Random access and massive wireless networks

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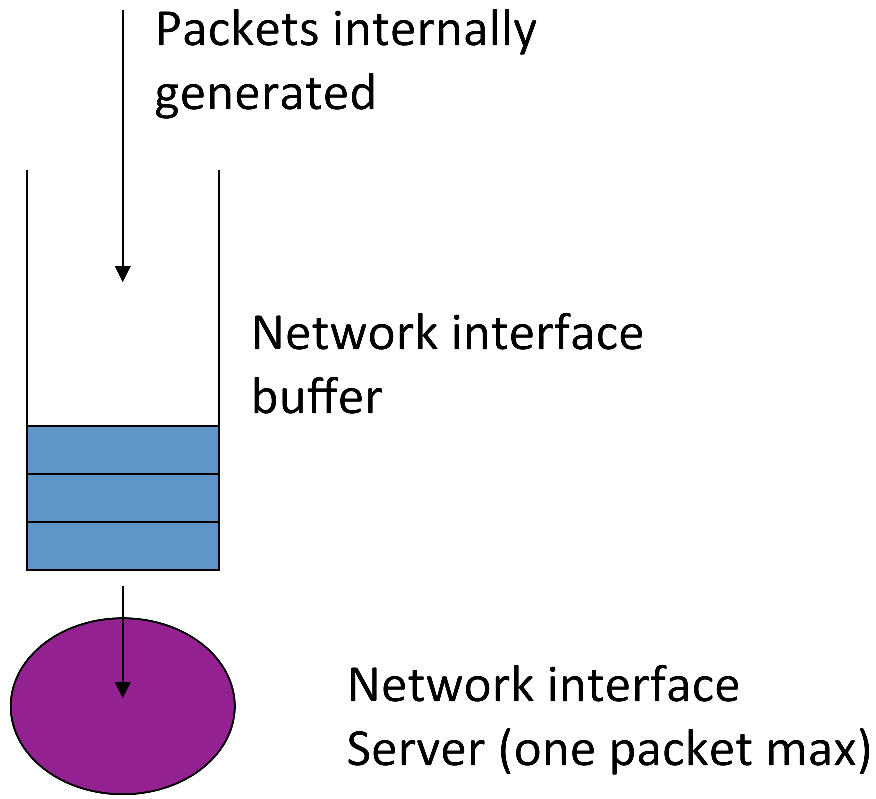
Terminal network interface model



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Traffic Poisson model

- Finite population model
  - Poisson model traffic λ_i per slot for node i
  - $\lambda = \sum_i \lambda_i \quad eg \quad \lambda_i = \frac{\lambda}{N}$
  - Infinite buffer
  - Maximum stable capacity $\lambda_{\max}^{F}$
- Infinite population model
  - N=∞
  - Buffer limited to 1 unit.
  - A Poisson generation of node with one packet
  - Maximum stable capacity $\lambda_{\max}^{I} \leq \lambda_{\max}^{F}$

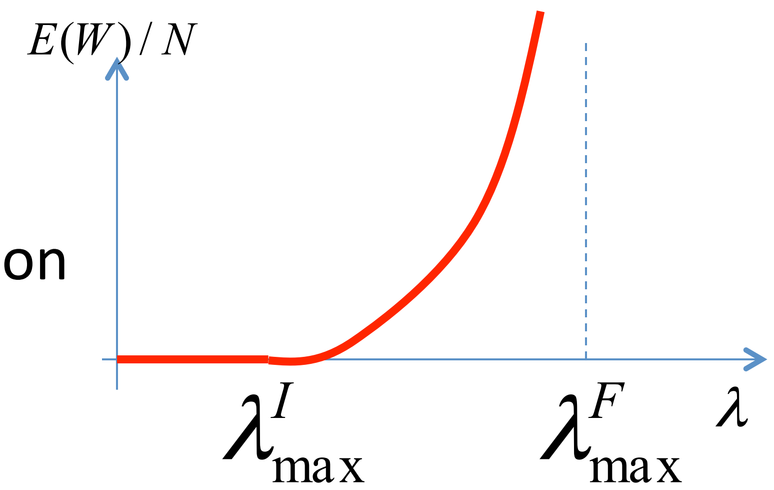
Why infinite population model is interesting

- When $\lambda < \lambda_{\max}^{I}$
  - Average delivery delay is finite

$$E(W) < \infty$$

  Independent of N

- When $\lambda_{\max}^{I} \leq \lambda < \lambda_{\max}^{F}$
  - We expect $E(W) = O(N)$
  - $\lambda_{\max}^{F}$ may vary with input repartition
  - Starvation effects
  - Problem when N increases
    - *Eg* N=10^6 or larger



ALOHA Performance

- Packet generation over all nodes
  - Poisson process, cumulated rate λ packet per slot
- ALOHA Packet transmission attempt process: Two model cases:
  - infinite population: nodes transmit only one packet and die;
  - Finite population nodes are permanent and manage a queue of packets

$$P(\text{slot is empty}) = e^{-\rho}$$

$$P(\text{slot is success}) = \rho e^{-\rho}$$

$$P(\text{slot is collision}) = 1 - (1+\rho)e^{-\rho}$$

  - Poisson process, cumulated rate ρ packet per slot

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Aloha and infinite population

- Is unstable for all λ>0: $\lambda^{I}_{\max} = 0$
  - Take B large number of waiting packets:
  $$P(\text{slot is success}) = (1-p)^{B}\lambda e^{-\lambda} + Bp(1-p)^{B-1}e^{-\lambda} < \lambda$$
  - System diverges: B(t) at time t
  $$E(B(t+1) - B(t) \mid B(t) = B) = \lambda - P(\text{slot is success})$$
  - $\lambda^{I}_{\max} = 0$ for binary exponential backoff (Ethernet, Wifi)

Aloha and finite population

- N nodes
  - In this case max{B(t)}=N
  - System is stable when B=N and

$$P(\text{slot is success}) = Np(1-p)^{N-1} > \lambda$$

  - When $p = O(\frac{1}{N})$

$$P(\text{slot is success}) \approx Np\exp(-Np)$$

    - And max throughput

$$\lambda_{\max}^{F} = e^{-1} \approx 0.36787\cdots$$

Stack collision resolution in infinite population

- Stack algorithm
  local procedure

```
    C←0;
 While packet to transmit{
   if (C=0) then {
          transmit;
          if collision then C←rand(0,1)}
   else {
          if listen=collision then C←C+1;
          else C←C-1
 }
```

Stack algorithm stability condition

ABC	AB	-	AB	A	B	C
	C	AB	C	B	C	
		C		C		

$$\lambda_{\max} \approx 0.360177\cdots$$

Ternary Stack collision resolution

- Ternary Stack algorithm
  local procedure

```
   C←0;
While packet to transmit{
  if (C=0) then {
        transmit;
        if collision then C←rand(0,1,2)}
  else {
        if listen=collision then C←C+1;
        else C←C-1
}
```

$$\lambda_{\max} \approx 0.401599\cdots$$

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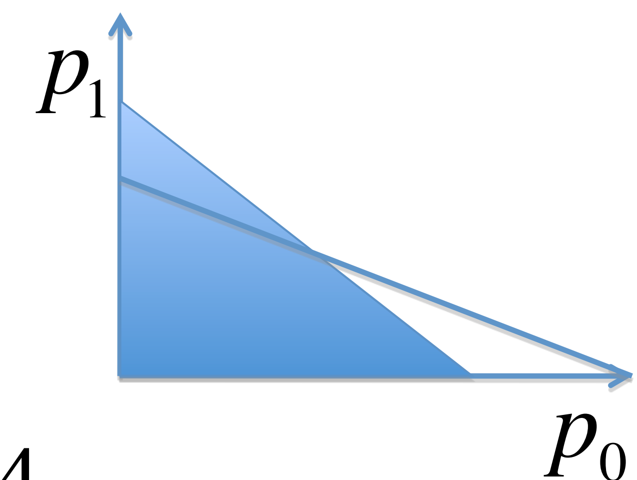
Upper bound on colision resolution algorithms stability

