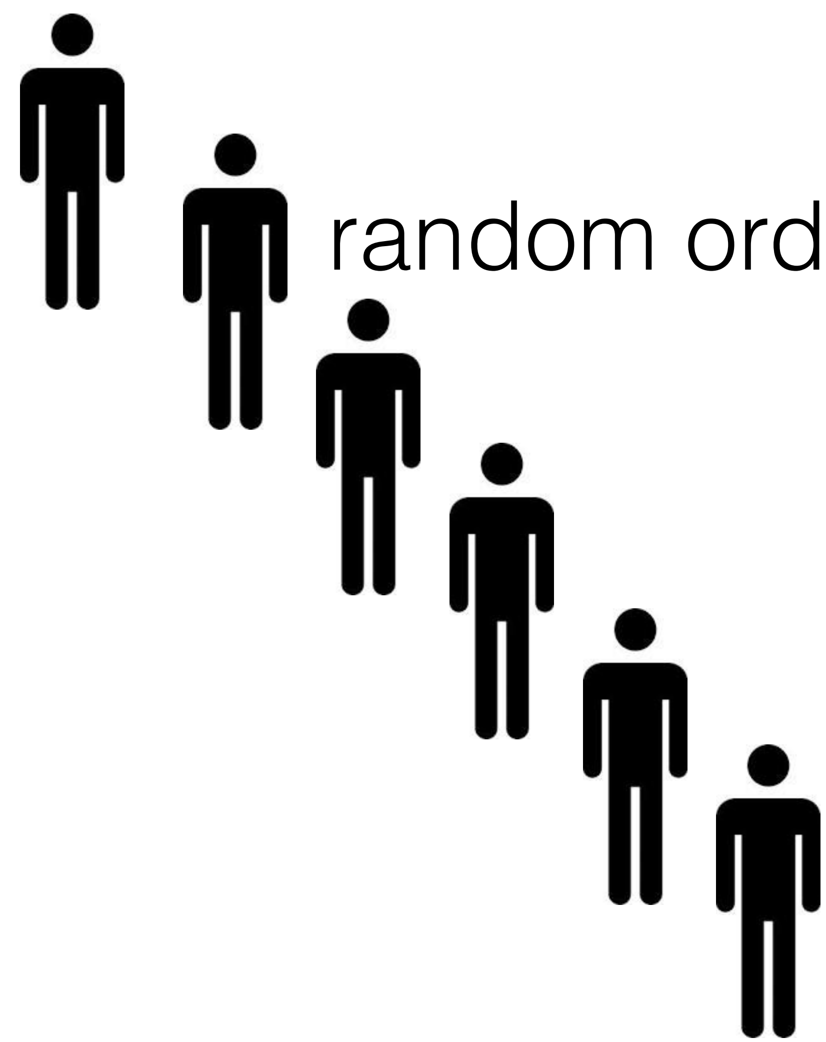
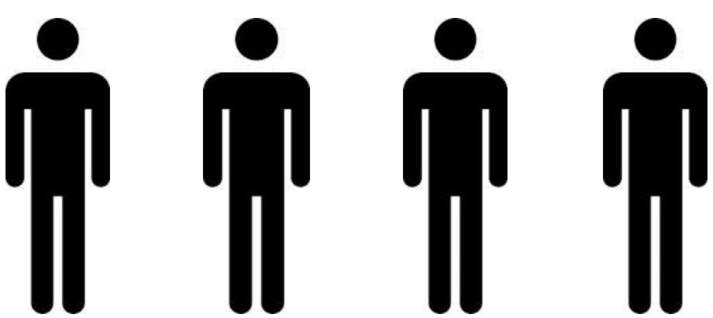
Hiring staff - not adversarial



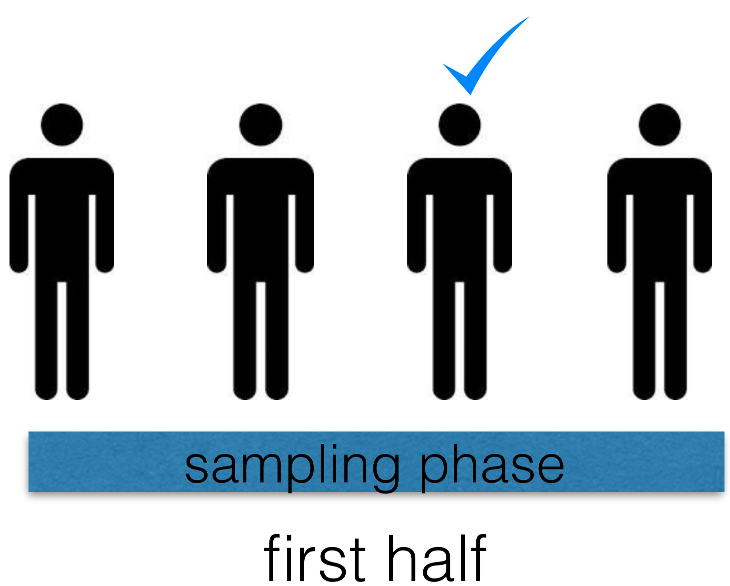
random order

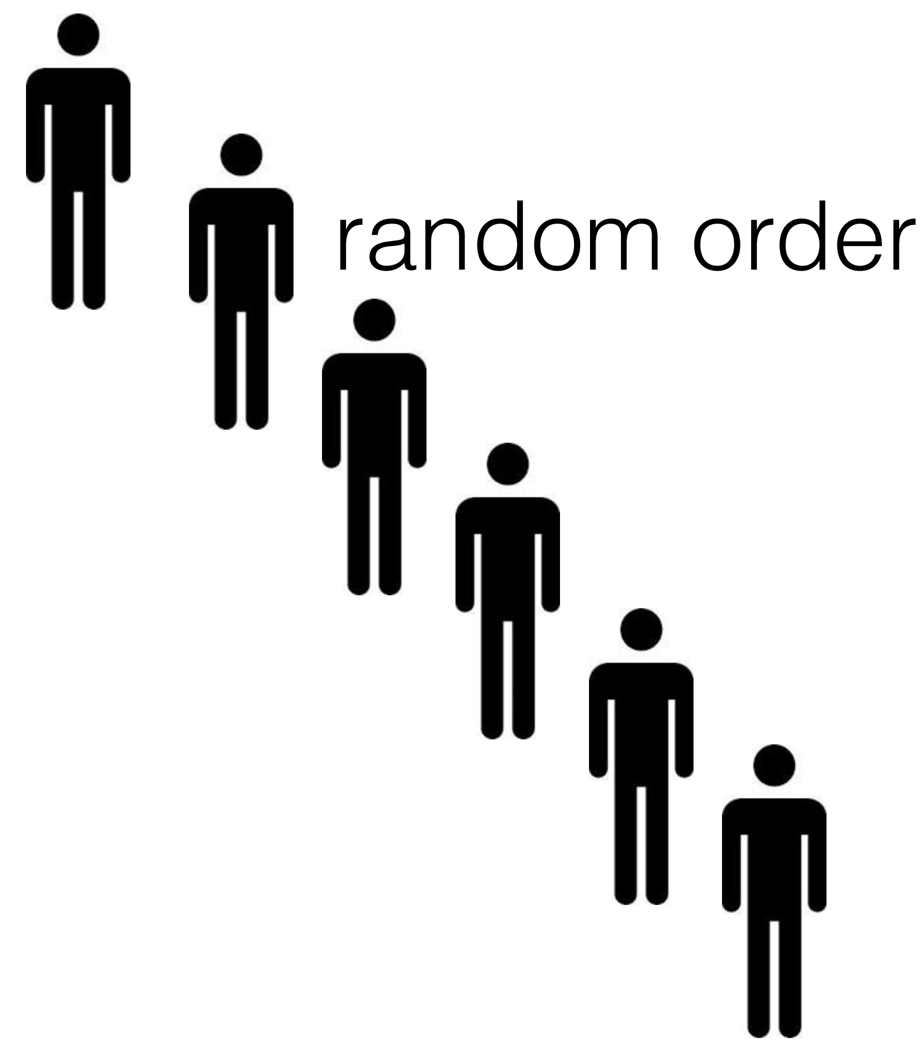


sampling phase

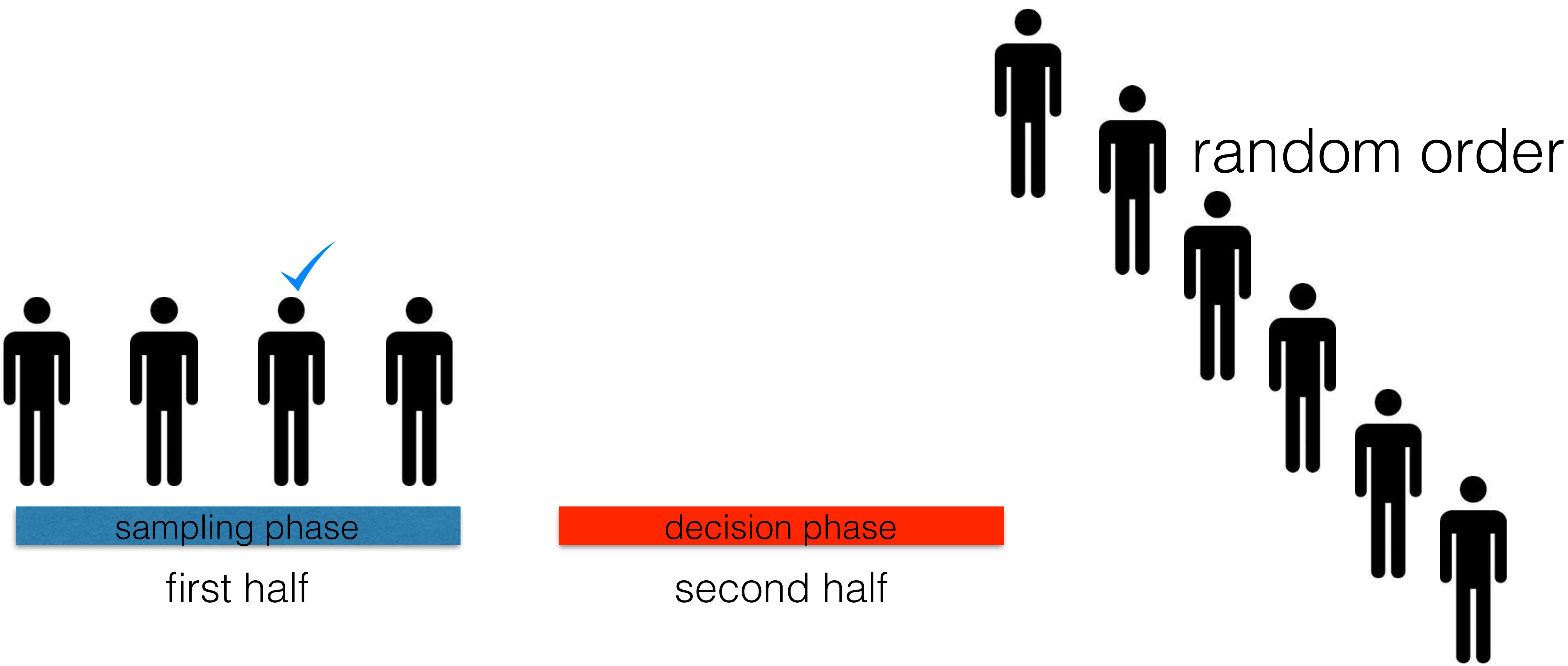
first half

Hiring staff - not adversarial





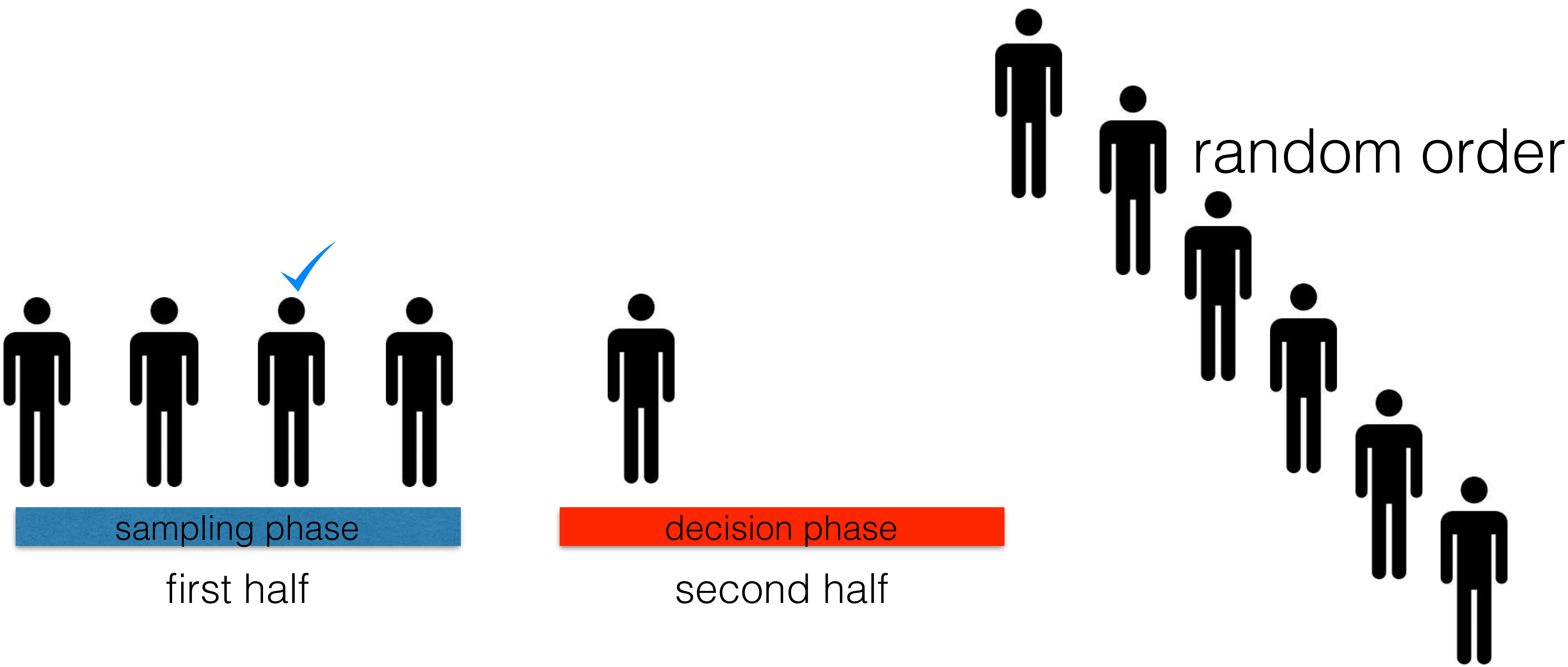
Hiring staff - not adversarial



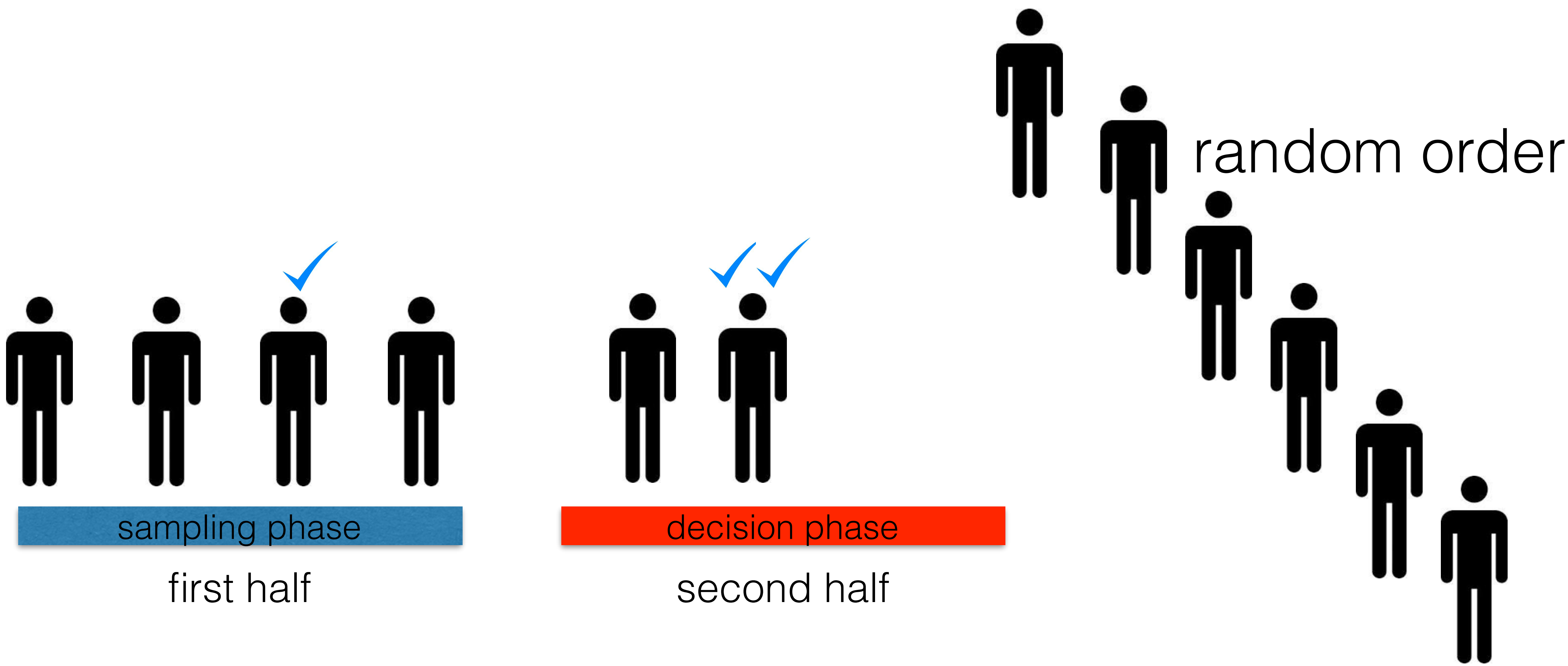
first half

second half

Hiring staff - not adversarial

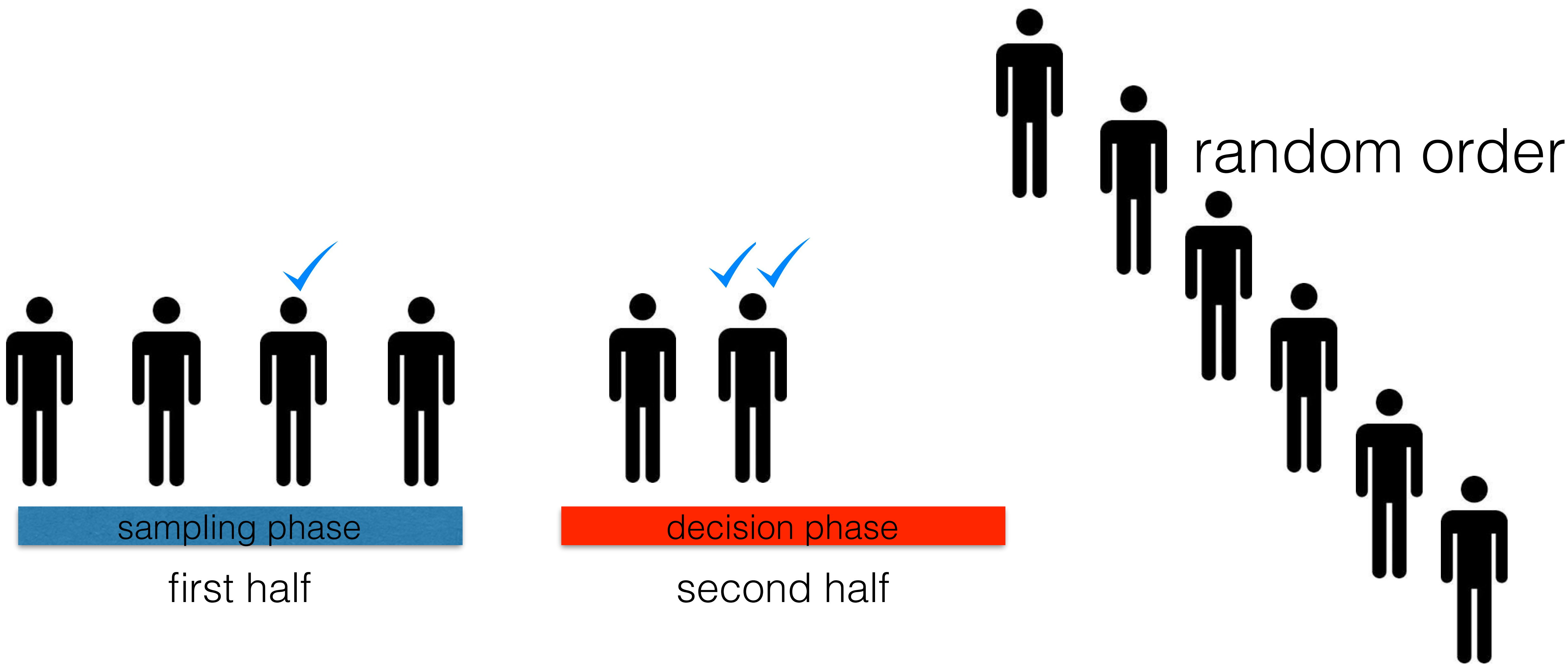


Hiring staff - not adversarial

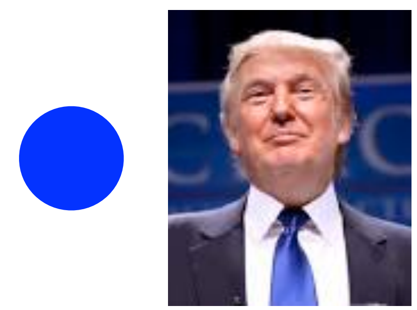


Hiring staff - not adversarial

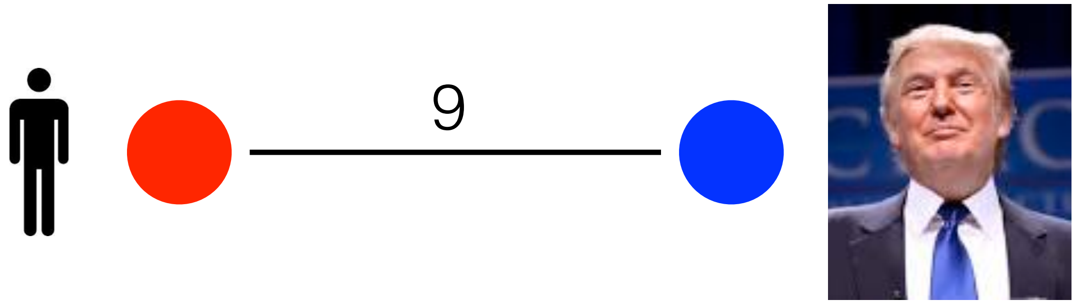
Success with prob > 1/4



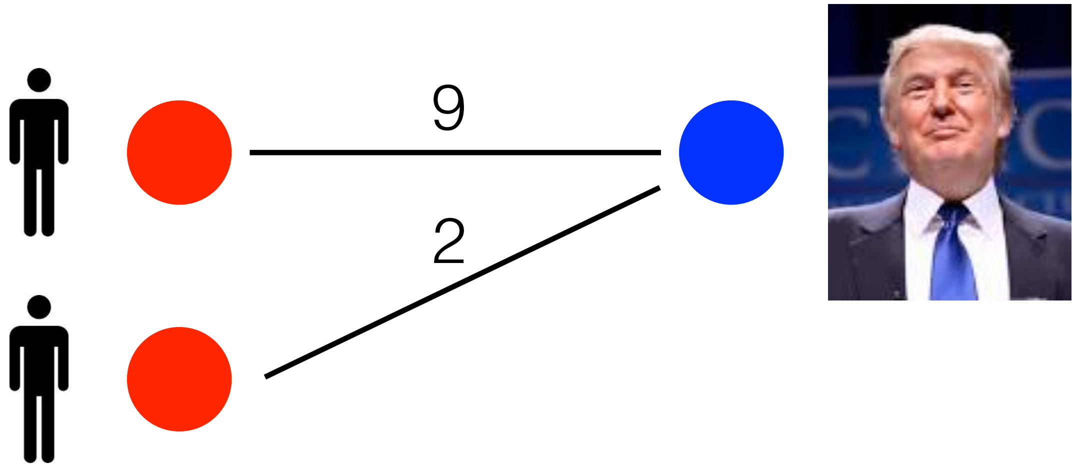
Actually Matching



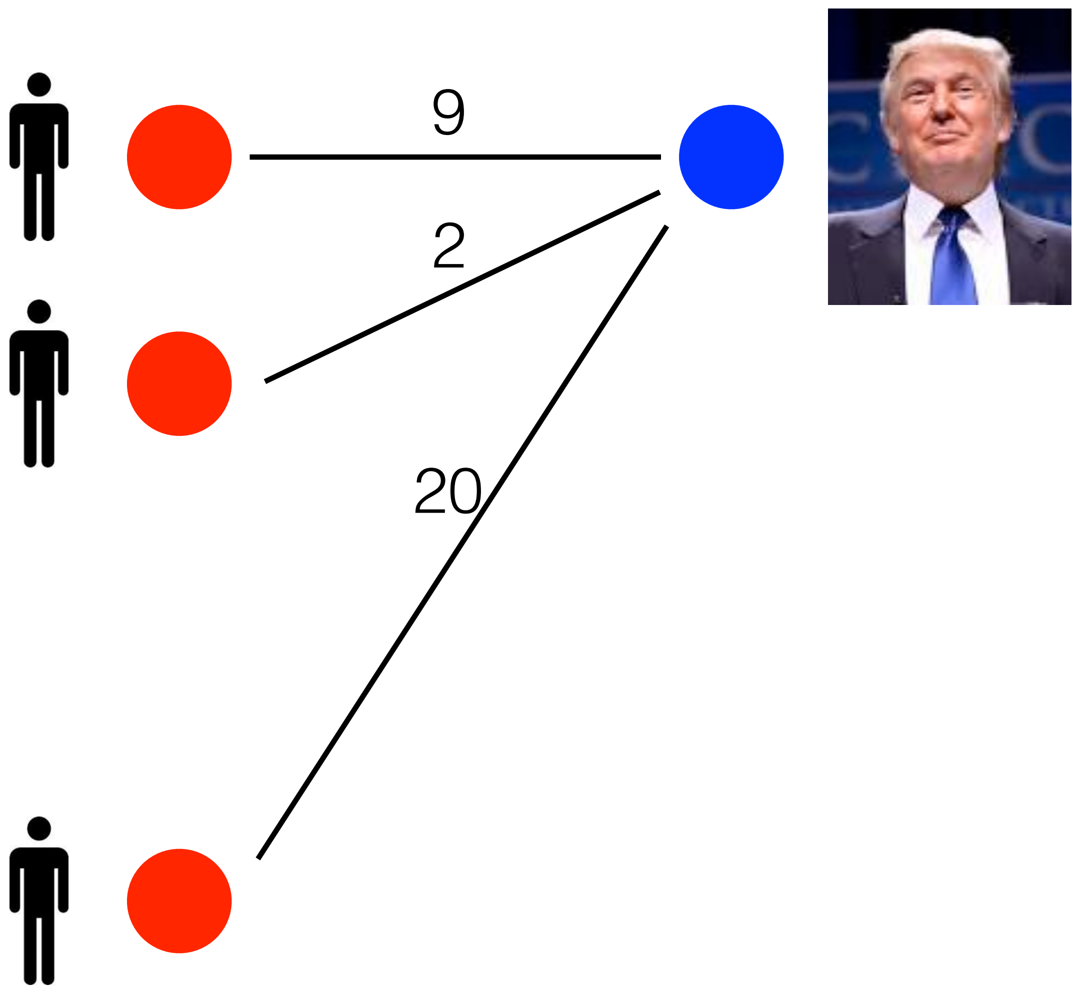
Actually Matching



Actually Matching

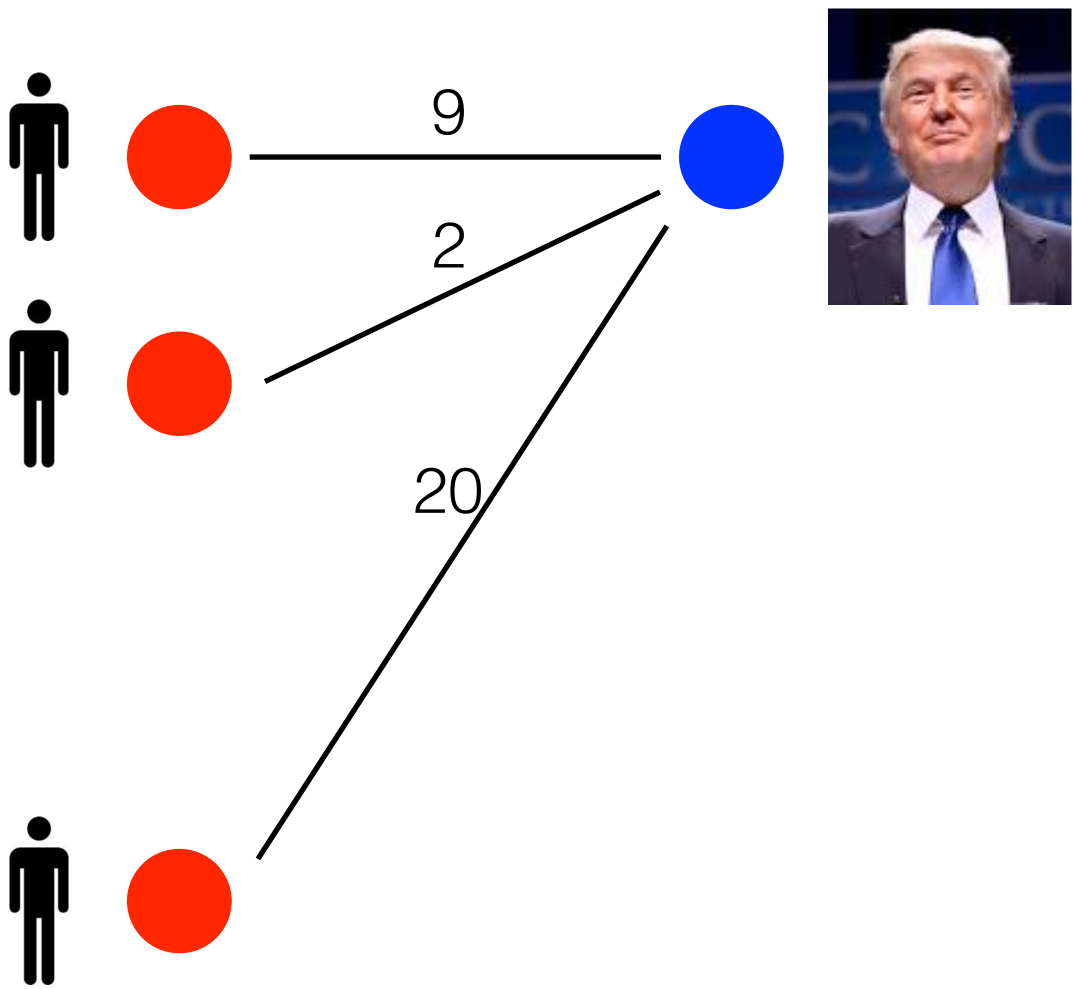


Actually Matching



Actually Matching

accept the edge with the largest weight instantaneously

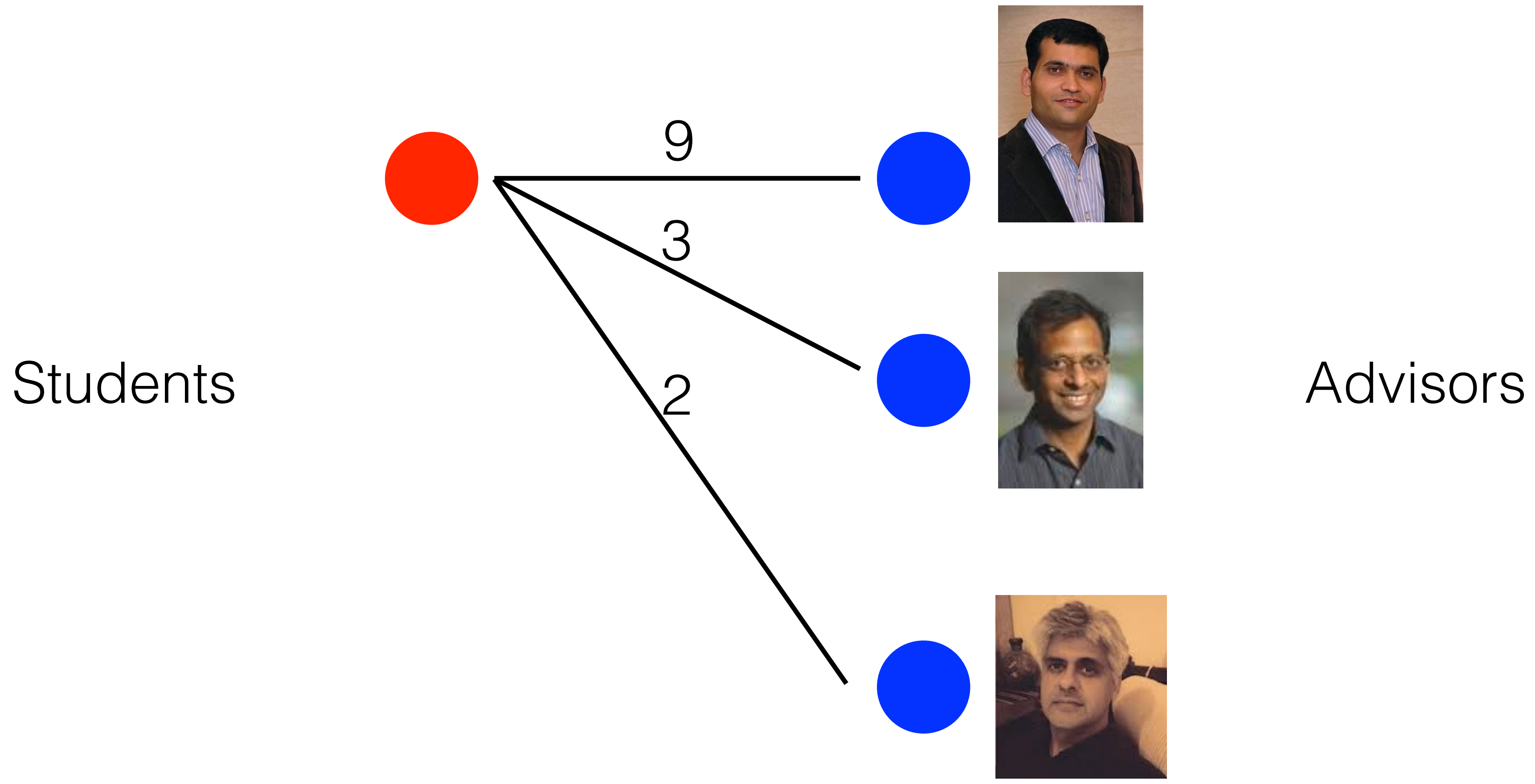


Natural Generalization

Students

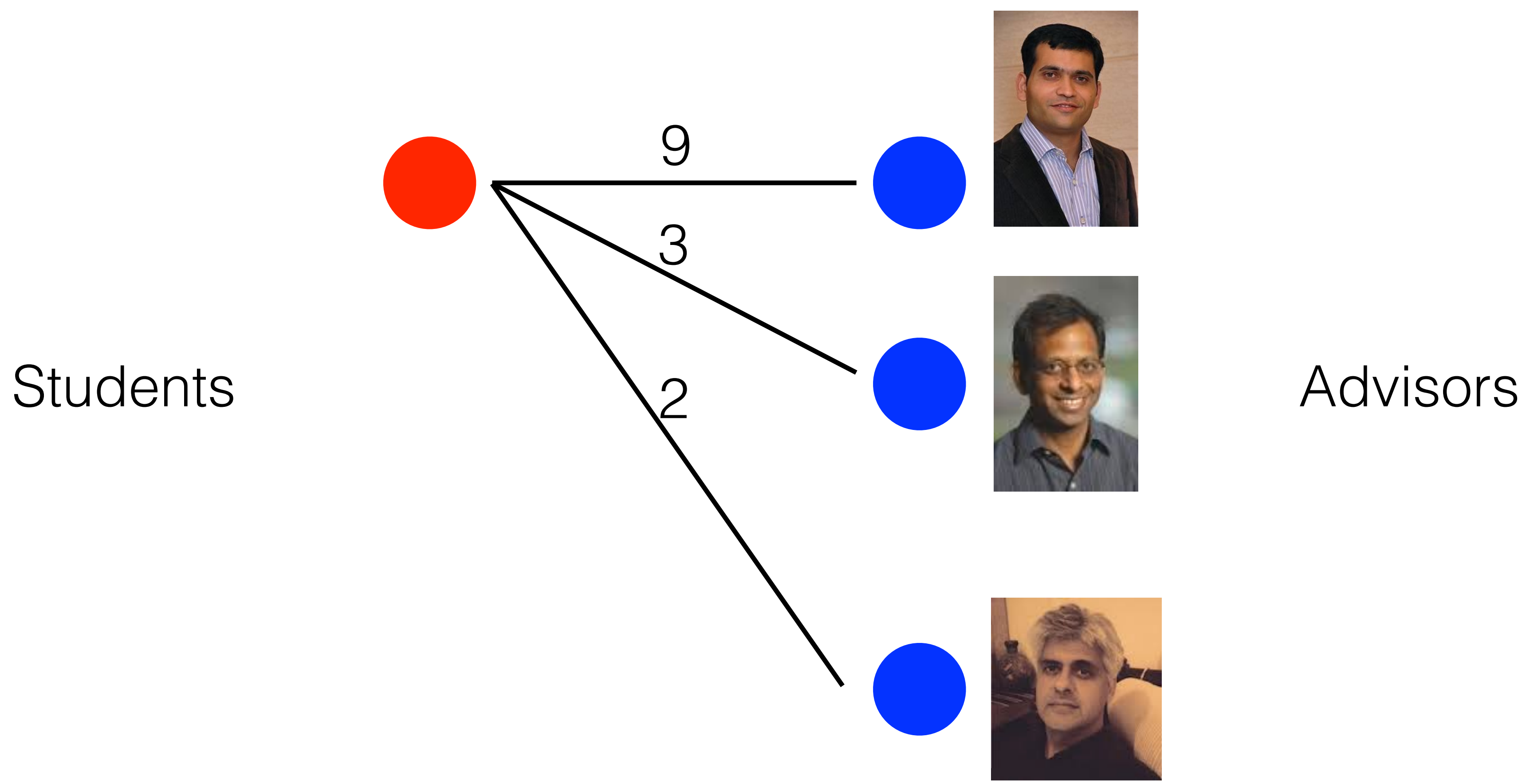
Advisors

Natural Generalization



Natural Generalization

each advisor gets at most one student



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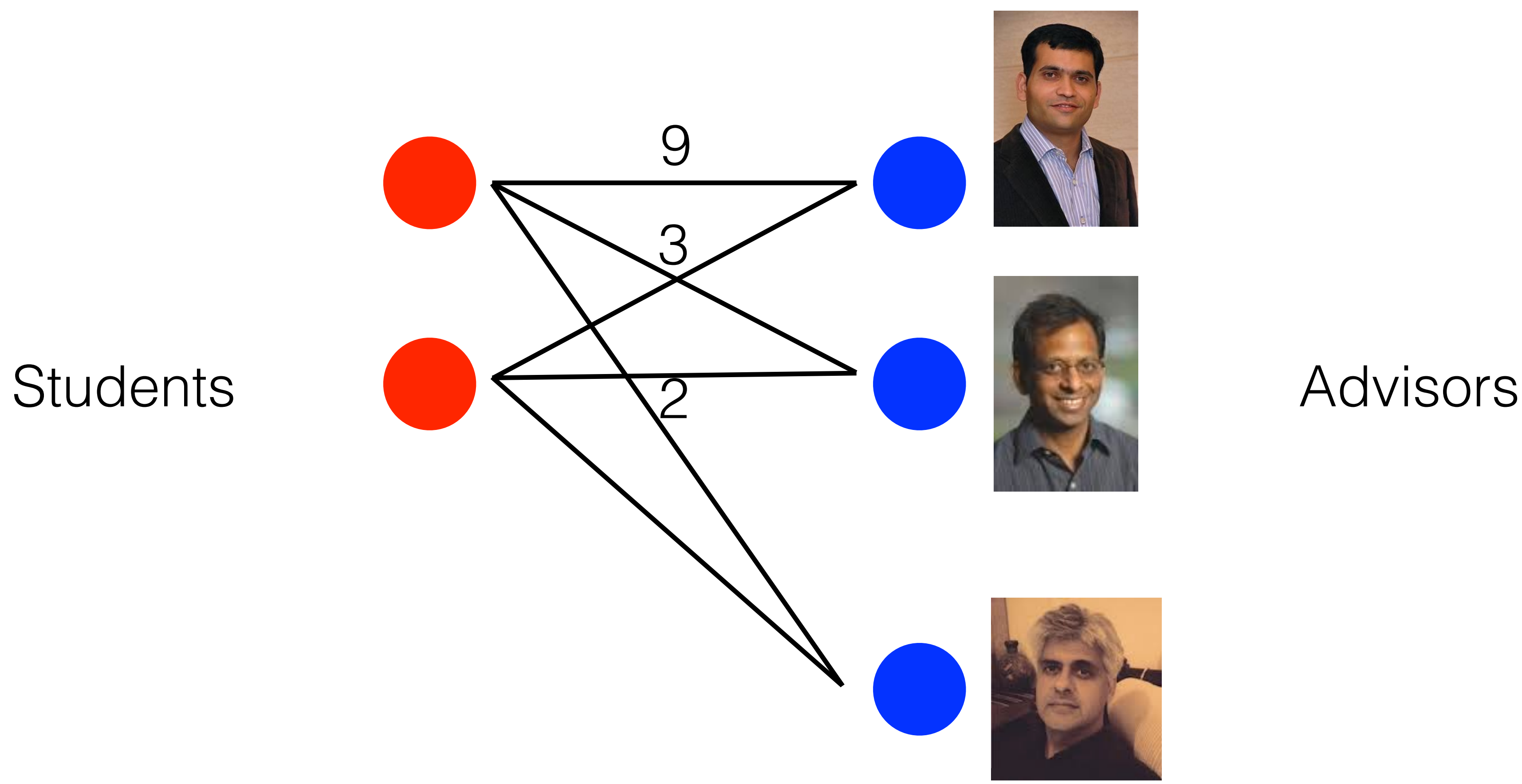

