Convergence Rates for Stochastic Dual Descent Algorithm

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Stochastic Resource Allocation Problem

$$(\mathbf{P}_1) \qquad (\mathbf{x}^\star, \{\mathbf{p}_t^\star\}_{t\in\mathbb{N}}) = \arg\max f_0(\mathbf{x}) \tag{1}$$

$$\text{s. t. } \mathbb{E}\left[\mathbf{s}_t(\mathbf{p}_t, \mathbf{x})\right] \geq 0 \tag{2}$$

$$\mathbf{x} \in \mathcal{X}, \mathbf{p}_t \in \mathcal{P}_t \tag{3}$$

- **Goal** is to determine the resource allocation variable $\mathbf{x} \in \mathbb{R}^n$ and policy $\mathbf{p}_t \in \mathbb{R}^p$ for all $t \in \mathbb{N}$

Dual Descent Based Solution

- **Dual descent using sample averages**

$$\{\mathbf{x}_t, \mathbf{p}_t\} \in \arg \max_{\mathbf{x}\in\mathcal{X}, \mathbf{p}(.)\in\mathcal{P}} f(\mathbf{x}) - \boldsymbol{\lambda}^T \mathbb{E}\left[\mathbf{s}_t(\mathbf{p}, \mathbf{x})\right] \tag{4a}$$

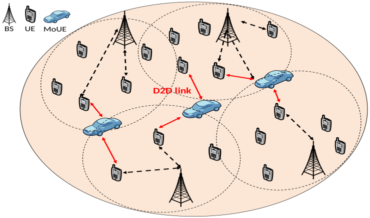
$$\boldsymbol{\lambda}_{t+1} = \left[\boldsymbol{\lambda}_t - \epsilon \mathbb{E}\left[\mathbf{s}_t(\mathbf{p}_t, \mathbf{x}_t)\right]\right]_+ \tag{4b}$$

- **Stochastic Dual Descent Algorithm**

$$(\mathbf{x}_t, \mathbf{p}_t) = \arg \max_{\mathbf{x}\in\mathcal{X}, \mathbf{p}\in\mathcal{P}_t} f_0(\mathbf{x}) + \boldsymbol{\lambda}_t{}^T \mathbf{s}_t(\mathbf{p}, \mathbf{x}) \tag{5a}$$

$$\boldsymbol{\lambda}_{t+1} = \left[\boldsymbol{\lambda}_t - \epsilon \mathbf{s}_t(\mathbf{x}_t, \mathbf{p}_t)\right]_+ \tag{5b}$$

System model for Edge Caching



Proposed Algorithm (Repeat for $t = 1, 2, \ldots,$)

Step 1. Collect download costs c_t^i and γ_t^i from mobile caches $i = 1, 2, \ldots, M_t$

Step 2. **Primal update:** Find "winning" cache i_t and allocated power p_t^i

$$r_t \in \underset{0 \le r \le r_{max}}{\arg\max}\ U(r) - \lambda_t r, \qquad \{p_t^i\}_{i=1}^{M_t} \in \underset{\{p^i\}_{i=1}^{M_t} \in \mathcal{P}_t}{\arg\max} \sum_{i=1}^{M_t} \left[\lambda_t R_i(p^i, \gamma_t^i) - c_t^i p^i\right].$$

Step 3. Download at rate $R_{i_t}(p_t^{i_t}, \gamma_t^i)$ from user i_t

Step 4. **Dual update:** $\lambda_{t+1} = \mathcal{P}_\Lambda\left(\lambda_t - \epsilon\left[R_{i_t}(p_t^{i_t}, \gamma_t^i) - r_t\right]\right)$