$$p_0 = P(\text{algo returns } 0)$$

$$p_1 = P(\text{algo returns } 1)$$

$$\begin{cases} p_0 \lambda e^{-\lambda} + p_1 e^{-\lambda} = \lambda \\ p_0 + p_1 \leq 1 \end{cases}$$



$$\lambda^I_{\max} \leq e^{-\lambda^I_{\max}} : \lambda^I_{\max} \leq 0.56714\cdots$$

largest known $0.487\cdots$

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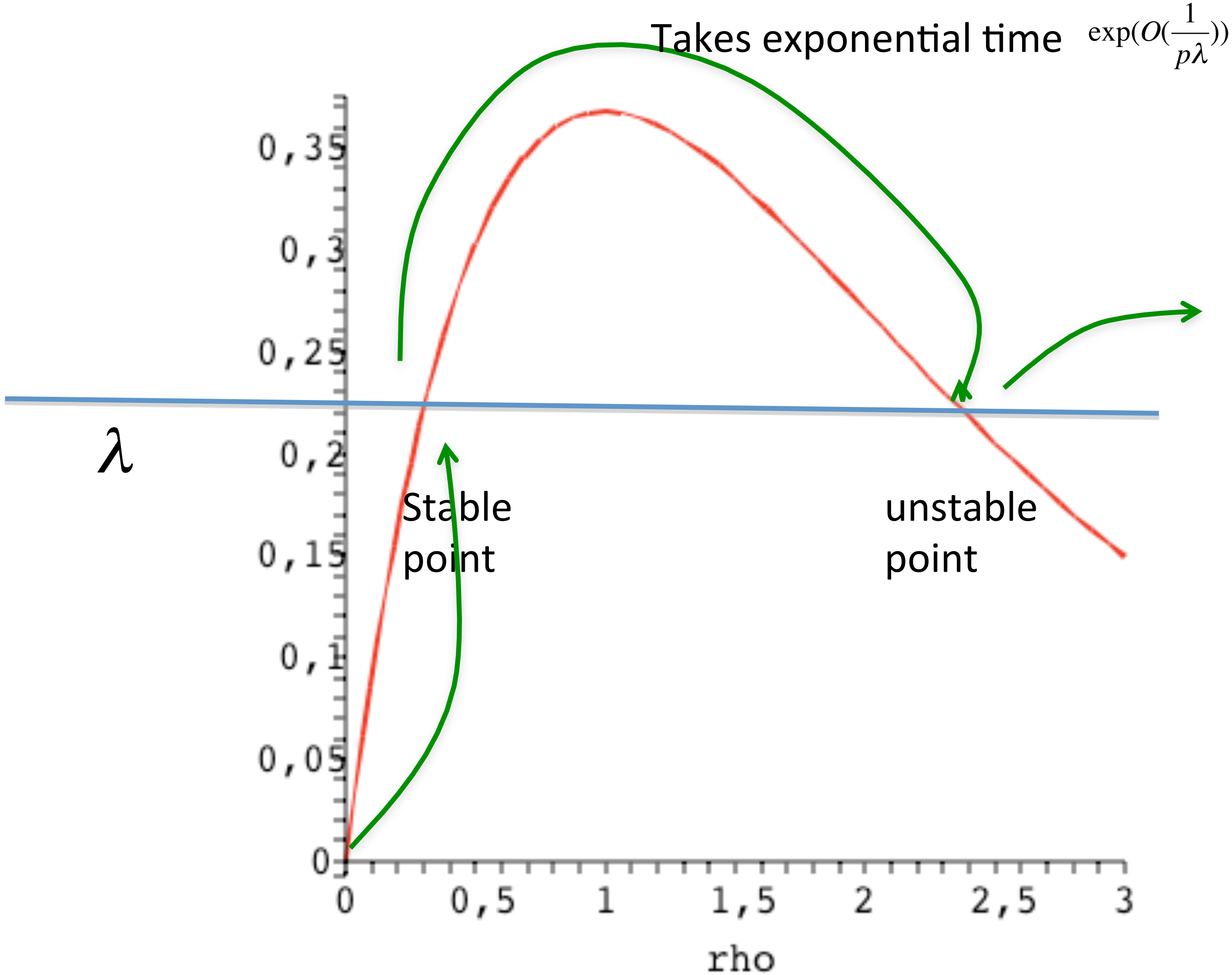
Aloha under small load

- Infinite population with $\lambda << e^{-1}$
- Transmission and retransmission is a Poisson process
  - cumulated rate ρ packet per slot

$$P(\text{slot is empty}) = e^{-\rho}$$
$$P(\text{slot is success}) = \rho e^{-\rho}$$
$$P(\text{slot is collision}) = 1-(1+\rho)e^{-\rho}$$

  - Equilibrium equation:

$$\lambda = \rho e^{-\rho}$$



Takes exponential time $\exp(O(\frac{1}{p\lambda}))$
0,35
0,3
0,25
0,2
0,15
0,1
0,05
0
λ
Stable point
unstable point
0
0,5
1
1,5
2
2,5
3
rho