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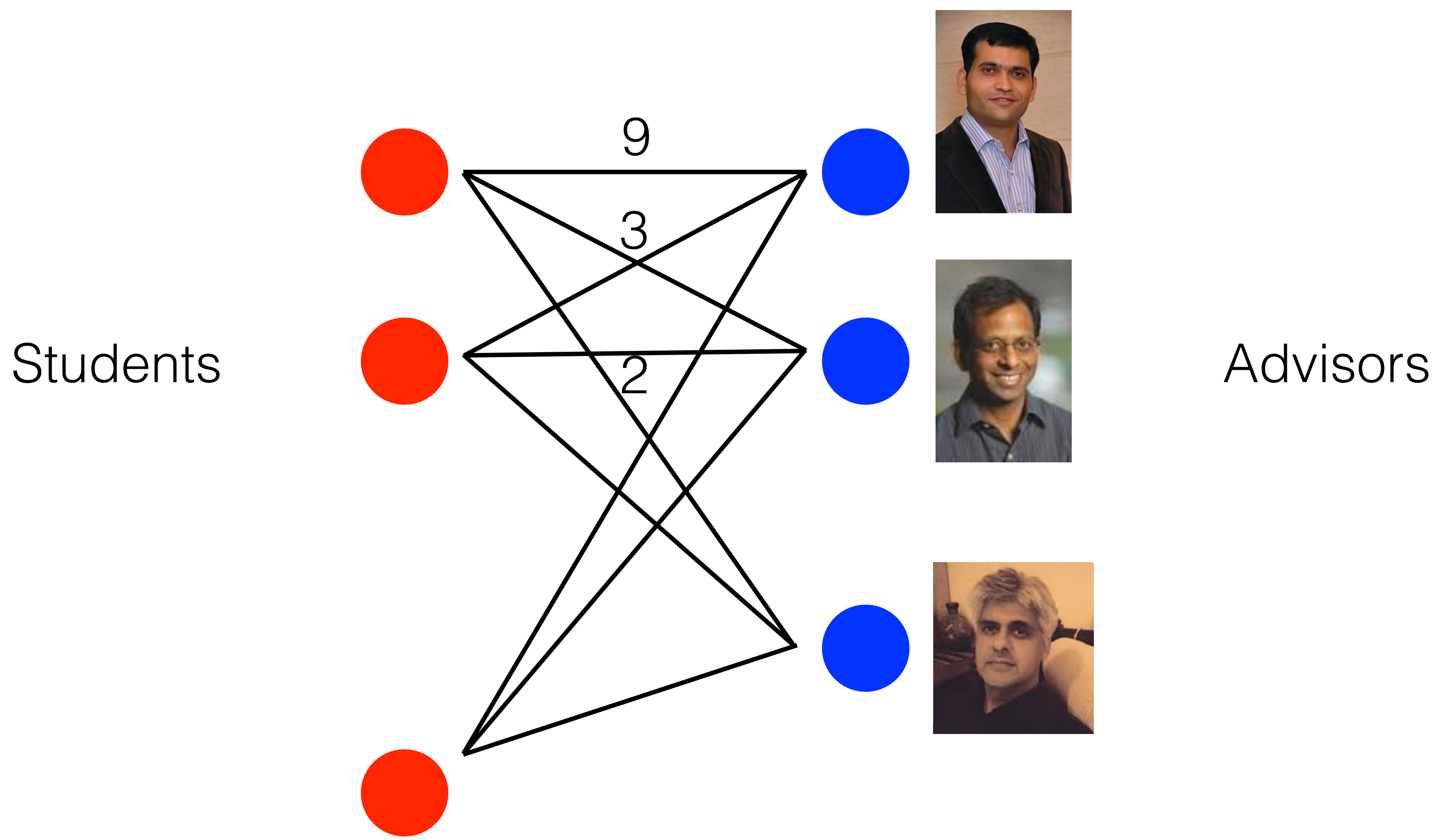
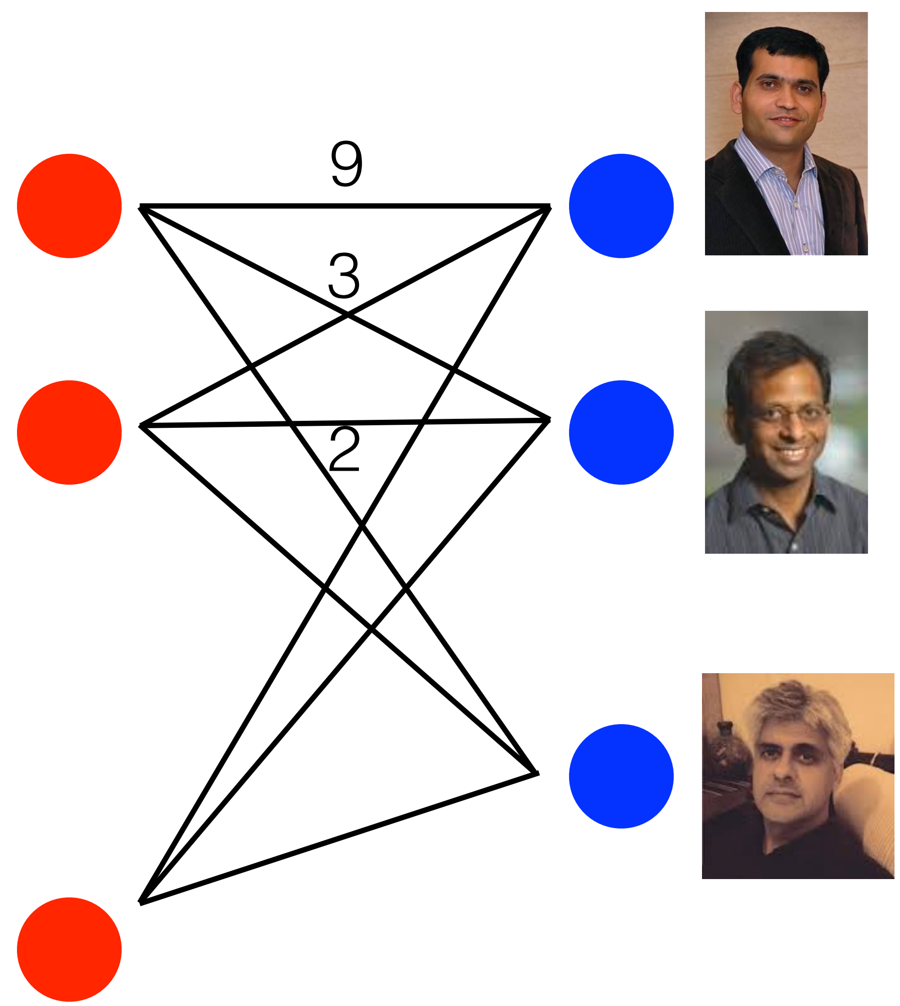


Natural Generalization

each advisor gets at most one student - allocation made by **DEAN**

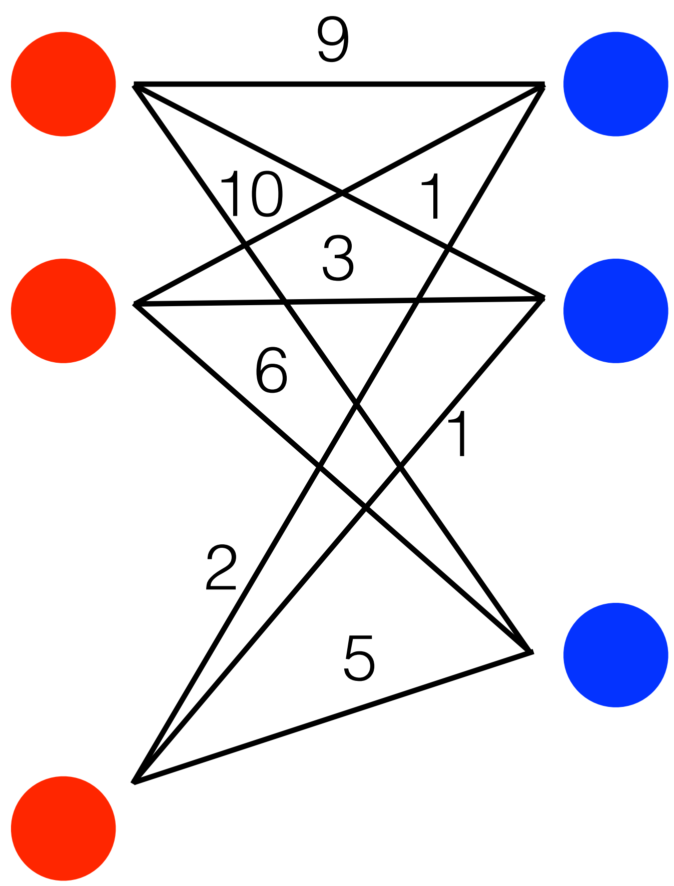
Students



Advisors

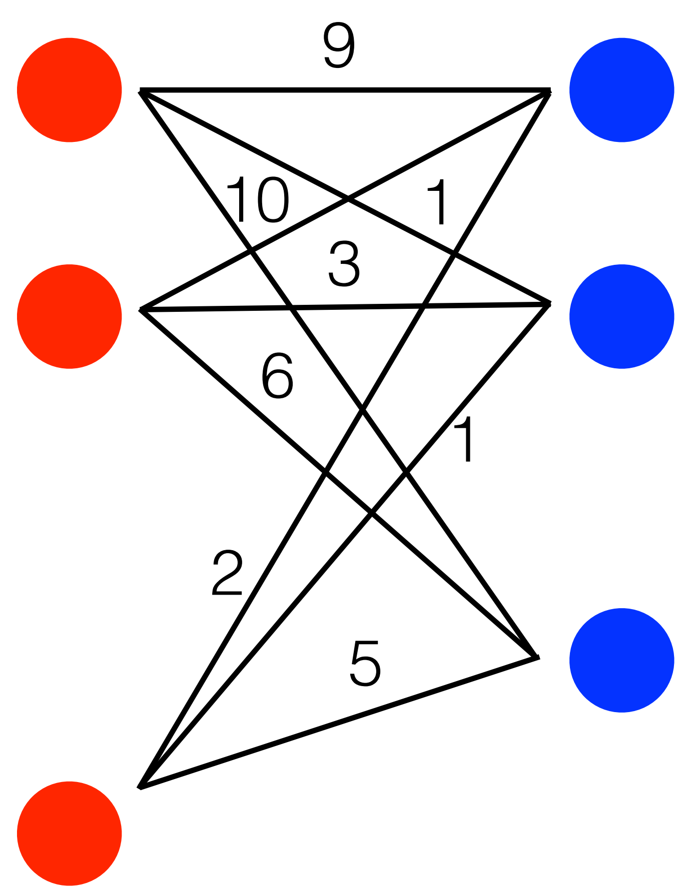
Objective: Matching with largest sum weight

Example



Example

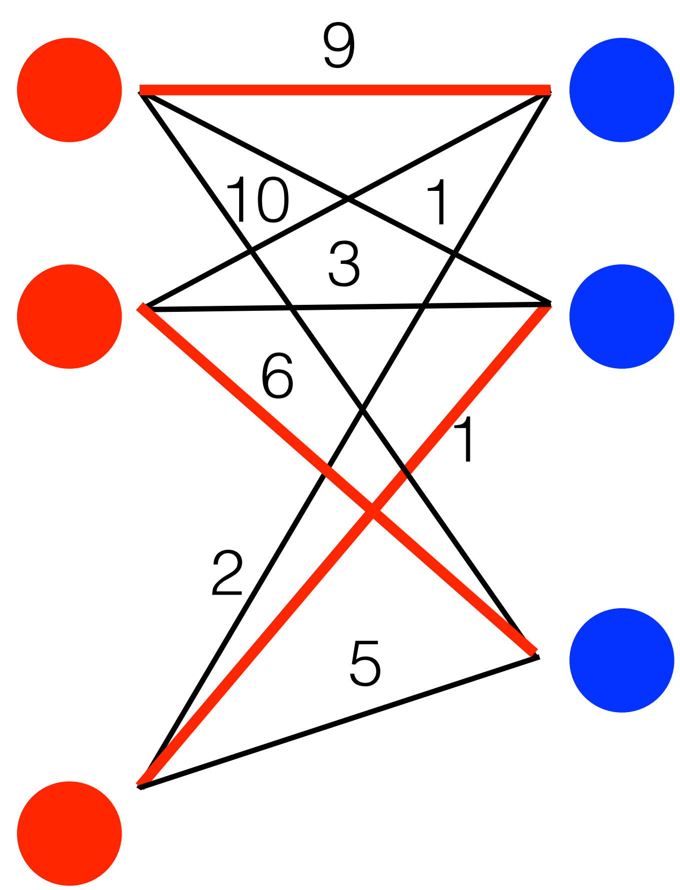
at most one accepted edge



Objective: Matching with largest sum weight

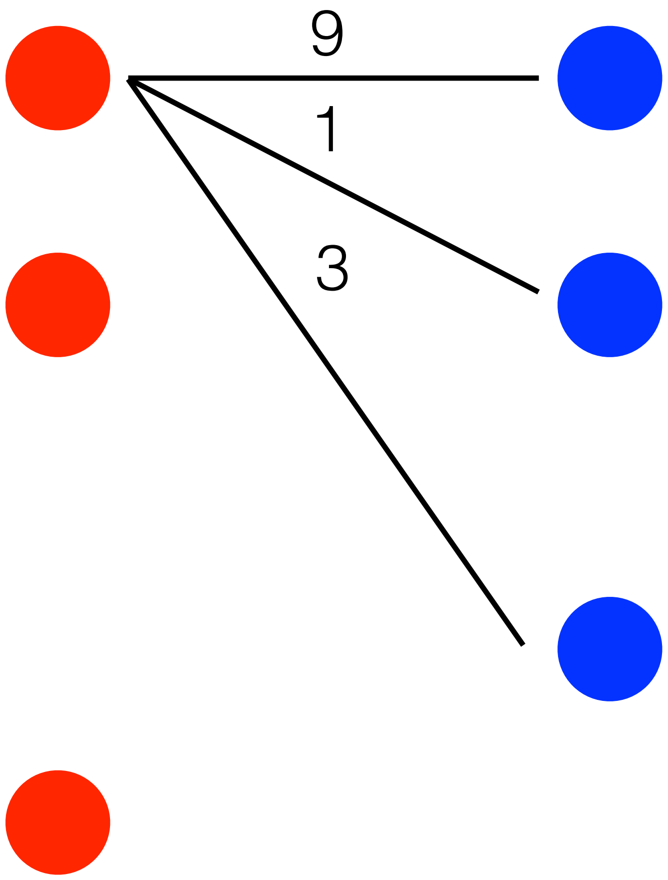
Example

at most one accepted edge



Objective: Matching with largest sum weight

How to solve this ONLINE



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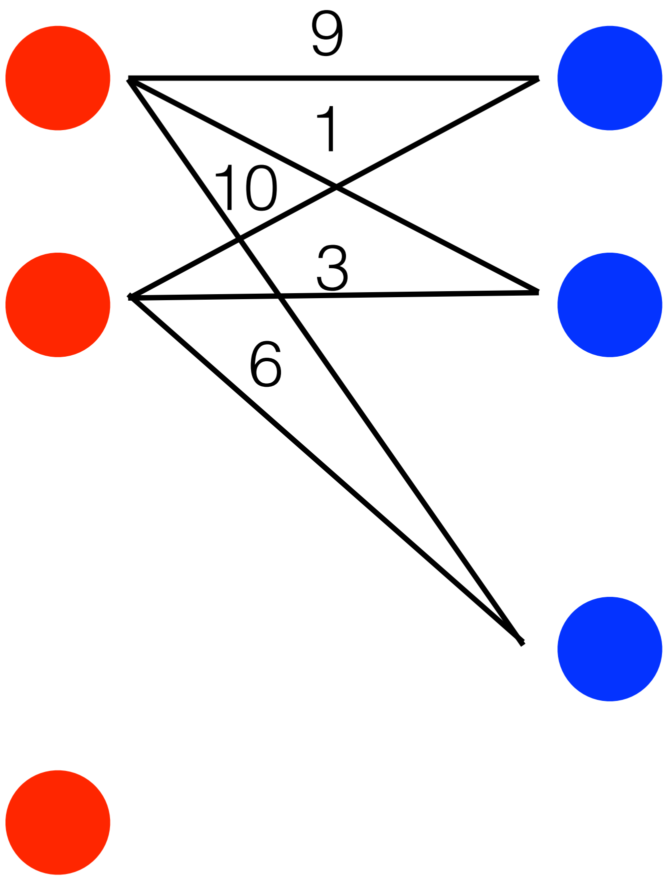
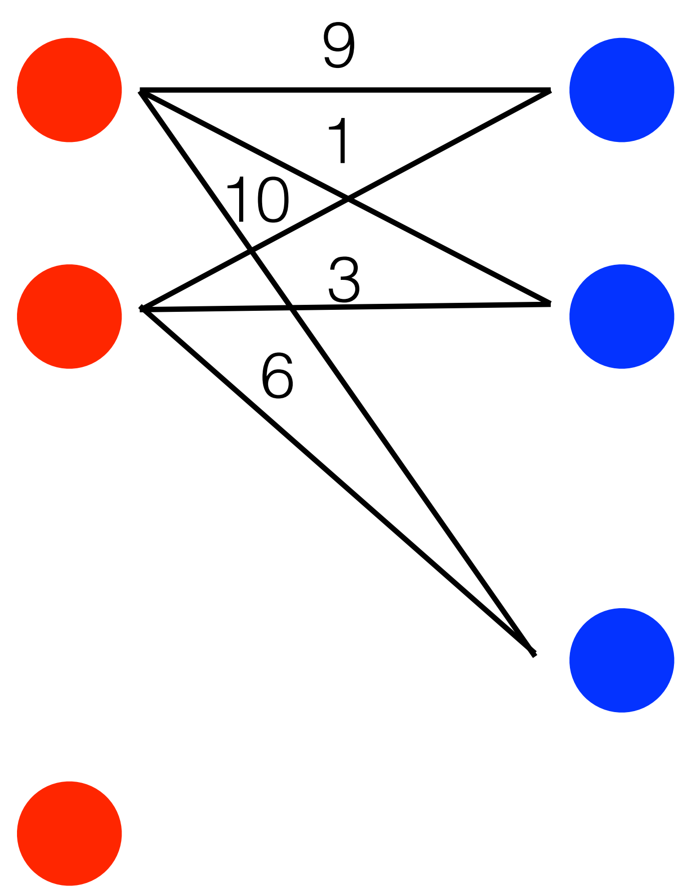


How to solve this ONLINE

Sampling idea as before



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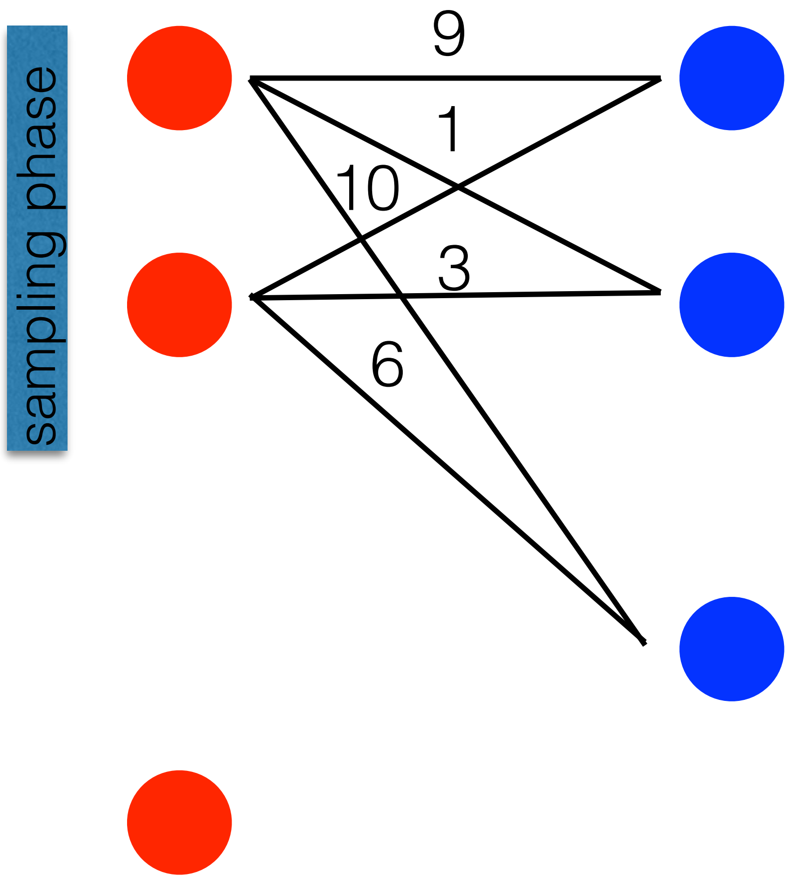
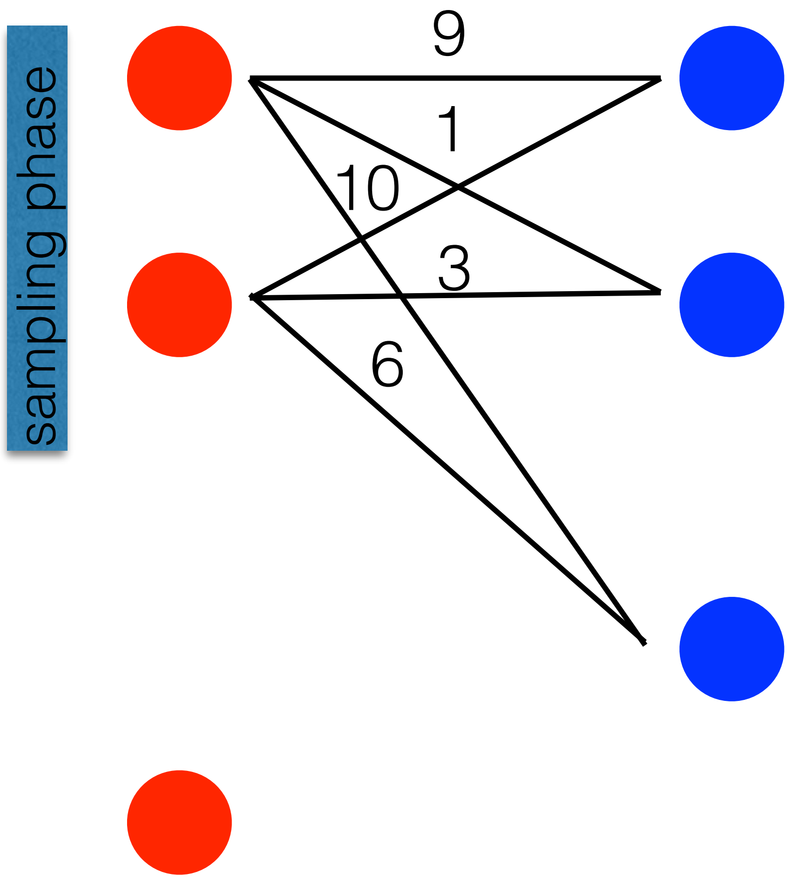


How to solve this ONLINE

Sampling idea as before

Find best matching



How to solve this ONLINE

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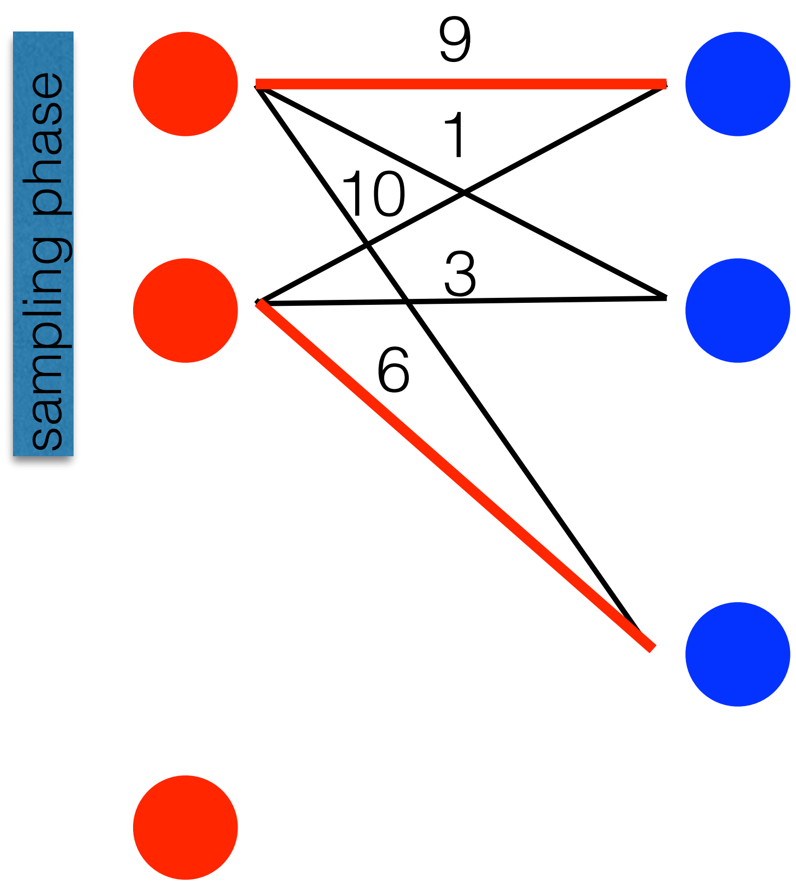

How to solve this ONLINE

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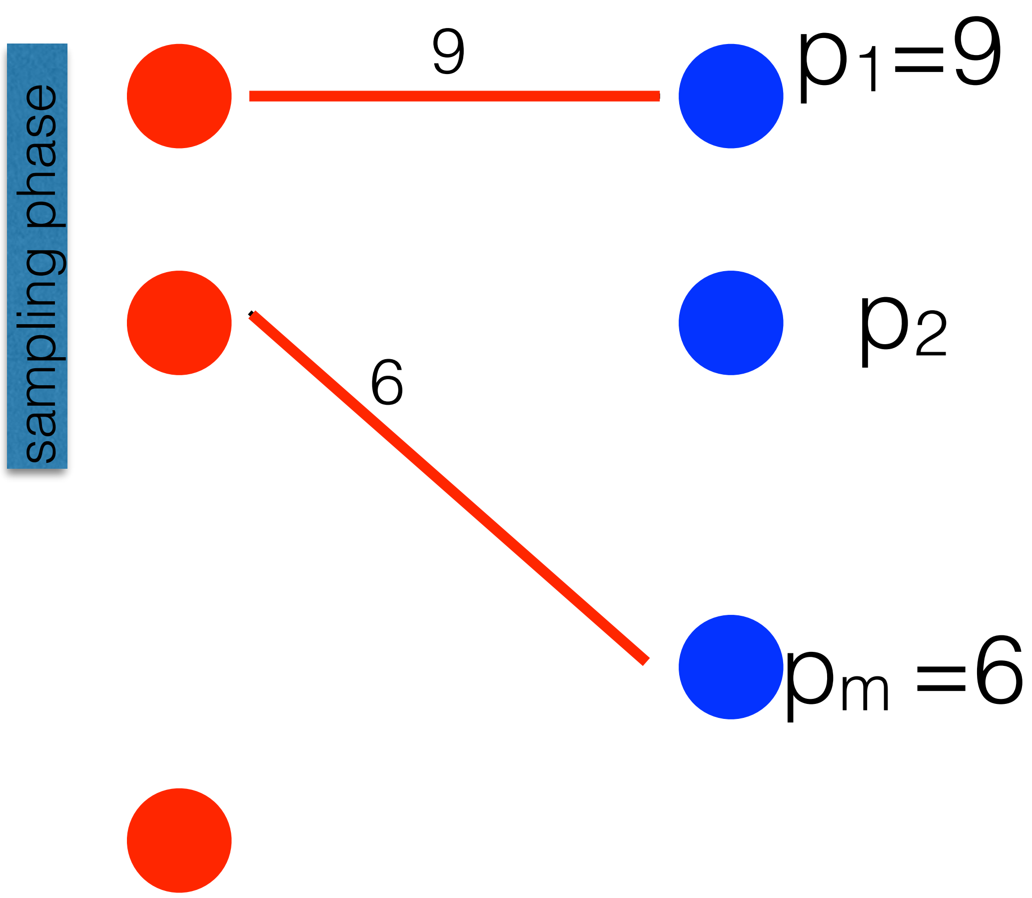

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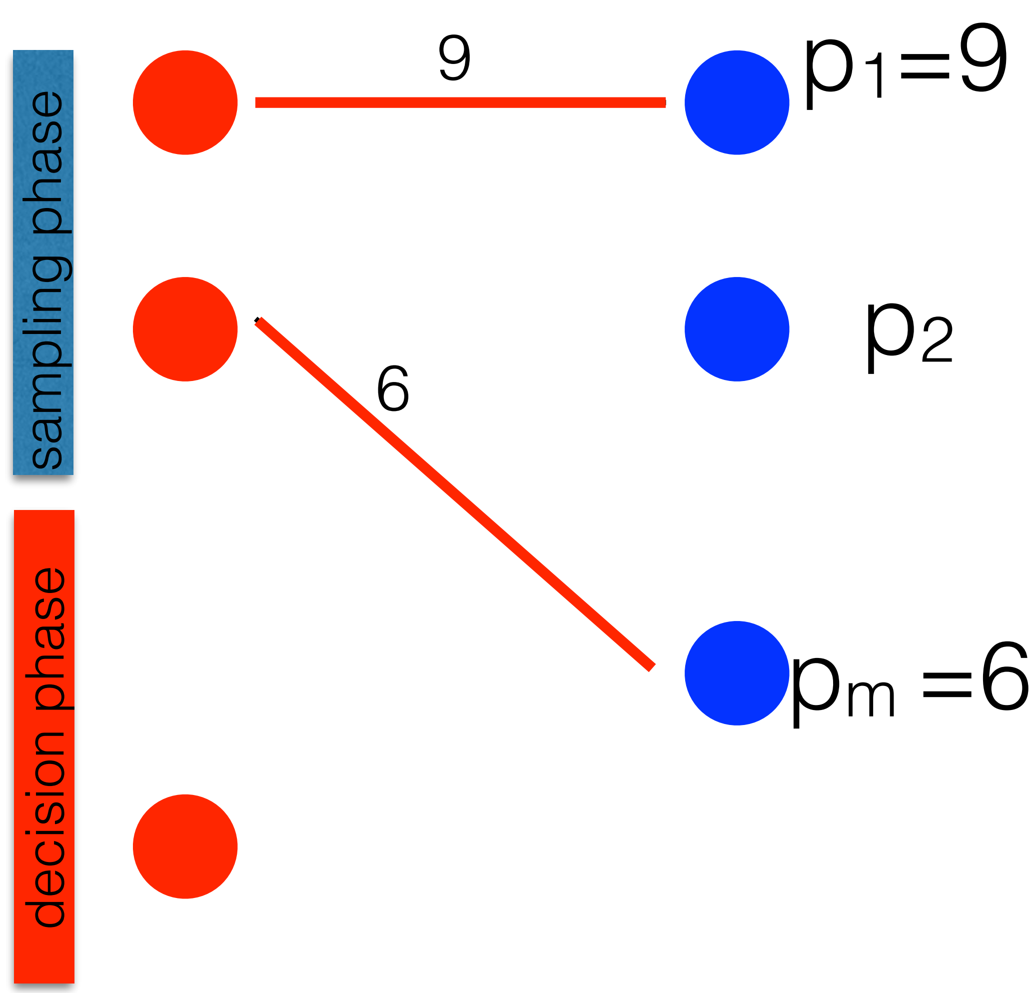

How to solve this ONLINE

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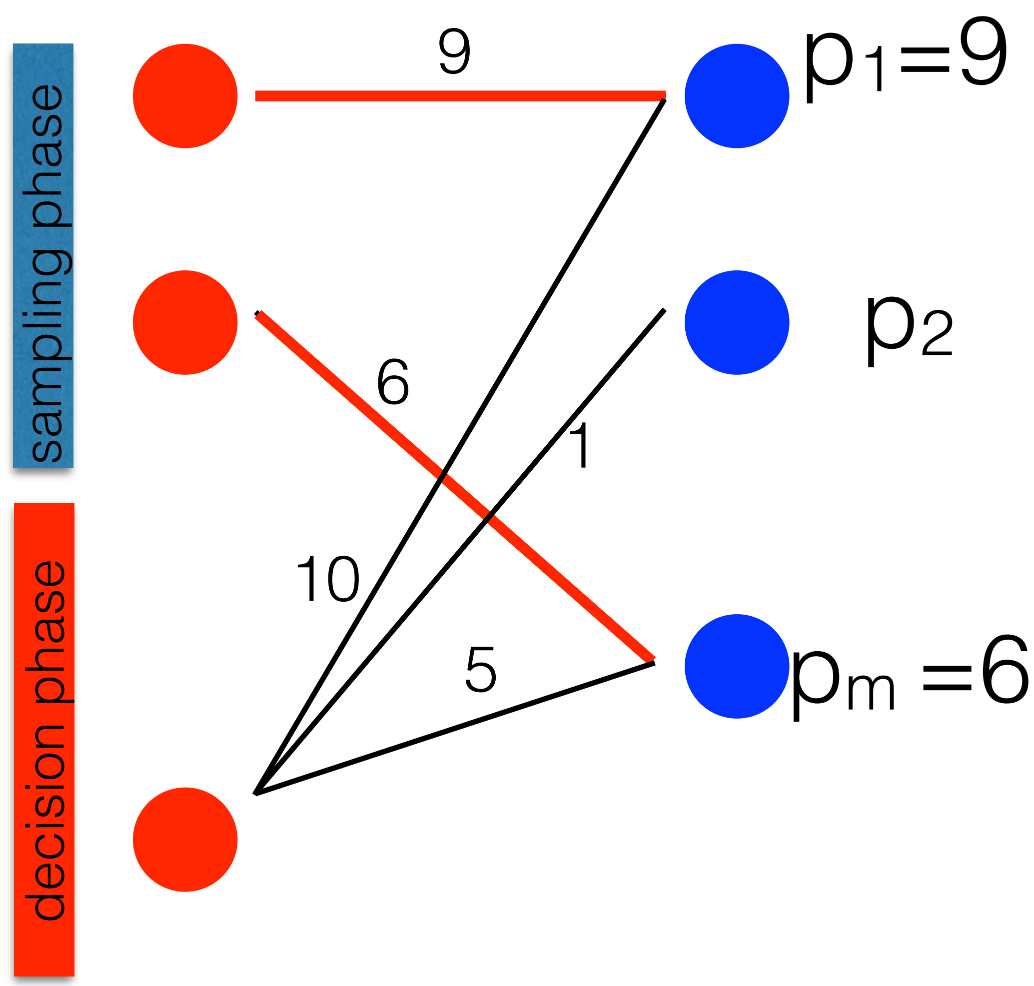
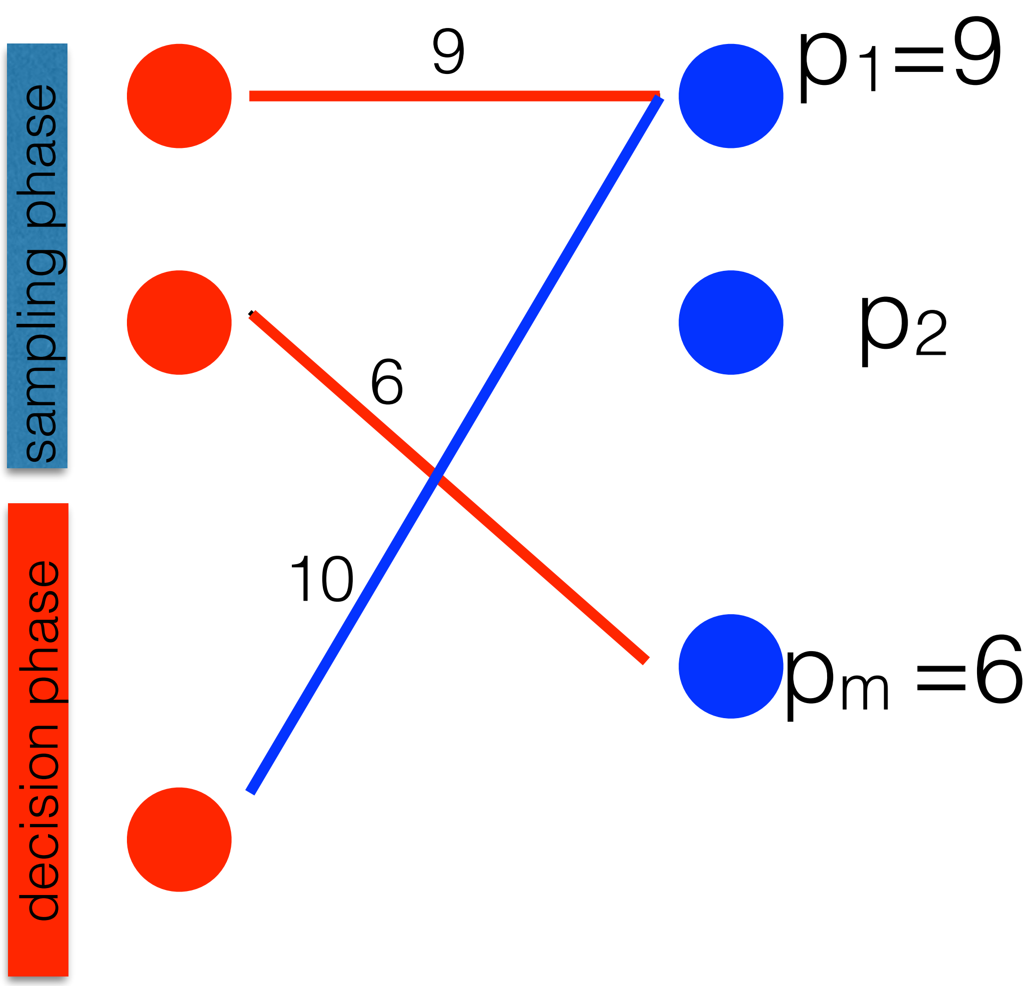


How to solve this ONLINE

Sampling idea as before

Find best matching

match the largest weight
more than price



How to solve this ONLINE

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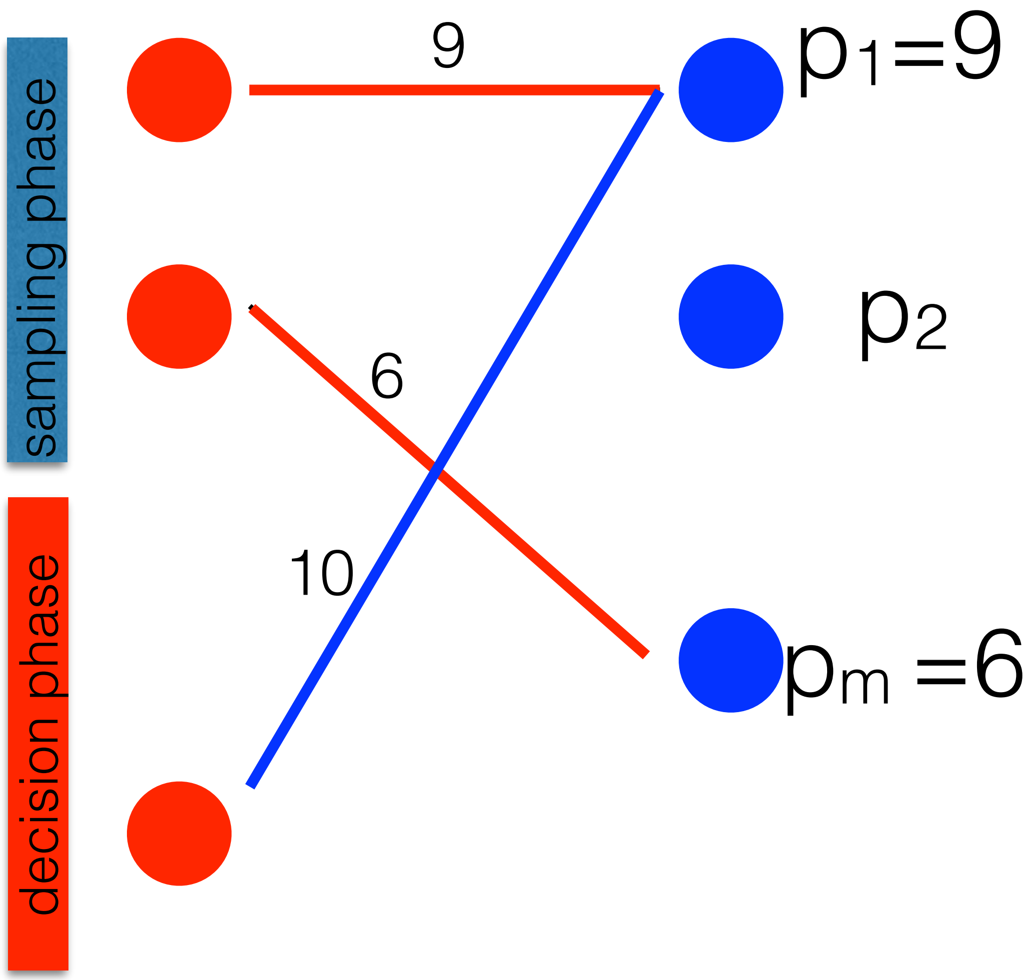
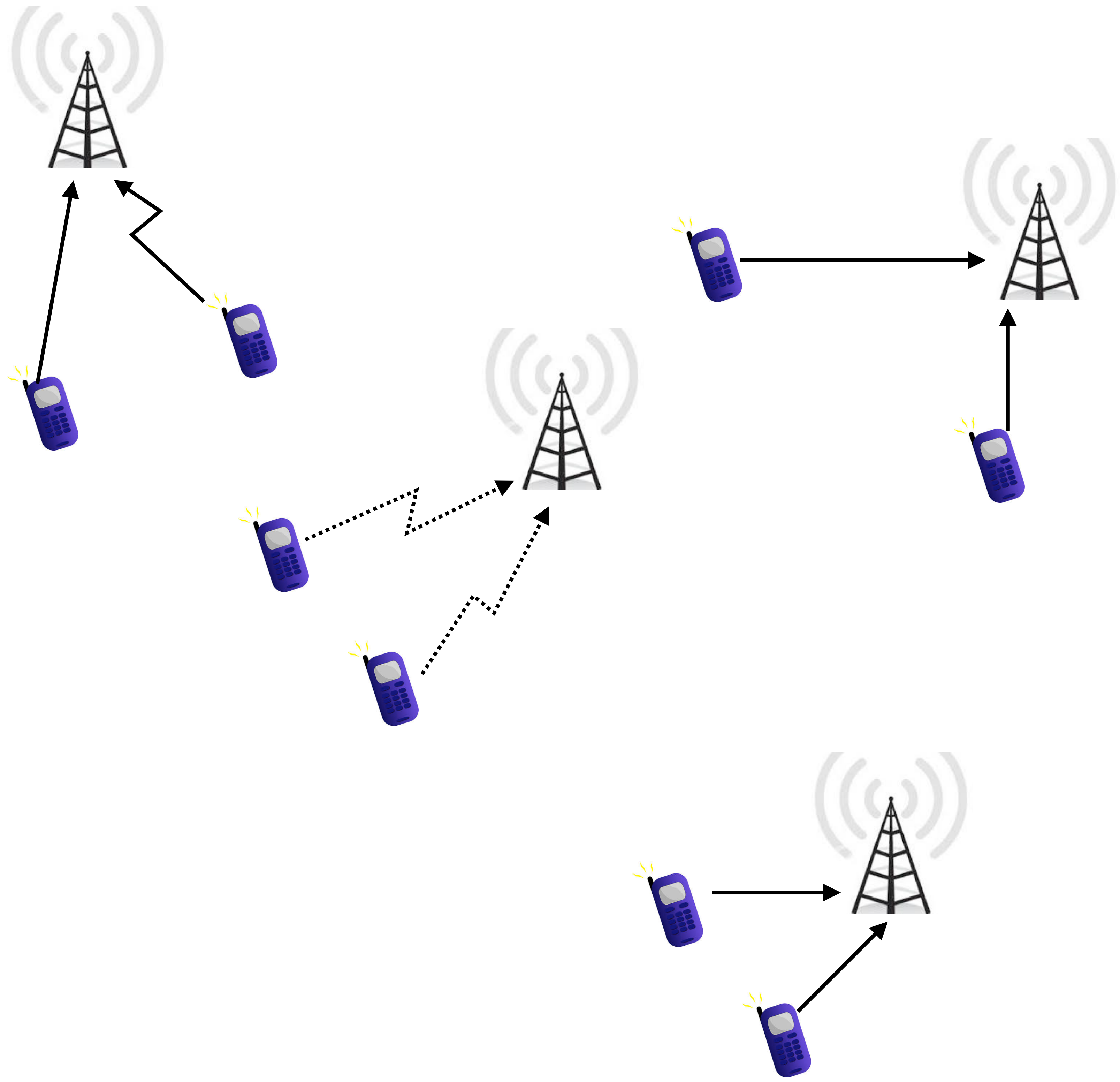