ALOHA in finite population

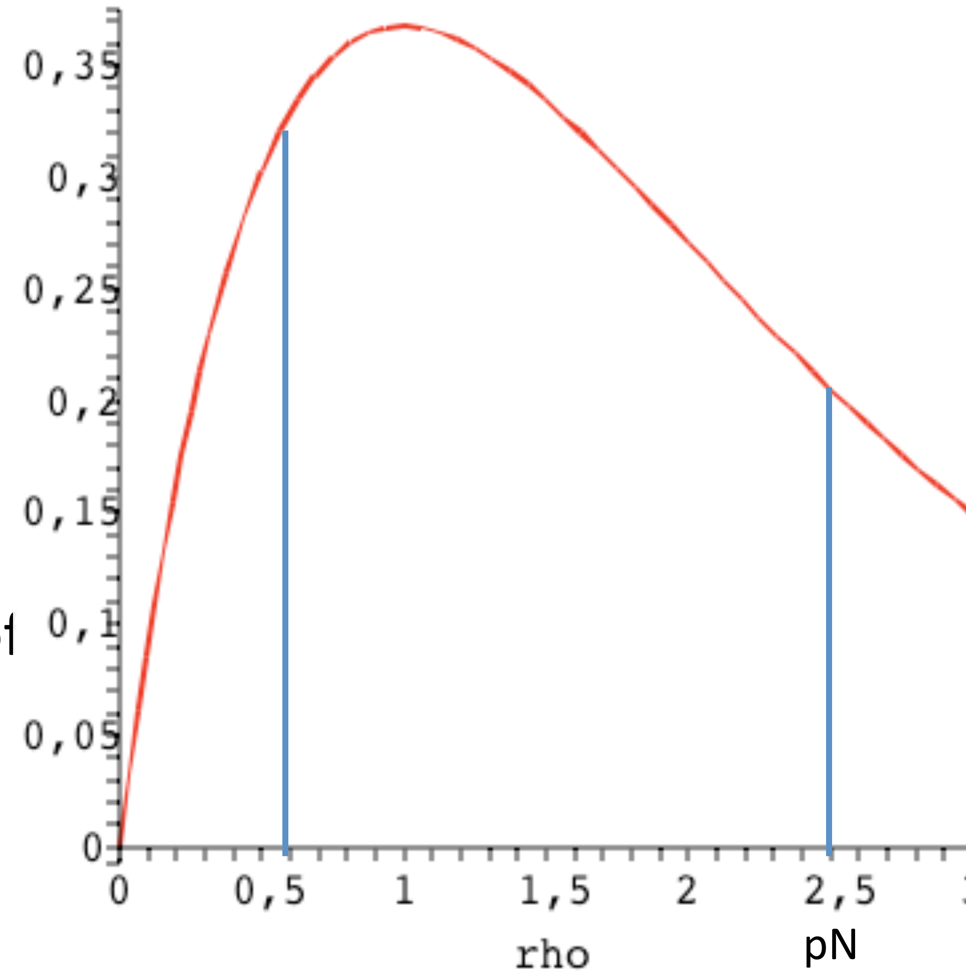
- Maximum throughput

$$\lambda^F_{\max} = pNe^{-pN}$$

  - All buffers full

$$\rho = pN$$

  - Takes long to attain except if big burst of traffic
  - If pN<1: starvation



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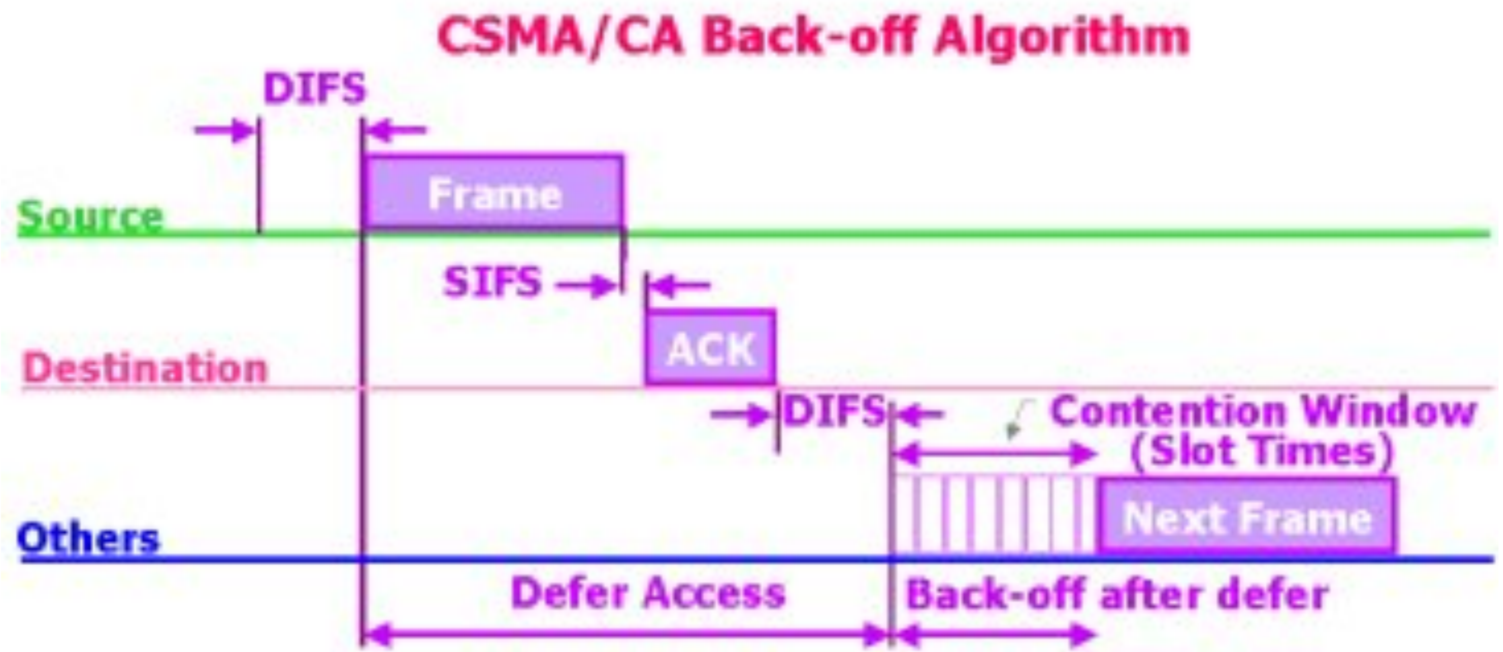
Protocol CSMA (Wifi)

- Mini-slots



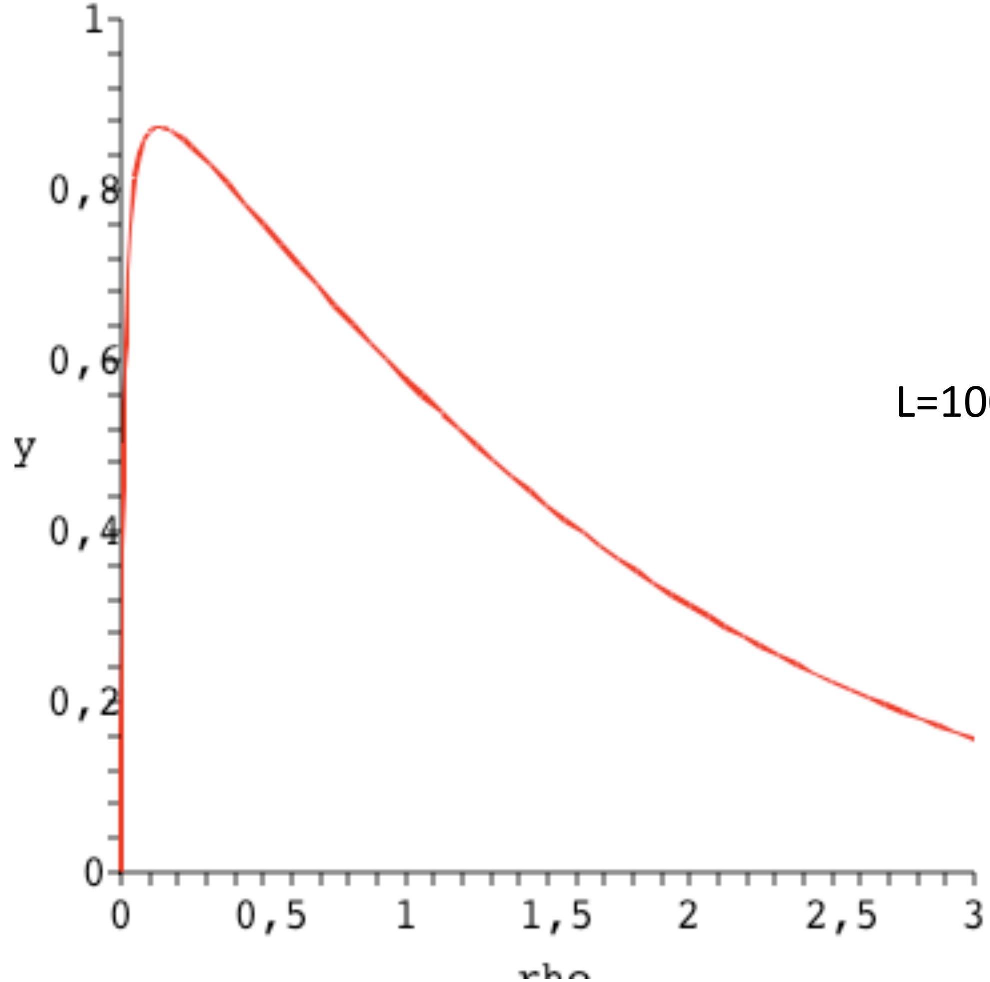
Performances of stationary CSMA

- Poisson model:
  - ρ: per mini-slot load
  - L: packet length (in mini-slots)