$$u(M_{\mathrm{ON}}) \geq \frac{u(M_{\mathrm{OFF}}(\mathrm{half}))}{2}$$

Result: 8-competitive/optimal [*Korula, Pal' 08***]**

Modern Problem Device-2-Device Communication



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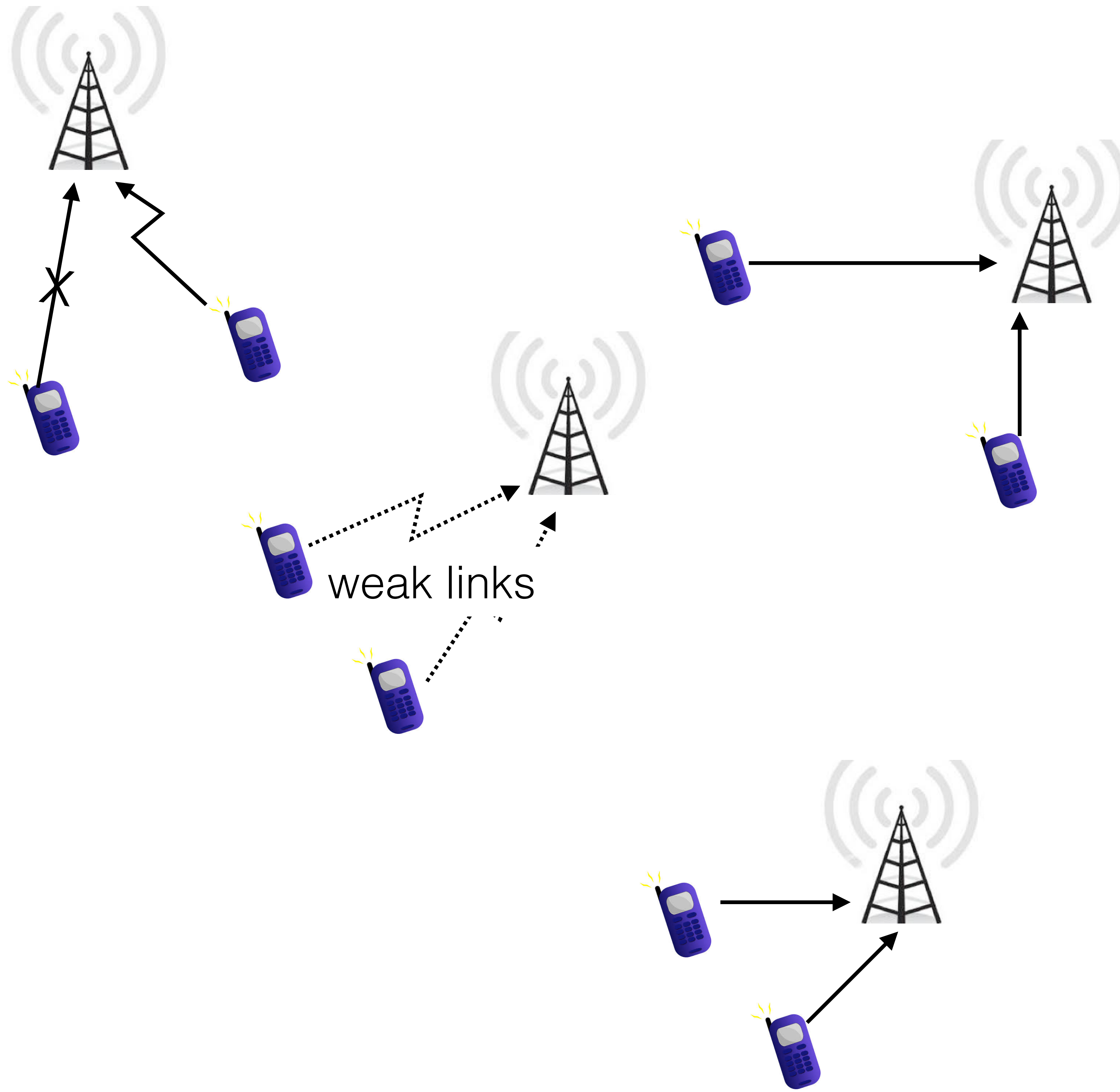
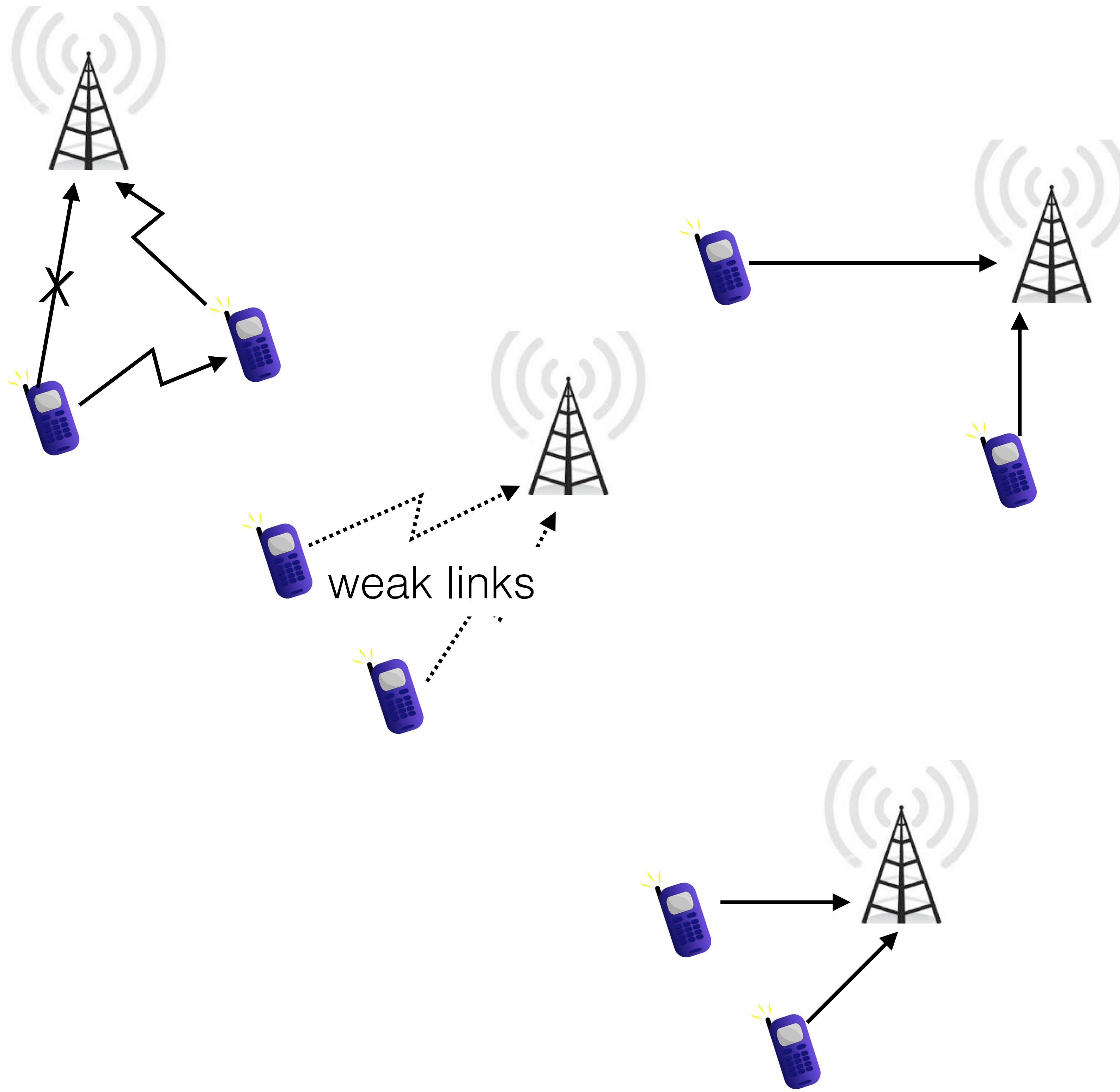


Modern Problem Device-2-Device Communication



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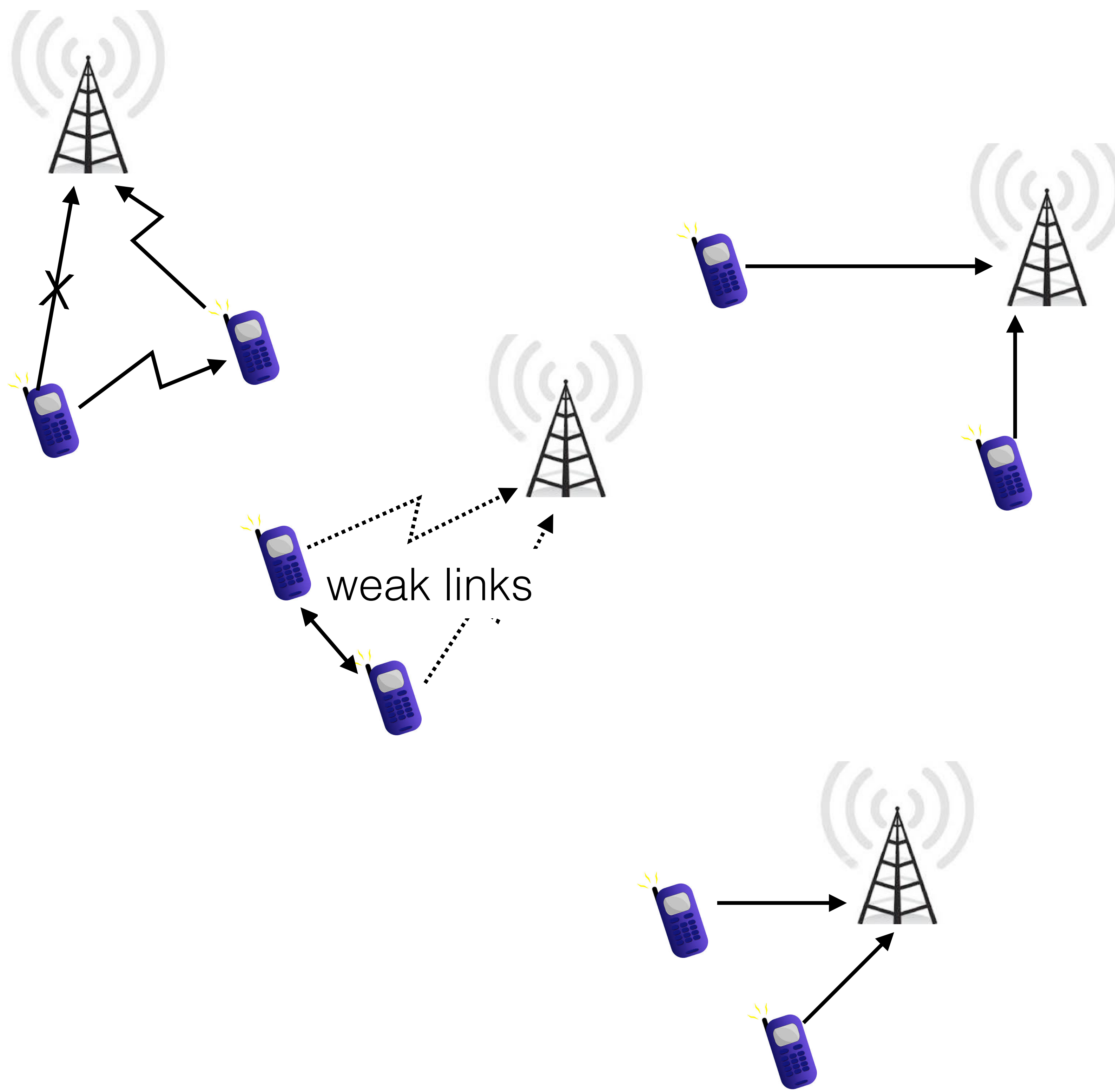

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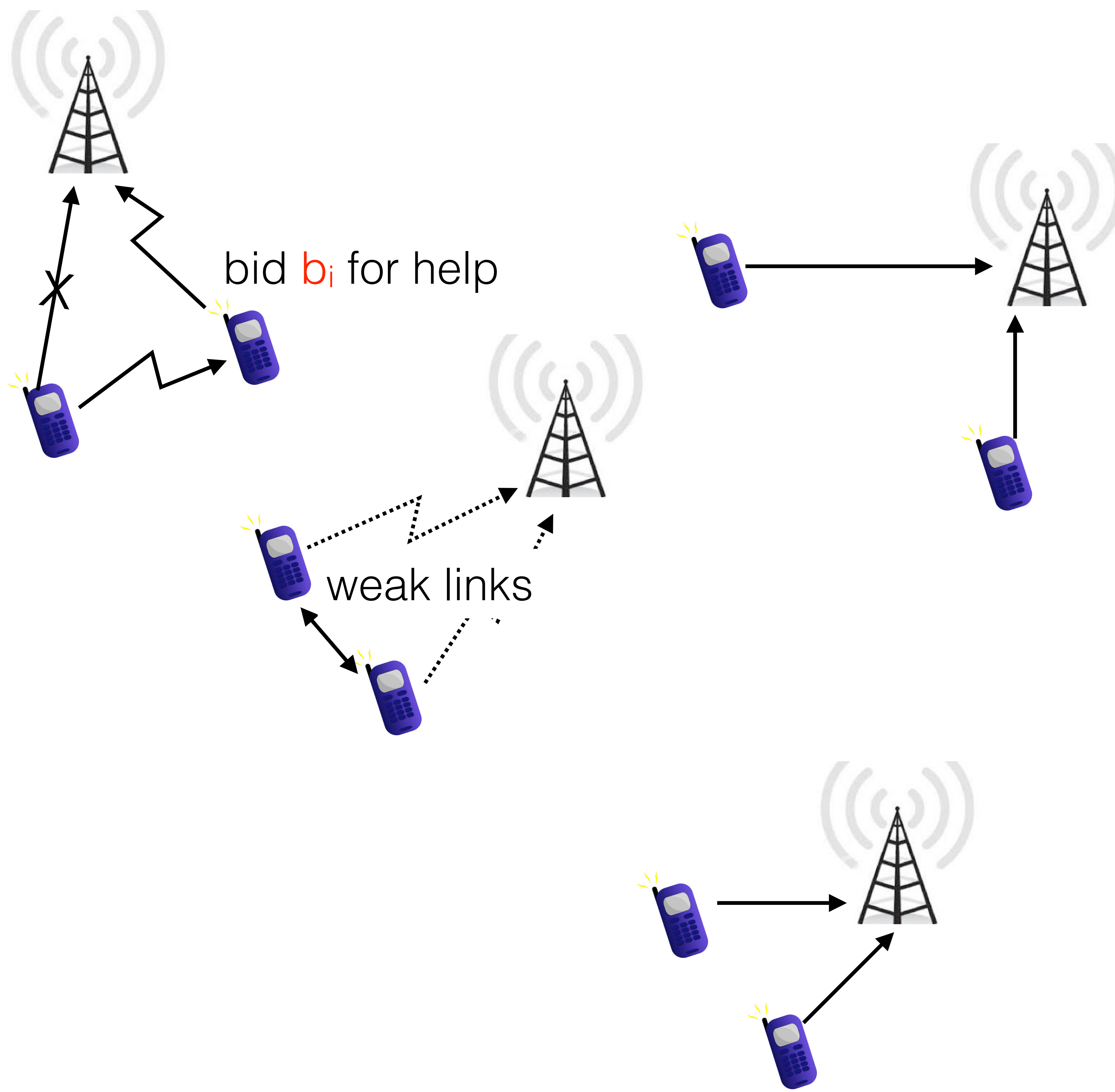

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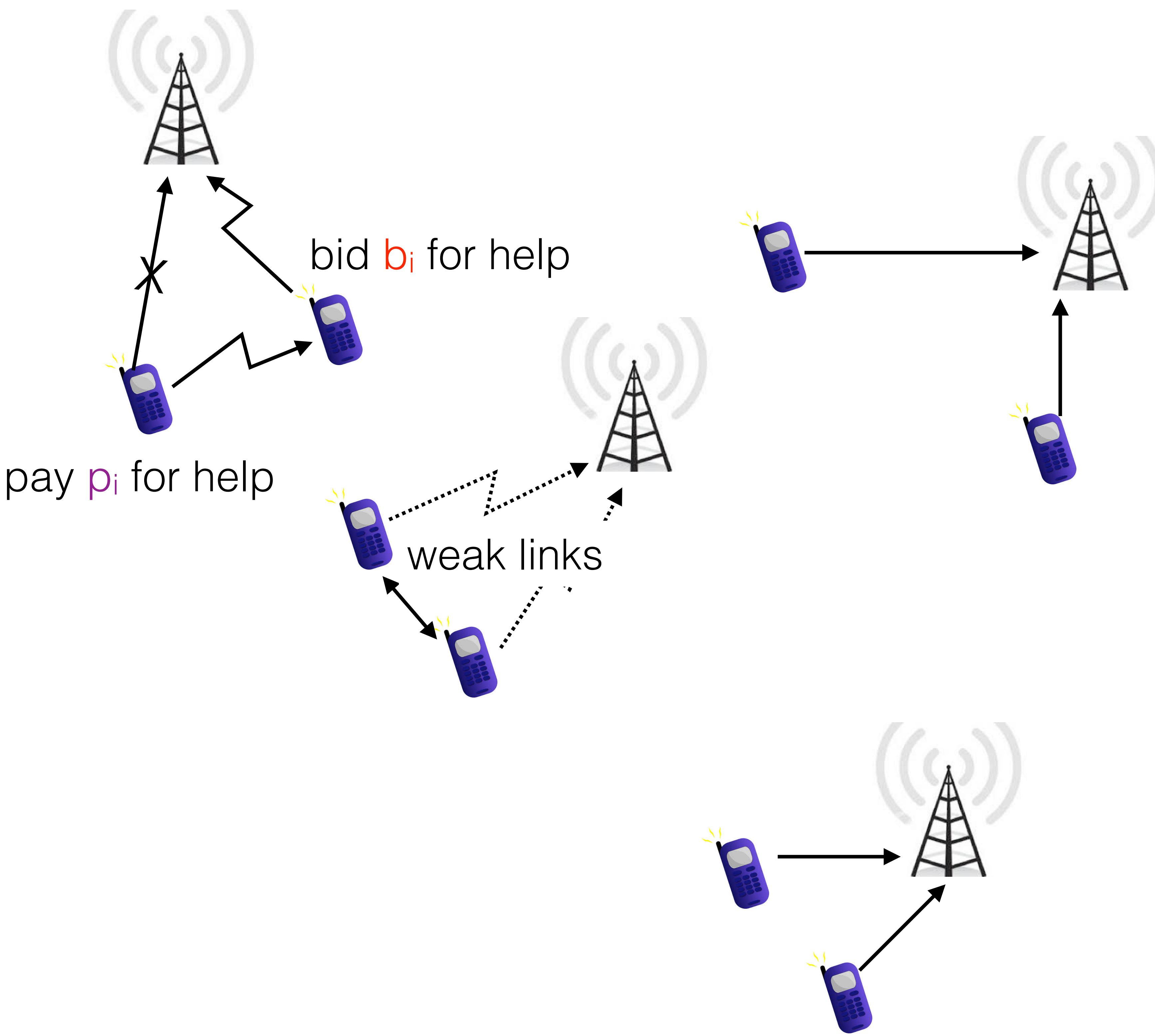
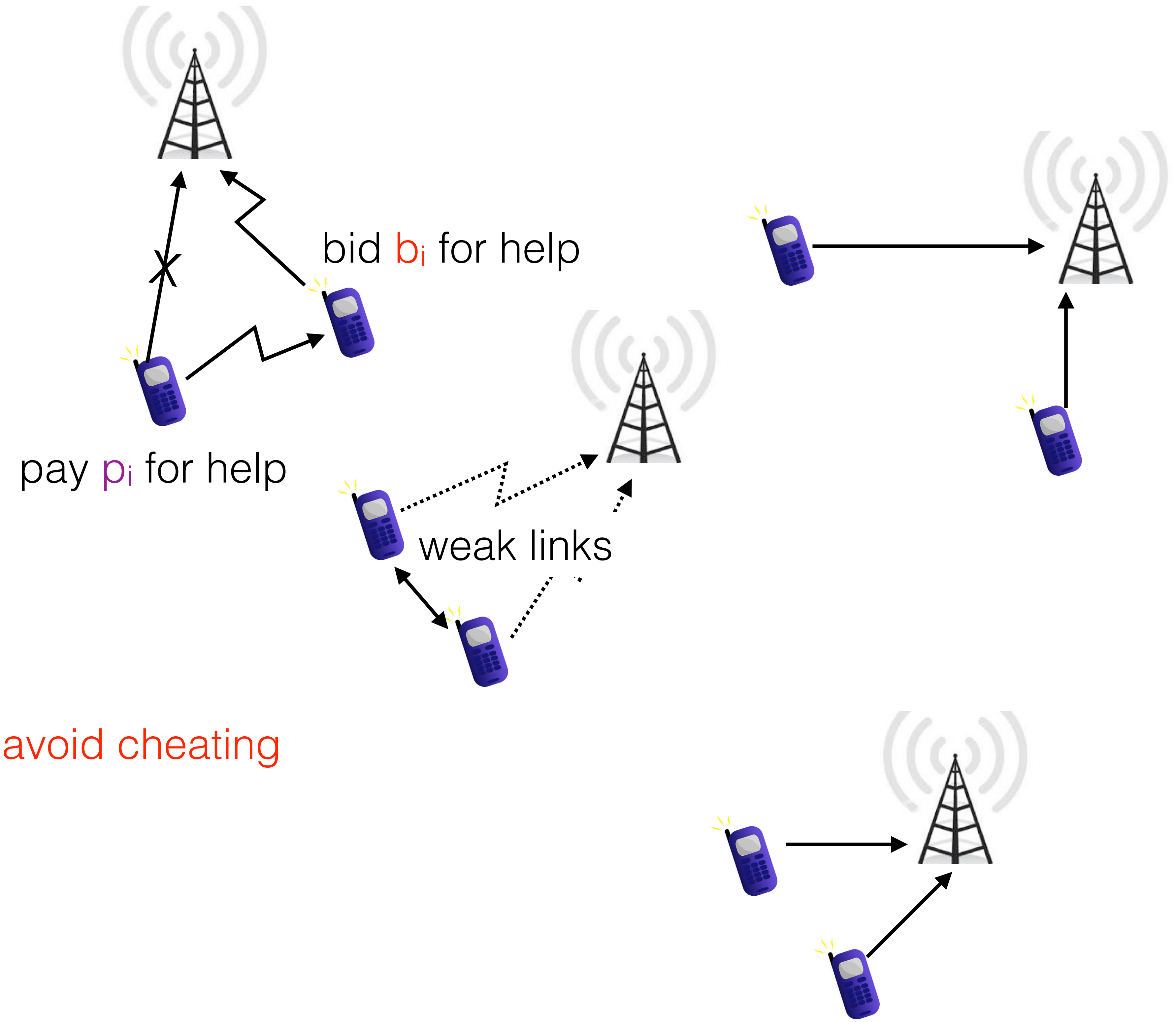
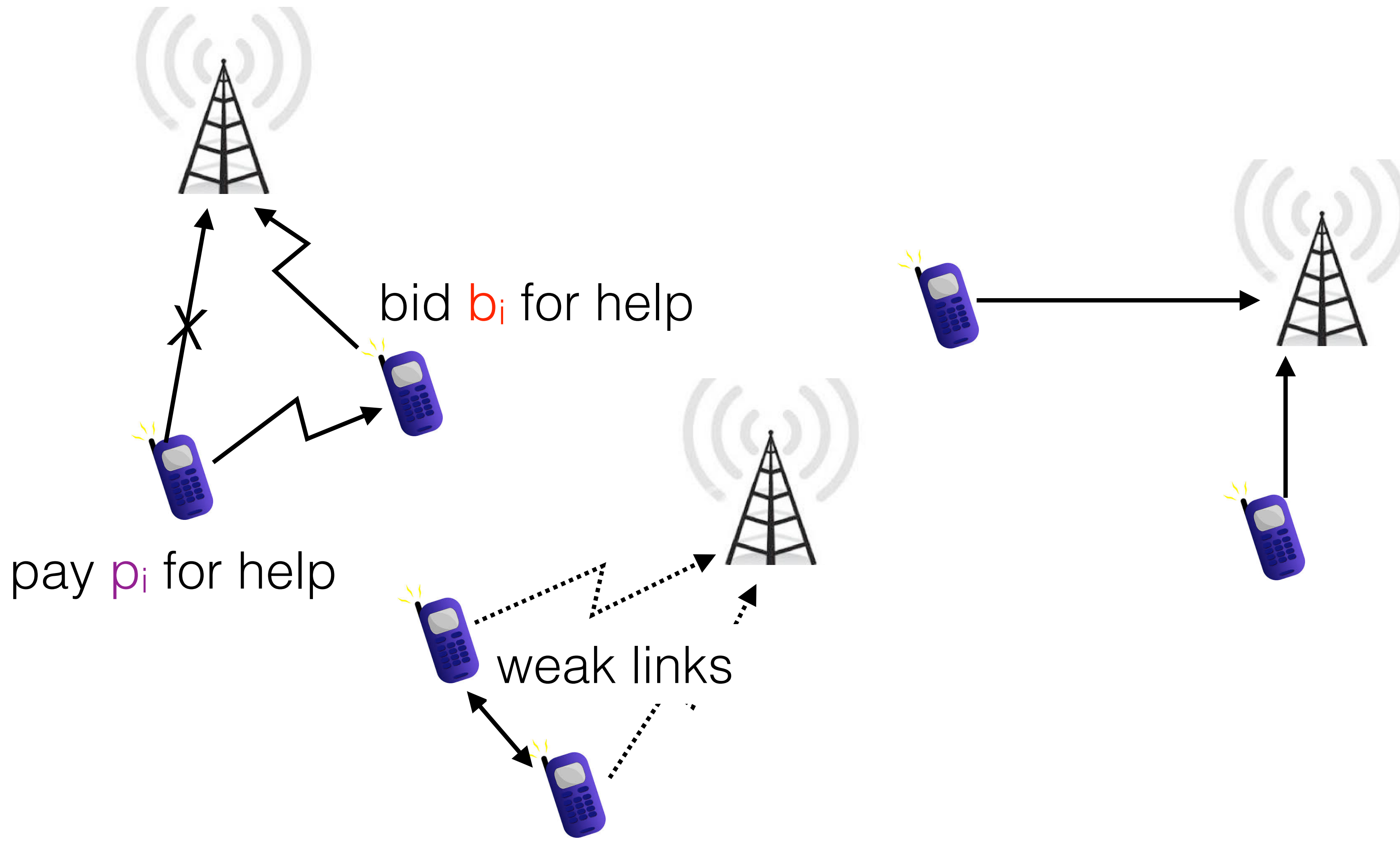


Modern Problem Device-2-Device Communication



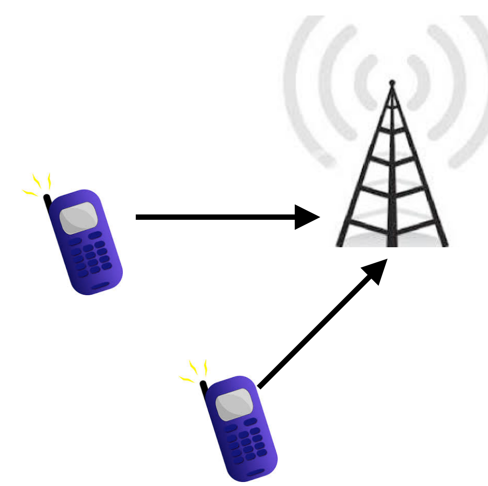
mechanism to avoid cheating

Modern Problem Device-2-Device Communication

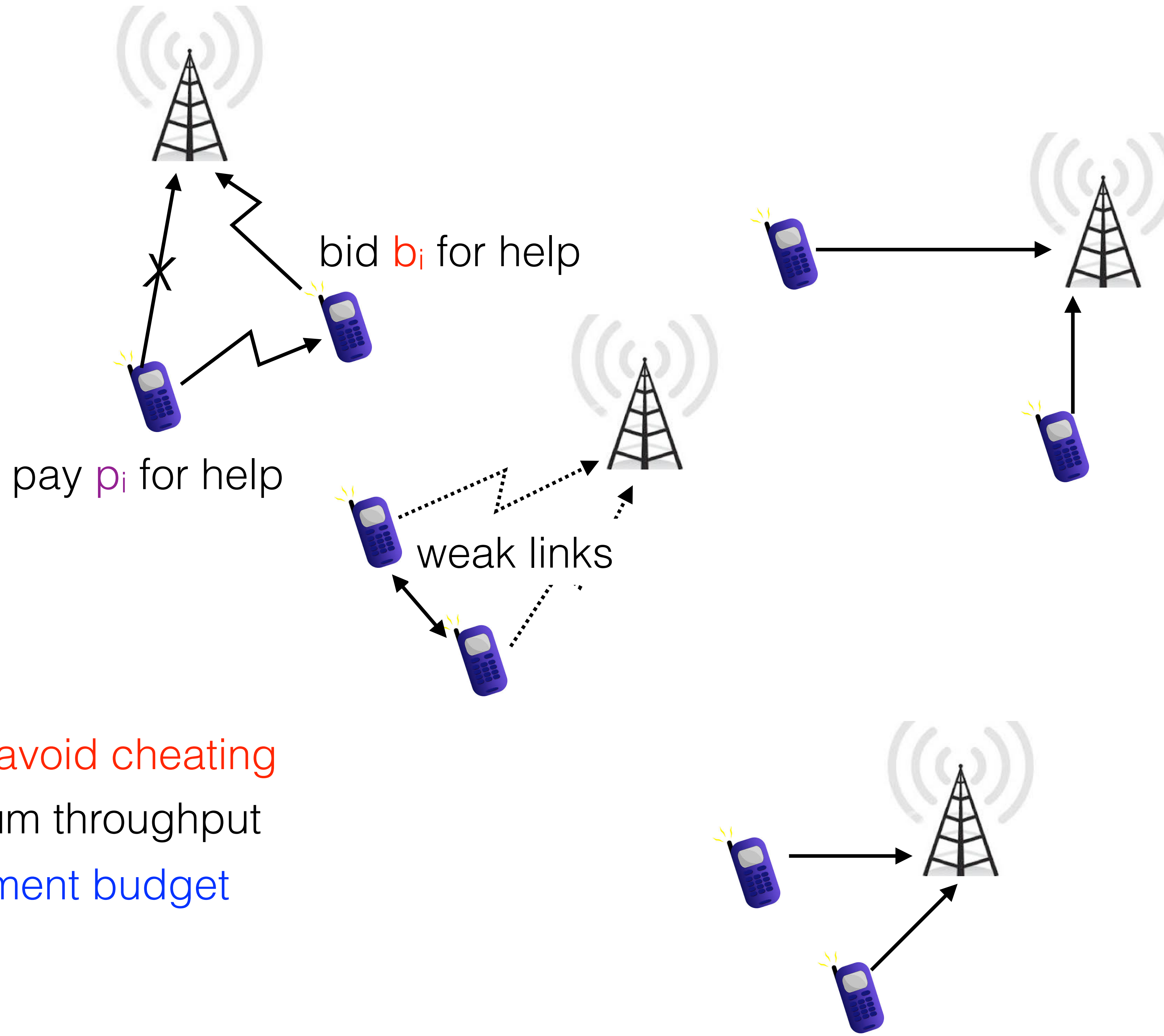


mechanism to avoid cheating

ensure maximum throughput



Modern Problem Device-2-Device Communication

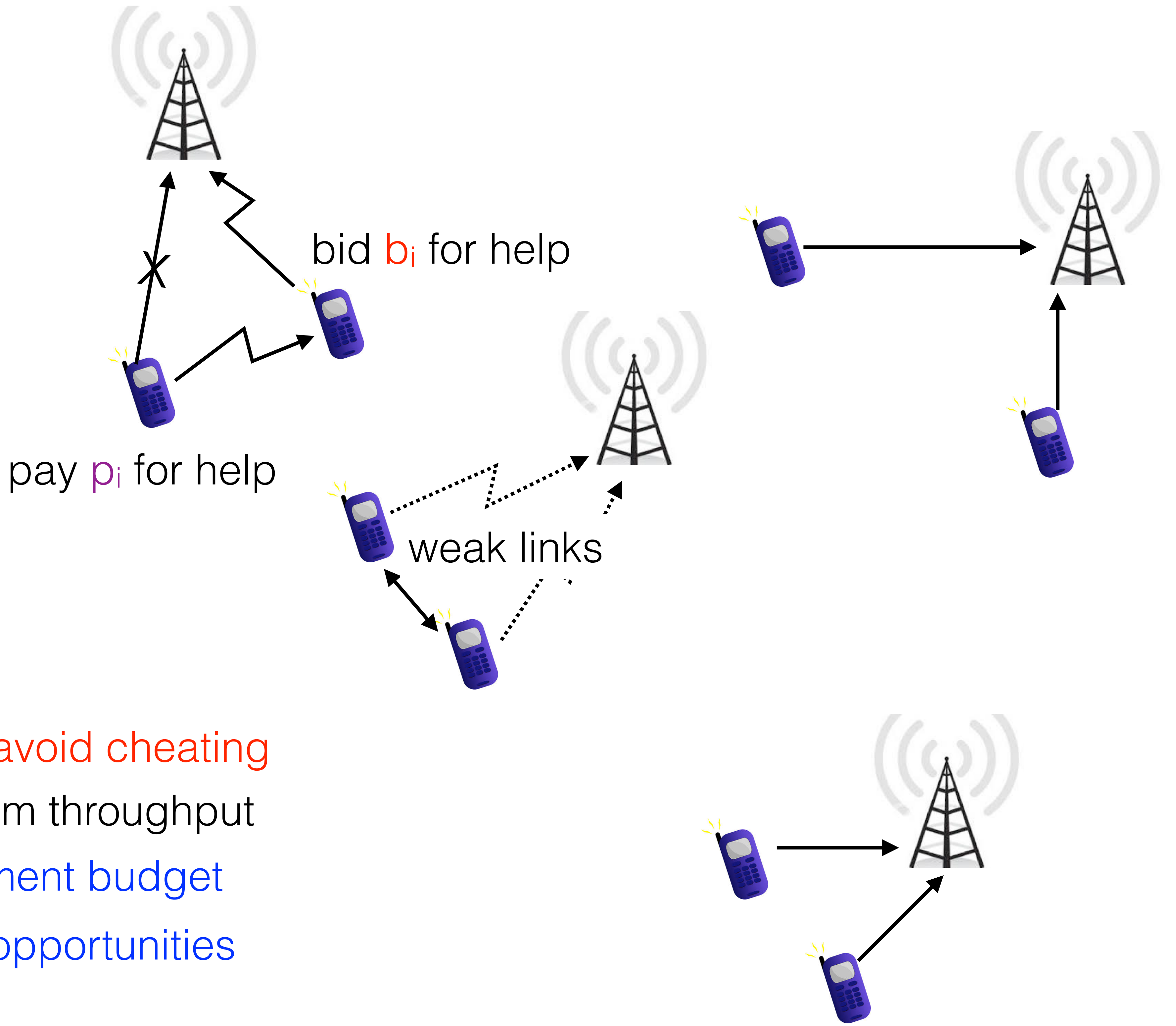


mechanism to avoid cheating

ensure maximum throughput

subject to payment budget

Modern Problem Device-2-Device Communication



mechanism to avoid cheating

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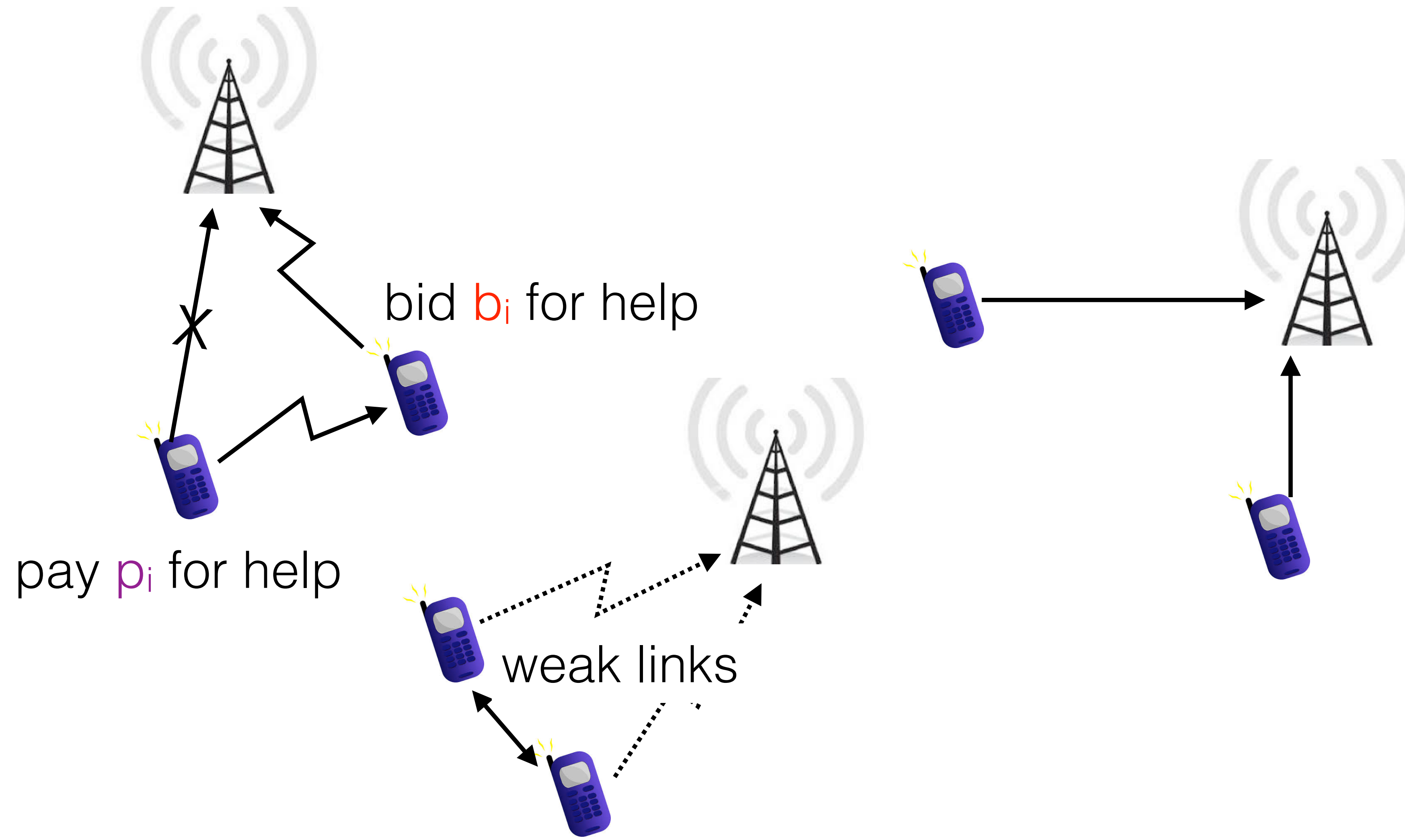
unknown help opportunities

Modern Problem Device-2-Device Communication



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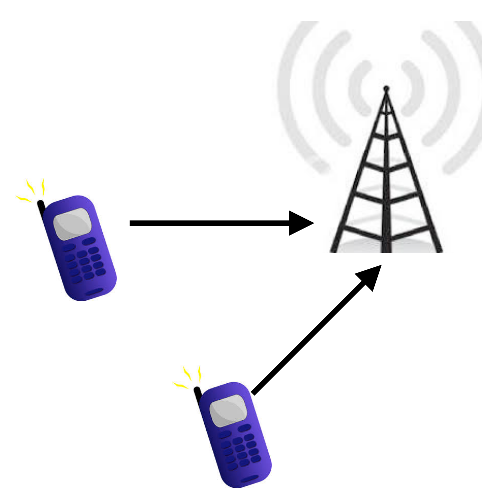
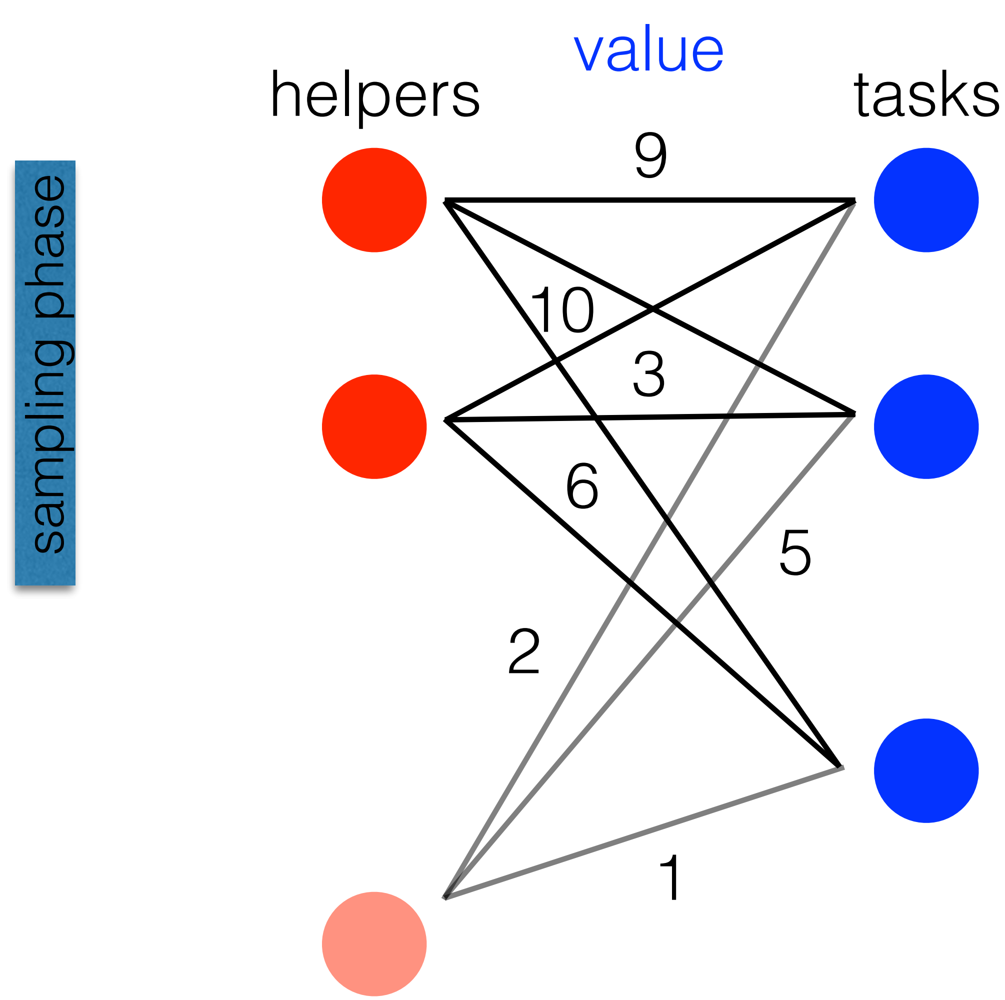


Find optimal helper association and incentive rule that is truthful

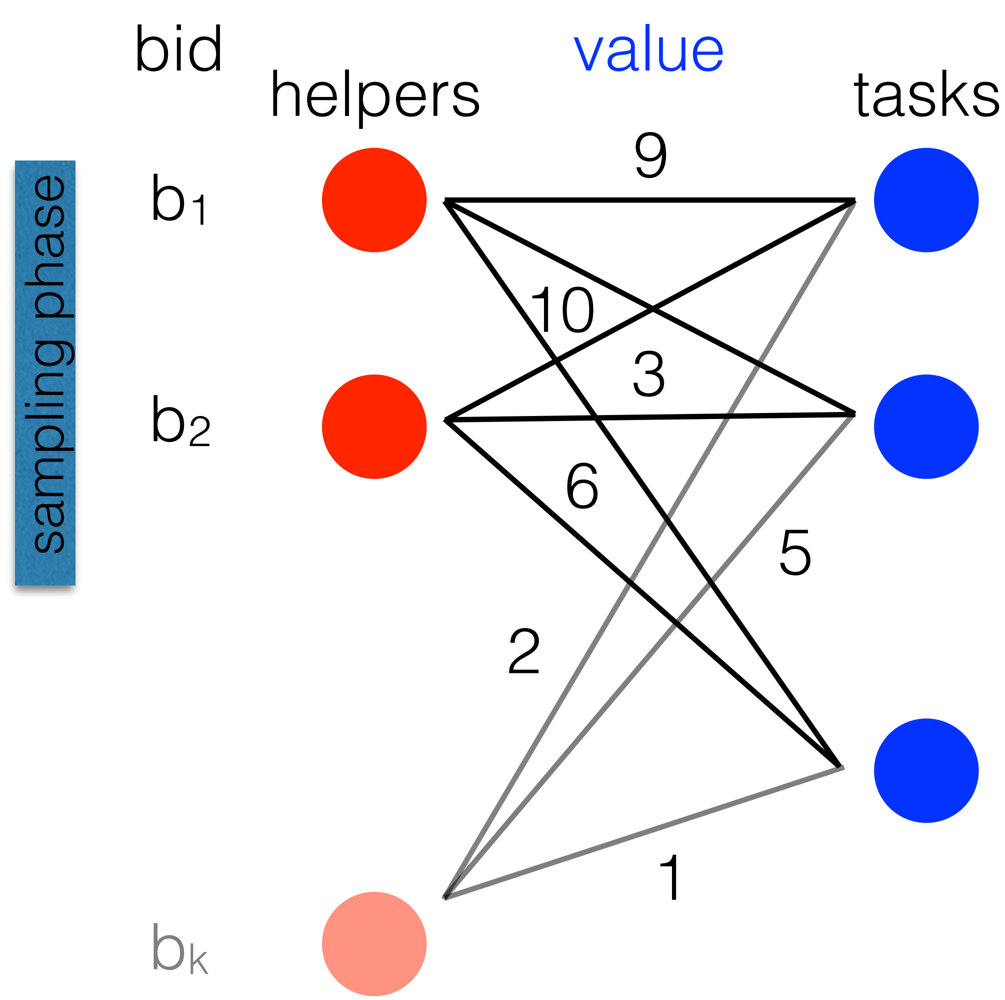
Possible Strategy for Our Problem



Objective: Truthful Matching with largest sum weight under a budget constraint

Possible Strategy for Our Problem

Offline Soln



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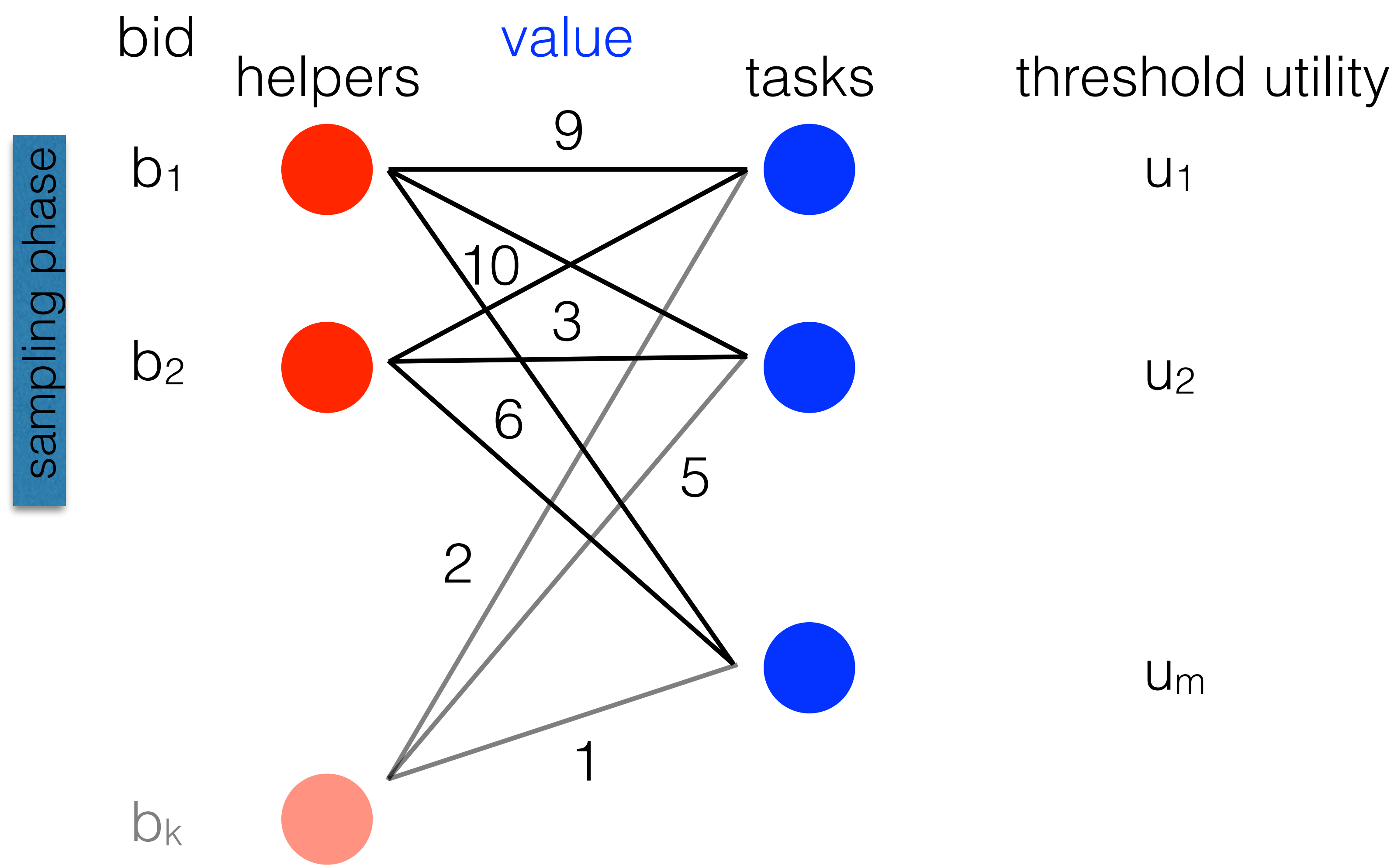

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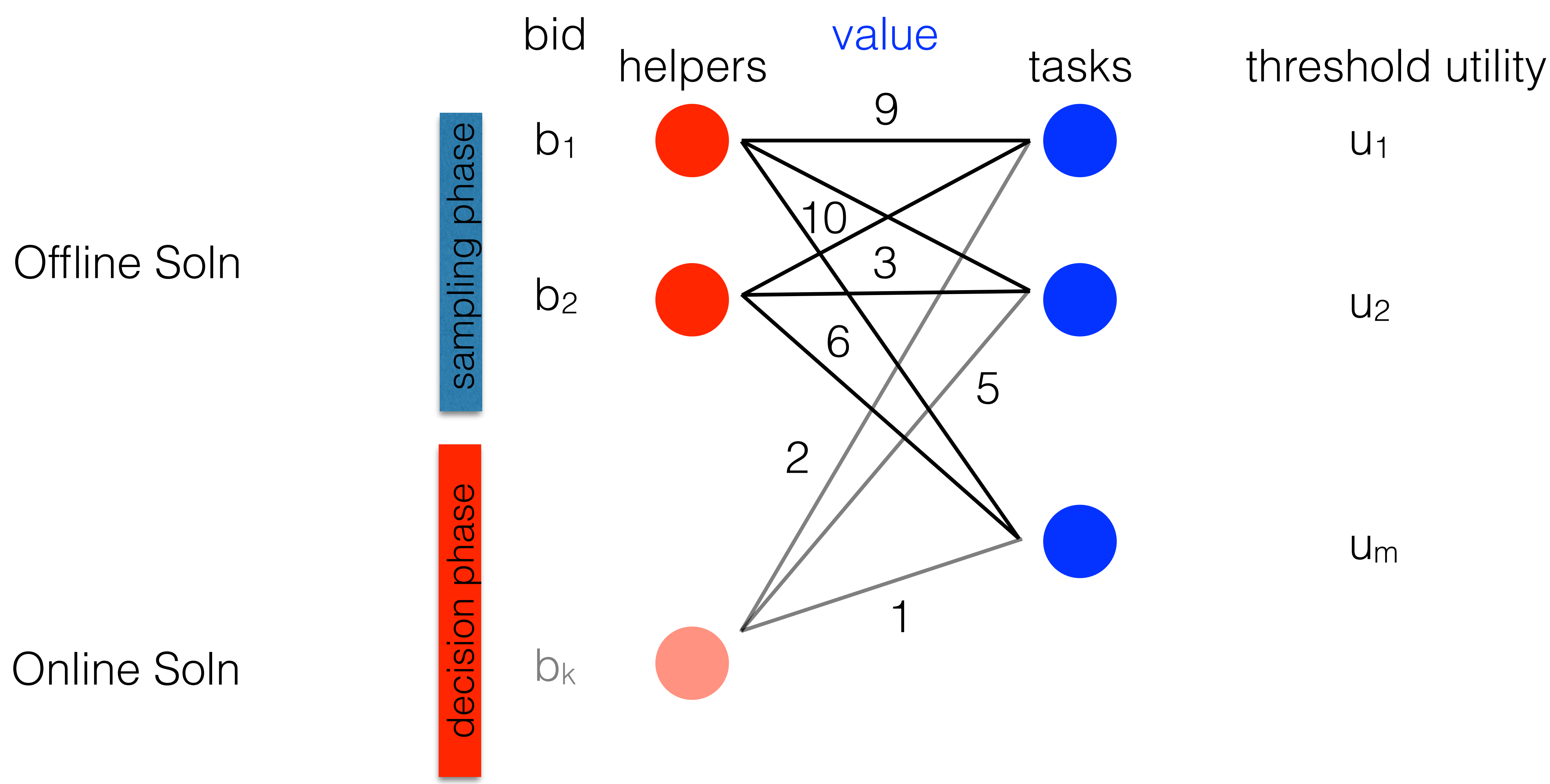

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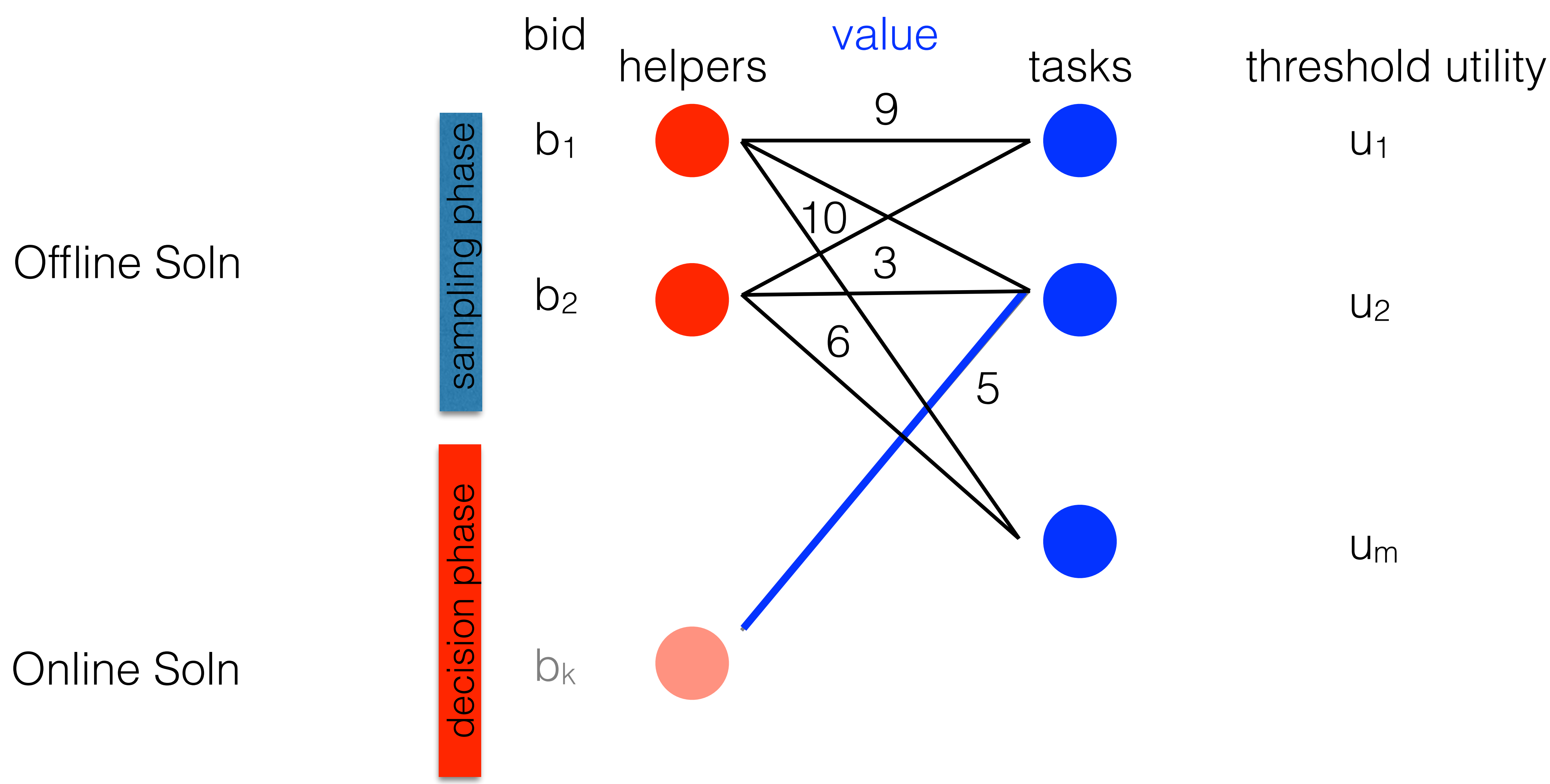

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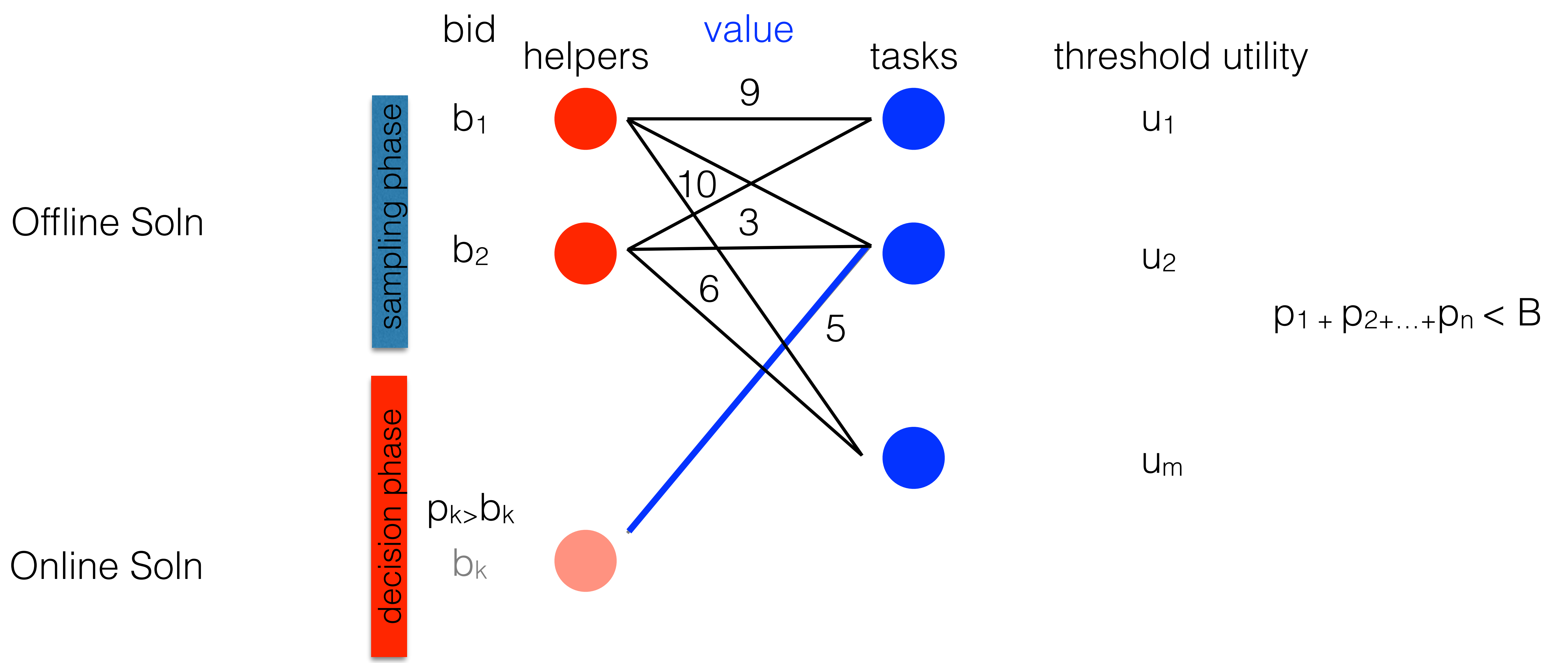

Objective: Truthful Matching with largest sum weight under a budget constraint

Known Offline Algorithm with Truthfulness

Known Offline Algorithm with Truthfulness

bid to benefit ratio of an edge $\frac{b(e)}{v(e)}$

Known Offline Algorithm with Truthfulness

bid to benefit ratio of an edge $\frac{b(e)}{v(e)}$

Algorithm *[Goel, Nikzad, Singla ‘ 14]*

M be greedy matching over G

Remove any edge e with $\frac{b(e)}{v(e)} > \frac{B}{u(M)}$

$G = G \backslash \{e\}$ **go to Step 1**

Known Offline Algorithm with Truthfulness

Problem

bid to benefit ratio of an edge $\frac{b(e)}{v(e)}$

All edges part of big graph and not necessarily in small graph

Algorithm *[Goel, Nikzad, Singla ' 14]*

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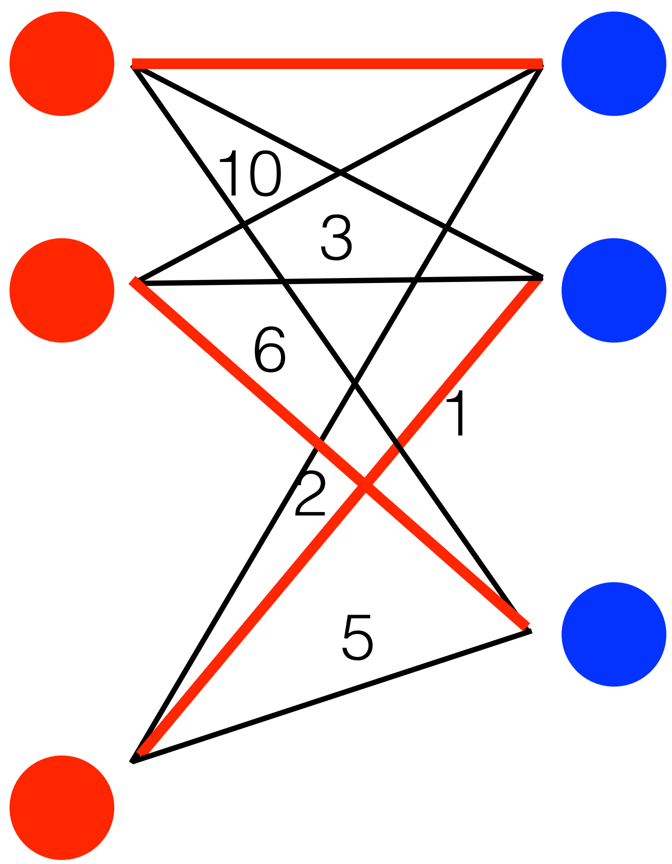

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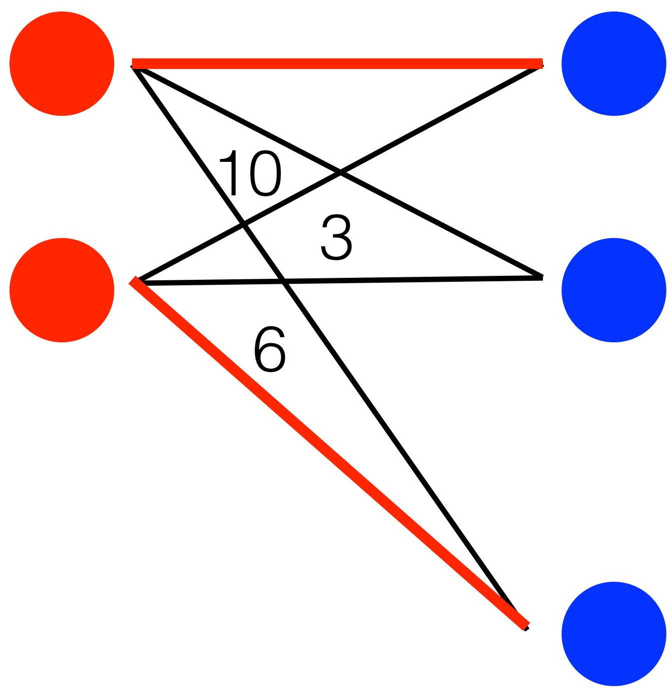

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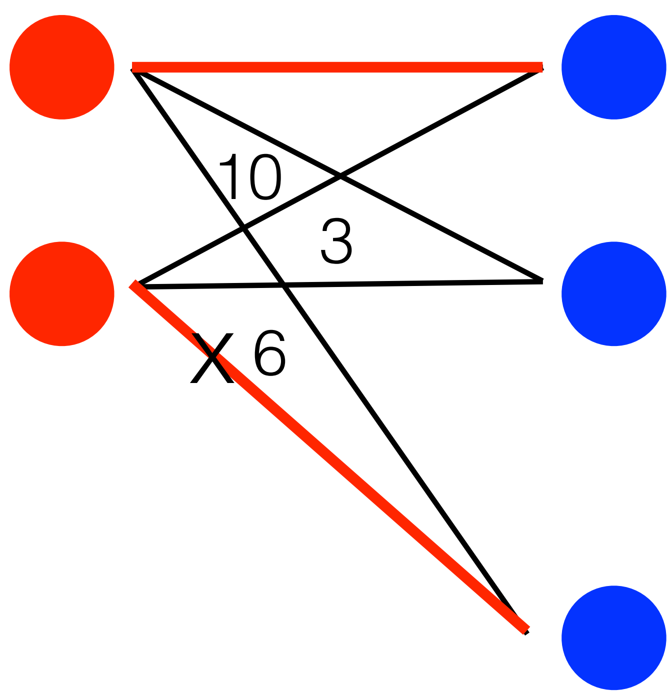

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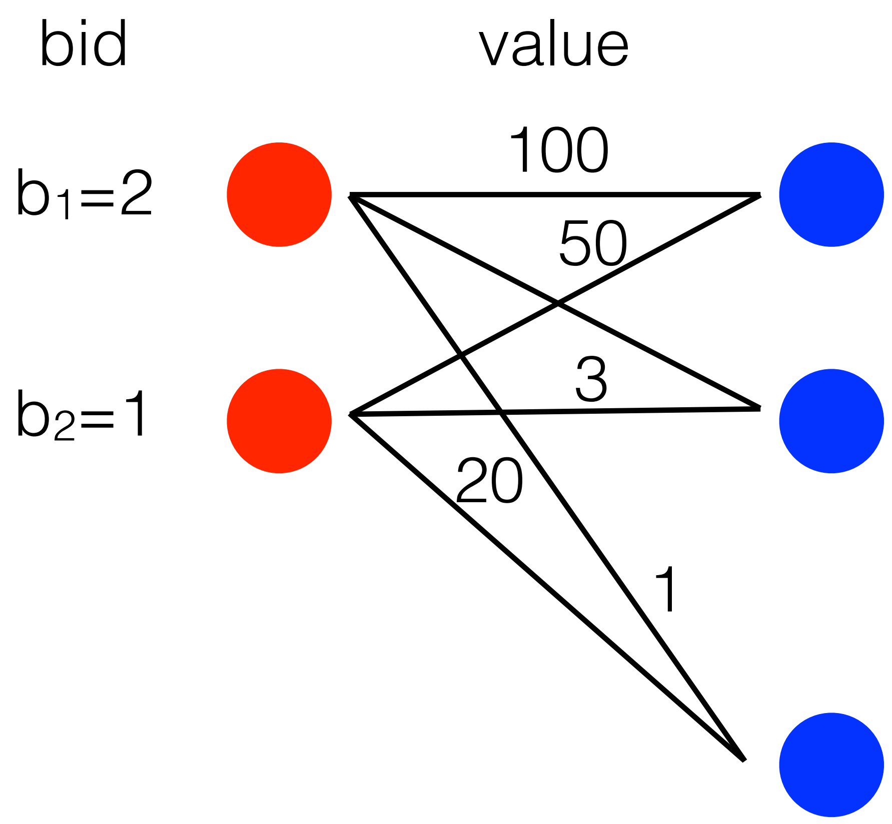
All edges part of big graph and not necessarily in small graph

Our Offline Algorithm **OFF**



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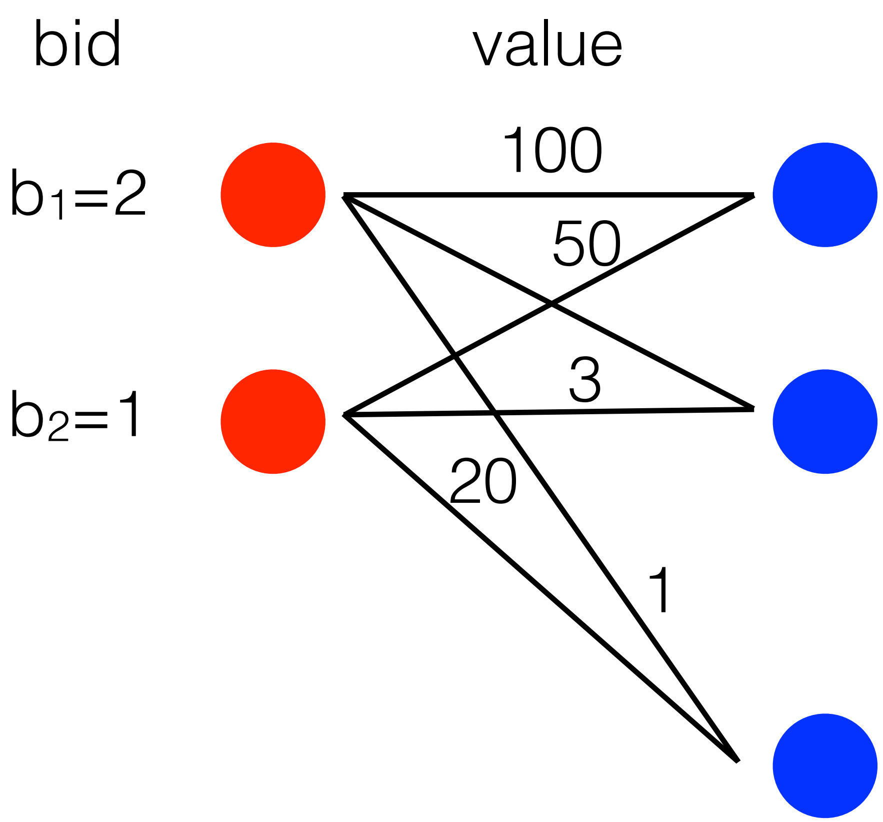
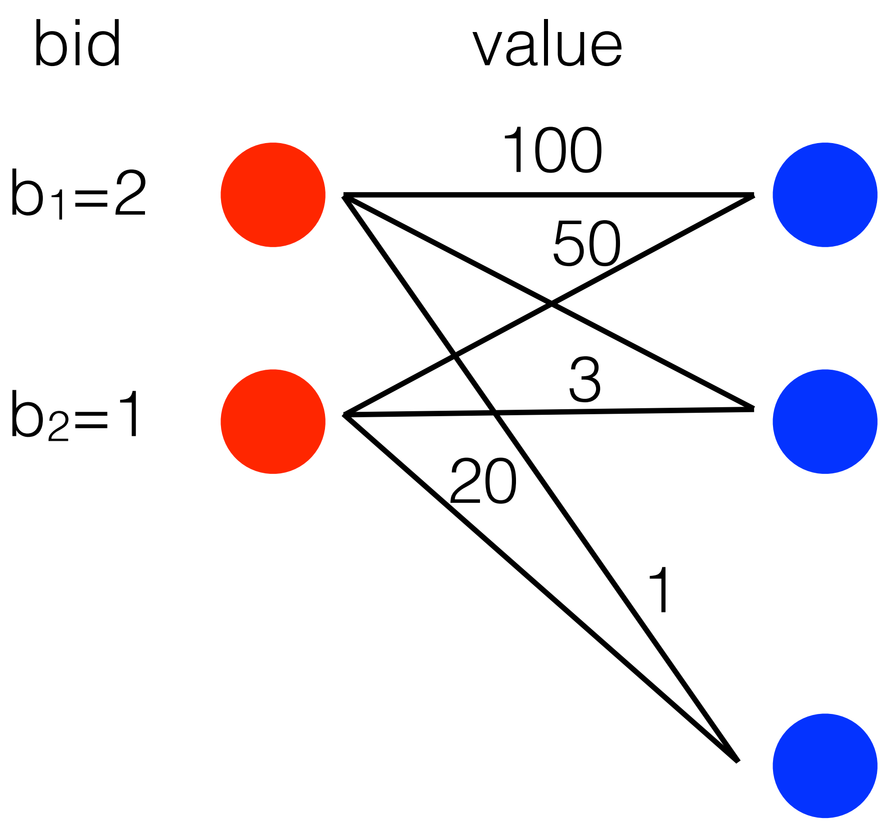


bid to benefit ratio of an edge $\frac{b(e)}{v(e)}$

Our Offline Algorithm `OFF`



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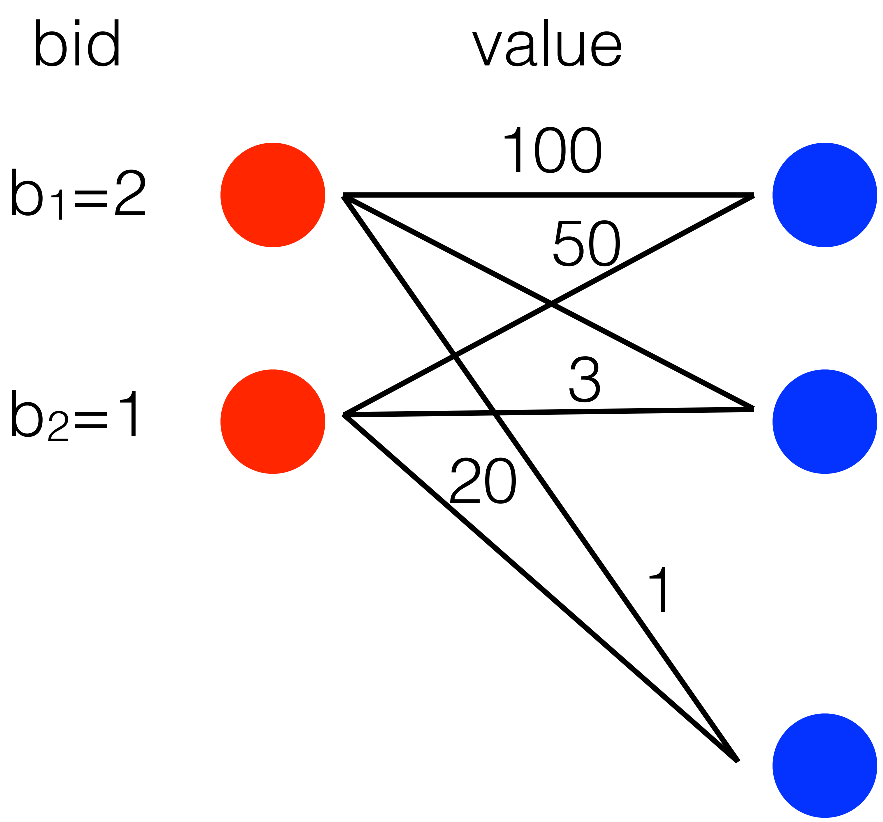

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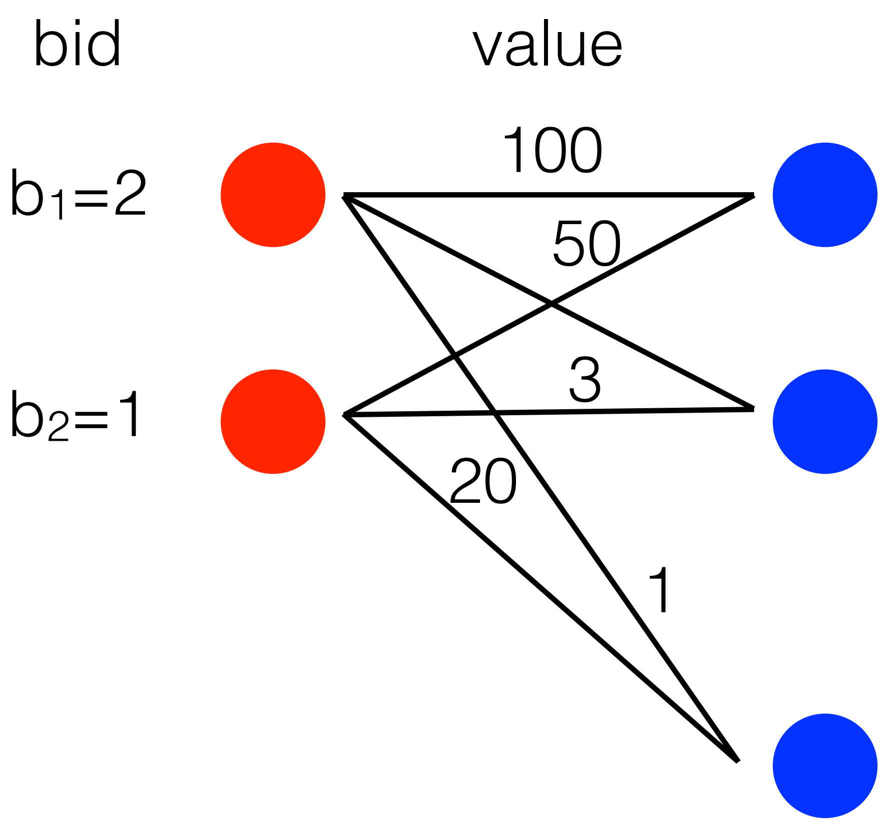

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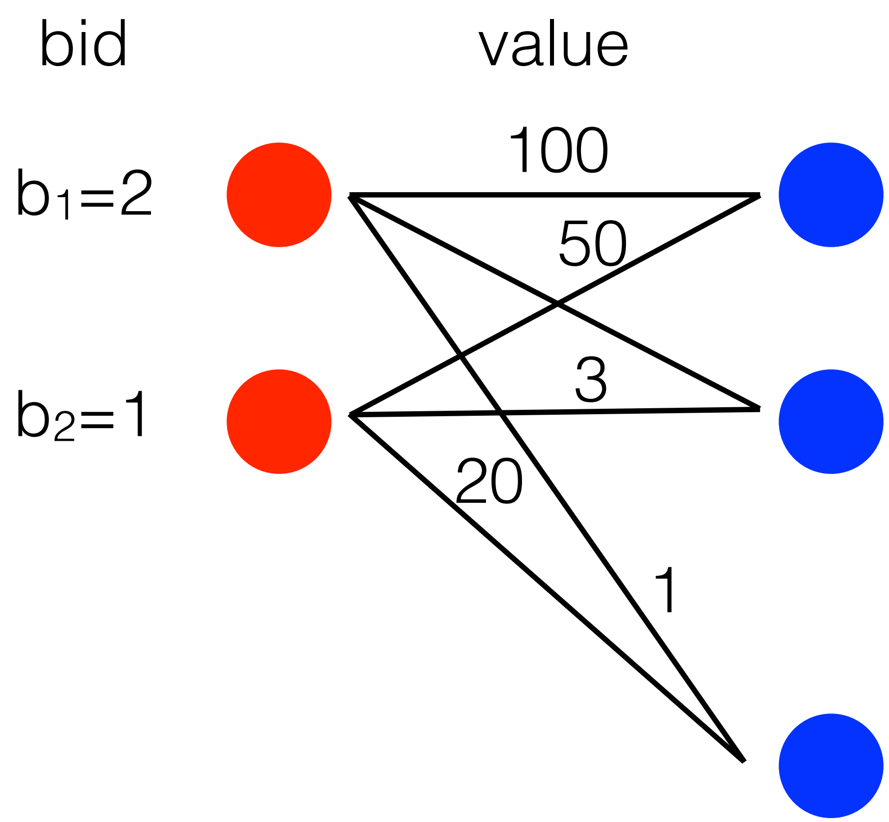

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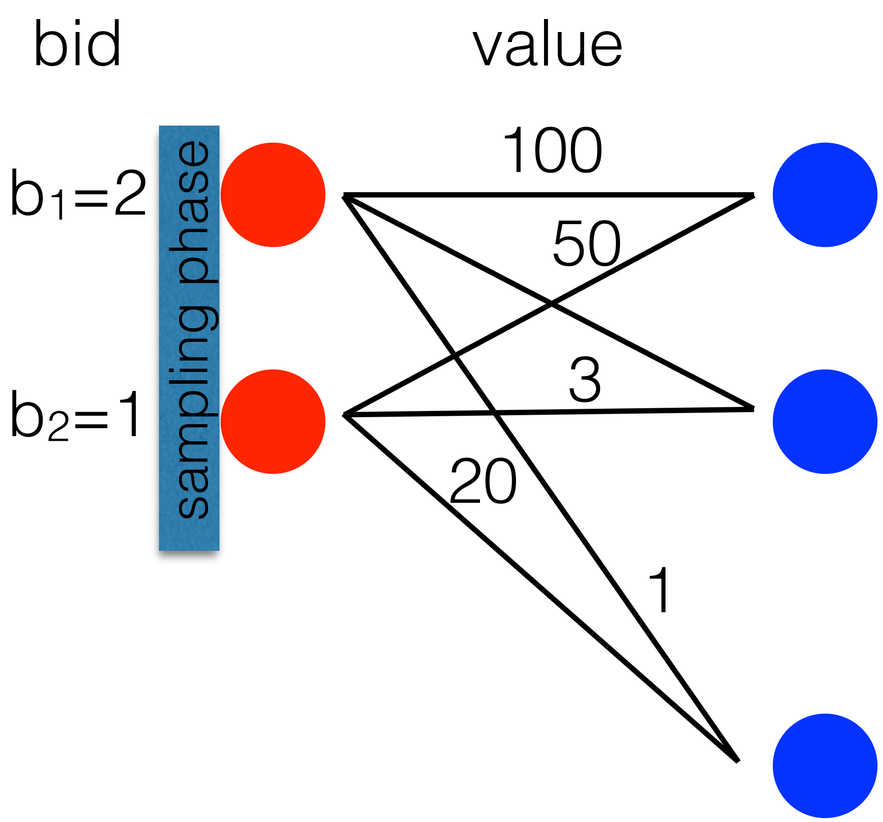

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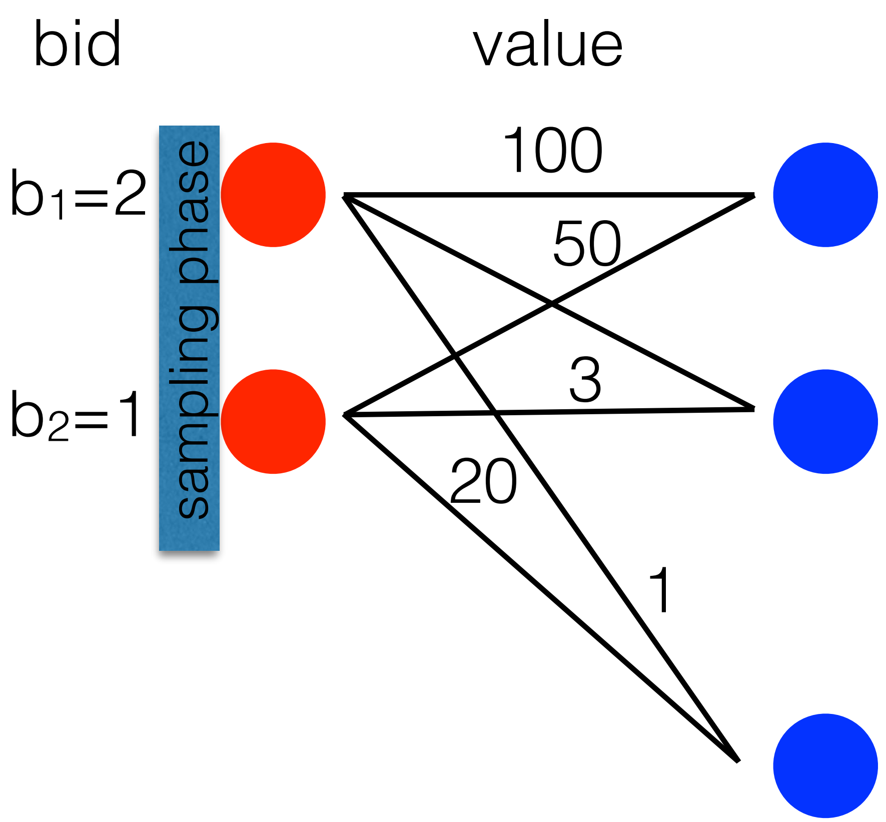

$\gamma = 0.1$

bid to benefit ratio of an edge $\frac{b(e)}{v(e)}$

Algorithm

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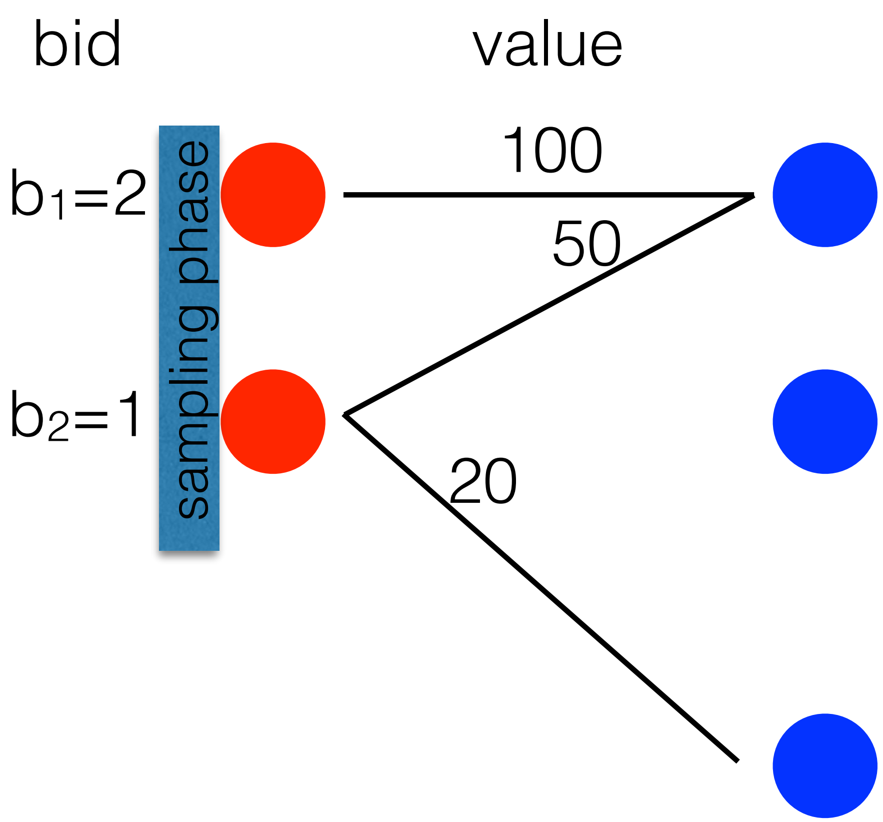