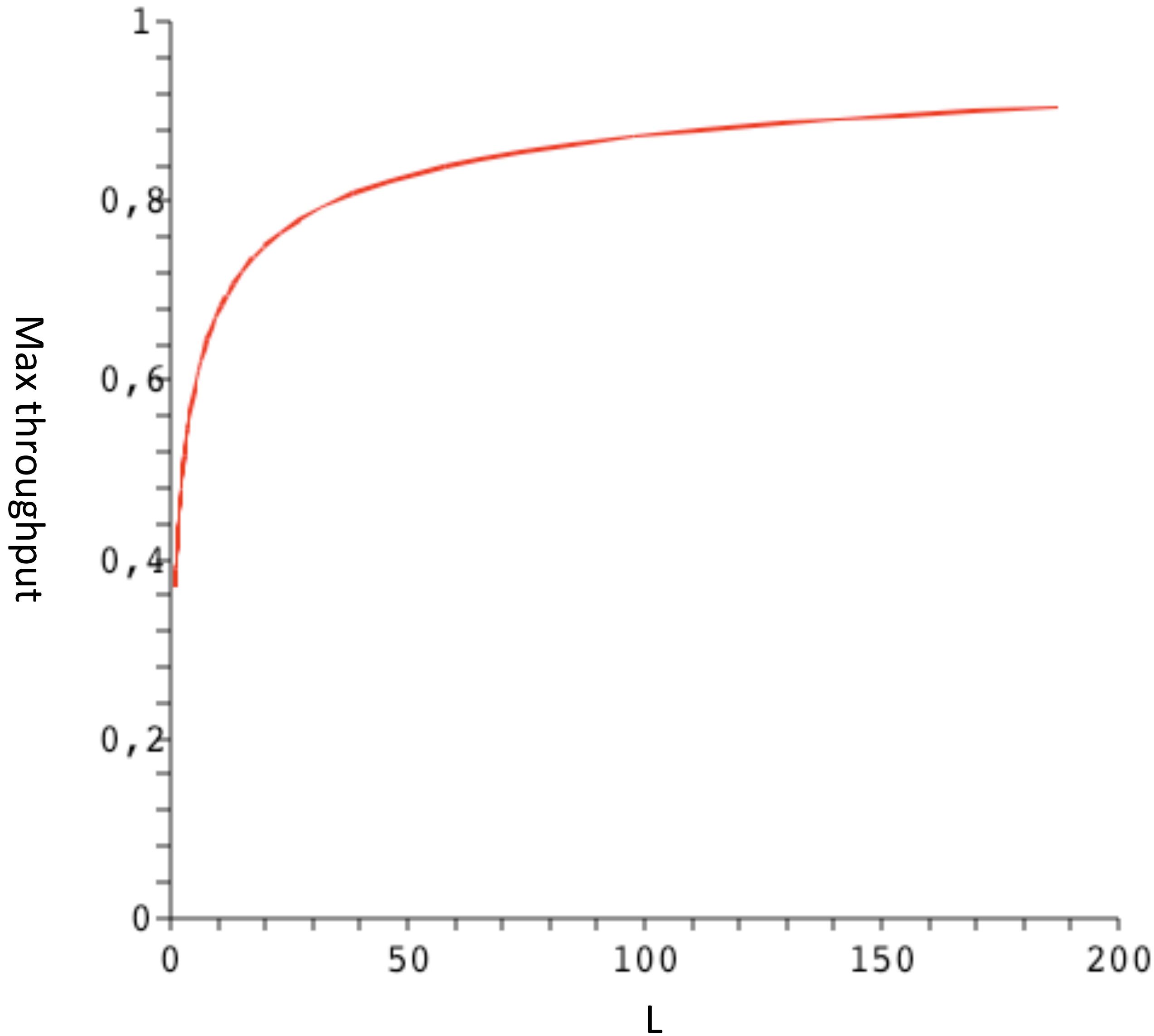
- Net throughput

$$\frac{L\rho}{1+(e^{\rho}-1)L}$$

$$\max \approx 1-\sqrt{\frac{2}{L}}$$

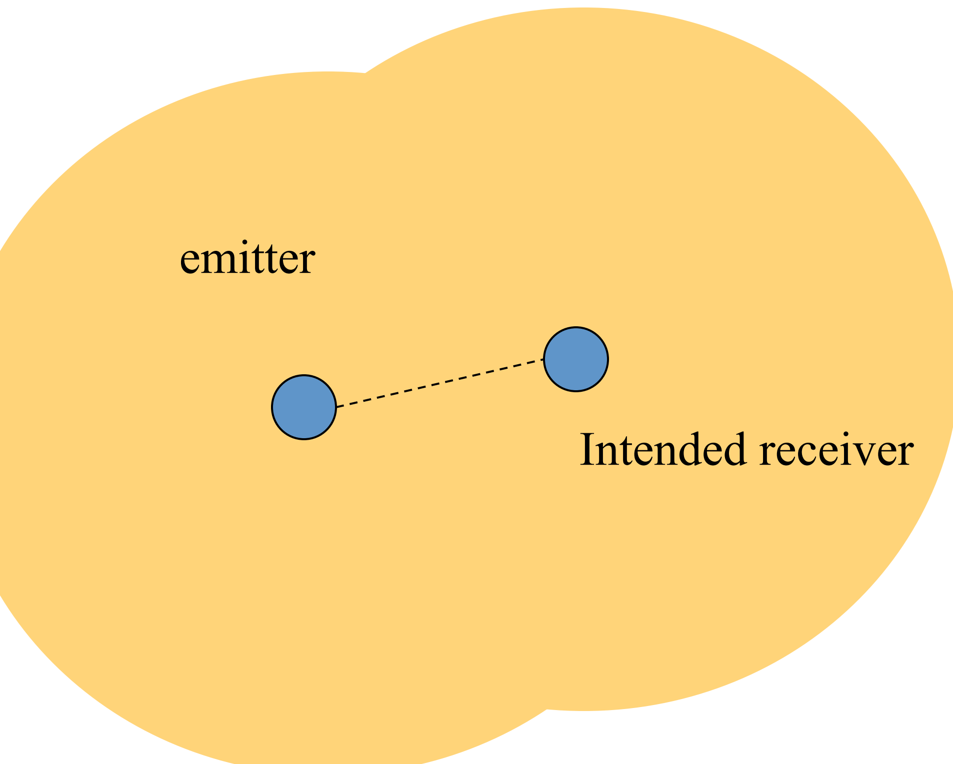


L=100

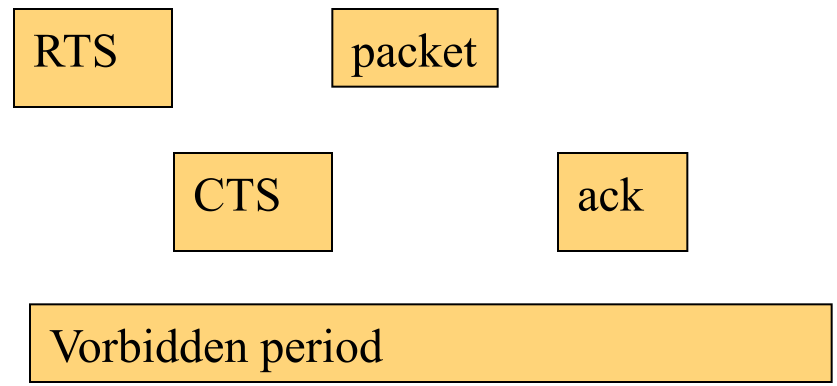


1
0,8
0,6
0,4
0,2
0
0
50
100
150
200
L
Max throughput

RTS-CTS



emitter
Intended receiver



RTS
packet
CTS
ack
Vorbidden period

CSMA/CA performances

- Net throughput with RTS-CTS

$$C_{\max} = \frac{\rho L}{1 + \rho L + (e^{\rho} - 1)R}$$

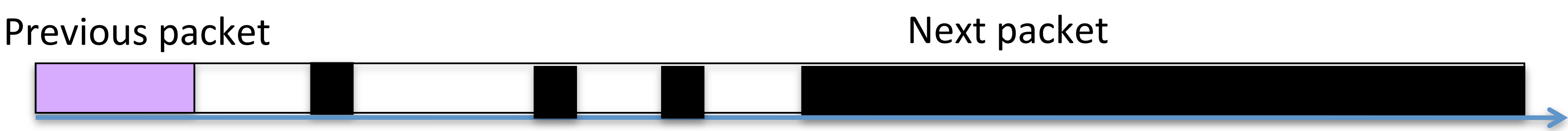
$$C_{\max} = \frac{1}{1 + \frac{\beta(R)}{L}} \approx 1 - \frac{\beta(R)}{L}$$

Green contention

- Hypothesis: we know an upper bound of the population.
- Quasi channel transparency
  - Delay are sublinear function of N.