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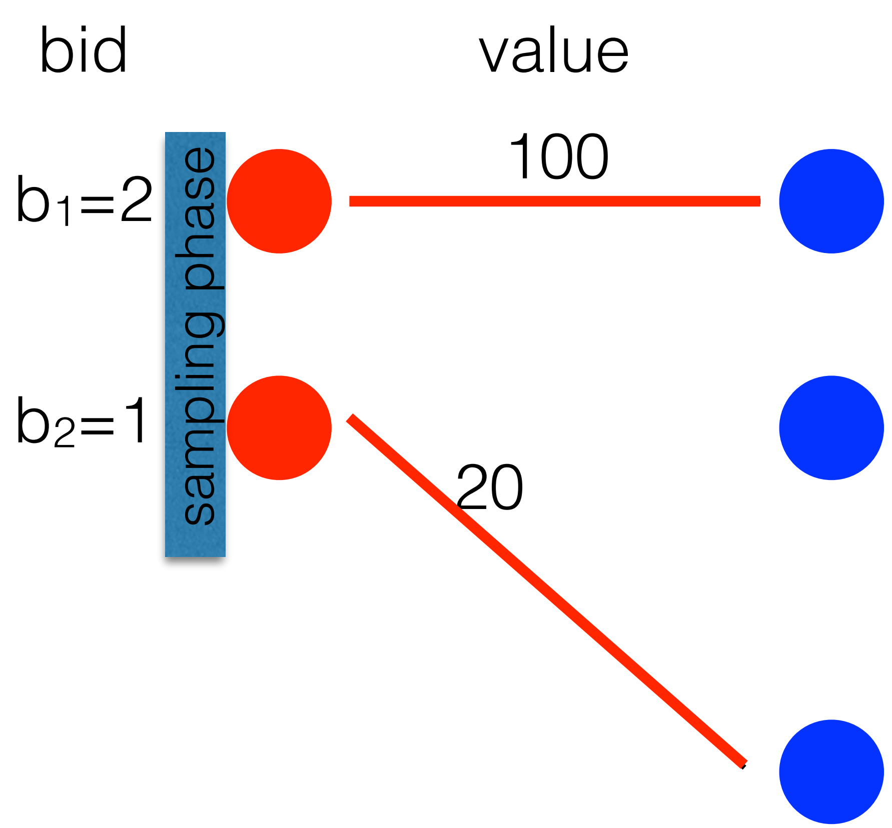

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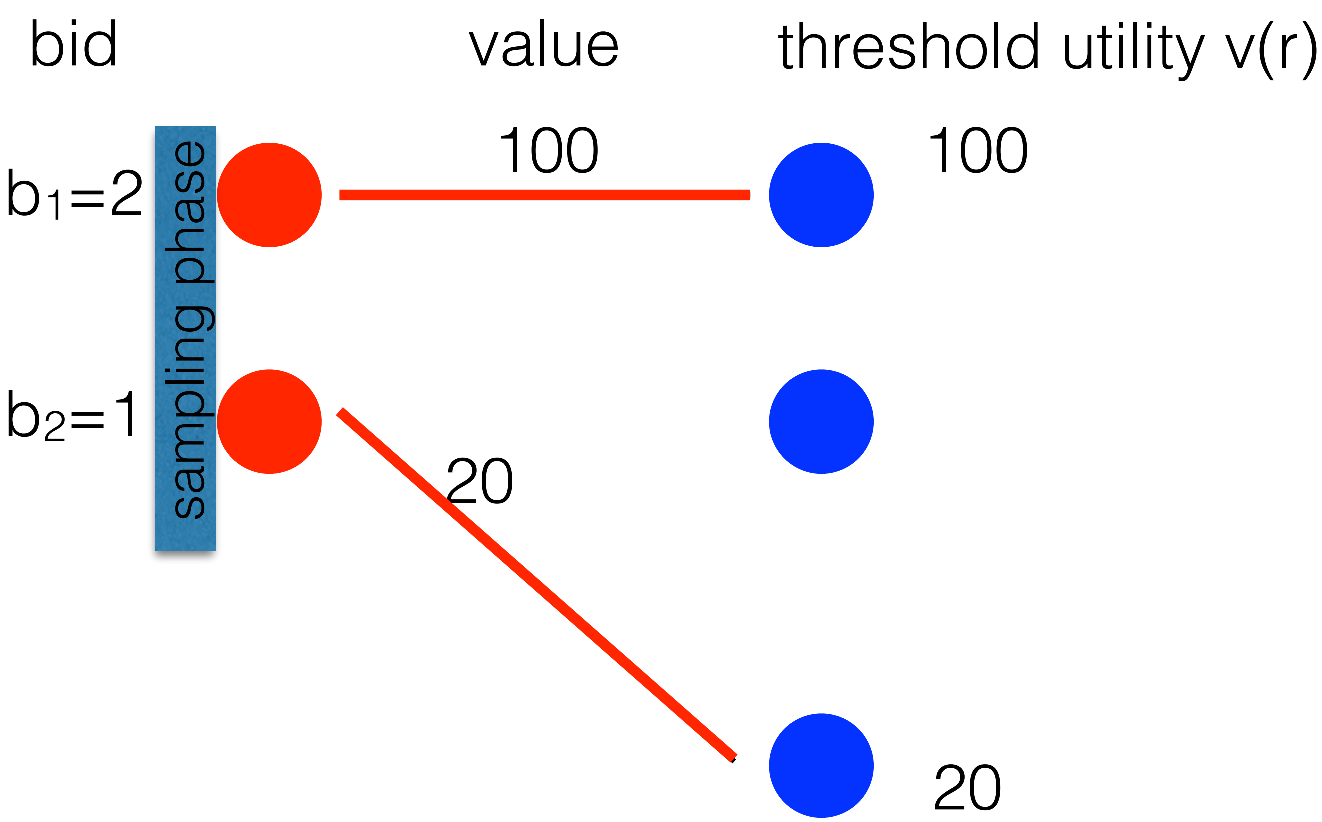
Our Offline Algorithm $\texttt{OFF}$



utility threshold v(r) of each blue node
to be its value in offline Matching

bid to benefit ratio of an edge $\frac{b(e)}{v(e)}$

Algorithm

good Graph $\mathrm{G}(\gamma) = \left\{ e \in G : \frac{b(e)}{v(e)} < \gamma \right\}$

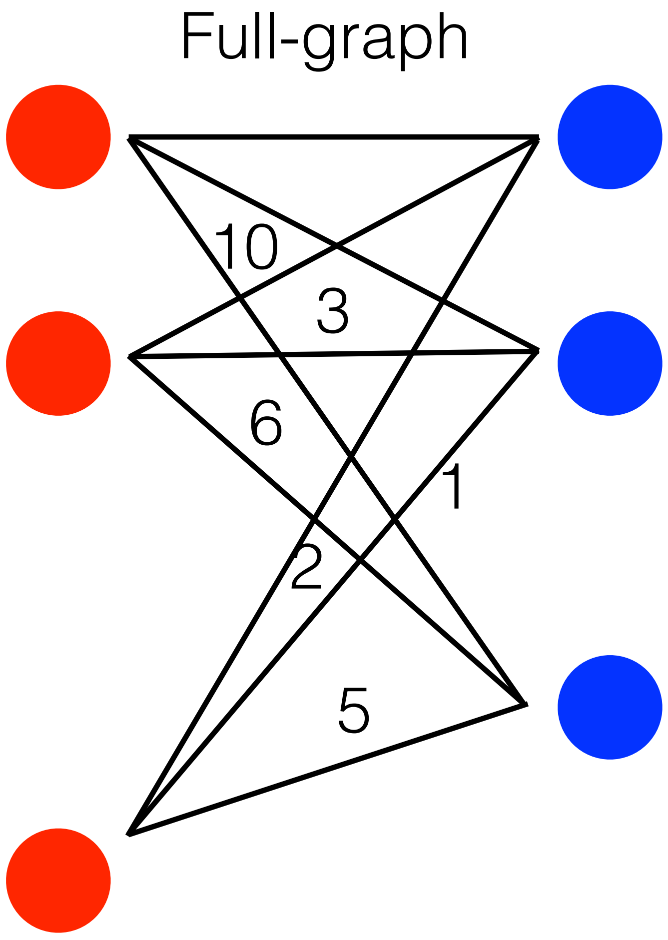
$\mathrm{M}(\gamma)$ be greedy matching over $G(\gamma)$

Find largest $\gamma \sum_{e \in M(\gamma)} v(e) \leq B$

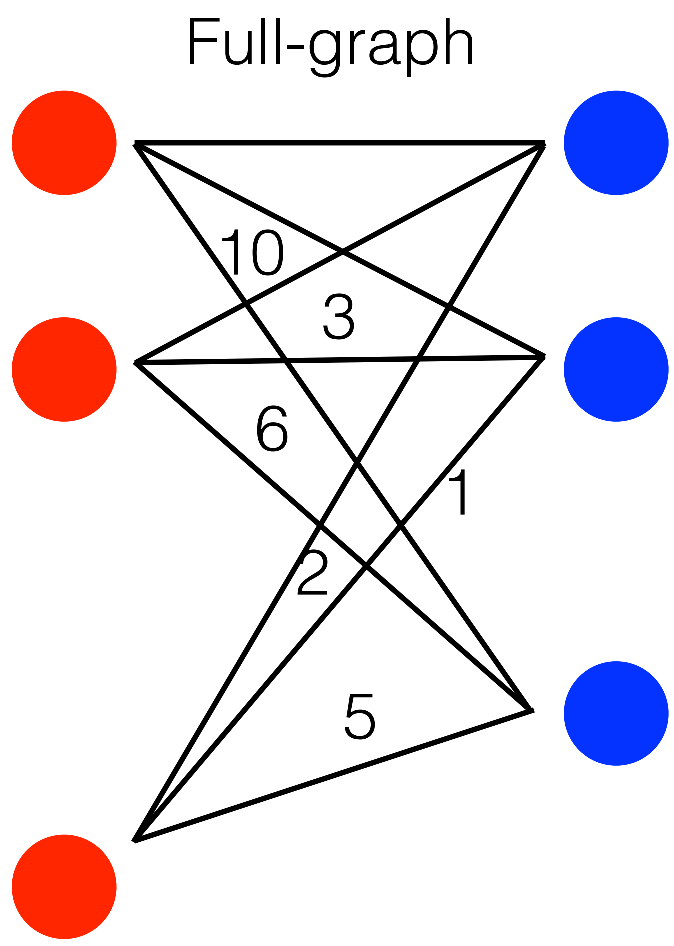
Claim : $\texttt{OFF}$ is 12 - approximate in expectation

Properties of `OFF`

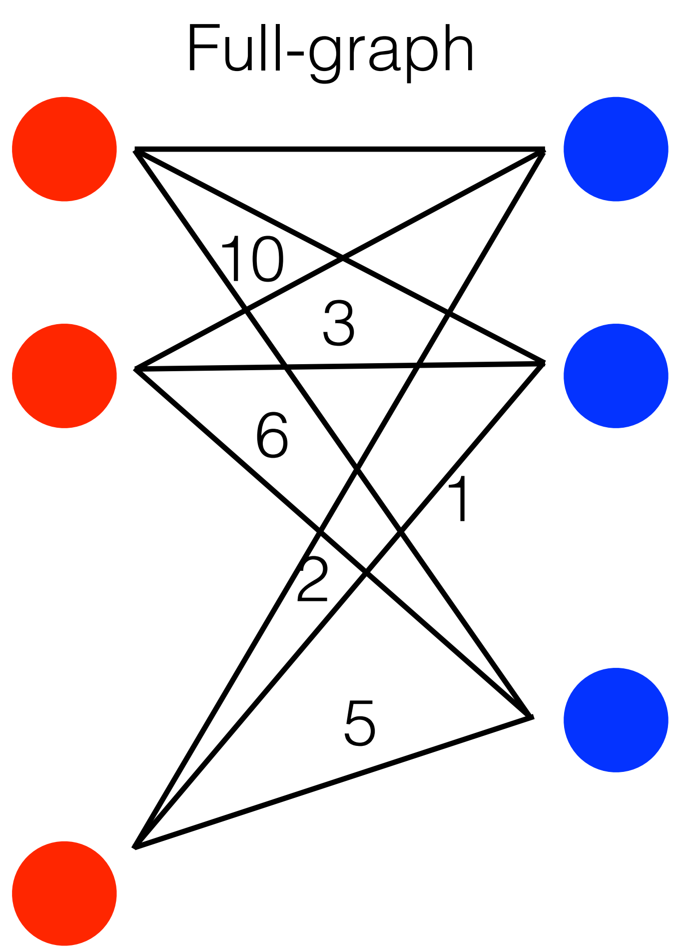
Properties of **OFF**



Properties of `OFF`



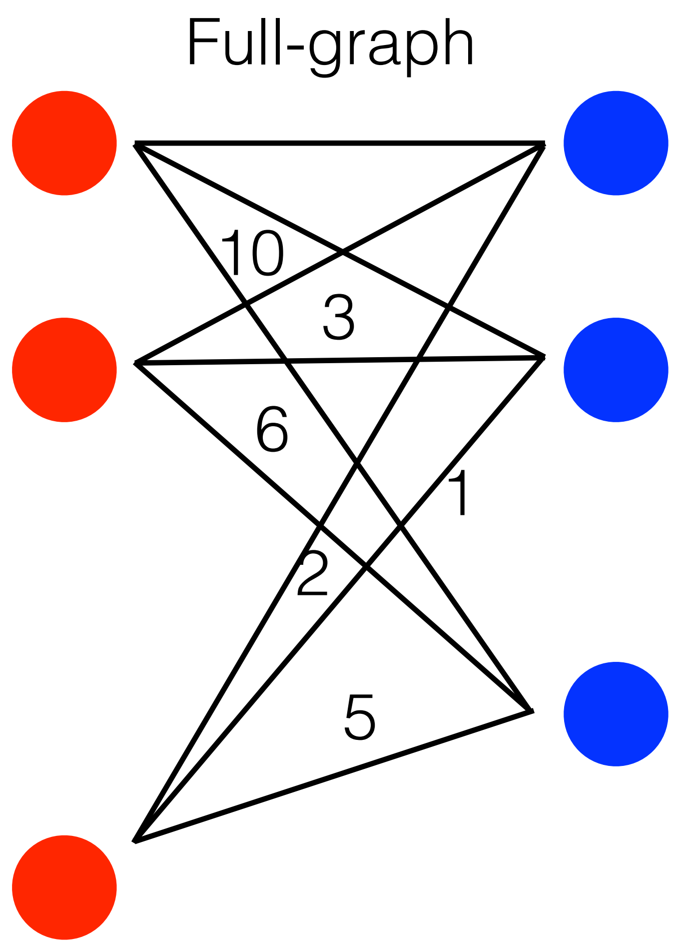
Properties of `OFF`



Greedy matching with edges

$$\boxed{\frac{b(e)}{v(e)}} < \gamma$$

Properties of `OFF`



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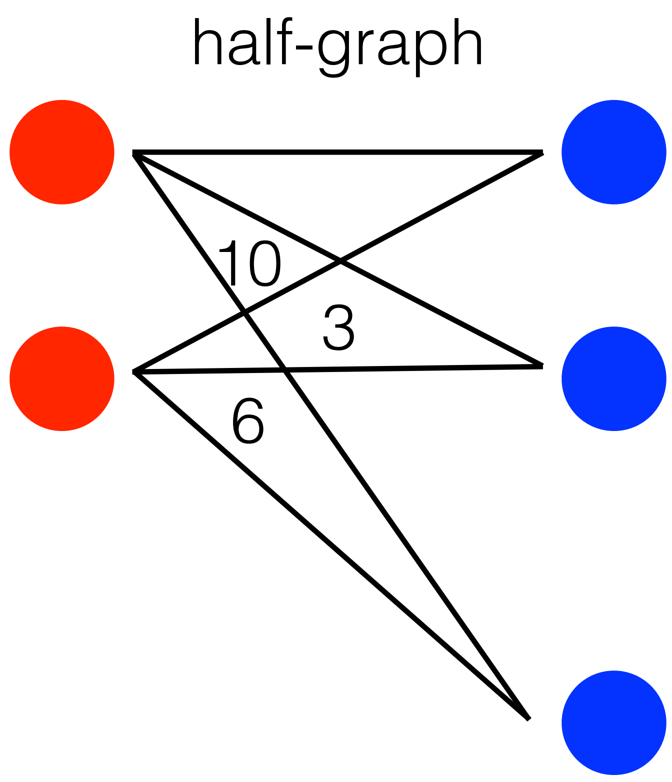
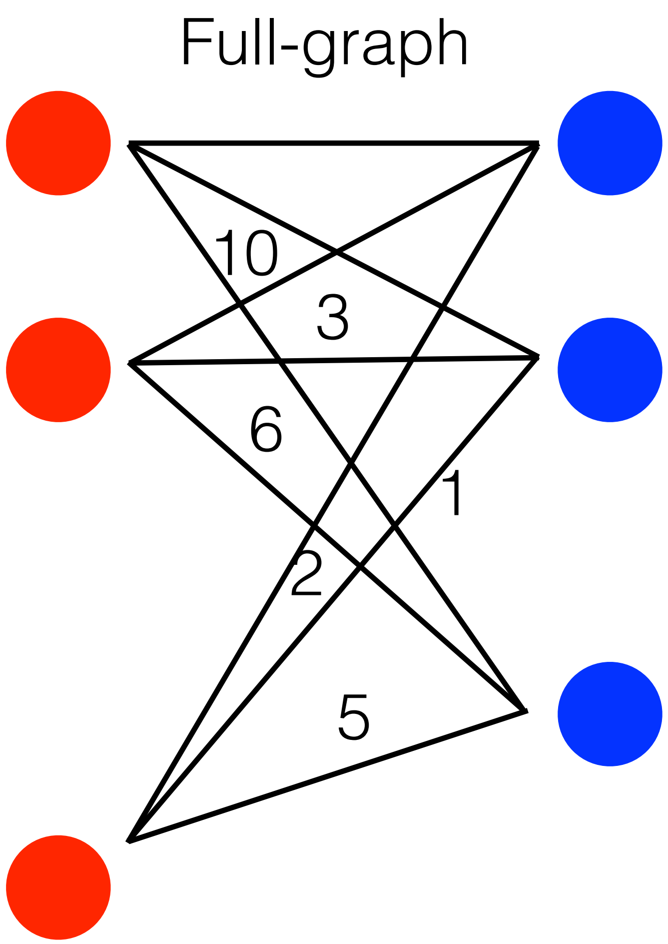


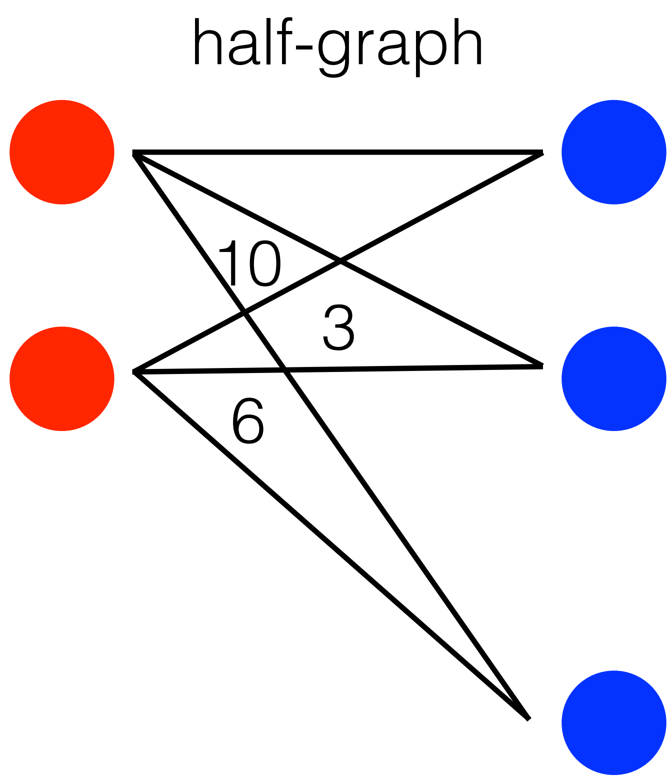
Properties of **OFF**



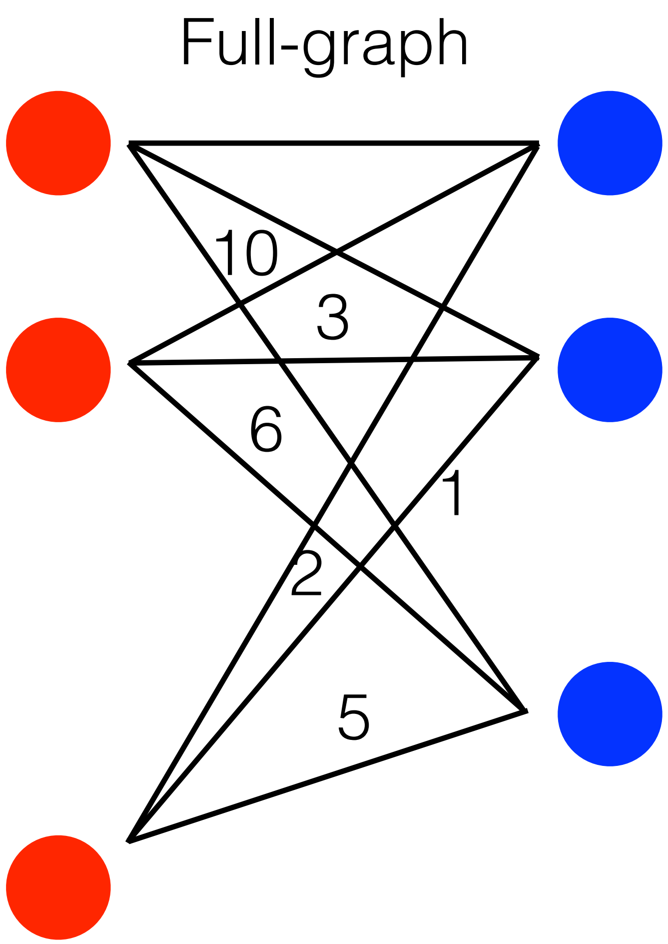
Greedy matching with edges

$$\frac{b(e)}{v(e)} < \gamma$$

$$\gamma_f \leq \gamma_h$$



Properties of **OFF**



Greedy matching with edges

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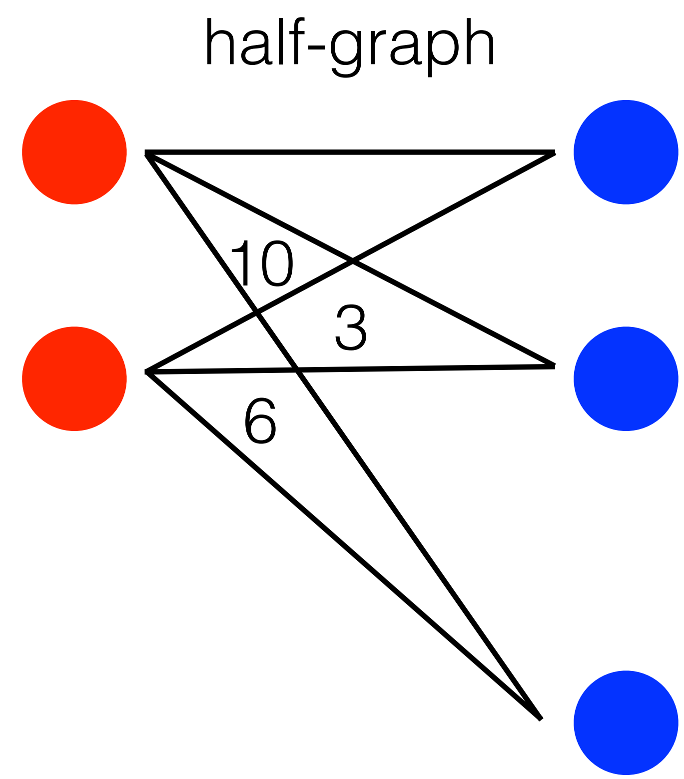
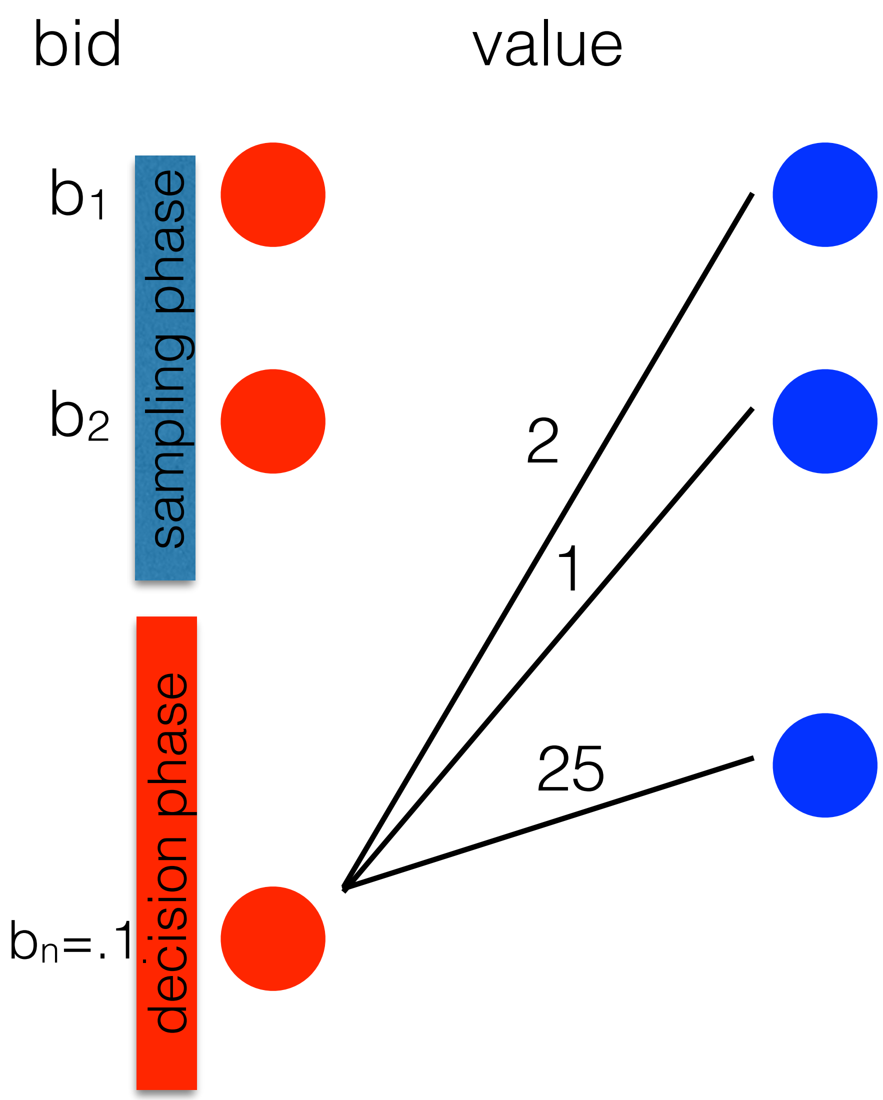


So all eligible edges in big graph are also eligible in small graph

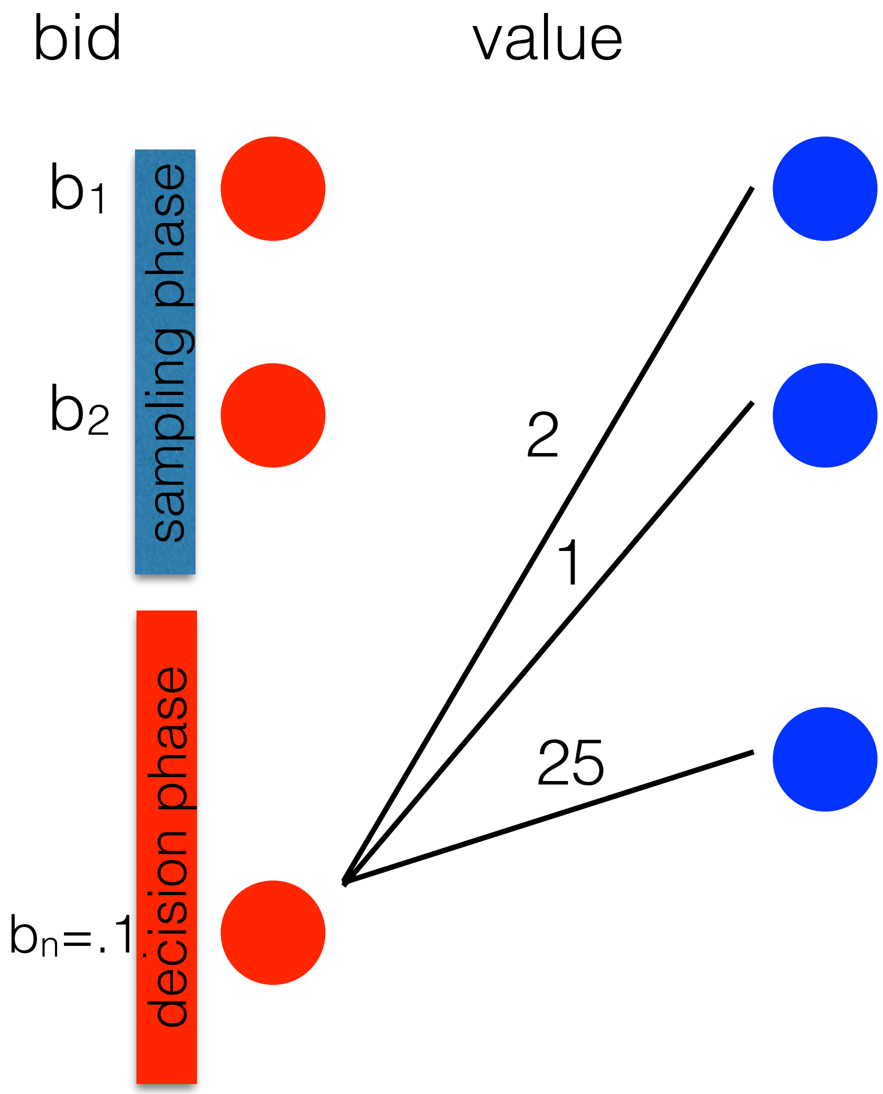
Algorithm - `ON`



bid to benefit ratio of an edge $\frac{b(e)}{v(e)}$

Algorithm - `ON`

γ and threshold utility v(r) obtained from `OFF` **with budget** B/β



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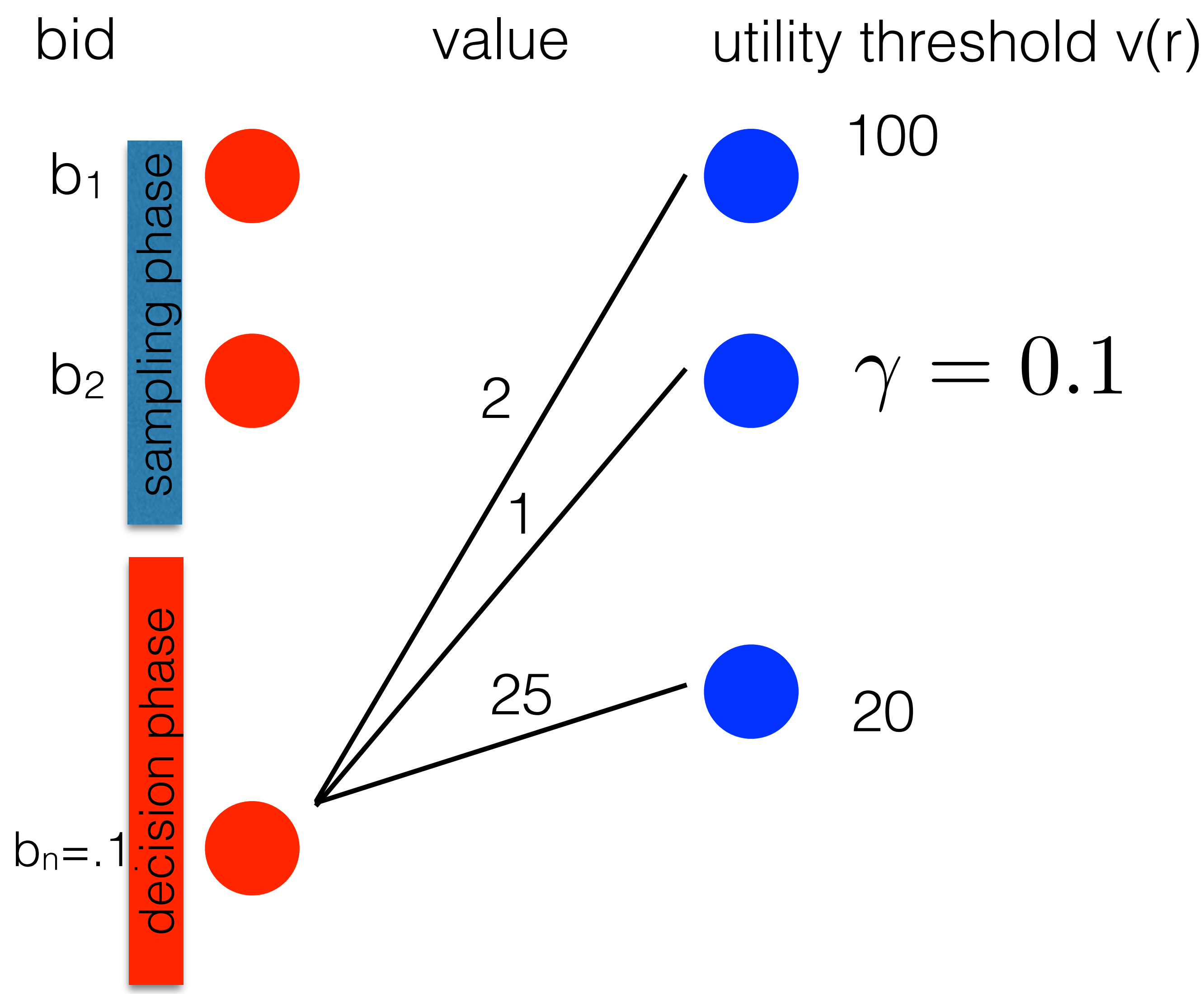

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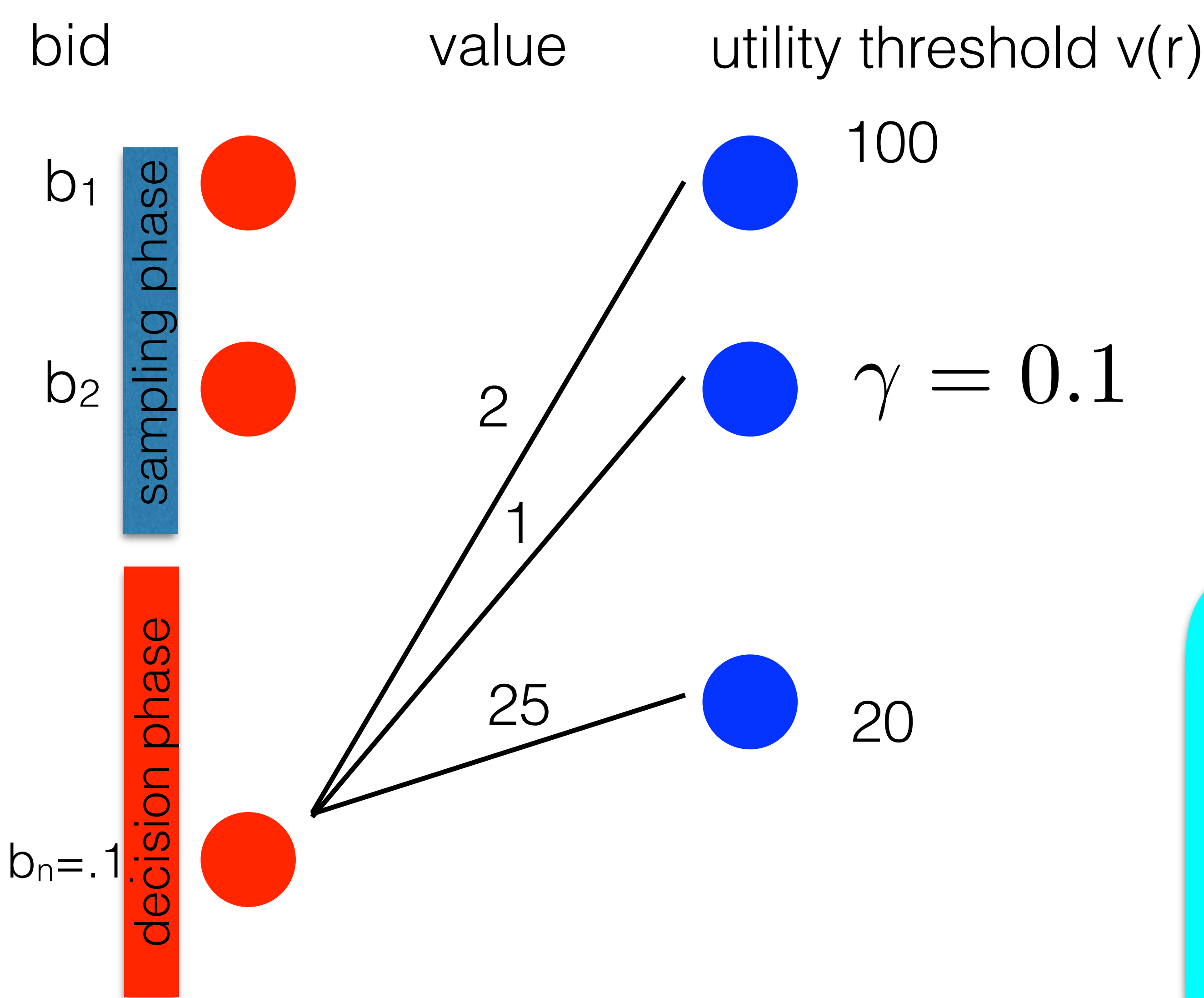
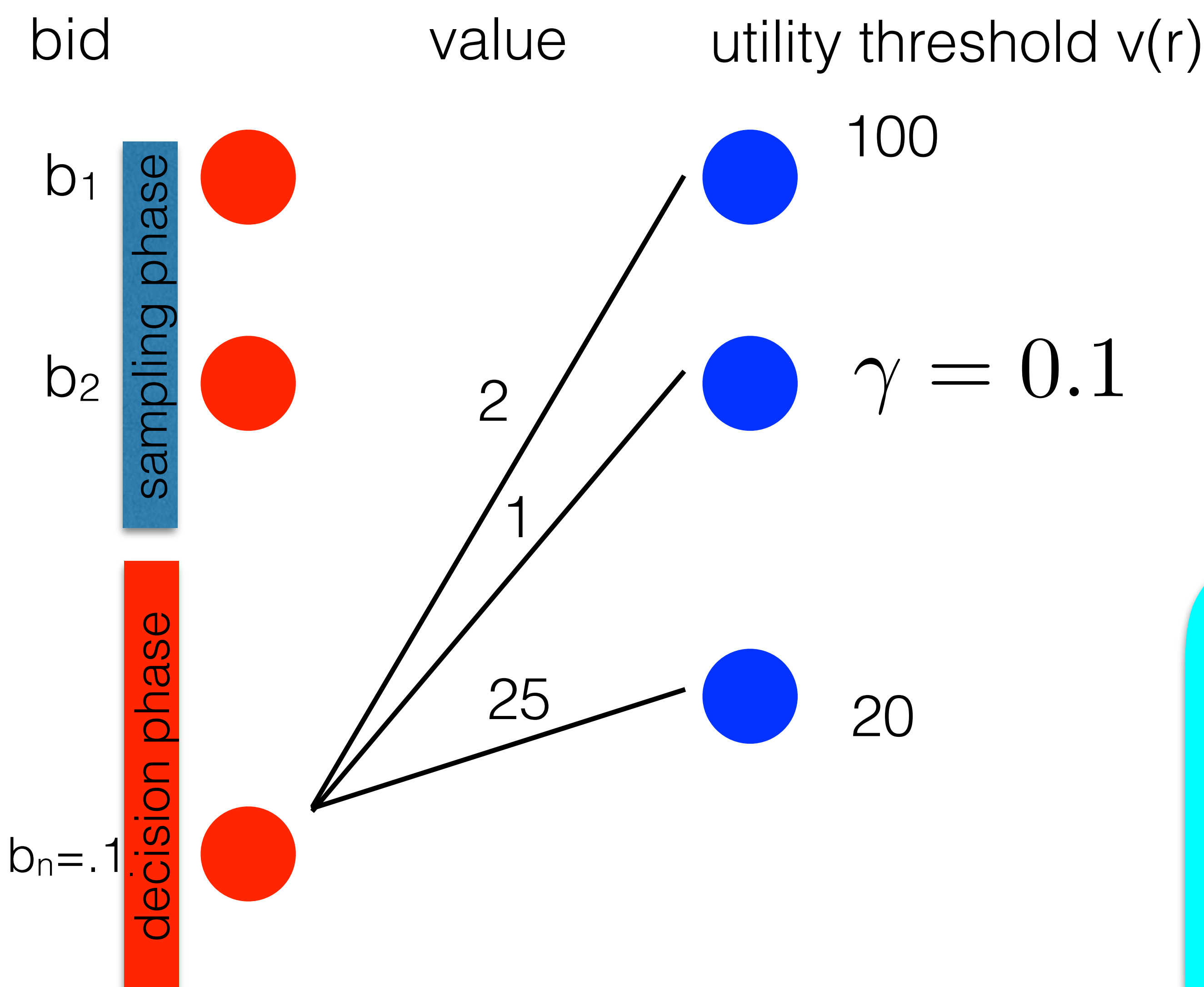


bid to benefit ratio of an edge $\frac{b(e)}{v(e)}$

on arrival of a new left vertex

Algorithm - ON

γ and threshold utility v(r) obtained from OFF **with budget** B/β



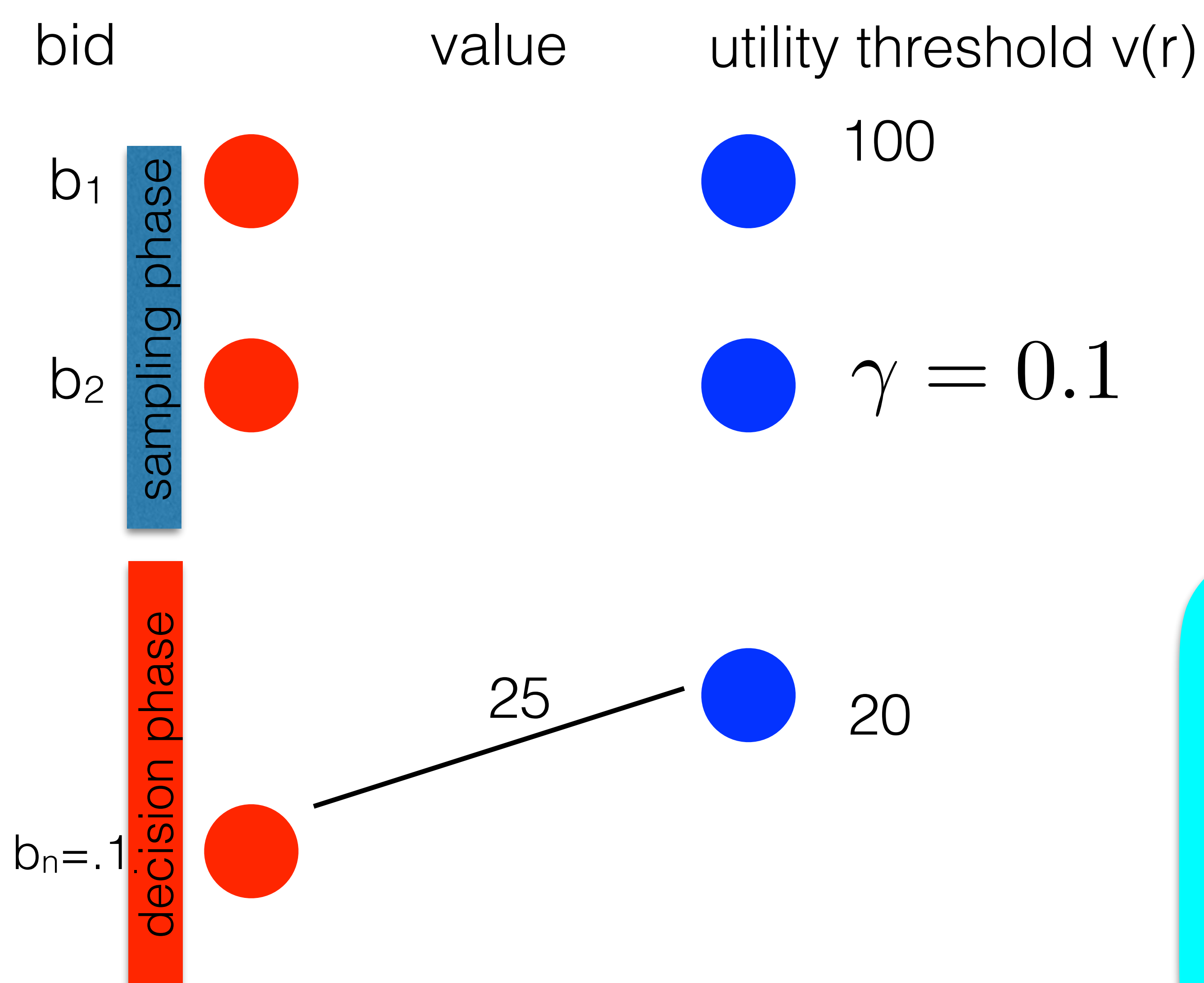
bid to benefit ratio of an edge $\frac{b(e)}{v(e)}$

on arrival of a new left vertex

remove all edges with **bid to benefit** ratio > γ

Algorithm - `ON`

γ and threshold utility v(r) obtained from `OFF` **with budget** B/β



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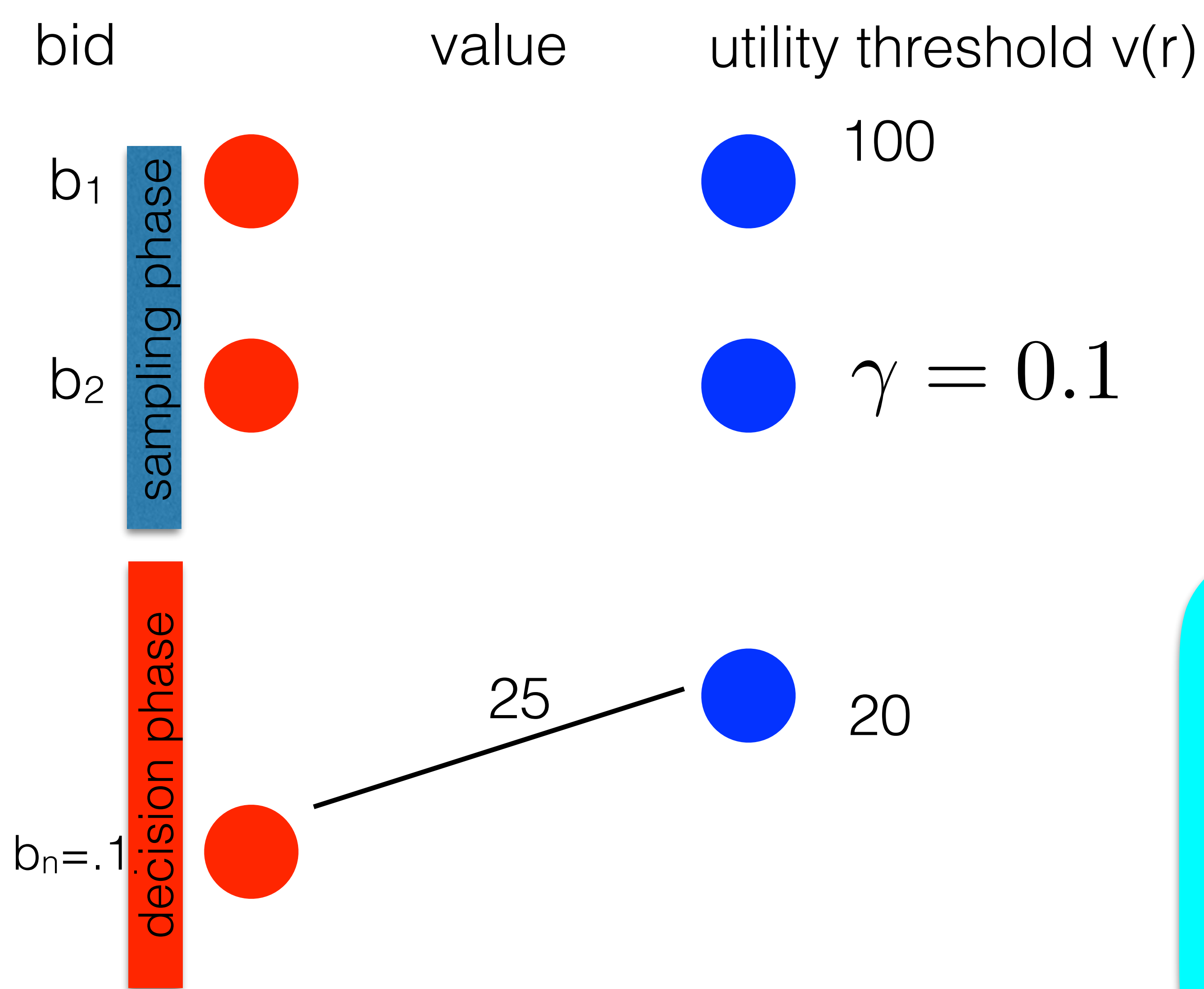


bid to benefit ratio of an edge $\frac{b(e)}{v(e)}$

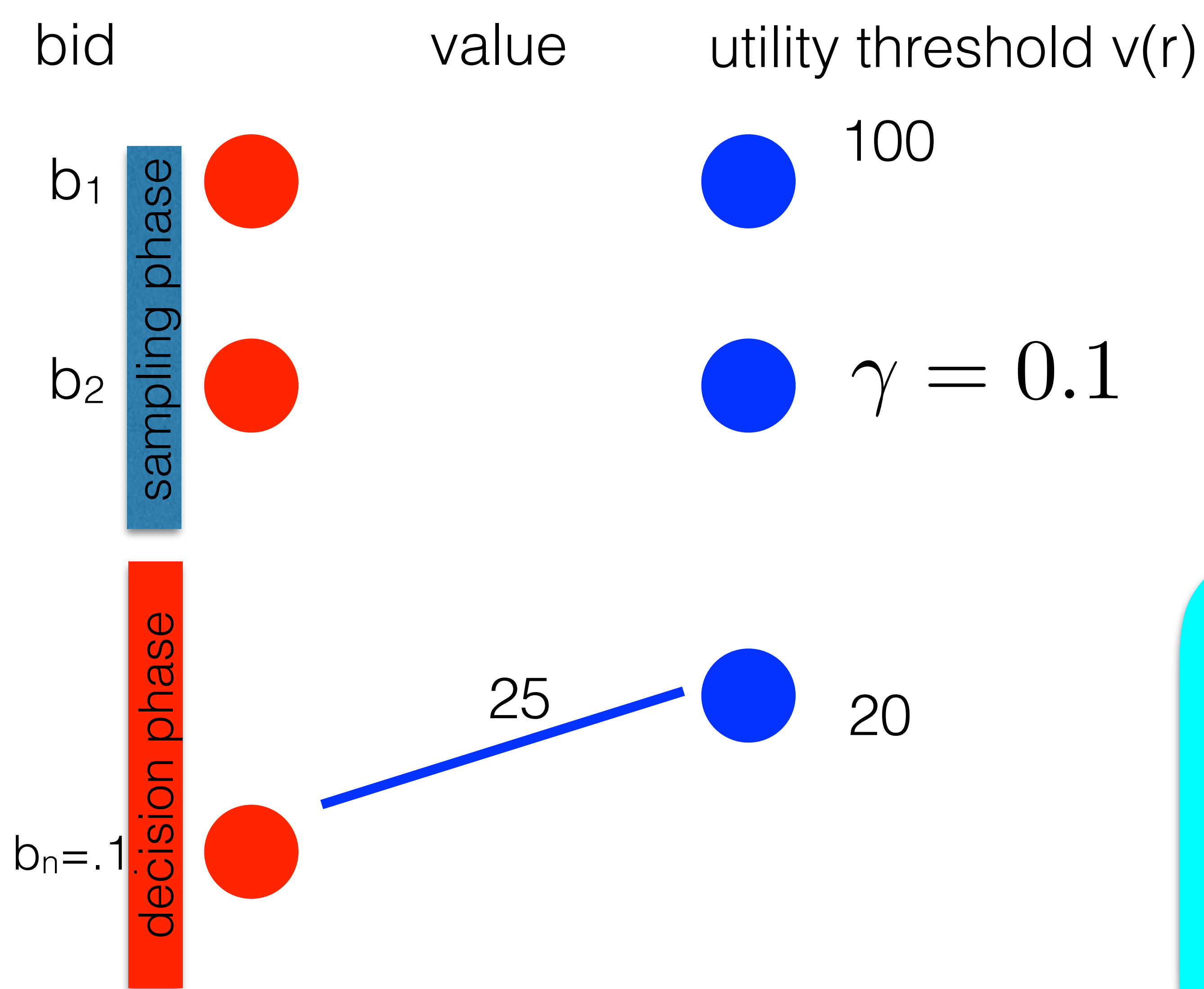
on arrival of a new left vertex

remove all edges with **bid to benefit** ratio > γ

match the right vertex with max w > utility thres.

Algorithm - ON

γ and threshold utility v(r) obtained from **OFF** **with budget** B/β



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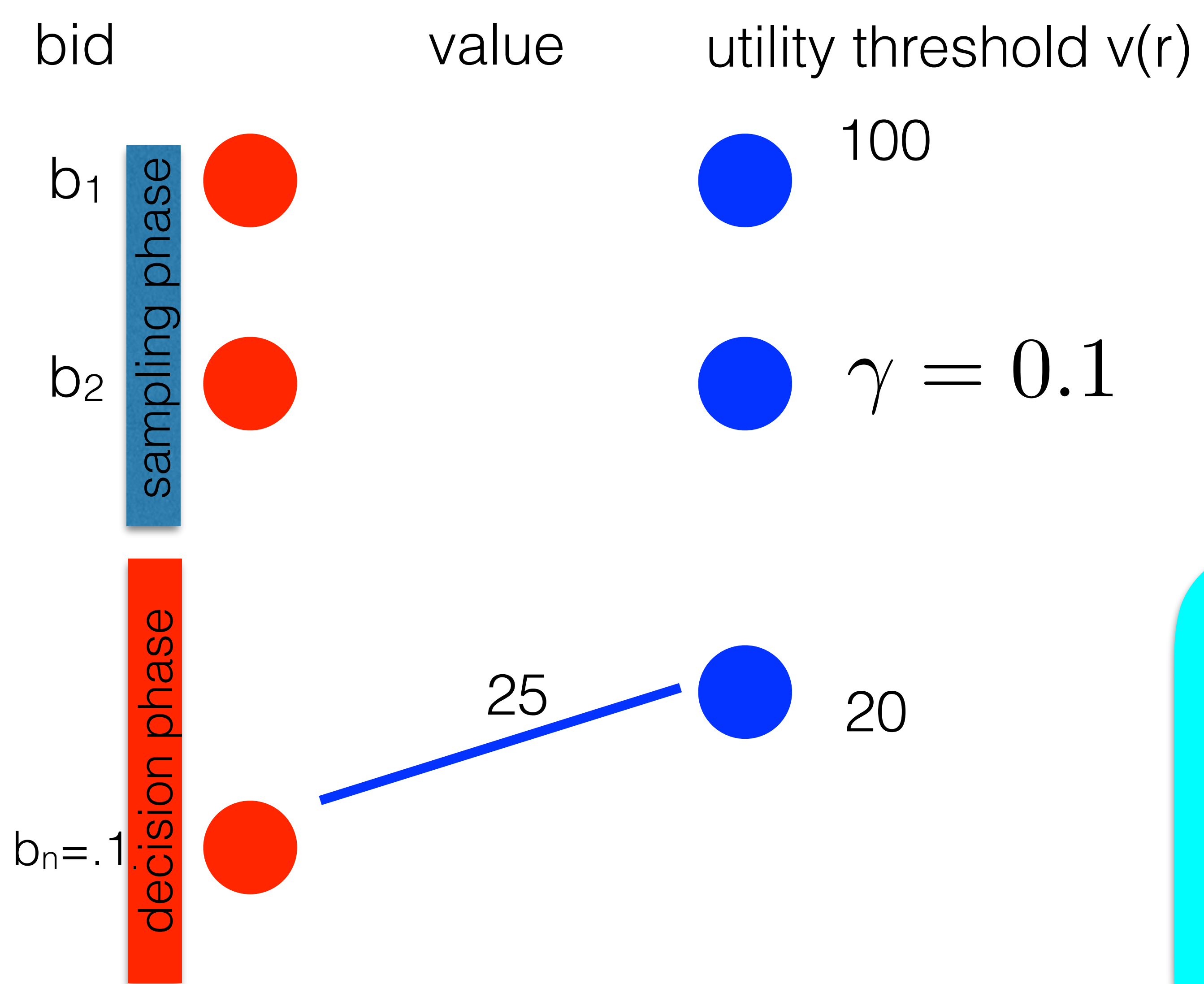
Algorithm - **ON**

γ and threshold utility v(r) obtained from **OFF** with budget B/β



bid to benefit ratio of an edge $\frac{b(e)}{v(e)}$

on arrival of a new left vertex

remove all edges with **bid to benefit** ratio > γ

match the right vertex with max w > utility thres.

Payment for selected left vertex $p_\ell = \beta\gamma v(r)$

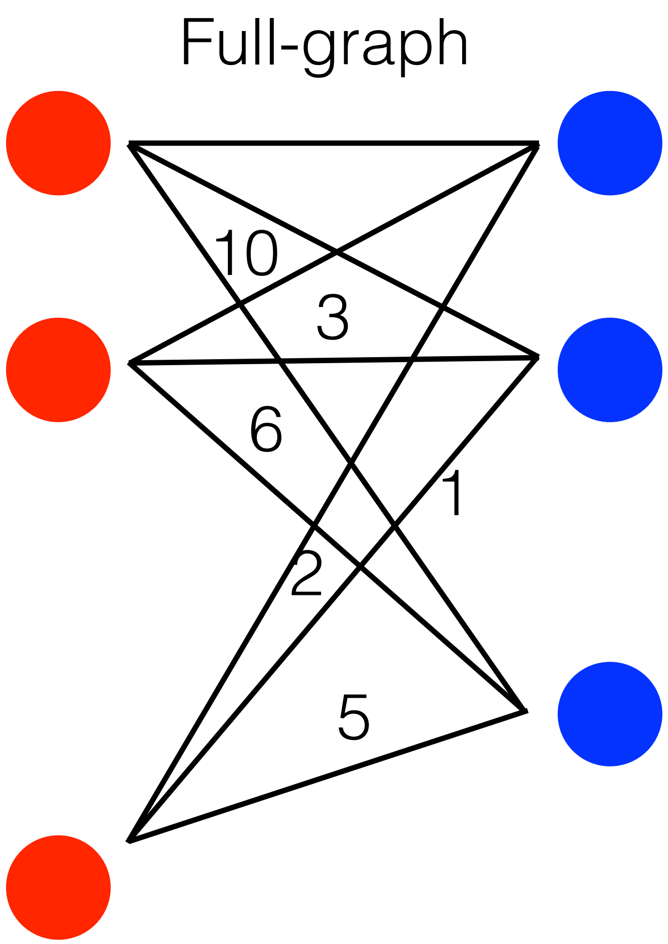
Properties of **`ON`**

24β - competitive in expectation

is truthful

satisfies the payment budget constraint

Properties of $\texttt{OFF}$



$$u(M_{\mathrm{OFF}}(\mathrm{full})) \geq \frac{u(\mathrm{OPT_{full}})}{3}$$

Algorithm

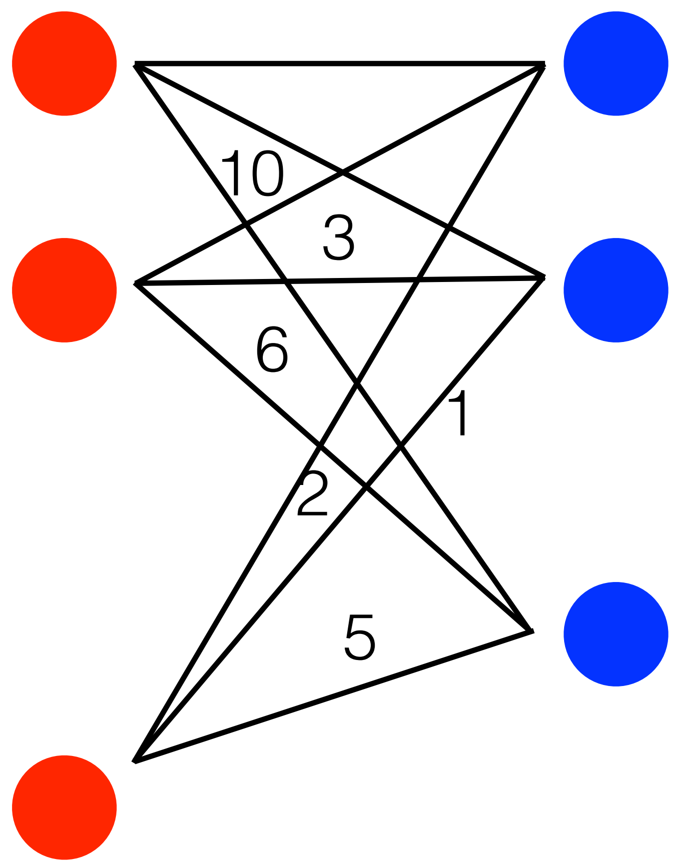
good Graph $\mathrm{G}(\gamma) = \left\{ e \in G : \frac{b(e)}{v(e)} < \gamma \right\}$

$\mathrm{M}(\gamma)$ be greedy matching over $G(\gamma)$

Find largest $\gamma \sum_{e \in M(\gamma)} v(e) \leq B$

Properties of **OFF**

Full-graph



$$u(M_{\text{OFF}}(\text{full})) \geq \frac{u(\text{OPT}_{\text{full}})}{3}$$

$$\text{OPT} = \{\text{OPT}^+, \text{OPT}^-\}$$

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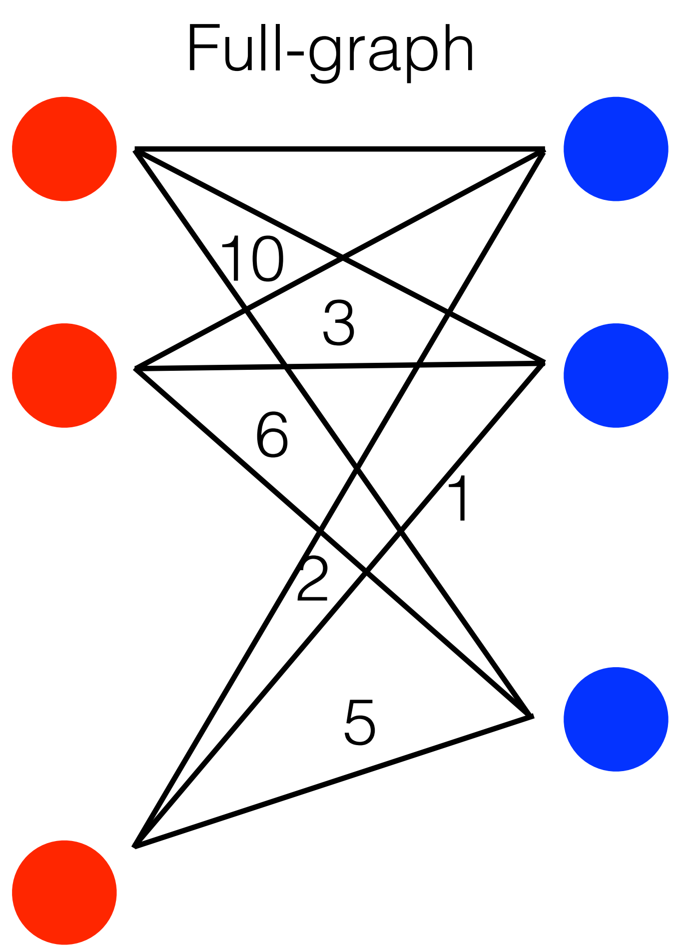

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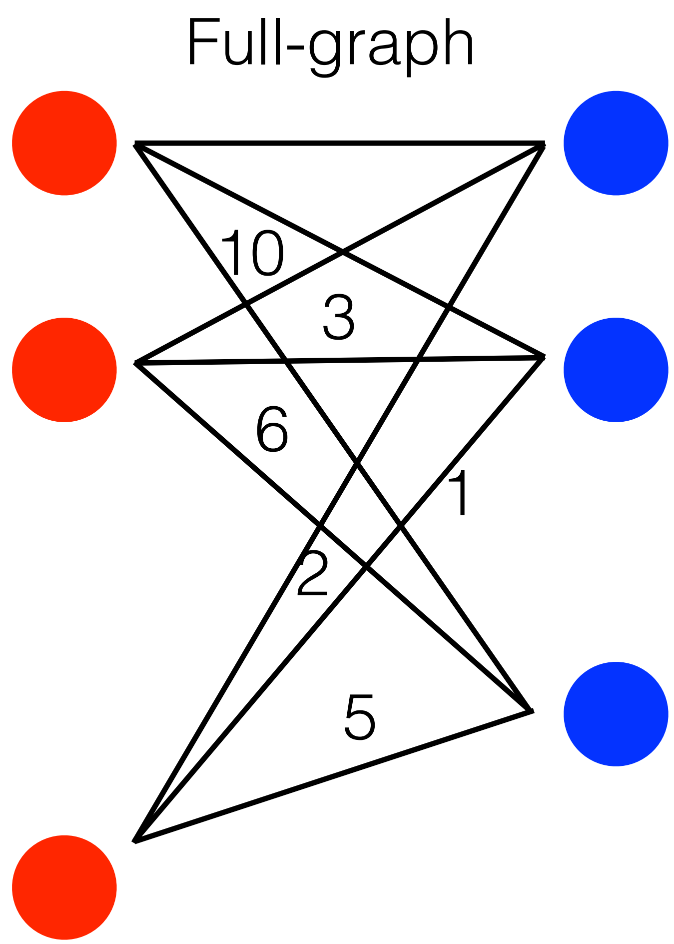

$$u(M_{\text{OFF}}(\text{full})) \geq \frac{u(\text{OPT}_{\text{full}})}{3}$$

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$M_{\text{OFF}}(\text{full})$ is a greedy matching over the same set

Algorithm

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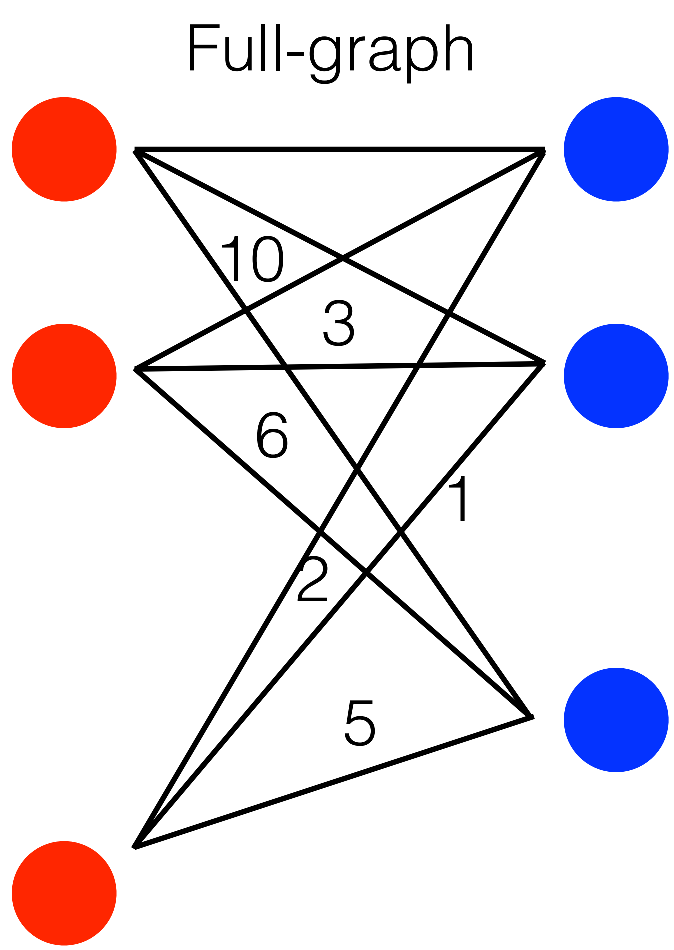
Properties of OFF



$$u(M_{\text{OFF}}(\text{full})) \geq \frac{u(\text{OPT}_{\text{full}})}{3}$$

$$\text{OPT} = \{\text{OPT}^{+}, \text{OPT}^{-}\}$$

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$M_{\text{OFF}}(\text{full})$ is a greedy matching over the same set

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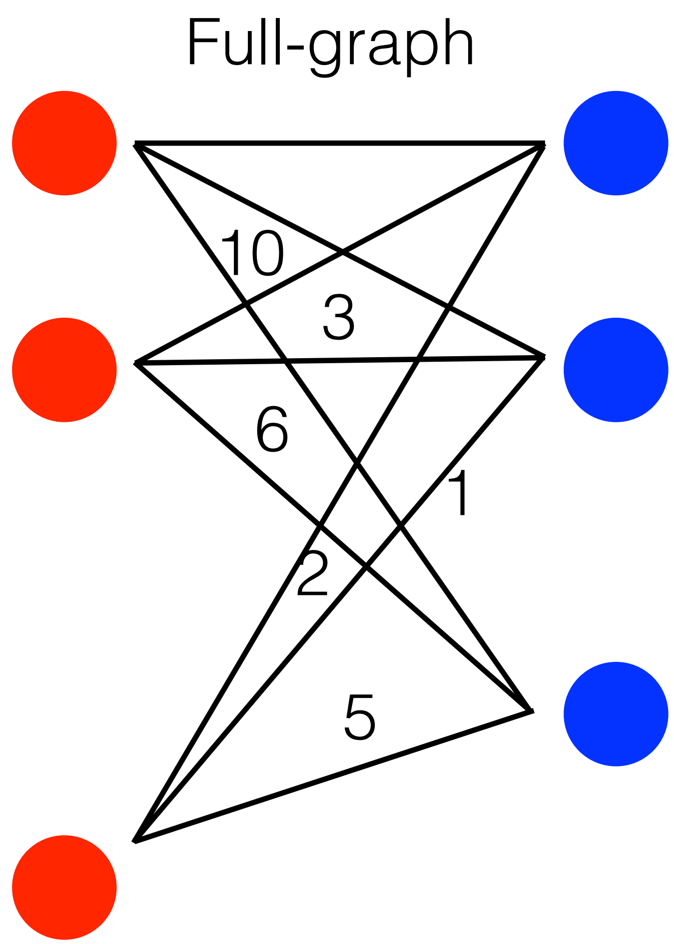

$$\color{red}{u(M_{\text{OFF}}(\text{full})) \geq \frac{u(\text{OPT}_{\text{full}})}{3}}$$

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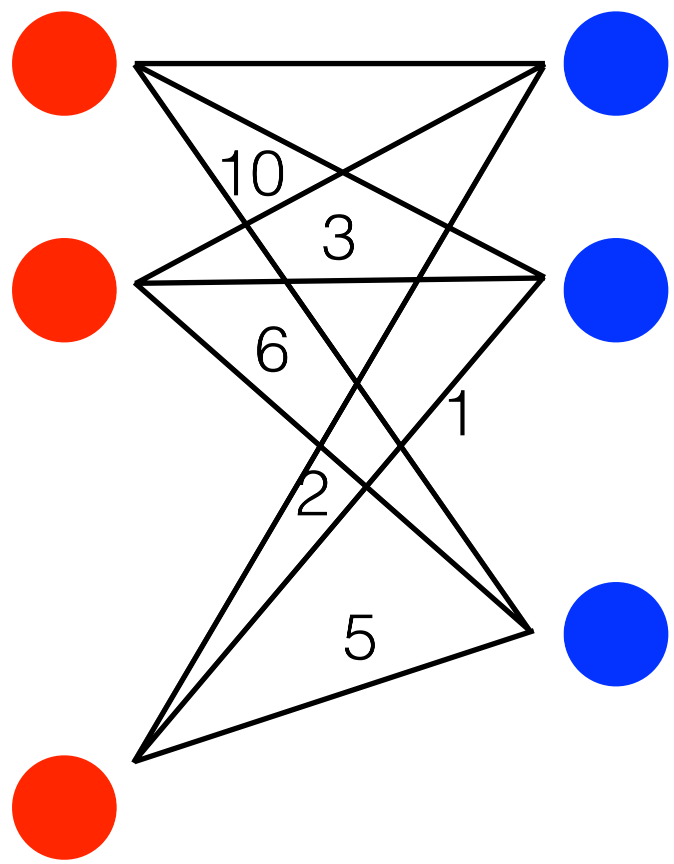

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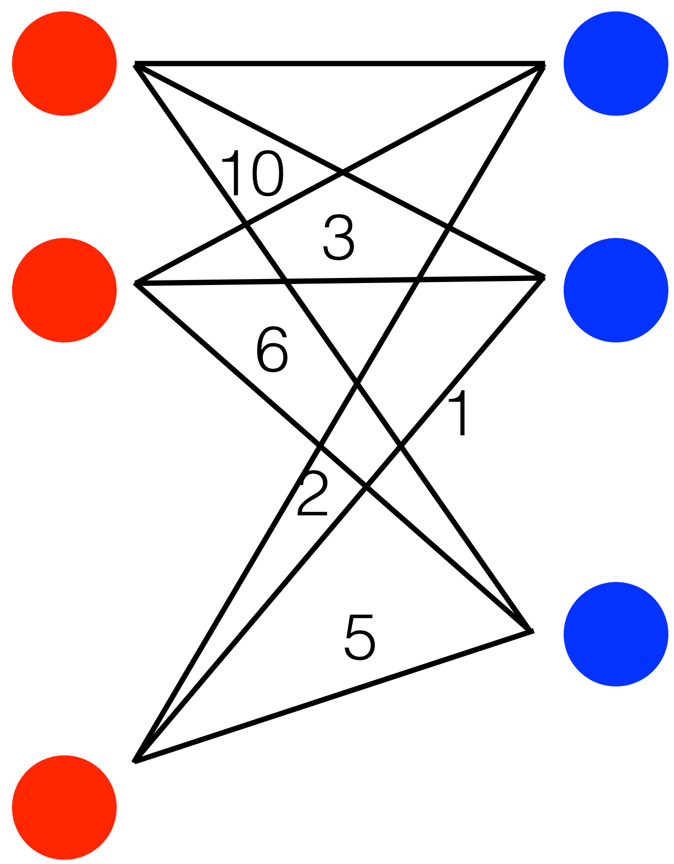
Properties of **OFF**



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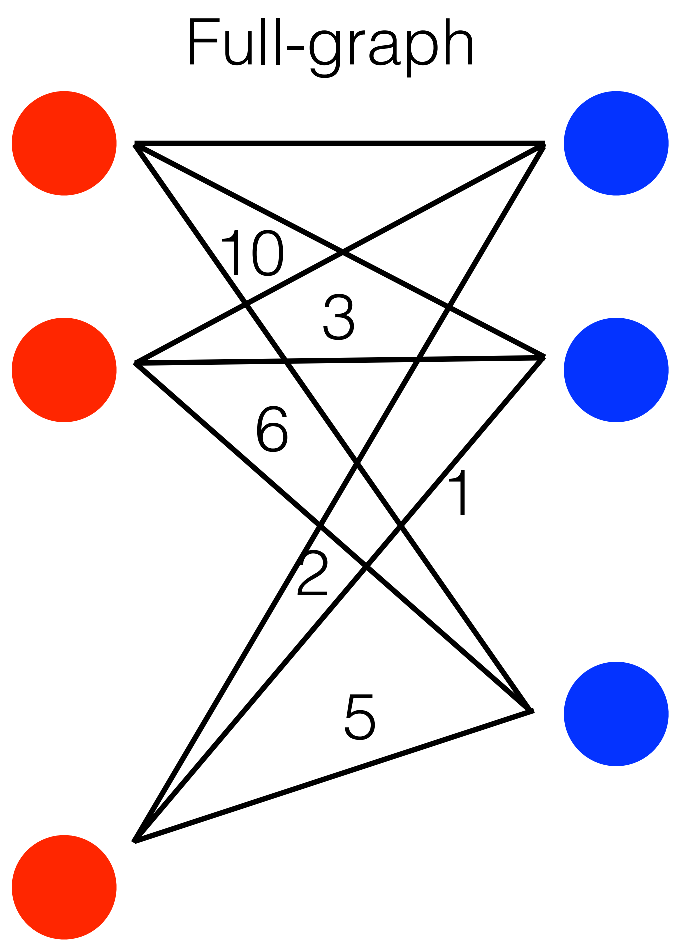
Properties of OFF



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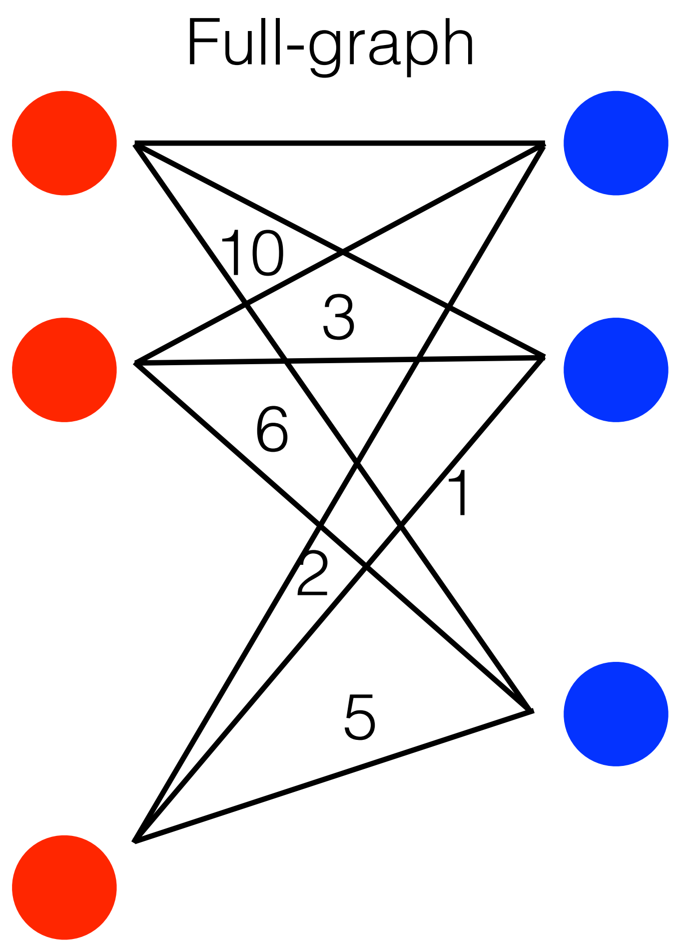
$$= u(M_{\mathrm{OFF}}(\text{full}))$$

Properties of **OFF**



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Properties of `OFF`

12 - approximate in expectation in offline phase

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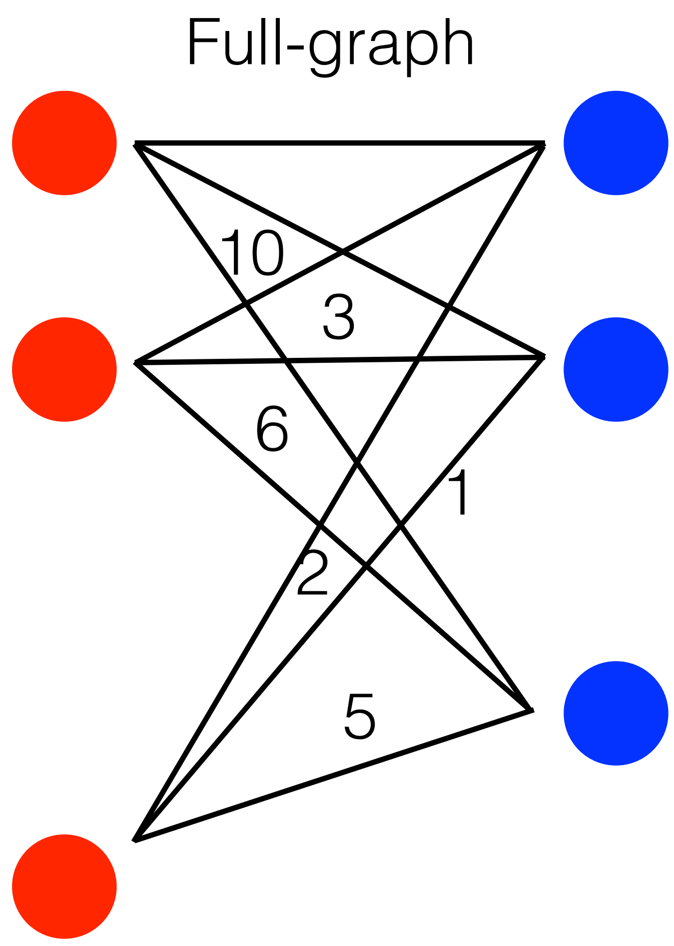

Properties of **OFF**

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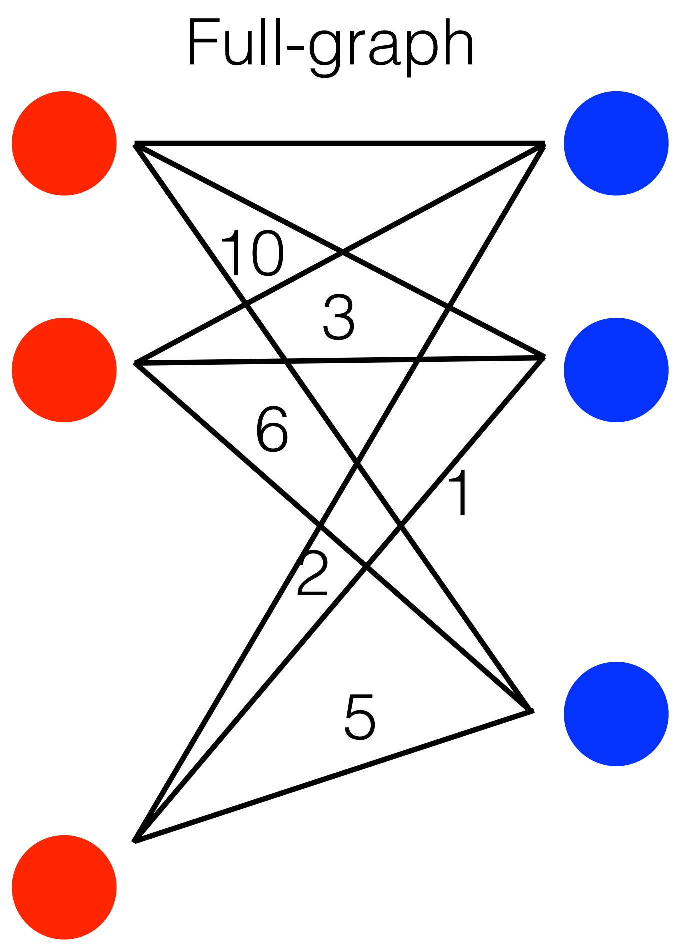

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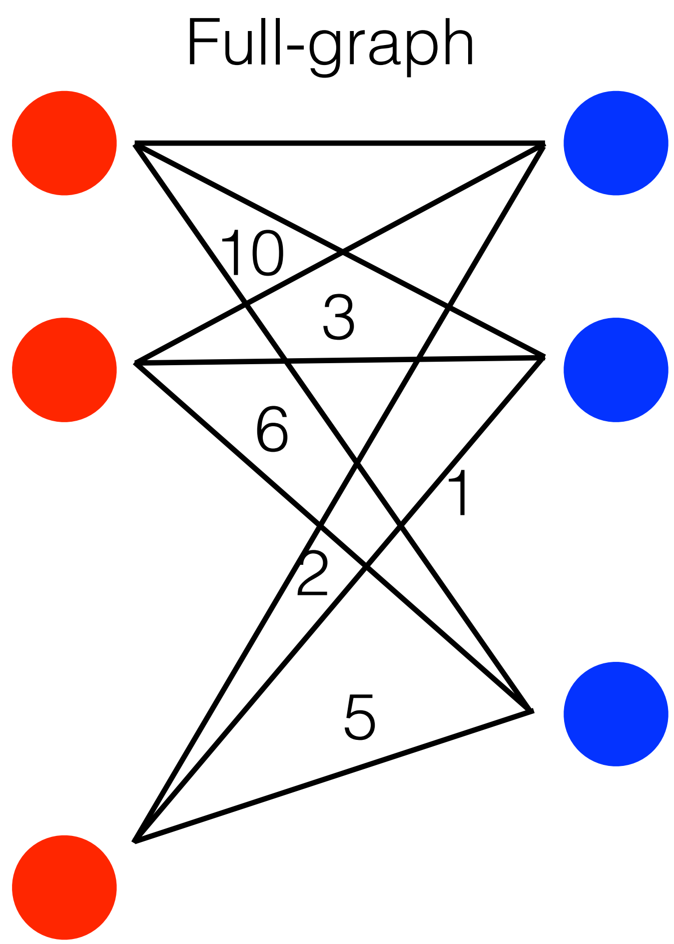