Improvement to CSMA: Bursty Preamble transmission

- Each primary transmits sequence of burst before packet transmission



  - Bursts used to resolve contentions

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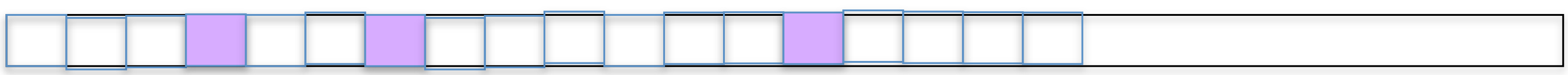
Access keys

- Divide preamble in mini-slots
  - Binary access pattern of a primary contender
    - « 1 »: contender transmits a burst
    - « 0 »: contender listens the slot



  - Let integer k be the ratio frame/mini-slot (eg k=10)
    - Access keys are constrained (0,k-1) sequences
    - Run of zeros should not exceeds k-1
      - to avoid desynchronisation
    - Sequence of super-alphabet $A_k = \{1, 01, 001, \ldots, 0^{k-1}1\}$

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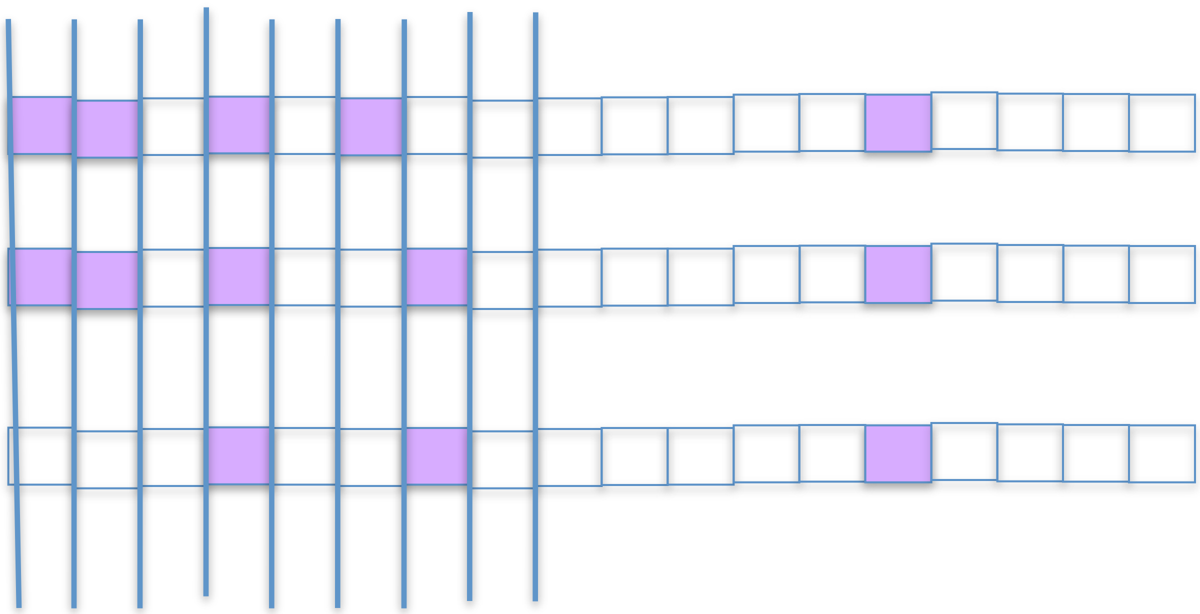
Contention resolution (leader election)

- Contenders set their access keys before slot 1
- On i-th slot
  - surviving contenders with a « 1 » as i-th bit
    - Transmit a burst
  - Surviving contenders with a « 0 » as i-th bit
    - Listen to the slot
    - If burst detected, the contender aborts contention
      - Defer for the next election.

Contention resolution (leader election)



Access keys management

- Deterministic:
  - The access keys are derived from node ID and are unique (over N nodes)
    - In fact optimal packing with $\log_{\frac{1}{\rho}} N$ super-symbols with $\sum_{i=1}^{k} \rho^i = 1$

    P. Jacquet, P. Mu¨hlethaler, ”Cognitive networks: anew access scheme which introduces a Darwinian approach” Wireless Days, 2012

    Fairness obtained by round robin-like protocol.
- Probabilistic:
  - The access keys can be probabilistic
  - Eg super-symbols are drawn uniformly on A_k
  - Residual collision may exist
    - Rate can be made negligible
    - Add to radio loss rate.

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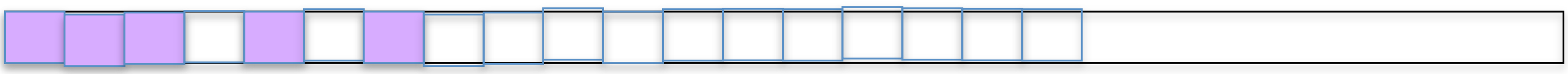
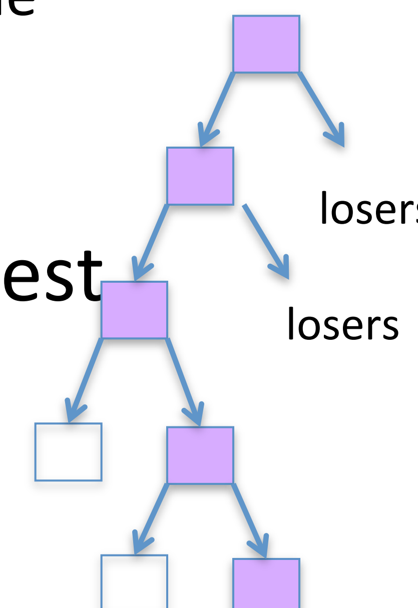
Part and try algorithm

- Case k=2 is the part and try algorithm

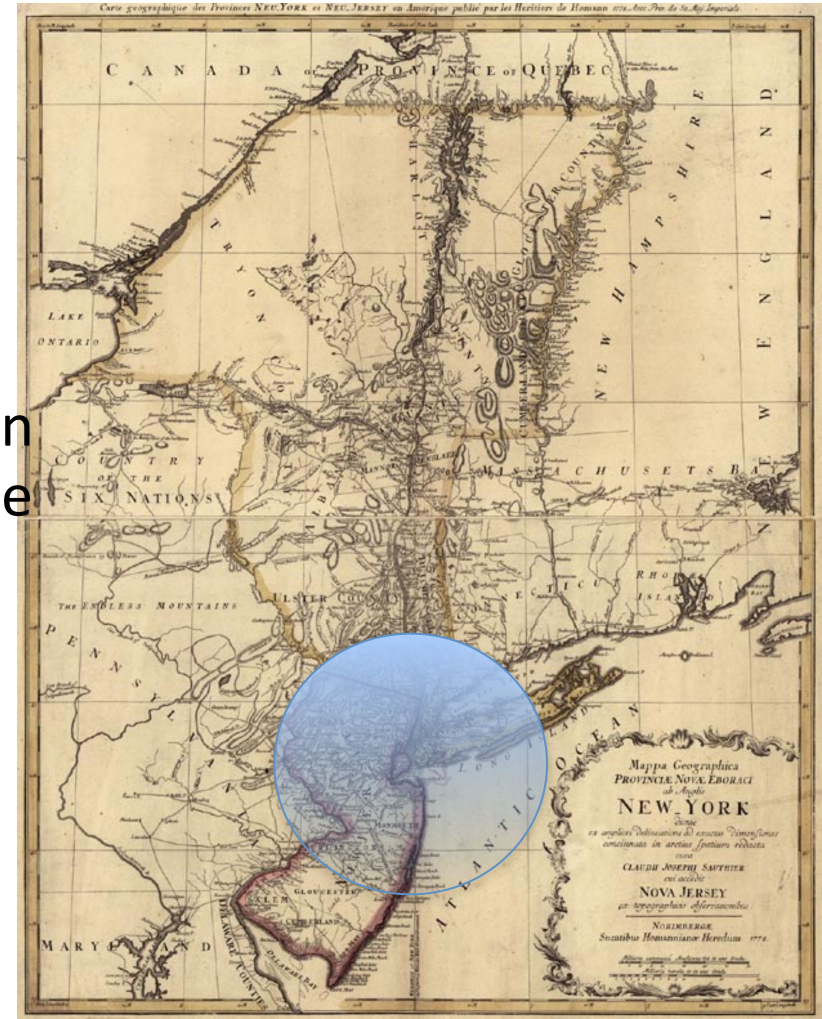


  - BS Tsybakov, VA Mikhailov ”Random multiple packet access: part-and-try algorithm” Problemy Peredachi Informatsii, 1980.
- The winners are those with the largest binary sequence
  - Average election duration $\log_k n$