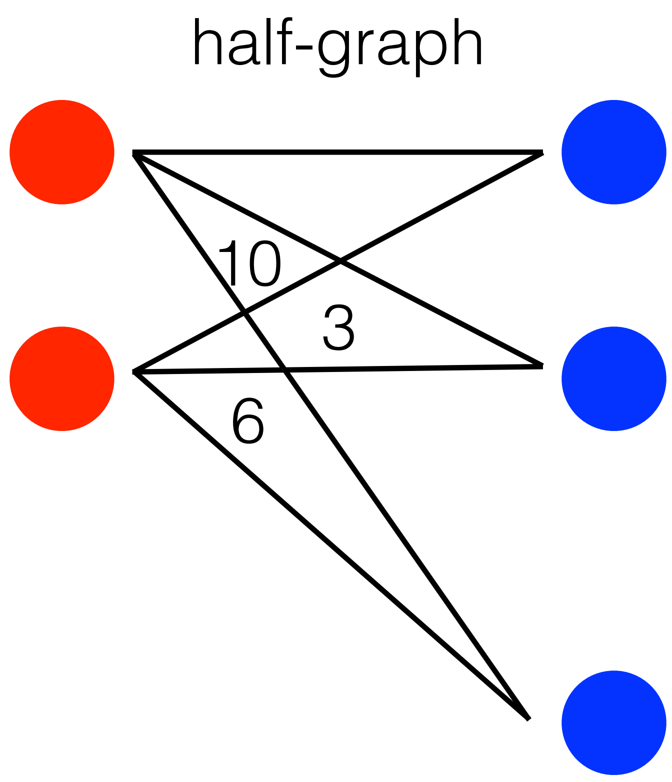

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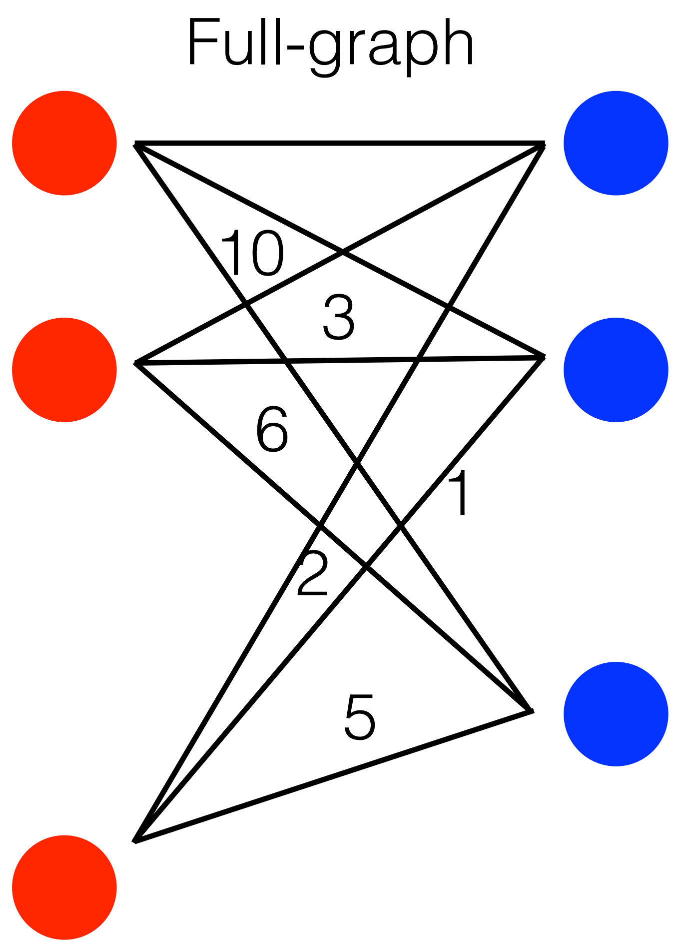

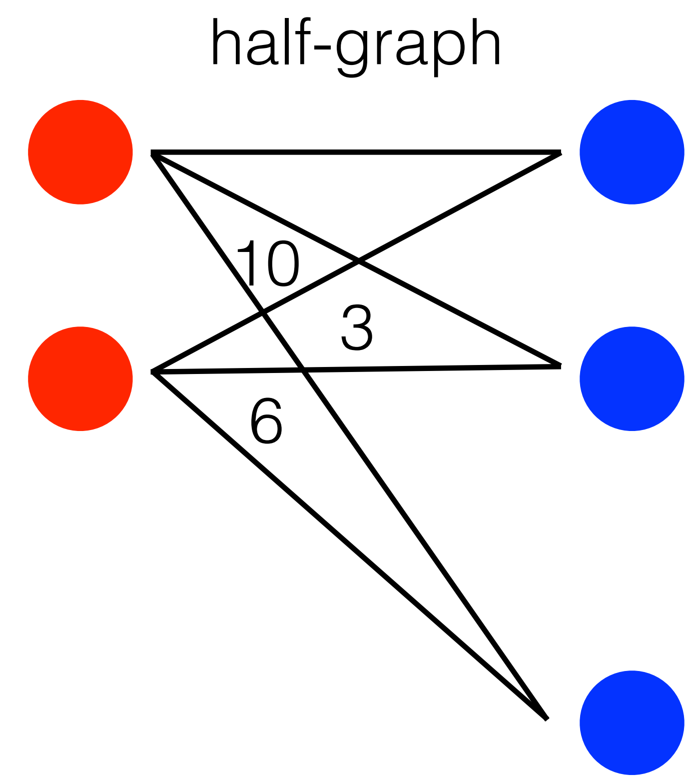

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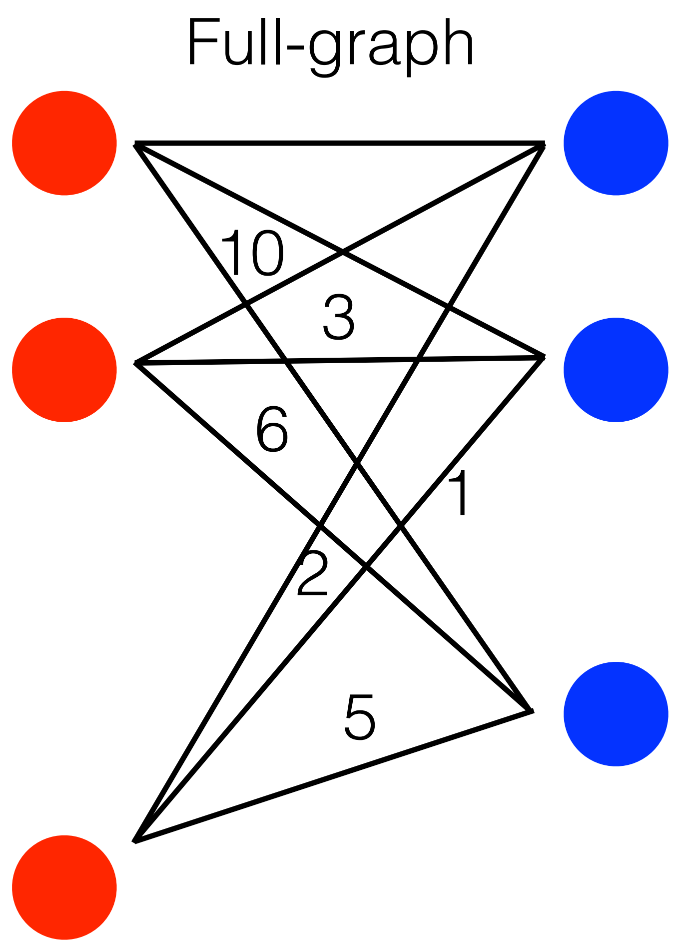

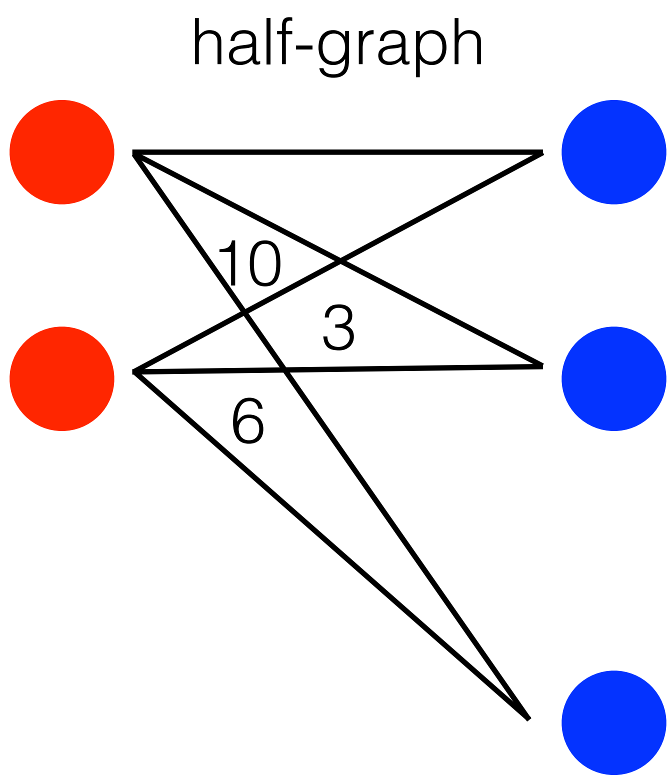

$$u(\mathrm{OPT}_{\mathrm{half}}) \geq u(M_{\mathrm{OFF}}(\mathrm{full}, \mathrm{half}))$$

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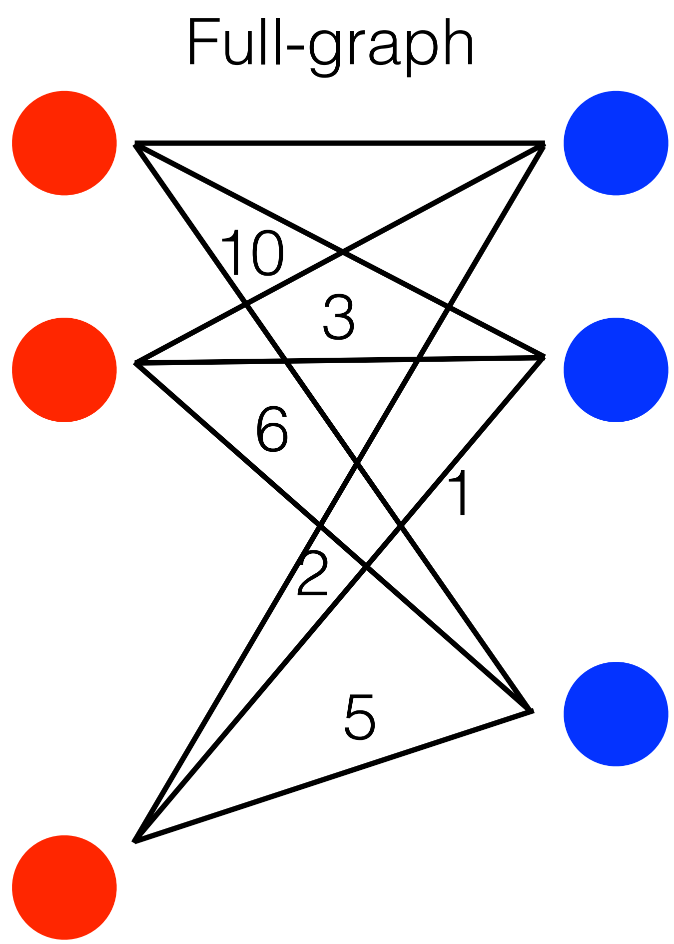

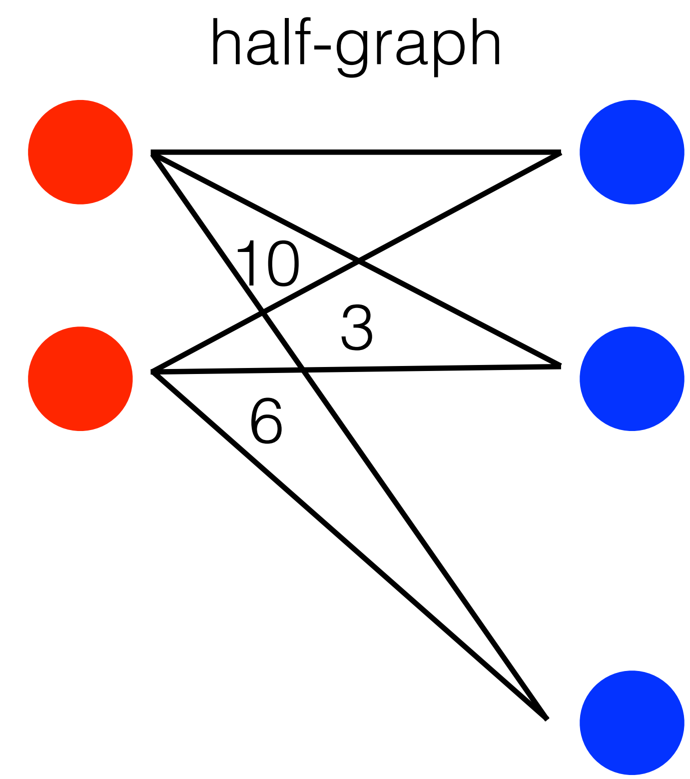

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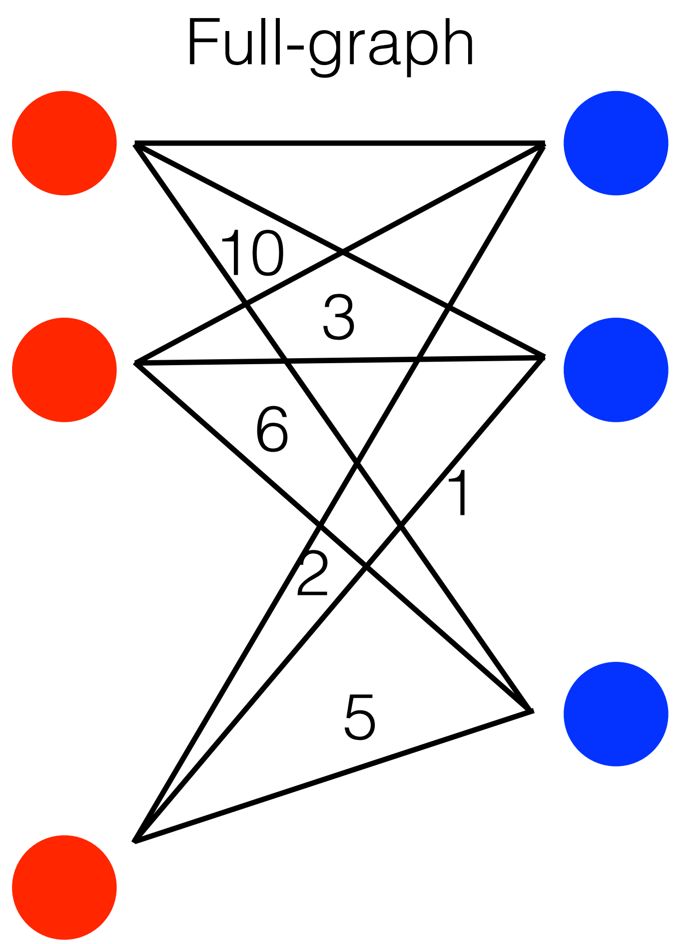
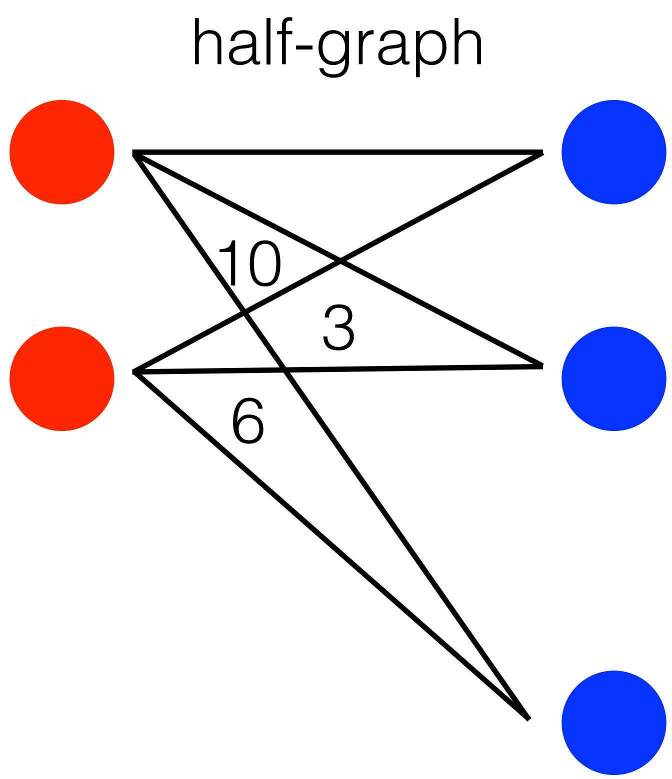


$\gamma_f \leq \gamma_h$



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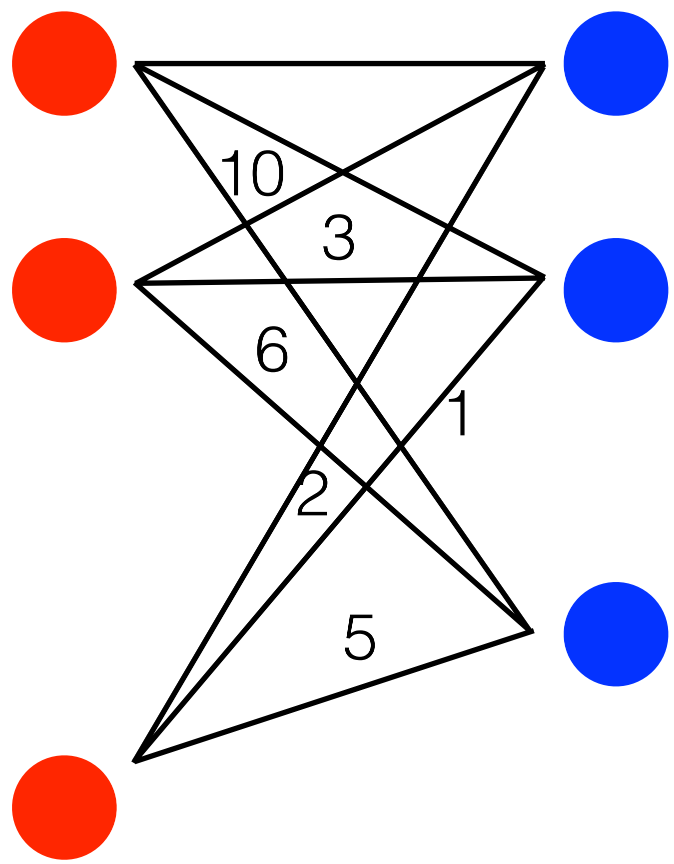

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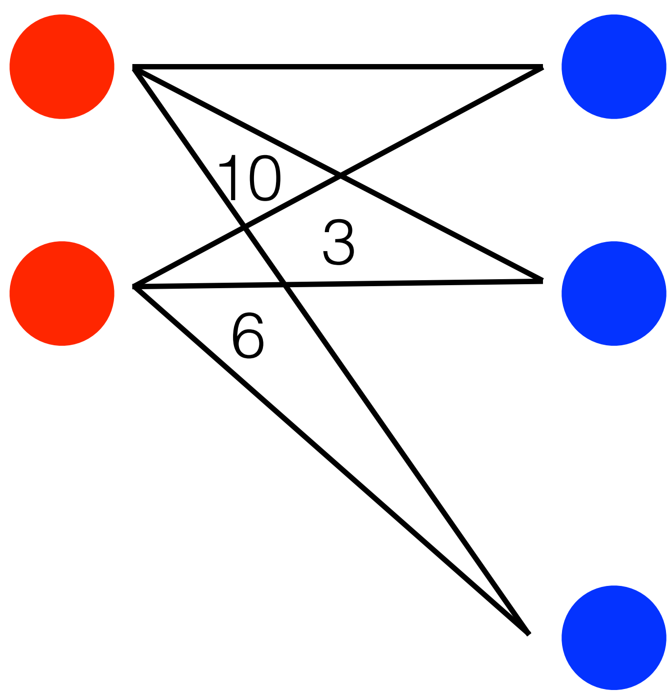

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Properties of `OFF`

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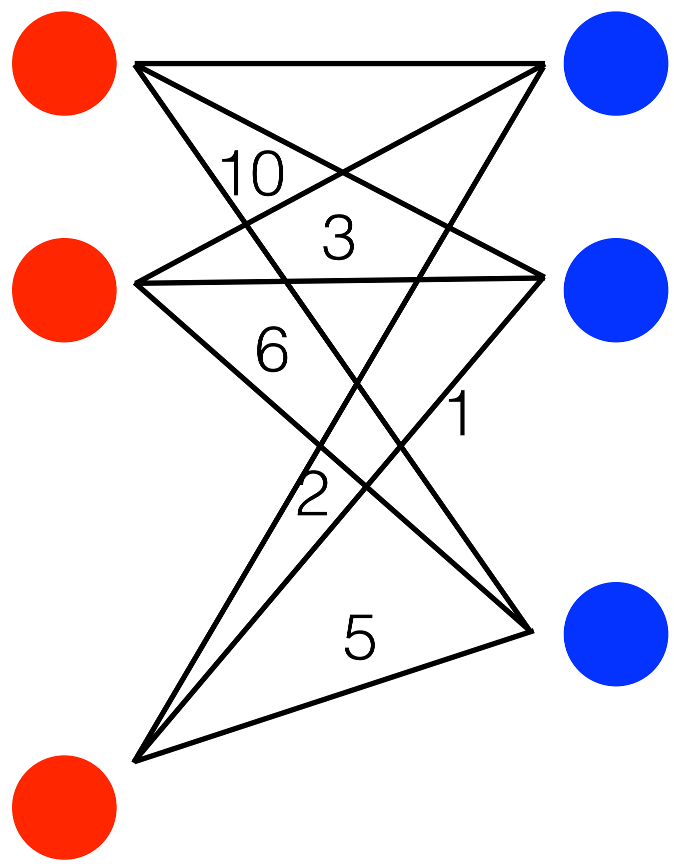
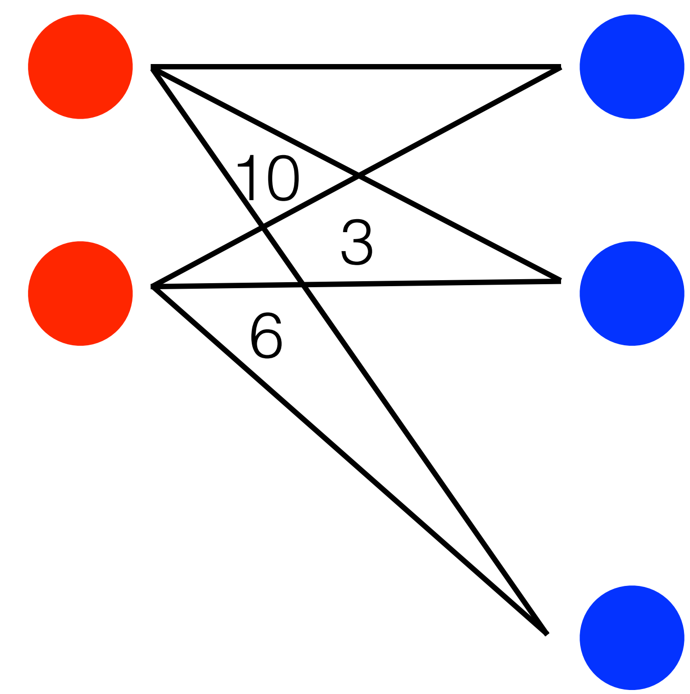
Full-graph



$\gamma_f \le \gamma_h$

half-graph



$$u(\mathrm{OPT_{half}}) \ge u(M_{\mathrm{OFF}}(\mathrm{full},\mathrm{half}))$$

$$\mathbf{E}\left\{u(M_{\mathrm{OFF}}(\mathrm{full},\mathrm{half}))\right\} \ge \frac{u(M_{\mathrm{OFF}}(\mathrm{full}))}{2}$$

$$u(M_{\mathrm{OFF}}(\mathrm{half})) \ge \frac{u(\mathrm{OPT_{half}})}{2}$$

$$u(M_{\mathrm{OFF}}(\mathrm{full})) \ge \frac{u(\mathrm{OPT_{full}})}{3}$$

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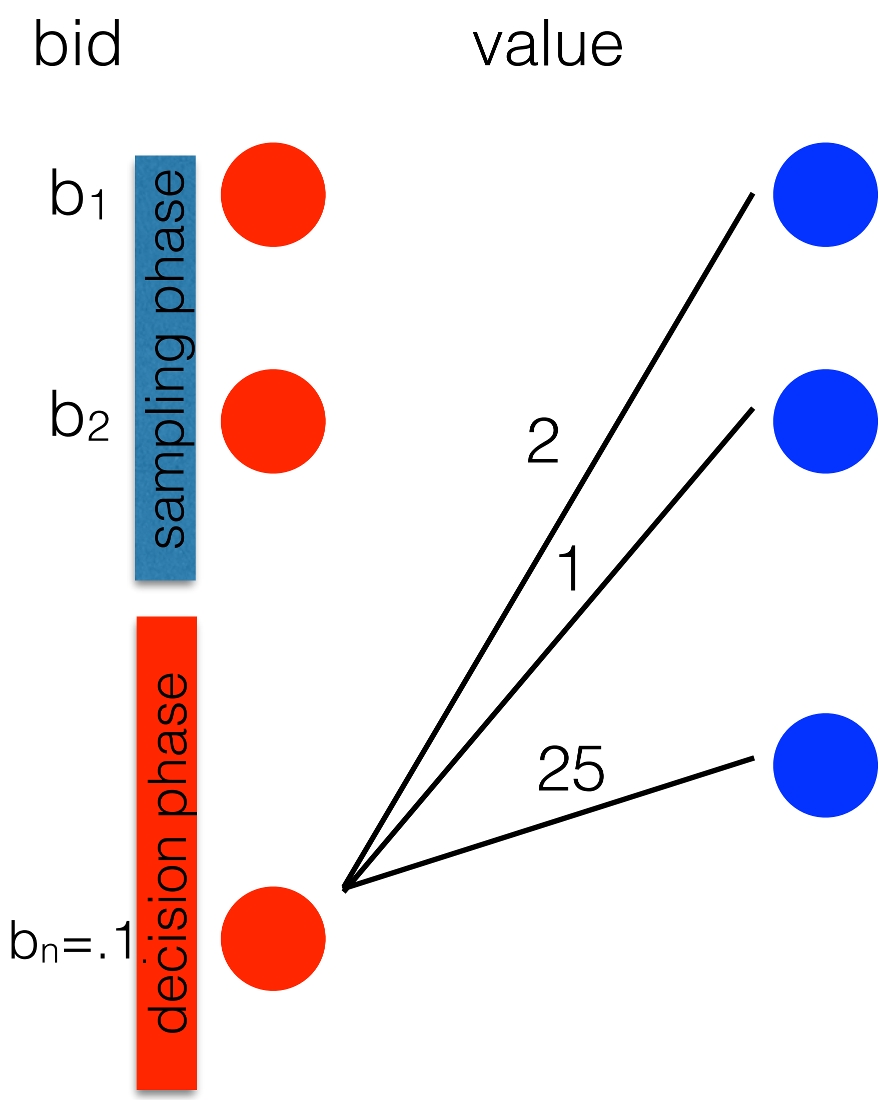
Properties of ON



on arrival of a new left vertex

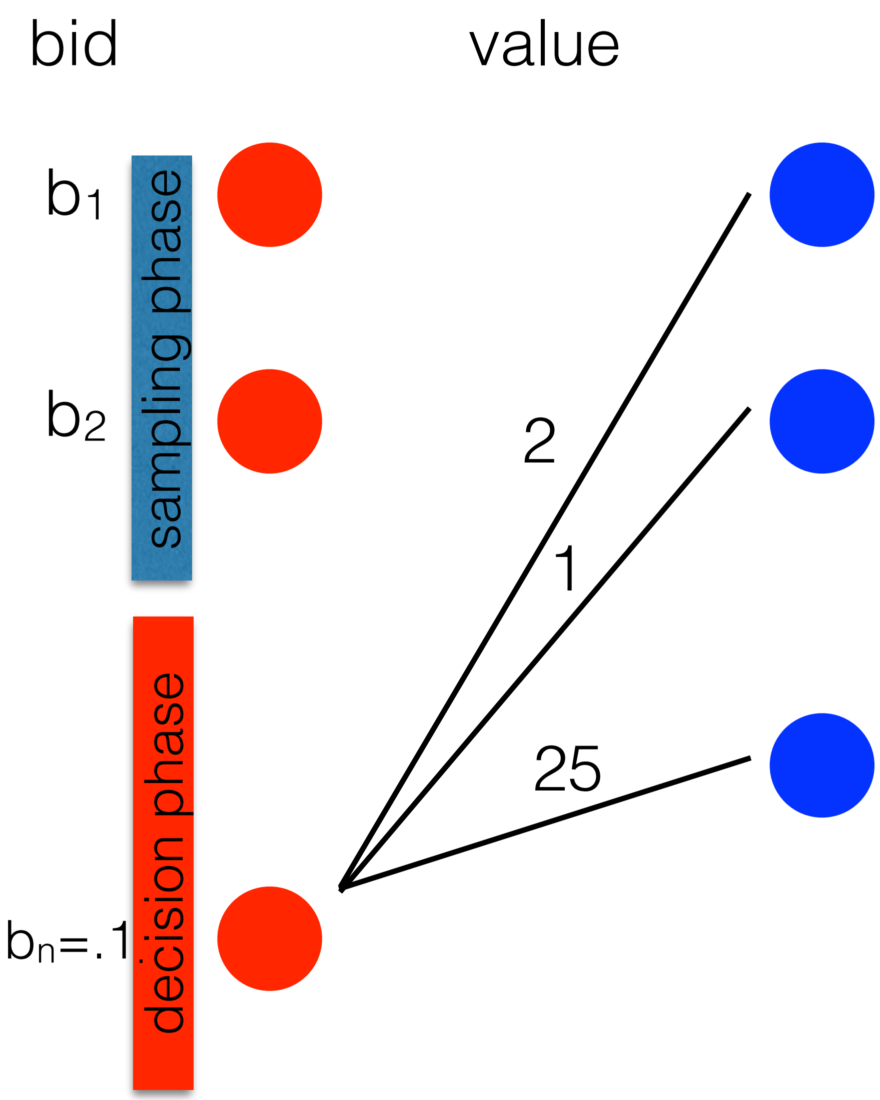
remove all edges with **bid to benefit** ratio > γ

match the right vertex with max w > utility thres.

Payment for selected left vertex $p_\ell = \beta\gamma v(r)$

Properties of **ON**

γ and threshold utility v(r) obtained from **OFF**



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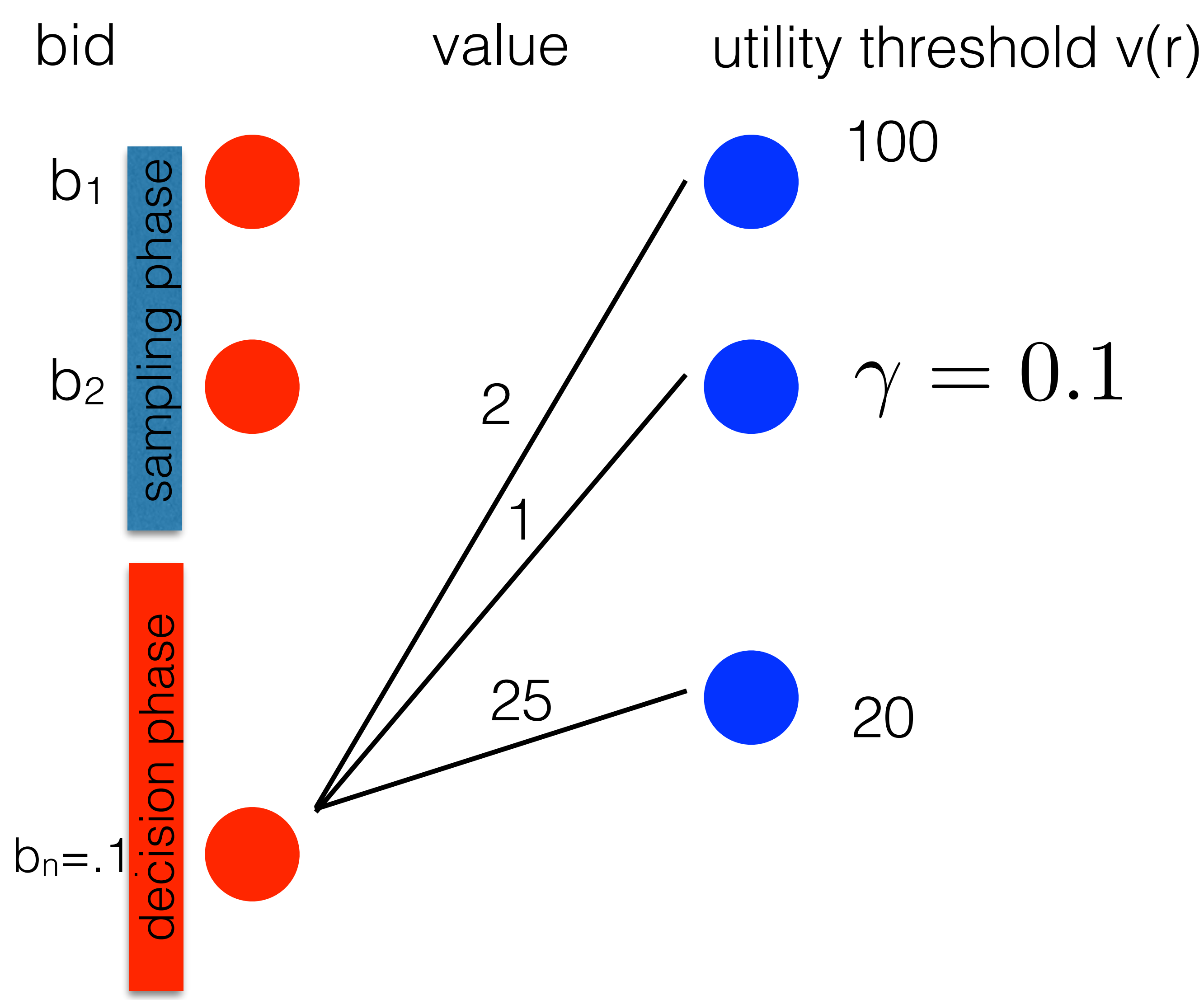

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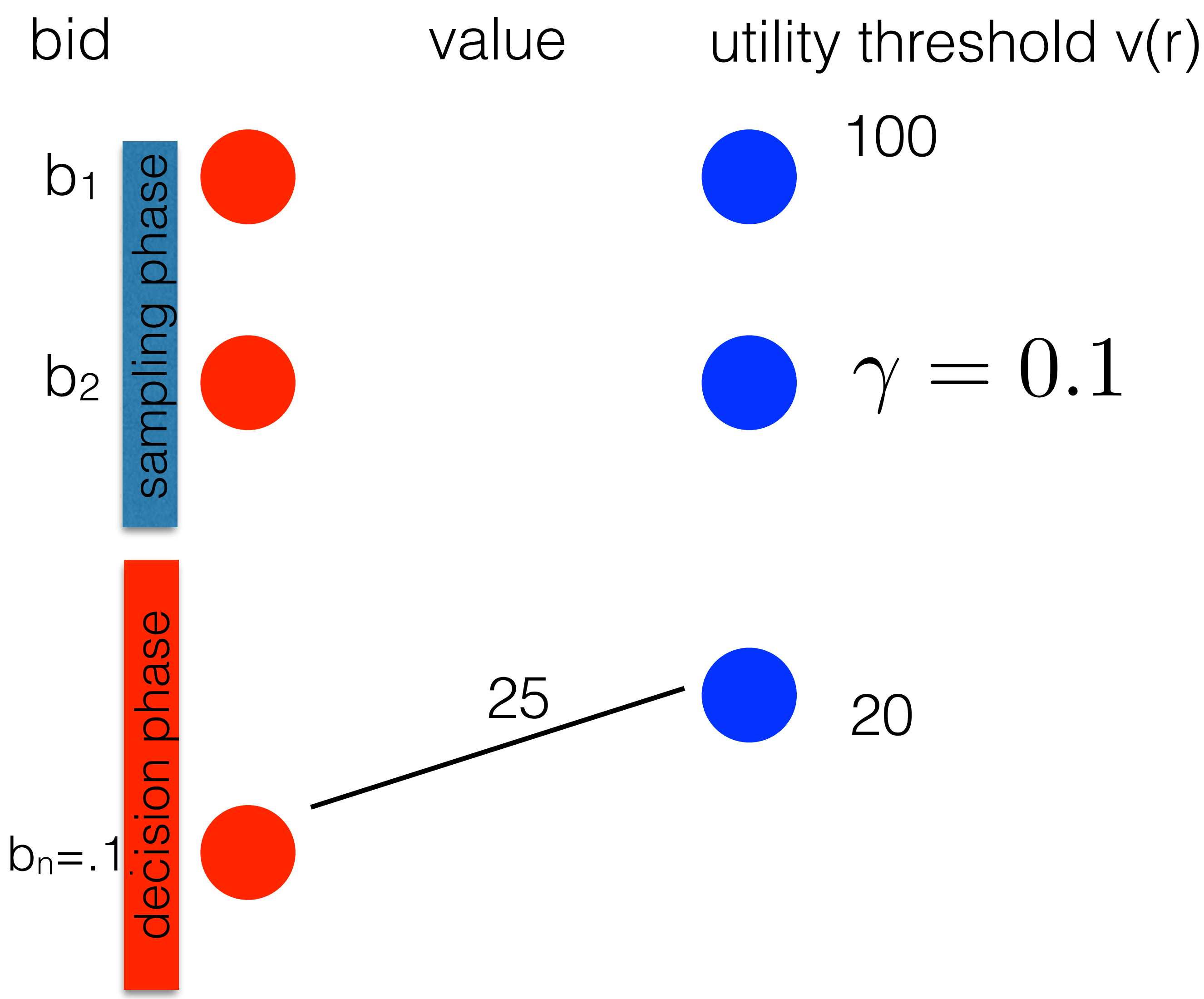

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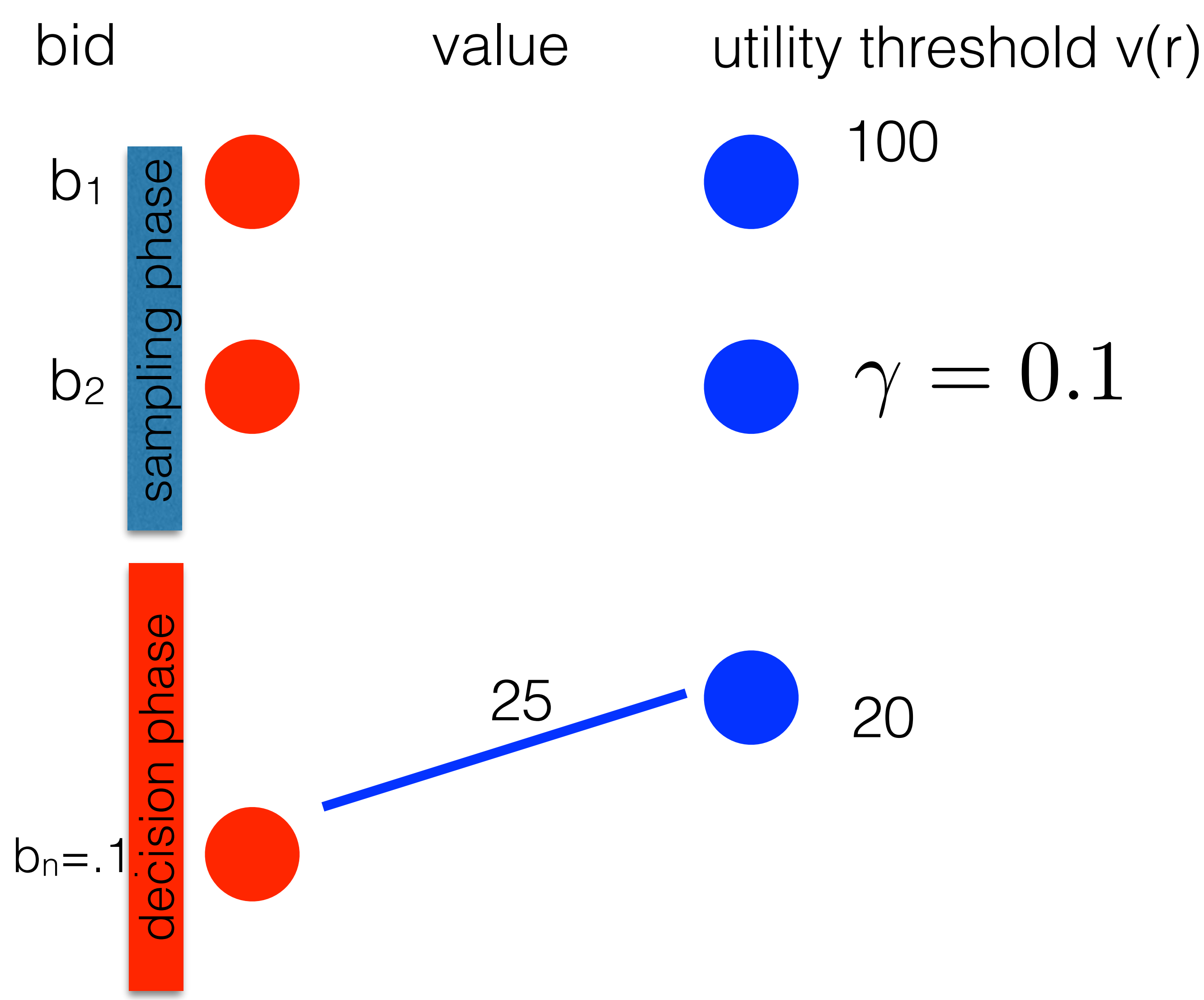

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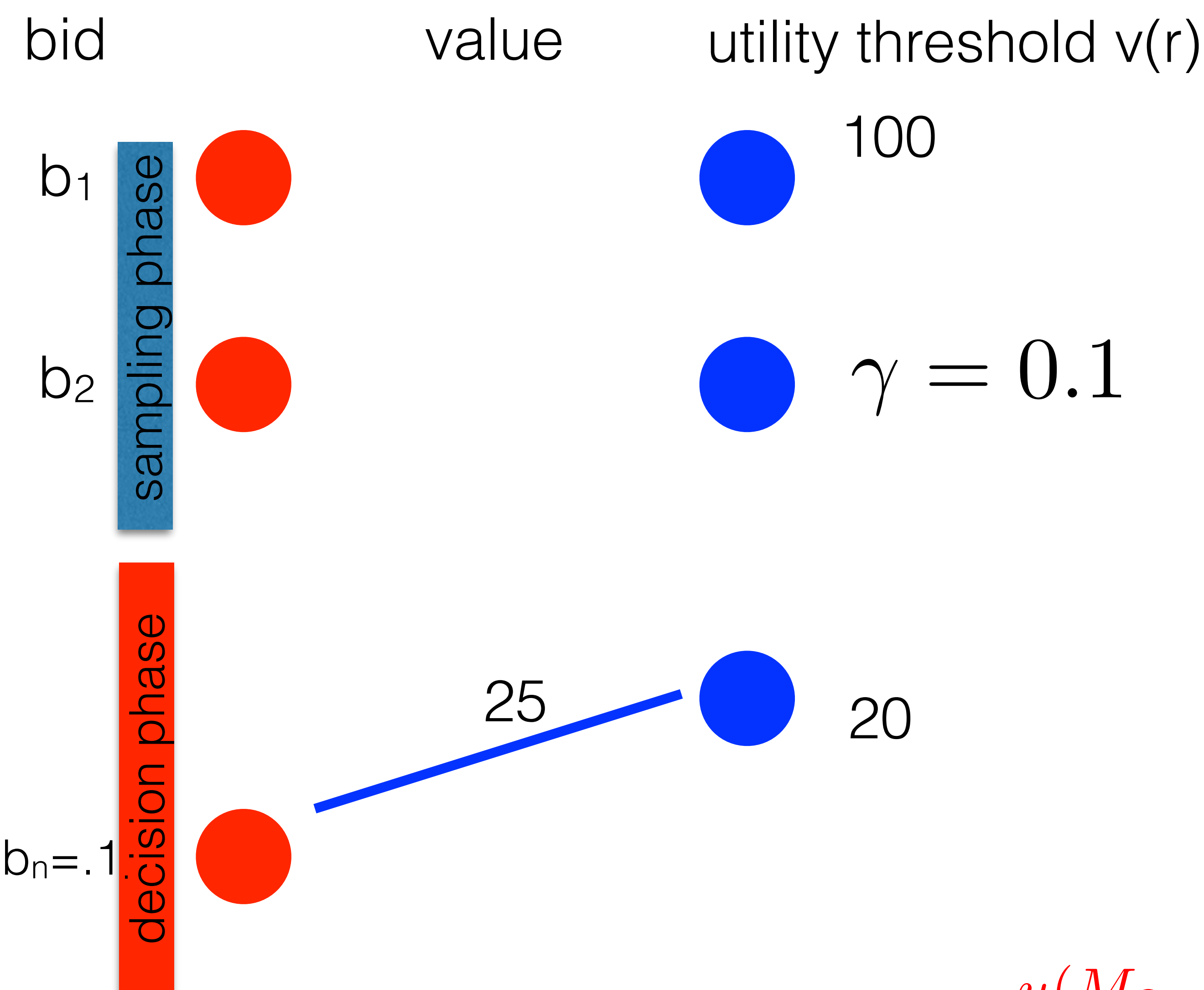

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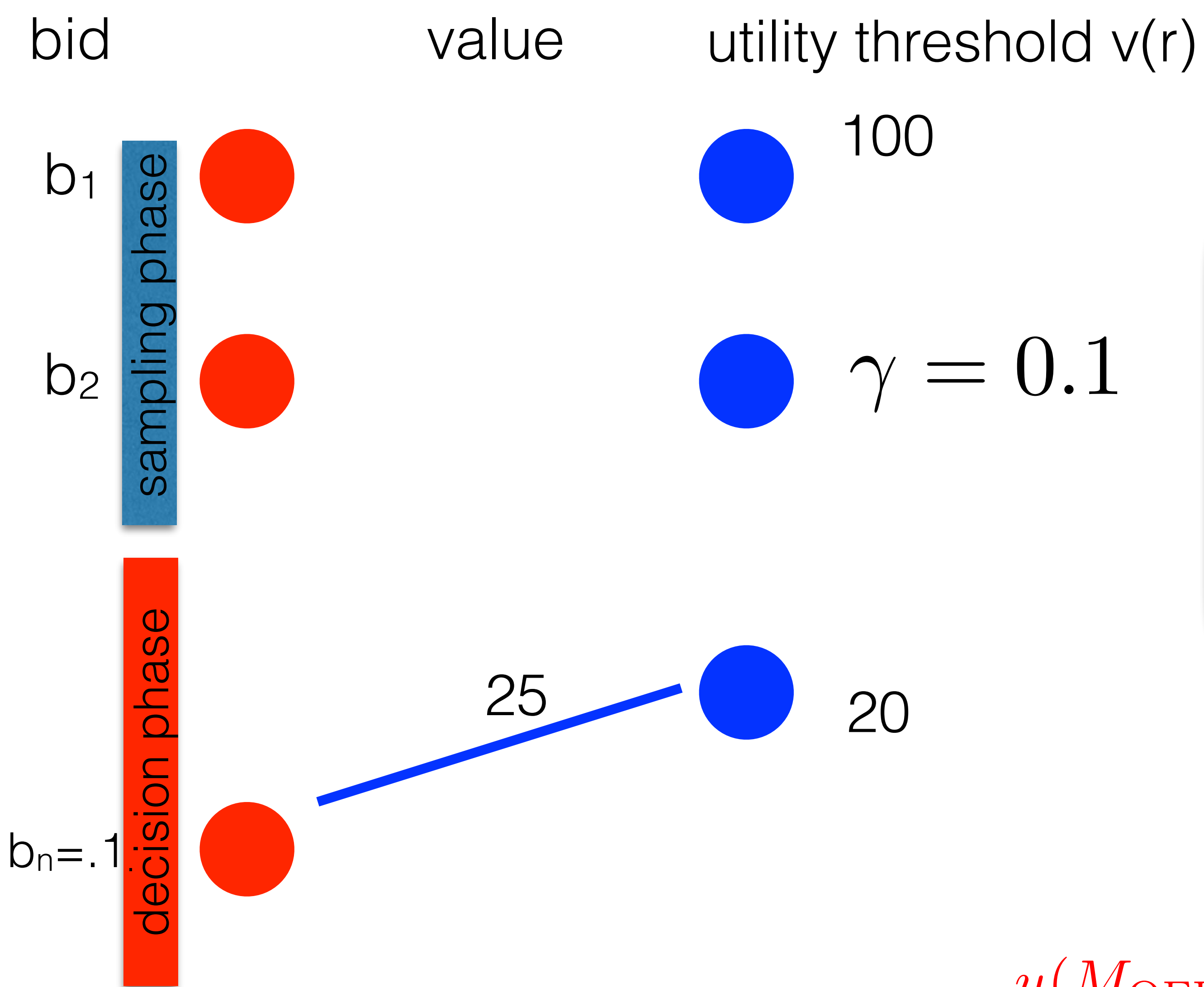
$$u(M_{\mathrm{ON}}) \geq \frac{u(M_{\mathrm{OFF}}(\mathrm{half}))}{2}$$ [Korula, Pal ‘ 08]

Properties of ON

γ and threshold utility v(r) obtained from **OFF**



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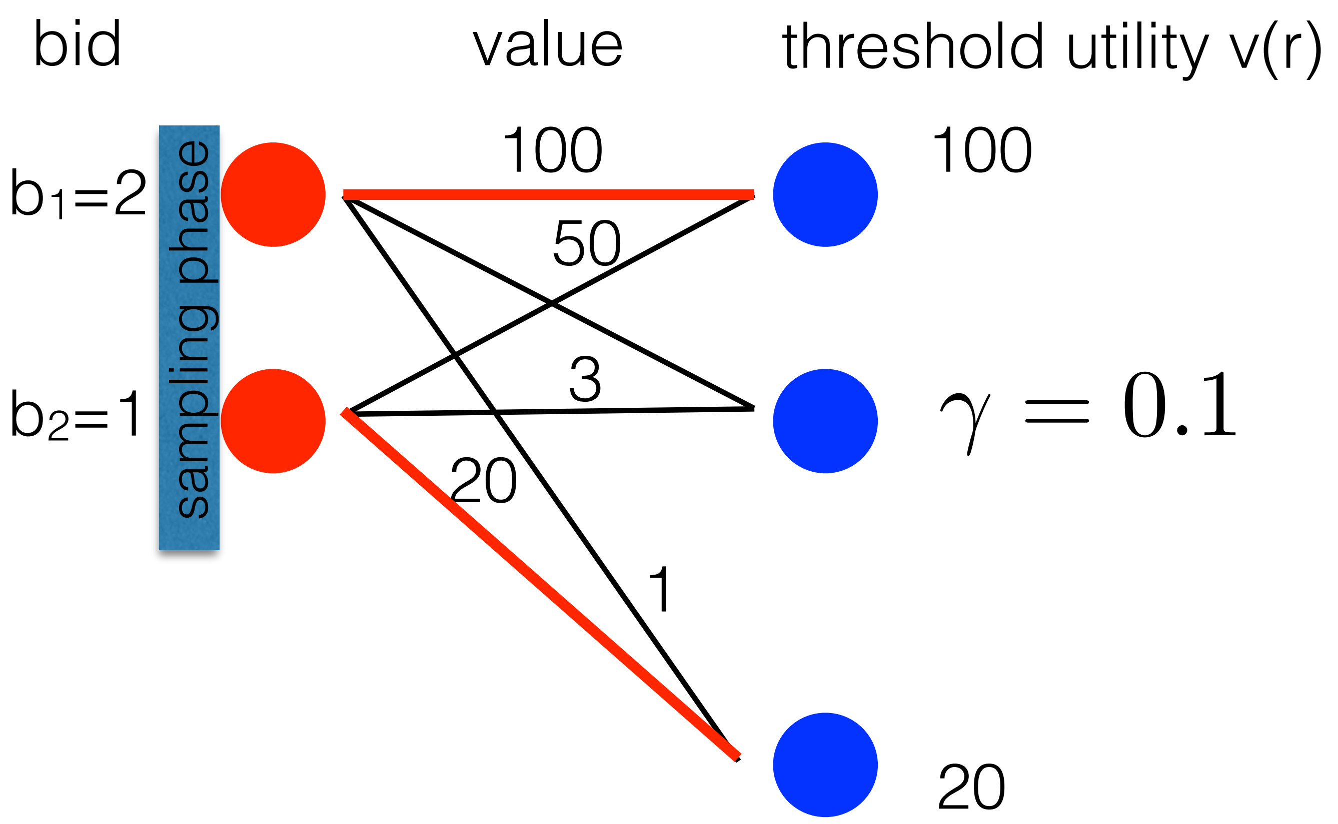
$$u(M_{\mathrm{ON}}) \geq \frac{u(M_{\mathrm{OFF}}(\mathrm{half}))}{2}$$ [Korula, Pal ' 08]

Since we run OFF with budget B/β $\quad u(M_{\mathrm{ON}}) \geq \frac{\mathrm{OPT}_{\mathrm{full}}}{24\beta}$

Properties of **ON**



Algorithm - OFF

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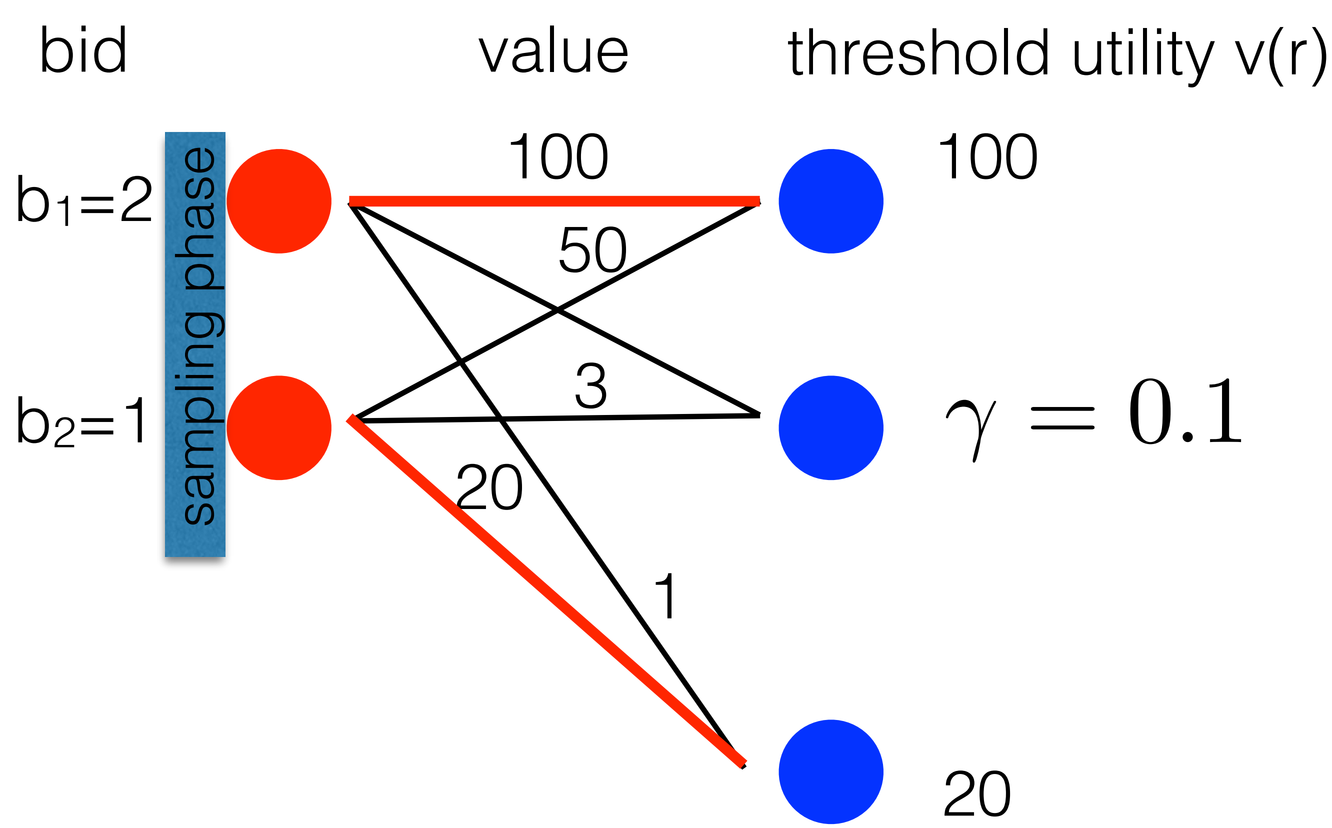
$\mathrm{M}(\gamma)$ be greedy matching over $G(\gamma)$

Find largest $\gamma \sum_{e \in M(\gamma)} v(e) \leq B$

Properties of **ON**

satisfies the payment budget constraint



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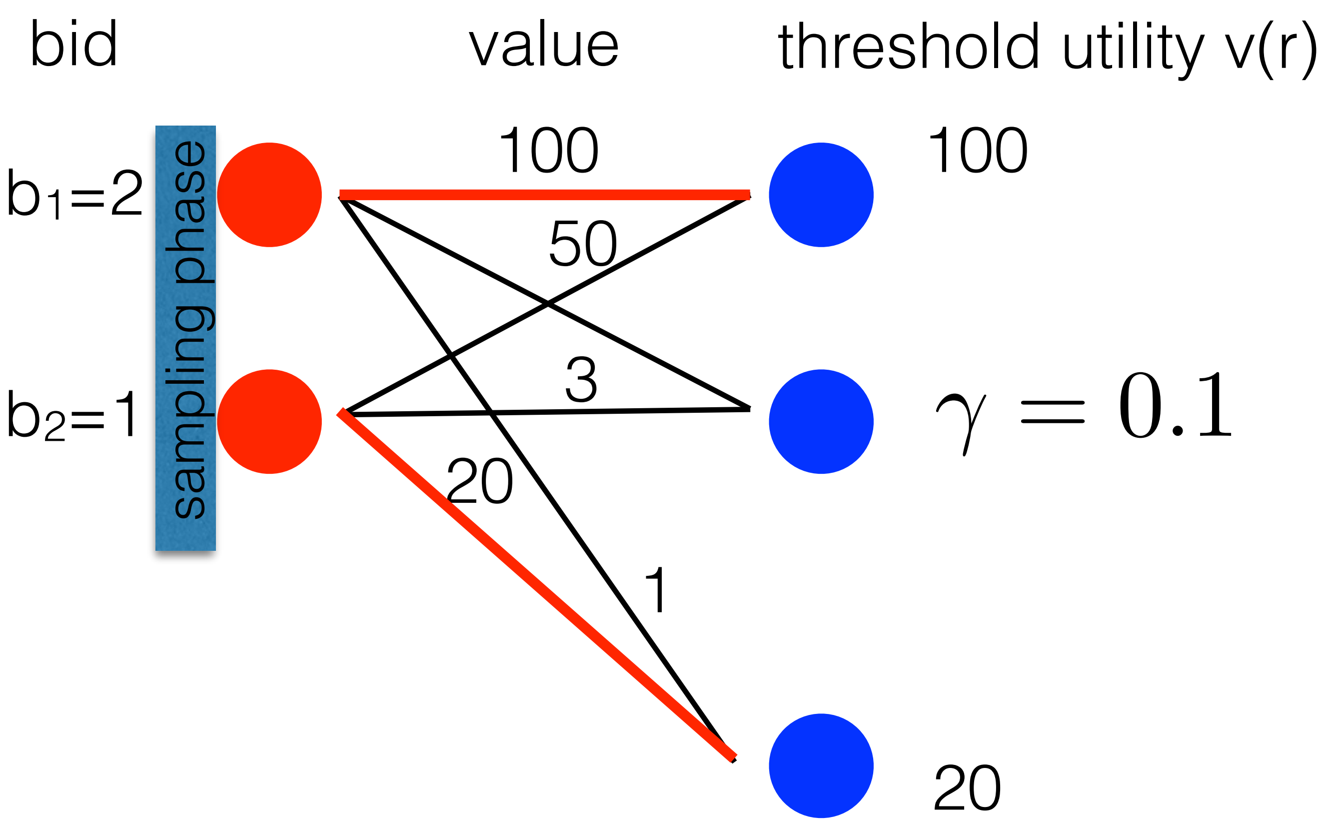

$\gamma = 0.1$

$\gamma \sum_{e \in M_{\mathrm{OFF}}} v(r) \leq B/\beta$

Algorithm - `OFF`

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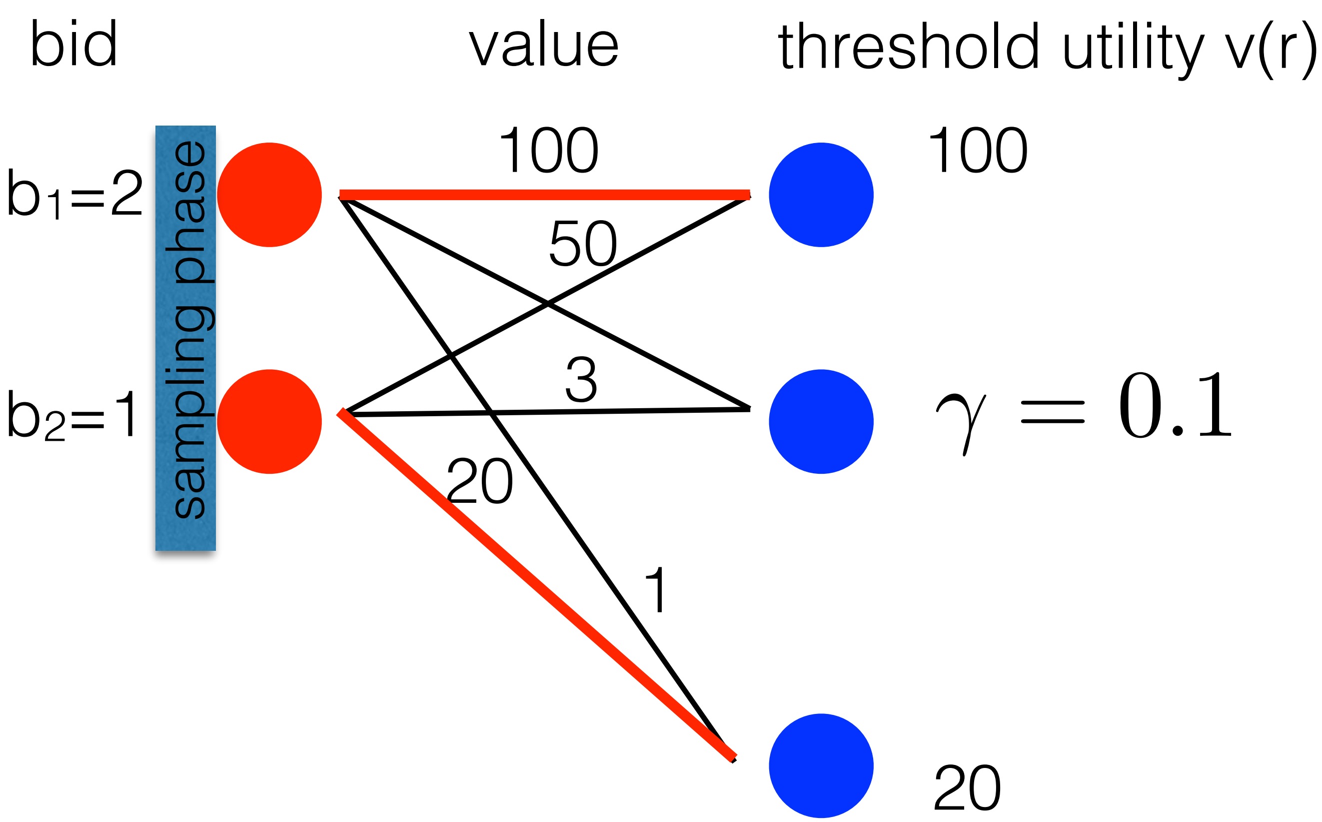

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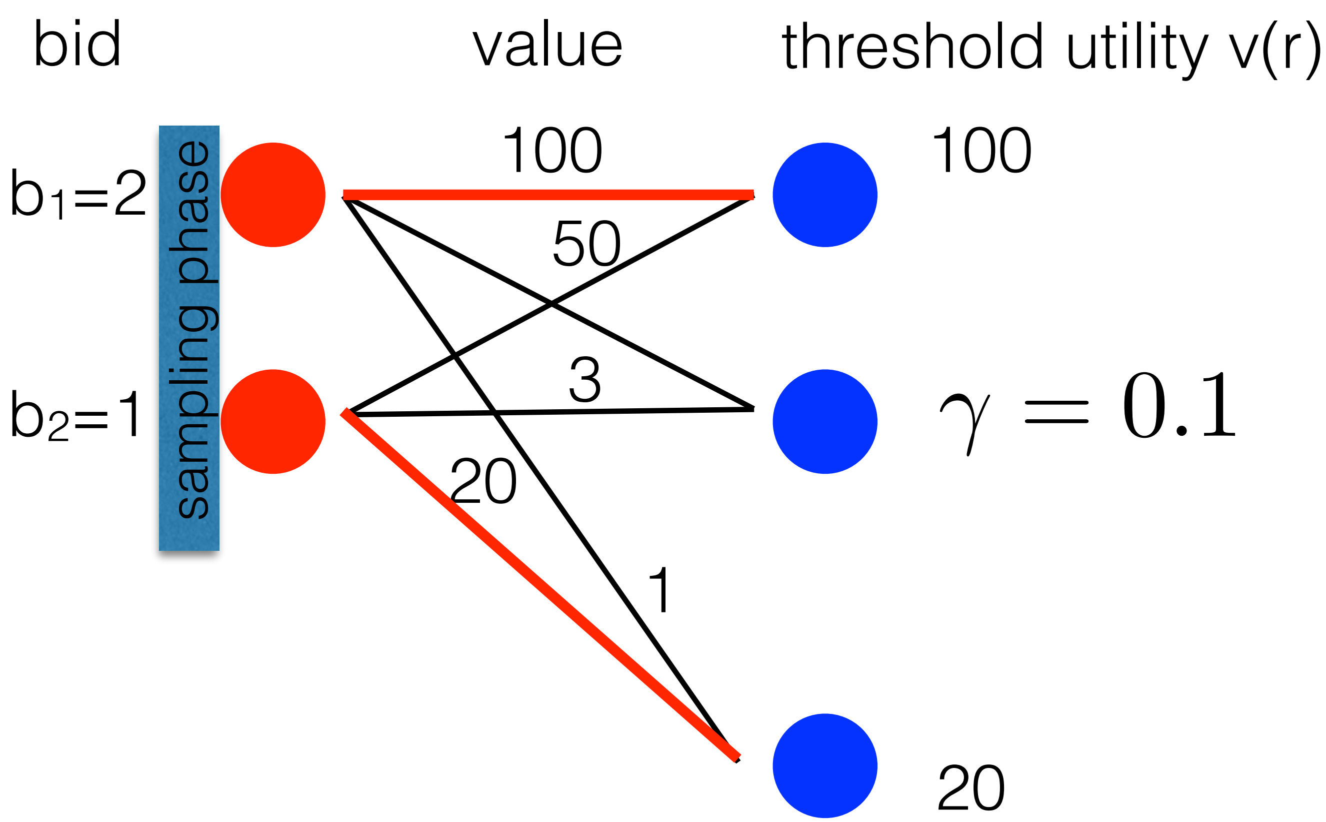

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Since we are looking for matching

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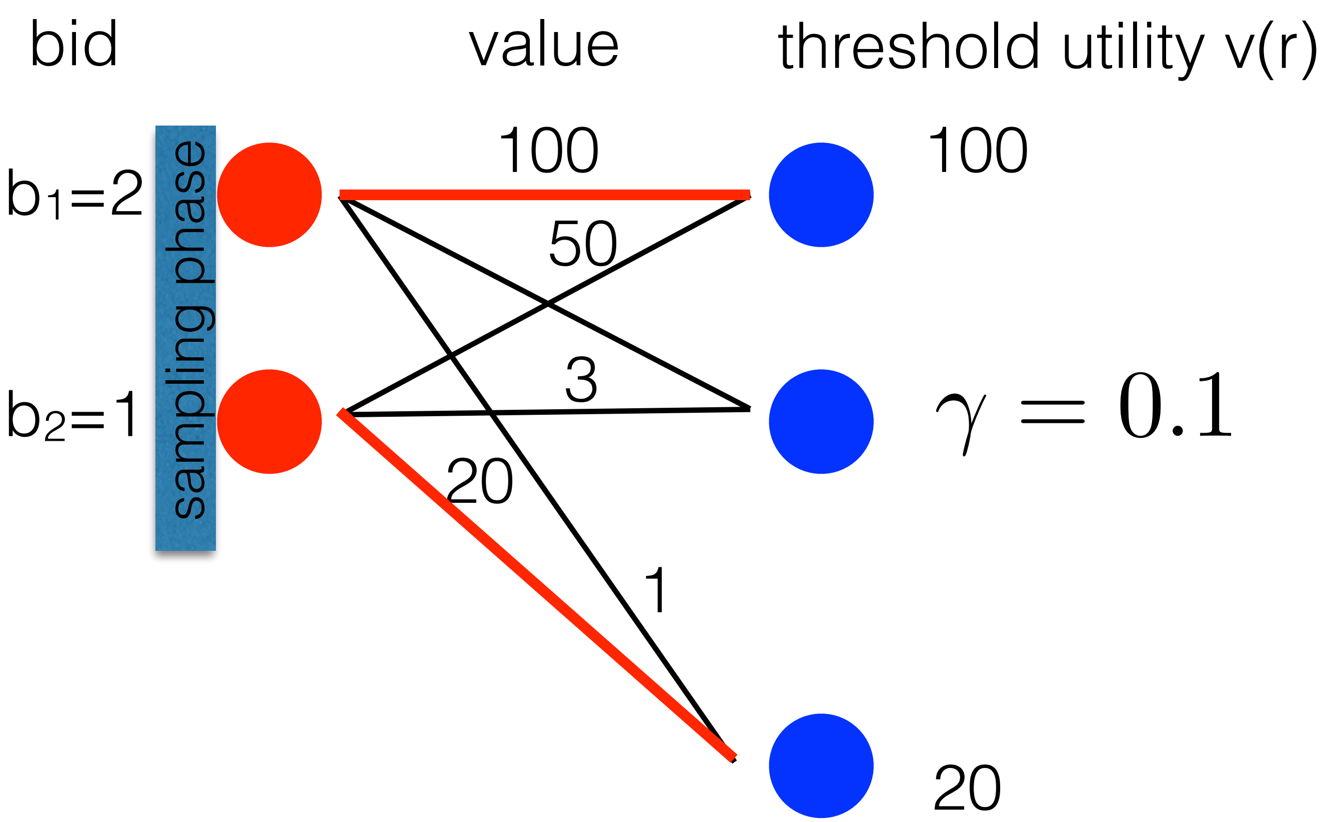
Properties of **ON**

satisfies the payment budget constraint



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$$p_\ell = \gamma \beta v(r)$$

Since we are looking for matching

$$\sum_{e=(\ell,r) \in M_{\text{ON}}} p_\ell \leq \gamma\beta \sum_{e \in M_{\text{OFF}}} v(r) \leq B$$

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Properties of **ON**

For truthfulness we just check

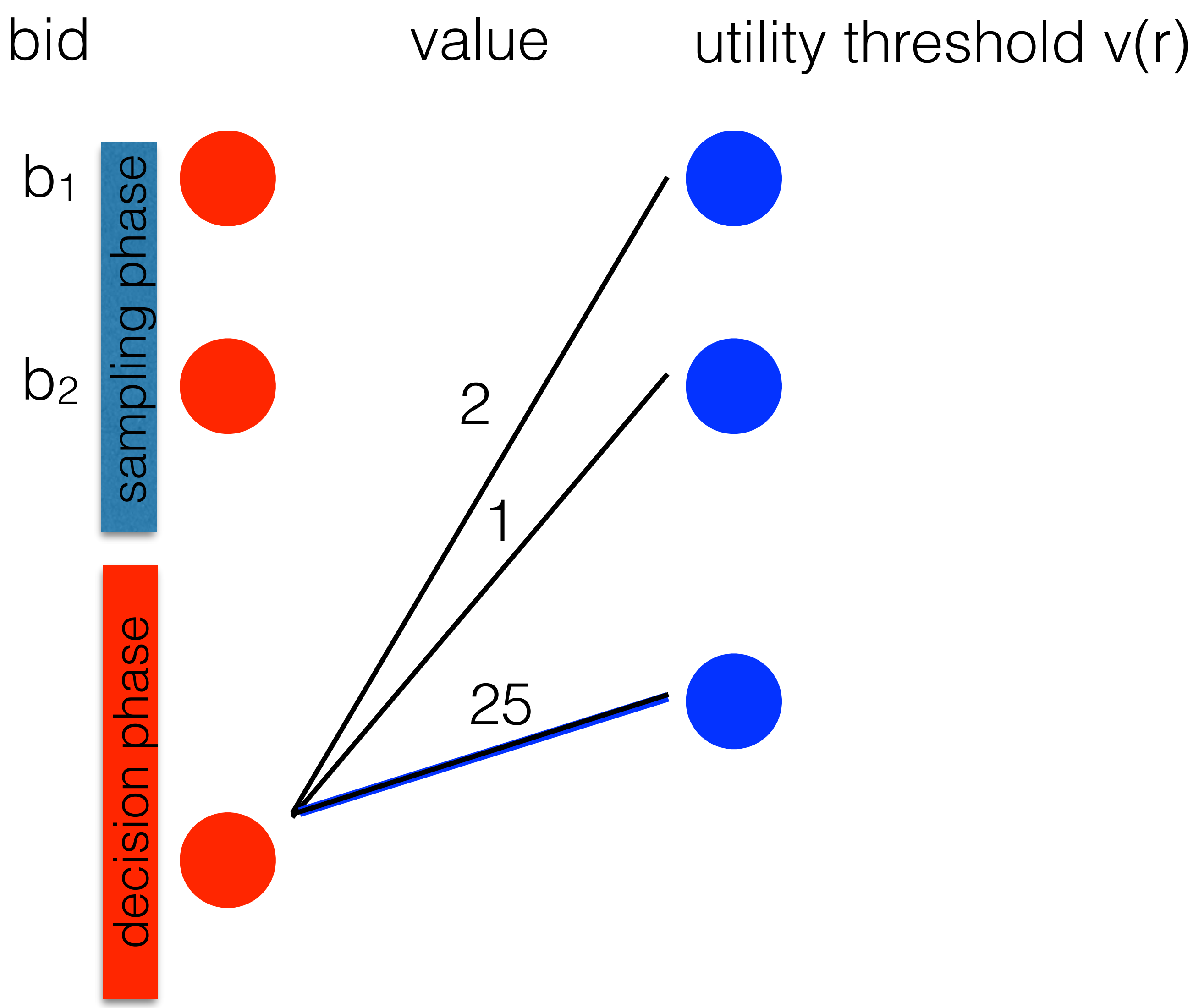
R. Myerson

Monotonicity - if an agent is selected with bid b, then he is always selected if he bids below b

Critical Price - there exists a threshold price such that if an agent bids above it, he is never selected

Properties of `ON` – truthfulness

γ and threshold utility v(r) obtained from **`OFF`**



on arrival of a new left vertex

remove all edges with **bid to benefit** ratio > γ

match the right vertex with max w > utility thres.

Payment for selected left vertex $p_\ell = \beta\gamma v(r)$

Properties of **ON** – truthfulness

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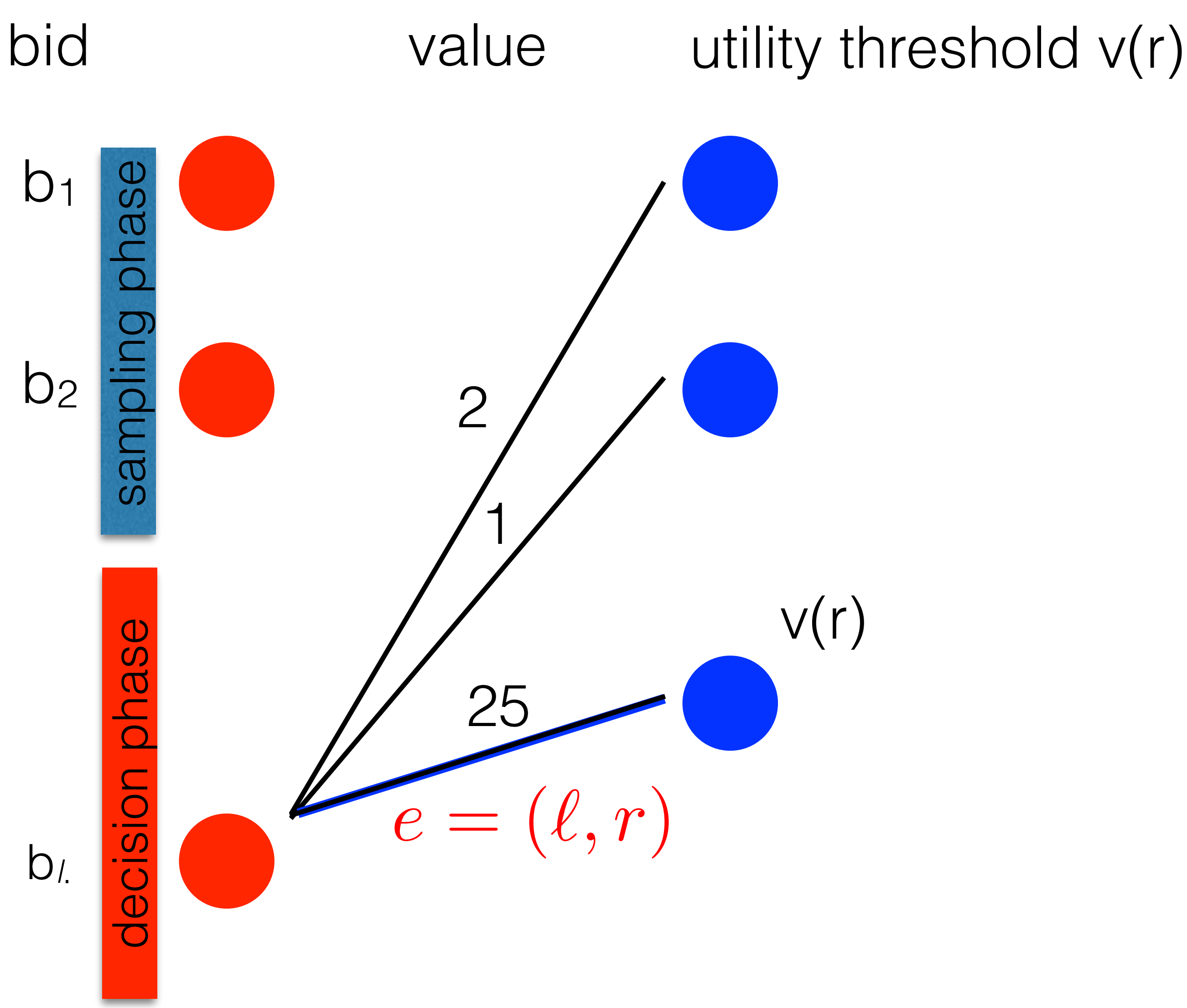

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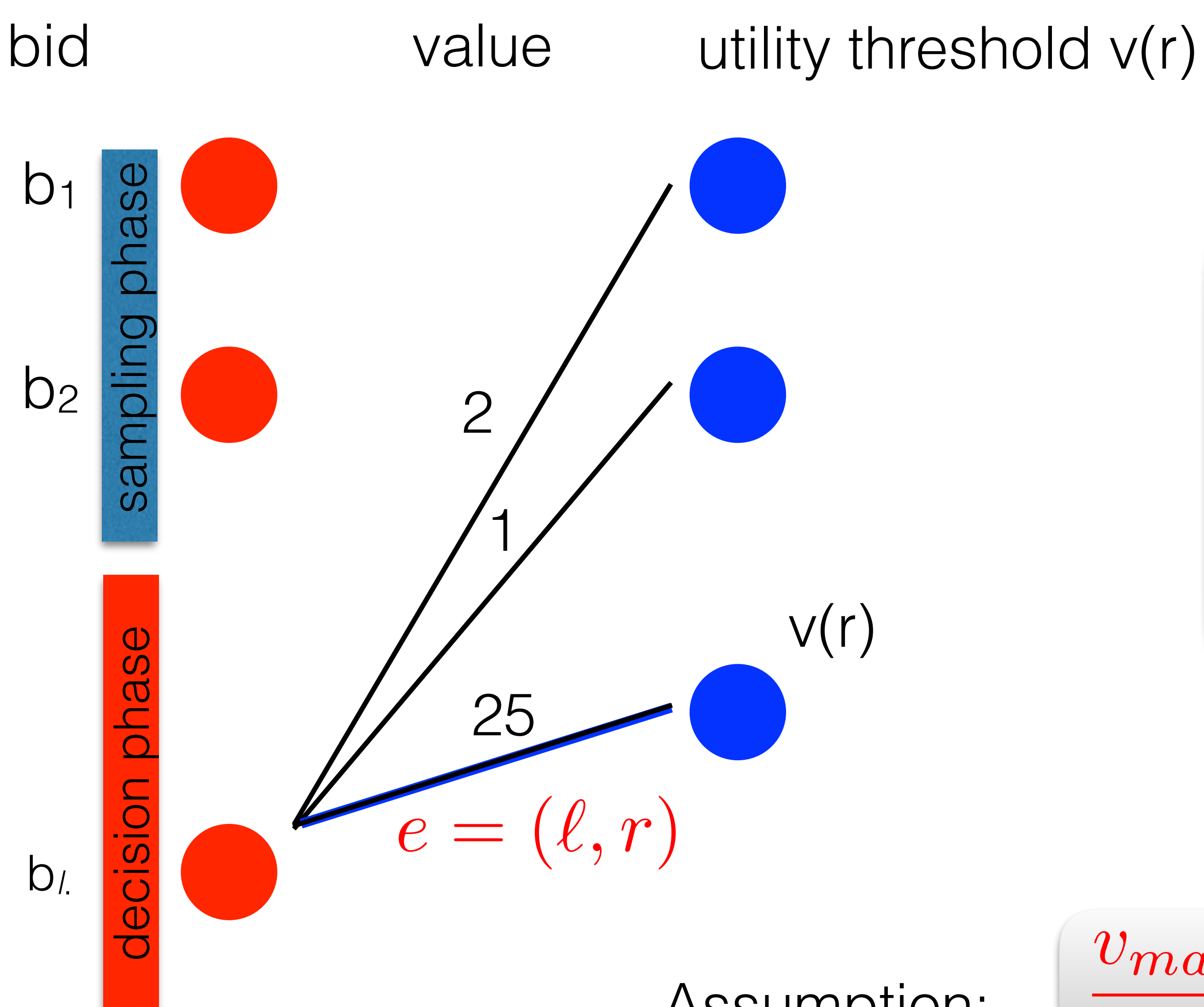

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Assumption: $\frac{v_{max}}{v_{min}} \leq \beta$ $\quad u(e) \leq \beta v(r)$

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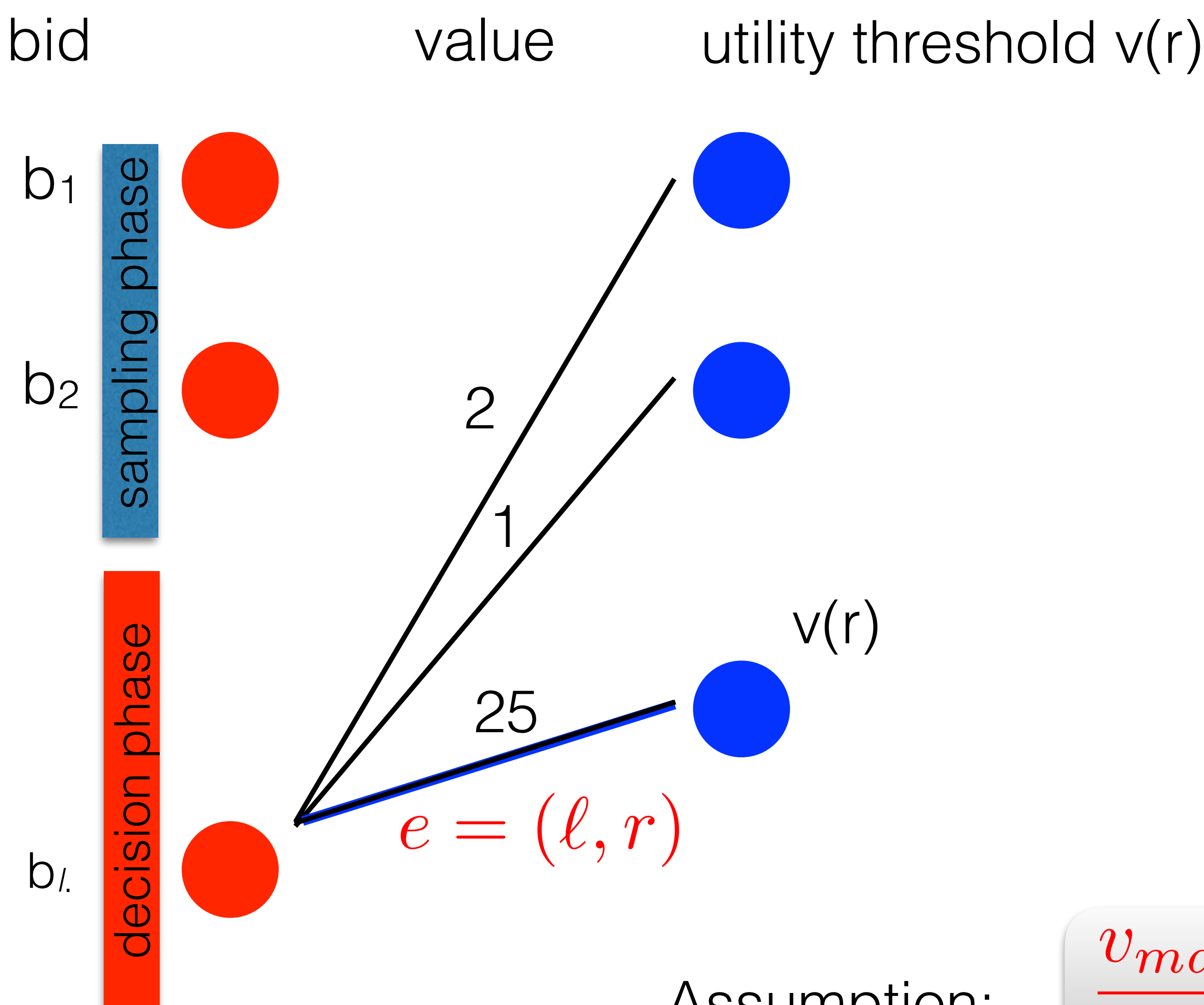

on arrival of a new left vertex

remove all edges with **bid to benefit** ratio $> \gamma$

match the right vertex with max w > utility thres.

Payment for selected left vertex $p_\ell = \beta\gamma v(r)$

Assumption: $\boxed{\frac{v_{max}}{v_{min}} \leq \beta}$ $\qquad u(e) \leq \beta v(r)$

If $b_l > p_l$ then bid to benefit ratio of that edge $\frac{b_\ell}{u(e)} > \gamma$

Important Extension

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If instead of hard budget constraint, its an expected budget constraint

$$\mathbf{E}\{p_{1} + p_{2} + \ldots + p_{m}\} < B$$

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- a small variation of proposed algorithm is constant (288) competitive

Peynet Museum' Antibes