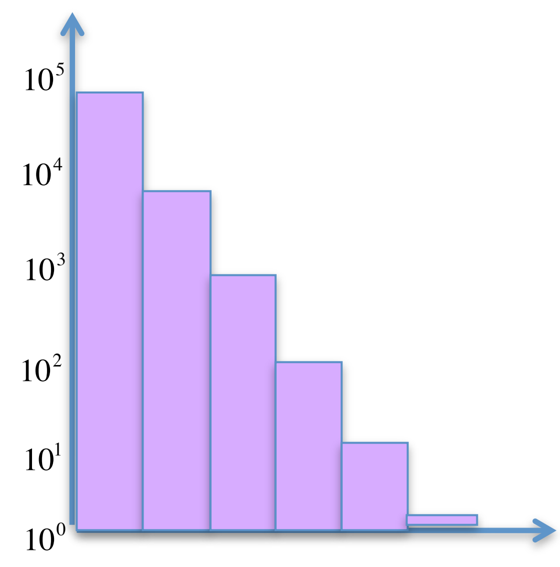
Energy cost issue

- Take the first burst.
  - In average n/2 transmit & collide
  - If n is of order the million (urban area)
    - The flash of the first burst can create of 100 km interference radius
- Further bursts
  - n/4, n/8, etc.
  - Average global energy cost per election is n



Energy cost

- A global energy cost of one million bursts per packet transmission is unacceptable
  – Would run-down batteries in seconds



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Energy cost saving (green) leader election algorithm

- Algorithm performs election for n≤N
  - Average duration $k \log_k \log N$ minislot
  - Average global energy cost in $O\left(\sqrt[k]{N}\right)$
    - N is a maximum network size.
  - Residual collision rate bounded
    - Can be made arbitrary small
  - Example with k=10, N=1,000,000
    - Duration 30 minislots
    - Energy cost 5.5
    - Collision rate less than 1%

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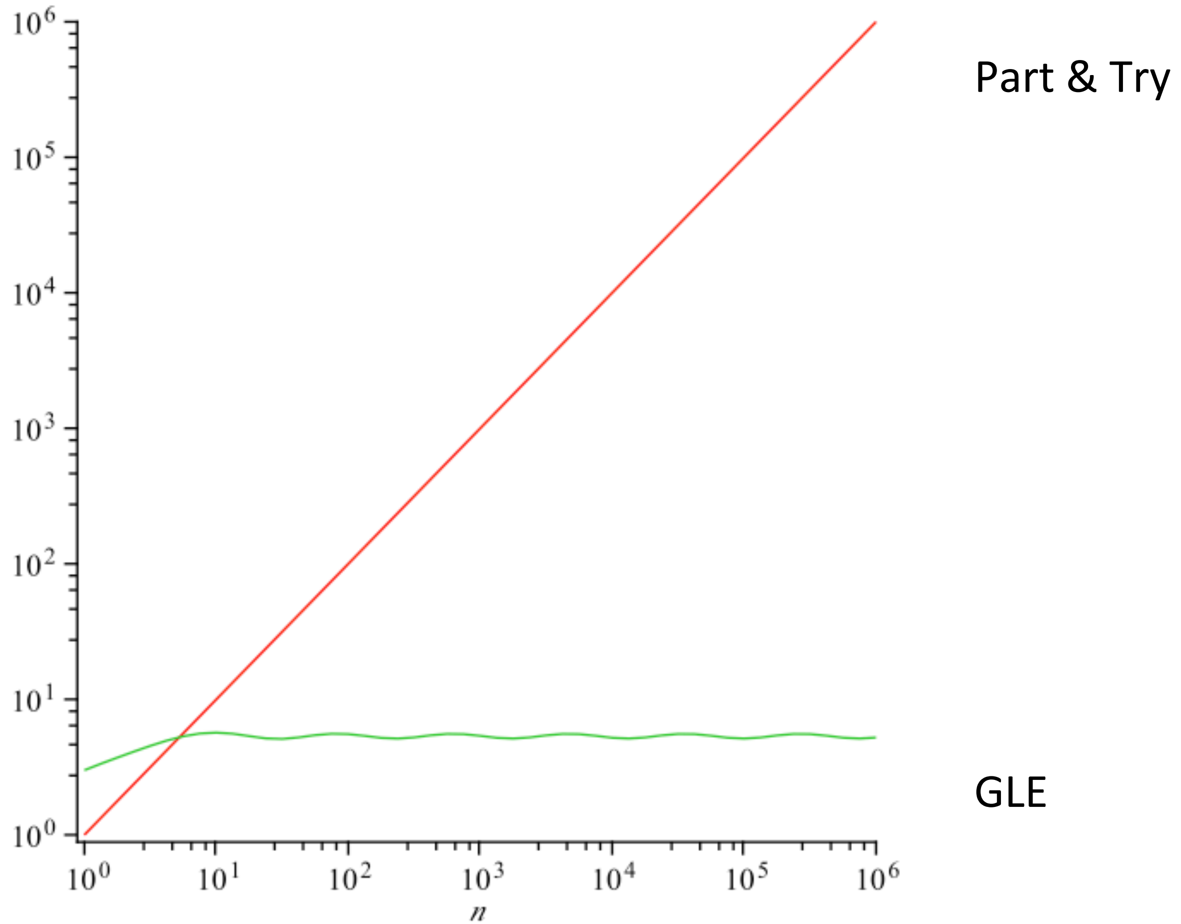
green leader election algorithm



n

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Energy cost saving (green) leader election algorithm

- Contender access key computation
  - Access key is made of $L_N = \log_k \log N + O(1)$ super-symbols
    - Say $L_N = 3$
  - Scalar p shared by all nodes
    - Say $p = 0.02$
  - Every contender selects a random integer X
    - X is geometric with probability rate $1 - p$

$$P(X \geq m) = (1 - p)^m$$

  - The access key is k-ary translation of

$$\max\{k^{L_N} - X - 1, 0\}$$

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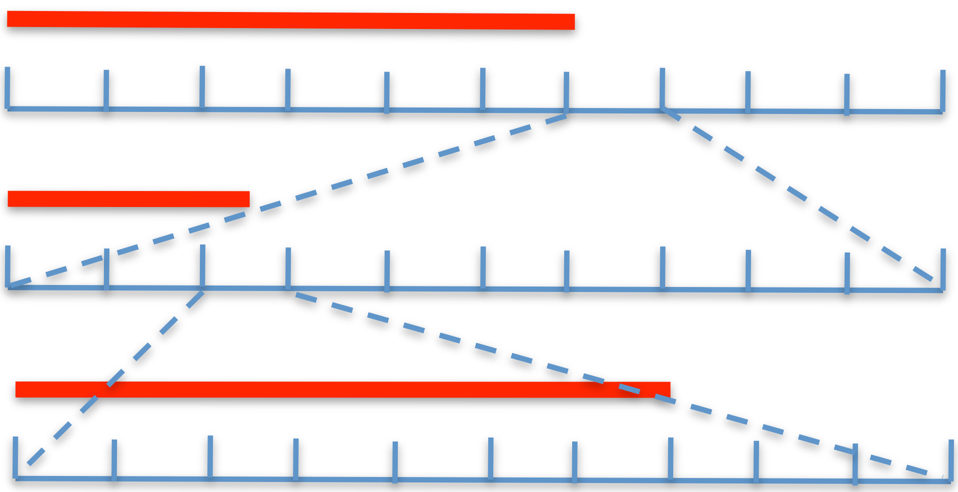
Green leader election algorithm

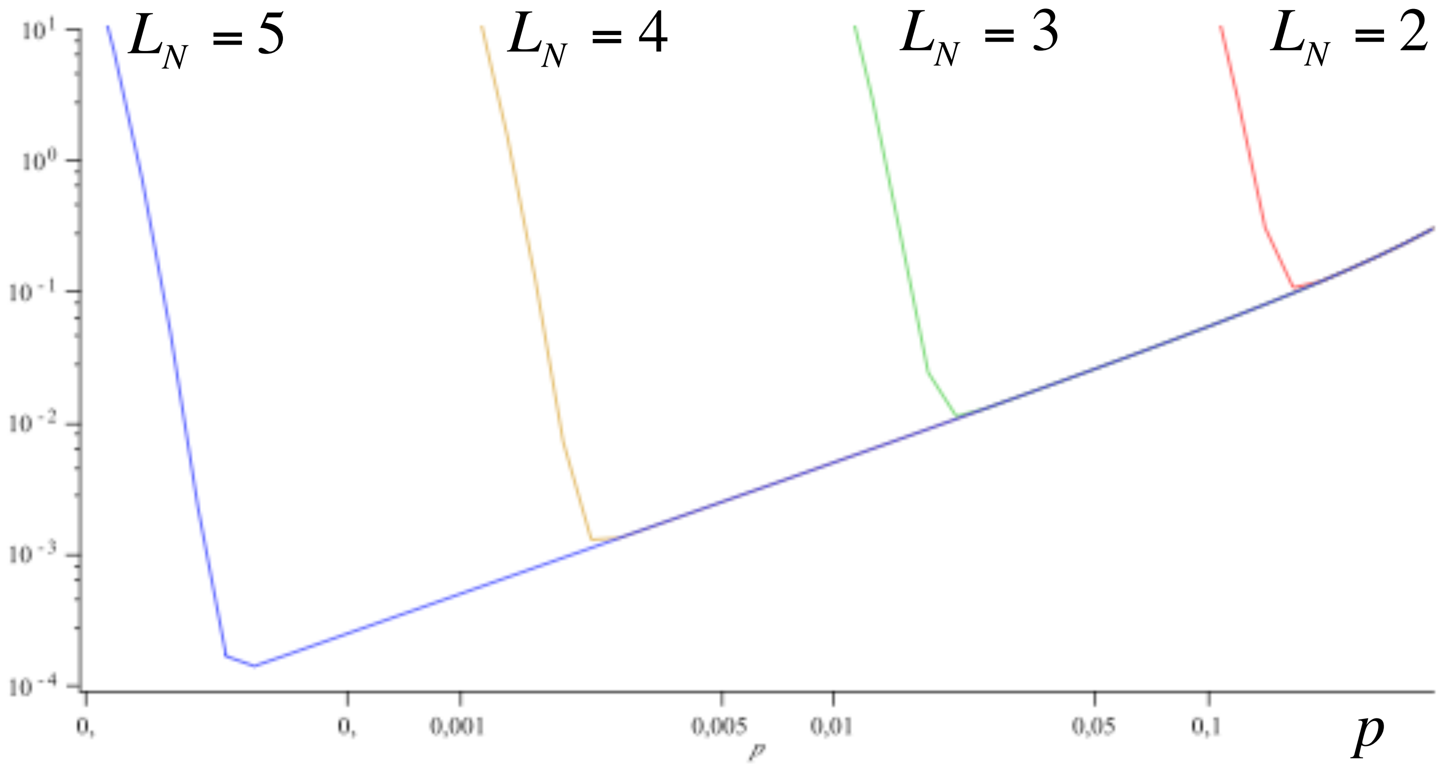
$$X = 372 = 999 - 627$$



$$\text{access key}:\ 0^6 1 0^2 1 0^7 1 = 0000001001000000001$$

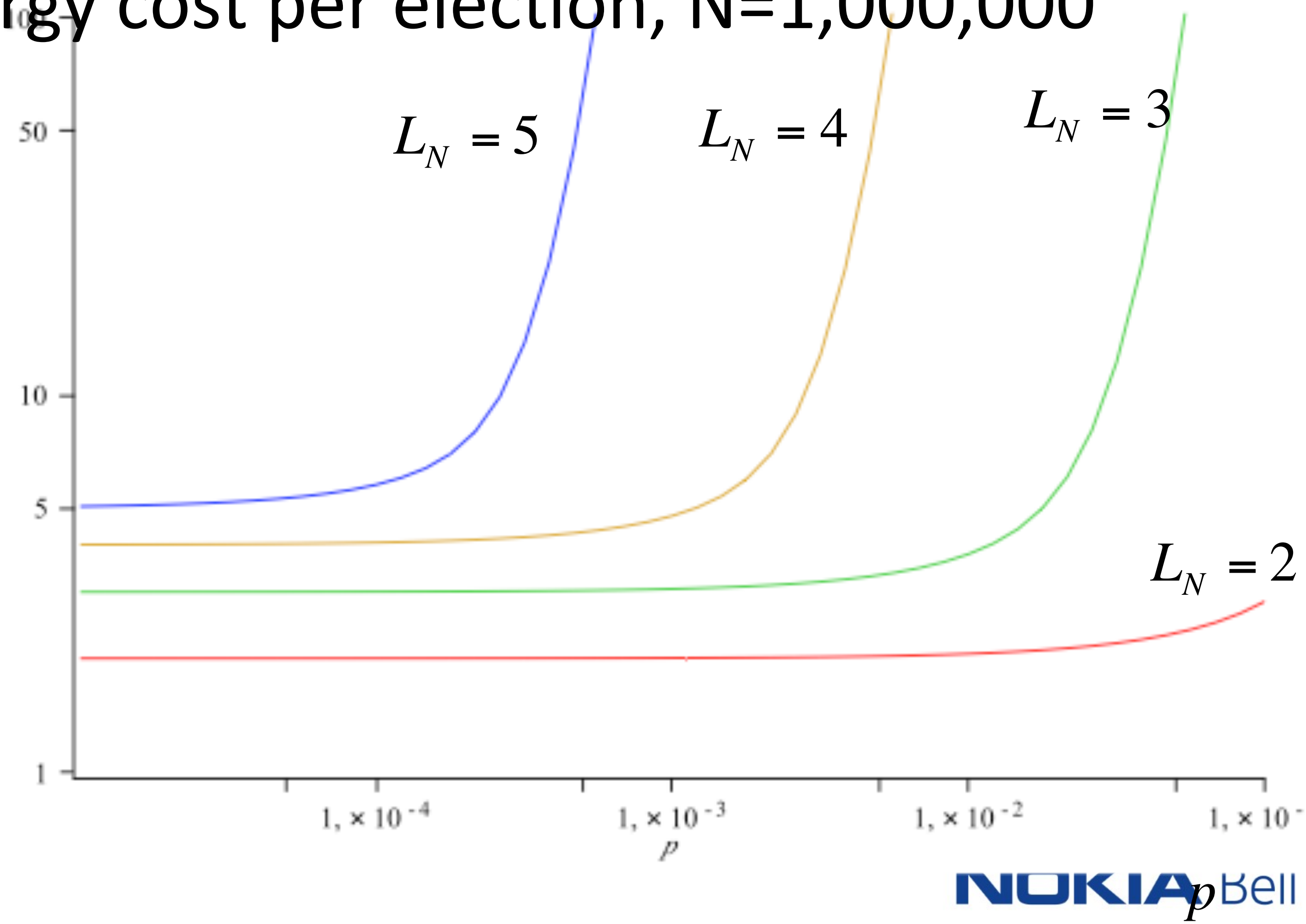
Parameters of the algorithm

- Residual collision rate, N=1,000,000



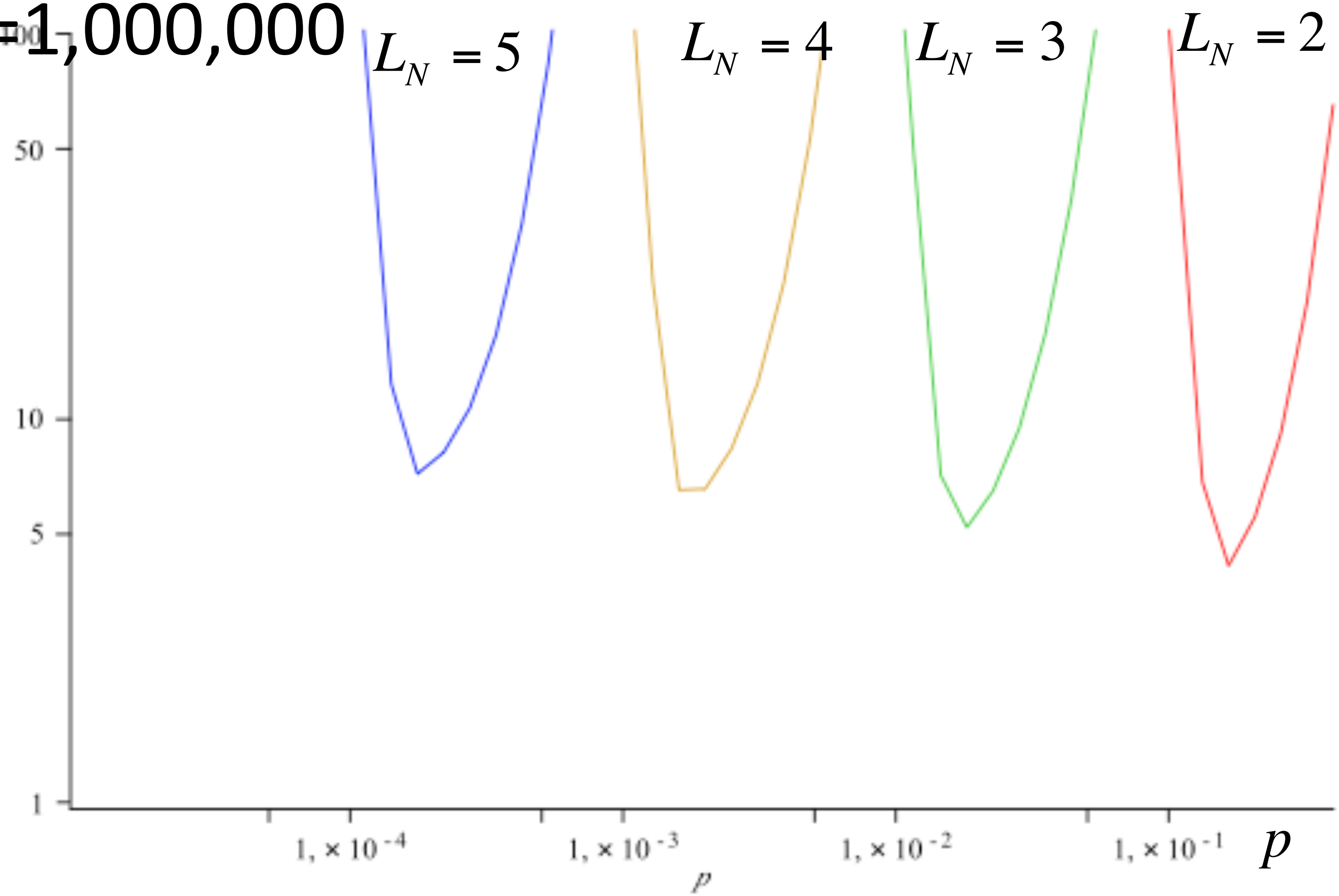
Parameters of the algorithm

- Energy cost per election, N=1,000,000



Parameters of the algorithm

- Energy cost per successful election, N=1,000,000



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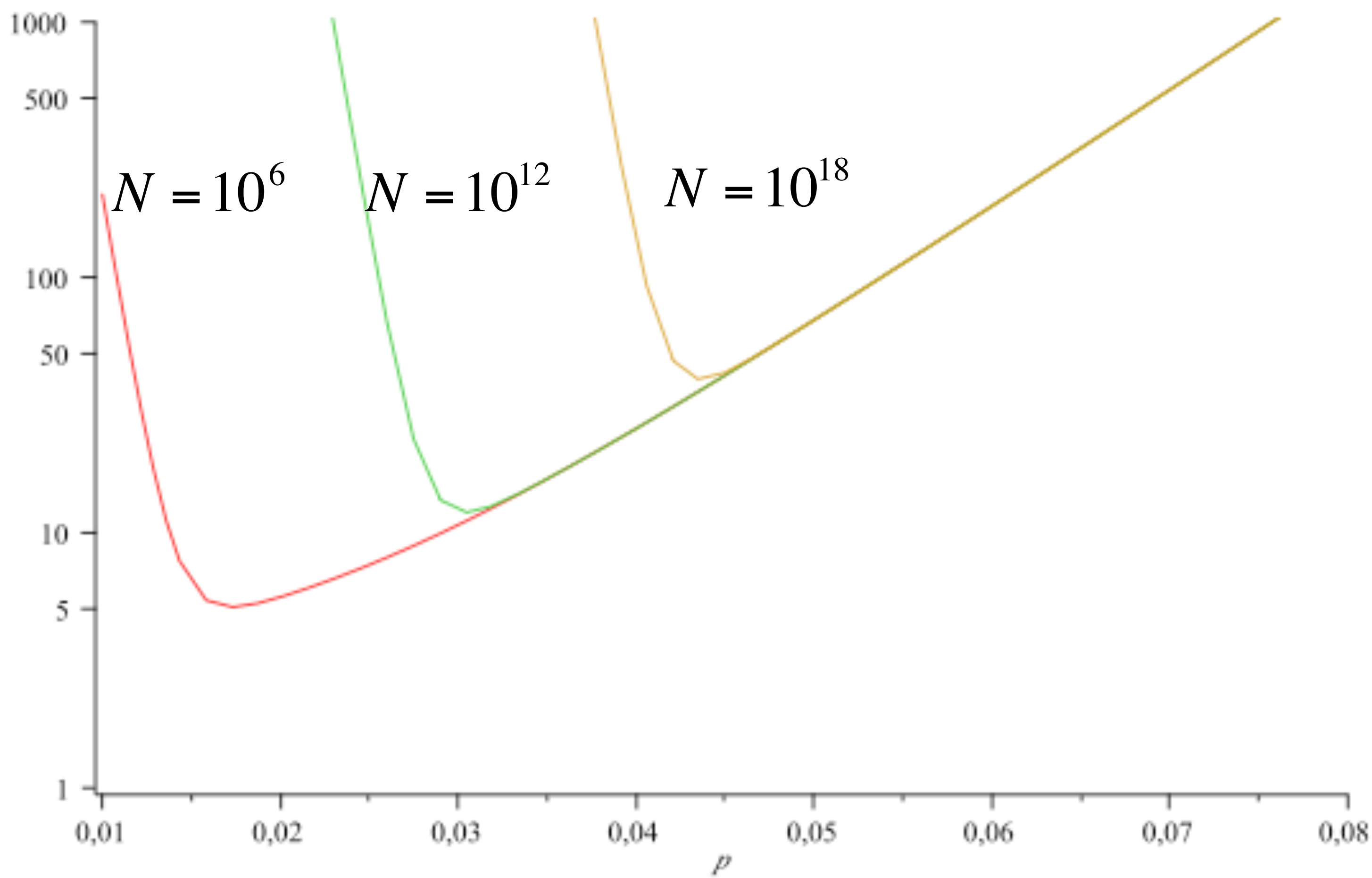
Extensibility of the Algorithm

Energy cost per successful election for $L_N = 3, N = 10^6,\ 10^{12},\ 10^{18}$



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Conclusion

- Random access
  - Infinite population model
    - channel transparency
  - Finite population model
    - Packet Delay proportional to N
    - Queues form on node,
    - unfairness and starvation may occur.
- **Green** collision resolution and leader election
  - Need a known upper bound N of population size
  - Intermediate with channel transparency
    - Packet Delay proportional to loglog N
    - Energy per packet proportional to $O\left(\sqrt[10]{N}\right)$

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